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Application of the Boundary Element Method to the Analysis of Bridge Abutments

A. Martínez Cutillas (*), E. Alarcón (**),
M.S. Gómez-Lera (**), F. Chirino (***)
(*) E.T.S. Ingenieros Aeronáuticos, Universidad Politécnica de Madrid, Spain
(**) E.T.S. Ingenieros Industriales, Universidad Politécnica de Madrid, Spain
(***) E.T.S. Ingenieros Industriales, Universidad de Las Palmas, Spain

ABSTRACT

The B.E. technique is applied to an interesting dynamic problem: the interaction between bridges and their abutments. Several two-dimensional cases have been tested in relation with previously published analytical results. A three-dimensional case is also shown and different considerations in relation with the accuracy of the method are described.

INTRODUCTION

The repeated failure of bridges under seismic actions has launched an increased interest to understand their behaviour. In short to medium span bridges the earth embankments are a very significant part of the structure. For instance, an overcrossing with a 6 m. high abutment and an approaching slope of 4% needs a length of 150 m. The usual practice of considering rigid end connections is not realistic, specially in bridges with integral abutments where the motions act directly against the embankment. The correct modelling of both the stiffness and damping properties will affect the natural frequency of the global system as well as its dissipative properties. Both of them are important magnitudes in the seismic response so the problem appears as a very good occasion to demonstrate the advantages of BEM to model actual engineering problems.

NUMERICAL TREATMENT BY BEM

The integral representation for the steady state harmonic response of a linear viscoelastic solid Ω may be written if initial conditions and volume forces are zero, as:

$$\mathbf{c}\,\mathbf{u}(P) = \int_{\Gamma} [\mathbf{U}^*(P,Q)\mathbf{t}(Q) - \mathbf{T}^*(P,Q)\mathbf{u}(Q)] \qquad (1)$$

$$P \in \Omega \subset \Re^3 \quad \Gamma \equiv \partial\Omega \quad Q \in \partial\Omega$$

where Q are the boundary points over which the integrals extend. $\mathbf{u}$(Q) and $\mathbf{t}$(Q) are the displacement and traction components at Q. P is a point in the solid.

c is a coefficient which value depends on P belonging to the boundary or P being an inner or outer point to the boundary as well as on the geometry of the boundary at P.

$\mathbf{U}^*(P,Q)$ and $\mathbf{T}^*(P,Q)$ are the displacement and traction components of the fundamental solution at point Q when the unit point load is applied at the collocation point P.

The expression of these kernels may be found in the literature [3, 5, 7, 8].

In order to compute the integrals the boundary Γ is divided into elements where an evolution of $\mathbf{u}$(Q) and $\mathbf{t}$(Q) is assumed.

The present research has been done under the assumption of constant elements i.e. displacements and tractions are supposed to be constant inside each element and the resulting value is adscribed to the centroid of the elements.

Once the discretization is chosen it is possible to reduce the integral equation to a linear system of equations.

CALIBRATION OF THE PROPOSED METHOD

In order to test the validity of BEM in the new proposed problems J.H.Wood [12] and H.Tajimi [10] works were selected as benchmarks.

These benchmarks allow the establishment of discretization criteria for the solution of the new problems.

Wood's Model:

The dynamic analytical solutions were always obtained based on the free vibration modes of the model shown in Fig.1.a-b. The upper horizontal boundary

