EXTERNAL HEATING OF A FLAT PLATE IN A CONVECTIVE FLOW

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Abstract—The steady-state and transient processes of the external heating of a flat plate under a convective flow is studied in this paper, with inclusion of the axial heat conduction through the plate. The balance equations reduce to a single integro-differential equation with only one parameter, α, denoting the ratio of the ability of the plate to carry heat in the streamwise direction to the ability of the gas to carry heat out of the plate. The two limits of a good conducting plate ($\alpha \to \infty$) and a bad conducting plate ($\alpha \to 0$) are analysed through the application of a regular perturbation procedure for the first case and a singular perturbation technique for the latter. The existence of two boundary layers at both edges of the plate is shown and their structure are analysed. The evolution of the temperature of the plate is then obtained for a constant external energy flux input.

NOMENCLATURE

- $B(i)$ beta function
- C_s specific heat of the plate
- C_p specific heat at constant pressure
- f Blasius function
- h thickness of the plate
- K kernel of the integral relation defined in equation (2)
- L length of the plate
- Pr Prandtl number, $\mu C_p/\lambda$
- q_e external heat flux
- T temperature
- t time
- u longitudinal Cartesian component of fluid velocity
- x longitudinal Cartesian coordinate.

Greek symbols

- α parameter defined in equation (6a)
- $\Gamma(i)$ gamma function
- ζ inner stretching coordinate
- η similarity variable defined in equation (46)
- θ non-dimensional temperature defined in equation (5)
- λ coefficient of thermal conductivity
- μ dynamic viscosity
- ξ non-dimensional stretching coordinate defined in equation (23)
- ρ density
- τ non-dimensional time defined in equation (5)
- ϕ non-dimensional variable introduced in equation (27)
- χ non-dimensional coordinate defined in equation (5)
- ψ stretching variable defined in equation (22)
- Ω variable defined in equation (45).

Subscripts

- s plate
- ∞ free stream
- l leading edge.

INTRODUCTION

The study of coupled forms of heat transfer has received considerable attention in the literature in the past few years. For a flat plate boundary layer flow there are several works involving these coupled forms of heat transfer [1–6]. Luikov [3] and Payvar [4] analysed the problem where the lower surface of the plate is maintained at a constant and uniform temperature. At the other surface of the plate there is a boundary layer flow. Heat is then transferred to the gas through the plate. Luikov [3] made two approximate solutions, one based on a differential analysis with a low Prandtl number and the second based on an integral analysis with polynomial velocity and temperature profiles through both boundary layers, mechanical and thermal, respectively. He concluded that for Brun numbers greater than 0.1 the plate thermal resistance can be neglected. The Brun number is defined as the ratio of the thermal resistance of the plate to that of the fluid. Payvar [4] used the Lighthill approximation [7] to obtain an integral equation which was solved numerically. For large and small Brun numbers, he obtained asymptotic solutions for the problem. In both these works, axial conduction was not taken into account. Heating (or cooling) a flat plate in a convective flow was analysed by Sohal and Howell [5] and Karvinen [6]. Both works used the Lighthill approximation to obtain an integro-differential equation. Sohal and Howell [5] used a numerical scheme to solve the equation. Karvinen used an iterative method to solve the integro-differential equation for both steady-state and transient cases. He obtained good agreement with experimental results.

The objective of the present work is, using perturbation methods, to obtain the solution to the integro-differential governing equation resulting when the plate is externally heated in a convective flow. For values of α much larger than unity (α being the ratio of the ability of the plate to conduct heat to the ability of the gas to carry heat out from the plate), regular perturbation methods are used to obtain the evolution of the temperature of the plate with time at different positions from the leading edge. For small values of α, a singular perturbation technique (asymptotic matching) is used to solve the governing equations. Similarity solutions are obtained for zero axial heat transfer through the plate.

FORMULATION

The physical model analysed is shown in Fig. 1. A gas flows over a flat plate of finite thickness. The plate is heated through the lower surface with a known constant external energy flux. The initial temperature is assumed to be equal to the temperature of the free stream. At time $t = 0$, the plate is heated in such a way that the temperature of the plate varies along the longitudinal coordinate because of the finite plate thermal conductivity. The energy balance equation in the plate is given by [8]

$$q_e - \rho_s C_s h \frac{\partial T}{\partial t} + \lambda_s h \frac{\partial^2 T}{\partial x^2} = C_p f''(0) Pr^{-2/3} \frac{\sqrt{(u_\infty \rho_\infty \mu_\infty)}}{\sqrt{(2x)}} \times \left\{ T_1 - T_\infty + \int_{T_1}^{T} K(\bar{x}, x)\, d\bar{T} \right\}, \quad (1)$$

where the Lighthill approximation [7] for a high Prandtl number (Pr) has been used. This approximation, however, gives good results for Prandtl numbers of order unity. The kernel of equation (1) is given by [8]

$$K(\bar{x}, x) = \left\{ 1 - \left(\frac{\bar{x}}{x} \right)^{3/4} \right\}^{-1/3}. \quad (2)$$

In the integro-differential equation, equation (1), a quasi-steady behaviour in the gas-phase was assumed; the gas follows the ideal gas law with heat capacity and Prandtl number as constants. In the above equations ρ_s, C_s and λ_s denote the density, heat capacity and thermal conductivity of the plate, respectively; h corresponds to the thickness of the plate; x is the longitudinal coordinate with the origin at the leading edge of the plate; T and T_1 correspond to the temperature of the plate and that given at the leading edge, respectively; u_∞, ρ_∞, μ_∞ and T_∞ are the velocity, density, viscosity and the temperature of the free stream, respectively; C_p is the heat capacity at constant pressure of the gas; $f''(0)$ denotes the second derivative of the Blasius function evaluated at the plate (0.4695) and q_e represents the external heat flux per unit surface per unit time added to the plate. It is further assumed that the temperature of the plate is uniform in the transversal coordinate, that is

$$\frac{3\lambda_s \sqrt{L}\, Pr^{2/3}}{C_p h \sqrt{(u_\infty \rho_\infty \mu_\infty)}} \gg 1, \quad (3)$$

where L corresponds to the length of the plate.

Assuming both edges of the plate are adiabatic and the initial temperature of the plate is equal to that of the free stream, the initial and boundary conditions are given by

$$T = T_\infty \quad \text{for} \quad t < 0,$$
$$\frac{\partial T}{\partial x} = 0 \quad \text{at} \quad x = 0 \text{ and } L. \quad (4)$$

It is convenient to introduce the following non-dimensional variables

$$\theta = \frac{C_p \sqrt{(u_\infty \rho_\infty \mu_\infty)} f''(0) Pr^{-2/3}}{\sqrt{(2L)} q_e} (T - T_\infty),$$
$$\chi = \frac{x}{L}, \quad (5)$$
$$\tau = \frac{C_p \sqrt{(u_\infty \rho_\infty \mu_\infty)} f''(0) Pr^{-2/3}}{\sqrt{(2L)} \rho_s C_s h} t.$$

The energy balance equation, equation (1), then transforms to

$$-\frac{\partial \theta}{\partial \tau} + \alpha \frac{\partial^2 \theta}{\partial \chi^2} + 1 = \frac{\theta_1}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}} \int_{\theta_1}^{\theta} K(\bar{\chi}, \chi)\, d\bar{\theta}. \quad (6)$$

Equation (6) shows only one free parameter, α, which represents the ratio of the ability of the plate to carry heat in the streamwise direction to the ability of the gas to carry heat from the plate. This parameter is defined as

$$\alpha = \frac{\sqrt{2}\, \lambda_s h\, Pr^{2/3}}{f''(0) \sqrt{(u_\infty \rho_\infty \mu_\infty)} C_p L^{3/2}}. \quad (6a)$$

The non-dimensional initial and boundary conditions

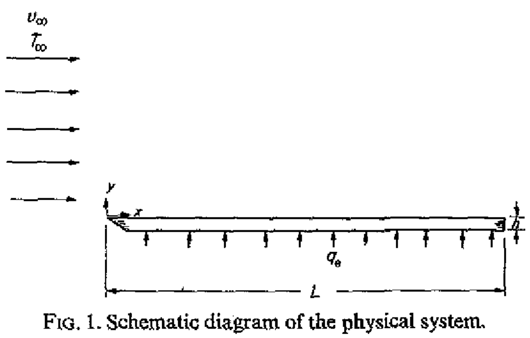

FIG. 1. Schematic diagram of the physical system.

are also transformed to

$$\theta = 0 \quad \text{for} \quad \tau < 0,$$
$$\frac{\partial \theta}{\partial \chi} = 0 \quad \text{at} \quad \chi = 0 \text{ and } 1. \tag{7}$$

Solving equations (6) and (7) gives the evolution of the temperature of the plate. The temperature distribution on the plate is strongly affected by α, denoting the streamwise heat conduction through the plate. In the following sections, three different limits for α are analysed; $\alpha \gg 1$, $\alpha \ll 1$ and α of order unity. First, the steady-state analysis is undertaken in order to obtain the equilibrium temperature distribution for different values of α. The transient analysis is left to the second part of this paper.

STEADY-STATE ANALYSIS

In this section the steady-state analysis of the above mentioned problem is made for different values of α. The steady-state forms of the energy balance equation, equation (6), with the adiabatic boundary conditions are

$$\alpha \frac{d^2\theta}{d\chi^2} + 1 = \frac{\theta_1}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}} \int_{\theta_1}^{\theta} K(\bar{\chi}, \chi)\, d\bar{\theta}, \tag{8}$$

and

$$\frac{d\theta}{d\chi} = 0 \quad \text{at} \quad \chi = 0 \text{ and } 1. \tag{9}$$

In the following lines we seek solutions to equations (8) and (9) for different limits of α.

Good conducting plate ($\alpha \to \infty$)

If α is large enough, the temperature of the plate varies little in the longitudinal coordinate. Thus, solution to equations (8) and (9) can be obtained through a regular perturbation procedure with $1/\alpha$ as the small parameter of expansion. The temperature of the plate is then assumed to be

$$\theta(\alpha, \chi) = \sum_{n=0}^{\infty} \frac{1}{\alpha^n} \theta_n(\chi). \tag{10}$$

Substituting equation (10) in equations (8) and (9) gives the following set of equations

$$\frac{d^2\theta_0}{d\chi^2} = 0, \tag{11}$$

$$\frac{d^2\theta_1}{d\chi^2} = \frac{\theta_{01}}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}} \int_{\theta_{01}}^{\theta_0} K(\bar{\chi}, \chi)\, d\bar{\theta} - 1, \tag{12}$$

$$\frac{d^2\theta_{n+1}}{d\chi^2} = \frac{\theta_{n1}}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}} \int_{\theta_{n1}}^{\theta_n} K(\bar{\chi}, \chi)\, d\bar{\theta}_n, \quad \text{for} \quad n \geq 1, \tag{13}$$

and

$$\frac{d\theta_n}{d\chi} = 0 \quad \text{at} \quad \chi = 0 \text{ and } 1 \text{ for all } n. \tag{14}$$

Solving equations (11) and (14) gives

$$\theta_0 = \frac{1}{2}. \tag{15}$$

This constant was found after integrating equation (12) over the whole length of the plate. Thus, the solution to equation (12) is

$$\theta_1 = \theta_{11} + \frac{2}{3}\chi^{3/2} - \frac{1}{2}\chi^2, \tag{16}$$

where θ_{11} is found again after integration of the next equation. This gives

$$\theta_{11} = \frac{4}{15} B(8/3, 2/3) - \frac{1}{3} B(2, 2/3), \tag{17}$$

where $B(i, j)$ denotes the beta function. In the same way we obtain

$$\theta_2 = \theta_{21} + \frac{4}{3}\theta_{11}^{3/2} + \frac{2}{9} B(2, 2/3)\chi^3 - \frac{16}{105} B(8/3, 2/3)\chi^{7/2}, \tag{18}$$

with

$$\theta_{21} = -\frac{2}{3}\theta_{11} - \frac{8}{63} B(2, 2/3) B(4, 2/3) + \frac{4}{45} B(8/3, 2/3) B(14/3, 2/3). \tag{19}$$

The results obtained in this section give a very good approximation for values of $\alpha \geq 5$.

Bad conducting plate ($\alpha \to 0$)

In this limit, which in fact is a singular limit, the longitudinal heat conduction through the plate can be neglected in the whole length of the plate except in regions close to the leading and trailing edges. Therefore, the solution to equations (8) and (9) shows the existence of two boundary layers in both edges of the plate. In regions outside these boundary layers (denoted as the outer zone), the solution can be found from equation (8) without the diffusion term. That is

$$1 = \frac{1}{\sqrt{\chi}} \int_0^{\theta_e} K(\bar{\chi}, \chi)\, d\bar{\theta}_e. \tag{20}$$

Here, the subscript e denotes the outer zone. After inversion, equation (20) has the solution

$$\theta_e = b\sqrt{\chi}, \quad \text{with} \quad b = \frac{\sqrt{3}\Gamma(4/3)\Gamma(1/3)}{2\pi\Gamma(5/3)} = 0.7305. \tag{21}$$

Here $\Gamma(i)$ represents the gamma function. Close to the leading edge ($\chi \to 0$), the adiabatic boundary condition can be achieved only if the heat diffusion term is retained. To study this boundary layer, we introduce the following stretching variables

$$\psi = \frac{\theta}{\alpha^{1/3}}, \tag{22}$$

and

$$\xi = \frac{\chi}{\alpha^{2/3}}. \tag{23}$$

The energy balance equation, equation (8), then reduces to the following parameter free form

$$1+\frac{d^2\psi}{d\xi^2} = \frac{\psi_1}{\sqrt{\xi}} + \frac{1}{\sqrt{\xi}}\int_{\psi_1}^{\psi} K(\bar{\xi},\xi)\,d\bar{\psi}. \tag{24}$$

The adiabatic boundary condition is reduced to

$$\frac{d\psi}{d\xi} = 0 \quad \text{at} \quad \xi = 0. \tag{25}$$

The other boundary condition is dictated by matching with the outer solution given by equation (21), that is

$$\frac{d\psi}{d\xi} \sim \frac{b}{2\sqrt{\xi}} \quad \text{as} \quad \xi \to \infty. \tag{26}$$

A solution to equations (24)–(26) can be obtained numerically. Thus, ψ_1 is selected in such a way that matching between inner and outer solutions is assured. The behaviour of ψ for large values of ξ is found to be very sensitive to ψ_1. For slightly different values of the critical one that makes possible the matching, the solution diverges strongly as $\xi \to \infty$. In Fig. 2 the structure of the leading edge boundary layer is shown.

On the other hand, close to the trailing edge of the plate ($\chi \to 1$), exists a thin zone of order $\alpha^{3/5}$, where the heat conduction term has to be retained in order to obtain the adiabatic boundary condition. We define the following inner variables

$$\theta = b\sqrt{\chi} - \frac{b}{2}\left(\frac{3}{4}\right)^{1/5}\alpha^{3/5}\Phi, \tag{27}$$

and

$$\zeta = \frac{1-\chi}{\alpha^{3/5}}\left(\frac{4}{3}\right)^{1/5}. \tag{28}$$

For this inner zone, the energy balance equation, equation (8), reduces to

$$\frac{d^2\Phi}{d\zeta^2} = \int_0^{\Phi} \frac{d\bar{\Phi}}{(\bar{\zeta}-\zeta)^{1/3}}. \tag{29}$$

The adiabatic boundary condition is given by

$$\frac{d\Phi}{d\zeta} = -1 \quad \text{at} \quad \zeta = 0. \tag{30}$$

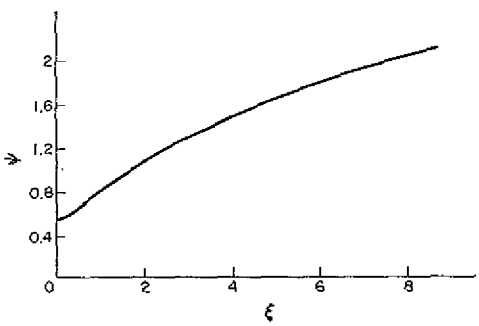

FIG. 2. Structure of the leading edge boundary layer.

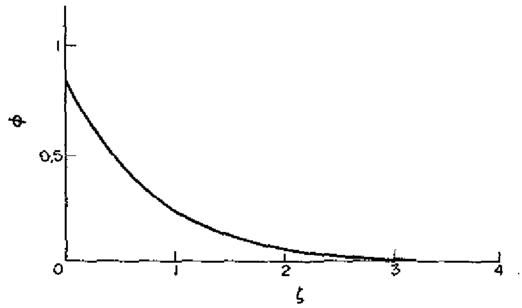

FIG. 3. Structure of the trailing edge boundary layer.

The other boundary condition comes from matching with the outer zone, and is given as

$$\Phi \to 0 \quad \text{as} \quad \zeta \to \infty.$$

Solutions to equations (29) and (30) are obtained numerically. To facilitate the numerical integration, it is convenient to deduce the asymptotic behaviour for large values of ζ. This is given by

$$\Phi \sim \exp(-1.2\zeta) \quad \text{as} \quad \zeta \to \infty. \tag{31}$$

Thus, the right hand term of equation (29) can be changed in the following form

$$\int_0^{\Phi} \frac{d\bar{\Phi}}{(\bar{\zeta}-\zeta)^{1/3}} = \int_{\infty}^{\zeta_f} \frac{d\bar{\Phi}}{d\bar{\zeta}}\frac{d\bar{\zeta}}{(\bar{\zeta}-\zeta)^{1/3}} + \int_{\zeta_f}^{\zeta} \frac{d\bar{\Phi}}{d\bar{\zeta}}\frac{d\bar{\zeta}}{(\bar{\zeta}-\zeta)^{1/3}}, \tag{32}$$

where

$$\int_{\infty}^{\zeta_f} \frac{d\bar{\Phi}}{d\bar{\zeta}}\frac{d\bar{\zeta}}{(\bar{\zeta}-\zeta)^{1/3}} = \exp(-1.2\zeta_f) \times\left\{1 - \frac{1}{3.6(\zeta_f-\zeta)} + O(\zeta_f-\zeta)^{-2}\right\} \quad \text{for} \quad \zeta_f \gg 1. \tag{33}$$

Figure 3 shows the structure of the boundary layer at the trailing edge.

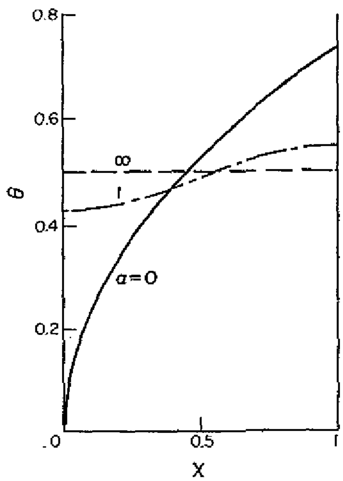

FIG. 4. Non-dimensional temperature profiles on the plate for different values of α.

For values of α of order unity, equations (8) and (9) are directly numerically integrated and the temperature profiles are shown in Fig. 4, for the cases of $\alpha = 0$, 1 and ∞.

TRANSIENT ANALYSIS

In this section the transient heating process is analysed, in order to obtain the evolution of the temperature of the plate with time, for the two limiting cases of a good conducting plate and an adiabatic plate.

Good conducting plate ($\alpha \to \infty$)

The solution to equations (6) and (7) can be obtained, for large values of α, through regular perturbation techniques. We assume a solution to the above mentioned equations in the following form

$$\theta(\alpha,\chi,\tau) = \sum_{n=0}^{\infty} \frac{1}{\alpha^n}\,\theta_n(\chi,\tau). \tag{34}$$

The energy balance equation, equation (6), with the associated initial and boundary conditions (7) transform to the following set of equations

$$\frac{\partial^2\theta_0}{\partial\chi^2} = 0, \tag{35}$$

$$\frac{\partial^2\theta_1}{\partial\chi^2} = \frac{\partial\theta_0}{\partial\tau} - 1 + \frac{\theta_{01}}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}}\int_{\theta_{01}}^{\theta_0} K(\bar{\chi},\chi)\,\mathrm{d}\bar{\theta}_0, \tag{36}$$

$$\frac{\partial^2\theta_{n+1}}{\partial\chi^2} = \frac{\partial\theta_n}{\partial\tau} + \frac{\theta_{n1}}{\sqrt{\chi}} + \frac{1}{\sqrt{\chi}}\int_{\theta_{n1}}^{\theta_n} K(\bar{\chi},\chi)\,\mathrm{d}\bar{\theta}_n \quad \text{for} \quad \eta \geqslant 1, \tag{37}$$

and

$$\theta_n(\chi,0) = 0; \qquad \partial\theta_n/\partial\chi = 0 \quad \text{at} \quad \chi = 0 \text{ and } 1 \text{ for all } n. \tag{38}$$

