Abutments influence in the dynamic response of bridges

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SUMMARY – *A simplified analytical model of a short span bridge is proposed. The inertial interaction effects of pier foundations and abutments has been included in order to evaluate the response sensitivities to different soil-structure interaction variables. The modification of natural frequency and damping properties is shown for typical short span bridges of the integral deck-abutment type for longitudinal vibrations or general bridges for the transverse ones.*

KEYWORDS: Brige abutments; dynamic soil-structure interaction; seismic response of bridges; boundary element method.

1. Introduction

During the modelling of bridges that have to be analyzed under seismic loading a great deal of attention is dedicated to the careful representation of the details of the superstructure while the interaction with the soil is usually represented in a less strict way. This is specially true for the abutments where no much experience is available, while for the pier footings it is possible to use formulas that were developed for other uses (machine foundations, nuclear power plants, buildings, etc.). Paradoxically some Codes, [1], recommend the introduction of the abutment dynamic properties in the model and their study was considered a worthwhile one since the very beginning of the systematic research on seismic bridge behavior, [3].

Except for the case of bridges which deck is monolithic with the abutments (the so called integral deck-abutments bridges), their infuence is not very large in the longitudinal response. On the contrary for transverse or even vertical displacements, taking account the interaction effect can modify largely the results, specially in short span bridges. That modification is related to the stiffness properties but also to the damping that the radiation of waves to the surrounding soil can introduce in the global behavior of the superstructure. This can be observed in a typical highway overcrossing analyzing, in a qualitative way, the relative dimensions embankment-abutment as it is shown in Fig. 1.

In that sense it is very instructive to see the studies developed to identify the mechanical properties of different models made to understand the data registered in actual bridges that were subjected to seismic actions. A detailed compilation of them has been made in our previous work [9]. One of them, J.C. Wilson and B.S. Tan [18], [19], includes a first attempt to make a simplified representation of the approaching embankments and it presents numerical values of the damping and stiffness that would he neccessary to understand the values registered in an actual structure. Among those results two of them are appealing: first the apparent reduction of the abutment stiffness with respect to the static values (of the order of 50%) and simultaneously a damping ratio (from 25 to 45%) very high in comparison with the generally accepted ones. The structure studied is of the integral

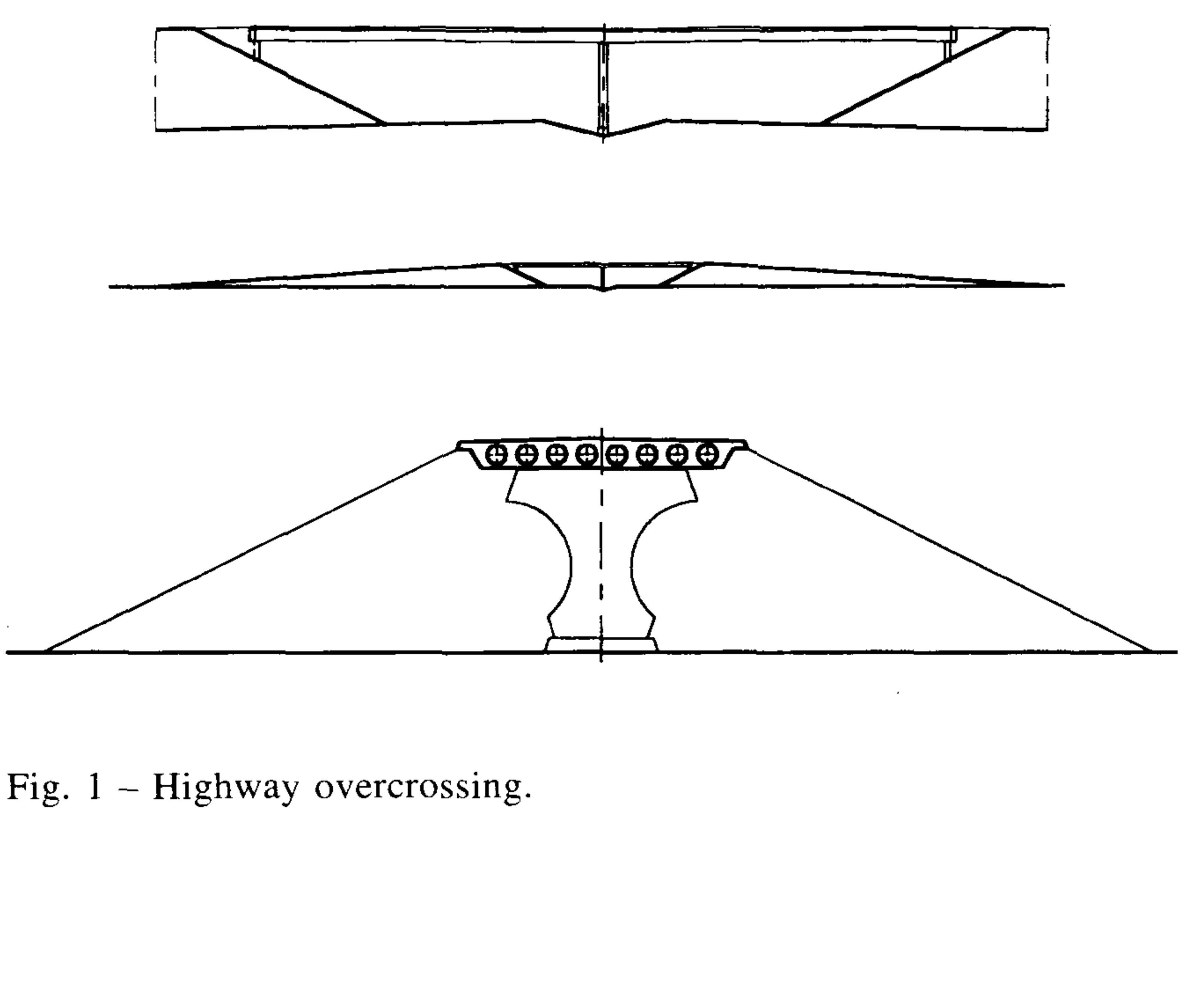

Fig. 1 – Highway overcrossing.

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type so the interaction effects are specially important.

From the point of view of the dynamic soil-structure interaction both phenomena could be interpreted as the effect of the mobilized embankment mass in the effective stiffness and the radiation damping respectively. None of the two effects are included in the simplified model of J.C. Wilson et al., [18], [19]. There is a general reluctance to use linear models to analyze soil-structure interaction in walls. Nevertheless several authors J. Wood [21], H. Tajimi [15] and A.S. Veletsos et al. [16] have tried to improve the comprenhension of the problem using them and stablishing their limits of applications.

Motivated by the above mentioned ideas a research was launched to improve the existing models using a dynamic formulation. Plane longitudinal and transverse models for the abutments were studied introducing layering of the soil, computing the complex components of the dynamical impedances under the assumption of a massless and rigid abutment.

The results confirm the above mentioned effects of stiffness reduction and increase of damping in proportions promisingly similar to the experimental measurements, E. Alarcón et al. [2].

On the other hand it is well known the difficulty of modelling the damping with plane models so the study was extended with a simplified three dimensional model of the abutment. In both cases the numerical technique was the Boundary Element Method (B.E.M.) in the frequency domain which is specially well suited for the treatment of viscoelastic semiinfinite media A.M. Cutillas et al. [6], [7], [8], [9].

Once the possible focus of the discrepancies detected by J.C. Wilson et al. [18] was localized, we have tried to quantify the importance of the soil-structure interaction using a simple model that is developed below.

2. Modelling of the structure

In order to study the relative importance of the different parameters which are involved in the dynamic reponse of bridges, a simple model will be considered. This model will take into account the inertial, stiffness and damping properties from the deck, abutments, piers and foundations.

Only *inertial interaction* effects will be considered in the abutments and pier foundations.

The displacements considered in the soil include the *kinematic interaction* effects or their influence will be so small to consider negligible.

With this model all the parameters influence will be analyzed in a single degree system whose response will be condensed in:

- The natural frequency of vibration.
- The equivalent damping ratio.

A single dynamical degree of freedom system will be considered to analyze the influence of different interaction parameters. This model will be similar to those employed by J.P. Wolf [20], D.R. Somaini [11], [12], [13], Spyrakos [14] and Maragakis et al. [5]. This model is shown in Fig. 2.

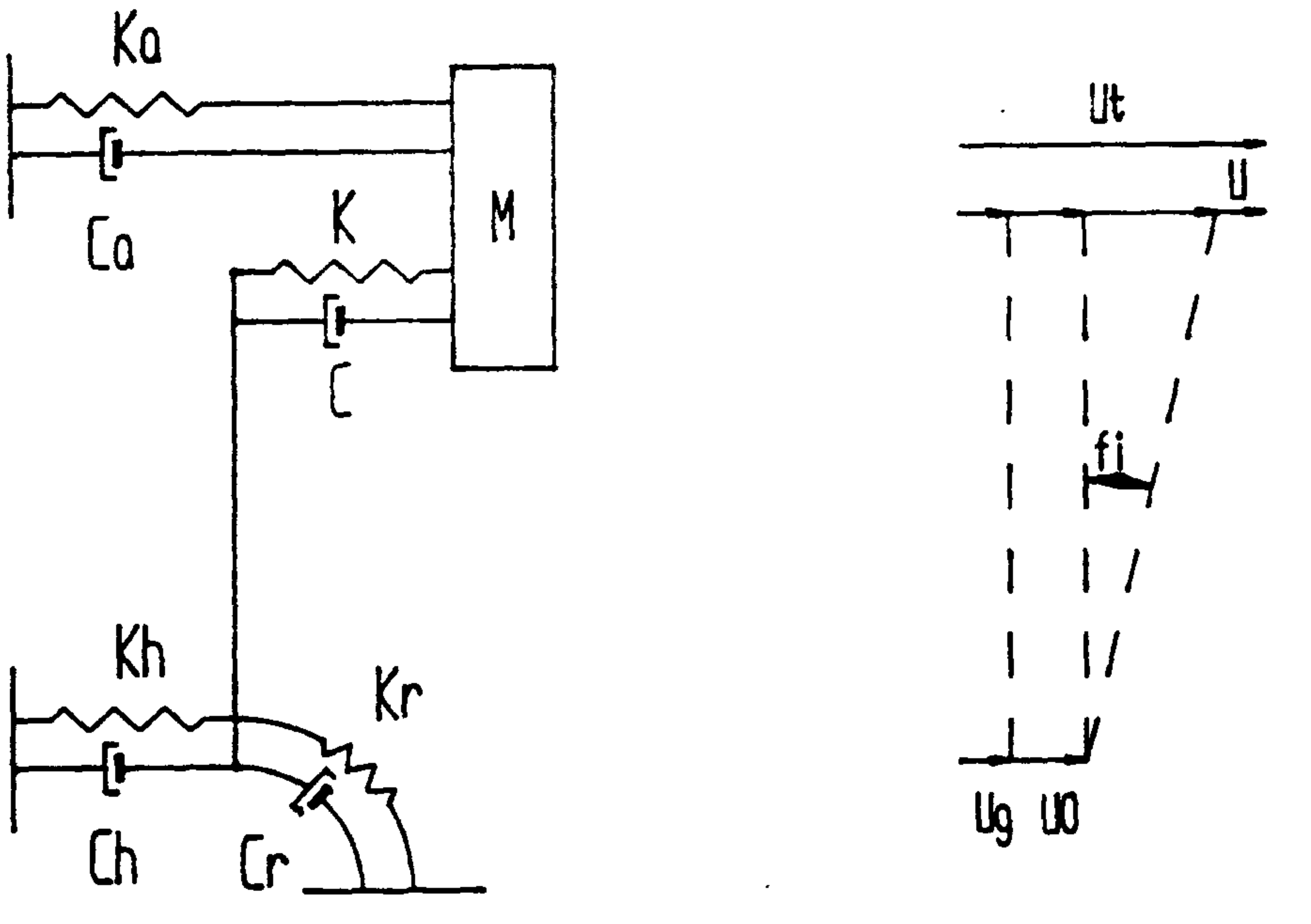

Fig. 2 – Dynamical model.

The deck and piers mass is m, pier stiffness and damping are k and c respectively. The dynamic stiffness matrices of the soil are represented in a simple way by the pairs (k_h, c_h), (k_r, c_r); for the piers foundations and (k_a, c_a) for the abutments.

The dynamical degree of freedom is the horizontal relative displacement u between the pier foundation and the deck. Depending on the stiffness and damping parameters involved, this displacement could be the longitudinal or transverse one.

After establishing the main assumptions in the analysis, the equation of motion will be performed using the substructures method in soil-structure interaction. As the model is a single degree of freedom system, only a single displacement is needed to obtain its equivalent frequency and damping characteristics:

2.1. Basic assumptions

The basic assumptions in the behavior of deck, piers foundations and abutments are:

- *Linear elastic behavior* of the materials: Bridge deck, piers and the soil interaction with the piers foundations and the abutments.
- *Rigid behavior of the deck* in the horizontal plane. Horizontal stiffness of usual bridge decks are greater than the piers ones and the whole abutment-embanlement system.
- *Ractangular decks.* The bearing axis and the longitudinal deck axis are perpendicular. *Skew decks* are not considered in this study. With this assumption it is possible to uncouple the longitudinal and transverse movements and to study them independently.
- *The whole mass of the bridge is concentrated on the deck.* The mass in a bridge is formed by the self weight of the structural elements and superimposed loads such as surfacing, railings etc. located on the deck. The piers mass is usually smaller than the deck mass so the mass center is very close to the deck.
- *Pier deck connection,* is usually done by elastomeric bearings. The horizontal stiffness of the bearings is included in piers stiffness. With respect the rotational stiffness, two extreme assumptions will be done:

– *Hinged connection* between the pier and the deck. This assumption considers the rotational stiffness of the bearings negligible in comparison with the piers ones.

Built-in connection between the pier and the deck. In this situation there are no elastomeric bearings. The stiffness of the deck is greater than the piers ones.

• *The degrees of freedom* considered are those which produce horizontal motions on the bridge deck:

– *Horizontal displacement of the pier foundation*: u_0.

– *Rotation of the pier foundation*: φ.

– *Relative horizontal displacement of the pier*: u.

• *Only Inertial soil-structure interaction* has been considered.

Kinematic soil-structure interaction is considered small enough to be neglected.

The length of the deck will be considered small in order to assume identical soil displacements in the piers foundations and in the abutments.

• *Dynamic stiffness matrices of the soil* will be considered uncoupled. Frequency dependent pairs of values (k, c), will be defined in each degree of freedom:

– In the pier foundation (k_h, c_h) will be the stiffnesses for the horizontal displacement and (k_r, c_r) for the rotation.

– In the abutment (k_a, c_a) will be considered for the horizontal displacement.

• Energy dissipation will be considered with an *hysteretic damping* coefficient c or ζ_s concentrated in the piers.

2.2. Equations of motion

The equilibrium equations in frequency domain will be:

$$S(\omega)u(\omega) = P(\omega) \tag{1}$$

where

$$S(\omega) = K + i\omega C - \omega^2 M \tag{2}$$

The application of *substructures method* that have been explained in [9] leads to the next equations of motion for a seismic excitation:

$$\begin{bmatrix} S_{ss} & S_{sb} \\ S_{bs} & S^s_{bb} + S^g_{bb} \end{bmatrix} \begin{Bmatrix} u^t_s \\ u^t_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ S^g_{bb} \quad u^g_b \end{Bmatrix} \tag{3}$$

doing *the kinematic interaction* and *inertial interaction* decomposition and neglecting the kinematic part, the equations of motion become:

$$\begin{bmatrix} S_{ss} & S_{sb} \\ S_{bs} & S^s_{bb} + S^g_{bb} \end{bmatrix} \begin{Bmatrix} u^i_s \\ u^i_b \end{Bmatrix} = \omega^2 \begin{bmatrix} M_{ss} & M_{sb} \\ M_{bs} & M^s_{bb} \end{bmatrix} \begin{Bmatrix} u^k_s \\ u^k_b \end{Bmatrix} \tag{4}$$

Because of all nodes are in contact with the soil it can be expressed as:

$$[S^s_{bb} + S^g_{bb}]\{u^i_b\} = \omega^2[M^s_{bb}]\{u^k_b\} \tag{5}$$

In Appendix A the dynamic stiffness matrix and load vector will be obtained both in total and relative motion coordinates.

2.3. Displacement solution

Expressed the equations of motion for each frequency it is possible to obtain the displacement for each degree of freedom.

In order to employ dimensionless parameters:

$$\omega_s^2 = \frac{k}{m} \quad \omega_h^2 = \frac{k_h}{m} \quad \omega_r^2 = \frac{k_r}{mh^2} \quad \omega_a^2 = \frac{k_a}{m} \tag{6}$$

these are frequency parameters and

$$\zeta_h = \frac{c_h\omega}{2k_h} \quad \zeta_r = \frac{c_r\omega}{2k_r} \quad \zeta_a = \frac{c_a\omega}{2k_a} \tag{7}$$

are the damping ratios.

The two types of pier deck connections will be studied: hinged and built-in situation.

Hinged pier

From eq. 46, obtained in Appendix A:

$$S(\omega)u = P \tag{8}$$

dividing by m, and ω^2, and with the new parameters defined above, the dynamic stiffness matrix $S(\omega)$ will be expressed as:

$$\begin{bmatrix} \frac{\omega_s^2}{\omega^2}(1+2\zeta_s i) + \frac{\omega_a^2}{\omega^2}(1+2\zeta_a i) - 1 & \frac{\omega_a^2}{\omega^2}(1+2\zeta_a i) - 1 & \frac{\omega_a^2}{\omega^2}h(1+2\zeta_a i) - h \\ -\frac{\omega_s^2}{\omega^2}(1+2\zeta_s i) & \frac{\omega_h^2}{\omega^2}(1+2\zeta_h i) & 0 \\ -\frac{\omega_s^2}{\omega^2}h(1+2\zeta_s i) & 0 & \frac{\omega_r^2}{\omega^2}h^2(1+2\zeta_r i) \end{bmatrix}$$

with the displacement and load vectors:

$$u = \begin{Bmatrix} u \\ u_0 \\ \varphi \end{Bmatrix} \qquad P = u_g \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

Solving the equations, the displacement u is:

$$\left\{(1+2\zeta_s i) + \frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i) - \frac{\omega^2}{\omega_s^2} + \left[\frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i) - \frac{\omega^2}{\omega_s^2}\right]\frac{\omega_s^2}{\omega_h^2}\frac{1+2\zeta_s i}{1+2\zeta_h i} + \left[\frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i) - \frac{\omega^2}{\omega_s^2}\right]\frac{\omega_s^2}{\omega_r^2}\frac{1+2\zeta_s i}{1+2\zeta_r i}\right\} u = \frac{\omega^2}{\omega_s^2}u_g \tag{9}$$

With this solution the system can be studied for different variation of stiffness and damping parameters.

This is the only way to obtain the system response when the stiffness and damping coefficient from the dynamic matrices of the soil are frequency dependent.

Built-in pier

In this case, from eq. 47 (obtained in Appendix A), dividing by m and ω^2, the dynamic stiffness matrix $\boldsymbol{S}(\omega)$ will be expressed as:

$$\begin{bmatrix} \frac{\omega_s^2}{\omega^2}(1+2\zeta_s i)+\frac{\omega_a^2}{\omega^2}(1+2\zeta_a i)-1 & \frac{\omega_a^2}{\omega^2}(1+2\zeta_a i)-1 & \frac{\omega_a^2}{\omega^2}\frac{h}{2}(1+2\zeta_a i)-\frac{h}{2} \\ -\frac{\omega_s^2}{\omega^2}(1+2\zeta_s i) & \frac{\omega_h^2}{\omega^2}(1+2\zeta_h i) & 0 \\ -\frac{\omega_s^2}{\omega^2}\frac{h}{2}(1+2\zeta_s i) & 0 & \frac{\omega_s^2}{\omega^2}\frac{h}{12}(1+2\zeta_s i)+\frac{\omega_r^2}{\omega^2}h^2(1+2\zeta_r i) \end{bmatrix}$$

with the same displacement and load vector than the previous case, the displacement u will be:

$$\Bigg\{(1+2\zeta_s i)+\frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i)-\frac{\omega^2}{\omega_s^2}+$$

$$+\left[\frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i)-\frac{\omega^2}{\omega_s^2}\right]\frac{\omega_s^2}{\omega_h^2}\frac{1+2\zeta_s i}{1+2\zeta_h i}+$$

$$+\left[\frac{\omega_a^2}{\omega_s^2}(1+2\zeta_a i)-\frac{\omega^2}{\omega_s^2}\right]\frac{\omega_s^2(1+2\zeta_s i)}{\frac{\omega_s^2}{3}(1+2\zeta_r i)+4\omega_r^2(1+2\zeta_r i)}\Bigg\}u=\frac{\omega^2}{\omega_s^2}u_g \quad (10)$$

2.4. Equivalent frequency

An interesting approach to understand the new results is to obtain the properties of an equivalent single-degree of freedom system.

The response u of a single-degree of freedom system with natural frequency $\bar{\omega}$, damping ratio $\bar{\zeta}$ fixed in the soil with a ground motion $\bar{u}_g$ will be:

$$\left[1+2\bar{\zeta}i-\frac{\omega^2}{\bar{\omega}^2}\right]u=\frac{\omega^2}{\bar{\omega}^2}\bar{u}_g \quad (11)$$

in the frequency domain.

Hinged pier

From eq. 9 it is possible to obtain the equivalent frequency $\bar{\omega}$, considering the system without any damping $\zeta_s=\zeta_h=\zeta_r=\zeta_a=0$. The excitation frequency which makes null the u displacement coefficient, $\omega=\bar{\omega}$ is the equivalent frequency. Strictly speaking, the frequency which make singular the response for the system with no damping is not the frequency which maximizes the response of the system with damping. However both frequencies are very close and for practical purposes may be considered the same.

The equivalent frequency expression becomes:

$$\bar{\omega}^2=\omega_a^2+\frac{1}{\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}} \quad (12)$$

When the frequency of the abutment cancels, $\omega_a=0$, the equivalent frequency corresponds to a system in which the compliances of the foundation and the structure are added.

Built-in pier

In this case the system with no damping from eq. 10, is considered obtaining a new expression for the equivalent frequency:

$$\bar{\omega}^2=\omega_a^2+\frac{1}{\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\frac{\omega_s^2}{3}+4\omega_r^2}} \quad (13)$$

2.5. Equivalent damping

It is possible to obtain an equivalent damping ratio $\bar{\zeta}$ which summarizes the influence of different damping ratios in the global response of the system studied.

If eqs. 9 and 10 are *linearized* with the following assumptions:

$$\frac{1+2\zeta_s i}{1+2\zeta_h i}=\frac{(1+2\zeta_s i)(1-2\zeta_h i)}{1+4\zeta_h^2}\sim 1+2\zeta_s i-2\zeta_h i$$

$$\frac{1+2\zeta_s i}{1+2\zeta_r i}=\frac{(1+2\zeta_s i)(1-2\zeta_r i)}{1+4\zeta_r^2}\sim 1+2\zeta_s i-2\zeta_r i$$

because the damping ratios are smaller than unity and the products $\zeta_i\zeta_j \ll 1$ can be neglected for every pairs of values of the indexes i, j.