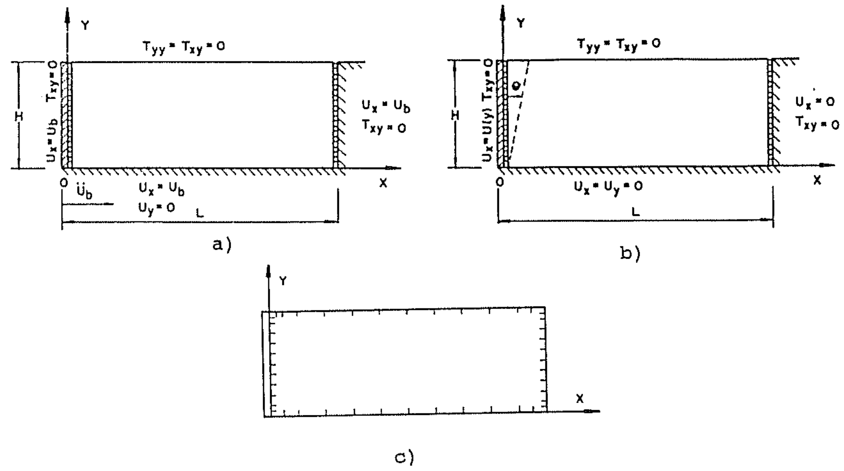

Figure 1:

is a free one while the lower is rigid. The vertical boundaries represent smooth rigid walls. A linear elastic behaviour of the soil is assumed.

Two forced solutions were analyzed: horizontal harmonic forcing on rigid boundaries and rotating harmonic forcing of a wall.

Horizontal harmonic forcing:

A horizontal harmonic displacement is imposed on the rigid horizontal boundary and on the vertical smooth rigid walls Fig.1.a. This horizontal displacement derives from a harmonic aceleration of the base. A constant aceleration in frequency domain implies a frequency dependent displacement in this domain:

$$\ddot{u}_b(t) = \alpha e^{i\omega t} \qquad (2)$$

$$u_b(t) = A(\omega) e^{i\omega t} \qquad (3)$$

where α is the constant amplitude of the aceleration and A(ω) is the frequency dependent displacement amplitude.

$$\ddot{u}_b(t) = -A(\omega)\omega^2 e^{i\omega t} \qquad (4)$$

$$A(\omega) = -\frac{\alpha}{\omega^2} \qquad (5)$$

Dissipation effects were taken into account in Wood's analysis by the addition of viscous damping terms. These terms include non linear behaviour within the soil structure and the radiation of energy from the system owing to the fact that in general the boundaries are not perfectly rigid.

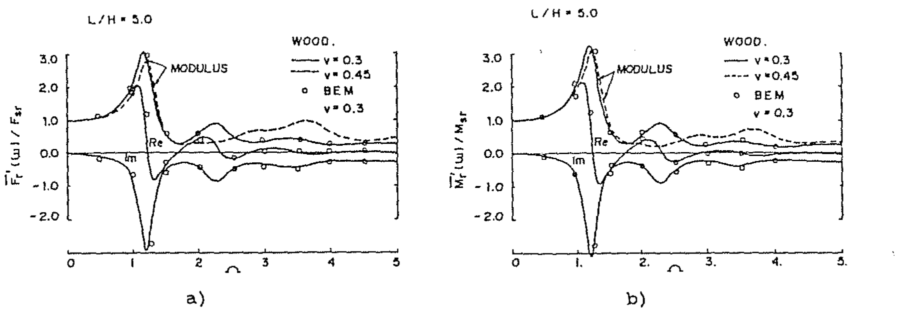

Figure 2:

To duplicate the results a constant boundary element mesh was built. Special care must be taken with the corners to obtain accurate results Fig.1.c.

Dissipation effects are naturally taken into account in the viscoelastic formulation with an hysteretic damping coefficient.

Although viscous and hysteretic damping are different coefficients, near resonant frequency are both the same (see Crandall [4]).

Complex amplitude ratios of resultant forces and moment of the horizontal stresses behind the rigid wall against dimensionless frequency $\Omega = \frac{\omega}{\omega_s}$ are shown in Fig.2. ω is the angular frequency of the imposed displacement $\omega_s = \pi\frac{c_s}{2H}$ is the natural angular frequency of the lowest pure shear mode of an infinite stratum, and $cs = \sqrt{\frac{G}{\rho}}$ is the shear waves propagation velocity.

Resultant forces and moments are normalized to the resultant of forces and moments to one g static horizontal loading.

Rocking harmonic forcing:

A rigid rotational deformation of the wall about its base is considered Fig.1.b.

Stress distribution behind the rigid wall both in the statical and dynamical case is shown in Fig.3. BEM results agree with Wood's ones.