Solving equations (35) and (38) shows that θ_0 is only a function of τ. This function is obtained after integration of equations (36) and (38) along the whole length of the plate. This gives

$$\theta_0 = \frac{1}{2}\{1-\exp(-2\tau)\}. \tag{39}$$

Following the same procedure with the next equation (37), we obtain

$$\theta_1 = \theta_{11} + \frac{4}{3}\theta_0\chi^{3/2} - \theta_0\chi^2, \tag{40}$$

with

$$\theta_{11} = A - A\exp(-2\tau) - (0.2+A)\exp(-2\tau)\tau, \tag{41}$$

and

$$A = \frac{4}{15}B(8/3, 2/3) - \frac{1}{3}B(2, 2/3). \tag{42}$$

The other functions of the expansion can be obtained in the same form without difficulty.

Adiabatic plate ($\alpha = 0$)

For this concrete case, the energy balance equation, equation (7), is reduced to

$$\frac{\partial\theta}{\partial\tau} = 1 - \frac{1}{\sqrt{\chi}}\int_0^{\theta} K(\bar{\chi},\chi)\,\mathrm{d}\bar{\theta}. \tag{43}$$

Equation (43) can be transformed to a self-similar form given by

$$\Omega - 2\eta\frac{\mathrm{d}\Omega}{\mathrm{d}\eta} = 1 - \frac{1}{\sqrt{\eta}}\int_0^{\Omega} K(\bar{\eta},\eta)\,\mathrm{d}\bar{\Omega}, \tag{44}$$

with

$$\Omega(\eta) = \frac{\theta(\chi,\tau)}{\tau}, \tag{45}$$

and

$$\eta = \frac{\chi}{\tau^2}. \tag{46}$$

The asymptotic behaviour of Ω for small and large values of η can be found to be given by

$$\Omega \sim b\sqrt{\eta} \quad \text{for} \quad \eta \to 0, \tag{47}$$

and

$$\Omega \sim 1 - \frac{1}{2\sqrt{\eta}} \quad \text{for} \quad \eta \to \infty. \tag{48}$$

Equation (47) determines the steady-state equilibrium solution of the heating process and is equivalent to equation (21). Equation (48) is dictated by the initial condition. It is convenient to introduce ϕ as

$$\phi = \Omega - b\sqrt{\eta}, \tag{49}$$

with the associated conditions given by

$$\phi(0) = 0; \quad \phi \sim -b\sqrt{\eta} \text{ as } \eta \to \infty. \tag{50}$$

Equation (44) then transforms to

$$\phi - 2\eta\frac{\mathrm{d}\phi}{\mathrm{d}\eta} = -\frac{1}{\sqrt{\eta}}\int_0^{\phi} K(\bar{\eta},\eta)\,\mathrm{d}\bar{\phi}. \tag{51}$$

Equation (51) with $\phi(0) = 0$ allows an infinite number of solutions, including the trivial one, denoting the point $\eta = 0$ as a nodal point. However, there is only one solution that satisfies the asymptotic behaviour for large η. For small values of η it can be shown that

$$\phi \sim c\exp\left\{-\frac{a}{\eta^{3/4}}\right\} \quad \text{as} \quad \eta \to 0,$$

where

$$a = \{\Gamma(5/3)\}^{3/2},$$

and c is a free constant which in fact gives the infinite number of solutions. The value of c is determined by the asymptotic solution for large η. In Fig. 5, Ω is plotted as a function of η obtained from the numerical solution of equation (51) with the aid of equation (49).

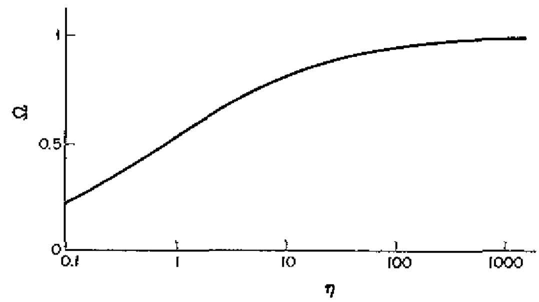

FIG. 5. Similarity parameter, Ω, as a function of η for an adiabatic plate ($\alpha = 0$).