With these assumptions, expressions for the equivalent damping ratio can be obtained in the following way:

$$\bar{\zeta}=F_s\zeta_s+F_h\zeta_h+F_r\zeta_r+F_a\zeta_a \quad (14)$$

The coefficient F_s, F_h, F_r and F_a are the participation factors of the damping ratios from the structure, translational and rotational ones from the pier foundations and from the abutment, respectively, in the total equivalent damping ratio.

These coefficients have been obtained when the excitation frequency is equal to the equivalent frequency. In this frequency range the damping ratio has a more significant influence in the dynamic response of the system.

The expression for the participation factors are different depending on pier-deck connection:

Hinged pier

$$F_s = \frac{1}{\omega_s^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{1}{1+\frac{k}{k_h}+\frac{k}{k_r}}$$

$$F_h = \frac{1}{\omega_h^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{\frac{k}{k_h}}{1+\frac{k}{k_h}+\frac{k}{k_r}} \qquad (15)$$

$$F_r = \frac{1}{\omega_r^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{\frac{k}{k_r}}{1+\frac{k}{k_h}+\frac{k}{k_r}}$$

$$F_a = \omega_a^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right] = \frac{k}{k_a}\left[1+\frac{k}{k_h}+\frac{k}{k_r}\right]$$

Built-in pier

$$F_s = \frac{\frac{1}{\omega_s^2}+\frac{3\omega_s^2}{(\omega_s^2+12\omega_r^2)^2}}{\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{3}{\omega_s^2+12\omega_r^2}} = \frac{1+\frac{3}{\left(1+12\frac{k_r}{k}\right)^2}}{1+\frac{k}{k_h}+\frac{3}{1+12\frac{k_r}{k}}}$$

$$F_h = \frac{1}{\omega_h^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{3}{\omega_s^2+12\omega_r^2}\right]} = \frac{\frac{k}{k_h}}{1+\frac{k}{k_h}+\frac{3}{1+12\frac{k_r}{k}}} \qquad (16)$$

$$F_r = \frac{\frac{36\omega_r^2}{(\omega_s^2+12\omega_r^2)^2}}{\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{3}{\omega_s^2+12\omega_r^2}} =$$

$$= \frac{36\frac{k_r}{k}}{\left(1+12\frac{k_r}{k}\right)^2}\frac{1}{1+\frac{k}{k_h}+\frac{3}{1+12\frac{k_r}{k}}}$$

$$F_a = \omega_h^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{3}{\omega_s^2+12\omega_r^2}\right] =$$

$$= \frac{k}{k_a}\left[1+\frac{k}{k_h}+\frac{3}{1+12\frac{k_r}{k}}\right]$$

Table 1 – Variation of parameters

	SOIL CONDITIONS		
STRUCTURE		GOOD	POOR
SMALL	h/L k/kh ka/k	2.40 0.05 3.50	1.20 0.10 3.50
BIG	h/L k/kh ka/k	1.40 0.15 0.70	0.80 0.30 0.70

2.6. Equivalent ground acceleration

From eq. 11 the equivalent ground acceleration for an equivalent degree of freedom system can be obtained:

$$\bar{u}_g = \frac{\bar{\omega}^2}{\omega_s^2}u_g \qquad (17)$$

This value can be calculated once the equivalent frequency is known.

3. Equivalent frequency and damping

Once the main parameters which have a significant influence in the equivalent frequency and damping have been obtained, their dependence with the stiffness and damping characteristics will be studied.

Firstly a study of range of variation of different variables involved in *Highway overcrossings* will be done. With these variables and additional assumptions a parametric study will be made.

Many experimental data and analysis from the seismic response of Meloland Road Overcrossing in California have been studied by Werner et al. [17], Levine et al. [4] and Wilson and Tan [18], [19]. An experimental validation of the results can be done employing their data.

3.1. Range of variation of different variables

To obtain reasonable values of the variables involved, a table with extreme parameters for the soil and the structure in usual highway overcrossing has been built (Table 1). Different assumptions have been made:

• For the *structure*: deck and piers, two extreme situations for two spans highway overcrossing will be considered:

– A *small structure* in which one carriageway road overcrosses a highway without any central reserve (15 m. long span and 12 m. wide).

– A *big structure*, two carriageways road overcrossed a highway with a central reserve (25 m. long span and 23 m. wide).

A hinged pier-deck connection will be considered.

• *A footing type* pier foundation will be considered.

The dynamic stiffness of the footings are frequency dependent. Because this dependence is small, as a first approach the following constant values may be considered [20], [10]:

$$k_h = \frac{8GL}{2-\nu} \qquad c_h = \frac{4.6GL^3}{(2-\nu)c_s} \tag{18}$$

$$k_r = \frac{8GL^3}{3(1-\nu)} \qquad c_r = \frac{0.4GL^4}{(1-\nu)c_s} \tag{19}$$

G is the transverse modulus of elasticity of the soil, L is the footing radius, ν is the Poisson's ratio and c_s is the shear wave velocity of the soil.

If the stiffnesses are expressed in an usual way employing the statical component:

$$k_h = k_{st},_x(k_x + ia_0c_x)$$

$$k_r = k_{st},_\varphi(k_\varphi + ia_0c_\varphi) \tag{20}$$

the expressions shown above are equivalent to:

$$k_x = 1 \qquad c_x = 0.575 \tag{21}$$
$$k_\varphi = 1 \qquad c_\varphi = 0.150 \tag{22}$$

where the total stiffness component is equivalent to the statical one.

As there is a relation between the rocking and horizontal stiffness in circular footings, the pier foundation participation factors in the equivalent damping may be joined into the factor F_{so}:

$$\bar{\zeta} = F_s\zeta_s + F_{so}\zeta_h + F_a\zeta_a \tag{23}$$

• The *soil conditions in the pier foundations* will be compatible with footing type foundations.

Two different soil conditions have been analyzed:

– *good soil*, with a shear wave velocity $c_s = \sqrt{\frac{G}{\rho}} = 200 m/s$.

– *poor soil*, with a shear wave velocity around 100 m/s.

In both cases the footing dimensions have been obtained using an admissible settlement of 0.01 m.

• *The soil conditions in the embankment*, behind the abutment are between the two extreme soil conditions mentioned above because the embankment properties are independent from foundation soil conditions.

The values k_a used correspond to the statical longitudinal component in the half-space case:

$$k_a = 6.07\frac{GH}{2-\nu} \tag{24}$$

where H is the abutment height. For the study purposes, the abutment and the piers will have the same dimensions $H = h$.

3.2. Parametric study

The classical dimensionless parameters to analyze soil-structure phenomena may be expressed [20]:

• With respect the characteristic dimension of the pier foundation $L = a$:

$$\bar{h} = \frac{h}{a} \quad \bar{m} = \frac{m}{\rho a^3} \quad \bar{s} = \frac{\omega_s h}{c_s} \tag{25}$$

• With respect the height of the abutment H:

$$\bar{H} = \frac{h}{H} \quad \bar{M} = \frac{m}{\rho H^3} \quad \bar{S} = \frac{\omega_s H}{c_s} \tag{26}$$

Where

h : is the piers height.

a : is a characteristic dimension of the foundation: footing radius.

ρ : is the soil density.

c_s: is the shear wave velocity of the soil.

H : is a characteristic dimension of the aboutment: its height.

The stiffness and damping ratios may be expressed as functions of those parameters in order to obtain the equivalent frequency and equivalent damping of the system:

$$\frac{k_a}{k} = \frac{k_{xa}}{\bar{S}^2\bar{M}} \quad \frac{k}{k_h} = \frac{\bar{s}^2\bar{m}}{8}\frac{(2-\nu)}{\bar{h}^2k_x} \quad \frac{k}{k_r} = \frac{\bar{s}^2\bar{m}}{8}\frac{3(1-\nu)}{k_\varphi} \tag{27}$$

$$\zeta_h = \frac{c_xa_0}{2k_x} = \frac{c_x}{2k_x}\frac{\bar{s}}{\bar{h}}\frac{\omega}{\omega_s}$$

$$\zeta_r = \frac{c_\varphi a_0}{2k_\varphi} = \frac{c_\varphi}{2k_\varphi}\frac{\bar{s}}{\bar{h}}\frac{\omega}{\omega_s} \tag{28}$$

$$\zeta_a = \frac{c_{xa}\bar{a}_0}{2k_{xa}} = \frac{c_{xa}}{2k_{xa}}\bar{S}\frac{\omega}{\omega_s}$$

k_x, k_φ, c_x y c_φ: are the Veletsos' coefficients of the piers footings. They are usually frequency dependent from the dimensionless frequency $a_0 = \omega a/c_s$, but they can he considered constant in a wide range of its values.

k_{xa} y c_{xa} are the Veletsos' coefficients of the abutment. They are frequency dependent from the dimensionless frequency $\bar{a}_0 = \omega H/c_s$.

The damping ratios depend on the excitation frequency ω. These ratios have a significant influence in the system response when $\omega = \bar{\omega}$ so only the values at this frequency will be considered.

In order to obtain the frequency dependent abutment stiffness coefficient a simple non linear problem in the form:

$$\begin{cases} \dfrac{\overline{\omega}}{\omega_s} = f(k_{xa}) \\ k_{xa} = g\left(\dfrac{\overline{\omega}}{\omega_s}\right) \end{cases} \tag{29}$$

must he solved to get the equivalent frequency. The solution is obtained in an iterative way.

The basic parameters studied are the relative slenderness of the pier $\overline{h}$ and the relative stiffness of the abutment $k_a\,/k$ against the relative stiffness soil-structure using the parameters $\overline{s}$.

The mass parameter $\overline{m}$ and dimensionless stiffness pier foundation coefficients will be constant. For these coefficients, circular footings values given in the previous section will be used.

These studies have been done assuming the two extreme pier-deck connections: hinged and built-in connection (Figs. 3 to 6):

• The variation with relative slenderness parameter $\overline{h}$ is shown in Figs. 3 and 4 with the values $k_a\,/k = 0.7$ and $\overline{m} = 10$. Interaction effects are more important for squat piers (h decreases) and when the structure is stiffer than the soil ($\overline{s}$ increases).

Interaction effects decrease the equivalent frequency and increase the equivalent damping of the system. These variations are bigger in the hinged pier case than in the built-in one because in the last one the displacements are smaller.

• The variation with relative abutment stiffness k_a / k is shown in Figs. 5 and 6). With $\overline{h} = 2$ and $\overline{m} = 10$ it can be seen how relative frequency is bigger when relative stiffness is smaller. Its maximum value will be reached when there is no connection between the deck and the abutment, that is $k_a\,/k = 0$.

Otherwise equivalent damping ratio increases when the abutment relative stiffness increases. The connection between the pier and the deck has a greater influence in this case. The hinged pier produces bigger deck displacements and therefore bigger damping ratios by the presence of the abutment than in the built-in case.

3.3. MELOLAND ROAD OVERCROSING

The results obtained may he compared with the conclusions reached by Wilson and Tan [18], [19] about the movement measurements in Meloland Road Overcrossing in California submitted to the Imperial Valley Earthquake in 1979.

Some of these conclusions may be explained with the new results:

• Important reduction in the stiffness of the abutment-embankment system was detected during the motion, around a 50%.

This stiffness reduction was explained by the non linear behavior of the soil during the motion.

As it can be obtained, the dynamic stiffness components in a standard three dimensional abutment decrease with frequency. Reductions with respect the statical component around 50% or even greater can be reached:

– The transverse mode is one of the most important with a frequency of 2.5 Hz. Taking as soil property $c_s = 67$ m/s and for the abutment $H = 5$ m the dimensionless frequency will be $\overline{a}_0 = 1.2$, the transverse stiffnesses of the abutment on a rigid base (Y axis) will be [9]:

$$k_y \in [0.1, 0.5] \qquad c_y \in [0.3, 0.8] \tag{30}$$

depending on the rigid base depth.

– These values show a stiffness reduction around 50 and 90% and damping ratios in the next interval:

$$\zeta_y \in [0.30, 0.50] \tag{31}$$

• System identification techniques detect damping ratios in the abutment-embankment system between 25 and 45%. These values are included in the interval obtained above.

• The equivalent damping ratios of the whole sys-

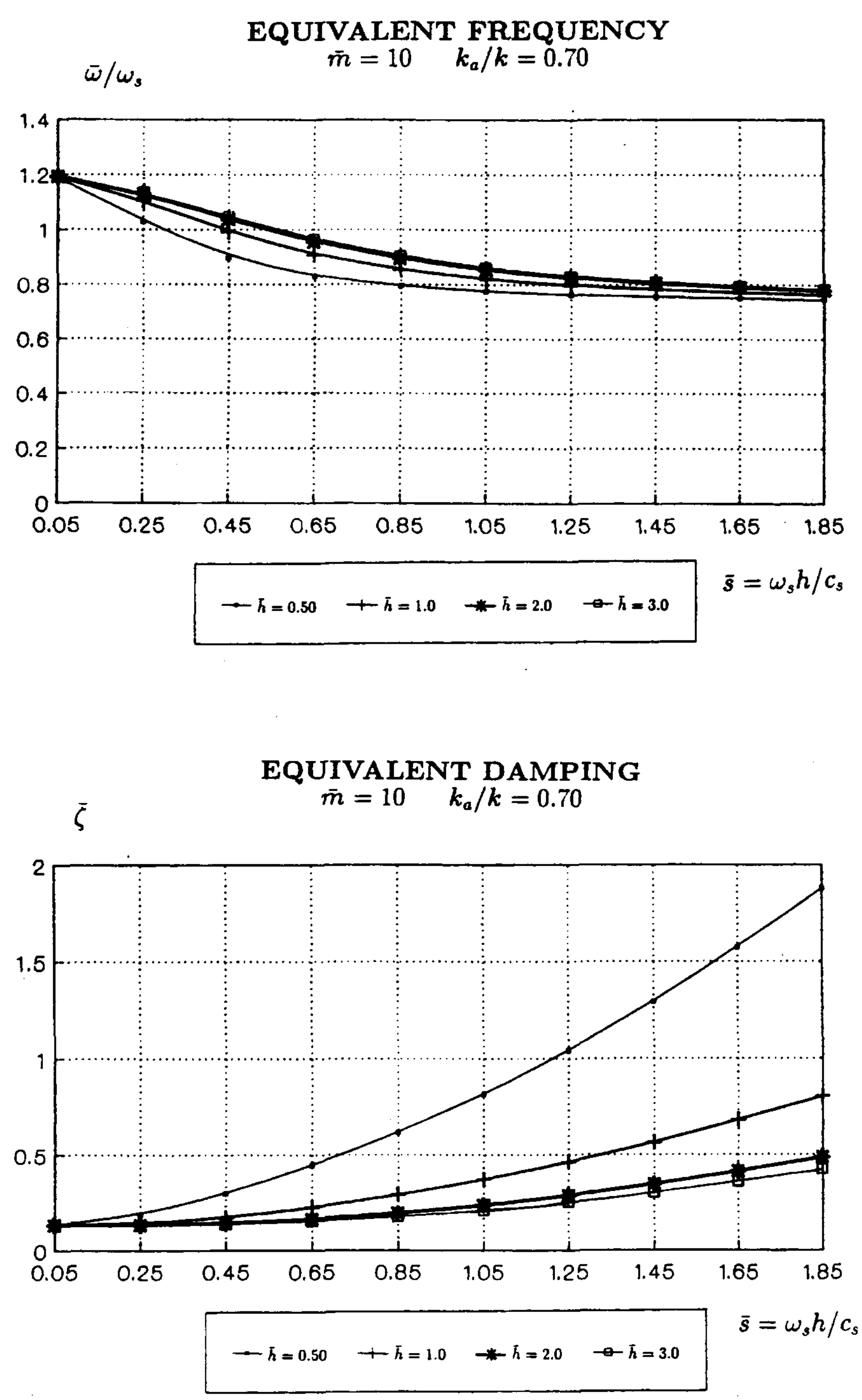

Fig. 3 – Equivalent frequency and damping ratio. $\overline{h}$ dependence. Hinged pier.

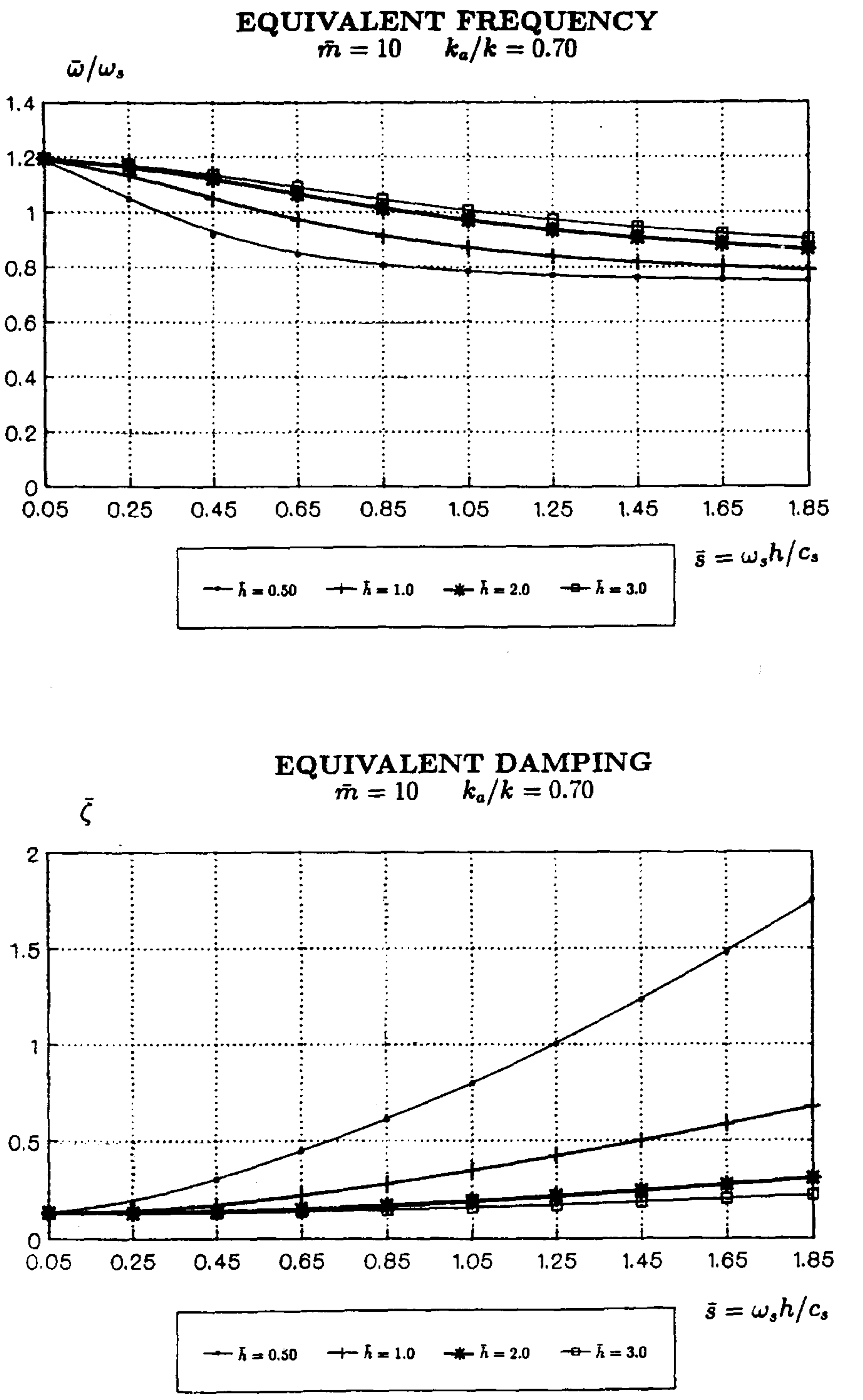

Fig. 4 – Equivalent frequency and damping ratio. $\bar{h}$ dependence. Built-in pier.

tem, including abutments effects, are between 3 and 12%.

If the next values are considered for the structure:

$$\bar{h} = 40 \quad \bar{m} = 200000 \quad \bar{M} = 4 \qquad \zeta_s = 0.05 \quad (32)$$

and no interactions effects in the pier foundations are considered, equivalent frequency and damping ratio for the system can he obtained for different abutment stiffness ratios in Fig. 7.

If a built-in pier deck connection is assumed, very close to the real Overcrossing situation, a range of damping ratios between 3 and 10% can be obtained.

4. Conclusions

The main conclusions of this work may be summarized as follows:

• A single degree of freedom model of a bridge structure including inertial interaction effects, in pier foundations and abutments, have been proposed.

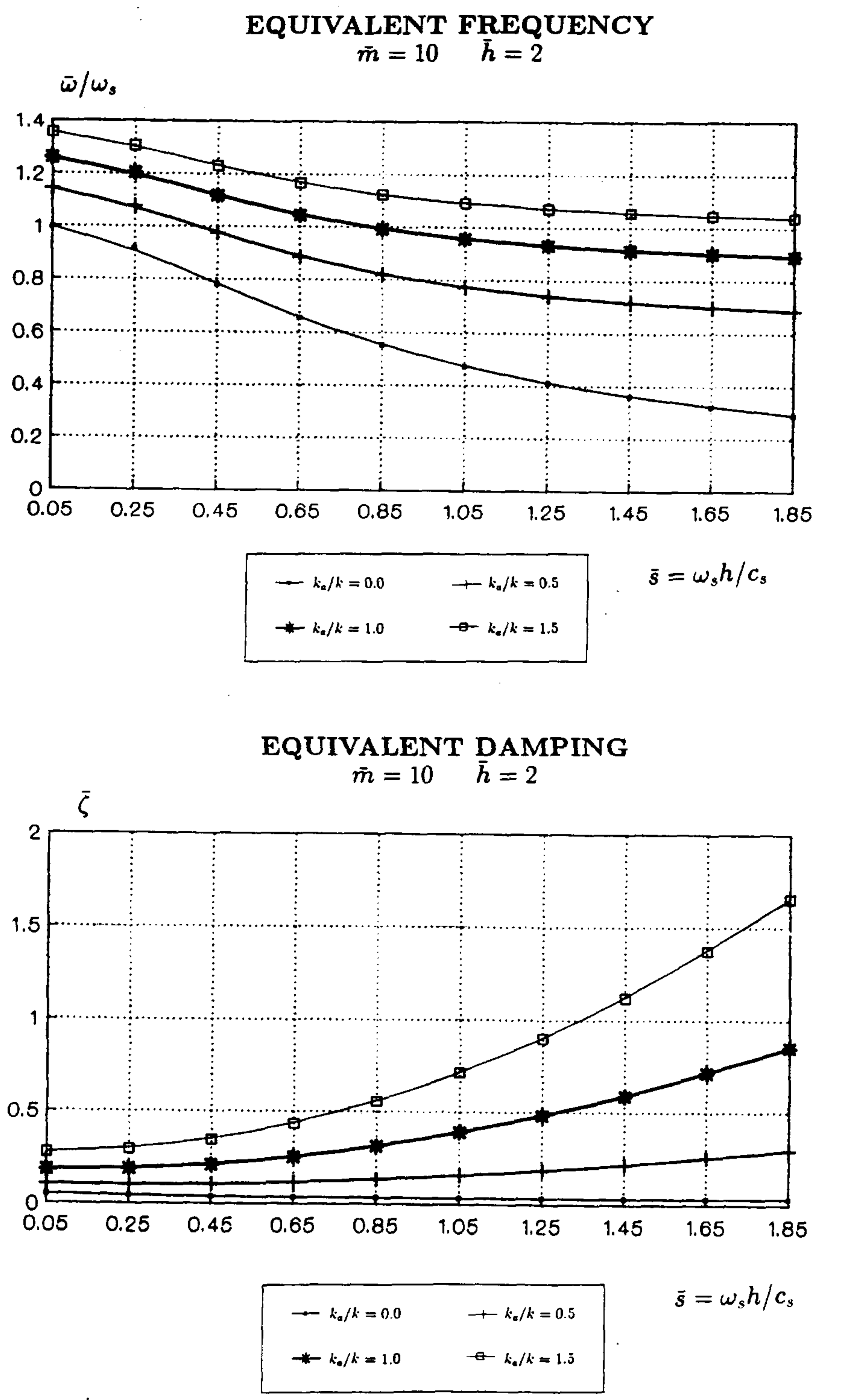

Fig. 5 – Equivalent frequency and damping ratio. k_a/k dependence. Hinged pier.

Analytical expressions of equivalent frequency and damping ratio may be obtained under some assumptions, depending on pier-deck connection: pinned or built-in connection.

• Important variations in the equivalent frequency may be obtained in rigid structures with low abutment to structure stiffness ratios.

• Equivalent damping ratios increase if the stiffness of the structure increases and the abutment to the structure stiffness ratio increases.

• Mechanical and geometrical characteristics of an instrumented bridge may be implemented in the analytical model in order to explain the response records to moderate seismic motions.

Reduction of the soil-abutment stiffness detected may he explained with the frequency dependent dynamic stiffness analysed.

Dynamic stiffnesses may explain the measured values of local damping ratios in the abutments and in the whole deck-abutment system.

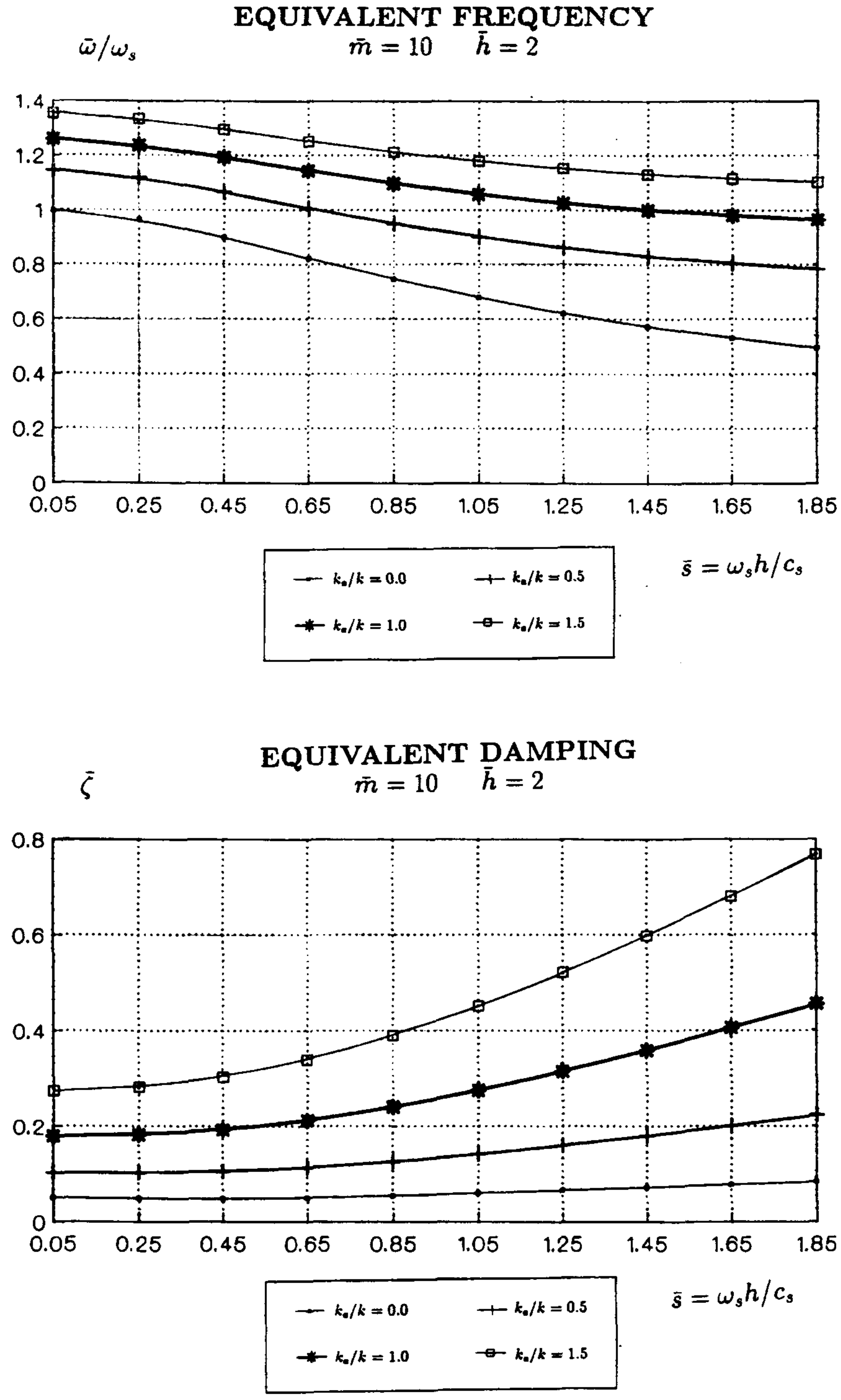

Fig. 6 – Equivalent frequency and damping ratio. k_a/k dependence. Built-in pier.

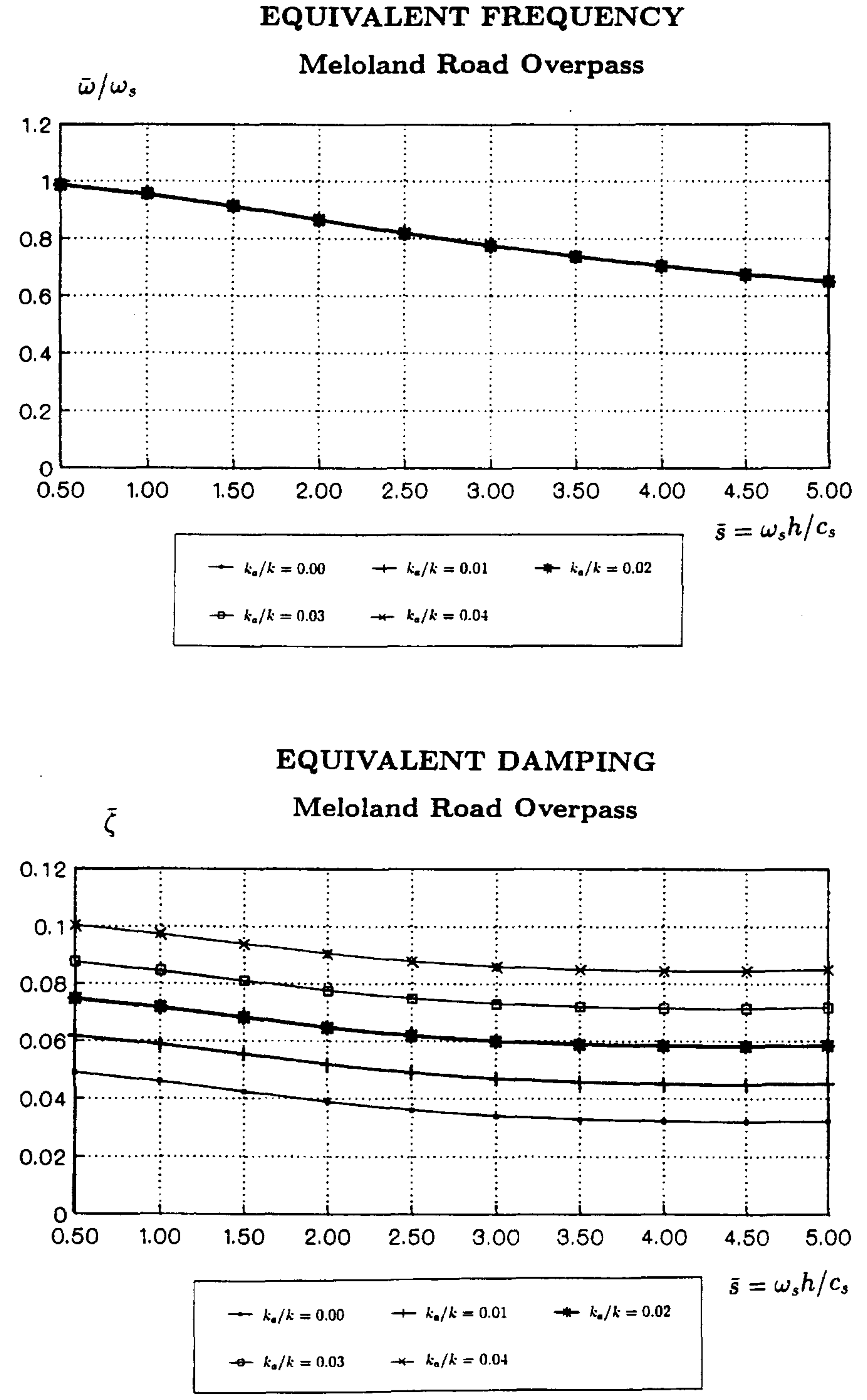

Fig. 7 – Equivalent frequency and damping. k_a/k dependence. Meloland Road Overcrossing.

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Appendix. A Equations of motion

In this Appendix a detailed obtention of the stiffness matrices and load vectors of the dynamical model from Fig. 2 will be done. The equation of soil-structure interaction will be initially written in total motion coordinates and they will be transformed into relative motion coordinates.

From eq.5:

$$[S^s_{bb} + S^g_{bb}]\{u^t_b\} = \omega^2[M^s_{bb}]\{u^k_b\} \tag{33}$$

The degrees of freedom of the total motion will be:

$$u^t_b = \{u^t_b\}^T = [u_2 \quad u_1 \quad \varphi_1] \tag{34}$$

where:

u_2: is the total displacement of the deck and the top of the pier.

u_1: is the total displacement of the pier foundation.

φ_1: is the total rotation in the pier foundation.

The dynamic stiffness matrices of the soil will be:

• In the pier foundation:

$$S^g_{bb1} = \begin{bmatrix} k_h + i\omega c_h & 0 \\ 0 & k_r + i\omega c_r \end{bmatrix} \tag{35}$$

with

$$u^T_{bb1} = [u_1 \quad \varphi_1] \tag{36}$$

• In the abutment:

$$S^g_{bb2} = [k_a + i\omega c_a] \tag{37}$$

with

$$u_{bb_2} = \{u_2\} \tag{38}$$

The mass matrix of the structure will be:

$$M_{bb} = \begin{bmatrix} m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \mathrm{diag}\,[m \quad 0 \quad 0] \tag{39}$$

The stiffness matrix of the structure will be different depending on the pier-deck connection:

• If the pier is hinged on the top part, (the bending moment is null) condensating the beam element degrees of freedom:

$$K^s_{bb} = \begin{bmatrix} k & -k & -kh \\ -k & k & kh \\ -kh & kh & kh^2 \end{bmatrix} \tag{40}$$

where k is the force necessary to produce a unit displacement in a built-in-hinged beam $k = \frac{3EI}{h^3}$:

– E: modulus or elasticity of the pier.
– I: inertia sectional modulus of the pier.
– h: pier height.

• If the pier is built-ill on the top part (the rotation is null):

$$K^s_{bb} = \begin{bmatrix} k & -k & -\frac{kh}{2} \\ -k & k & \frac{kh}{2} \\ -\frac{kh}{2} & \frac{kh}{2} & \frac{kh^2}{2} \end{bmatrix} \tag{41}$$

in this case, k is the force necessary to produce a unit displacement in a beam built-in at both ends $k = \frac{12EI}{h^3}$

where E, I are the mechanical characteristics of the pier mentioned above.

The soil displacements will be, if kinematic interaction is not considered and the motion of the abutment and the pier foundations are identical:

$$\boldsymbol{u}_b^k = \boldsymbol{u}_g \begin{Bmatrix} u_g \\ u_g \\ 0 \end{Bmatrix} \tag{42}$$

It is suitable to make a change of variables in which the total displacement of the deck is trasformed into the relative displacement Between the top and bottom part of the pier:

$$[u_2 \quad u_1 \quad \varphi_1] \rightarrow [u \quad u_0 \quad \varphi]$$

The change of variables equations will be:

- For the hinged pier (Fig. 2):

$$\begin{aligned} u_2 &= u_1 + h\varphi_1 + u \\ u_1 &= u_0 \\ \varphi_1 &= \varphi \end{aligned} \tag{43}$$

- For the built-in pier:

$$\begin{aligned} u_2 &= u_1 + \frac{h\varphi_1}{2} + u \\ u_1 &= u_0 \\ \varphi_1 &= \varphi \end{aligned} \tag{44}$$

In these new variables is very easy to obtain equilibrium equations directly.

The new stiffness matrices referred to the new displacement vector:

$$\boldsymbol{u}(\omega) = \boldsymbol{u}_b^T = [u \quad u_0 \quad \varphi] \tag{45}$$

will be:

- The hinged pier with $k = 3EI/h^3$ and calling ζ_s as the hysteretic damping coefficient:

$$\boldsymbol{S}(\omega) = -\omega^2 \begin{bmatrix} m & m & mh \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} k(1+2\zeta_s i)+k_a & k_a & k_a h \\ -k(1+2\zeta_s i) & k_h & 0 \\ -k(1+2\zeta_s i)h & 0 & k_r \end{bmatrix} + $$

$$+ i\omega \begin{bmatrix} c_a & c_a & c_a h \\ 0 & c_h & 0 \\ 0 & 0 & c_r \end{bmatrix} \tag{46}$$

- The built-in pier with $k = 12EI/h^3$:

$$\boldsymbol{S}(\omega) = -\omega^2 \begin{bmatrix} m & m & m\frac{h}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} k(1+2\zeta_s i)+k_a & k_a & k_a\frac{h}{2} \\ -k(1+2\zeta_s i) & k_h & 0 \\ -k(1+2\zeta_s i)\frac{h}{2} & 0 & k(1+2\zeta_s i)\frac{h^2}{12}+k_r \end{bmatrix} +$$

$$+ i\omega \begin{bmatrix} c_a & c_a & c_a\frac{h}{2} \\ 0 & c_h & 0 \\ 0 & 0 & c_r \end{bmatrix} \tag{47}$$

The load vector will be in both cases:

$$\omega^2 \boldsymbol{M}_{bb} \boldsymbol{u}_b^k = \omega^2 m u_g \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{48}$$