Evaluation of the mechanical properties of self-compacting concrete using current estimating models
Estimating the modulus of elasticity, tensile strength, and modulus of rupture of self-compacting concrete

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ABSTRACT

This study includes an analysis of the applicability of current models used for estimating the mechanical properties of conventional concrete to self-compacting concrete. The mechanical properties evaluated are: modulus of elasticity, tensile strength, and modulus of rupture. An extensive database which included the dosifications and the mechanical properties of 627 mixtures from 138 different references, was used. The models considered are: ACI, EC-2, NZS 3101:2006 (New Zealand code) and the CSA A23.3-04 (Canadian code). The precision in estimating the modulus of elasticity and tensile strength is acceptable for all models; however, all models are less precise in estimating the modulus of rupture.

Keywords:
Self-consolidating concrete
Estimating models
Tensile strength
Modulus of elasticity
Modulus of rupture

1. Introduction

Compared with conventional concrete, SCC mixtures generally have higher powder content, high content of high range water reducing agents (HRWRA), lower gravel content, smaller maximum gravel diameter, and a higher paste volume [3]. These modifications in the composition of the mixture affect its behavior in the fresh state, but also its mechanical properties in the hardened state.

It is generally considered that the mechanical properties of SCC and conventional vibrated concrete (CC) are similar. However, the opinions on this subject are various, and further research is still needed. Evaluating numerous studies regarding this topic, it is clear that the conclusions regarding the mechanical properties of SCC in comparison to conventional concrete are not unanimous. For example, Attiogbe et al. [1] concludes in his study that conventional and SCC have equivalent modulus of elasticity. On the other hand, Holschemacher and Klug [2] indicate that the modulus of elasticity of SCC is lower than that of conventional concrete. Regarding the tensile strength, New Zealand code NZS 3106:2006 [11] reports that SCC and conventional concrete have equivalent tensile strength, while Martí et al. [4] indicates that the tensile strength of SCC is higher. For the modulus of rupture, Leemann and Hoffmann [5] determine that it is similar for both concretes, while Turcny et al. [6] found that it is higher for SCC. The differences in the mechanical properties of conventional concrete and SCC can be attributed to three main characteristics of SCC: modifications in the composition of the mixture, improvement of the microstructure of the concrete, and the no vibration of the concrete when poured.

The modifications in the composition of the SCC refers to the high paste content and fine material, the lower water/cement ratio and water/powder ratio, lower gravel content, and lower maximum diameter size of the gravel, the use of HRWRA, and viscosity modifying agents (VMA). The improvement in the microstructure can be attributed to the characteristics of the paste and the lower porosity of the transition zone between the aggregate and the paste. The lower water/powder ratio, necessary together with the HRWRA to obtain adequate flowability, favors a more compact and homogeneous transition zone, which in turn, improves the mechanical characteristic of the concrete. In addition, since in SCC there is no need to apply external mechanical compaction, such as vibration, during pouring, the problems that may result from this process, such as segregation of the mixture, or the formation of voids, are avoided.

Considering that the mechanical properties of SCC may vary from those of conventional concrete, and that the various estimating models for calculation theses properties have not been modified for their application in SCC, it is necessary to confirm
their applicability to this type of concrete. The mechanical properties evaluated are: modulus of elasticity, tensile strength, and modulus of rupture. Most current codes include estimation models for calculating these characteristics in the design phase of a project. However, these models have been developed and adjusted for conventional concrete.

As part of the study, it was necessary to build an extensive database so that the analysis results are not based on a few experimental results, but rather on a large and representative sample. This study intends to evaluate the applicability of the existing models using an extensive database, which permits to analyze the results using a wide range of statistical tools, necessary in these types of studies.

1.1. Database

The compiled database includes the dosifications and mechanical properties from 138 different references. Most references are from articles published in scientific publications, publications of research centers, conferences and symposiums, and doctoral theses. The database includes a total of 627 mixtures for compressive strength, 193 mixtures for modulus of elasticity, 165 mixtures for indirect tensile strength, and 59 dosifications for modulus of rupture. The objective was to compile the widest database possible, and to include most of the publications on SCC, up to date. The criteria for including a mixture in the database, was that the complete dosification and at least one of the mechanical properties analyzed has been included in the reference.

