Technical Challenges in the Construction of Gothic Vaults: The Gothic Theory of Structural Design

SANTIAGO HUERTA

The construction of a Gothic vault implied the solution of several technical challenges. The literature on Gothic vault construction is quite large and its growth continues steadily. The main challenge of any structure is that, during and after construction, it must be "safe", that is, it must not collapse. Indeed, it must be amply safe, able to support different loads for long periods of time. Masonry architecture has shown its structural safety for centuries or millennia. The Pantheon of Rome stands today after almost 2,000 years without having needed any structural reinforcement (of course, the survival of any building implies continuous maintenance). Hagia Sophia in Istanbul, finished in the 6th century AD, has withstood not only the dead loads but also many severe earthquakes. Finally, the Gothic cathedrals, with their appearance of weakness, are more than a half millennium old.

The question arises of what the source of this amazing strength is and how the illiterate master masons were able to design such daring and safe structures. This question is usually evaded in manuals of Gothic architecture. This is quite surprising, the structure being a fundamental part of Gothic buildings. The present article aims to give such an explanation, which has been studied in detail elsewhere. In the first part, the Gothic design methods will be discussed. In the second part, the validity of these methods will be verified within the frame of the modern theory of masonry structures. References have been reduced to a minimum to make the text simpler and more direct.

The Gothic scientia of structures

The building of Gothic churches and cathedrals was not an amateur task. Medieval builders were "masters". Gothic structures justify this title, and even today, with a well-developed structural theory, very few architects or engineers, if any, would dare to sign similar projects (and this is a problem in restoration work and structural expertise).

The science of statics was not sufficiently developed in the Middle Ages to allow a scientific structural design; in fact, scientific structural theory originated in the 17th century (Galileo, Hooke), but began to be applied only in the second half of the 18th century. How is it possible, then, that the Gothic masters built such magnificent

Fig. 1 | Collegiate church of Berlanga de Duero, 1526–1530
structures? Was the design of Gothic cathedrals a matter of pure chance, the result of a blind trial-and-error process? Is, therefore, the history of Gothic architecture plagued with collapses and ruined buildings? The truth is that this was not so. There were collapses, but very few in comparison with the number of successes. Besides, there were so many mutations, entirely new types of structures, as to invalidate completely a purely Darwinist theory based on the survival of the more apt designs. The development of Gothic was revolutionary, an explosion of structural creativity.

The Gothic master builders had a *scientia*, a theory, a body of knowledge which permitted them to design and build safe structures. This *scientia* was not scientific in the sense we give today to this word; it was not deduced from general laws and scientific principles, it was not an applied science. The set of rules and procedures were deduced empirically, from the observation of existing buildings. This empirical approach is not altogether unscientific. Each building was a successful experiment and the observation of ruins and collapses was also very informative. Finally, during the building process, the masonry structure moves and shakes, adapting itself to the different phases of construction. These movements suggest corrections to improve the stability of the work and may lead to new patterns of equilibrium.

What was then, precisely, the nature of this medieval *scientia* of structures? This is a difficult question to answer. It must have been a wide and complex body of knowledge. The construction of a Gothic cathedral involved many different operations: surveying, soil mechanics, foundation design, centring, buttress and vault design, stereotomy, carpentry, lifting devices, labour organisation, etc. These are the modern keywords for some of the activities involved. The architect, master of the work, had to make decisions in all these aspects, which were probably intertwined in a complex way. The depth of understanding in all its aspects could be best judged from the results. Consider, for example, Beauvais Cathedral: one can feel a security of design, an absence of doubts, a determination, which could arise only from a mastery of the building processes.

Buildings are, then, our primary source and any hypothesis concerning the nature of the medieval *scientia* of structures must account for the evidence of so many churches and cathedrals that have survived over centuries. Literary sources from the Gothic period are scarce and only very few Gothic manuscripts about building design have survived, most of them from the Late Gothic period. Not very much information from which to infer the nature of a knowledge that, as has been already said, was rich and complex.

Only the album of Villard de Honnecourt pertains to the classic Gothic era, the age of wonder when the "best" cathedrals were built. The album of Villard is rich on technical information, but Villard is silent on structural matters. However, a lot of information on structural matters can be found in certain Late Gothic manuscripts. Some of them could be called treatises as they contain information about all aspects involved in the design of a Gothic church. Others treat only particular aspects: the design of gables or pinnacles, or the solution of certain geometrical problems. Finally, some expertise concerning structural problems has also survived and is an invaluable source for understanding Gothic structural thinking as Milan, Chartres or Gerona have been analysed many times. Nevertheless, many documents still remain unpublished or unnoticed.

The structural knowledge was codified in the form of practical rules. There were rules to obtain, for example, the size of buttresses or the cross-sections of the ribs. These rules were a mere register of right dimensions for different structural elements. By their very nature they are specific and pertain to certain structural types. The application of Gothic rules to a Renaissance building, for example, will lead to disaster: The thrust of a Gothic cross vault could be less than one half the thrust of a Renaissance barrel vault. Periods of transition were critical and, indeed, there is documentary evidence both in treatises and in the registers of many churches of damage associated with the use of the wrong rules.

In this paper, only some specific structural rules are investigated, particularly those rules for vault and buttress design, with some comments also on tower design. We are going to consider, then, only one aspect of the whole process of vault design and construction. This separation is arbitrary; building is not the sum of several independent activities.
Late Gothic German rules

Several architectural manuscripts of the 15th and early 16th century have survived. Some were already known in the 19th century. Stieglitz, Hoffstadt, Reichensperger and Ungewitter studied them carefully.6 Their content was important in the development of neo-Gothic architectural design methods.

However, it was not until the end of the 20th century that a complete diplomatic transcription was published by Ulrich Coenen.7 Three of them are true architectural treatises and contemplate the whole process of church design: Unterweisungen (Instructions) by Lorenz Lechler (1516), Von des Chores Maß und Gerechtigkeit (about 1500) and Wiener Werkmeisterbuch (1400–1450).8 Coenen calls them “Werkmeisterbücher”, books of the “magister operis“ or master of the work. This name seems more appropriate than others like “Musterbücher“ or “Steinmetzbücher“? The three treatises contain a rich set of structural rules, to size the main structural elements: walls, buttresses, vaults and towers. Here, we shall outline the character of the rules, giving brief examples. The rules have been studied in detail elsewhere9 and our purpose is to discuss their logic and validity as safe structural rules.

Choir wall

In the three treatises all the dimensions depend on the span of the choir: this is the great module that controls the general dimensions and proportions of the building. The wall of the choir is a fraction of the span. Typically, one-tenth (%), but Lechler also cites other proportions (%) and recommends corrections depending on the quality of the masonry. The rules are given as recipes.

In Von des Chores Maß it is stated: “If the Choir has 20 feet span, its wall should be 2 feet thick. For 30 feet span, 3 feet thick”10 and Unterweisungen recommends: “A Choir has 20 feet span, and the masonry is good, then make the wall 2 feet thick. If it is made of ashlar masonry, reduce 3 inches, if made of rubble add 3 inches.” And again, as a general rule: “If you want to find the thickness of building, you should divide it in ten parts, and the size of one part, this should be the thickness of the wall.”11 Finally, suggests Wiener Werkmeisterbuch: if the work “has 40 feet span the wall should have 4 feet: if it has 30 feet, 3 feet.”12

There rules may be checked with the numerous plans of churches in the Akademie der bildenden Künste Wien. Most times the 10% rule is applied, but other proportions are present.13 Figs. 2, 3

There are rules also for the wall of the main aisle and of the lateral aisles. The thickness could be either the same as that of the choir wall Fig. 4 or ¾ of the choir wall.

Buttresses

The buttress that resists the thrust of the vault consists of part of the wall and the counterfort, the projection of the masonry that reinforces the wall. The thickness of the wall being known, it is needed to define the projection of the buttress and its width. There are several rules.
The most-quoted consists in giving to the projection of the buttress twice the thickness of the wall, and to its width the thickness of the wall. In Unterweisungen one finds: "and as the width of the counterfort, twice should be the projection".\(^1\) The same rule is repeated, almost word by word, in the other two treatises.

If we call \(t\) the thickness of the wall, which is \(\frac{1}{3}\) of the span \(s\) \((t = \frac{1}{3}s)\); the buttress' breadth is equal to the wall thickness. Figs. 2-4 This leads to a dimension \(c = 3t = 3\frac{1}{3}t = \frac{3}{3}s\) (at the base); this basic dimension could be diminished or increased depending on the quality of the masonry. This is the thickness at the base, which diminishes in height with taluses.

The proportions could be found in many churches of this period and, also, in some of the surviving plans. However, not all the plans adjust to the above cited rule. As it has been said, there were other rules, and, in any case, a true master would have felt free to deviate from the established rules taking into account the particular circumstances of the building in question.

