The Islamic Crossed-Arch Domes in Cordoba:
Geometry and Structural Analysis
of the “Capilla de Villaviciosa”

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Crossed-arch domes are a singular type of ribbed vaults in which the ribs do not cross in the centre, but define a polygon so that the centre is left open. The first known domes appear in the maqsurah of the Great Mosque of Cordoba, and they were built as part of al-Hakam II’s extension, between the years 961 and 965. Later, further domes appear not only in Spain, but also in the North of Africa, Armenia and Persia. Some examples also exist in European Gothic. These domes are built continuously until the 16th century. In the 17th and 18th centuries Guarini and Vittone use them in some of their buildings. Even in the 20th century Luis Moya builds them in Spain.

This study focuses on the domes built by al-Hakam II in the Mosque of Cordoba and, in particular, on the dome over the former Abd-al-Rahman II’s mihrab, known as the Chapel of Villaviciosa (Fig. 1). This dome was most likely the first one built in this extension and, according to Nieto Cumplido (2007), the project dates from the year 961. Three other crossed-arch domes were built in the new maqsurah, one in front of the mihrab, and one more on either side. These four domes feature three different layouts. The plan of the dome in front of the mihrab is a rectangle with close to equal sides. The geometry is defined by eight arches, which describe two rotated squares in plan. They spring from the vertices of an octagon inscribed in the rectangular layout and define a second octagon in the middle, which is covered by a pumpkin dome. The vaults on either side of the central dome have the same geometrical pattern defined by eight arches. These arches spring from the vertices of an octagon inscribed in the floor plan and form a new central octagon. The three domes feature a drum that makes the transition from the square to the octagonal plan. The vault covering the Chapel of Villaviciosa is unique, different from the more common designs for crossed-arch domes. The floor plan of the chapel is clearly rectangular and there is no drum to achieve the octagonal layout of the dome. Four arches parallel to the sides are intertwined, forming a square. A further four arches describe a rhombus inscribed in the floor.
plan and cross the previous ones at the vertices of the square, thus generating an intersection of three arches. A pumpkin dome stands on this central square. The webs between the arches are decorated with shells, with the exception of the four corners. Three of them feature a small crossed-arch dome, while the fourth features a pumpkin dome. Each of the four sides has four windows. At present, the windows on two of the sides are blocked up.

The geometry of the Chapel of Villaviciosa

Despite being the first known crossed-arch dome, the dome over the Chapel of Villaviciosa has the most complex geometry. Though this article does not explore in depth the details of the plan of the design, it is important to emphasize that this design is not simple. Bearing in mind that the rectangle was defined by the dimensions of Abd-el-Rahman II's old mosque, there is only one possible geometry for which the eight arches define a central square and, in addition, meet in groups of three at a point.

The Chapel of Villaviciosa has been surveyed with a laser total station. A total of 2193 points were collected from six different locations. The extrados of the dome has been measured up with a measuring tape and a laser distance meter. The intrados and extrados measurements could be related through the south wall windows. Two horizontal profiles of the dome have been measured, one immediately underneath and one immediately above the cornice. It is not possible to survey the base because the cornice casts a shade. Figure 2 shows the layout of the dome. As it can be seen, the rectangle is practically perfect. The dimensions of the outermost rectangle shown in Figure 2 have been measured at a height of 9.80m from the floor level, along the underside of the cornice. The central square, defined by arches 1, 2, 3 and 4 has a side length of 3.63m, and is obtained with great accuracy from the surveyed points. The central octagon is not perfectly regular, since there is a maximum deviation of six cm from a regular octagon inscribed in the square. The central dome has a radius of 1.62m.

at the base, and its center is very close to the center of the square, with a deviation of only 1.39cm.