The cement content varied between 133 and 665 kg/m³, with a mean value of 374. The water/cement value varied between 0.26 and 1.34, with a mean value of 0.51. The additions content varied between 0 and 490 kg/m³ with a mean value of 158 kg/m³. The slump flow of the mixtures varied between 381 and 864 mm, with a mean value of 699 mm. Ref. [7] includes detailed information regarding the references used, and the dosifications and mechanical characteristics of all the mixtures included in the database.

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1.2. Description of the estimating models

The following are the models evaluated to estimate the mechanical properties of SCC: ACI [8,9], Eurocode 2 [10], the New Zealand code NZS 3106:2006 [11], and the Canadian code CSA A23.3-04 [12]. These models were chosen in such a way as to represent codes of worldwide reference (ACI and EC-2), and two additional codes from Anglo-Saxon countries, but different for the other two. Table 1 includes the estimating equations of the different models.

It is important to indicate that in order to estimate the same property in all the models, it is necessary to adjust some of the models. The EC-2 model and the CSA A23.3-04 code both estimate the direct tensile strength. In order to convert the value to indirect tensile strength, it was considered that the direct tensile strength is 90% the indirect tensile strength. In the case of the EC-2 model, it was also necessary to convert the characteristic strength, \( f_{ck} \), to mean compressive strength, \( f_{cm} \), using the following expression [10]:

\[
F_{ck} = f_{cm} - 8 \text{ MPa}
\]  

(1)

In the EC-2 model, the modulus of rupture is defined in terms of the mean tensile strength and the mean height of the beam element. The mean tensile strength is transformed into the mean compressive strength using the following equation:

\[
F_{cm} = 0.3(f_{cm} - 8 \text{ MPa})^{2/3}
\]  

(2)

A specimen height of 150 mm is considered. This is substituted into the original equation.

2. Analysis of the mechanical properties of SCC in comparison with the different estimating models

2.1. Modulus of elasticity

In Fig. 1 the modulus of elasticity for all the mixtures included in the database and the corresponding best fitted curve is represented in terms of the compressive strength. The figure also includes this relationship for the different estimating models. It can be observed that the curve corresponding to the ACI 318-08 adjusts well to the best fitted curve of the experimental values, but only for compressive strength lower than 50 MPa. For values higher than 50 MPa, this model slightly overestimates the modulus of elasticity of the SCC mixtures.

The EC-2 overestimates the modulus of elasticity for compressive strength lower than 90 MPa. The overestimation magnitude decreases as the compressive strength of the concrete increases. In the case of the NZS 3101:2006, the modulus of elasticity is underestimated. However, the estimation is more precise for lower compressive strength, than for higher strength concrete.

The CSA A23.3 model fits well the best fit curve of the experimental values, slightly underestimating the modulus of elasticity for compressive strength lower than 60 MPa.

Fig. 2 shows the compressive strength versus the relationship between modulus of elasticity of the various models relative to

![Table 1](https://example.com/table1.png)

Table 1

<table>
<thead>
<tr>
<th>Mechanical property</th>
<th>Code</th>
<th>Estimating model</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>ACI 318-08</td>
<td>( E_c = 4700 \sqrt{f_c} )</td>
<td>( E_c ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>EC-2</td>
<td>( E_c = 22(f_{cm}10)^{1/3} )</td>
<td>( E_c ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>NZS 3101:2006</td>
<td>( E_c = 3320 \sqrt{f_c} - 6900 )</td>
<td>( E_c ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3-04</td>
<td>( E_c = 4500 - \sqrt{f_c} )</td>
<td>( E_c ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>ACI 363R-08</td>
<td>( f_{crp} = 0.59(f_{cm} - 8 \text{ MPa})^{1/3} )</td>
<td>( f_{crp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>EC-2</td>
<td>( f_{crp} = 0.54f_{cm} )</td>
<td>( f_{crp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>NZS 3101:2006</td>
<td>( f_{crp} = 0.67f_{cm} )</td>
<td>( f_{crp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3-04</td>
<td>( f_{crp} = 0.94f_{cm}^{1/2} )</td>
<td>( f_{crp} ) (MPa)</td>
</tr>
<tr>
<td>Indirect tensile strength</td>
<td>ACI 363R-08</td>
<td>( f_{tp} = 0.435f_{cm} - 8 \text{ MPa}^{2/3} )</td>
<td>( f_{tp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>EC-2</td>
<td>( f_{tp} = 0.8f_{cm} )</td>
<td>( f_{tp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>NZS 3101:2006</td>
<td>( f_{tp} = 0.6f_{cm} )</td>
<td>( f_{tp} ) (MPa)</td>
</tr>
<tr>
<td></td>
<td>CSA A23.3-04</td>
<td>( f_{tp} = 0.6f_{cm} )</td>
<td>( f_{tp} ) (MPa)</td>
</tr>
</tbody>
</table>

