Burnup Uncertainty Analysis for PWR-HFP Exercise I-1

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Burn-up credit analyses are based on depletion calculations that provide an accurate prediction of spent fuel isotopic contents, followed by criticality calculations to assess $k_{eff}$.

Different systems coupling a neutron transport code with an isotopic inventory code are being applied:

- MCNP
- ORIGEN
- MONTEBURNS
- SCALE 6
- NEWT
- KENO
- ORIGEN-S

Need of evaluating uncertainties in isotopics for spent fuel and assess their potential impact on reactivity:

- assumptions made in the calculation models, coupling, …
- uncertainties in nuclear data: cross section, fission yields and decay data
Sources of uncertainties in a depletion calculation

The influence of all these sources should be investigated in order to understand and quantify the uncertainties associated with computer code predictions for spent fuel isotopics:

\[
\frac{dN}{dt} = [\lambda]N + [\sigma_{\text{eff}}^{\text{eff}}]\cdot \Phi N + [(\gamma \sigma_{\text{fiss}}^{\text{eff}})]\cdot \Phi N = A \cdot N
\]

\[
N = N(\lambda, \sigma_{\text{eff}}^{\text{eff}}, \Phi) = N(\lambda, \gamma, \sigma^g, \phi^g (E), \Phi)
\]

- Uncertainties in decay constants: $\Delta \lambda$
- Uncertainties in one-group effective xs: $\Delta \sigma^{\text{eff}}$
  \[
  \sigma^{\text{eff}} = \sum_g \sigma^g \phi^g / \sum_g \phi^g
  \]
  - uncertainties in the evaluated nuclear xs data: $\Delta \sigma^g$
  - uncertainties in the flux spectrum obtained from the transport calculation: $\Delta \phi^g$
- Uncertainties in the integrated neutron flux: $\Delta \Phi$
Propagation of uncertainties in burn-up calculations

“Brute force” random sampling method

Same sequence that the coupled calculation scheme to infer an error propagation procedure throughout the time

Simultaneous random sampling of the PDF of all the input parameters

For each isotope, \( N_i \):

Sample of \( M \) vectors \([N]_M\) of isotopic concentrations

INPUT

\( \sigma_i \)

\( [N]_0 \)

\( [N]_1 \)

\( [N]_M \)

MCNP calculations

\( \phi_M \)

\( \phi_1 \)

\( \phi_{M_1} \)

depletion 1, with flux from MCNP

depletion M, with flux from MCNP

depletion

OUTPUT

Burnup steps (S)

PDF

\( N_i \)
Propagation of uncertainties in burn-up calculations

Procedure based on a first order Taylor series approach

\[ N_i(\sigma^{eff}) = N_i(\hat{\sigma}^{eff}) + \sum_{j=1}^{R} \left[ \frac{\partial N_i}{\partial \sigma_j} \right]_{\hat{\sigma}^{eff}} (\sigma_j^{eff} - \hat{\sigma}_j^{eff}) + \ldots \]

Sensitivity coefficient \( \rho_{ij} \)

\[ \varepsilon_j =\sum_{g=1}^{G} \phi_g^j \left( \sigma_j^{g} - \hat{\sigma}_j^{g} \right) + \sum_{g=1}^{G} \sigma_j^g \left( \phi_g - \hat{\phi}_g \right) = \phi^T \varepsilon_{\sigma_j} + \sigma_j^T \varepsilon_{\phi} \]

Sensitivity coefficient error in the 1-G effective xs

\[ \varepsilon_j \]

errors due to uncertainties in the multigroup xs

\[ [COV_{\sigma_j}] \]

to be processed from the uncertainty libraries

errors due to uncertainties in the multigroup flux spectrum

\[ [COV_{\phi}] \]

to be obtained from a single MCNP calculation
Propagation of uncertainties in burn-up calculations

\[ N(\sigma^{\text{eff}}) - N(\hat{\sigma}^{\text{eff}}) \approx S \varepsilon \]

\[ \text{var } N \approx S \{ \text{COV}_{\sigma_{\text{eff}}} \} S^T \approx S \left[ \begin{array}{cc}
\mathbf{0} & \phi^T \mathbf{0} \\
\phi^T \mathbf{COV}_{\phi} \phi & \mathbf{0}
\end{array} \right] + \left[ \begin{array}{cc}
\mathbf{0} & \hat{\sigma}_j^T \mathbf{COV}_{\phi} \hat{\sigma}_j \\
\hat{\sigma}_j^T \mathbf{COV}_{\phi} \hat{\sigma}_j & \mathbf{0}
\end{array} \right] S^T \]

- Propagates the multigroup xs uncertainties when there is no statistical flux errors
- Propagates statistical flux errors when there is no multigroup xs covariances

Best-estimated calculation
\[ \sigma_0 = (\sigma_{10}, \ldots, \sigma_{j0}, \ldots, \sigma_{R0}) \]

\[ [N]_0 \]

\[ \sigma_0 \quad \phi_1 \quad E \quad \text{MCNP} \]

\[ [N]_1 \quad \frac{dN}{d\sigma} \]

\[ \sigma_0 \quad \phi_2 \quad E \quad \text{MCNP} \]

\[ [N]_2 \quad \frac{dN}{d\sigma} \]

Burnup

OUTPUT
Sensitivity matrix along burnup

depletion with flux from MCNP representative of step 1

depletion with flux from MCNP representative of step 2
Propagation of uncertainties in burn-up calculations

“Hybrid Monte Carlo Method”

Best-estimated calculation
\[ \sigma_0 = (\sigma_{10}, ..., \sigma_{j0}, ..., \sigma_{m0}) \]

\[ [N]_0 \]
ACAB
\[ [N]_1 \]
step 1
step 2

MCNP
MCNP
Burnup

Uncertainty calculations

\[ \text{PDF} \]

For each isotope, \( N_i \):

\[ \Delta N_i \]

Sample of \( M \) vectors \([N]\) of isotopic concentrations

\[ \text{PDF} \]

- PWR-HFP Burnup: 4 cycles taken from Phase-IB (case C). Burnup ~ 65 GWd/tMU
- Fluctuations in power density: transport-inventory coupling, time step, statistical errors, …
Istotopic predictions: Actinides

- Major and minor actinides versus burnup
Fission Products versus burnup

![Graph showing the mass of material over time for different isotopes and power density against time. The graph includes lines for Nd-143, Nd-145, Nd-148, Cs-133, Cs-134, Cs-135, Cs-137, Sm-147, Sm-149, Sm-151, Sm-152, Eu-153, Eu-154, and Tc-99. The x-axis represents time in days, and the y-axis represents mass of material in grams. The power density is shown on the right side.]
Fission Products versus burnup
**Prediction of $\Delta k/k$ using TSUNAMI**

- $\Delta k/k$ (%) predicted with TSUNAMI (SCALE6.0) and the most important contributions
- Here, NO uncertainties in the isotopic inventory are taking into account
Sensitivity $\Delta k/k/\Delta N/N$ predicted with TSUNAMI (SCALE6.0) and the most important contributions by isotopes

Prediction of sensitivities using TSUNAMI
\( \Delta N/N \) (%) predicted with Hybrid Monte Carlo Method due to uncertainties in XSs (EAF2007/UN)
\(\Delta N/N\) (%) predicted with Hybrid Monte Carlo Method due to uncertainties in XSs (EAF2007/UN)
\(\Delta N/N\) predicted with Hybrid Monte Carlo Method due to uncertainties in FYs (JEFF-3.1.1)
Prediction of $\Delta k/k$ due to $\Delta N/N$

- $\Delta k/k$ (%) due to the uncertainties in the isotopic inventory in comparison with the $\Delta k/k$ (%) calculated by TSUNAMI/SCALE6.0 without isotopic inventory uncertainties

- Nuclear data uncertainties: cross-sections and fission-yields

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Uncertainty (%)</th>
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<tbody>
<tr>
<td>$^{235}\text{U}$</td>
<td>3.2 %</td>
</tr>
<tr>
<td>$^{239}\text{Pu}$</td>
<td>18.1 %</td>
</tr>
<tr>
<td>$^{240}\text{Pu}$</td>
<td>15.1 %</td>
</tr>
<tr>
<td>$^{241}\text{Pu}$</td>
<td>7.1 %</td>
</tr>
<tr>
<td>$^{149}\text{Sm}$</td>
<td>11.7 %</td>
</tr>
<tr>
<td>$^{154}\text{Eu}$</td>
<td>4.4 %</td>
</tr>
<tr>
<td>$^{155}\text{Eu}$</td>
<td>24.1 %</td>
</tr>
<tr>
<td>$^{135}\text{Xe}$</td>
<td>12.5 %</td>
</tr>
<tr>
<td>$^{135}\text{Xe}$</td>
<td>94.5 %</td>
</tr>
<tr>
<td>$^{149}\text{Sm}$</td>
<td>2.2 %</td>
</tr>
</tbody>
</table>
In summary …

We have carried out a **Burnup Uncertainty Analysis** for the PWR-HFP Exercise I-1 Benchmark (Burnup ~ 65 GWd/tMU)

1) Assuming **no uncertainties in the isotopic inventory**, TSUNAMI/SCALE6.0 predicts $\Delta k/k$ (%) at BOC:~ 0.5% and EOC :~ 0.8%

At EOC, the most important reactions are: Pu239(nubar), U238(n,gamma), U238(n,n'), Pu239(fission) and Pu239(fission-capture)

2) To take into account **uncertainties in the isotopic inventory**, an Hybrid Monte-Carlo methodology that links transport and inventory calculations is presented

It enables to estimate the impact of nuclear data (neutron cross section and fission yields) uncertainties on the inventory in transport-burnup combined problems.

At EOC, we predict the values of $\Delta k/k$ (%) due to $\Delta N/N$:

- Cross-sections for actinides:~ 0.3% and for fission products :~ 0.2%
  The most important isotopes:Pu239 and P240; Eu-155, Xe135 and Sm149

- Fission yields: ~ 0.2%
  The most important isotopes: Xe135