World Congress on shell and spatial structures: 20th anniversary of IASS
Madrid (Spain) 1979

GENERAL REPORT FOR THEME 2

Dynamic Studies of Shell and Spatial Structures

ENRIQUE ALARCON [1]

INTRODUCTION

For the past 20 years, dynamic analysis of shells has been one of the most fascinating fields for research.

Using the new light materials the building engineer soon discovered that the subsequent reduction of gravity forces produced not only the desired shape freedom but the apperance of ecologic loads as the first factor of design; loads which present strong random properties and marked dynamic influence.

On the other hand, the technological advance in the aeronautical and astronautical field placed the engineers in front of shell structures of non-conventional shape and able to sustain substantialy dynamic loads.

The response to the increasingly challenger problems of the last two decades has been very bright; new forms, new materials and new methods of analysis have arosen in the design of off-shore platforms, nuclear vessels, space crafts, etc.

Thanks to the intensity of the lived years we have at our disposition a coherent and homogeneous amount of knowledge which enable us to face problems of inconceivable complexity when IASS was founded.

The open minded approach to classical problems and the impact of the computer are, probably, important factors in the Renaissance we have enjoyed these years, and a good proof of this are the papers presented to the previous IASS meetings as well as that we are going to consider in this one.

Particularly striking is the great number of papers based on a mathematical modelling in front of the meagerness of those treating laboratory experiments on physical models.

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STUDY ON DYNAMIC BEHAVIOURS OF ELASTIC SHELL-TYPE OFFSHORE TOWERS SUBJECTED TO RANDOM OCEAN WAVES
Y. TANAKA T. HAMAMOTO
F. ISHIKAWA S. SHIRAISHI

STUDY ON DYNAMIC RESPONSE ANALYSIS OF ROTATIONAL SHELL WITH EDGE RING
TOSHIO NISHIMURA
KIYOSHI SHINGU

STATIC AND DYNAMIC ANALYSIS OF SPIRAL SHELLS WITH EDGE BEAMS
TOSHIO NISHIMURA
KATSUHIKO KAMIZONO

ON THE WIND-INDUCED DYNAMIC BEHAVIOURS OF SUSPENDED ROOFS
KAZUO UCHIYAMA
YASUSHI HIGUCHI
YASUSHI UEMATSU

DYNAMIC RESPONSE OF GUYED MASTS TO WIND ACTION
FISCHER Ondřej,
NÁPRSTEK Jiři
PIRNER Miroš

Plate 1

DYNAMIC BEHAVIOUR OF A NONCIRCULAR CYLINDRICAL PANEL UNDER INITIAL STRESS STATE
C. MASSALAS
A. LEONTITSIS
G. TZIVANIDIS
K. SOLDATOS

ON THE FINITE BREATHING MOTION OF LAYERED CYLINDRICAL SHELLS
AUBAR ERTEPINAR

STABILITY OF SUPPORTING CORES SUBJECTED TO ACCIDENTAL LOADINGS
B. GOSCHY

ON THE ACCURACY OF THE FINITE ELEMENT ANALYSIS OF THE VIBRATIONAL CHARACTERISTICS OF SHALLOW CYLINDRICAL TANKS
Dr. CELAL N. KOSTEM
JOSEPH W. TEDESCO

ANALISIS DINAMICO DE LAMINAS CILINDRICAS A BASE DE UNA MALLA RETICULAR DE PERFILES METALICOS PARA CUBIERTAS INDUSTRIALES
JUAN ROVIRA SOLER
SALVADOR GRAU PASCUAL

Plate 2

The universal entering of the computer into almost every phase of our lifes, and the cost of physical models, are —may be— reasons for this lack of experimental methods. Nevertheless they continue offering useful results as are those obtained with the shaking-table in which the computer plays an essential role in the application of loads as well as in the instantaneous treatment of control data.

Plates 1 and 2 record the papers presented under dynamic heading, 40 % of them are from JAPAN in good correlation with the relevance that japanese research has traditionally showed in this area. Also interesting is to find old friends as profesors Tanaka, Nishimura and Kostem who presented valuable papers in previous IASS conferences.

As we see there are papers representative of all tendencies, even purely analytical!

Better than discuss them in detail, which can be done after the authors presentation, I think we can comment in the general pattern of the dynamical approach as sumarized in plate 3.

2. MODELLING ACTIONS

The random character of actions, even the self-weight, is generally recognized, but the matter is that, in some cases, the random approach is the only sound possibility we have.

Wind, earthquakes, blasts, etc., only admit a probabilistic representation and the fact of the existence of deterministic codes only reflects the meagerness of our knowledge and the well-known conservatism of builders.

In plate 4 we present the general features of the probabilistic description of random-processes as well as three well known power spectral density functions (p.s.d.).

Dynamic studies

Summary

1. **Modelling actions**
   - **1.1. Deterministic approach**
   - **1.2. Probabilistic approach**
2. **Modelling the structure**
   - **2.1. Physical**
   - **2.2. Mathematical**
     - **2.2.1. Analytical**
     - **2.2.2. Numerical**
3. **Response analysis**
   - **3.1. Normal mode method**
   - **3.2. Frequency response methods**
   - **3.3. Step by step methods**

Plate 3

The technique has been also applied to the study of roughness of roads, railway tracks, etc. in order to predict the response of cars container's structures.