**Vault ribs**

A Gothic vault is composed of ribs, keystones and webs (curved masonry that fills the voids between ribs). Only the ribs are mentioned. It is said specifically that the cross ribs are semicircular; other instructions referring to the geometry of the other ribs are difficult to interpret because of the absence of drawings. The instructions contained in the three "Werkmeisterbücher" are complicated and sometimes contradictory, but, in general, from them emerges a consistent method of design. Only Unterweisungen contains explicative drawings;\(^1\) we shall use them to explain the design procedure.\(^1\)

The templates, that is, the form of the sections of the vault ribs, are obtained from the thickness of the wall. The template was cut in a sheet of wood or metal and was the indispensable device to cut the voussoirs of the ribs. Templates were obtained from the wall thickness, following a mixed arithmetical and geometrical procedure. A square with sides of the wall thickness was built.\(^1\) Fig. 5b This was made in full size, as the objective is to produce the form of the template (for spans between, say, 20 and 40 feet, the sides will be 2-4 feet, 0.6-1.2\(\text{m}\), which is a convenient size). The sides of the square are divided into three parts and by tracing parallel lines to the sides, nine squares are obtained; the central square is the basic square from which the ribs are designed. Within this square another is built tracing lines from the middle points of the sides; the diagonal of this last square is precisely \(\frac{1}{5}\) of the wall thickness (the breadth of the rib is always \(\frac{3}{5}\) of its thickness). Note that in the original drawing of the manuscript the central square has been rotated to ease the drawing; we have drawn the inner square in Fig. 5c in the position it occupies in the basic wall square.

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**Fig. 4** | Proportions of wall and buttresses in a three-aisle church

**Fig. 5** | Rib vault design, fol. 42r in: Lechler, Unterweisungen, 1516: (a) original drawing; (b) basic wall square; (c) inner square (rotated 45°)

**Fig. 6** | Rib vault design, fol. 42r in: Lechler, Unterweisungen, 1516: original drawing and enlarged lower part

**Fig. 7** | The little (modern) and the great (old) cross ribs, fol. 41r in: Lechler, Unterweisungen, 1516
In fact, in the inner square, the window mullions are drawn, but Lechler is explicit that the same sizes correspond to rib vaults. Other drawings from Lechler make this clear. In Fig. 6, the same procedure is explained. In the enlarged detail, it is explained how from the wall square different elements are obtained: mullions, impost, columns, moldings, etc. In the upper part of the drawing there is a geometric procedure to obtain the actual cross rib from the old, great cross rib; after the drawing the relationship is 1 to 22, but Lechler gives also the more simple arithmetical rule 5 to 7. Indeed, ¾ is a good approximation to \( \sqrt{2} \) (1 per cent error). In another figure, Lechler draws both ribs with a numerical scale, Fig. 7. However, another part of the manuscript states that the little rib is \( \frac{1}{3} \) of the great rib.

Then, in Fig. 8, we have reproduced the last of Lechler’s drawings on rib and mullion design. The drawing contains two parts with different scales, though the lines show no interruption. In the lower part, the wall square is represented and from there some impost design is obtained. Note that both the little and great cross rib are drawn to scale: \( \frac{1}{3} \) of the wall thickness for the great rib (drawn vertical, in the middle) and \( \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \), nearly \( \frac{1}{4} \), for the small rib (drawn horizontal, in the left corner). In the upper part of Fig. 8, the relationship between little and great mullions/ribs is shown again. Some drawings of the same type have survived. In Figs. 9–11, three of them are shown; the first from the collection of the Akademie and the third from the “Wiener Architektur-Musterbuch” in the Albertina. It is evident that different masters obtain from the basic wall square slightly different dimensions from ribs and mullions. In the three
cases, the wall square has been rotated, however it is evident that some elements are derived from Lechler’s inner central square. Thus, in Fig. 9, the rib on the lower part is 1/3 of the side, following the rule, but the mullion is the side of the octagon, i.e. $1/(1 + \sqrt{2}) = \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$, nearly. On the contrary, in Fig. 11, the mullion is 1/3 and the rib 1/3 of the wall thickness.

The ribs so obtained are the cross ribs of the choir vault, and they are 1/3 the wall thickness, that is 1/30 of the choir span. The other ribs are obtained from them. Thus, the transverse ribs should be 1/3 larger than the cross ribs, which is around 1/2 of the span. The ribs of the central and lateral aisles are also obtained from the choir span. The rules may be summarised in the following table (the table is illustrative as some paragraphs of the manuscript allow for different interpretations):

<table>
<thead>
<tr>
<th>location</th>
<th>cross rib</th>
<th>transverse rib</th>
</tr>
</thead>
<tbody>
<tr>
<td>choir/main nave</td>
<td>span/30</td>
<td>span/22.5</td>
</tr>
<tr>
<td>lateral nave</td>
<td>-</td>
<td>span/30</td>
</tr>
</tbody>
</table>

Finally, the question arises about the practical use of these rules. Coenen has studied certain Late Gothic churches and found a very good agreement. 20

Towers

High towers surmounted with spires are as typical of Gothic architecture as flying buttresses and cross vaults. The relevant parameter, given the plan and general proportion of the tower (relation between the side and the height), is the wall thickness. Two of the treatises gave the same rule: the wall thickness of the tower should be $\frac{1}{30}$ of its height.

In Unterweisungen it is suggested: “If you want one tower two hundred feet high, so give the wall thickness ten feet; if the tower is three hundred feet high, take fifteen feet for the wall.” 21
And *Von des Chores Maß* recommends: “The tower thickness is ruled by the height of the tower. If the height is 200 feet, give it 10 feet; if it is 300 feet high, give 15 feet.”

If the tower has counterforts, these were to have the same depth as the wall thickness, and a breadth ⅓ the wall thickness. Fig. 12 The first rule for the wall thickness must have been a common rule in Germany because Albrecht Dürer used it in his *Unterweisung der Messung* (Geometrical Instructions) when he explains the design of a city tower of 300 feet in height; he gives 15 feet to the wall (without citing any rule), i.e. ⅓ of its height. Fig. 14

Dürer gives the general dimensions: diameter at the base 40 feet, height until the upper gallery, at the springing of the dome, 200 feet, wall thickness at the base 10 feet, wall thickness at the gallery 5 feet. Therefore, without mentioning it, Dürer is applying the ⅓ Gothic rule. The details of the design are noteworthy. The thickness diminishes with height, and at the gallery the diameter is ⅓ less than at the base and “das stet im wol an und tregt starck” (has a good effect and makes the tower strong). Dürer stresses also the overall proportion: the height is five times the diameter at the base, “machen thuren von unten auf biss unter die dachung zwehundert schuch hochm so wilt er seiner untersten weyter fünfter hoch”.

Now we may compare the dimensions with an actual Gothic tower of approximately the same height, the Campanile of Florence. Fig. 15 It was designed by Giotto in 1334 (finished 1359). The height to the upper balustrade is 260 feet and the uniform wall thickness is 10 feet. The ratio thickness/height is ⅙. Indeed, the Late Gothic Italian campaniles were very slender.

A century later, in his treatise *De re aedificatoria*, written circa 1450, Alberti gave a more conservative rule for tower design: ⅜ of the height. It is no surprise that Renaissance rules were more conservative.
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Geometrical rules for Gothic buttresses

Other Gothic rules have survived through Renaissance or Baroque treatises of stereotomy or architecture in Spain and France. Two of them are important for their diffusion. Both rules refer to the dimensioning of Gothic buttresses.

**Geometrical Rule no. 1**

The first, which we shall call Rule no. 1, permits to obtain the buttress for a cross vault using the profile of the transverse arches. The rule appears for the first time in the stonemasonry treatise of the Spanish architect Martínez de Aranda of circa 1600. The treatise was never published. The rule was published for the first time by François Derand in 1643 in his *L'Architecture des voûtes*. The geometrical form is different, but the two rules are the same, being based on the division of the transverse arch in three parts. However, the rule can be traced back, at least, to the first half of the 16th century in the lost treatise of Baccojani published in 1546. The rule was published in the architectural treatise of François Blondel, and in many books it is called "Blondel’s rule". During the whole 18th century it appeared once, and again in architectural and engineering treatises. The rule can be tracked also in many building manuals all through the 19th- and the beginning of the 20th-centuries. Even in the second half of the 20th century, the rule reappears in the 1960s. Since the beginning of the 17th century it has been misinterpreted and appears in manuals applied to size the buttresses of simple arches and barrel vaults. However, there is no doubt that it is a Gothic rule and applies to Gothic buttresses.

The rule is as follows: in Fig. 17, the semicircular arc AD is divided into three equal parts by the points B and C. The line CD is then prolonged so that CD = DF. The point F defines the outer edge of the buttress FG. In the top drawing in Fig. 16, Martínez de Aranda’s construction is simpler. Again the arc is divided into three equal parts by two points. Trace a perpendicular from one of them, a, to the springing line to obtain point b. The distance bc is the thickness of the buttress. Derand remarks that the rule is orientative, explicitly says that it applies to vault buttresses and not to arch buttresses: "[...] il n’est pas toujours nécessaire que les susdites épaisseurs trouvées par la pratique [...] se gardent en toute l’estendue des murs qui portent les voûtes: ainsi il suffira de les conserver à l’endroit des arcs principaux, où elles formeront des avances, lesquelles se nomment vulgairement, corps saillants ou arc-boutants." The mention of arc-boutants (flying buttresses)
buttresses) is a proof of the Gothic origin of the rule. However, the best proof is that the buttress dimension that gives (nearly) ¼ of the span at the springings corresponds to Gothic vault buttresses. The proportion is clearly insufficient for arch or barrel vault buttresses, which need at least ½ of the span.