Tajimi's Model:

Tajimi obtained a solution for the harmonically forced wall problem in a quarter space using two dimensional elastic wave propagation theory Fig.4.a. In the static case (excitation frequency equals to zero) his results agree with those obtained by W.D.L.Finn [6].

The comparison with the BEM model is shown in Fig.4.b.

Horizontal translation of the wall:

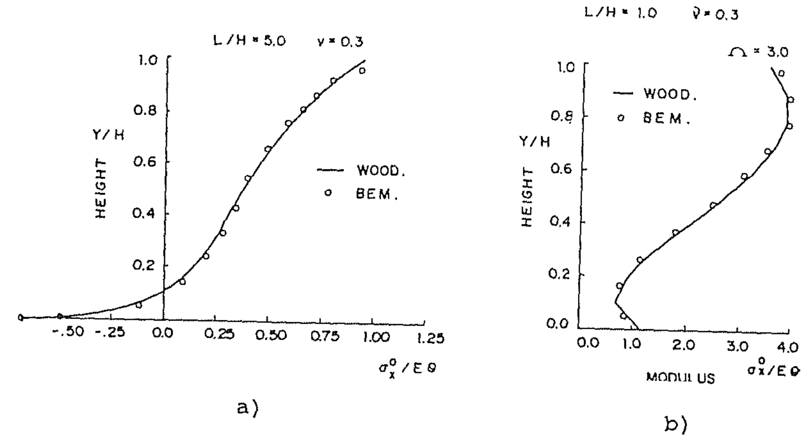

Figure 3:

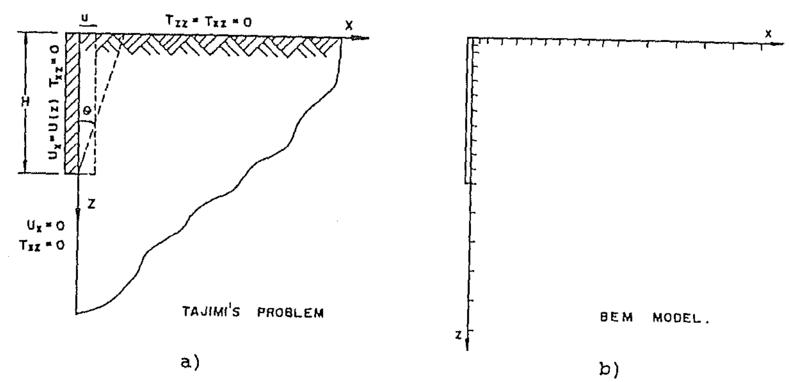

Figure 4:

The numerical results of total pressure distribution is expressed in the form of dimensionless functions S_1 , S_2. Fig.5.a.

$$-\sigma\mid_{x=0}=\frac{G}{H}U_0e^{i\omega t}[S_1(\omega,z)+iS_2(\omega,z)] \tag{6}$$

and the dimensionless frequency $\frac{\omega H}{v_s}$, $v_s=c_s=\sqrt{\frac{G}{\rho}}$ for a velocity ratio of shear to longitudinal waves of $\frac{v_s}{v_p}=1/3$ which correspond to a Poisson's ratio $\nu=0.4375$.

The resultant force acting behind the wall can be determined from:

$$-\int_0^H \sigma\mid_{x=0}dz=GU_0e^{i\omega t}[K_1(\omega)+iK_2(\omega)] \tag{7}$$

The good agreement between analytical (Tajimi's) and numerical results (BEM) can be seen in Fig.5.a.

Rocking of the wall:

The total pressure distribution in this case can be expressed as:

$$-\sigma\mid_{x=0}=\frac{G}{H}U_0e^{i\omega t}[R_1(\omega,z)+iR_2(\omega,z)] \tag{8}$$

The resultant moment about the bottom is determined from:

$$-\int_0^H (H-z)\sigma\mid_{x=0}dz=GU_0He^{i\omega t}[M_1(\omega)+iM_2(\omega)] \tag{9}$$

with the same parameters used in the horizontal translation of the wall.

The results are shown in Fig.5.b.