PROBABILISTIC DESCRIPTION

Random processes: $X(t)$ $\quad S(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-iw\tau} d\tau$

Wiener-Khinchin theorem $R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega) e^{iw\tau} d\tau$

stationary & ergodic assumption

$$R(\tau) = E[X(t), X(t + \tau)]$$
$$R(0) = E[X(t)^2] = \overline{X(t)}^2$$

$$\text{if } E[X] = 0 \rightarrow \sigma_x^2 = \overline{X(t)}^2 = R(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)$$

DAVENPORT along-wind gustiness

$$S(\nu) = 4k\overline{V}_{10}^2 \frac{X}{(1 + x^2)4/3}$$
$$W = 2\pi\nu$$

$\overline{V}_{10}$ = mean hourly wind speed h = 10 m
k = surface drag coef.
$x = \frac{\nu L}{\overline{V}_{10}}$; L = 1200 m

PIERSON-MOSKOWITZ oceanic-wave height

$$S(w) = \frac{\alpha g^2}{w^5} \exp\left[-\beta\left(\frac{\omega_0}{\omega}\right)^4\right]$$

α = $8.1 \cdot 10^3$
β = 0.74
w_0 = $g/\overline{V}$
$\overline{V}$ = mean wind speed at 19 5 m
g = acceleration of gravity

KANAI-TAJIMI seismic firm soil acceleration

$$S(W) = \frac{1 + 4\zeta_0^2\left(\frac{\omega}{w_0}\right)^2}{\left[1 - \left(\frac{w}{w_0}\right)^2\right] + 4\zeta_0^2\left(\frac{w}{w_0}\right)^2}$$

ζ_0 = 0.6
w_0 = 15.6
S_0 = intensity factor

Plate 4

In this conference TANAKA et. al, the spectral representation is used for presenting results on velocity as well as on wind pressure near a hanging roof, and also to visualize its displacement response.

On its part the deterministic approach combines the previous experience and, often, statistical treatment of data.

A lucid exposition of the deterministic method limitations was contained in the SPANISH Code H. A. 61, called «Eduardo Torroja Code» in honour of his collector and founder of our Association.

In plate 5 a popular deterministic device, the pseudo velocity spectrum, is presented. As the non stationarity of earthquakes records complicates the probabilistic approach the deterministic one on the basis of previous experience has received a wide acceptance an is a current tool in every day engineering.

The relation between p.s.d. and Newmark spectrum has been treated in several studies wich are outside of the object of this presentation.

3. MODELLING THE STRUCTURE

When the actions are modelled we have to build a mathematical or physical representation of the structure.

Linearity or nonlinearity is the first bifurcation point. The first is generally dictated by computational savings but, sometimes, it is impossible to avoid nonlinearity. Typical examples are inestability, large deformations, non-hookean material, etc.

These last two points are treated in the paper by ERTEPINAR et al., while GOSCHY uses an elastoplastic model and the cable nonlinearity is present in FISHER paper.

On their part, MASSALAS et al. present a rich comparison between dynamic and inestability phenomena.

Linear models are still valid, and in them the everyday methods of analysis are founded. On the other hand nonlinear models are usually analyzed as piece-wise linear ones.

Also critical is the election between a physical or a mathematical model. The experimental tendency, predominant 20 years ago, has strongly suffered the computer competency. The laboratories are indispensable but nowadays a great amount of research is carried out with mathematical models and the laboratory is only used in very expensive structures. As we said, the so called hybrid models are spacially interesting.

UCHIYAMA et al. & ROVIRA et al. present experimental results, while KOSTEM & FISHER use them as motivation and comparison.