When applied to some single-nave Gothic buildings, the rule shows good concordance. Fig. 18 This does not necessarily mean that precisely this rule was used; it proves only that the rule is Gothic. The simplicity of the rule may be surprising. Nothing is said about the thickness of the vault, the height of the buttresses, the breadth of the bay, etc. The rule relates simply the overall form of the vault, as represented by its transverse arch, with the buttress thickness at the springing. Karl Mohrmann thought that it was a good procedure to design the buttresses of neo-Gothic churches,33 introducing some precision. In fact, the rule gives a good estimation that will be corrected by the master depending on the particular circumstances of the building.

Geometrical Rule no. 2
The second geometrical rule for buttress design, Rule no. 2, was discovered by the author in the architectural treatise of Hernán Ruiz el Joven, a Spanish architect of the 16th century.34 Hernán Ruiz gives the rule as a method to obtain the abutment for simple arches, but it is, again, a Gothic rule for buttress design. The same construction appears in the first edition of Ungewitter of 1859 as a rule to size the buttresses of a polygonal Gothic apse.35 Fig. 19 Ungewitter says nothing of its origin, but it is very probable that both have the same Gothic origin. The appearance of the same rule in such different places and epochs is a demonstration of their importance and diffusion.
The rule is as follows: Figs. 20, 21 consider a drawing of half the transverse arch of a Gothic vault with its thickness. Draw the chord of the semi-arc, then trace a parallel line tangent to the extrados; the point where this line cuts the horizontal line of the arch springings defines the thickness of the buttress. The rule is cited three times in the manuscript, which is a proof of its importance. The results are similar to those obtained with the previous rule.

The structural rules of Rodrigo Gil de Hontañón

The most complete set of Late Gothic rules appear in the manuscript of the architectural treatise of Rodrigo Gil de Hontañón (1500–1577), the most important and prolific Spanish architect of the 16th century. The son of a famous Gothic master builder, Juan Gil de Hontañón, he inherited the tradition of Gothic construction, but during his life he assimilated also the new vocabulary of the Renaissance. He participated to a greater or lesser degree in the construction of nine cathedrals (Astorga, Salamanca, Segovia, Plasencia, Santiago, etc.) and built many parish churches and civil buildings. Between 1544 and 1554 he wrote a treatise of architecture that was copied by Simón García in his Compendio de Arquitectura of 1681. There are two facsimile editions and an English translation by Sergio Luis Sanabria. In what follows, all the English quotations to the manuscript are Sanabria’s translations. References to the pages of the original manuscript are in brackets.

The manuscript treats in a systematic way the different aspects of the design of a Late Gothic church. In particular in chapter 6, he treats specifically the sizing of structural elements using certain general rules (“reglas generales”). It is this last part that converts the manuscript into something unique. In no other Gothic source does a conscientious separation of the structural skeleton appear. In spite of this, the rules have not received great attention: only Kubler, Sanabria and the author have studied them in detail.

The rules could be divided into two groups:
1) rules for the design of the structural elements of a Gothic church;
2) rules to investigate the buttress for an arch in a Renaissance arcade.

It is important to make this distinction, which is justified by their location in the manuscript and, above all, by their different goals:
practical in the first case, of research in the second. (Kubler and Sanabria make no distinction between the rules.)

In the 16th century, most of the churches built in Spain were covered by a special type of Gothic vault, the "bóvedas baídas". These vaults are of domical form and the ribs are very nearly disposed in the surface of a sphere, which has as diameter the diagonal of the bay (cross ribs are perfect semicircles). All the examples in the manuscript correspond to this type of vault.

Gil de Hontañón explains along four pages (fols. 24r–25v) the process of construction of the vaults; it is the only description from a Gothic master that has survived. However, he remarks that "[...] these things may be difficult to understand if one lacks experience and practice, or if one is not a stone mason, or has never been present at the closing of a rib vault" (24r).41

First, a platform is built at the level of the tas-de-charge (a little above of the springings). Fig. 22. There the plan of the vault is drawn over it and the keystones are placed in position above wooden struts. Then, centring between the keystones are constructed, the ribs are built and finally the masonry web between the ribs is laid. The rib skeleton functions as a permanent centring and ribs and keystones should have certain dimensions so that this skeleton remains in equilibrium, not only at the end, but during the whole building process.

After defining the general proportions of the church, Gil de Hontañón exposes his general rules. They refer to the sizing of piers, buttresses, ribs and keystones of the vault, and the walls of towers.
Fig. 23 | Rodrigo Gil de Hontañón’s rule for buttress design; left: the rule expressed algebraically; right: the slenderness of the buttresses c/s, for different proportions height/span, for spans (7.5–20 m)
**Piers**

Gil de Hontañón gives a rule to obtain the diameter (piers were usually cylindrical) of the interior piers. The rule is arithmetical and contains a square root but it is exposed discursively, by writing: "Returning to the thickness of the piers, I say that the width of a nave bay, 40 feet, should be added to the length, 30, which is 70. To this should be added the height of the column, 40 feet, which is 110. The square root of 110 is 10.89, half of this is 5.45, and this should be the diameter of the column on the lower part. This is the closest to what is right." (17v). The rule can be expressed algebraically:

\[ d = \frac{1}{2} \sqrt{h + w + l} \]  

where \( h \) is the height of the pier, and \( w \) and \( l \) are the width and length of that bay.

The rule is not dimensionally correct and to obtain good results the data should be introduced in Castilian feet (0.28 m); if we introduce the dimensions in metres the proportion \( \frac{1}{2} \) is multiplied nearly by a factor of two. This rule is easy to verify in actual buildings; the author has checked the rule in the church of Villacastín, near Madrid, and the agreement is very good. In general, it can be said that the dimensions obtained by the rule agree quite well with those seen in published plans.

**Buttresses**

Another arithmetical rule is given to determine the size of the vault buttresses. Gil de Hontañón gives first the rule in a general way and then applies it to a vault of certain dimensions. It is an important rule and he wanted, possibly, that no error could be committed.

The text says: "To find the necessary projection of the pier buttress, add up the feet of circumference (i.e. the perimeter) of the ribs supported by the buttress. By this is to be understood half of the length of the ribs, which is the lengths of the tiercerons to their keystones, the lengths of the diagonal ribs to their central bosses and half of the length of the transverse arch. Having added up all this, subtract one third, which is what is normally taken up by the mouldings. Should the mouldings take up more or less, subtract more or less accordingly. Now measure the height of the buttress, and add it to the remainder of the previous operation. Take the square root, and divide it by three. One of these thirds will be the width of the buttress, and the remaining two thirds its length, including the engaged half column, the wall thickness and the external projection." (17v) The formula reads algebraically as follows:

\[ c = \frac{1}{2} \sqrt{h + \frac{2}{3} \sum N_i} \]  

where \( c \) is the total thickness of the buttress (including the wall) at the level of the springings of the vault, \( h \) is the height of the buttress and \( \sum N_i \) is the sum of the lengths of the ribs converging on the buttress, measured from the springing to their respective keystones. The breadth of the buttress is \( \frac{1}{3} \). After giving a detailed numerical example, Gil de Hontañón affirms: "This is the right size to hold the thrust of the arches. The workman can add somewhat more, because it is better to have too much than too little, although this size will be sufficient, as was stated." (18r).

Gil de Hontañón remarks that this is the depth of the buttress at the level of the springing of the vaults, but that downwards it will be increased, forming "steps" at intervals. In Fig. 23 left, the way to use the rule is represented; at the right, the relationship \( \frac{1}{3} \) has been plotted for different relations height to span, \( \frac{2}{3} \), and different spans (the figures within the squares, in metres). The buttresses become slenderer as the span grows.

The rule is cited again twice in other parts of the manuscript. The first time at the beginning of chapter 2, where he discusses several church designs, here he simply applies the rule without explanation, as a routine calculation (5r). It appears again at the end of chapter 6, where Gil de Hontañón remarks strongly the excellence of the rule: "Thus seeking the intrinsic reasons and irreproachable causes, it is necessary first to study the elevation of the temple to determine which members are thrusting against the buttress [...]. Having followed all the various instructions discussed above, the result will be strong, safe, beautiful and proper." (22r, 22v. Author's italics)

**Vaults: ribs and bosses**

The sizing of ribs and bosses, the big stones where the ribs meet, is treated together. Gil de Hontañón stresses the importance of the problem: "It is good to know the correct size and thickness of the ribs and bosses of rib vaults, since we have seen many ruined either because their bosses were too heavy and thus much larger than what the ribs could hold, or else much too light so that the weight of the ribs lift them." (22v) Gil de Hontañón alludes, probably, not to the completed vault, but to the vault under construction, as we shall see later.
For the ribs, he gives simple arithmetical formulae. It is interesting that he tries to reconcile older Gothic geometrical rules with the design by analogy with the human body: Fig. 24 "Now in order to have a general rule, which is what we want, we must understand that the thumb may be viewed as the transverse arch, the index and ring fingers as tiercerons, the middle finger as the diagonal rib, and the little finger as the formeret. To determine the proportions of the fingers to the hands, take half the ounces of these fingers, which is the length of each fingernail." (23v) Gil de Hontañón uses this proportion divided by two to obtain the thickness of the ribs: "[...] dividing the length, or side, or a bay in 20 parts, one part shall be the height of the voussoirs of the transverse rib. The length of the bay divided in 24 parts shall be the height of the diagonal rib. The tiercerons will be $\frac{1}{8}$, and the formeret $\frac{1}{10}$. Thus shall they be proportioned, in accordance with the work they do." (23v) It is another example of his desire to relate the Gothic rules with the proportions of the human body.