Some considerations must be done about the BEM mesh:

- Special care must be taken with corners. Small elements of the same size must be employed around each one or special integral rules have to be implemented.
- There is a singularity in the stress field at the bottom of the wall so a careful discretization must be used.
- As two of the boundaries are unbounded, element mesh must be truncated in a sensible way to guarantee the accuracy.
  - In the static case ($\omega=0$), good results have been obtained for L/H=20 and 50 constant elements.
  - In the dynamic case ($\omega\neq 0$),,good results have been obtained with an adaptive mesh depending on the frequency with $L=\lambda_s/4$ where λ_s is the wave length for shear waves and the size of the element less than $\lambda_s/6$ (see [1]).

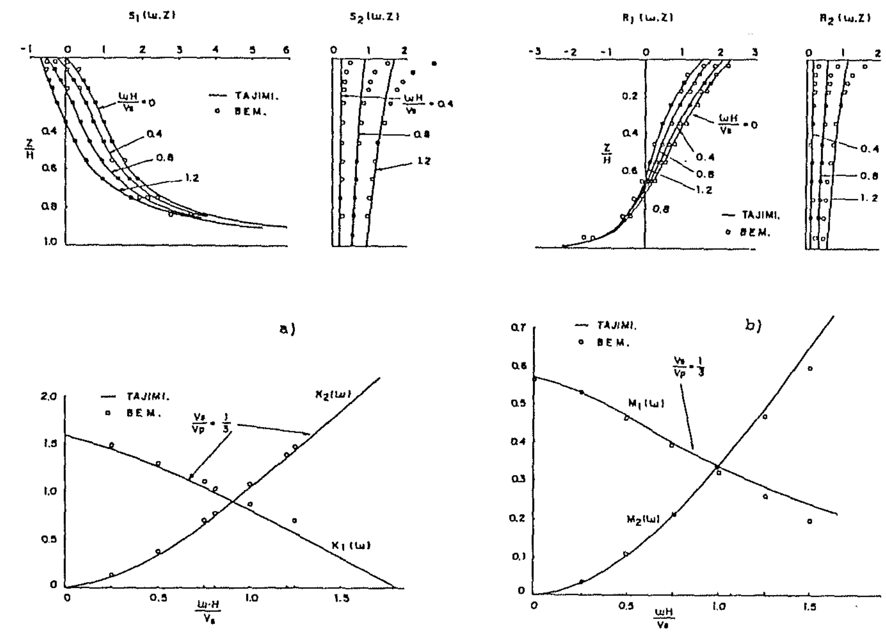

Figure 5:

DYNAMIC STIFFNESS OF BRIDGE ABUTMENTS

The above mentioned numerical techniques have been applied to the determination of the dynamic stiffness of bridge abutments.

The importance of the abutment contribution to the seismic response of bridges has been studied by Wilson et al [11], Maragakis et al [9] and E.Alarcon et al.[2]

With Boundary Element techniques more realistic geometries may be considered both in 2D and 3D problems.

Parametric studies can be performed analyzing the influence of Poisson's ratio, the existence of a rigid base below the wall or the existence of layers with different properties.

2D problems:

Very wide abutments may be treated as 2D problems. If a longitudinal section is studied, horizontal and rocking stiffness may be obtained.

Dynamic stiffness may be expressed in the well known dimensionless form

$$K = K_{st}(k + ia_0 c) \tag{10}$$

where K_{st} is the static stiffness when exists, and a_0 is the dimensionless frequency defined as

$$a_0 = \frac{\omega H}{c_s} \tag{11}$$

where ω is the excitation frequency, c_s the shear wave propagation velocity and H the height of the wall.

Two different cases have been considered. A rigid wall in a half-space (Fig.6) and a rigid wall with a rigid base (Fig.7). For the numerical model the same discretization techniques as those used in Tajimi's model have been employed.

Several parametric studies have been run analyzing Poisson's ratio (ν) dependence for the half-space. In this case $\frac{1}{1-\nu}$ variation has been observed in k and c terms for the horizontal and rocking dynamic stiffness in equation 10 as it is shown in Fig.6.