DETERMINISTIC SPECTRUM

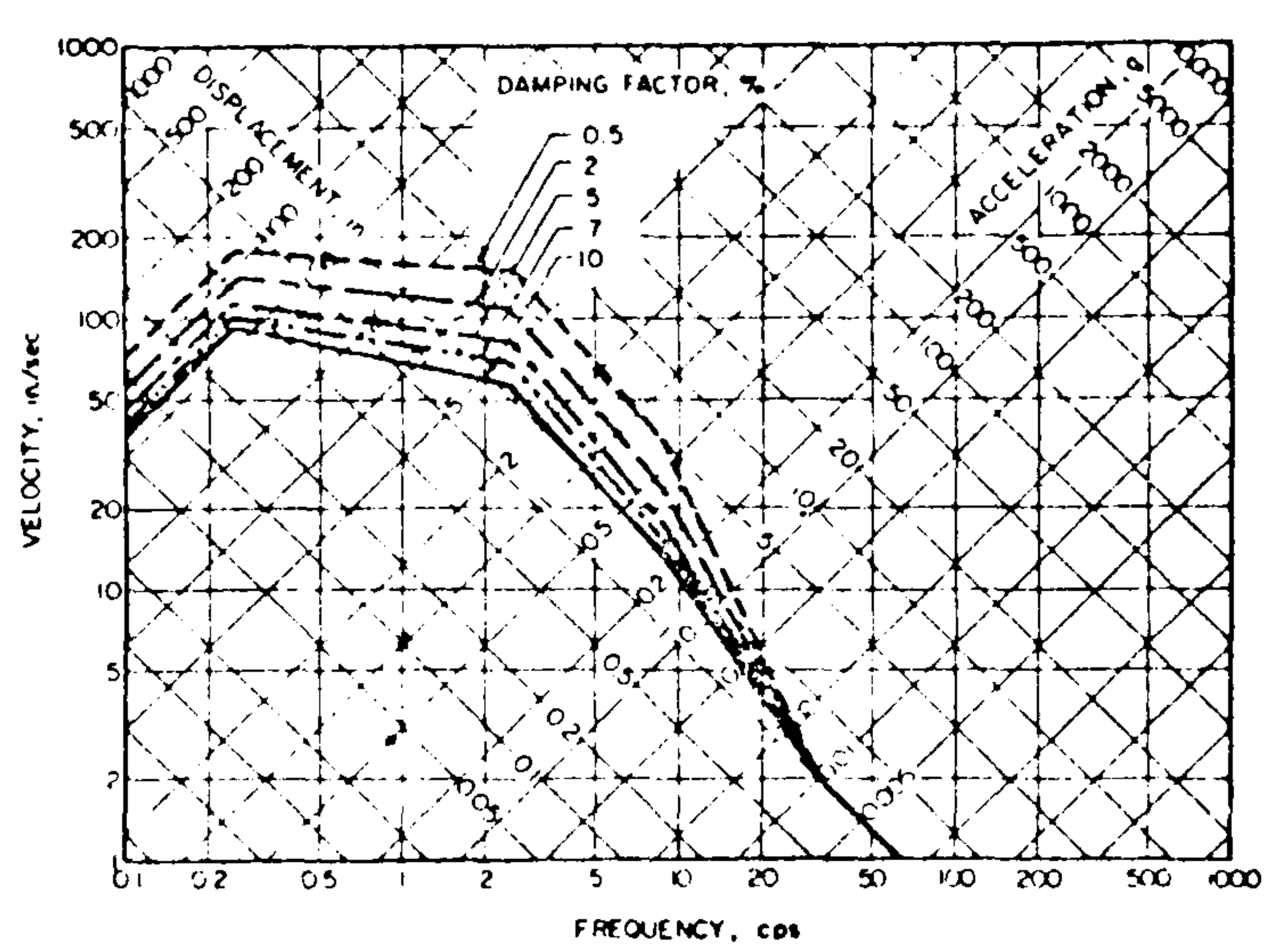
$$PSV = \max \left| \int_0^t \ddot{x}(\tau) \exp\left[-\zeta w(t-\tau)\right] \operatorname{sen} w_d(t-\tau) d\tau \right|$$

$$\text{max response} = \frac{PSV}{w}$$

$\ddot{x}(\tau)$ = accelerogram

ζ = damping ratio

$w_d = w\sqrt{1-\zeta^2}$



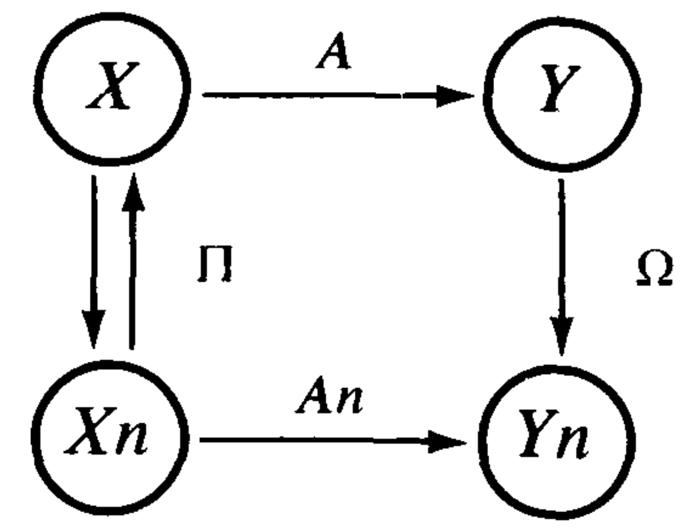
--Design Spectra for Various Damping Factors
Blume - Newmark - Kapur

Plate 5

Mathematical modelling is generally accomplished by a set of partial differential equations and boundary and initial conditions.

MODELLING THE STRUCTURE

$$Au = f$$



$u \in X$
$f \in Y$

$$An\,Un = \Omega f$$

$$U^n = \Pi Un \approx U^*$$

Proyective methods

$$U^* = \sum_1^{\infty} a_j \psi_j$$

$$\Pi Un = \sum_1^{n} a_j \psi_j$$

Petrov-Galerkin

$$(AU, \psi_i^*) = (f, \psi_i^*) \qquad i = 1, 2, \ldots, n$$

$$\Downarrow$$

$$\sum_1^n a_j (A\psi_j, \psi^*) = (f, \psi_i^*) \qquad i = 1, 2, \ldots, n$$

$$\uparrow An \qquad \uparrow \Omega y$$

$$\mathscr{P}Au = \mathscr{P}f \rightarrow \mathscr{P}A\Pi Un = \mathscr{P}f$$

Plate 6

In general, the analytical treatment, is impractical and this explains the relevance of experimental methods 20 years ago and numerical ones nowadays. Plates 6 and 7 summarize the mean features of the most popular projective methods whose compact formulation and mathematical respetabilization has been one of the most important facts of this days.