The thickness of the ribs in function of the span $s$ are:

- transverse ribs $\frac{1}{8}$
- cross ribs $\frac{1}{8}$
- tiercerons $\frac{1}{8}$
- formerets $\frac{1}{8}$

Gil de Hontañón explains the application of the rule to several practical cases. First, he remarks that the rule is for height of the nave, until the springings of the vaults, equal to the span. If the height is greater then, the thickness of the ribs should be increased proportionally: "Note that we give this rule assuming that the bay elevation to the capitals is equal to its side. If the elevation should be greater or smaller, add or subtract using the rule of three." (23v)

If the vault is surbased, basket handle, then the rib thickness should be increased as the height of the vault decreases: "Nonetheless, should the elevation consist of basket handle arches, sizing should be increased as the arch is lowered, this also using rule of three." (23v)

Finally, Gil de Hontañón remarks that when the bay is rectangular, the most common case, "[...] do not take either the long or the short sides but add them and divide by two. For example, suppose a bay has 20 feet to one side and 30 feet to the other. Together, they add up to 50, half is 25, and upon this base shall the distribution of the member sizes be computed." (23v) Thus, all the cases of the application of the rule have been considered.

For the keystones the rule is again arithmetical. It is one of the most difficult rules to interpret. The rule gives the weight of the keystones in "quintales" (a quintal = 46 kg or, approximately, the weight of a cubic foot of a medium stone). In the formula, one should enter again the lengths of the ribs but a distinction should be made between those members that "sustain" and those that "are sustained": "Those that are sustained must be subtracted from those that sustain. They can be told apart because those that sustain spring from the tasse-de-charge, and those that are sustained spring from bosses. There are also sustaining and sustained bosses. Those found along the lengths of the diagonal rib or tiercerons are sustained. Those that are on the ends of the diagonal ribs or tiercerons sustain all others." (23v, 24r)

Then, Gil de Hontañón gives his formula, which can be written algebraically:

$$Q = P \sqrt{\sum R_i - \sum S_i}$$

where $Q$ is the weight of the boss in quintales, $P$ is the weight of the cross rib (quintales/foot), $\sum R_i$ is the sum of the lengths of the ribs that sustain and $\sum S_i$ is the sum of the lengths of the ribs that are sustained.

The rule is, again, dimensionally incorrect. To use the rule correctly we should enter the data in Castilian feet and quintales, and the
result will be in quintals. The keystones serve, obviously, to solve a complicated stereotomic problem (the union of different ribs), but they play also a fundamental role stabilising the rib skeleton during the construction of the masonry webs (see below).

Towers

Gil de Hontañón also treats the structural design of towers. The problem is discussed in two parts of the manuscript. The general proportions of the tower are obtained using the analogy with the human body. Fig. 25

The tower signifies a whole body without arms; the arms are the church or temple. We already know that from one shoulder to the other the width is 2 faces, and the height to the shoulders is $8 - \frac{1}{2}$ faces. This third is from the ankles down, and signifies the depth of the foundations. The remaining height is proportioned to the width as 4:1. The head adds yet another $\frac{1}{3}$, which is the appropriate height for the crowning and steeple or pyramid. This is shown in the following figure, which will allow us to understand the general rule for proportioning any tower and its steeple (9r-9v).\footnote{\textsuperscript{53}}

The relationship side to height is $\frac{1}{3}$, the spire, dome or steeple should have $\frac{1}{3}$ of the side. The foundations should be excavated to $\frac{1}{3}$ of the height, and in the drawing it is stated that they should be made of rubble masonry or irregular ashlar.\footnote{\textsuperscript{53}}

The rules to size the wall thickness and the counterforts of the towers are given in the context of a minute description for the design of a church. The rules are arithmetical and are given discursively in the manuscript. For the wall thickness: “To determine the thickness of the upper part of the wall, take the square root of the 120 feet height and divide it by two. The said square root is 11 feet, thus placing this in the angle yields 5-\frac{1}{2} feet.” (5v)\footnote{\textsuperscript{54}} Expressed algebraically:

$$t = \frac{1}{2} \sqrt{h}$$  \hspace{1cm} (4)

where $t$ is the wall thickness and $h$ is the tower height, both in Castilian feet.

In this case, a tower 120 feet high, the relationship wall thickness to height is $\frac{32}{90}$, nearly $\frac{1}{3}$. If we apply the rule to the tower of Segovia Cathedral with a height of nearly 322 Castilian feet (90 m), the rule gives 9 Castilian feet (or 2.5 m); the actual thickness at the base is 10 feet (2.8 m). The rule gives a relationship thickness/height of $\frac{1}{3}$ ($\frac{1}{3}$ in the actual tower). We may compare these results with the German rule of $\frac{1}{2}$; the wall thickness would have been $\frac{32}{90} = 16$ feet, in-
In the manuscript, we find evidence of the practical application of these rules. Chapter 75 of García's *Compendio* has the title *General conditions to rebuild a ruined building* (fols. 135r–137r). The ruined building in question is a tower and the text is a report written by Gil de Hontaño describing carefully the demolition of the ruin and the construction of a new tower (the plan and elevation are those marked with A in Fig. 28). The tower was to have a height of 120 feet, as before. Gil de Hontaño does not cite any rule but recommends as wall thickness 5 feet and as buttress thickness 7 feet; he is rounding the results of the application of his rules. Thus, there is no doubt that he used his rules in practice.

**Rules for the buttresses of Renaissance arcades**

Rodrigo Gil de Hontaño manifests no doubts in designing Gothic vaults, buttresses and towers. His rules were an empirical adjustment of the data of many buildings, data which he would have inherited from his father and obtained in the archives of the many cathedrals and churches in which he worked. But when it comes to designing the buttress for a single arch, Gil de Hontaño confesses himself at a loss. He commences the corresponding section by saying: “I have tried many times to account for the buttress that any arch may need, but I have never found any rule to be sufficient. I have also discussed this with both Spanish and foreign architects, and none seems to have been able to verify such a rule: but all follow their own judgement. When I ask how do we know that so much is sufficient for a buttress, the answer is that it needs that much, but no reason is given. Some give it ½, and others design it by means of certain orthogonal lines, and then they dare to entrust themselves to this, believing the buttress firm.” (18v)55

The word “reason” here does not refer to a certain scientific theory; reason, “razón” in Spanish, means also “the order and method to do something”. Gil de Hontaño wanted a set of verified procedures, like those he used in the design of Gothic structures. A simple barrel vault was an alien structure to him (as far as I know he built none) and he was perplexed.56

The section, then, has the character of a research. Gil de Hontaño gives four different geometrical rules and one arithmetical rule. In Fig. 26, the four geometrical rules are reproduced. There is no room here to discuss the types and evolution of the rules but their experimental character is evident. Sanabria has even suggested that the first two rules may be a register of actual experiments with real arches, and there are many arguments in favour of this hypothesis.57

In any case, it is evident that Gil de Hontaño knew the specific character of the Gothic rules and he does not even try to apply them to the new structural type.

The definite rule that represents the conclusion of Gil de Hontaño’s research appears later in the manuscript (fol. 59r) with the
text: “This demonstration serves to know the buttress depth of any kind of arch.” The rule is much simpler than the others and is easy to use (an essential requisite of any practical rule). The elevation of the intrados of the arch or barrel vault with its piers is drawn. Then a line is traced joining the keystone of the arch with base of the pier. The intersection of this line with the horizontal impost line gives a point. The distance of this point to the vertical axis of symmetry is the thickness of the buttress. Pointed arches need less buttress than semicircular ones, and these, in turn, less than surbased arches, as is seen in the figure.

It is easy to obtain the algebraic expression:

$$\frac{c}{s} = \frac{h}{2(s+h)}$$  \hspace{1cm} (6)

where $c$ is the buttress thickness, $s$ is the span, $h$ the height to the impost line and $f$ is the height of the arch.

For a height equal to the span ($\frac{h}{s} = 1$), the buttress for a semicircular arch ($\frac{h}{f} = \frac{1}{2}$) will be exactly $\frac{1}{3}$ of the span. The ratio increases with the height of the abutments: if the height is 1.5 times the span ($\frac{h}{s} = 1.5$), the ratio buttress/span is $\frac{5}{12}$; if it is two times the span, the ratio is $\frac{1}{2}$. This agrees perfectly with the Renaissance rule for arches and barrel vaults: the buttress should be at least $\frac{1}{2}$ of the span.

There is an interesting limit for a flat arch: the buttress will be half the span. Gil de Hontañón did indeed succeed in devising a safe and practical rule for arches and barrel vaults.

**Expertises of Segovia: stability during construction**

That a Gothic cathedral is in a state of safe equilibrium is, literally, evident: these monuments have stood for centuries. But the building was also in stable equilibrium during the whole process of construction. Prior to the closing of the finishing of the walls, the closing of the vaults, etc., the state of equilibrium would have been much more delicate. This matter has been rarely considered in the literature on Gothic construction.

