Parameterization of the atmospheric boundary layer for offshore wind resource assessment with a limited-length-scale k-ε model

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Abstract

The structure of the atmospheric boundary layer (ABL) is modelled with the limited-length-scale k-ε model of Apsley and Castro. Contrary to the standard *k-ε* model, the limited-length-scale *k-ε* model imposes a maximum mixing length which is derived from the boundary layer height, for neutral and unstable atmospheric situations, or by Monin-Obukhov length when the atmosphere is stably stratified. The model is first verified reproducing the famous Leipzig wind profile. Then the performance of the model is tested with measurements from FINO-1 platform using sonic anemometers to derive the appropriate maximum mixing length.

1. Introduction

In operational wind resource assessment it is standard practice to use numerical models based on Monin-Obukhov similarity theory for the surface boundary layer. Such approach is justified in onshore wind energy sites, which are normally subjected to moderate to high winds, typical of neutral atmospheric conditions.

Using surface boundary layer models for offshore wind resource assessment produces unsatisfactory results in stable atmospheric conditions [1]. A very shallow stable boundary layer develops, due to the very low roughness of the sea surface, producing low-level jets of high wind shear. The increasing size of modern wind turbines further contributes to extend the area of interest beyond the surface layer, regardless of the terrain or wind conditions.

This paper investigates the possibilities of using various ABL turbulence closure schemes for offshore wind resource assessment. The proposed models aim at providing a more realistic description of the average ABL structure in the wind turbine rotor area. The modelling approach also allows a more comprehensive assimilation of mesoscale data as main forcing of the model. This is particularly useful in the offshore environment, where surface measurements are expensive to obtain and mesoscale models perform well.

2. The ABL model

Under horizontally-homogeneous conditions, the Reynolds averaged equations for the dry ABL, depend on the time *t* and the height above ground level *z*. In this context, the one-dimensional boundary layer equations for the horizontal velocity components (*U*,*V*) and potential temperature *Θ* are:

$$\begin{aligned} \frac{\partial U}{\partial t} &= f_c(V - V_g) - \frac{\partial \langle uw \rangle}{\partial z} \\ \frac{\partial V}{\partial t} &= -f_c(U - U_g) - \frac{\partial \langle vw \rangle}{\partial z} \\ \frac{\partial \Theta}{\partial t} &= -\frac{\partial \langle w\theta \rangle}{\partial z} \end{aligned} \qquad (1)$$

where U_g and V_g are the horizontal components of the geostrophic wind, which are derived from the hydrostatic relation:

$$\left(U_g, V_g\right) = \frac{1}{\rho f_c}\left(-\frac{\partial P}{\partial y}, \frac{\partial P}{\partial x}\right) \qquad (2)$$

with f_c=2Ωsinλ, the Coriolis parameter (Ω is the Earth's rotational speed and λ is the latitude).

The kinematic shear stress components <*uw*> and <*vw*> and kinematic heat flux <*wθ*> are modeled through eddy-diffusivity coefficients that relate the turbulent fluxes with the mean velocity and potential temperature gradients:

$$\left(\langle uw\rangle,\langle vw\rangle\right) = -\nu_t\left(\frac{\partial U}{\partial z},\frac{\partial V}{\partial z}\right)$$
$$\langle w\theta\rangle = -\frac{\nu_t}{\sigma_t}\frac{\partial \Theta}{\partial z} \qquad (3)$$

σ_t is the turbulent Prandtl number, taken as 0.74 in [4]. The velocity and potential temperature profiles depend directly on the turbulent viscosity ν_t profile, which has to be modeled in order to close the set of equations for the ABL.

The turbulent viscosity is related to the turbulent kinetic energy k (TKE) and the mixing length l_m by:

$$\nu_t = l_m\left(\alpha k\right)^{\frac{1}{2}} \qquad (4)$$

where $\alpha = C_\mu^{1/2}$ is the ratio of the surface shear stress to the TKE.