The series representation of solution is used by MASSALAS while the weighted residual methods is used by ERTEPINAR.

30 % of the papers use the Finite Element Method which is no doubt the most powerful accomplishment of these 20 years.

Already in the fascinating Symposium of 1959 in Copenhagen (fascinating by the topics as well as by the unbelieavable amount of geniune masters who meet there) Prof. Tottenham presented a general treatment of numerical methods. We can say then, thant the so called «modern methods» are nothing but classical problems focused from an unexpected viewpoint.

Weighted residual method

$$(A\Pi Un,\ \psi_i^*) = (f,\ \psi_i^*) = (Au,\ \psi_i^*) \qquad i = 1, 2, \ldots, n$$

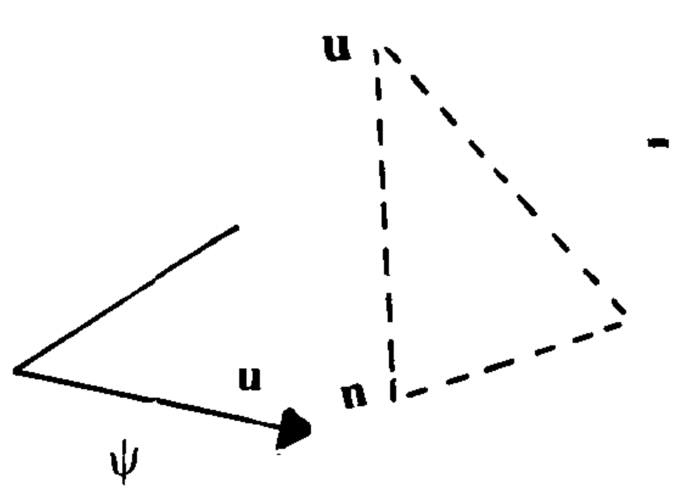
$$[A(\Pi Un - u),\ \psi_i^*] = 0 \Rightarrow (\varepsilon,\ \psi_i^*) = 0 \qquad \varepsilon_n \to 0 \quad n \to \infty$$

Rayleigh-Ritz method

$$\psi_i = \psi_i^*$$

$$\Sigma a_j(A\psi_j,\ \psi_j) = (f,\ \psi_{ii})$$

Least-squares method



Energy inner product

$$(\cdot\,,\ \cdot)_A = (A\cdot\,,\ \cdot)$$

$$\Sigma a_j(\psi_j,\ \psi_i)_A = (u,\ \psi_i)_A$$

F.E.M.

Rayleigh-Ritz + polinomial splines of compact support.

Boundary methods

$$(Au,\ \psi_i^*)_{\mathscr{D}} = a(u,\ \psi_i^*)_{\mathscr{D}} + b(u,\ \psi_i^*)_{\partial\mathscr{D}} = (f,\ \psi_i^*)_{\mathscr{D}}$$

$$(u,\ A\psi_i^*) = a(u,\ \psi_i^*)_{\mathscr{D}} + b'(u,\ \psi_i^*)_{\partial\mathscr{D}} = (f^*,\ u)_{\mathscr{D}}$$

$$(Eu,\ N\psi_i^*)_{\partial\mathscr{D}} = (Nu,\ E\psi_i^*)_{\partial\mathscr{D}} + (f,\ \psi_i^*)_{\mathscr{D}} - (f^*,\ u)_{\mathscr{D}}$$

$f^* = 0 \quad \rightarrow$ TREFFTZ methods

$f^* = \delta(x_j) \rightarrow$ B. I. E: M :

Plate 7

Plate 8 **presents the** dynamic discretization procedure which **can be easely obtained** by recognizing the nature of **the load** vector.

DYNAMIC DISCRETIZATION

$$f = q + q_D + q_I$$

$$q_D = -\underline{c}\,\dot{u}$$

$$q_I = -\underline{m}\,\ddot{u}$$

$$(Au,\ \psi_i^*) = (q,\ \psi_i^*) - (c\dot{u},\ \psi_i^*) - (m\ddot{u},\ \psi_i^*) \qquad i = 1, 2, \ldots, n$$

$$\Pi Un = \sum_1^n a_j\psi_j$$

$$\sum_1^n \ddot{a}_j(m\psi_j,\ \psi_i^*) + \sum_1^n \dot{a}_j(c\psi_j,\ \psi_i^*) + $$

$$+ \sum_1^n a_j(A\psi_j,\ \psi_i^*) = (q,\ \psi_i^*)$$

$\underline{m} \rightarrow m_{ij} = \int_{\mathscr{D}} \psi_i^*\, m\psi_j$ mass matrix

$\underline{c} \rightarrow c_{ij} = \int_{\mathscr{D}} \psi_i^*\, c\psi_j$ damping matrix

$\underline{k} \rightarrow k_{ij} = \int_{\mathscr{D}} \psi_i^*\, A_j$ stiffness matrix

$\underline{f} \rightarrow f_i = \int_{\mathscr{D}} \psi_i^*\, q$ force vector

$$\underline{m}\,\ddot{\underline{x}} + \underline{c}\,\dot{\underline{x}} + \underline{k}\,\underline{x} = \underline{f}$$

Plate 8

About the election of close or numerical methods one has nothing to say but remember the report presented by Dr. A. L. PARME (USA) in the 1966 Conference of Leningrade. Although, no doubt, the impetus is on numerical methods, it would be deplorable if developments along classical lines are ignored. For it is only by the insight given by generalized solutions that a keen appreciation of the fundamental bahaviour of shells can be rich comparison between dynamic and inestability phenomena.