**Villard’s evidence after Choisy**

The first author to address the matter of Gothic building processes is Auguste Choisy in a brief paragraph in his *Histoire de l’architecture: Aperçu de la marche générale d’un chantier gothique* (general process of the construction of a Gothic church). Choisy bases his argument on the interpretation of one of the drawings contained in Villard de Honnecourt’s sketchbook, fol. 31v, reproduced in Fig. 27. The drawing contains an elevation from outside and a cross longitudinal section of the main nave of Reims Cathedral during construction. In the elevation, the buttresses appear unfinished, reaching only a few feet above the roof of the lateral nave. The capitals to receive the heads of the flying butt-
tresses are in the wall. In the sections, it is possible to see, at the level of the tas-de-charge, a rectangle with a cross inside that Choisy interprets as a wooden horizontal tie (in this, Choisy is following Viollet-le-Duc's interpretation of the "chainages" found in several French Gothic cathedrals Fig. 28); the vaults have not yet been built. Choisy explains the process thus: "From this authentic document it turns out that the sequence of construction was as follows: they raised the piers of the high vaults; they erected the roof; and it is under its protection that they built the high vaults. The flying buttresses were built at the same time as the vaults, and the tie rods resisted the consequent thrusts while awaiting the completion of the final decisive abutment. The roof itself, during this period of the work, was a valuable feature of consolidation. Not only did it add to the stability of the piers by its own weight, but its ties above the vault added a role equivalent to that fulfilled by the tie rods at the springings."63

Choisy's text is a little confusing as it appears that the tie rods, "tirants", should work in tension rather than in compression. However, when he says, referring to the upper "tirant", that "on les entretoisit au niveau des naissances par des tirants provisoires" he is explicitly saying that the "tirants" may be working in compression as "entretoises".64

Viollet-le-Duc describes the existence of "tirants", "chainages", in the lower side-aisle vaults of some cathedrals (he cites Amiens and Reims). He is explicit about their temporary character: the "tirants" would have been useful to resist the thrust of the side-aisle vaults until the building of the nave wall; John Fitchen cites Viollet-le-Duc's interpretation of the function of these ties:65 "They were placed during the course of erection [...] and were left in place until the building was completed; that is to say, until the moment at which the interior piers were charged to the point where the builders no longer needed to fear any buckling produced by the thrust of the side-aisle vaults."66

Two expertises on the construction of Segovia Cathedral published recently give new evidence and demonstrate the attention played by the Gothic masters on the order of construction.67 Figs. 29, 30

Enrique Egas (1532)
The report of Enrique Egas, architect of Toledo Cathedral, describes the state of the work at the date of the visit and, then, he judges the plans ("traças") to continue the work.68 His opinion is completely positive: "having considered all the details of what has been built, the work is good and very well done with all the elements of enough dimension, as the work requires, and the work is made following the plans made for it, and this is my opinion before God and my own conscience."69

For the purposes of the present contribution, the most interesting part is his response to a question posed by Juan Rodríguez, the fabri-
quire of the cathedral (the man named by the chapter to direct the construction). Rodríguez asked first about the order of building the vaults: “To build the vaults of the nave and of the two side-aisles which are together, the above-cited Mr. Juan Rodríguez asked which of the three vaults should be built first, as they buttress each other and it is difficult to build them.”

Egas answered that before proceeding with the building of the vaults of the side-aisles it is necessary to construct the wall of the main nave and prepare the springings of the main vaults until the tas-de-charge: “I say that once the tas-de-charge of the two side-aisle vaults has been built, and built the main pillars of the central nave until the tas-de-charge, also, and being the stones of these tas-de-charge in place, before the building of any of the ribs of the side and main vaults.”

Then, thick horizontal struts should be placed between the opposite pillars at the height of 1/3 of the height of the transverse arches (marked A in Fig. 31; diagonal bracing conjectural). After this, the lateral vaults may be built and, finally, the main vault: “[...] it is necessary to place some horizontal struts of sufficient cross-section between the pillars on their feet at the height where the thirds of the ribs of side vaults thrust, and in this way, having put the struts well, it is possible to build the vaults of the side-aisles, and once they have been finished, you can build the vaults of the main nave.”

The construction of both vaults will follow a procedure as described by Rodrigo Gil de Hontañón in his treatise (see above). The exterior buttresses will be built at the same time as the wall and the flying buttresses will be set on centring (B in Fig. 31; centring conjectural). These centring will not be removed until the main vault is closed: “[...] taking care that the flying buttresses are well designed and that their centring is not removed until the vault of the main nave is finished.”

Francisco de Colonia (1536)
Three years later, Francisco de Colonia, architect of Burgos Cathedral, wrote another expertise on the work. At this time, the ten lateral chapels, until the crossing, the exterior buttresses on top of the

Fig. 29 | Plan of Segovia Cathedral, 1525–1607

Fig. 30 | Transverse section of Segovia Cathedral after J. M. Merino de Cáceres

Fig. 31 | Process of building of Segovia Cathedral suggested by Enrique Egas in 1532: A horizontal strut; B flying buttresses, centring conjectural
walls between the chapels were commenced, and the pillars of the main nave were also built until the level of the tas-de-charge of the lateral aisles. Therefore, everything was prepared to continue the work and to vault the lateral and main aisles. Francisco de Colonia describes the form and heights of the ribs, and comments many details of the construction. As for the order of building, the same question that was answered by Egas, he remarks: "I say that in my opinion the vaults of the two side-aisles may be built before the vault of the main nave, with the condition that after building the arches between the nave pillars, the wall resting on them should be built up to the level of the base of the windows, and not before because the weight of the walls over the arches is sufficient as a buttress to build the lateral vaults, without causing damage to the nave pillars."

Francisco de Colonia thinks that the wall of the nave should be built only until the level of the windows (line C–C in Fig. 31), before decentering the vaults of the side-aisles. His opinion is less conservative than that of Egas. Anyway, both use the same device to buttress the vaults: increasing the load upon the nave pillars.

**Buttressing by loading**

The device of buttressing the nave pillars by adding weight is cited also in manuals of the 16th century. For example, the Spanish engineer Cristóbal de Rojas in his *Tratado de fortificación* (1598) says that the buttress for a semicircular arch should be ⅓ of the span, but he remarks that in some cases it is possible to reduce the buttress to ⅔ of the span if the pillars are heavily loaded: "The arch being semicircular, one third of the span is enough and sometimes one fourth will be enough, when a great weight is loading the pillars." 75

Christopher Wren explains the device in detail in his report on Westminster Abbey (published in the *Parentalia*). 77 With reference to Fig. 32, Wren’s text constitutes the best explanation: “Let ABC be an Arch resting at C, against an immovable Wall KM, but at A upon a pillar AD, so small as to be unable to a sufficient Butment to the Pressure of the Arch AB: what is then to be done? I cannot add FG to it to make it a Butment, but I build up E so high, as by Addition of Weight, to establish it so firm, as if I had annexed FG to it to make it a Butment: it need not be enquired how much E must be, since it cannot exceed, provided AD be sufficient to bear the Weight imposed on it."

In fact, Wren was concerned with securing the main pillars of the crossing by adding on them the weight of a tower, but he is explaining a Gothic solution to the problem of buttressing a pillar of insufficient depth.

In conclusion, Villard’s drawing of the nave of Reims during construction and the comments of both Enrique Egas and Francisco de Colonia are consistent. The vaults of the side-aisles cannot be built (or centred) without providing some kind of buttressing. Viollet-le-Duc and Choisy suggested the use of wooden ties, based on evidence found in some Gothic churches and cathedrals. The use of ties in the lateral aisles or struts at the level of the tas-de-charge over the central aisle is a possibility. The recommendation of Egas, wooden elements working in compression, rather than ties working in tension is simpler and more economical. But, of course, the most economical way is to avoid the use either of ties or struts, and this

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Fig. 32 | Buttressing of a pillar by adding weight, p. 301 in: Wren, *Parentalia*, 1750

Fig. 33 | Semicircular masonry arch in a safe state of equilibrium
may be achieved by organising the construction so that the permanent masonry elements act as a buttress through their own weight: this is the objective of Ega's and Colonia's condition of building the nave walls to a sufficient height before decentering the side-aisle vaults.

Validity of the rules

As we have seen, the Gothic master builders used empirical rules for the design of the structural elements of their buildings. The rules were only a part of a more complex body of knowledge, and could not be used safely but by a master builder. These rules had a great diffusion, geographical and chronological, and there is abundant evidence of their use throughout Europe.78

Proportional rules

A great majority of the structural rules for masonry are “proportional”, that is to say, they produce “similar” forms in a geometrical sense. They give, for example, the depth of the buttress for an arch depending on its curve of intrados but regardless of its size. In other words, they implicitly believe in the existence of a “law of similitude”: a valid structural form continues to be correct independently of its size (see for example Fig. 18, drawn to the same scale).

Galileo argued the impossibility of the existence of this kind of principle. In structures supporting as the main load their own weight, the dead load rises as the cube of the linear dimensions while the section of the structural members rises as the square; therefore, the stresses rise linearly with the size (the so-called “square-cube law”). Galileo's argument is valid only when the criterion of strength governs the design.79 As Jacques Heyman has pointed out many times, this is not the case with masonry structures: the most restricted condition is that of stability.80 A masonry structure will be safe if it is possible to find a system of compressive internal forces in equilibrium with the loads. This is a geometrical condition, which depends on the form of the structure but not on its size. The case of a simple arch may be used as an example: in Fig. 33, the semicircular arch is in a state of safe equilibrium with the line of thrust comfortably within the middle half of its thickness, and this leads to $r = \frac{1}{4}$. This state is independent of the scale and the rule will be valid for arches, say, up to 1 km span, when Galileo's law will begin to govern the design.81 It is these kinds of rules that were used in Gothic rib design.