Two turbulent closures are considered in this paper, based on: a first-order *k-l* mixing length model and a second-order *k-ε* model. Both models have in common the solution of the transport of turbulent kinetic energy *k*.

$$\frac{\partial k}{\partial t} = G - \varepsilon + \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial z}\right) \qquad (5)$$

where *G* represents the production of TKE due to shear and thermal stratification:

$$G = G_k + G_b$$
$$G_k = -\langle uw\rangle\frac{\partial U}{\partial z} - \langle vw\rangle\frac{\partial V}{\partial z} \qquad (6)$$
$$G_b = \beta g\langle\theta w\rangle$$

where β is the thermal expansion coefficient and g the gravity.

The mixing length in (4) can be formulated diagnostically, through and algebraic function of *z*, or using a prognostic equation for the turbulent dissipation rate ε (TDR), characterized by a dissipation length scale:

$$l_d = \frac{\left(\alpha k\right)^{3/2}}{\varepsilon} \qquad (7)$$

and assuming that l_m=l_d, so that the turbulent viscosity can be expressed as:

$$\nu_t = C_\mu\frac{k^2}{\varepsilon} \qquad (8)$$

In the second-order closure, a transport equation for ε is required:

$$\frac{\partial\varepsilon}{\partial t} = C_{1\varepsilon}G\frac{\varepsilon}{k} - C_{2\varepsilon}\frac{\varepsilon^2}{k} + \frac{\partial}{\partial z}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial z}\right) \qquad (9)$$

C_μ=0.09, C_ε=1.44, $C_{2\varepsilon}$=1.92, σ_k=1 and σ_ε=1.3 are the default constants of the Jones and Launder (1972) *k-ε* Standard model. For ABL, Rodi (1980) showed that it is necessary to prescribe the following relationship in order to fulfil the equilibrium in the near wall region (*G=ε*).

$$\sigma_\varepsilon\sqrt{C_\mu}\left(C_{2\varepsilon} - C_{1\varepsilon}\right) = \kappa^2 \qquad (10)$$

Other higher-order closures exist but they will not be considered herein. Probably, the most popular closure beyond the *k-ε* 2^{nd} order is the 2.5-order closure from Mellor and Yamada [7], in which a prognostic equation for the product *kl* is used. The complexity introduced by a larger number of constants can make the model less efficient than the simpler *k-l* or *k-ε* approaches.

2.1 k-l mixing length model

The application of mixing length theory to ABL can be attributed to Blackadar (1962) who proposed that l_m was proportional to the height above ground level and approached a maximum constant value l_{max}=0.00027|U_g|/f_c (|U_g| is the geostrophic wind modulus). In neutral conditions:

$$\frac{1}{l_m} = \frac{1}{\kappa z} + \frac{1}{l_{\max}} \qquad (11)$$

Several variations of Blackadar's parameterization have been proposed ever since. A stability correction can be included if the stability function $\phi_m(z/L_0)$ is introduced:

$$\frac{1}{l_m} = \frac{\phi_m\left(\frac{z}{L_0}\right)}{\kappa z} + \frac{1}{l_{\max}} \qquad (12)$$

where:

$$\phi_m\left(\frac{z}{L_0}\right)=\begin{cases}\left(1-a\dfrac{z}{L_0}\right)^p & \dfrac{z}{L_0}\le 0\\ 1+b\dfrac{z}{L_0} & \dfrac{z}{L_0}>0\end{cases} \quad (13)$$

And a=16, b=4.7 and p=-1/4 are the classical Bussinger-Dyer (1971) coefficients for the surface layer. Apsley and Castro [2] proposed the same parameterization for neutral conditions and a limiting-length-scale that depends on the surface Monin-Obukhov length L_0 for stable conditions:

$$l_{\max}=\frac{\kappa}{b}L_0 \quad (14)$$

where constant b comes from the stability function (13). For stable conditions Delage (1974) introduced a second limiting length scale, function of the local M-O length L. This parameterization is used by Weng and Taylor [5] for the stable ABL.

$$\frac{1}{l_m}=\frac{1}{\kappa z}+\frac{1}{l_{\max}}+\frac{b}{\kappa L}$$
$$L=-\frac{\left(\langle uw\rangle^2+\langle vw\rangle^2\right)^{3/4}}{\kappa\beta g\langle w\theta\rangle} \quad (15)$$