Linear models are still valid and in them are founded the everyday methods of analysis. On **the** other hand nonlinear models are usually **analyzed** as piece-wise linear ones.

4. RESPONSE ANALYSIS

The actions and the structure modelled one has to obtain the response.

In experimental modelling the work is limited to the analysis of registered results and its integration and representation as can be seen in the papers of UCHIYAMA & ROVIRA.

For the mathematical modes there are fundamentally 3 approaches. The normal mode method is contained in plate 9 where special emphasis is put in the eigenvalue solver know as «subspace iteration» which is, perhaps, one of the most important achievements of these years. The method is specially suited to seismic cases (plate 10) in which only a limited number of eigenvectors is needed; and is the one used by ROVIRA et al. who also presents an interesting mode correlation in a way similar to the «component mode synthesis».

RESPONSE ANALYSIS

Modal Analysis

$$\underline{m}\,\ddot{\underline{x}} + \underline{c}\,\dot{\underline{x}} + \underline{k}\,\underline{x} = \underline{F}$$

eigenvalue problem $[-w_i^2\,\underline{m} + \underline{k}]\underline{\phi}_i = 0$

$$\underline{x} = \underline{\phi}\,\underline{\xi}$$

$$(\underline{\phi}^T\underline{m}\,\underline{\phi})\ddot{\underline{\xi}} + (\underline{\phi}^T\underline{C}\,\underline{\phi})\dot{\underline{\xi}} + (\underline{\phi}^T\underline{k}\,\underline{\phi}) = \underline{\phi}^T\underline{F}$$

Subspace iteration

$$\underset{n\times 1}{\underline{\phi}} = \underset{n\times q}{\underline{\psi}^*}\cdot\underset{q\times 1}{\underline{a}} = \sum_{i=1}^{q} a_j\psi_i^*$$

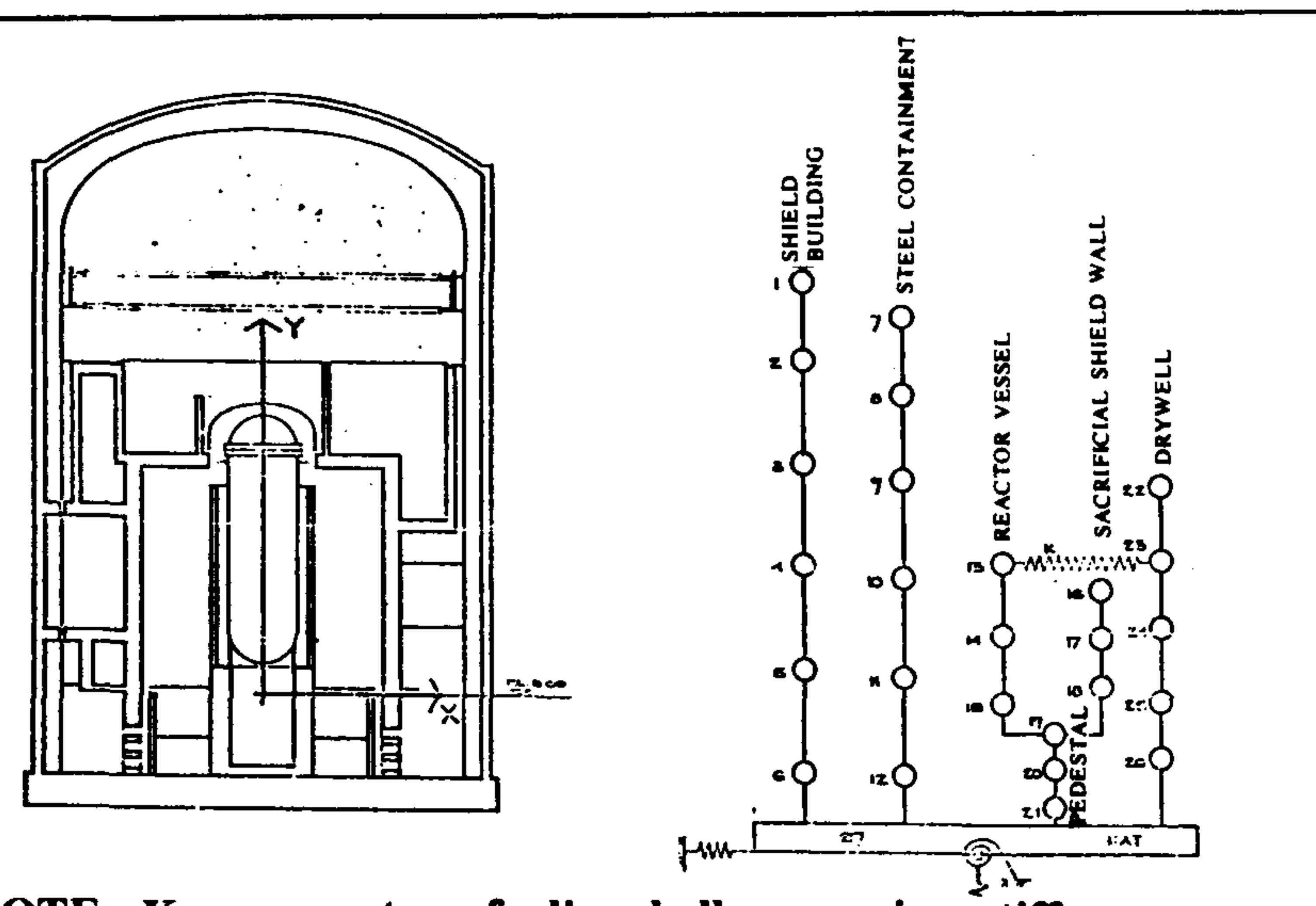
$$A \rightarrow \underline{k} - w_i^2\underline{m}$$

$$\sum_1^q a_j[(\underline{k} - w_i^2\,\underline{m})\psi_j^*,\ \psi_i^*] = 0$$

$$\left.\begin{array}{l}k^* = \underline{\psi}'^{*T}\underline{k}\,\underline{\psi}^*\\ \underline{m}^* = \underline{\psi}^{*T}\underline{m}\,\underline{\psi}^*\end{array}\right\} \rightarrow \underset{(q\times q)}{\underline{k}^*}\cdot\underset{(q\times 1)}{\underline{a}} = \underline{m}^*\underline{a}$$

Starting vectors	$\underline{\psi} = [\underline{\psi}_1\,\underline{\psi}_2\,\ldots\,\underline{\psi}_q]$
Inverse iteration	$\underline{K}\,\underline{\psi}^* = \underline{M}\,\underline{\psi}$
Projection	$\underline{k}^*\underset{(q\times q)}{\underline{Q}} = \underline{m}^*\underset{(q\times q)}{\underline{Q}}\,\underline{\Lambda}$
Inverse image	$\underset{(n\times q)}{\underline{\phi}} = \underset{(n\times q)}{\underline{\psi}^*}\,\underset{(q\times q)}{\underline{Q}}$

Plate 9



NOTE: K represents refueling bellows spring stiffness: case 1, K = 1200 K/FT; case 2, K = 0.

Plate 10

Another method used these years has been the frecuency response method (plate 11) which was possible only after the F. F. T. algorithm. The dynamic problem is reduced to a series of statistics ones which are superimposed «a posteriori». The method is valid only for linear situations.

FREQUENCY RESPONSE METHOD

$$\underline{m}\,\ddot{\underline{x}} + \underline{c}\,\dot{\underline{x}} + \underline{k}\,\underline{x} = \underline{f}(t)$$

$$\underline{F}(\alpha) = \mathscr{F}\underline{F} = (\mathscr{F}F_1, \mathscr{F}F_2, \ldots, \mathscr{F}F_n)_T$$

$$X(\alpha) = \mathscr{F}\underline{x}$$

$$(-\alpha^2\underline{m} + i\alpha\underline{c} + \underline{k})\,\underline{x}(\alpha) = \underline{F}(\alpha)$$

s.d.o.f. $x(\alpha) = H(\alpha, w)f(\alpha)$

$$H(\alpha, w) = \frac{1}{k\left[1 - \left(\frac{\alpha}{w}\right)^2 + i2\zeta\left(\frac{\alpha}{\omega}\right)\right]}$$

random vibration s.d.o.f.

$$s_x(\alpha) = H(\alpha, w)\bar{H}(\alpha, w)S_f(\alpha) = |H|^2 S_f$$

m.d.o.f.

$$R_x^i(0) = \frac{1}{2\pi}\phi_i\left[\int_{-\infty}^{\infty}\underline{\bar{H}}(\alpha)\underline{S}_f(\alpha)\underline{H}(\alpha)d\underline{\alpha}\right]\underline{\phi}_i^T$$

$$\underline{S}_f(\alpha) = (\underline{\Omega}^2)^{-1}\underline{\phi}\,\underline{S}_F(\alpha)\underline{\phi}^T(\Omega^2)^{-1}$$

a) Elimination of out of phase contributions.

b) Ditto of cross products H_iH_j.

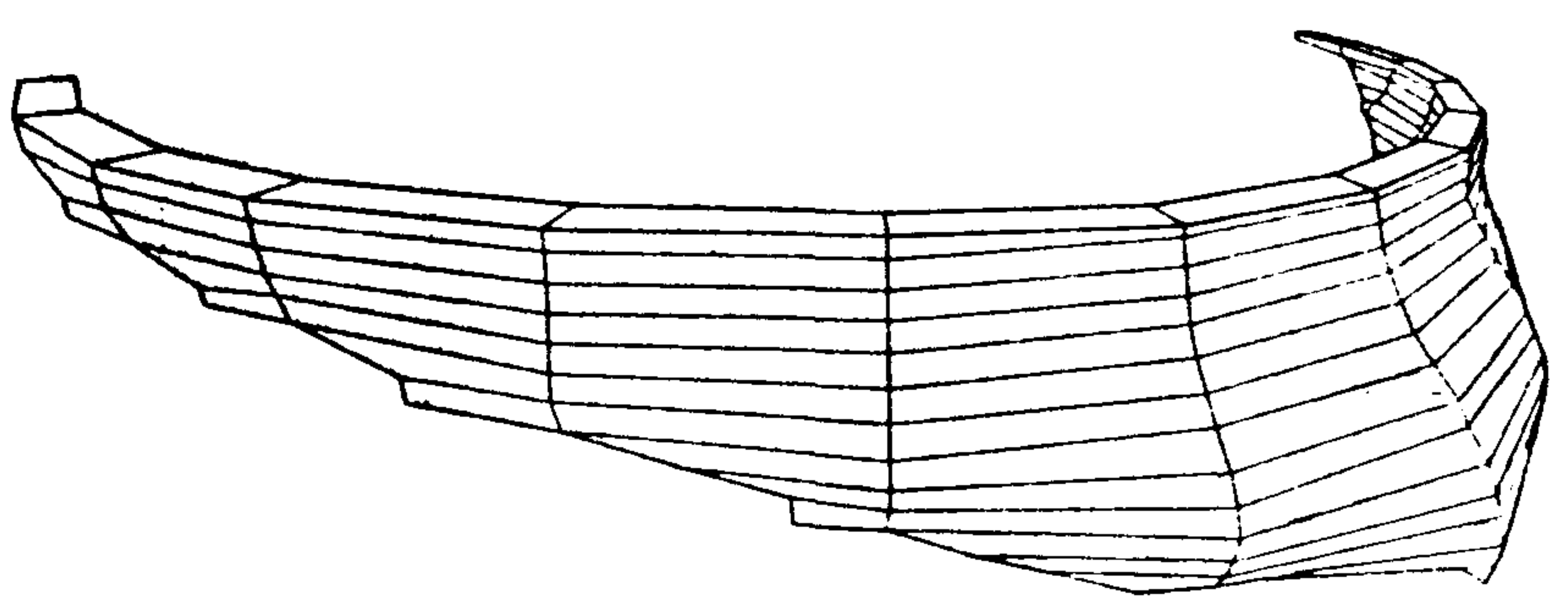
$$\bar{x}^2 = \sum_{r=1}^{n}\underline{\phi}_{-r}\frac{1}{w_r^4 m_r}\frac{1}{2\pi}\int_{-\infty}^{\infty}|H_r(\alpha)|^2 S_F(\alpha)d$$

