Simplified modelling of bridge abutments-soil interaction under seismic effects

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ABSTRACT: A simplified model is proposed to show the importance that the dynamic soil-abutment interaction can have in the global behavior of bridges submitted to seismic loading. The modification of natural frequency and damping properties is shown in graphic form for typical short span bridges of the integral deck-abutment type for longitudinal vibrations or general ones for transverse vibrations.

1 INTRODUCTION

During the modelling of bridges that have to be analyzed under seismic loading a great deal of attention is dedicated to the careful representation of the details of the superstructure while the interaction with the soil is usually represented in a less strict way. This is specially true for the abutments where no much experience is available, while for the column footings it is possible to use formulas that were developed for other uses (machine foundations, nuclear power plants, buildings, etc

Paradoxically some Codes, AASHTO (1983), recommend the introduction of the abutment dynamic properties in the model and their study was considered a worthwhile one since the very beginning of systematic research on seismic bridge behavior, ATC (1979).

Except for the case of bridges which deck is monolithic with the abutments (the so-called integral deck-abutment bridges) their influence is not very large on the longitudinal response. On the contrary, for transverse and even vertical displacements, taking into account the interaction effect can modify largely the results, specially in short span bridges.

That modification is related to vibration modes but also to the damping that the radiation of waves to the surrounding soil can inout in the global behavior of

superstructure

instructive to see

developed to identify the mechanical properties of models built to understand the data registered in actual bridges that were subjected to seismic actions. One of them J.C. Wilson et al (1990) is interesting on the one hand because it includes a first try to make a simplified representation of the approaching embankments but also because it presents numerical values of the damping and stiffness that would be neccessary to interpret the values registered in an actual structure. Among those results two of them are appealing : first the apparent reduction of the abutment stiffness with respect to the static values (of the order of 50 %) and simultaneously a damping ratio (from 25 to 45%) very high in comparison with the generally accepted ones. The structure is of the integral type so the interaction effects are specially important.

From the point of view of the dynamic soil-structure interaction both phenomena could be interpreted as the effect of the mobilized embankment mass in the effective stiffness and the radiation damping respectively. None of the two effects are included in the simplified model of J.C. Wilson et al (1990).

There is a general reluctance to use linear models to analyze soil-structure interaction in walls. Nevertheless several authors J. Wood (1973), H. Tajimi (1973) and A.S. Veletsos et al (1994) have tried to improve the comprehension of the problem using them and to stablish their limits of applications.

Motivated by the above mentioned ideas a research was launched to improve the model of J.C. Wilson et al (1990) using a dynamic formulation. Plane longitudinal and transverse models for the abutments were studied introducing layering of the soil and

computing the complex components of the dynamics impedances under the assumption of a massless and rigid abutment.
The resulting curves present the above mentioned effect of stiffness reduction and increase of damping in proportions promisingly similar to the experimental measurements E. Alarcon et al (1992).
On the other hand it is well known the difficulty of modelling the damping with plane models so the study was repeated with a simplified three dimensional model of the abutment. In both cases the numerical technique used was the Boundary Element Method (B.E.M.) in the frequency domain that is specially well suited for the treatment of viscoelastic semiinfinite media A.M. Cutillas et al (1992), A.M. Cutfflas (1993).
Once the possible focus of the discrepancies detected by J.C. Wilson et al (1990) was localized we have tried to quantify the importance of the soil-structure interaction using a simple model that is developed below.

2 MODELLING OF THE STRUCTURE

In order to calibrate the importance of the effect a simple type of bridge has been chosen: a continuous deck over a central pier and integral abutments (Notice that a similar model can be used to treat transverse vibrations of the bridge). In this way it is possible to include the soil interaction effect both in the pier and in the abutments. Following the line of D.R. Somaini (1984), J.P. Wolf (1985) & C.C. Spyrakos (1990) the deck is modelled as a concentrated mass and the pier as an elastic beam. The connection with the soil is done

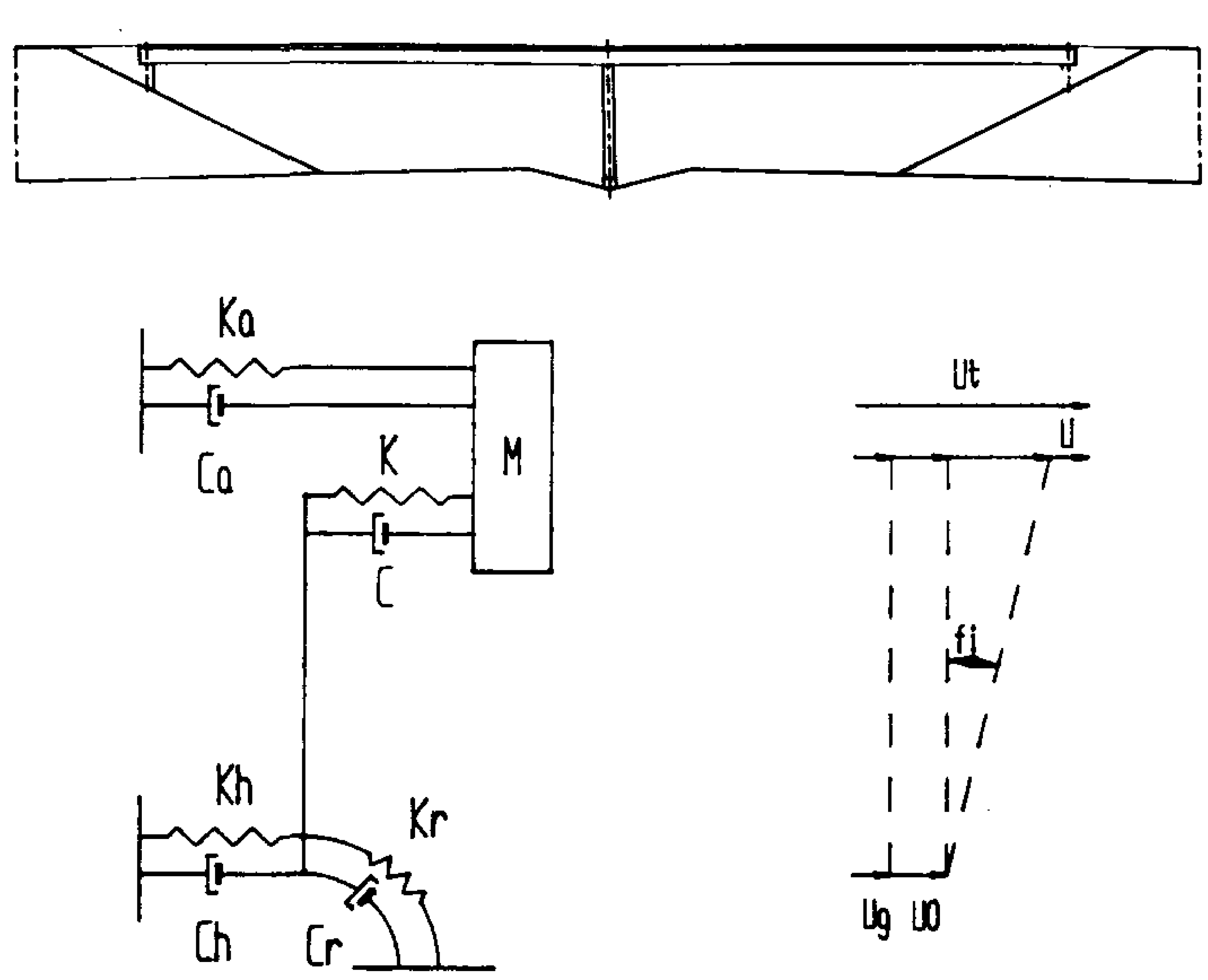

Fig. 1. Simple model

by springs and deshpots joining on the one end the soil and the pier footing while the mass is joined to the soil to represent the abutment effect (Fig. 1).
As it is interesting to see the influence of the degree of fixity between the pier and the deck, two cases have been analyzed: hinged end and built-in one.
The simplicity of the model allow the establishment of closed form solutions for the frequency and for the damping ratio as shown below.

3 EQUATIONS OF MOTION

The equation of motion in frequency domain may be expressed as:

$$\left[-\omega^2\,\mathbf{M} + i\omega\,\mathbf{C} + \mathbf{K}\right]\mathbf{u}(\omega) = \mathbf{P}(\omega) \qquad \mathbf{S}(\omega)\mathbf{u}(\omega) = \mathbf{P}(\omega) \tag{1}$$

where

$$co^2\,\mathrm{M} + ico\,\mathrm{C} + \mathrm{K} = \mathrm{K}_{,,,}(\mathrm{K}^* + ico\,C) \tag{2}$$

is the dynamic stiffness matrix or impedance matrix.
If substructures method is used, the equations of motion of inertial soil-structure interaction in frequency domain may be expressed in a general form as:

$$\begin{bmatrix}\mathbf{S}_{ss} & \mathbf{S}_{sb}\\ \mathbf{S}_{bs} & \mathbf{S}^s_{bb}+\mathbf{S}^g_{bb}\end{bmatrix}\begin{Bmatrix}\mathbf{u}^i_s\\ \mathbf{u}^i_b\end{Bmatrix} = \omega^2\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{M}_{sb}\\ \mathbf{M}_{bs} & \mathbf{M}^s_{bb}\end{bmatrix}\begin{Bmatrix}\mathbf{u}^k_s\\ \mathbf{u}^k_b\end{Bmatrix} \tag{3}$$

where the nodes 's' belong to the structure only; nodes 'b' belong to the soil-structure interface; S_{bb} represent soil dynamic stiffness matrix and $u^8{}_b$ the ground motion.
Because of all the nodes are in contact with the soil it can be expressed as:

$$[S'_w + SL]k\} = oi^2[Mi]\{u^*\} \tag{4}$$

The evaluation the soil dynamic stiffness is one of the steps in every soil-structure interaction analysis.
Constant boundary elements in frequency domain have been employed. E.Alarcdn et al.(1992), A.M. Cutillas et al (1992), A.M. Cutillas (1993).
The results may be expressed in the dimensionless form.

$$Ş'M = K\hat{}foU) + kb<vM] \tag{5}$$

where
Ky^{**} is the static stiffness.

$a_0 = \dfrac{\omega H}{C_s}$: dimensionless frequency

$C_s = \sqrt{\dfrac{G}{\rho}}$: shear wave velocity of the soil

H: Height of the wall.

dimensionless frecuency dependent stiffness

$c_{ij}(a_0)$: dimensionless frecuency dependent damping

$i = \sqrt{-1}$

As result of numerical studies the static stiffness for the longitudinal and transversal degree of freedom may be expressed respectiveley as:

$$\mathrm{K}_x = 6.07\frac{GH}{2-\nu} \qquad \mathrm{K}_y = 4.90\frac{GH}{2-\nu} \tag{6}$$

with G: transverse modulus of elasticity and ν: Poisson's ratio of the soil.

The dimensionless dynamic stiffness are shown in A.M. Cutillas (1993). The results confirm small dependance of Poisson's ratio ν.

If kinematic interaction is neglected it will be assumed:

$$\mathbf{u}_b^i = \mathbf{u}_b^t = \mathbf{u} \quad \mathbf{u}_b^k = \mathbf{u}_g \tag{7}$$

Introducing the frequency parameters:

$$\omega_s^2 = \frac{k}{m} \quad \omega_h^2 = \frac{k_h}{m} \quad \omega_r^2 = \frac{k_r}{mh^2} \quad \omega_a^2 = \frac{k_a}{m} \tag{8}$$

where h is the height of the pier, and the damping ratios:

$$\varsigma_h = \frac{c_h\omega}{2k_h} \quad \varsigma_r = \frac{c_r\omega}{2k_r} \quad \varsigma_a = \frac{c_a\omega}{2k_a} \tag{9}$$

The dynamic stiffness matrix S(ω) of Eq.1 for the hinged pier-deck connection, becomes:

$$\begin{bmatrix} \frac{\omega_s^2}{\omega^2}(1+2\varsigma_s i)+\frac{\omega_a^2}{\omega^2}(1+2\varsigma_a i)-1 & \frac{\omega_a^2}{\omega^2}(1+2\varsigma_a i)-1 & \frac{\omega_a^2}{\omega^2}h(1+2\varsigma_a i)-h \\ -\frac{\omega_s^2}{\omega^2}(1+2\varsigma_s i) & \frac{\omega_h^2}{\omega^2}(1+2\varsigma_h i) & 0 \\ -\frac{\omega_s^2}{\omega^2}h(1+2\varsigma_s i) & 0 & \frac{\omega_r^2}{\omega^2}h^2(1+2\varsigma_r i) \end{bmatrix} \tag{10}$$

with

$$\mathbf{u} = \begin{Bmatrix} u \\ u_o \\ \varphi \end{Bmatrix} \qquad \mathbf{P} = u_g\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{11}$$

The displacement u may be expressed as:

$$\left\{\begin{aligned} &(1+2\varsigma_s i)+\frac{\omega_a^2}{\omega_s^2}(1+2\varsigma_a i)-\frac{\omega^2}{\omega_s^2}+\left[\frac{\omega_a^2}{\omega_s^2}(1+2\varsigma_a i)-\frac{\omega^2}{\omega_s^2}\right] \\ &\frac{\omega_s^2}{\omega_h^2}\frac{1+2\varsigma_s i}{1+2\varsigma_h i}+\left[\frac{\omega_a^2}{\omega_s^2}(1+2\varsigma_a i)-\frac{\omega^2}{\omega_s^2}\right]\frac{\omega_s^2}{\omega_r^2}\frac{1+2\varsigma_s i}{1+2\varsigma_r i} \end{aligned}\right\} u = \frac{\omega^2}{\omega_s^2}u_g \tag{12}$$

Similar expression may be obtained for the built-in situation A.M.Cutillas(1993).

4 EQUIVALENT FREQUENCY AND DAMPING

In order to obtain the properties of an equivalent single-degree of freedom system, with their nodes fixed in the soil, which equation of motion is

$$\left[1+2\bar{\varsigma}i-\frac{\omega^2}{\bar{\omega}^2}\right]u = \frac{\omega^2}{\bar{\omega}^2}\bar{u}_g \tag{13}$$

$\bar{\omega}$ and $\bar{\varsigma}$ is the natural frequency and damping ratio of the single degree of freedom system.

From Eq.12 the equivalent frequency for the proposed model can be obtained as:

$$\bar{\omega}^2 = \omega_a^2 + \frac{1}{\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}} \tag{14}$$

Expressed as a linear combination of each damping ratio, the equivalent damping ratio for the model is:

$$\bar{\varsigma} = F_s\varsigma_s + F_h\varsigma_h + F_r\varsigma_r + F_a\varsigma_a \tag{15}$$

with the dimensionless participation constants:

$$\begin{aligned} F_s &= \frac{1}{\omega_s^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{1}{1+\frac{k}{k_h}+\frac{k}{k_r}} \\ F_h &= \frac{1}{\omega_h^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{\frac{k}{k_h}}{1+\frac{k}{k_h}+\frac{k}{k_r}} \\ F_r &= \frac{1}{\omega_r^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right]} = \frac{\frac{k}{k_r}}{1+\frac{k}{k_h}+\frac{k}{k_r}} \\ F_a &= \omega_a^2\left[\frac{1}{\omega_s^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2}\right] = \frac{k}{k_a}\left[1+\frac{k}{k_h}+\frac{k}{k_r}\right] \end{aligned} \tag{16}$$

To obtain the equivalent damping in a simple way, linearization of the expression.

$$\begin{aligned} \frac{1+2\varsigma_s i}{1+2\varsigma_h i} &= \frac{(1+2\varsigma_s i)(1-2\varsigma_h i)}{1+4\varsigma_h^2} \approx 1+2\varsigma_s i-2\varsigma_h i \\ \frac{1+2\varsigma_s i}{1+2\varsigma_r i} &= \frac{(1+2\varsigma_s i)(1-2\varsigma_r i)}{1+4\varsigma_r^2} \approx 1+2\varsigma_s i-2\varsigma_r i \end{aligned} \tag{17}$$

may be done. Due to damping ratios are lesser than unity, the products $\varsigma_i\varsigma_j << 1$ may be negligible.

Table 1. Variation of parameters

	SOIL CONDITIONS		
STRUCTURE		GOOD	POOR
SMALL	h/L	2.40	1.20
	k/kh	0.05	0.10
	ka/k	3.50	3.50
BIG	h/L	1.40	0.80
	k/kh	0.15	0.30
	ka/k	0.70	0.70

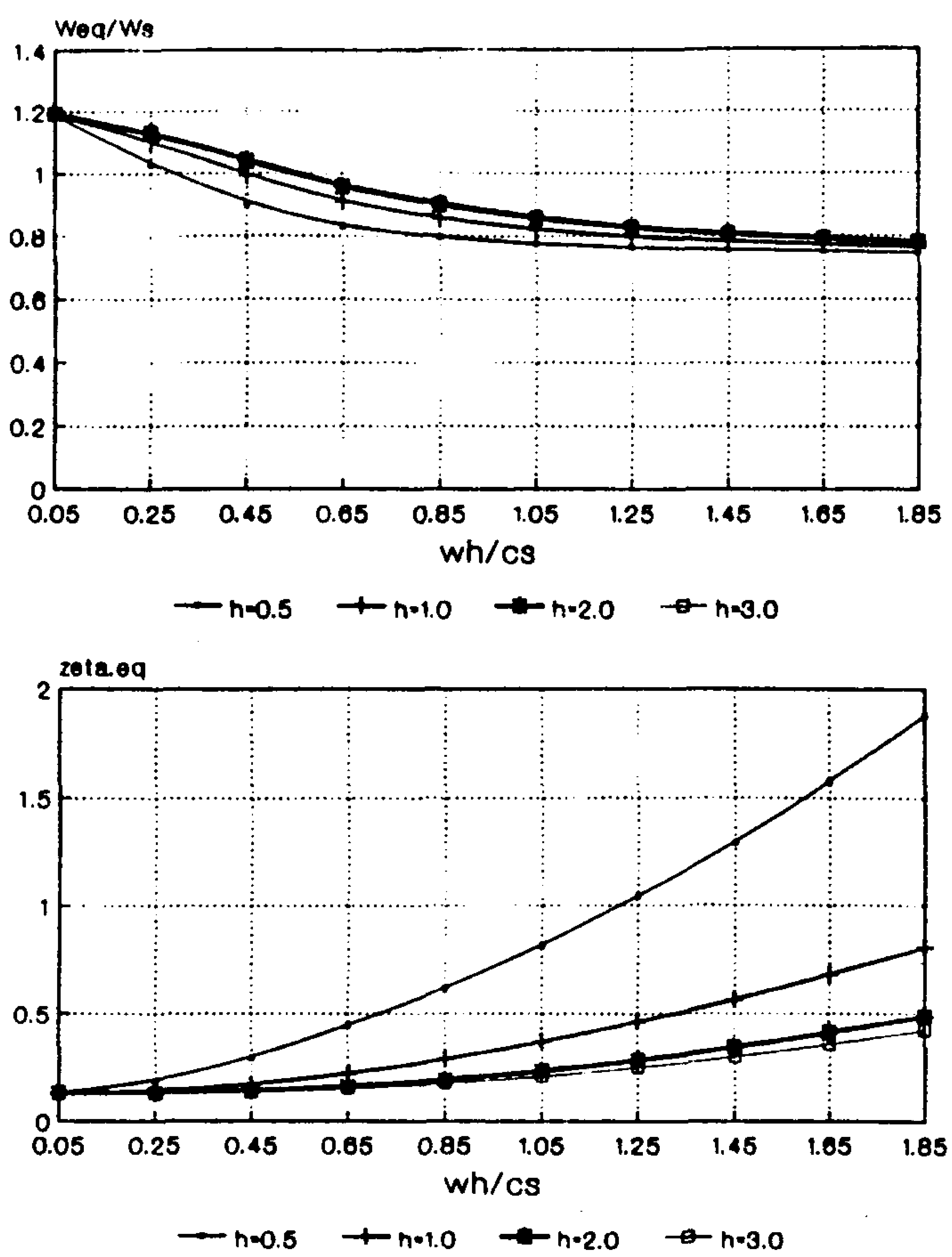

Fig. 2 Equivalent frequency and damping ratio. $\bar{h}$ dependence. Hinged pier.

5 RANGE OF VARIATION OF THE DIFFERENT VARIABLES

There are many paraments involved in bridge abutments soil-structure interaction analysis. It is very important to study the range of variation of the main variables in usual highway overcrossings. Two kind of structures (with their corresponding deck and piers) have been considered (Table 1).
A "small" structure in which one carriageway road overcrosses a highway without any central reserve (15 m. long span and 12 m. wide).
A"big" structure, two carriageways highway overcrosses a highway with a central reserve (25 m. long span and 23 m. wide).
Two different soil conditions have been analyzed, good and poor ones both of them with footing type foundation.
The range of the variables for these extreme situations are shown in table 1.

6 PARAMETRIC STUDY

The classical dimensionless parameters to analyze soil-structure phenomena may be expressed with respect the characteristic dimension "a" of the pier foundation as:

$$\bar{h}=\frac{h}{a}\qquad \bar{m}=\frac{m}{\rho a^{3}}\qquad \bar{s}=\frac{\omega_s h}{c_s} \tag{18}$$

and with respect the height of the abutment:

$$\bar{H}=\frac{h}{H}\qquad \bar{M}=\frac{m}{\rho H^{3}}\qquad \bar{S}=\frac{\omega_s H}{c_s} \tag{19}$$

The stiffness ratios may be expressed as functions of those parameters:

$$\frac{k_a}{k}=\frac{k_{xa}}{\bar{S}^2\bar{M}}\qquad \frac{k}{k_h}=\frac{\bar{s}^2\bar{m}}{8}\frac{(2-\nu)}{\bar{h}^2 k_x}\qquad \frac{k}{k_r}=\frac{\bar{s}^2\bar{m}}{8}\frac{3(1-\nu)}{k_\varphi} \tag{20}$$

The damping ratios in a similar way:

$$\varsigma_h=\frac{c_x a_o}{2k_x}=\frac{c_x}{2k_x}\frac{\bar{s}}{\bar{h}}\frac{\omega}{\omega_s}$$

$$\varsigma_r=\frac{c_\varphi a_o}{2k_\varphi}=\frac{c_\varphi}{2k_\varphi}\frac{\bar{s}}{\bar{h}}\frac{\omega}{\omega_s} \tag{21}$$

$$\varsigma_a=\frac{c_{xa} a_o}{2k_{xa}}=\frac{c_{xa}}{2k_{xa}}\bar{S}\frac{\omega}{\omega_s}$$

k_x, k_φ, c_x, c_φ are the dimensionless frequency dependent dynamic stiffness of pier foundations. Sometimes they can be considered constant for a wide range of the dimensionless frequency a_0.

k_{xa} and c_{xa}, are the dimensionless dynamic stiffness of the abutment.

In order to fix some of the variables a circular footing has been considered. In this case for a wide range of the dimensionless frequency the following values can be considered (J.P.Wolf 1985):

$$k_x=1\qquad c_x=0.575$$
$$k_\varphi=1\qquad c_\varphi=0.150$$

Assuming that the damping ratios have importance in the response of the system when

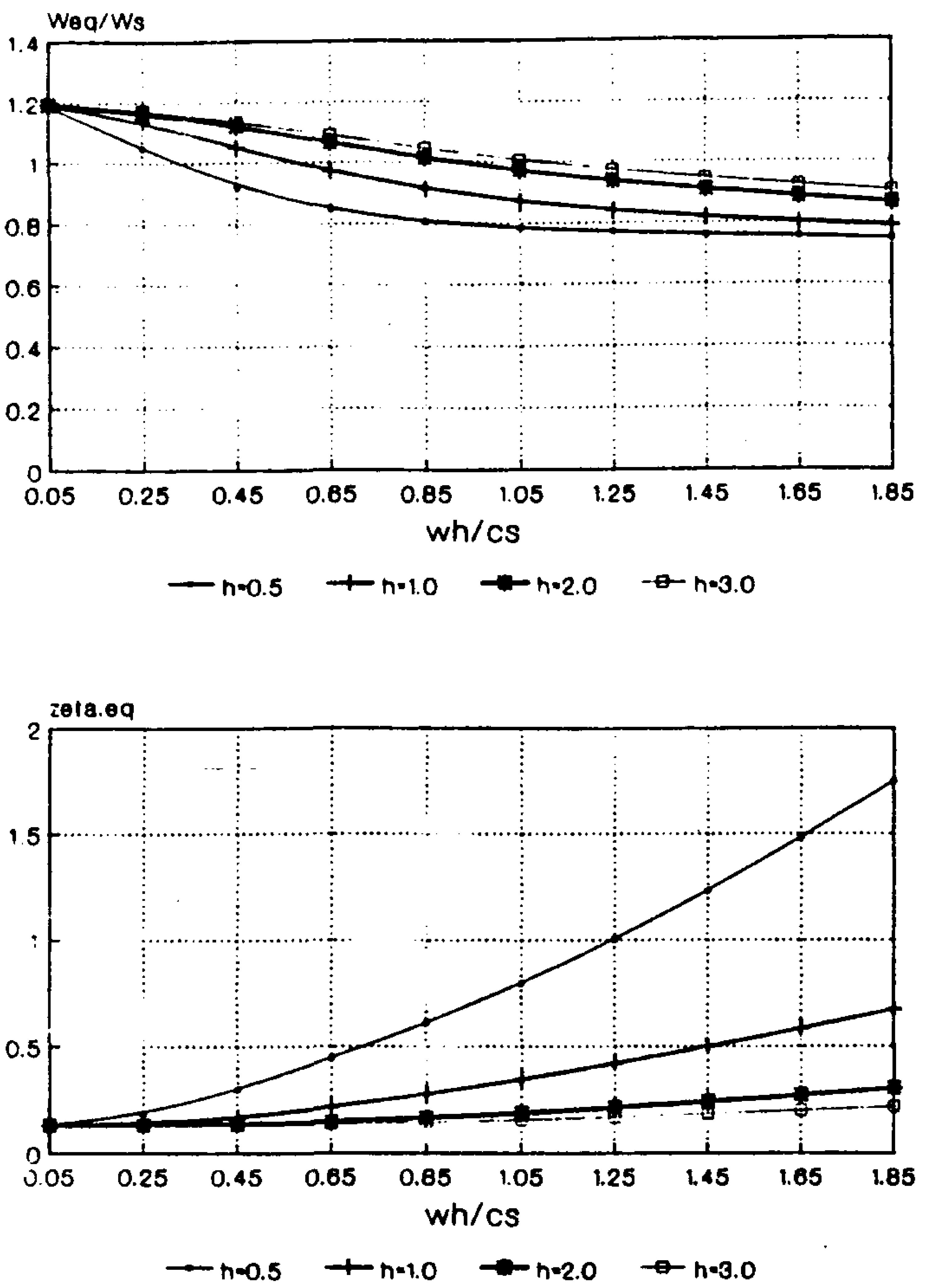

Fig. 3 Equivalent frequency and damping ratio. $\bar{h}$ dependence. Built-in pier

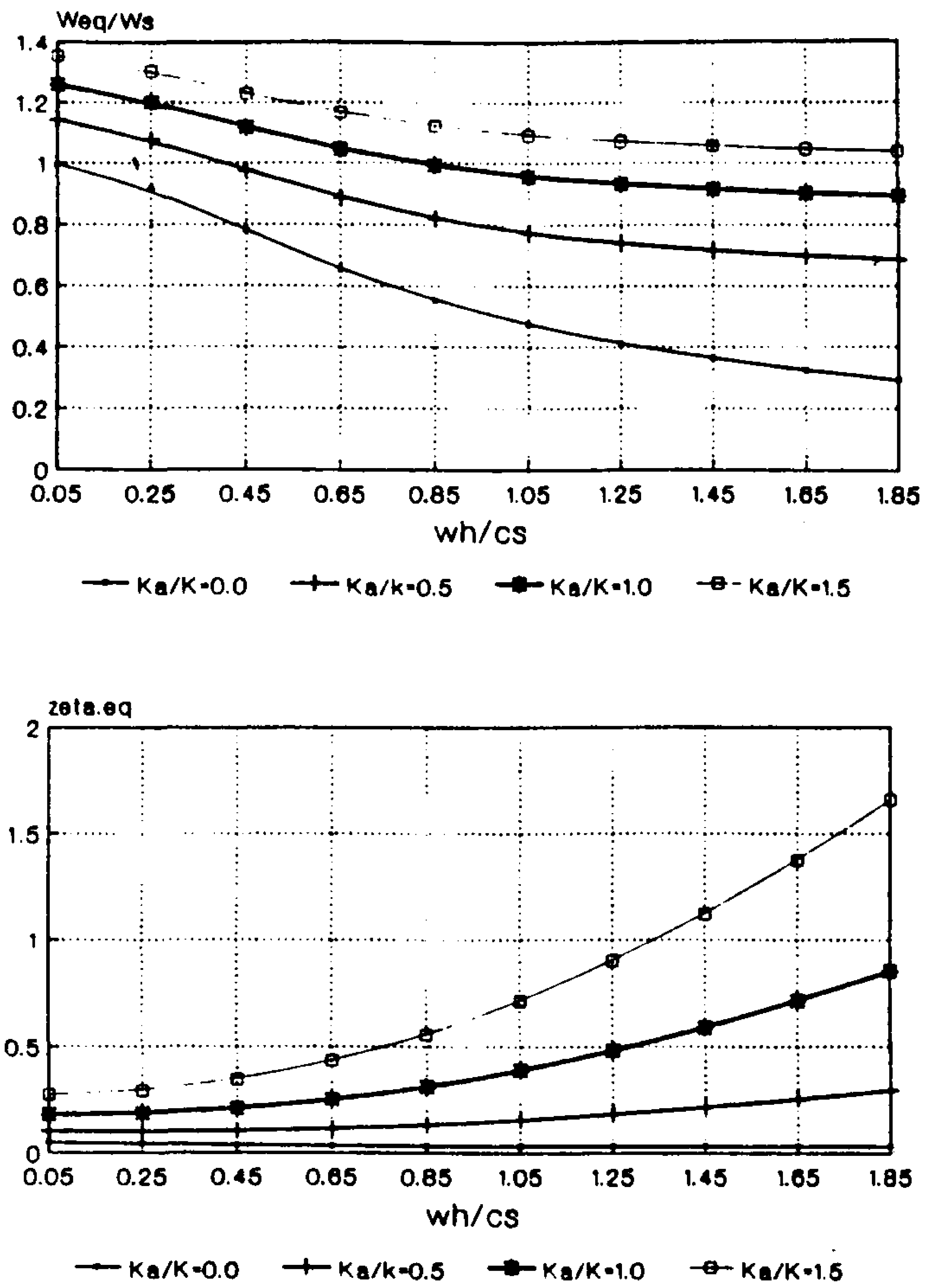

Fig. 4 Equivalent frequency and damping ratio. k_a/k dependence. Hinged pier.

m = 6), it is possible to solve the non linear problem in a simple way in order to obtain the equivalent frequency and damping of the system for the frequency dependent parameters of the abutment.

In Figs. 2 and 3. we show the variation of the equivalent frequency ratio with respect the natural frequency of the system without any interaction phenomena *co I a>$_s$* and the equivalent damping ratio, for different values of *h*, with $k_a/k=0.7$ and *m* = 10. As it could be seen interaction phenomena are more important for squat piers *(h* decreasing) and when t

increasing).

equivalent frequency

and the equivalent <

significantly. These variations are bigger in i hinged pier case than in the built-in one bees the last one the displacements are smaller.

Variation of bridge abutment stiffness can seen in Figs. 4 and 5.expressed with the rati

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relative frequency are bigger when relative stiffness is smaller. Its maximum value will be reached when there is no connection between the deck and the abutment, that is $k_a/k=0$.

Otherwise equivalent damping ratio increases when the abutment relative stiffness increases. The connection between the pier an the deck has greater influence in this case.

7 CONCLUSIONS

Recent data in instrumented bridges in seismic areas have shown the importance of soil-structure interaction phenomena in pier foundations and bridge abutments.

Dynamic stiffness of bridge abutments and embanknents may be obtained by Boundary Element Method (B.E.M.) techniques.

High damping ratios and reduction of the abutment stiffness that have been detected may be explained with simple models in which the dynamic stiffness of the abutment is properly taken into account.

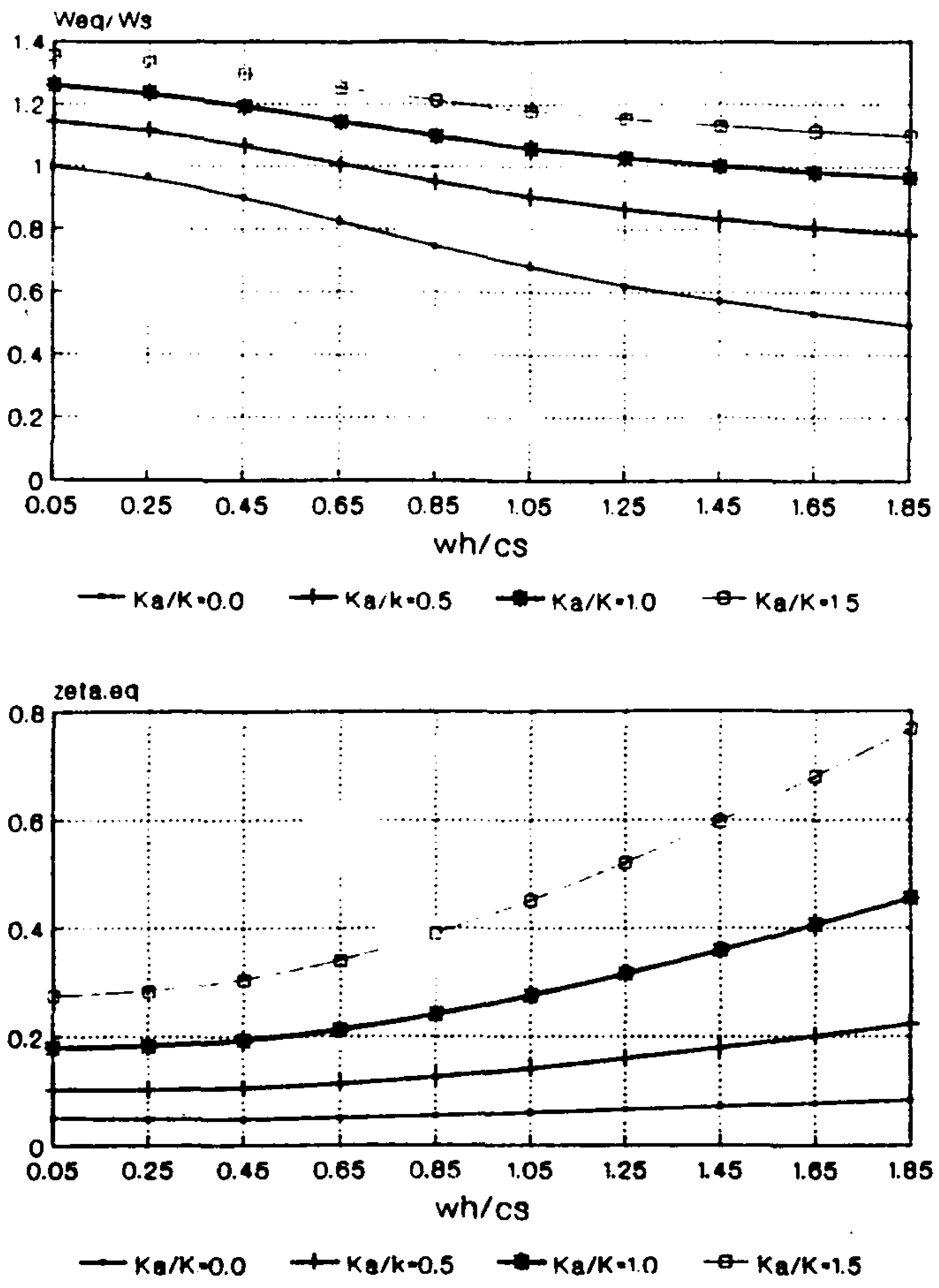

Fig. 5 Equivalent frequency and damping ratio. k_a/k dependence. Built-in pier

REFERENCES

AASHTO. 1983 (Rev. 1988). "Guide Specifications for Seismic Design of Highway Bridges".

ATC 1979. "Proceedings of a Workshop on Earthquake Resistance of Highway Bridges".

Alarc6n,E.,Martinez Cutillas, A., Gdmez Lera,M[a].S. 1992 "Dynamic impedance of bridge abutments[n].*Proc. 10th World Conf. on Earth. Eng.* Madrid.

Martinez Cutillas, A. ,Alarcdn,E. ,G6mez Lera,M[a].S.,Chirino,F. 1992 "Applications of Boundary Element Methods to the analysis of bridge abutments",*Proc. 14th., International Conference on Boundary Element Methods.* Sevilla.

Martinez Cutillas,A. 1993 "Dynamic Stiffness of Bridge Abutments". *PhD. Thesis.* Universidad Polit&nica de Madrid.(in Spanish).

Martinez Cutillas, A., Gdmez Lera, M.S., Alarcdn, E. 1994. "Dynamic soil-structure interaction in bridge abutments". *Proc. 8th Int. Conf on Comp. Meth. and Adv. in Geomechanics,* pp. 905-910.

Spyrakos, C.C. 1990. "Assesement of SSI on the longitudinal seismic response of short span bridges" *Eng. Struct.,* Vol. 12, pp 60-66.

Somaini,D.R. 1984 "Parametric study on soil-structure interaction of bridges with shallow foundations".*Proc. 8th. World Conf. on Earth. Eng.* San Francisco.

Tajimi, H. 1973. "Dynamic earth pressures on basement wall". *Proc. 5th World Conf. on Earth. Eng.* Rome.

Veletsos, A.S., Youman, A.H. 1994 "Dynamic soil pressures on rigid walls" *Earth Eng. & Struct. Dynamic,* Vol. 23 pp 275-301.

Wilson, J.C; Tan, B.S. 1990. "Bridge abutments: formulation of simple model for earthquake response analysis". *J. Engrg. Mech., ASCE,* Vol. 116(8), pp 1828-1837.

Wilson, J.C, Tan, B.S. 1990. "Bridge abutments: assesing their influence on earthquake response of Meloland Road Overpass" *J. Engrg. Mech., $ASCE_y$* Vol. 116(8), pp 1838-1856.

Wolf, J.P. 1985. *Dynamic Soil-Structure Interaction.* Prentice Hall.

Wood, J. 1973. "Earthquake induced soil pressures on structures. *PhD. Thesis.* Caltech Pasadena