Fatigue damage modelling of steel structures

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ABSTRACT: In the present work a constitutive model is developed which permits the simulation of the low cycle fatigue behavior in steel framed structures. In the elaboration of this model, the concepts of the mechanics of continuum medium are applied on lumped dissipative models. In this type of formulation an explicit coupling between the damage and the structural mechanical behavior is employed, allowing the possibility of considering as a whole different coupled phenomena. A damage index is defined in order to model elastoplasticity coupled with damage and fatigue damage

1 INTRODUCTION

The modern approach to the seismic design of structures accounts for dissipation of the seismic energy input through plastic deformations. A ductile response is characterized by the structure's ability to undergo large inelastic displacements without loss in the load carrying capacity. The evaluation of the structural performance requires the definition of parameters to characterize the structural damage. Traditionally, ductility has been employed as the principal criterion for design. However ductility does not account for the duration of ground shaking which is very important in inelastic design since the combined effects of ductility and energy absorption may lead to failure even at modest ductility demands. To suitably account for the performance of the structures in the design procedure, it is necessary to assess accurately the damage accumulation which progressively reduces the mechanical properties of the structural components subjected to plastic strains under earthquakes.

As an alternative to ductility based design , energy may be used as the basis for design. The simplest way to evaluate the cumulative damage using an energy approach consists on summing the inelastic deformations. However, this approach does not take into account the fact that the damage due to a large number of small plastic deformations may be less than one due to a smaller number of large plastic deformations. To overcome this problem, another way of thinking about energy is to use the concept of low cycle fatigue. Since the deformation histories are composed of random cycles, the structural damage is governed both by the maximum plastic displacement and by the dissipated energy. Then the low cycle fatigue approach appears to be a very interesting approach. It is possible to express the duration effects of an earthquake in terms of a effective number of cycles of loading and, in a similar way, to consider the energy absorption capacity in terms of a number of displacement cycles.

On the other hand, in the last years the fatigue study has been reorientated through its incorporation in the Continuum Damage Mechanics (CDM) (Lemaitre and Chaboche 1985; Lemaitre 1993). The same concepts used in CDM to model ductile failure can be extended to the low cycle fatigue damage processes, where plasticity is the key mechanism for crack initiation. Damage Mechanics deals with damage as a continuum variable and, because of it, CDM models including plasticity and damage can predict ductile crack initiation. An extension of themselves including the number of cycles could be suitable to simulate the low cycle fatigue damage. According to it, Chaboche (1985) developed a formulation for damaged materials where the fatigue phenomenon was incorporated in the CDM. However, only harmonic loads were considered being the hypotheses of fatigue cumulative damage suitable.

In this paper, a non linear damage model extended to consider fatigue effects is proposed. The model is formulated according to the concepts and theories of the CDM and is particularly well suited in the case of steel structures where damage accumulation due to local buckling and low cycle fatigue reduces the mechanical behavior of the steel structural components.

2 ELASTOPLASTIC DAMAGE MODEL

2.1. *Constitutive Equations*

Damage in Continuum Damage Mechanics takes into account the degradation of materials resulting in a stiffness reduction.

According to the Strain Equivalence Principle proposed by Lemaitre (1971) and using the Kachanov's definition of effective stress, the stiffness of a damaged material can be obtained as E(1-d) being E the initial Young's modulus and d a scalar representing the isotropic damage. Assuming a damaged elastic material, the strain due to damage can be obtained as (Ortiz 1985; Ju 1989):

$$\varepsilon^d = \frac{\sigma d}{E(1-d)} \tag{1}$$

which is consistent with the response of steel under uniaxial monotonic loading.

Equation 1 can be applied to a member of constant area A sujected to an axial load:

$$\delta^d = \frac{NL}{EA}\frac{d_a}{1-d_a} \tag{2}$$

where N is the axial force and δ^d the elongation due to the axial damage d_a.

Equation 2 can be generalized in order to take into account the flexural damage effects in a frame member. Thus , we consider an element where the stress distribution is described by a three component vector, $q=[M_i,M_j,N]^T$, collecting the bending moments at the two ends and the axial force (Fig. 1), which is associated to the corresponding kinematic variables $u=[\theta_i,\theta_j,\delta]^T$. The constitutive equations expressing the relations between the flexural moments and the corresponding rotations due to damage, $u^d = \left[\theta_i^d,\theta_j^d,\delta^d\right]$, are obtained as:

$$\theta_i^d = \frac{d_i}{1-d_i}\frac{L}{4EI}M_i \tag{3}$$

$$\theta_i^d = \frac{d_i}{1-d_i}\frac{L}{4EI}M_i \tag{4}$$

being di and dj the damage variables due to flexural effects at both ends of the member. Therefore, the damage vector for each member will be defined as $D^t = (d_i \quad d_j \quad d_a)$.

More details about the formulation of the constitutive equations for this model can be found in (Florez-Lopez 1995; Perera *et al* 1998).

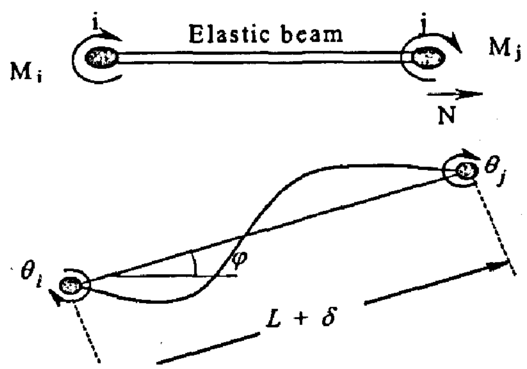

Figure 1. Generalized stresses and strains for the model

2.2. *Plastic Dissipative Potential*

In order to completely define the model, the evolution laws for plastic strains and damage must be specified. In order to obtain the plastic evolution law it is necessary to define a plastic dissipative potential. For it, by analogy with the effective stress concept proposed by Rabotnov in 1968, the three component stress vector q proposed in the last paragraph can be redefined as an effective generalized stress vector using the following expression:

$$\tilde{q} = \left(\frac{M_i}{1-d_i} \quad \frac{M_j}{1-d_j} \quad \frac{N}{1-d_a}\right)^t \tag{5}$$

According to the strain equivalence principle, any constitutive equation for a damaged material may be derived in the same way as for a virgin material replacing the usual stress by the effective stress (Lemaitre 1971).

Therefore the plastic dissipation potential for each plastic hinge of the member may be expressed using the same expression employed for undamaged materials replacing the moment by the corresponding effective moment.

Then, when damage occurs, if we do not consider the effect of the axial plastic strains, the plastic function can be written at each end as:

$$f_i = \left|\frac{M_i}{1-d_i} - X_i\right| - M_y \tag{6}$$

where X_i is the kinematic hardening term and M_y is the yield moment.

To define the evolution of X, the following expression is proposed:

$$X = X_\infty(1 - e^{-\alpha\theta^p}) \tag{7}$$

being X_∞ and α parameters to be identified for each material and geometry; from the expresion it can be observed that X tends to saturate to some value X_∞ with a velocity controlled by the value of α.

Being defined the plastic potential, the Principle of Maximum Plastic Dissipation implies the normality of the plastic flow rule in the generalized stress space:

$$du^p = d\lambda^p \frac{\partial f}{\partial q} \tag{8}$$

where $d\lambda^p$ is a plastic parameter which can be obtained enforcing the plastic consistency condition.

3 CUMULATIVE DAMAGE LAW

To completely define the model, the damage evolution law has to be specified. In cyclically loaded materials it is convenient to use cumulative damage models. Since seismic loads induce severe inelastic cycles at relatively large ductilities, the concept of using low-cycle fatigue theories to model damage is logical.

Assuming linear damage accumulation, the total cyclic fatigue damage may be obtained using the principle formulated by Palmgren (1924) and Miner (1945). Damage functions due to each individual cycle are summed until fracture occurs. Failure is assumed to occur when these damage functions sum up to or exceed unity:

$$D = \sum \frac{n_i}{N_f} \geq 1 \tag{9}$$

where n_i is the number of cycles for the current amplitude and N_f is the number of cycles to failure for this amplitude.

The quantification of the number of cycles to failure N_f is usually performed through the Manson-Coffin relationship (1953):

$$N_f = C(\Delta\delta^p)^K \tag{10}$$

where $\Delta\delta^p$ is the plastic strain amplitude of the hysteretic cycles (Fig. 2) and C and K are parameters depending on the materials. Some authors (Kunnath *et al* 1997; Koh and Stephen 1991) suggested that total strain amplitude could be used instead of plastic strain.

Damage values evaluated according to Equation 9 through the loading history are introduced in the elastoplastic damage model pre-sented in Section 2. For it, first of all, it is necessary to evaluate the plas

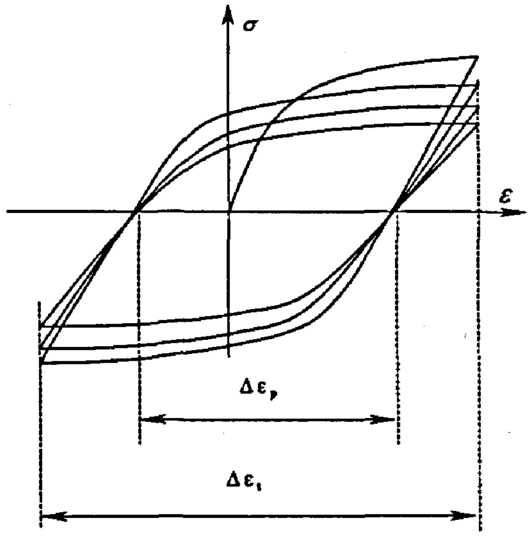

Figure 2. Total and plastic strain amplitude

tic strain increment at every load step using the flow rule of Equation 8 and enforcing the plastic consistency condition. With the plastic strain amplitude obtained and using Equations 9 and 10 the progressive damage increments are obtained.

Therefore, we need to determine the Manson-Coffin law in order to complete the model. Then, the keypoint of the cumulative damage law is related to the identification of the structural damage parameters C and K appearing in Equation 10.

Usually, through experimental tests performed on beams made of different profiles some results are obtained to calibrate the Manson-Coffin relationship (Ballio & Castiglioni 1994). The specimens are subjected to displacement cycles of constant amplitude up to collapse. The results obtained allow the definition of a relationship between the amplitude of the displacement cycles imposed and the number of cycles performed to reach the failure (N_f). Performing the tests for different amplitudes a linear relationship amplitude-N_f is obtained on a log-log scale which allows the determination of the parameters C and K.

These results, combined with the Miner law, may be useful as a criterion to predict the failure of structural elements. However, in the model proposed in Section 2, the damage is defined as an internal variable affecting the mechanical behavior and, basically, incorporatig the gradual loss of stiffness. Therefore, the limiting value d=1.0 of the damage variable may be identified with complete loss of stiffness. Due to it, in the definition of the parameters appearing in the Manson-Coffin law and, therefore, in the damage evaluation would be more convenient to keep the consistency with the definition of the damage index in the model as a variable measuring the progressive loss of stiffness.

Krawinkler & Zohrei (1983) performed several experimental tests of constant amplitude cyclic loading on steel structures in order to characterize the cumulative damage.In the experimental work developed, they consider damage associated to several different phenomena such as strength deterioration, energy dissipation and, as in Continuum Damage Mechanics, stiffness deterioration. The constant amplitude tests of several wide flange shapes (W 6x9) of ASTM A36 steel provided the relationship between damage increment per reversal (in terms of stiffness deterioration), and plastic rotation range. This relation is assumed to be constant within a certain range of the number of reversals. For it, three deterioration ranges were identified according to the deterioration rate. In the first and third ranges, deterioration grows rapidly while in the second range deterioration proceeds at a slow and almost constant rate. More details about it can be found in Krawinkler & Zohrei (1983).

For each range, the rate of *stiffness deterioration* per reversal, Δd_k, for constant amplitude cycling is expressed by a function of the form:

$$\Delta d_k = A(\Delta\theta_p)^a \tag{11}$$

where A and a are determined through experimental tests and $\Delta\theta_p$ is the plastic rotation range. From Equation 11, assuming linear damage accumulation for reversals with variable amplitude, the accumulated damage can be expressed as:

$$d = \sum_{i=1}^{n}(\Delta d_k)_i = A\sum_{i=1}^{n}(\Delta\theta_p)^a{}_i \tag{12}$$

where n is the number of reversals.

Denoting as K_o and K the undamaged and damaged stiffnesses, respectively, the rate Δd_k represented by Krawinkler & Zohrei (1983) corresponds to the relation $(K_o\text{-}K)/K_o$. In order to employ Equation 11 in the model presented in Section 2 a suitable relationship between the rate Δd_k defined in this equation and the rate Δd corresponding to the model has to be deduced. After some calculations, the following expression for the damage in the model is deduced:

$$d = \frac{4(1 - K/K_o)}{4 - K/K_o} \tag{13}$$

from which the following relationship is obtained

$$\Delta d = \frac{(4-d)^2}{12}\Delta d_k \tag{14}$$

or, applying Equation 11:

$$\Delta d = \frac{(4-d)^2}{12}A(\Delta\theta^p)^a$$

Therefore Equation 15 will be employed in our model to evaluate the damage rate per reversal.

On next section, to check the efficiency of the model proposed in this paper, numerical results are compared with the experimental results presented in Krawinkler & Zohrei (1983).

4 NUMERICAL SIMULATION

Figure 3 shows the results of one experimental test performed on a beam with W6x9 section subjected to a cyclic loading of constant amplitude equal to 1.7 in.

From the test, in deterioration range II the following rate of deterioration per reversal has been obtained:

$$\Delta d_k^{II} = 0.446\left[\Delta\theta^p\right]^{1.415} \tag{16}$$

where range II includes from cycle 10 to cycle 40. This rate has been employed in Equation 14 to evaluate the damage increment in the numerical simulation.

In the same way, the following values have been employed in the plastic dissipative potential: M_y=23 kN m; X_∞ =20 kN m; α=150.

Figure 4 shows the results obtained in the numerical simulation. As it has been commented before, only the function corresponding to the wider range of number of reversals has been used, which implies a certain deviation from the experimental results for the first and the last cycles.

In Figure 5, the damage evolution through the number of cycles obtained numerically is represented. The last numerical value (d=0.6) can be compared with the last experimental value (d=0.65) which has been obtained through Equation 13 measuring the relation K/K_o in the last cycle of range II. As it is logical, the numerical value is a little smaller than the experimental value since in the numerical results the range I, for which the deterioration proceeds at high rate, has not been considered.

5 CONCLUSIONS

The results obtained are very hopeful. The model performs very well under cyclic loading of constant amplitude. The model appears to be very interesting since it applies the concepts of the CDM in a simplified way to simulate the cumulative damage.

The approach presented is amenable of further generalizations and it would be convenient to obtain experimental results for more complex loading histo

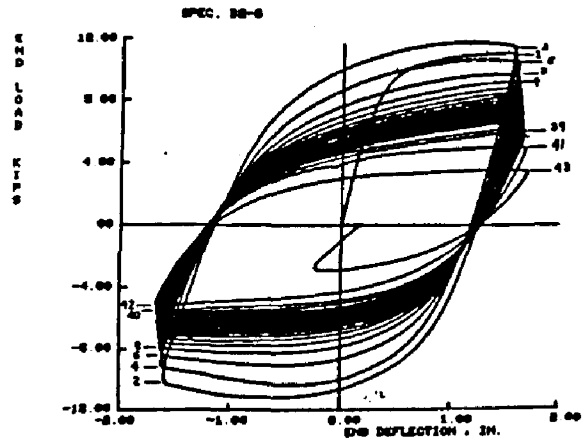

Figure 3. Experimental results

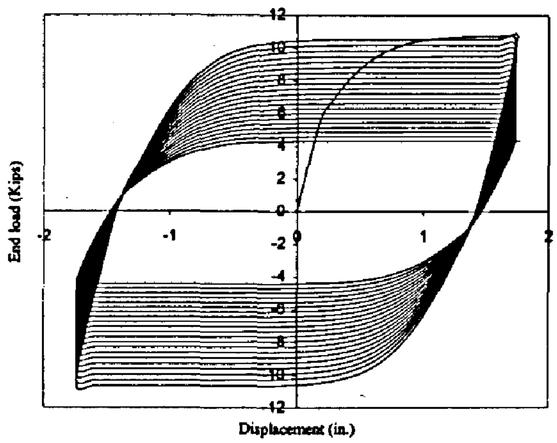

Figure 4. Numerical results

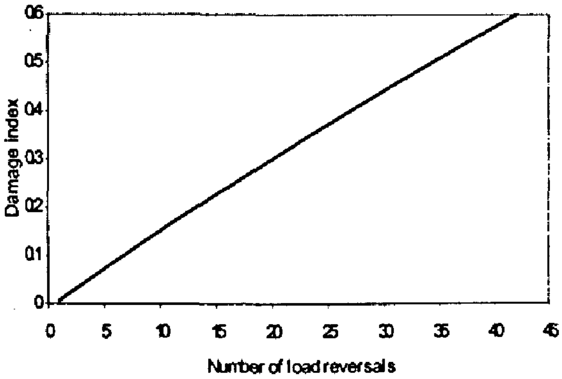

Figure 5. Damage evolution

ries (cyclic loading with variable deflection amplitudes, seismic loading) in order to check the efficiency of the model in more realistic loading cases.

In the model proposed, damage is related to the stiffness degradation. A very interesting possibility would be to try performing through some experimental tests a calibration of this damage index in order to formulate a failure criterion of the structure.

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