$$m_r = \underline{\phi}_r^T\underline{m}\,\underline{\phi}_r$$

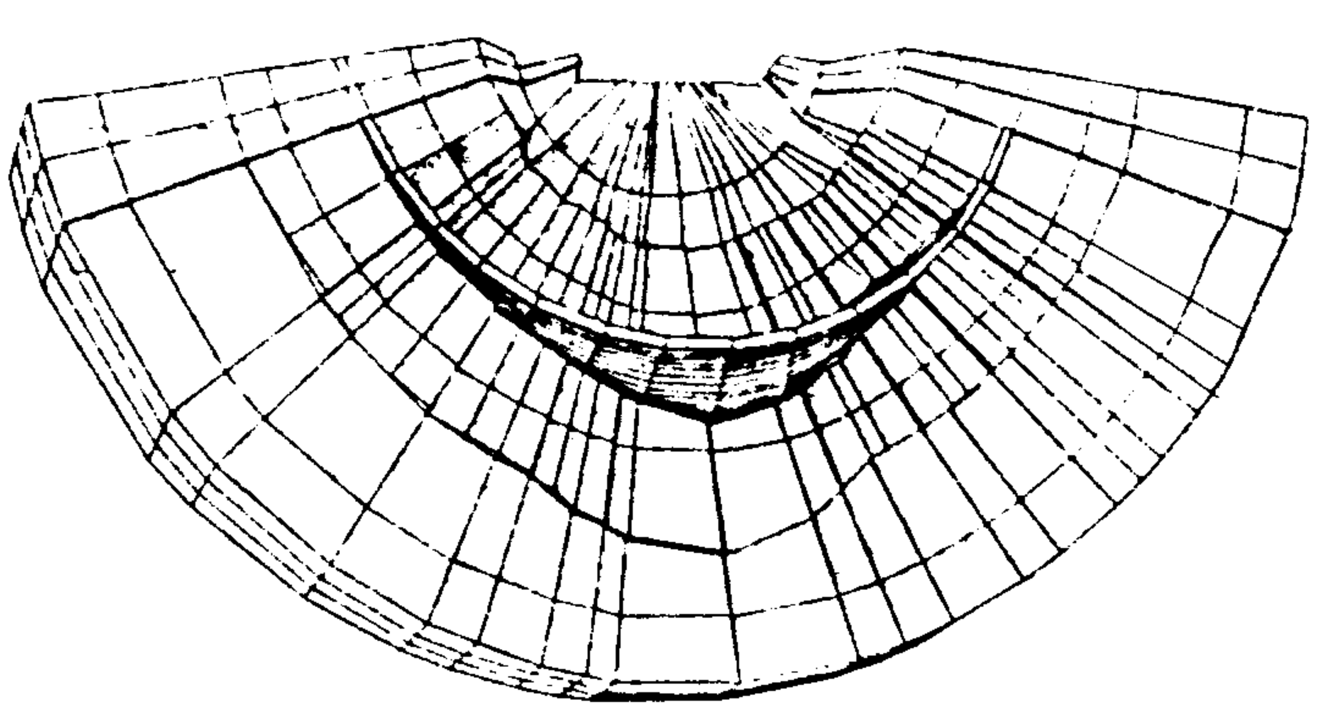
Plate 11

A tipical field of application is the soil-structure interaction problems, in which the soil is modelled by equivalent elements (springs, dashpots, etc.) whose characteristics are frecuency dependent. The response is obtained through a substructure approach (Plate 12). In these problems B. I. E. M. are specially indicated because they allow en easy treatment of infinite boundaries.

Also important is its use for treating the random analysis. The aproximation shown in plate 11 is used by TANAKA et. al. in their paper.



Small finite element grid



Large finite element grid including foundation
BALSARA & NORMAN (EESDJ. 1975)

Plate 12

The step by step method (Plate 13) is used in very broad circumstancies; as a rule we can say that its use is unavoidable when the system under study is non linear and its is neccesary to get the complete response record. Generally, they are costly methods whose main problems are those of convergence and stability.On the other hand they are usually simple and the resolution of the final systems can be simplified taking advantage of the special form of the matrix. NISHIMURA et al. have used this method in some of the problems whose results they present.

STEP BY STEP METHODS

Central difference method

$$\underline{x}_{t+\Delta t} = \underline{x}_t + \Delta t \dot{x}_t + \frac{\Delta t^2}{2}\ddot{x}_t$$

$$\underline{x}_{t-\Delta t} = \underline{x}_t - \Delta t \dot{\underline{x}}_t + \frac{\Delta t^2}{2}\ddot{x}_t$$

Adding, substracting

$$\ddot{\underline{x}}_t = \frac{1}{\Delta t^2}(\underline{x}_{t+\Delta t} - 2\underline{x}_t + \underline{x}_{t-\Delta t})$$

$$\underline{x}_t = \frac{1}{\Delta t}\frac{\underline{x}_{t+\Delta t} - \underline{x}_{t-\Delta t}}{2}$$

$$\underline{m}\,\ddot{\underline{x}}_t + \underline{c}\dot{\underline{x}}_t + \underline{k}\,\underline{x}_t = \underline{f}(t)$$

$$\left(\underline{m} + \frac{\Delta t}{2}\underline{c}\right)\underline{x}_{t+\Delta t} = \Delta t^2 \underline{f} +$$

$$+ [2\underline{m} - \Delta t^2 \underline{k}]x_t + \left[\frac{\Delta t}{2}\underline{c} - \underline{m}\right]\underline{x}_{t-\Delta t}$$

Newmark's method

$$\left[\underline{m} + \frac{\Delta t}{2}\underline{c} + \beta\Delta t^2 \underline{k}\right]\underline{x}_{t+\Delta t} =$$

$$= \Delta t^2[\beta \underline{f}_{t+1} + (1 + 2\beta)\underline{f}_t + \beta \underline{f}_{t-1}] +$$

$$+ [2\underline{m} - \Delta t^2(1 - 2\beta)\underline{k}]\underline{x}_s -$$

$$- \left[\underline{m} - \frac{1}{2}\Delta t \underline{c} + \beta \Delta t^2 \underline{k}\right]\underline{x}_{s-1}$$

Plate 13

FUTURE TRENDS

When comenting the failure of «Fronton Recoletos» shell the late Prof. TORROJA presented perhaps the first analysis of a dynamic failure. Quoting for the 1957 conference proceedings... (During the war) – the shell withstood quite satisfactorily the various hits by cannon shells...» «However in 1939, a bomb dropped by an aircraft outside the area covered by the shell roof, must have subjected the roof to very substancial deflections...» etc.

Since then, dynamic effects are increasingly important as we said and —no doubt— more research will be developed in the future.

Experimental models will probably continue, but the most important experimental results will be those obtained with life-sized models in conjunction with the systems identification theory in order to produce a better understanding of the dynamical phenomenon and a closed evaluation of parameters.

Also computers will continue to play an important role as was also predicted by Prof. TORROJA in the quoted Conference (1957)... «It is hoped that the rational use of electronic computers will permit the practical solution of many problems and will make the work more easier for the designers of shells»

It is clear that sophisticated new methods as the B. I. E. M.; the substructure deletion one, or combinations of known ones as, for instance, the substructure subspace iteration method presented recently, or the refinements introduced in random vibration theory will make the profession more powerfull and... difficult to learn and teach.