Following similar reasoning, Gryning et al. [6] recently proposed another way of limiting the mixing length introducing also the distance from the boundary layer height z_i.

$$\frac{1}{l_m}=\frac{\phi_m}{\kappa z}+\frac{1}{l_{MBL}}+\frac{1}{\kappa(z_i-z)} \quad (16)$$

where l_{MBL} depends on roughness length and stability and influences the middle of the boundary layer. Peña et al. [8] stated that for offshore ABL, the l_{MBL} term can be neglected and z_i can be parameterized with the Rossby and Montgomery formula:

$$z_i=0.12\frac{u_*}{|f_c|} \quad (17)$$

for neutral and stable conditions.

Mahrt and Vickers [11] proposed a mixing length parameterization that depends on the local values of M-O length L and friction velocity u_* (so called hybrid similarity theory). The local non-dimensional wind shear function $\Phi(z/L)$ can be expressed as:

$$\Phi_m(z/L)=\frac{\kappa z}{u_*}\frac{\partial u}{\partial z}=\begin{cases}1+b\dfrac{z}{L} & \dfrac{z}{L}\ge 0\\ \left(1-a\dfrac{z}{L}\right)^p & \dfrac{z}{L}<0\end{cases} \quad (18)$$

where b, a and p are not necessarily the same values of the surface layer. A correction for boundary layer depth is also introduced:

$$l_m=\begin{cases}\dfrac{\kappa z}{\Phi_m}\left(1-\dfrac{1}{\Phi_m}\dfrac{z}{z_i}\right)^{3/2} & \dfrac{z}{L}\ge 0\\ \dfrac{\kappa z}{\Phi_m}\left(1-\dfrac{z}{z_i}\right)^{3/2} & \dfrac{z}{L}<0\end{cases} \quad (19)$$

Again, the boundary layer depth can be estimated from (17).

The various forms of mixing-length profile put in evidence that the ABL turbulence scales are not only limited by the distance to the ground, as in the surface layer, but also by stable stratification and by the the boundary layer depth.

Figures 1, 2 and 3 show a comparison of the different mixing length parameterizations for typical offshore (z_0=0.2mm) neutral, stable and unstable ABL conditions. The 'Blackadar' profiles are obtained with the stability correction (12) for stable ABL. In unstable conditions both 'Apsley' and 'Delage' parameterizations take the neutral form of 'Blackadar' (11). The surface mixing length formulation ($l_m=\kappa z$) is also shown as a reference.

The main difference between the models is found in the upper part of the ABL, where some models progressively reduce l_m to zero as $z\rightarrow z_i$, while others approach the limiting value l_{max}. 'Mahrt' parameterization has a similar behaviour as 'Delage's for neutral and stable conditions and also presents local scaling characteristics in unstable conditions. 'Gryning' also presents local scaling at all stability regimes but with larger mixing lengths acting in a wider portion of the ABL.

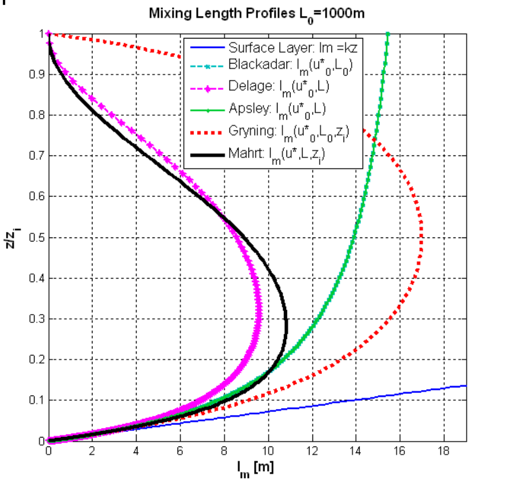

Figure 1: Mixing length profiles for neutral ABL.