\( E_c \): Modulus of elasticity at 28 days. \( f_c \): Characteristic compressive strength of concrete at 28 days. \( f_{cm} \): Mean compressive strength of concrete at 28 days. \( f_{crp} \): Indirect tensile strength of concrete at 28 days. \( f_{tp} \): Modulus of rupture of concrete at 28 days. \( f_{tp} \): Modulus of rupture of concrete at 28 days (EC-2).
2.2. Compressive strength

In Fig. 3 the relationship between the compressive strength and the tensile strength of the mixtures included in the database and the corresponding best fitted curve are shown. The figure also includes this relationship for the different estimating models. It can be observed that the ACI 363R-08 overestimates the tensile strength for compressive strength lower than 50 MPa. However, for higher strength concrete, \( f_c > 70 \text{ MPa} \), the estimated values coincide well with the experimental values. For the NZS 3101-2006, the calculated values coincide well with the tensile strength of SCC when the compressive strength is lower than 50 MPa. From this point, the model tends to underestimate the tensile strength of the concrete. Regarding the CSA A23.4-04, it can be observed that this model generally overestimates the tensile strength of SCC.

In Fig. 4 the compressive strength versus the relationship between the tensile strength of the various models relative to the best fit curve of the experimental values are shown. It can be observed that the EC-2 model is the most accurate in estimating the tensile strength of SCC. For compressive strength higher than 40 MPa, the difference between the calculated values and the best fit curve for the experimental values ranges from 0%–5%. For lower strength concrete this difference can increase up to 13%, which is still a reasonable value. In the case of the ACI 363R-08 the overestimation of the tensile strength for lower strength SCC can reach a value of 17%, and the underestimation for high strength SCC can reach 12%. The CSA A23.4-04 model overestimates the tensile strength 32% for lower strength SCC, while the NZS 3101-2006 underestimates high strength SCC, a maximum of 20%.

2.3. Modulus of rupture

In Fig. 5 the relationship between the modulus of rupture and the compressive strength for the mixtures included in the database and the corresponding best fitted curve are shown.

In the case of the EC-2 model, for compressive strength below 60 MPa, the estimated values are below the best fit curve. For higher strength concrete, \( f_c > 70 \text{ MPa} \), the estimated values coincide well with the experimental values. For the NZS 3101-2006, the calculated values coincide well with the tensile strength of SCC when the compressive strength is lower than 50 MPa. From this point, the model tends to underestimate the tensile strength of the concrete. Regarding the CSA A23.4-04, it can be observed that this model generally overestimates the tensile strength of SCC.

The best fit curve of the experimental values. Ideally, this curve should be a horizontal line with a value of 1. It can be observed that in general the EC-2 overestimates the modulus of elasticity, and that the overestimation is especially high for lower compressive strength, with a value of approximately 20%. This overestimation decreases as the compressive strength of the concrete increases. The NZS 3101:2006, code, in general, underestimates the modulus of elasticity. However, in this case, the underestimation increases, to values of approximately 10%, for higher compressive strength values. The CSA A23.3 and ACI 318-08 models best estimate the modulus of elasticity of SCC, with the ACI 318-08 being more accurate for lower compressive strength values, and the CSA A23.3 for higher compressive strength values.

2.2. Tensile strength

In Fig. 3 the relationship between the tensile strength and the compressive strength for mixtures included in the database and the corresponding best fitted curve are shown. The figure also includes this relationship for the different estimating models. It can be observed that the ACI 363R-08 overestimates the tensile strength for compressive strength lower than 50 MPa. However, for higher strength concretes the model gradually underestimates the tensile strength. The difference between the best fit curve and the calculated values increases as the compressive strength increases.
The NZS 3101-2006 model overestimates the lowest strength values lower than 70 MPa. For high strength values, this model experimental values.

Fig. 6. The compressive strength versus the relationship between the calculated concrete, and up to 39% for the highest strength concrete.