Geometry of the ribs

The geometry of the ribs is explained in Tables 1 and 2. A hypothetical geometry has been derived from the surveyed points, assuming a perfect rectangle in plan so that arches 1, 2, 3 and 4 would be equal and parallel two by two, and all four diagonal arches would be equal. Arches 1 and 2 are in fact segmental arches. The center is located at a height of 10.03m. from floor level. The extrados is irregular and it is different depending on what side we view it from. This is so because what we actually see is not the extrados of the arch, but the point of intersection of the arch with the web. Arches 3 and 4 are comprised of three circular arcs, which are not tangential, but form kinks at their intersections [points A, B, C and D in Figure 2]. The central section has the same radius as arches 1 and 2, and the centre at the same height, and so the four arches form a square in plan. In fact, the change in curvature occurs at the intersection with the other two arches. This has to be necessarily so to assure the geometrical compatibility and intersection of the tree arches at a point.

To obtain the radii of these parts, the starting point is the intersection between arches 2 and 4 [point C]. To the right of arch 4, the best-
fitting arch to the surveyed points diverges by only six mm from the intersection point. An arch is drawn through the intersection and through the data points. This gives a radius of 2.49m. At the other end of the arch [intersection with arch 1, point B] and in arch 3 [points A and D], the data points yield an arch that does not reach the point of intersection with other arches. The actual distances obtained are: 8.8cm at point A [intersection between arches 1 and 3]; 13cm at point B [intersection between arches 1 and 4]; 0.58cm at point C [intersection between arches 2 and 4]; and 6.3cm at point D [intersection between the arches 2 and 3]. The difference between the surveyed circumferences and the ideal ones is given in Tables 1 and 2. In Figure 3, arch 4 is shown as an example to compare the surveyed points and the assumed ideal geometry. On systematising the geometry, arch 3 was assumed to be symmetrical, and arch 4 to be equal to it. The question that arises from this information is: why do the points surveyed on arches 3 and 4 yield an arch that is lower than the point of intersection with the other arches? The explanation is not evident. A possibility is that the arch does not have a horizontal intrados. This could respond to an adjustment introduced during construction so that the arches would intersect at a point.

Arches 5, 6, 7 and 8 are slightly horseshoe shaped, with the centre located at 1/17 of the arch radius. The points surveyed at the base seem to indicate the horseshoe shape more than those on the top, which follow the hypothetical geometry quite closely. It is possible that the horseshoe shape was accentuated at the base using plaster. Arches 6 and 7 are asymmetric. Undoubtedly, the bottom of these two arches, as well as the cornice, was cut off when the Gothic arch was built to obtain its desired height (Fig. 1).

In Tables 1 and 2 we show the discrepancies between the idealized arches and those obtained interpolating a circumference through the survey points.

**Central dome**

The central dome is a pumpkin dome, resting on a 12-side polygon, and has a pendant in the centre. On measuring the base, different radii have been obtained in plan for the different segments of the dome, in the region between 0.38 and 0.42m. The average is 0.40m. This measurement is very close to 1/4 of the radius of the circular base, and is the value used in the reconstruction of the ideal geometry. As for the dome itself, four sections have been surveyed. Two different radii have been obtained in each section, which means

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that the central dome is not spherical, but that a circular arc springs on either side of the central pendant. The dome is slightly raised.

**Extrados**
The geometry on the extrados is very irregular. The dome rests on a platform that is approximately horizontal. A prism rises in the centre, coincident with the central square defined in the intrados by the four arches parallel to the side walls. A second prism sits on this one. It has an octagonal base and it is well defined. A further prism, also octagonal but with rougher edges, sits on the second prism. On this final prism stands the base of a circular row of stones that support the central dome. The central dome has a very irregular extrados. A radius of 1.89m. has been obtained by measuring the circumference at the base. Part of the masonry of the web, delimited by the diagonal arches and those parallel to the side walls, is visible on the sides of this prism. There is another part of the web that lies under this horizontal platform, hidden by the fill. The web found at the four corners has a flat base that supports a small dome.
Structural analysis

Construction, materials and loads
There are many opinions about the materials this dome is built in, as well as about its structural behaviour. In 1927 Felix Hernández (1972), in a letter to Henri Terrasse, said that he does not think the arches are made of wood, in allusion to the work of Girault de Pangrey. He believes the arches to be made of stone, partly because that is what Velázquez Bosco said when he restored the dome and also because of the intersections of the ribs. In his reply, Terrasse disagreed with the idea of the arches being made of stone and added: "...que leurs nervures soient à briques ou de bois enduit, je pense que les coupoles à nervures omeyades ont un sens avant tout décoratif et que ce n’est que dans l’Espagne chrétienne que on a traité ces coupoles dans un sens architectural." In recent years, archeologist Marfil Ruiz has performed a study of these four domes and states that the arches are supported on a system of crossing wooden beams, and stresses the difference between the internal structure and the external aspect (Marfil Ruiz 2004). Fernández Puertas (2009) calls them “decorative domes,” based on Marfil Ruiz’s work. In our visits to the extrados of the Chapel of Villaviciosa we could not see any timber structures. The extrados of the dome is coated with very hard mortar and, as a result, it is not easy to see what material lies underneath. At points where the mortar is damaged we could see stone and brick, but we did not find an evident construction system. The central dome seems to be made of stone, and the joints can in fact be seen in some places. A mix of both stone and brick is clearly seen in the web. The arches, where they appear on the extrados, seem to be made of stone, but we cannot firmly assert this. It is certainly a masonry dome and structurally it is analysed as
The mosque is built in limestone. The specific weight for an average limestone, which is around 22 kN/m$^3$, has been used in the analysis. In those areas where brick and stone are mixed, an intermediate specific weight has been assumed with a value of 20 kN/m$^3$. The fill has a height of 3.20m. from the base of the dome. A specific weight of 18kN/m$^3$ has been adopted for the fill.

**Theoretical frame**

The theory of Limit Analysis of Masonry Structures, as developed mainly by Heyman in the last years (Heyman 1995; 1999), has been applied in this study. The structure is considered to be made of a rigid-unilateral material, which resists compressions but does not resist tensions. That is to say, we imagine the masonry as a set of rigid blocks in dry and direct contact supported by its own weight. Further assumptions are that stresses are low, and there is no risk compressive failure of the material, and that friction between the stones is sufficiently high to prevent sliding. These three hypotheses comprise to the Principles of Masonry Limit Analysis: masonry has an infinite compressive strength, it does not resist tension and sliding failure is impossible.

These hypotheses lead to the conclusion that the key issue of masonry structures is not the stress, but the equilibrium. The condition of stability of a masonry structure that fulfils the above principles requires that the path of the forces, the "line of thrust," is contained inside the structure; that is, for every hypothetical section of the structure the resultant one of the forces must be contained in inside its profile; only hereby is it possible to obtain equilibrium exclusively under compressive stresses.

The Safe Theorem states that, given a structure, if it is possible to find a state of equilibrium compatible with the loads that does not violate the yield conditions, the structure will not collapse. Applied to masonry this means that if it is possible to draw a thrust line contained inside the structure, the structure will not collapse. The power of the Theorem lies in the fact that the line of thrust, that is to say, the state of equilibrium, can be chosen freely. Once a line is chosen, we will be able to apply the safety conditions to each of the sections it crosses and obtain, thusly, a lower bound for the geometric safety factor: we know that the structure has at least that safety factor.

**Analysis of the vault**

We are going to assess the stability of the dome and, as an example, that of the south wall, for being the thinnest of the short walls supporting the thrust of the longest arches [3 and 4]. In order to make the analysis, we have divided the vault into parts. On the one hand we have the central dome. This dome exerts a weight and a horizontal thrust. We have considered that the horizontal thrust is taken by the web and the weight is taken by arches 1, 2, 3 and 4, acting as a uniform load on these four arches. We have also distributed the web among those four arches too. We have considered that the diagonal arches only support their self weight. The eight arches thrust against the side walls of the chapel. These walls can be considered as a standing up flat arch that transfers the thrust to the corners, where again the thrust divides between the walls that meet at the corner. The north wall has a thickness of 2.22m., the south wall of 1.10m., the west wall of 1.24m. and the east wall of 1.08m. The central pumpkin dome has been analysed by the slicing technique and a thrust of 2.45 kN per slice has been obtained. We have calculated the minimum thrust of each arch. Arches 1 and 2 exert a horizontal thrust of 48 kN, arches 3 and 4 of 71.8 kN and the diagonal arches [5, 6, 7 and 8] of 5.4 kN (Fig. 7).

The south wall has been analysed. The section and elevation drawn by Ewert (1968) have been used to calculate the weight of the wall. If a section is drawn under the cornice, the thrust of arches 3 and 4, 7 and 8, as well as the thrust of the central vault, which represents 2.45kN per segment, act together with the weight of the wall. The weight of the fill acts as well. As mentioned above, 2/3 of the fill are considered to act directly on the wall, and the other third on the arch. These forces act on the internal face of the wall. The thrust generated by the arches is transferred to the central line of the wall, on the section taken at nine m. A horizontal force $H^t$=51.1kN is needed for arches 3 and 4. Since the horizontal component of the thrust of arches 3 and 4 is 71.8 kN, a total 20.7kN remain ($H^t$ in Fig. 8). These horizontal forces $H^t$
Fig. 7: Thrust lines for the arches.

Fig. 8: Equilibrium of the south wall. Left: equilibrium of the vertical section; right: equilibrium of two horizontal sections, at 10.3 and 9.00 m, respectively.

and \( H' \) go through the wall, which would behave as a standing up flat arch at two different heights (Fig. 8). The thrust of arches 7 and 8 must also be accounted for. In order that the resultant forces from these arches went through the centre of the wall, a force larger than the horizontal components of the said resultants would be required. They are therefore left in their original position and will be taken into account in the analysis of the vertical section of the wall. Moments are taken at a distance \( x \) from the centre of the column to find the location of the resultant on the ground. The resultant is situated two cm from the centre of the column. It is not possible to know how the total weight of the wall is distributed between the columns. To calculate the stress it has been supposed to split in four, obtaining an average stress at the base of the columns of 2.85 N/mm².

The Chapel of Villaviciosa has the first known crossed-arch dome and, in spite of the fact that many authors have looked for its origin in the East, such theory has not been yet verified. The geometry of the original design of the dome may not be the idealised proposal in the current work, but it enables us to come somehow closer to what the original project could have been. It most certainly enables us to carry out a structural analysis. The safety of masonry depends primarily on the general form of the structure, and slight variations do not substantially change the results. The Chapel of Villaviciosa is evidently stable. There are no visible cracks neither in the intrados nor the extrados. In spite of the discussion that has taken place about the materials and the structural behaviour, it is a masonry dome and behaves as such. The ribs support the dome and are definitely not decorative, but truly structural.

Acknowledgements

I would like to thank architect Gabriel Ruiz Cabrero, as well as the Chapter of the cathedral of Córdoba, for facilitating the access to survey and visit the extrados of the Chapel of Villaviciosa.
Notes

1. Few authors have written on this topic, and constructive and structural issues have rarely been discussed. Among the studies that do discuss these, it is worth to note those of Lambert (1939) and Torres Balbás (1940; 1946). Other more recent studies are those of Giese-Vogeli (2007), Fuentes (2009) and Fuentes and Huerta (2010).

Reference list


LAMBERT, E., 1939. La croisée d'ogives dans l'architecture islamique. Recherche 1, 57-71.


TORRES BALBÁS, L., 1940. La bóveda gótico-morisa de la capilla de Talaver de la Catedral Vieja de Salamanca. Al-Andalus 5 (1) [Crónicas Arqueológicas de la España musulmana, VI], 174-178.


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