CONCLUSIONS

Perturbation methods have been applied to solve integro-differential equations resulting from the balance equations of the external heating of a flat plate in a convective flow. For large values of α, regular perturbation methods are used with $1/\alpha$ as the small parameter of expansion. A three-term expansion is obtained, giving good results (within 1%) for $\alpha \geqslant 5$. The singular limit of $\alpha \to 0$ is analysed through matched asymptotic methods. In this case, the longitudinal heat conduction has to be retained in regions close to both edges of the plate, leading to the existence of two boundary layers. The leading term of the expansion is then obtained. This gives a fairly good approximation for values of α such as $\alpha \leqslant 0.05$. For $\alpha = 0$, the energy balance equation can be reduced to a free-parameter self-similar form which can be integrated numerically. Steady-state and transient analyses were made in order to obtain the evolution of the temperature of the plate with time and longitudinal position. These analyses can be extended to a turbulent flow with the corresponding changes in the parameters of the energy balance equation and in the kernel of the integral relation.

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REFERENCES

1. E. M. Sparrow and S. H. Lin, Boundary layers with prescribed heat-flux application to simultaneous convection and radiation, *Int. J. Heat Mass Transfer* **8**, 437–448 (1965).
2. M. Sakikibara, S. Moriand and A. Tanimoto, Effect of wall conduction on convective heat transfer with laminar boundary layer, *Heat Transfer—Jap. Res.* **2**, 94–103 (1973).
3. A. V. Luikov, Conjugated convective heat transfer problems, *Int. J. Heat Mass Transfer* **17**, 257–265 (1974).
4. P. Payvar, Convective heat transfer to laminar flow over a plate of finite thickness, *Int. J. Heat Mass Transfer* **20**, 431–433 (1977).
5. M. S. Sohal and J. R. Howell, Determination of plate temperature in case of combined conduction, convection and radiation heat exchange, *Int. J. Heat Mass Transfer* **16**, 2055–2066 (1973).
6. R. Karvinen, Some new results for conjugated heat transfer in a flat plate, *Int. J. Heat Mass Transfer* **21**, 1261–1264 (1978).
7. M. J. Lighthill, Contributions to the theory of heat transfer through a laminar boundary layer, *Proc. R. Soc.* **A202**, 359 (1950).
8. W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer* (2nd edn.). McGraw-Hill, New York (1980).

APPENDIX

CONVECTIVE COOLING OF AN ADIABATIC FLAT PLATE

In this appendix, the analysis of the cooling of an adiabatic flat plate in a convective flow is made. The initial temperature of the plate is denoted at T_i. The free stream temperature is T_∞. The energy balance equation for the plate is given by

$$-\frac{\partial\theta}{\partial\tau} = \frac{1}{\sqrt{\chi}}\int_0^{\theta} K(\bar{\chi},\chi)\,\mathrm{d}\bar{\theta}, \tag{A1}$$

where θ is now defined as

$$\theta = \frac{T-T_\infty}{T_i - T_\infty}. \tag{A2}$$

The initial condition is therefore

$$\theta(\chi, 0) = 1. \tag{A3}$$

Equation (A1) allows a similar solution, being the similarity variable given by

$$\eta = \frac{\chi}{\tau^2}. \tag{A4}$$

Thus, equation (A1) is transformed to

$$2\eta^{3/2}\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = \int_0^{\theta} K(\bar{\eta},\eta)\,\mathrm{d}\bar{\theta}. \tag{A5}$$

The initial condition then transforms to

$$\theta(\infty) = 1. \tag{A6}$$

The final equilibrium temperature is the same as that of the free stream, that is $\theta(0) = 0$. Close to $\eta \to 0$, the solution to equation (A5) shows, as found earlier for equation (51), that

$$\theta \sim c\exp\left\{-\frac{a}{\eta^{3/4}}\right\} \quad \text{for} \quad \eta \to 0. \tag{A7}$$

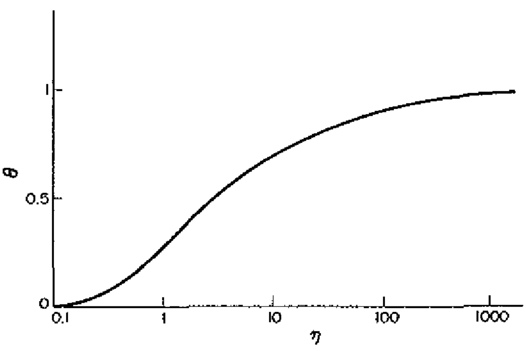

FIG. A1. Non-dimensional temperature of an adiabatic plate as a function of η for convective cooling.

There is only one value of c that enables the initial condition (A6) to be reached. After evaluation of equation (A5) in the limit $\eta \to \infty$, the asymptotic behaviour leads more precisely to

$$\theta \sim 1 - \frac{1}{\sqrt{\eta}} \quad \text{as} \quad \eta \to \infty. \qquad \text{(A8)}$$

Equation (A5) can be solved numerically. In Fig. A1, θ is plotted as a function of η, from which the temperature of the plate can be obtained at each χ position and time τ.

CHAUFFAGE EXTERNE D'UNE PLAQUE PLANE DANS UN ECOULEMENT CONVECTIF

Résumé—Les processus permanents et variables de chauffage externe d'une plaque plane soumise à un écoulement convectif sont étudiés en tenant compte de la conduction axiale dans la plaque. Les équations de bilan réduites à une seule équation intégro-différentielle avec un unique paramètre α qui dénote le rapport de l'aptitude de la plaque à conduire la chaleur dans la direction de l'écoulement à celle du gaz à arracher la chaleur de la plaque. Les deux limites d'une plaque très conductrice ($\alpha \to \infty$) et d'une plaque mauvaise conductrice ($\alpha \to 0$) sont analysées à travers la procédure d'une perturbation régulière pour le premier cas et d'une perturbation singulière dans le second cas. L'existence de deux couches limites aux deux bords de la plaque est montrée et leur structure est analysée. L'évolution de la température de la plaque est obtenue pour un flux thermique incident externe constant.

EXTERNE AUFHEIZUNG EINER FLACHEN PLATTE IN EINER KONVEKTIONSSTRÖMUNG

Zusammenfassung—In dieser Arbeit werden die stationären und instationären Prozesse der externen Aufheizung einer flachen Platte unter einer Konvektionsströmung untersucht, wobei die axiale Wärmeleitung durch die Platte mit berücksichtigt wird. Die Bilanz-Gleichungen reduzieren sich auf eine einzige Integro-Differential-Gleichung mit nur einem Parameter α, der das Verhältnis der Fähigkeit der Platte, Wärme in Stromrichtung zu leiten, zur Fähigkeit des Gases, Wärme aus der Platte zu transportieren, beschreibt. Die beiden Grenzfälle einer gut leitenden Platte ($\alpha \to \infty$) und einer schlecht leitenden Platte ($\alpha \to 0$) werden untersucht, wobei im ersten Fall ein reguläres Störungsverfahren und im zweiten eine singuläre Störungstechnik angewendet wird. Die Existenz zweier Grenzschichten an beiden Enden der Platte wird aufgezeigt, und ihre Struktur wird untersucht. Schließlich wird die Entwicklung der Plattentemperatur für konstanten Energiefluß von außen in die Platte erhalten.

ВНЕШНИЙ НАГРЕВ ПЛОСКОЙ ПЛАСТИНЫ ПРИ КОНВЕКТИВНОМ ТЕЧЕНИИ

Аннотация—Исследуются установившиеся и неустановившиеся режимы внешнего нагрева плоской пластины при конвективном течении с учетом аксиальной передачи тепла теплопроводностью через пластину. Уравнения баланса сводятся к одному интегродифференциальному уравнению с параметром α, представляющим отношение способности пластины проводить тепло по направлению течения к способности газа отводить тепло от пластины. Анализируются два предельных случая: хорошо проводящей ($\alpha \to \infty$) и плохо проводящей ($\alpha \to 0$) пластин с применением метода регулярных возмущений в первом случае и метода сингулярных возмущений во втором. Показано существование двух пограничных слоев на концах пластины и проведен анализ их структуры. Получено поле температур пластины при постоянной плотности подводимого потока энергии.