Concrete a maximum of 20%. The CSA A23.3-04 model underestimates the highest strength concrete, to 10% underestimation for highest strength concrete. In the case of the EC-2 model, this of the database for compressive strength lower than 40 MPa, how­

slightly underestimates the modulus of rupture. The EC-2 model.

In Fig. 6 the compressive strength versus the relationship be­

between modulus of rupture of the various models relative to the modulus of rupture for all concrete concretes. The NZS 3101-2006 model adjusts well to the best fit curve of the database for compressive strength lower than 40 MPa, however, for higher strength values the model underestimates the modulus of rupture. This difference increases as the compressive strength of concrete increases. The CSA A23.3-04 model substantially underestimates the modulus of rupture for all concrete strengths.

In Fig. 6 the compressive strength versus the relationship be­

between modulus of rupture of the various models relative to the best fit curve of the experimental values, are shown. In the case of the ACI 363R-08, there is a 23% maximum overestimation for low strength concrete and a 5% maximum underestimation for the highest strength concrete. In the case of the EC-2 model, this difference varies from 25% underestimation for the lowest strength concrete, to 10% underestimation for highest strength concrete. The NZS 3101-2006 model overestimates the lowest strength concrete a maximum of 3% and underestimates the highest strength concrete a maximum of 20%. The CSA A23.3-04 model underestimates the modulus of rupture from 22%, for the lowest strength concrete, and up to 39% for the highest strength concrete.

3. Comparison between calculated and measured values

This section includes a comparison between the measured values and the calculated values for each of the models and mechanical properties considered. Figs. 7, 9 and 11 include as reference the relationship $y = x$ which represents the condition of equal values for the calculated and measured characteristics, and a target devi­
ation limit of ±30% between the calculated and measured values. The accuracy of the models in estimating the mechanical properties of SCC is analyzed by comparing the measured values (experimental) and the calculated values.

In this type of analysis, the model that best estimates the mechanical properties, is the model that has most data points centered around the reference line $(x = y)$ and within the marked deviation margins of ±30%. The model underestimates the value if the majority of the points are located below the reference line and overestimates it if the majority of the points are above the reference line.

The analysis also includes the best fit line, calculated from linear regression analysis. In this analysis two conditions are considered: with and without the independent term. The second condition, without the independent term, calculates the best fit line that passes through the zero point. This gives the overall tendency of the model in comparison to the reference line. The first condition, with the independent term, permits to evaluate the capacity of the model in discriminating the variability of the results.

3.1. Modulus of elasticity

Fig. 7 includes the relationship between the measured and the calculated values for the different estimating models. Evaluating the distribution of the data points, it can be observed that in general for all models, most of the points are situated within the marked margins of ±30% deviation. In the case of the EC-2 and ACI models, there are some points above the +30% limit, meaning the calculated value is higher than the experimental values. In the case of the NZS 3101:2006 and CSA A23.2-04 models, there are some points below and some above these set limits.

Comparing the best fit line without the independent term, line 2, it can be observed that in the case of the EC-2 and ACI 318-08 models, the slope of the line is approximately 1. The CSA A23.2-04 model has a slope of 0.95, and the NZS 3101:2006 model has a slope of 0.89.

Regarding the best fit line with the independent term, it can be observed that for all models the slope is lower, and the line is more horizontal. This reflects the tendency of all the models to have results within a narrower range of values, in comparison with the experimental values which spread over a wider range of values. All the models have a limited capacity in estimating values that are not centered around the mean. This phenomenon can also be observed in Table 2.

The mean modulus of elasticity of the ACI 318-08 model is very close to the experimental mean, with values of 34.94 GPa and 34.57 GPa, respectively. The EC-2 model slightly overestimates the mean, with a value of 36.66 GPa, and the CSA A23.2-04 and NZS 3101:2006 models underestimate the measured mean, with values of 33.46 GPa and 31.58 GPa. For all models, the standard deviation of the calculated values is lower than that of the experimental value, which has a value of 6.73. An alternative method to evaluate the performance of the different estimating models is using the residual analysis. The residual value, $R$, is defined as the difference between the estimated and the measured value, $R = \text{estimated value} - \text{measured value}$.

In Fig. 8 the residuals of the SCC mixtures as a function of the experimental values for the different estimation models are
Fig. 7. Relationship between the measured modulus of elasticity and the calculated values for the different estimating models.

Fig. 8. Modulus of elasticity. Measured modulus of elasticity versus residual values.
plotted. Positive residuals indicate that the model overestimates the characteristic evaluated and negative residuals indicate that the model underestimates it. A model predicts better the mechanical property if the residuals are closely centered around the zero axis and if these are equally distributed in the positive and negative ranges. The distributions of the residuals in the negative and positive range for the different models are included in Table 2.

From Fig. 8, it can be observed that in all models the data points are centered around a negatively slopped line. This indicates that there is a direct negative relationship between the residuals and the measured modulus of elasticity. Ideally, the model should be independent of the experimental values, and there should be no relationship between the two. In this case, the models overestimate the value when the modulus of elasticity is low (positive residual) and underestimate it (negative residuals) when the modulus of elasticity is high. It is interesting to note that in all models this line crossed the x-axis (residuals of approximately zero), at around 35 GPa. This value corresponds approximately to the measured mean and the calculated mean, as shown in Table 2. It can also be noted that in the case of the EC-2 model, the points are less disperse than in the case of the other three models, as reflected also by its lower standard deviation.

When comparing the percentage of values with positive and negative residuals, it can be observed that the ACI 318-08 model shows the best and most balanced results, with 56% positive residuals. The EC-2 tends to overestimate the modulus of elasticity, with 66% positive residuals. The CSA A23.2-04 model slightly underestimates the modulus of elasticity, with 58% negative residuals, while the NZS 3101:2006 significantly underestimates it with 70% negative residuals.

Comparing the combined mean residual (positive and negative residuals) in Table 2, it can be observed that the ACI 318-08 model has the lowest mean residual (4.16 GPa), followed by the CSA A23.2-04, EC-2, and NZS 3101:2006 models, with values of 4.23 GPa, 4.71 GPa, and 4.80 GPa, respectively.

Considering the various analysis procedures applied, it can be observed that the most precise model in calculating the modulus of elasticity of SCC is the ACI 318-08 model, and the most inaccurate model is the NZS 3101:2006 model.

3.2. Tensile strength

Fig. 9 includes the relationship between the measured tensile strength and the calculated values for the different estimating models. Overall these results are similar to those of the modulus of elasticity analysis, included in the previous section.

Evaluating the distribution of the points, it can be observed that in general for all models most of the points are located within the marked margins of ±30%. In the case of the ACI 363R-08, EC-2 and NZS 3101:2006 models, there are some points below the +30% limit, meaning that in these cases the calculated values are lower than the experimental values. In the case of the CSA A23.2-04 model, there is a group of data points above the +30% deviation margin.

Comparing the best fit line without the independent term, line 2, it can be observed that the CSA A23.2-04, ACI 363R-08, and EC-2 models have slopes close to 1, with values of 1.06, 0.94, and 0.92.

![Fig. 9. Relationship between the measured tensile strength and the calculated values for the different estimating models.](image-url)
Fig. 10. Tensile strength. Measured tensile strength versus residual values.

Fig. 11. Relationship between the measured modulus of rupture and the calculated values for the different estimating models.
respectively. The NZS 3101:2006 model has the lowest slope value of 0.83, indicating an overall underestimation.

Regarding the best fit line with the independent term (best fit line 1), it can be observed that in all models the slope is less steep, and the line is more horizontal. Also in this case, the calculated results are within a narrower range of values in comparison with the experimental values, which spread over a wider range of values.

Comparing the parameters in Table 3, it can be observed that the mean tensile strength of the ACI 363R-08 and EC-2 models are very close to the experimental mean, with values of 4.18, 4.05 and 4.24 MPa, respectively. The CSA A23.3-04 model tends to overestimate the tensile strength, with 75% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals.

Comparing the combined mean residual (positive and negative residuals) in Table 3, it can be observed that the EC-2 model has the best and most balanced results, with 54% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals. The CSA A23.2-04 model tends to overestimate the tensile strength, with 75% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals.

In the case of the residual analysis for the tensile strength, it can also be observed in Fig. 10 that there is a direct negative relationship between the residuals and the experimental value. Also in this case, the models overestimate the value when the tensile strength is low (positive residual) and underestimate it (negative residuals) when the tensile strength is high. For all the models, the best fit line crossed the x-axis (residuals of approximately zero) at approximately the mean calculated value, as indicated in Table 3. It can also be noted that in the case of the EC-2 model, the data points are more disperse than in the case of the other three models. This is also reflected by its higher standard deviation of 0.84.

When comparing the percentage of values with positive and negative residuals, it can be observed that the ACI 363R-08 model shows the best and most balanced results, with 54% positive residuals, followed by the EC-2 model with 45% positive residuals. The CSA A23.2-04 model tends to overestimate the tensile strength, with 75% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals.

Comparing the combined mean residual (positive and negative residuals) in Table 3, it can be observed that the EC-2 model has the best and most balanced results, with 54% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals. The CSA A23.3-04 model tends to overestimate the tensile strength, with 75% positive residuals. The NZS 3101:2006 model underestimates the tensile strength, with 65% negative residuals.

Considering the various analysis procedures applied, it can be observed that the EC-2 and ACI 363R-08 models predict better the tensile strength of SCC than the NZS 3101:2006 and CSA A23.2-04 models.

### 3.3. Modulus of rupture

Fig. 11 includes the relationship between the measured modulus of rupture and the calculated values for the different estimating models.
Table 4
Modulus of rupture. Statistical parameters for evaluating the different estimating models.

<table>
<thead>
<tr>
<th>Modulus of rupture (MPa)</th>
<th>Calculated</th>
<th>ACI 363R-08</th>
<th>EC-2</th>
<th>NZS 3101:2006</th>
<th>CSA A23.3-04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>3.00</td>
<td>4.65</td>
<td>2.82</td>
<td>3.96</td>
<td>2.97</td>
</tr>
<tr>
<td>Maximum</td>
<td>10.30</td>
<td>9.14</td>
<td>8.51</td>
<td>7.78</td>
<td>5.83</td>
</tr>
<tr>
<td>Mean, m</td>
<td>6.74</td>
<td>6.97</td>
<td>5.69</td>
<td>5.93</td>
<td>4.45</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.68</td>
<td>0.99</td>
<td>1.26</td>
<td>0.84</td>
<td>0.63</td>
</tr>
<tr>
<td>Variation coefficient</td>
<td>0.25</td>
<td>0.14</td>
<td>0.22</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>^calculated ^measured</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Z+</td>
<td>0.23</td>
<td>-1.05</td>
<td>-1.81</td>
<td>-2.29</td>
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</tr>
<tr>
<td>Z+¡n+</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>n+</td>
<td>33.10</td>
<td>5.35</td>
<td>10.19</td>
<td>1.37</td>
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<tr>
<td>%</td>
<td>34.00</td>
<td>7.00</td>
<td>14.00</td>
<td>1.00</td>
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</tr>
<tr>
<td>E-</td>
<td></td>
<td></td>
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<tr>
<td>n—</td>
<td>-19.09</td>
<td>-67.82</td>
<td>-58.04</td>
<td>-136.74</td>
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</tr>
<tr>
<td>%</td>
<td>25.00</td>
<td>52.00</td>
<td>45.00</td>
<td>58.00</td>
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<tr>
<td>E-ln—</td>
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</tr>
<tr>
<td>n—</td>
<td>-0.79</td>
<td>-1.30</td>
<td>-1.29</td>
<td>-2.36</td>
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<tr>
<td>mean (&quot;Σ+&quot; + &quot;Σ—&quot;)/n</td>
<td>0.89</td>
<td>1.24</td>
<td>1.16</td>
<td>2.34</td>
<td></td>
</tr>
</tbody>
</table>

Even though, the overall behavior of the results for this mechanical property is somewhat similar to the results of the modulus of elasticity and tensile strength, some clear differences can be detected.

Evaluating Fig. 11, it can be observed that for the modulus of rupture there are less data points than for the other two cases, 59 data points, in comparison with 193 for the modulus of elasticity and 165 for the tensile strength.

Evaluating the distribution of the data points, in the case of the ACI 363R-08 model most points are situated within the marked margins of ±30% deviation, with the exception of a few points above the +30% limit. In the case of the EC-2 and NZS 3101:2006 models, most of the points are situated at the lower part of the marked range, and some of the points are below the marked –30% limit. In the case of the CSA A23.2-04 model, most points are outside and below the marked –30% limit.

Comparing the best fit line without the independent term, line 2, it can be observed that the ACI 363R-08 model has a slope of 1. The EC-2 and NZS 3101:2006 models underestimate the modulus of rupture with slopes of 0.83 and 0.85, respectively. The CSA A23.2-04 model significantly underestimates the modulus of rupture with a slope of 0.64.