A fixed value of $\nu = 0.4375$ has been considered to study the dependence of the dynamic stiffness on the depth of the rigid base (h). Static stiffness decrease with rigid base depth and damping term c increase with it. The natural frequency of the stratum may be observed in dynamic stiffness oscillation with a_0. In order to attenuate these oscillations an hysteretic damping $\zeta = 5\%$ has been considered (Fig.7)

3D problems:

The earth of the embankments that provide the traffic approach to the bridge have a very important three dimensional behaviour because of their height to width ratios.

In order to study different discretizations variables, three meshes have been considered in the BEM numerical model (Fig.8). In the first mesh (m-1) the free surface length (L) discretization is six times the wall height (L=6H) and the maximum side of an element (Le_{max}) is λ_s , the shear wave length for the maximum frequency. In the second mesh (m-2) those parameters have been selected as L=2H and $Le_{max} = 0.50\lambda_s$. In the third mesh (m-3) L=2H and $Le_{max} = 0.25\lambda_s$.

In Fig.9 comparisons among dynamic stiffnesses for the three meshes are made. The results are very similar except for the horizontal stiffness k in which the results are quite different for a dimensionless frequency a_0 greater than 1.5 for which $Le_{max} = 0.50\lambda_s$.

In any case the method seems to be ideally suited to the physical problems.

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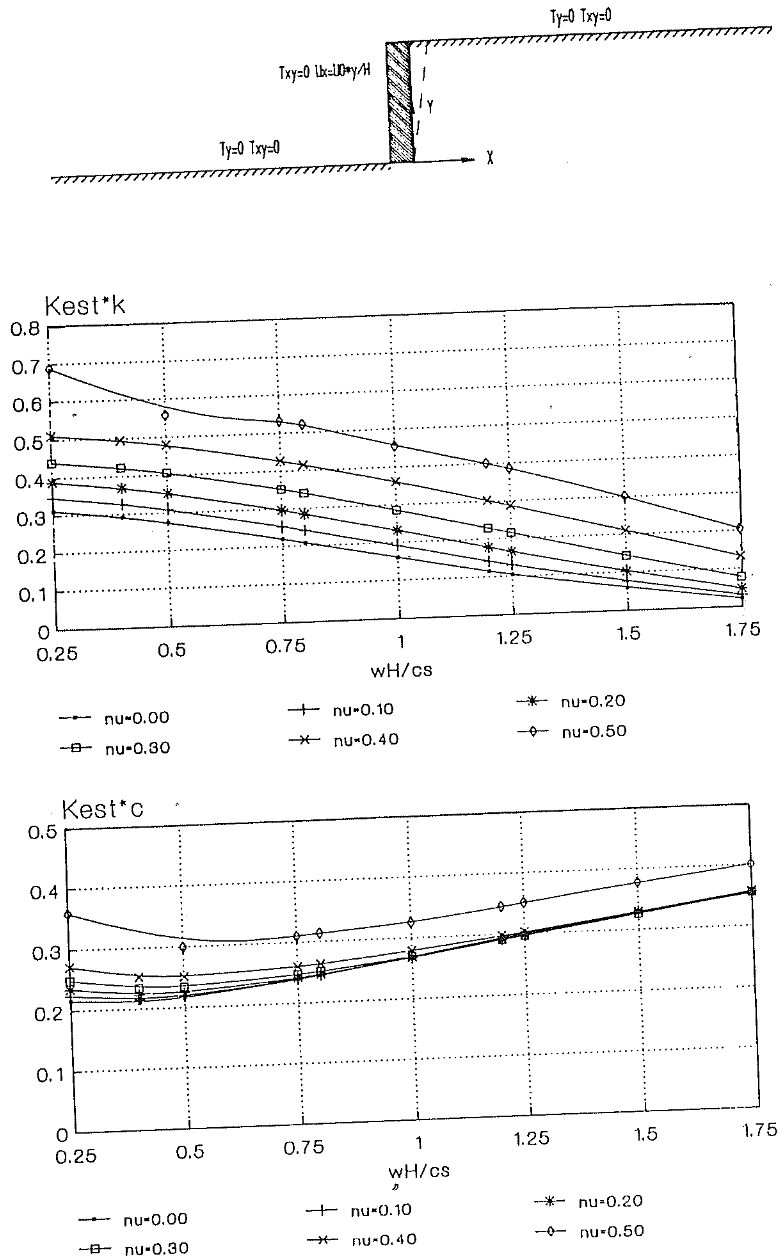