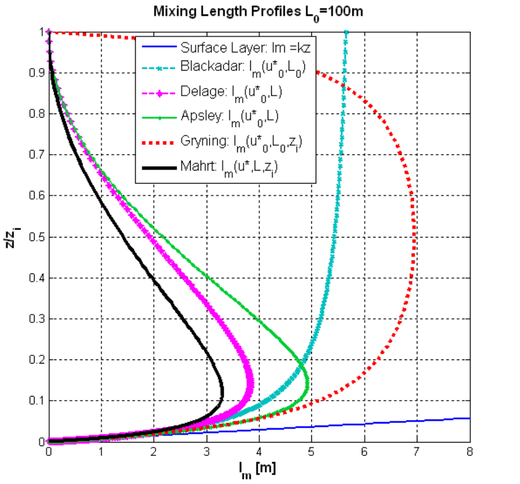

Figure 2: Mixing length profiles for stable ABL

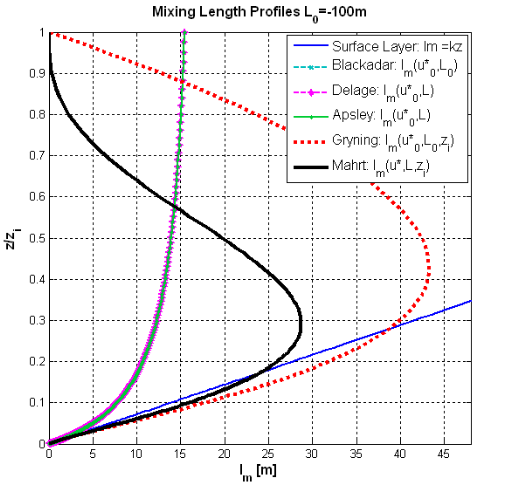

Figure 3: Mixing length profiles for unstable ABL

The surface layer parameterization appears to be only valid within the first 3% of the stable ABL and 20-30% of the unstable ABL. This corresponds to less than 10m and 100-150m in stable and unstable conditions respectively.

2.1 Limited-lengh-scale k-ε model

In 1985 Detering and Etling [3] showed that the ABL simulated with the standard k-ε model is severely affected by a monotonically increasing turbulent length scale, which leads to excessive turbulent mixing and very deep boundary layers.

To overcome this, several modifications of the *k-ε* Standard model have been proposed. The three models presented in this paper agree on introducing the mixing length in the ε equation in order to modify the production of TDR. This is done by modifying the $C_{1\varepsilon}$-term in (9) as follows:

- Detering [3]: $C_{\varepsilon 1}\dfrac{l_d}{h_i}G\dfrac{\varepsilon}{k}; \quad h_i = c_h\dfrac{u_*}{f}$ (20)
- Apsley [2]: $\left[C_{\varepsilon 1}+(C_{\varepsilon 2}-C_{\varepsilon 1})\dfrac{l_m}{l_{\max}}\right]G\dfrac{\varepsilon}{k}$ (21)
- Weng [4]: $C_{\varepsilon 1}\dfrac{C_\mu^{3/2}k^2}{l_m^2}$ (22)

The modification introduced by Apsley and Castro has the advantage that the model remains consistent with the log-law in the surface layer (l_m<<l_{max}).

2.3 Boundary conditions

In homogeneous flow conditions, it is sufficient to specify boundary conditions at the bottom (ground) and top (geostrophic) levels. At geostrophic level, the velocity components (U_g,V_g) and potential temperature Θ_g are prescribed and the gradients of turbulent quantities are set to zero. At the bottom, no slip conditions are imposed (*U*=*V*=0) and local equilibrium, i.e. production of TKE equals dissipation (*G*=*ε*) is imposed.

2.4 Numerical model

The set of constitutive equations (1) are solved numerically using the unconditionally positive scheme of Moryossev and Levy [9] for two-equation turbulence models.

For the solution of the quasi-stationary stable ABL, the strategy of Apsley and Castro [2] is adopted. A linear profile of the kinematic heat flux is explicitly defined from estimates of the surface heat flux and boundary layer depth.

$$\langle w\theta \rangle = \langle w\theta \rangle_0 \left(1 - \frac{z}{z_i}\right) \qquad (23)$$

where z_i can be estimated using the Zilitinkevich expression for the stable boundary layer height:

$$z_i \approx 0.4 \left(\frac{u_* L_0}{|f_c|}\right)^{\frac{1}{2}} \qquad (24)$$

This approach avoids solving the energy equation, for which non-stationary simulations are required, as shown by Weng and Taylor [5].

2. Model Verification

The numerical model is first verified with the solution of the Leipzig wind profile [10], a well known reference test case for 1D models of the ABL. A 3km high stretched grid with 184 vertical levels is used.

The Leipzig wind profile was measured under steady and horizontally homogeneous neutral (or slightly stable) conditions. The profile is defined by the following input parameters: U_g=17.5m/s, V_g=0m/s, f_c=1.13e-4s^{-1}, z_0=0.3m and u*=0.65m/s. The maximum mixing length, according to Blackadar expression is 36m.

Three models are tested: k-ε Standard, mixing-length using (11) and the modified k-ε model with the limited-length-scale approach of Apsley and Castro using (21).

Figure1 shows the comparison of the simulations with the experimental profiles and the simulations obtained by Detering and Etling [3] for the Standard k-ε and mixing-length models using the using the same constants of the referenced authors: Apsley used the default *k-ε* constants and Detering and Etling used: κ=0.4, C_μ=0.0256, $C_{1\varepsilon}$=1.13, $C_{2\varepsilon}$=1.9, σ_k=0.74 and σ_ε=1.3.

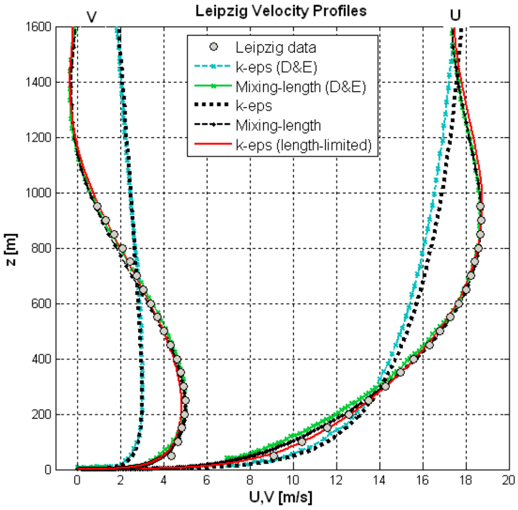

Figure 4: Leipzig velocity profiles

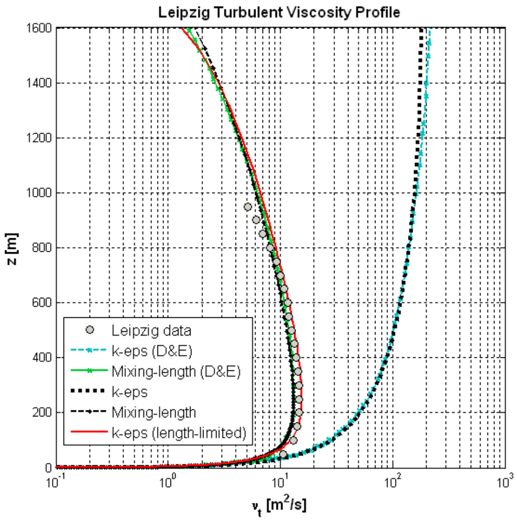

Figure 5: Leipzig turbulent viscosity profiles

The Standard k-ε model produces a monotonically increasing mixing length that introduces too much mixing in the upper part of the ABL producing very deep boundary layer. The mixing-length model is able to reproduce fairly well the Leipzig wind profile due to the limiting mixing-length introduced with (11). The boundary layer height is much lower and the Ekman velocity profile shows a supergeostrophic jet. The same effect is obtained with the limited-length-scale model of Apsley and Castro.

As stated by Apsley and Castro, in homogeneous flow conditions, there is no advantage of the two-equation model with respect to the mixing-length model. The advantages of the limited-length-scale k-ε model are met with non-equilibrium flows over topography, where the length scale don't follow an universal function but still is limited by a maximum eddy size. The difficulty, of course, is to determine this limiting length scale.