Proportional rules are therefore of the correct form and the old master builders possessed this all-important knowledge. The same property applies to much more complex structures, and in a Gothic cathedral, for example, the forms and dimensions of his elements allow a system of internal compressive forces, which transmit the loads within the masonry, in the same way as this occurs with the simple arch. Therefore, scaling up and down does not affect the safety of a masonry building.

The rules for buttress design register the proportion between the buttress and the span. Some rules, the known geometrical rules, consider the fact that the thrust is inversely proportional to the relation span/height of the vault. Surbase arches and vaults thrust more than semicircular or pointed arches or vaults. Of course, the rules can be applied only within the whole context of building: its deep meaning is understood only by the masters, who sometimes decide to deviate from them (compensating with other changes in the geometry).

At first sight, it appears that the rules should take into account the buttress height (as it occurs with Gil de Hontañón’s rule for Renaissance arches). The overturning moment of the vault thrust will grow linearly with the height. However, the line of thrust becomes almost vertical after a height equal to the span and it is possible to use simple rules relating buttress to span safely (in fact, there is a limit for thickness for an infinite height; the thrust line has a vertical asymptote).

The problem in buttress design is not the failure by overturning, but the possible leaning of the buttress. A very small leaning outwards of 0.5 to 1° would lead in a nave with buttresses 20 m high to an increase in span of 0.34 to 0.70 m! As a consequence, buttresses are much thicker than pure stability would require.82 The static analysis made by Léon Benouville on Beauvais Cathedral shows how near the thrust is to the centre at the base of the buttress.83 Figs. 34, 35
Technical Challenges in the Construction of Gothic Vaults:
The Gothic Theory of Structural Design

Fig. 34 | Beauvais Cathedral: static analysis, plate 160 in:
Benouville, Étude sur la cathédrale de Beauvais, 1891/92

Fig. 35 | Perspective of Beauvais Cathedral, fig. 15 in: Choisy,
Histoire de l'architecture, vol. 2, 1899

Fig. 36 | Scaling up and down does not affect the safety of a
masonry structure. Drawing, static analysis of Strasbourg
Cathedral, plate 42 (modified) in: Ungewitter, Lehrbuch der
gotischen Konstruktionen, rewritten by Mohrman, vol. 1, 1890
not excessive; less buttress would have led to the collapse of the high vaults with a very small leaning.

Thus, equilibrium in compression being the only requisite for stability, the use of scale models is completely valid. We know that the medieval masons built small models to learn how to cut the stones and, also, as part of the exams to become a master. Models of certain size would have been perfectly correct in order to ascertain the safety of the real structure. Besides, a small church could serve as scale model for a bigger church. Fig. 36

The problems of strength would occur, as for arches, with enormous dimensions. Benouville calculated the mean stress at the base of the nave piers in Beauvais (the highest vaults of Gothic, circa 47 m) as 1.3 N/mm². This is quite moderate; good masonry can resist, say, 5 times this value. This will lead to a height of more than 200 m. It is mechanically possible, economically impossible, and, in any case, what would have been the sense of such dimensions?

Non-proportional rules
Some problems led to non-proportional rules. This occurs in masonry structures when the forces acting on the structure grow at a different rate than the dead weight. For example, during the 19th century, non-proportional rules were used in bridge design: the effect
Technical Challenges in the Construction of Gothic Vaults:
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of a certain maximum load that is going to cross a certain bridge decreases rapidly as the bridge’s span grows, as the weight is growing (considering the breadth of the road constant) with the second power of the span.

Many of the rules of Rodrigo Gil de Hontañón are non-proportional, not even dimensionally correct. They have therefore been considered incorrect and nonsensical, or simply ignored. In fact, they refer to non-proportional problems. For reasons of space we shall consider only three cases: vault buttress design, wall design in towers and boss design.

Let us consider first the problem of buttress design. Fig. 37 In Late Gothic Spanish vaults, the thickness of the webs is very often constant: the minimum that can be practically built (150–200 mm of stone). In this situation, the weight, and therefore the thrust, of a Gothic vault rises with the square of its linear dimensions. However, the weight of the buttresses rises with the cube, as is evident looking at Fig. 37e. The ribs are very nearly the surface of a sphere with a radius half the diagonal of the bay (i.e. the radius of the cross-arches, Fig. 37b). We may, then, replace the real vault by an analogue: a semispherical shell with a thickness equal to the thickness of the webs. Then, using the equilibrium approach derived from the Safe Theorem of Limit Analysis, we may imagine the shell divided into elementary arches that are supported by the transverse arch. Fig. 37e The load on the transverse arch is very nearly uniform and it is easy to compute the vault thrust. Finally, the stability of the buttress is checked. In Fig. 37d, the buttress has been considered independent of the walls, which, of course, is a very safe assumption as there is always some connection between wall and buttresses. Anyway, the thrust line is contained within the masonry of the buttress and the geometric safety is ample.

The polygon of forces at the right side of Fig. 37d represents the equilibrium of the buttress. The weight of the vaults is $P_v$ and the weight of the buttress is $(P_b + P_d)$; it is evident from the force polygon that the buttress weight is almost 10 times the vault weight: buttresses not only assure the whole building, they also constitute 90 per cent (or more) of the structural masonry.

Now, if we scale up the church it will need buttresses proportionally slenderer (just the contrary of Galileo’s square-cube law). For a 1.5 increase, the vault weight will increase by a factor of $(1.5)^2 = 2.25$, while the buttress weight will increase by a factor of $(1.5)^3 = 3.37$. If we want to maintain the safe relationship of 1 to 10, obtained before, we may reduce the volume of the buttress ½. The economy is very important: we save 45 per cent of the total masonry.

The matter has been studied in more detail by the author elsewhere, following the rule by computing the buttress and comparing it with the result obtained by the calculated thrust of the vault, but here we want only to point out the essentially correct character of the rule.

It is interesting to note that in the tables for vault thrusts computed by Mohrmann, the weights and thrusts are obtained as function of the surface in plan of the vault and its thickness. That is, for a certain dimension and material of the web, the thrust will rise with the square of the span. Indeed, in other sets of tables for buttress design we may check this. Let us consider two vaults, one of double proportion as the other, of the same semicircular profile and made of the same material (½ feet brick). The results obtained are:

<table>
<thead>
<tr>
<th>Vault</th>
<th>Buttress</th>
</tr>
</thead>
<tbody>
<tr>
<td>span 4 m, height of abutment 5 m</td>
<td>buttress/span = $1.5^2 = 2.25$</td>
</tr>
<tr>
<td>span 8 m, height of abutment 10 m</td>
<td>buttress/span = $2.5^2 = 6.25$</td>
</tr>
</tbody>
</table>

The smaller vault needs a much larger buttress, in proportion to the span. However, Mohrmann was apparently unaware of the non-proportionality in buttress design.

We can summarise the results:
- proportional design: $A_3 = \text{constant} = A_1$
- Hontañón’s rule (equation 2): $A_2 = A_3 s^{1/2} = A_3 s^{0.5}$
- vault thickness constant: $A_4 = A_3 s^{2/3} = A_3 s^{0.66}$

where $c$ is the buttress thickness, $s$ is the span and $A_i$ are non-dimensional constants.

Gil de Hontañón's rule is of the correct form to provide an adjustment between the proportional design and the scientific calculation for vault thickness constant. It turns out that what seemed a non-sensical rule, a mathematical caprice, points to an essential aspect of Gothic buttress design.

The same occurs with high towers and spires. Here, the main load is by wind. The total thrust of the wind rises with the cross-
sectional surface of the tower, but its weight grows with the volume. Again, greater towers could have proportionally lesser thickness, and this property could easily be seen if we compare similar towers of different sizes. In this case, the calculations are quite easy.91

In Fig. 38, the relationship thickness/height ($\frac{t}{h}$) has been calculated by different heights. The dotted lines have been calculated so that the whole section is in compression (the resultant within the central nucleus of inertia at the base; masonry of specific weight 20 kN/m$^3$ and 1.5 kN/m$^3$ unit wind pressure) leading to thicknesses so thin as to be impossible to use in normal masonry building (only in Gothic spires may we find such orders of magnitude). It is evident that Gil de Hontañón's rule gives a much better adjustment than is given by the proportional rules.

For a tower 100 m high (nearly 360 Castilian feet), Gil de Hontañón's rule (equation 4 above) gives $t = 9.5$ feet. The Gothic rule $360/36 = 18$ feet and Alberti's rule $360/50 = 24$ feet. Gil de Hontañón's rule represents a reduction of the masonry to at least one-half. An enormous quantity of masonry is saved.

If we check the validity against the scientific calculation, we obtain:

- proportional design $\frac{t}{h} = \text{constant} = B_1$
- Hontañón's rule (equation 4) $\frac{t}{h} = B_2 h^{1/2}$
- scientific design $\frac{t}{h} = B_2 (\frac{t}{h}) = k_3 h^{1/2}$

where $B_1$ are non-dimensional constants.

Gil de Hontañón's rule gives a design that is a compromise between the exact calculation, which leads to extremely thin walls, and the proportional design, which leads to high towers of excessive thickness. Finally, Gil de Hontañón stressed the importance of the correct size for the heavy Gothic keystones. Gothic masters were well aware of the stabilising role of keystones in pointed arches, and there are many references to it.

In the construction of a Gothic vault, the rib skeleton must be stable during construction. Arch rib design is proportional and the rules are a fraction of the span. As we have seen, Gil de Hontañón insists on the correct sizing of ribs and bosses. With regard to the boss weight in particular, he remarks that it should be the right quantity because either "their bosses were too heavy and thus much larger than what the ribs could hold, or else much too light so that the weight of the ribs lift them, making movements." (22v)" The main danger lies, perhaps, in too light bosses. Web construction would have progressed from the perimeter to the centre of the bay. In this situation it is possible that the rib skeleton, loaded mainly in the haunches, could experience great movements by the rising of its central keystone. It is most interesting that Mohrmann comments on precisely this problem in the Lehrbuch: "It will be frequently observed that after partial covering of the compartments a movement occurs in the ribs, so that their upper ends with the keystone rises from the support. This occurs especially with a yielding centring and is a natural consequence of the lacking load at the middle at first, and when this is added the keystone rises." (The same problem occurs with pointed arches: they should be loaded at the keystone. Fig. 39)

Mohrmann, then, suggests loading the keystones during construction, using the same bricks already piled above: "But such great movements of the ribs are undesirable and should be prevented. This can be done by a careful propping of the keystone against the framework of the roof, but far better by a loading for which the bricks necessary for the compartments afford the natural means, and they may act directly on the keystone or be piled on planks enclosing it, indeed at first in an amount increasing with the increased height of the compartment."94

In Fig. 40, a quadripartite vault is under construction. First, the cross ribs have been built on a light centring. When the ribs are finished they are stable under their own weight. Fig. 41a The barrel webs have been constructed from the perimetral arches and rest on the cross arches (this is a very simplified example). The result is that the outer part of the cross ribs supports a heavy load and the rest of the ribs are free from load. As Mohrmann explained, the danger is that if there is some yielding of the cross rib centring, the outer part of the ribs will yield downwards, pushing the central part upwards.
Fig. 38 | Design of masonry towers. In solid lines the traditional rules; in dotted lines the results of scientific calculation of typical values for masonry and wind pressure.

Fig. 39 | Stabilising effect of the heavy keystones in pointed arches, p. 54 in: Ungewitter, Lehrbuch der gotischen Konstruktionen, rewritten by Mohrmann, vol. 1, 1890.

Fig. 40 | Quadripartite vault under construction. Only part of the webs have been built.
Fig. 41 | Effect of a slight yield of the cross rib centring

Fig. 42 | Stabilising function of the central keystone during the building of the vault
Five hinges form and only the centring stops the collapse of the cross rib. Fig. 41b

Gil de Hontañón’s advice is to place a boss, heavy enough to avoid the rising of the ribs. Indeed, heavy keystones placed on top of wooden struts were a passive weight that was used, if necessary, to stabilise the rib skeleton during construction. The static is evident and is explained in Fig. 42 (the ribs are supposedly “weightless”). In an intermediate phase of the building, without having finished the webs, they produced a load concentrated on the perimeter. The dotted line, completely outside the ribs, represents the situation without keystone, and the ribs will collapse inwards by raising the keystone.

In this case, precise calculations are difficult to make as they are subject to many possible variations, but the general form of the rule (equation 3 above) can be checked:

- proportional design \[ Q = C_1 s^3 \]
- Hontañón’s rule (equation 3) \[ Q = C_2 (s^2) (s^{1/2}) = C_2 s^{3/2} = C_3 s^{2.5} \]
- vault thickness constant \[ Q = C_3 s^2 \]

where \( s \) is the span of the vault, and \( C_i \) are non-dimensional constants.

Again, Gil de Hontañón’s empirical formula is of the right form: taking into account the weight of the ribs, which rises with the cube, the solution must be between the proportional design and the vault thickness constant.

Conclusions

The medieval master builders used empirical rules for the design of the structural elements of their buildings. These rules had a great diffusion, and we have found abundant evidence of their use throughout Europe. They most probably have a very ancient origin (Rome or Byzantium, even before), but the first documentary evidence of their use comes from the Late Gothic, and their appearance in architectural and building manuals continuous well into the 20th century.

The application of limit analysis to masonry structures confirms that the more restrictive condition for the design is not resistance but stability. For a structure to be stable, it should have certain dimensions depending on its geometrical form. This leads to valid “proportions” for the design of arches, vaults and buildings, independently of their size.

The empirical proportional rules of the ancient master builders afforded a means to fix these safe proportions. Therefore, they are a valid and rational method of design for this kind of structure. Of course, each rule has a certain field of application, but this is also true of the formulae and normatives of modern structural analysis.

Some problems in the design of masonry structures led to non-proportional solutions: that is the case of the counterforts and bosses of Gothic vaults, of bridges and of towers. These kinds of structures become stabler as they grow bigger, as is evident not only for the application of theory but also for the inspection of existing buildings. Proportional rules could be used in this case, too, when they represent an inferior limit for the design, but this leads to a great waste of material when sizes are considerable.

The wiser of the old master builders were aware of that and also gave non-proportional empirical rules for the above-cited cases. The application of these rules to the dimensioning of the keystone of the bridges is well documented, and so is the appearance and evolution of these rules between the 18th and the beginning of the 20th century. In the other two cases, although practical evidence of their use is in the existing buildings, the only set of rules that have survived is that of Rodrigo Gil de Hontañón.

The rules of Gil de Hontañón are exceptional in being the only survivors of an old Gothic practice. The set of rules is complete and permits the dimensioning of every structural element of a Late Gothic hall church. The verification of these formulae by means of modern limit analysis gives a surprisingly good coincidence with the results of calculus.

The use of empirical rules is not the only way to tackle the problem. Limit analysis also validates the use of scale models. Evidence of their use is abundant, but no written documentary proof demonstrates that they were used in this sense, although they most probably were. Of course, any existing building could be considered a “scale model” of a greater similar structure.

Lastly, we want to emphasise the possibility of making just a visual checking of the stability of an arch vault or buttress from a drawing to scale. In a profession where drawing is the most important means of expression and transmission of knowledge, this fact should not be underestimated. In fact, a drawing of a stable form also constitutes a kind of proportional rule, though it is not expressed in arithmetical form.
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8. Only the Instructions are dated. The other two dates after ibid.

9. Ibid., pp. 8-10.


12. “Item in khor der 20 Schuech weidt ist. Im liecht, vnd ist der stein guet, so macht die mauren zwenz Werkhschuech dicht. ist aber dier khor von Eydlen gehauen steinwerkh, so brich im ab 3 zoll, ist den faulier stein so gib im 3 zoll zue zu der dicht der Mauren.” The quotation is from Lechter’s manuscript in the Historisches Archiv der Stadt Köln, W* 276, p. 43v.

Transcription by Coenen 1990 (note 7), pp. 174-229. The rest of Lechter’s quotations derives from the same source.

13. “Item wer ein dinkt warkh anlegen will, der soll das werk liep teilen, in zechen teili, vnd als groß derselbe teili eines ist, also dink liep die mauer sein.” Historisches Archiv der Stadt Köln, W* 276 (note 12), p. 52r.

14. “Die törks! was xl stück weit ist, das sol haben iij schöeü maër, was xxx stück weit ist, sol das haben iiji schöeü maër.” Quoted in Coenen 1990 (note 7), p. 97.


17. The original manuscript by Lecher is lost. The copy of the Historisches Archiv der Stadt Köln (HS. W 276) is the only one with drawings.


24. Dürer 1525 (note 23), unpagd.

25. For a table of the main Italian campaniles indicating the height and the proportion of height to base, see: Willis, Robert. Remarks on the Architecture of the Middle Ages, especially of Italy. Cambridge, 1835, p. 186. The ratios are from 5 to 12; the last one corresponds to the Asinelli Tower in Bologna with a height of 312 feet and a basis of 26 feet.


27. Ibid., pp. 142-48.


33 Ungewitter 1890 (note 6), pp. 273–76. This part was written entirely by Mohrmann.


36 “Lo que le cabe de reóstro a cada arco en su Jenero. Al escarzano le cobe mas y tiene necesidad de mas reóstro al que de medio punto, y al apuntado menos que el de medio punto como se parece en las tres figuras aqui desheñadas.” Ms. Escuela de Arquitectura de Madrid, fol. 79v.


38 García, Simón. Compendio de arquitectura y simetría de las templos conforme a la medida del cuerpo humano, por Simón García, arquitecto natural de Salamanca. Año 1681. Ms. 8884, Biblioteca Nacional de Madrid.


41 “Aunque estas cosas, podrán ser difíciles de comprenderse, faltando en quien las procura la experiencia, la practica, la profesion de la cantería, y la ejecucion, o el abased allendo presente a algunos ciéres de cruceria.”

42 “Pues bolliendo a tratar de la grosseza de los pilarc digo, que se tomen los pies que tienen por el ancho la nave maior que son 40 y 30 que tiene la capilla de aya, y sumese y serán 70, junto con estos 70 lo que a de subir esta columna, que son 40 pies, y serán 110, la raiz quadrada de 110 serán 10 y 10/21 abos. Su mitad son 5, 5/21 abos, tanto tengas de diamenro la tal columna por la parte de abajo, y esto es lo mas cercano a razón.”


45 "Pues queriendo buscar la intrínseca razón, y la irreproducible causa, comendara mirar la manera de la montea que tal templo tiene, y que miembros ofenden a tal estrivo (...) y aciendo todas las circunstancias arriba dichas quedará fuerto, seguro y ermoso, como le toca.

46 "Por quanto bemos que en las capillas que acen de cruercia, es bien que se sepa la grand ense que an de tener las claves, y que gruesos los miembros, por quanto bemos que muchas se arruina, o por ser las claves muy pesadas, mas de lo que los miembros pueden sostener, o por ser tan libros que la gravedad de los miembros, las lebantán y acen sentimientos." 

47 "Pues para tener regla general (que es lo que pretendemos) se entenderá que el dedo polus, se tenga por el arco; y el index, y el anulo por terceletes, y el de en medio por cruero, y el auricul, por forma; y para saber que proprio tengan estos con la mano, son la mitad de las onzas de estos dedos, que es el largo de la ula."

48 "(...) adtibierase que esta Regla damos, subiendo la capilla de pie otro tanto como tuviero por lado. Y si mas subiero se le añada por Regla de 3, y si menos se le disminua." 

49 "No obstante que si la montea fuere a paynell por la mesma Regla de 3 se le acreciente según boaje."

50 "Si fuere perlondada no se tome, el lado maior, ni el menor mas junte, y partase por medio, y de aquello se saque esta Regla. Exemplo, supongo ser una capilla que tiene por un lado 20, y por otro 30, juntos son 50, la mitad son 25. Pues de esto se a de sacar, y repartir lo dicho."

51 "Porque los que son sustentados se an de restar de los que sustentan conosee en que los que sustentan, nacen de los zarjamientos, y los que son sustentados nacen de las claves. Tambien ay claves que sustentan; y otras que son sustentadas, las que estan en el arco del cruero, o tercelete, sin sustentadas. Y las que estan en los ultimos fines de los arcos de los terceletes, o cruero, sustentadas todas."

52 "La torre significa un cuerpo entero sin braços; los [fol. 9v] braços la iglesia o templo; pues siendo a y, ya sabemos que si medimos del un hombre a el otro que tiene 2 rostros. Y de allí bajo de los pies tiene 8 y 1/3, el cual terció que es de los tobillos abajo, significa para los zimentos. Y lo otro que es su altitud, que subirá quadrupla proporcion; lo que resta de alli a lo alto de la
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53 This depth for the foundation is too shallow. Fray Lorenzo de San Nicolás gives \( \frac{1}{4} \) of the side, depending on the nature of the soil: "y ahondaras fiendo la tierra firme, la tercera parte de su ancho" (if the soil is good you should excavate one third of the breadth). San Nicolás, Fray Lorenzo de Arte y Uso de Arquitectura. Primera parte. Madrid (1639), p. 115. For the rules of tower design in San Nicolás, see: Huerta 2004 (note 1), pp. 253–54. In fact, in the drawing in fig. 17, the depth has more or less this proportion. Of course, Gil de Hontañón is trying to accommodate the rules of construction to the proportion of the human body and it is not always possible to find complete agreement. However, all the overall proportions are mentioned and considered, and this is a remarkable fact.

54 "Para saber que grosseza tengan las piladas por lo alto de arriva, serán de estos 120 pies la raiz quadrada. Su mitad será lo que cave a cada grueso de pared la cual raiz son 11 pies. Por manera que puesto en el angulo le bien les 5 1/2."

55 "Probado he muchas veces a sacar Raon del estribio que abrá menester una cualquiera forma y nunca hallo regla que me sea sufitiente, y tambien le he probado entre arquitectos españoles y estrangeros, y ninguno parece alcanzar verificada regla, mas de un solo albedrio; y preguntando por que sabremos ser aquello bastante estrivo, se responde por que lo menester, mas no por que raon. Unos le dan el \( \frac{1}{4} \) y otros por ciertas lineas ortogonales lo hacen y se osan encomendar a ello, teniéndolo por firme."

56 The mention of other geometrical rules is quite interesting as it demonstrates their diffusion. The \( \frac{1}{4} \) proportion appears in the Geometrical Rule no. 1 for a semicircular transverse arch (fig. 17, drawing P). The allusion to some orthogonal lines may refer to Martín de Aranda’s procedure (fig. 16).


58 "Esta demostración sirve para saber lo que toca de estribio a cualquiera genero de arco."

59 The rule appeared once again after the 16th century. For example, Palladio cites it both in its manual and in some expertise, Huerta 2004 (note 1), pp. 193–94, 196–97, Fray Lorenzo de San Nicolás in the 17th century repeats and expands the rule, taking into account the material of the vault and the thickness of the walls, ibid., pp. 241–44.


64 Chabat, Pierre. Dictionnaire des termes employés dans la construction. Vol. 1, Paris, 1876, p. 504, "Entretiens: En général, pièce de u bois ou fer qui en relle deux autres et les maintient dans une position invariable". This definition implies that to avoid the movement the element can work either in tension or in compression, depending on the particulars of the construction in question.


69 "...I vistas todas las particularidades de todo lo que está hecho la obra es buena y muy cuerdamente labrada con todas sus fuerzas bastantes como se requiere para la tal obra esta la obra muy bien tratada conforme a la troça para ella esta fecha esto es lo que me paresce en Dios y en mi conciencia."

70 "Iten para cerrar la nave mayor y las dos colaterales que juntan con ella pregunta el dicho señor Juan Rodríguez qual de sus tres naves se cerrara primero pues son las unas estribos de las otras y tienen dificultad en el cerrarse."

71 "...I digo que enjardias las dos naves colaterales en los pilares torales en sus altos y enjardia las capillas de la nave mayor en los dichos pilares en sus altos que estando las jarjas en sus lugaras antes que se cierre ninguno de los arcos en las dos colaterales ni en la mayor."

72 "...I se han de hacer unas entivas de madera de vigas bastantes de pilar a pilar sobre sus pies en el alto donde an de estribar los tercios de los arcos de las dos naves colaterales y asy echadas las entivas a muy buen recabado se pueden cerrar los arcos y capillas de las naves colaterales y quando sean cerradas estas dos naves colaterales se pueden cerrar las capillas de la nave mayor teniendo miramiento que los arbotantes se cieren en su rason y que no les quiten las chinbrias fasta que las capillas de la nave mayor sean cerradas."

73 "...I teniendo miramiento que los arbotantes se cieren en su rason y que no les quiten las chinbrias fasta que las capillas de la nave mayor sean cerradas."


75 Iten digo, que me paresce que las dos naves laterales se pueden cerrar antes que la nave principal, con tal condicion que después de cerrados los arcos que van sobre los pilares torales y subidas las paredes que han de yr sobre ellos fasta donde han de comenzar las ventanas de la nave mayor, se cieren las dichas naves colaterales y no antes porque el pes de las paredes..."
76. "...siendo un arco de medio punto, le bastara por estribro la tercia parte de su hueco: y algunas vezes bastara la quarta parte, quando cargasse mucho peso sobre los pilares."


78. The matter of the validity of the traditional structural rules has been studied thoroughly in Huerta 2004 (note 1), Part 3, Validez de las reglas, pp. 387-515, and Conclusions, pp. 517-21.


81. That the limits of size in masonry structures are very far from the dimensions of actual constructions was well known. Leonardo da Vinci designed a bridge over the Golden Horn in Istanbul with a span of 240 m; Perronet, at the end of the 18th century, wrote a memoir on the possibility of building bridges to 500 feet (160 m) span; there were several contributions on the limit spans for masonry arches during the 19th century; at the beginning of the 20th century, Freyssinet claimed that bridges up to 1,500 m made of mass concrete could be built. For a discussion on the topic of limit spans of arches, see: Huerta 2004 (note 1), pp. 401-07. On the possibility of building skyscrapers with masonry, see: Mohrmann, Karl. Ist Eisen der alleinige Baustoff für die höchsten Bauwerke der Neuzeit? In: Deutsche Bauzeitung 26 (1892), pp. 357-359, 363-366.


85. In reality, the weight of the vault is the sum of the weight of the webs plus the weight of ribs and bosses. The last usually amounts to 10-15 per cent of the web weight. As the rib weight rises with the cube of the dimensions, the total weight rises slightly more than with the square of linear dimensions. However, the difference is small and can be ignored.

86. This is exactly true only if we maintain the relationship of buttress depth to span constant and diminish the buttress breadth. If the proportion of depth to breadth of the buttress remains constant, the relationship of the vault’s weight to the buttress’ weight varies slightly, but is still around 40 per cent.


89. "The fascination with the square root in all these arithmetic formulae indicates that it is a novel mathematical tool for master masons, and is being indiscriminately used.” Sanabria 1982 (note 40), p. 287.


92. "...por ser las clavas muy pesadas, mas de lo que los miembros pueden sustentar, o por ser tan libanas que la Gravedad de los miembros, las levantan y aycen sentimientos."


94. Ricker’s translation 1921 (note 6), p. 170. Ungewitter 1890 (note 6), p. 121: "Es sind derartige grössere Bewegungen aber für die Rippen unerwünscht, man sollte sie verhindern. Es kann dies geschehen durch ein behutsames Absteifen des Schlusssteines gegen das Dachgerüst, weit besser aber durch eine Belastung, zu welcher die oben zu den Kappen nötigen Ziegelsteine das natürliche Mittel bieten, dieselben werden direkt auf den Schlussstein oder auf denselben umgebende Bretter gepackt und zwar anfänglich in zunehmender Menge mit dem Höherwachsen der Kappe.”

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From Theories on Gothic Structures to Building Sites in the 19th Century

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