Regarding the best fit line with the independent term (best fit line 1), it can be observed that the slope of the regression lines of the ACI 363R-08, NZS 3101:2006, and CSA A23.2-04 models is less steep, and the line is more horizontal. In the case of the EC-2 model this line is steeper, indicating that the spread of the calculated values is more similar to the experimental spread.

Comparing the statistical parameters in Table 4, it can be observed that mean modulus of rupture of the ACI 363R-08 is the closest to the experimental mean, with values of 6.97 and 6.74 MPa, respectively. The NZS 3101:2006 and EC-2 underesti-
mate the mean modulus of rupture, with values of 5.93 and 5.69 MPa, respectively. The CSA A23.2-04 model significantly underestimates the mean modulus of rupture with a value of 4.45 MPa. For all models, except the EC-2, the standard deviation of the calculated values are significantly lower than that of the experimental values, which has a value of 1.68 MPa.

As in the case of the modulus of elasticity and the tensile strength, there is also a direct negative relationship between the residuals and the experimental value of the modulus of rupture, as shown in Fig. 12. As the value of the measured modulus of rupture increases, the magnitude of the residuals decreases. However, for the modulus of rupture, the majority of the residuals are negative in the case of the EC-2, NZS 3101:2006, and CSA A23.2-04 models. It can also be noted that in the case of the EC-2 model, the points are more dispersed than in the case of the other three models. This is also reflected by a higher standard deviation of 1.26.

When comparing the percentage of values with positive and negative residuals, it can be observed that the ACI 363R-08 model shows the best and most balanced results, with 58% positive residuals. The other three models, NZS 3101:2006, EC-2, and CSA A23.2-04 models significantly underestimate the modulus of rupture, with 76%, 88%, and 98% negative residuals, respectively.

Comparing the combined mean residual (positive and negative residuals) in Table 4, it can be observed that the ACI 363R-08 model has the lowest mean residual of 0.89 MPa, followed by the 363R-08, NZS 3101:2006, EC-2, and CSA A23.2-04 models, with values of 1.16, 1.24, and 2.34 MPa, respectively.

Considering the various analysis procedures applied, it can be observed that the ACI 336R-08 predicts better the modulus of rupture of SCC, and that the CSA A23.2-04 is the least precise model.

4. Conclusion

This study presents an extensive database of mechanical results for SCC and evaluates various estimating models and their applicability to this type of concrete. The models considered are the ACI, EC-2, NZS 3101:2006, and the CSA A23.2-0. A comparison between the experimental and calculated values and a detailed analysis of the results permits to establish the following conclusions:

- In terms of general applicability, all the models evaluated are suitable for the estimating the modulus of elasticity, tensile strength, and modulus of rupture of SCC.
- In general, the ACI models can be considered the most precise in estimating all the mechanical properties evaluated in this study. In the case of the indirect tensile strength, the EC-2 is of comparable precision.
- Modulus of elasticity. The ACI 318-08 model slightly overestimates the measured value. The EC-2 model also overestimates the measured modulus of elasticity. The NZS 3101:2006 and the CSA A23.3-04 underestimate the modulus of elasticity, the first one, to a greater degree.
- Tensile strength. The ACI 363R-08 model is the most precise. This model and the EC-2 model slightly under estimate the measured results. The NZS 3101:2006 underestimates the measured values to a greater degree, while the CSA A23.3-04 overestimates the measured values.
- Modulus of rupture. The ACI 363R-08 model is the most precise. The EC-2 and NZS 3101:2006 models slightly underestimate the measured values. The CSA A23.2-04 model significantly underestimates the modulus of rupture.
- Regarding the dispersion of the results, it is important to point out that in all cases the calculated values have less dispersion than the measured results. This indicates that the models do not detect the existing variability shown by the measured results of these mechanical characteristics. There is also a clear negative relationship between the residuals and the measured values. Similar studies carried out on conventional concrete show the same tendency of the models to adjust to mean values with a relatively low standard deviation, implying a lack in their discriminatory capability.
- It is to note that all models use the compressive strength to characterize the concrete, and even though the models are adequately adjusted to match the mean values, they do not consider other variables, which in the case of SCC may be important.
- For all models, the indirect tensile strength estimation is considerably more precise than the estimation of modulus of rupture. This is an indication that there is an inaccuracy in the conversion to a beam element. The only code that considers the element height is the EC-2, but the precision of this model is not better than the others.

References