3. Results

The objective of the validation is to verify if the parameterizations described previously can describe the structure of the ABL, with special focus on stable conditions.

3.1 Measurements at FINO-1 offshore platform

The FINO-1 offshore platform is located 45km off the Borknun Island. A 100-m mast fully equipped with meteorological and oceanographic instruments is measuring since 2003. Three sonic anemometers located at 40, 60 and 80m measure at 10Hz the 3D velocity components from which the turbulent fluxes are obtained using eddy-correlation techniques. Sonic anemometer measurements are available for the year 2006. The 60m sensor only works during the first half of the year.

Only open water conditions are considered, without mast distortion effects, reducing the analysis to the wind direction sector 190º-250º. Only velocities above 3m/s are considered to avoid large errors introduced by cup anemometer measurements. A stationary test [12] is run on 1hr intervals of sonic data in order to select profiles without the influence of low frequency mesoscale systems. This is necessary in order to be as close as possible to 'homogeneous' conditions, the main hypothesis of the above described models.

Seven stability classes are selected (Table 1), according to the 80m flux M-O length computed from (15), where the heat flux is corrected for humidity. At least 20 profiles are found in each class, producing averaged profiles with fair statistical convergence. 45% of the filtered profiles fall inside stable conditions.

Table 1: Stability classes

Stability Class		L	# profiles
Very Stable	vs	10<L<50	109
Stable	s	50<L<200	314
Slightly Stable	ss	200<L<500	145
Neutral	n	|L|>500	306
Slightly Unstable	su	-500<L<-200	182
Unstable	u	-200<L<-50	158
Very Unstable	vu	-50<L<-10	28
		Total	1242

3.2 Limited-length-scale k-ε model

Simulations of the stratified ABL are carried out with the limited-length-scale k-ε model formulated by Apsley and Castro [2], based on surface parameters and geostrophic wind. For sake of simplicity the maximum mixing length is parameterized using (11) with l_{max} to be obtained from the fitting to experimental data. As FINO-1 measurements are located above the surface boundary layer, it is necessary to make estimates of the surface parameters (L_0,$u*_0$ and $<w\theta>_0$). Besides, the geostrophic wind is also unknown. An iterative process is adopted with l_{max}, $|G|$ and L_0 as the most influential parameters to obtain the best fit to the velocity and turbulence intensity profiles. Friction velocity is indirectly determined by $|G|$ and l_{max}. The roughness length is link to the friction velocity by the Charnock relation:

$$z_0 = 0.0185 \frac{u_{*0}^2}{g} \qquad (25)$$

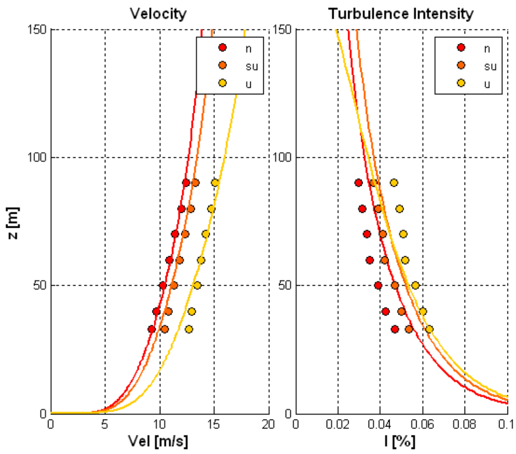

Figure 6: Simulations of Stable ABL

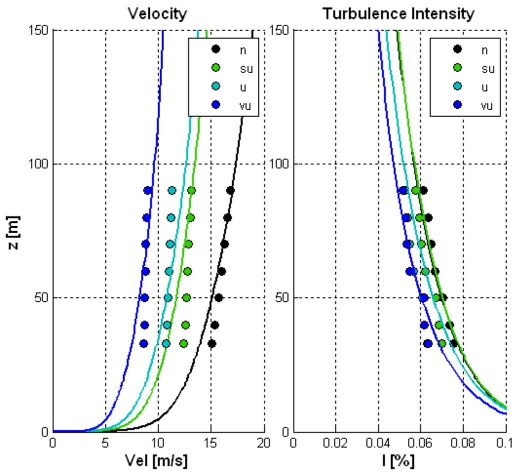

Figure 7: Simulations of Neutral and Unstable ABL

The resulting parameters are shown in Table 2 for each stability class.

Table 2: ABL parameters from k-ε model

	|G| [m/s]	$u*_0$	wt_0 [Km/s]	L_0 [m]	l_{max} [m]
vs	14	0.29	-0.0080	224	1.12
s	15.5	0.35	-0.0100	297	1.5
ss	18	0.43	-0.0200	295	2
n	23.0	0.58	0.0007	-2378	6
su	17	0.45	0.0120	-566	5.5
u	15.5	0.40	0.0120	-392	3.5
vu	11	0.28	0.0130	-121	2

The values of l_{max} obtained from the fitting process are much smaller than the ones observed by the sonic anemometers or predicted by the expressions found in the literature. It seems that the offshore ABL simulated by the k-ε model still produces a too deep boundary layer, making it necessary to reduce the value of l_{max} in order to reduce the turbulent intensity and increase the wind shear close to the ground.

3.3 Mixing-length model

An alternative to k-ε model, is found in the family of mixing-length models. From the various options introduced before, Mahrt parameterization is analyzed with FINO-1 data.

Figure 8 shows that using the local M-O length the non-dimensional wind shear expression (18) appears to be almost independent of the height. For the FINO1 dataset, the best fit in stable conditions is found using the coefficient b≈7, although the scatter spreads between 4.7 (Bussinger-Dyer value for the surface layer) and 10.

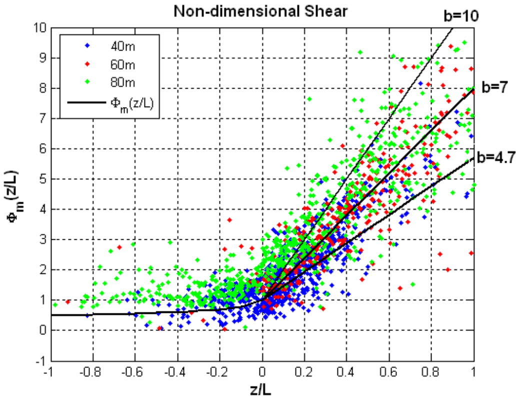

Figure 8: Non-dimensional wind shear versus local stability z/L (190º<WD<250º)

Using the friction velocities from the sonic anemometers and the velocity gradient from the two nearest cup anemometers one can compute the measured mixing length.

$$l_m = \frac{u_*}{\left|\frac{dU}{dz}\right|} \qquad (26)$$

The measured mixing length is compared in Figure 9 with the parameterized mixing length computed with (19) and p=3.5 as the best fit coefficient. A reasonable agreement is observed with increasing scatter as the mixing length increases due to less reliable velocity gradient measurements in neutral and unstable conditions and higher uncertainty on the boundary layer depth estimation.

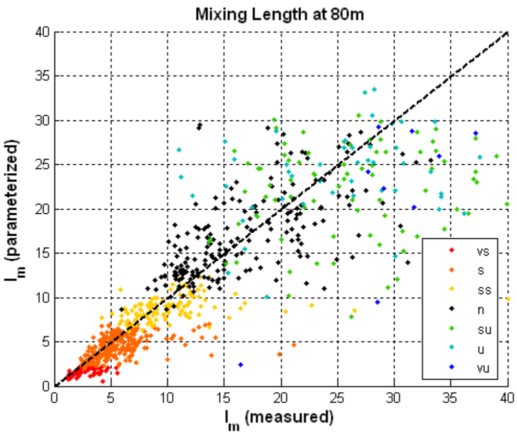

Figure 9: Measured vs parameterized mixing-length using (19) at 80m level

Conclusions

Several ABL parameterizations have been presented as an alternative to surface layer modelling. From preliminary simulations in the offshore environment of FINO-1, it seems that mixing-length models could be more competitive than limited-length scale k-ε models, as they are more flexible in the way the mixing length profile is defined and this can introduce swallower boundary layers.

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