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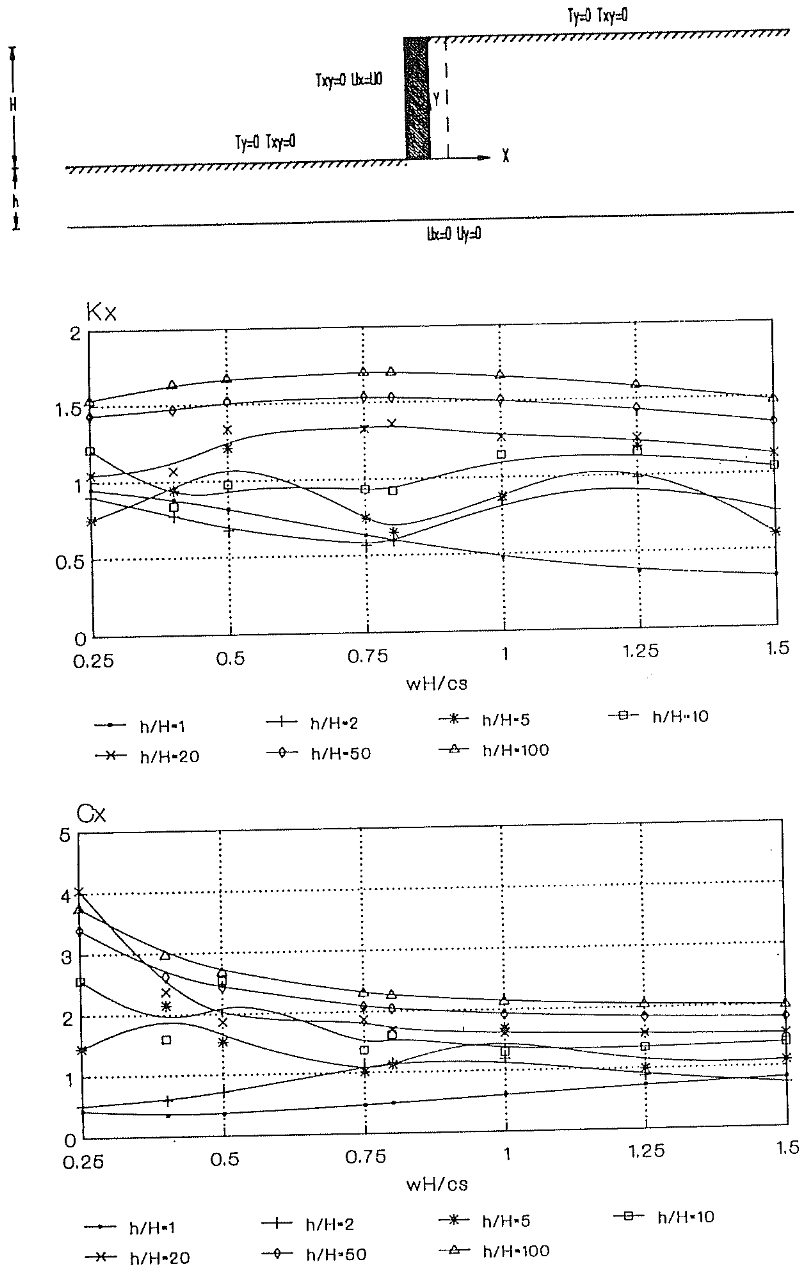

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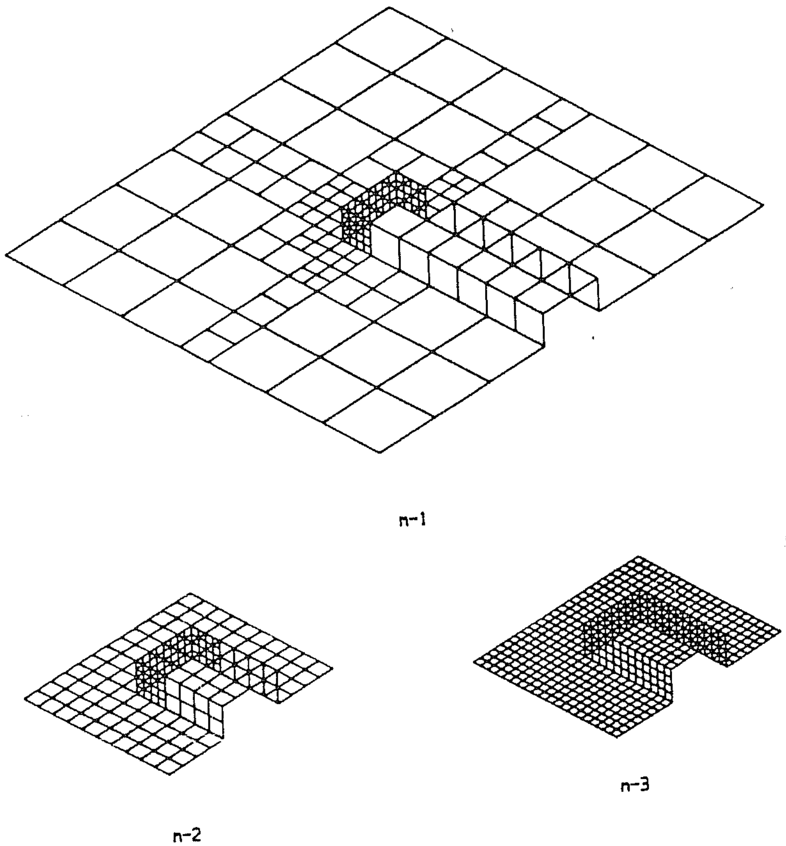

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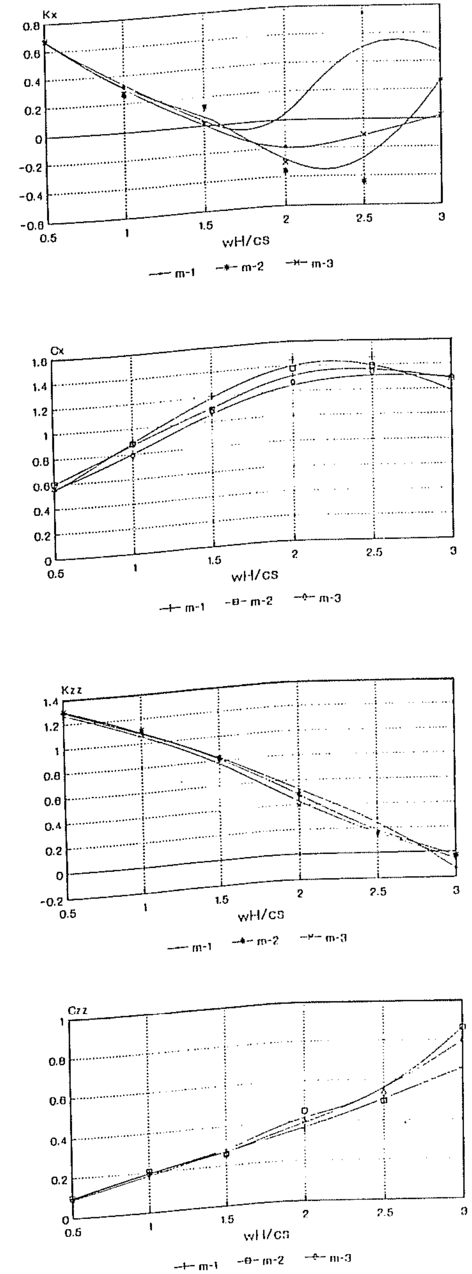

Figure 9: