SELF-SIMILAR MOTION OF LASER HALF-SPACE PLASMAS.
I DEFLAGRATION REGIME.

by

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NOTA:

El presente trabajo corresponde al Informe #2 del equipo i) Interacción del laser con la materia y generación de ondas de choque, del Subprograma de la JEN sobre Confinamiento Inercial, Proyecto Laser-Fisión-Fusión, descrito en el Informe JEN-351.
ABSTRACT

The one-dimensional self-similar motion of an initially cold, half-space plasma of electron density $n_o$, produced by the (anamalous) absorption of a laser pulse of irradiation $\phi=\phi_0 t/\tau (0< t \leq \tau)$ at the critical density $n_c (n_c / n_o \approx \varepsilon << 1)$, is considered; the analysis allows for electron heat conduction and ion-electron energy exchange (viscosities and ion conduction are found negligible) and retains three dimensionless numbers: $\varepsilon$, $Z_i$ (ion charge number), and a parameter $\alpha=(n_o^2 \tau/\phi_0)^{2/3}$. For $\alpha >> \varepsilon^{-4/3}$, a deflagration wave (where energy absorption occurs) develops, separating an isentropic compression with a shock bounding the undisturbed plasma, and an isentropic expansion flow to the vacuum. The structures of these three regions are completely determined; in particular, the Chapman-Jouguet condition is proved and the density behind the deflagration is found. The deflagration-compression thickness ratio is large (small) for $\alpha << \varepsilon^{-5/3}$ ($\alpha >> \varepsilon^{-5/3}$). There is a thin transition layer ahead of the deflagration, where density has a maximum and temperature a minimum.
I. INTRODUCTION

In a recent paper\(^1\) (called A hereafter), the authors analyzed the self-similar motion generated in a plasma (initially cold and filling the entire space with uniform density \(n_0\)) by the deposition, in a given plane, of a pulse of energy per unit area and time, \(\phi = \phi_0 t/T (t \leq T)\). The results showed in detail the importance of heat conduction and electron-ion temperature relaxation against convection of energy, and their dependence on a single parameter 
\[ \alpha = \left(\frac{n_0^2 T/\phi_0}{\bar{\alpha}}\right)^{2/3} \]

The existence of self-similar motion in a plasma with heat conduction and different ion and electron temperatures was noticed by Anisimov\(^2\); a broad discussion of conditions for self-similar plasma motion has been given recently\(^3\).

Laser fusion requires compressing a DT pellet to very high densities, heating only its core\(^4,5\); the radiation is usually absorbed at electron densities close to the critical value, \(n_c\) (much less than \(n_0\)) inside a hot, rarefied corona ablated by the laser. Hydrodynamics is an essential part of the phenomenon.

In this work we have considered an extension of A: the plasma occupies initially a half-space and the laser radiation is assumed absorbed uniformly in the plane where the electron density equals \(n_c\). Clearly, there are now two parameters, \(n_c/n_0\) and \(\alpha\); moreover, there is also a new effect, the corona expansion to the vacuum, competing with ion-electron relaxation and energy conduction and convection. The analysis includes, therefore, the essential hydrodynamics of laser fusion, and the results should have qualita-
tive (and in some parts quantitative) validity for the spherical geometry contemplated in laser fusion. Non-hydrodynamic effects, such as radiation pressure and emission, nuclear fusion and electron degeneracy are not considered in this work.

In this paper we limit ourselves to the case \( \alpha \gg (n_c/n_o)^{n/3} \); the complementary range for \( \alpha \) is considered in a companion paper. For such large \( \alpha \), a deflagration wave is found to develop. Deflagration waves in plasmas were first approximately discussed by Fauquignon and Floux and Bobin; related qualitative analysis have been carried out by a number of authors.

In Sec. II, we establish the self-similar variables and equations. Sections III-V deal with the three main regions which appear in the flow. A discussion of the results is given in Sec.VI. Finally, a thin, transition layer is analyzed in the Appendix.

II. STATEMENT OF THE PROBLEM

We consider an unmagnetized, fully ionized plasma, which at \( t = 0 \) is at rest in the half-space \( x > 0 \), having zero temperature and uniform electron density \( n_o \) and ion density \( n_o/Z_i \) (\( Z_i \) is the ion charge number). At that time, energy per unit time and area, \( \phi(t) \), starts being absorbed by the electrons in the plane where the electron density \( n_e \) equals the critical density \( n_c \). This process represents anomalous absorption of laser radiation inciding from the vacuum on the left; \( n_c \), assumed to be much less than \( n_o \), is given by
\[ \omega_{pe}(n_e) \equiv (4\pi n_e e^2/m_e)^{1/2} = \omega, \]

where \( \omega \) and \( \omega_{pe} \) are the laser and electron plasma frequencies respectively. We shall assume both a) quasineutrality and b) short enough ion and electron self-collision times, so that either species has a near-maxwellian distribution function; these assumptions, and the approximation of zero initial temperature, will be justified, for the conditions considered in this paper, in Sec. VI. We allow, however, different temperatures \( T_e \) and \( T_i \).

Since the resulting motion is onedimensional and the electric current must vanish in the undisturbed plasma far to the right, quasineutrality \( (n_e = Z_i n_i \approx n) \) leads to equal velocities, \( v_e = v_i \approx v \). The evolution of the plasma is then governed, as in A, by the equation of continuity for either species

\[ \frac{Dn}{Dt} = -n \frac{\partial v}{\partial x}, \quad \left( \frac{Dn}{Dt} = \frac{\partial \rho}{\partial t} + \nu \frac{\partial \rho}{\partial x} \right), \quad (1) \]

the momentum equation for the ion-electron fluid

\[ m_i n \frac{Dv}{Dt} = -\frac{\partial}{\partial x} \left[ nk(Z_i T_e + T_i) \right] + Z_i n \frac{\partial}{\partial x} \left[ (v_e + v_i) \frac{\partial \rho}{\partial x} \right], \quad (2) \]

and the entropy equation for each species

\[ nT_e \frac{D}{Dt} (k \ln \frac{T_e e^{3/2}}{n}) = \frac{\partial}{\partial x} \left( K_e \frac{\partial T_e}{\partial x} \right) + n \frac{\partial v}{\partial x} \left( \frac{\partial \rho}{\partial x} \right) - \frac{3}{2} kn \frac{T_e - T_i}{T_e i} \]

\[ + \phi(t) \delta(x-x_c), \quad (3a) \]
\[ \frac{n}{Z_i} \frac{T_i}{D_i} \frac{D}{Dx} \left( k \ln \frac{T_i}{n} \right) + \frac{4}{3} m_i \left( \frac{\partial v_i}{\partial x} \right)^2 + \frac{3}{2} k n \frac{T_e-T_i}{T_e} \]  

\( x_c \) being such that

\[ n(x_c) = n_c \]  

Because of quasineutrality, there is no electric field term in Eq. (2), where electron inertia has also been neglected \((m_e << m_i)\). The classical coefficients of viscosity and thermal conductivity, and the ion-electron energy relaxation time, \( \nu_j \), \( K_j \) and \( t_{ei} \), may be written as \( \nu_j = \overline{\nu_j} T_j^{5/2} \), \( K_j = \overline{K_j} T_j^{5/2} \), \( t_{ei} = \overline{t_{ei}} T_e^{-3/2} n^{-1} \); \( \overline{\nu_j} \), \( \overline{K_j} \) and \( \overline{t_{ei}} \) depend weakly on \( T_j \) and \( n \) through Coulomb logarithms.

We have initial conditions

\[ T_e(x,0) = T_i(x,0) = v(x,0) = 0, \quad n(x,0) = n_o \quad \text{for} \quad x > 0, \]  

and boundary conditions

\[ T_e(\infty,t) = T_i(\infty,t) = v(\infty,t) = 0, \quad n(\infty,t) = n_o, \]  

\[ n(x_v,t) = 0, \quad v(x_v,t) = x_v \]

where \( x_v(t) \) is the plasma-vacuum boundary on the left. In addition, conditions on \( T_j \) should be given at \( x_v \): for a collision dominated
plasma we should have

\[ T_e(x_v, t) = T_i(x_v, t) = 0 \]  \hspace{1cm} (6c)

since the mean free paths are proportional to \( T^{-2/n} \).

For the case

\[ \phi = \phi_0 t/t' \quad , \quad 0 < t < t' \]

the motion of the plasma is self-similar, as in A, if the Coulomb logarithms are taken to be constant. Defining

\[ \xi = x / [ \omega T (t/t')^{1/3} ] \quad , \quad u(\xi) = v / [ v_0 (t/t')^{1/3} ] \]

\[ \bar{n}(\xi) = n / n_0 \quad , \quad \Theta(\xi) = T_j / [ T_0 (t/t')^{2/3} ] \quad , \quad \alpha = 3 k T_0 / m_i v_0 w \]

and choosing the free parameters \( w, v_0 \) and \( T_0 \) so as to satisfy

\[ 3 v_0 = 4 w \quad , \quad \phi_0 = w k n_0 T_0 \quad , \quad \bar{\nu}_e T_0^{5/2} = w^2 T_0^k n_0 \]  \hspace{1cm} (7b)

Eqs. (1)-(4) become

\[ \frac{d\bar{n}}{d\xi} = \frac{\bar{n}}{\xi - u} \frac{du}{d\xi} \]  \hspace{1cm} (8)

\[ u - 4(\xi - u) \frac{du}{d\xi} = \alpha \frac{d}{d\xi} \left[ \bar{n}(Z_i \theta_e + \theta_i) \right] + \frac{Z_i}{n} \frac{d}{d\xi} \left[ (a_i \theta_e^{5/2} + a_2 \theta_i^{5/2}) \frac{du}{d\xi} \right] \]  \hspace{1cm} (9)
\[ \frac{\dot{n}}{Z_i} \left[ \theta_i \left( 1 + \frac{4}{3} \frac{d\epsilon}{d\xi} \right) - 2 (\xi - u) \frac{d\theta_i}{d\xi} \right] = a_4 \left( \theta_i \left( \frac{5}{2} \frac{d\theta_i}{d\xi} \right) \right) + a_3 \left( \frac{\theta_i - \theta_i}{\theta_i} \right)^{3/2} \]

\[ \dot{n}(\xi_c) = n_c / n_o \equiv \epsilon \ll 1 \]

with boundary conditions

\[ \begin{align*}
\theta_e(\infty) &= \theta_i(\infty) = u(\infty) = 0, & \quad \dot{n}(\infty) &= 1 \\
\dot{n}(\xi_v) &= \theta_e(\xi_v) = \theta_i(\xi_v) = 0, & u(\xi_v) &= \xi_v
\end{align*} \]

In Eqs. (8)-(10):

\[ a_1 = \frac{4k}{m_i} \frac{\nu_i}{K_e} = 4.9 \times 10^{-4} \quad \text{(for } Z_i = 1) \]

\[ a_2 = \frac{4k}{m_i} \frac{\nu_i}{K_e} = 3.9 \times 10^{-2} A_i^{-1/2} \quad \text{(for } Z_i = 1) \]

\[ a_3 = \frac{2m_i}{3k} \frac{K_e}{\epsilon_i} = 4.3 b(Z_i) \quad \text{(for } Z_i = 1) \]

\[ a_4 = \frac{K_i}{K_e} = 4.1 \times 10^{-2} A_i^{-1/2} \quad \text{(for } Z_i = 1) \]
where $A_i$ is the ion mass number, and $b(Z_i)$, ranging from 1 at $Z_i = 1$ to 4.2 at $Z_i = \infty$, may be found from Ref. 15. To obtain the numerical coefficients, all Coulomb logarithms have been assigned a common, arbitrary, value. Clearly, electron heat conduction is the dominant transport mechanism; in fact, we shall hereafter neglect ion conduction and ion and electron viscosities. We notice that, then, $\alpha$ and $Z_i$ are the only parameters involved in Eqs. (8)-(10); one must solve for $\bar{n}$, $u$ and $\theta_j$ as functions of $\xi$ and of the dimensionless numbers $\epsilon$, $\alpha$ and $Z_i$.

It will prove convenient to derive conservation laws, from Eqs. (8)-(10), for mass, momentum and energy.

$$\bar{n} = \frac{d}{d\xi} \left[ \bar{n}(\xi-u) \right], \quad (14a)$$

$$5\bar{n}u = \frac{d}{d\xi} \left[ 4\bar{n}u(\xi-u) - \alpha \bar{n}(Z_i\theta_e + \theta_{i_e}) \right], \quad (14b)$$

$$3\bar{n}(Z_i\theta_e + \theta_{i_e} + \frac{4u^2}{3\alpha}) = \frac{d}{d\xi} \left[ 2\bar{n}(\xi-u)(Z_i\theta_e + \theta_{i_e} + \frac{4u^2}{3\alpha}) \right]$$

$$- \frac{4}{3} \bar{n}u(Z_i\theta_e + \theta_{i_e}) + Z_i\theta_e \frac{5/2}{d\xi} \frac{d\theta_e}{d\xi} + Z_i\delta(\xi-\xi_c). \quad (14c)$$

In addition, with appropriate reference values for laser fusion

$$n_r = 4.7 \times 10^{22} \text{cm}^{-3}, \quad \phi_r = 10^{14} \text{W/cm}^2, \quad \tau_r = 10^{-9} \text{sec.},$$

Eqs. (7) lead to useful expressions for $v_o$, $T_o$ and $\alpha$. 

\[ v_0^2 = \frac{4}{3} w = \frac{4}{3} \left( \frac{K_e \phi_o^2}{k' n_o T} \right)^{1/9} = 1.73 \times 10^7 \left( \frac{b^2 \phi_o^5}{Z_i^2 \phi_r^5 \tau_r^2 n_r} \right)^{7/19} \text{ cm sec} \]

\[ T_o = \left( \frac{\phi_o^2 \tau_r}{k' n_o K_e} \right)^{2/9} \approx 1.05 \left( \frac{Z_i \phi_o^2 \tau_r n_r}{b \phi_r^2 \tau_r n_o} \right)^{2/9} \text{ keV} \]

\[ \alpha = \frac{9 \lambda}{4m_i} \left( \frac{\tau_k^2 n_o}{\phi_o K_e} \right)^{2/3} = \frac{12.9}{A_i} \left( \frac{Z_i \phi_r \tau_r n_o}{b \phi_o \tau_r n_r} \right)^{2/3} \]

also

\[ x = \xi \left( \frac{t}{\tau} \right)^{4/3} \left( \frac{K_e \phi_o^2}{k' n_o} \right)^{1/9} = 1.30 \times 10^{-2} \xi \left( \frac{t}{\tau} \right)^{4/3} \left( \frac{b^2 \phi_o^5 \tau_r n_r}{Z_i^2 \phi_r^5 \tau_r n_o} \right)^{7/19} \text{ cm} \]

We have now taken the Coulomb logarithms arbitrarily equal to 8; this approximation will be discussed in Sec. VI. In the next three sections we shall solve Eqs. (8)-(12) for, \( Z_i \) arbitrary, \( \varepsilon \) small and \( \alpha \gg \varepsilon^{-4/3} \).

III. \textbf{ISENTROPIC COMPRESSION REGION}

We expect the results of \( A \) for the dense plasma flow on the right side, to remain valid here. Then, for \( \alpha \gg 1 \), energy convection gives rise to a shock wave bounding the disturbed plasma; the flow behind the shock is isentropic, ion and electron temperatures being equal, and viscosity and heat conduction negligible. Actually, there is a very thin precursor\textsuperscript{17} ahead of the shock (see Sec. VI).
As in A the jump conditions across the shock, which may be directly obtained from system (14), give

\[ \bar{n}_f = 4, \quad u_f = 3\xi_f/4, \quad \theta_f = 3\xi_f^2/4(2\theta + 1) \alpha, \]

where subscript \( f \) labels conditions just behind the shock. Then, integrating Eqs. (14) between \( \xi_v \) and \( \xi_f \) leads to overall conservation laws

\[ \int_{\xi_v}^{\xi_f} \bar{n}d\xi = \xi_f, \quad (18a) \]
\[ \int_{\xi_v}^{\xi_f} 5\bar{n}ud\xi = 0, \quad (18b) \]
\[ \int_{\xi_v}^{\xi_f} 3\bar{n}(Z_i\theta + \theta_i + \frac{4u^2}{3\alpha}) d\xi = Z_i, \quad (18c) \]

The last equation, together with Eqs. (17), leads to \( \xi_f = \gamma \alpha^{1/3} \), where \( \gamma \) is an unknown constant, determining the shock position, which must be of the order of unity or smaller.

Introducing into Eqs. (8)-(10) new variables

\[ \eta = \xi/\xi_f, \quad \nu = \bar{n}/\bar{n}_f, \quad y = u/u_f, \quad z = \theta/\theta_f, \quad (19) \]

one can verify immediately that the heat conduction and viscosity terms, being of order \( \gamma^3 \alpha^{-3/2} \), may indeed be neglected, while the
ion-electron energy relaxation term is of order $\alpha^{3/2} \gamma^{-3}$, so that we do have $\theta_0 = \theta_1$. Then, adding Eqs. (10) the relaxation terms drop out and we get

$$2z(1 + dy/d\eta) - (4\eta - 3y) dz/d\eta = 0$$

while Eqs. (8) and (9) become

$$\frac{dv}{d\eta} = \frac{3v}{4\eta - 3y} \frac{dy}{d\eta}$$

$$v[y - (4\eta - 3y) \frac{dy}{d\eta}] = -\frac{d}{d\eta}(vz)$$

Eqs. (20)-(22) must be solved subject to boundary conditions

$$v(1) = y(1) = z(1) = 1$$

The electron absorption term has not been included in (20) since the critical density plane lies outside the region of validity of these equations.

A first integral of Eqs. (20)-(22) is

$$v^{7/3}(4\eta - 3y) z^{-2} = 1$$

which is the usual adiabatic integral of self-similar, isentropic flow$^{17}$. Next, eliminating $v$ between (21) and (22) leads to

$$y - (4\eta - 3y) \frac{dy}{d\eta} = -\frac{dz}{d\eta} + \frac{3z}{4\eta - 3y} \frac{dy}{d\eta}$$
which must be solved together with Eq. (20). Defining

\[ Y = \frac{y}{\eta}, \quad Z = \frac{z}{\eta^2}, \]

one is lead to a phase-plane \((Z, Y)\) equation. A detailed discussion of the integration of this equation is given in \(A\); final numerical results, obtained there for \(v, y\) and \(z\) versus \(\eta\) are represented in Fig. 1. In the neighborhood of a value \(\eta = 0.82\), the solution behaves as

\[ y = \frac{4}{3}\eta^{13/13} - \frac{2}{5}(\eta - \bar{\eta}), \quad z = B_1(\eta - \bar{\eta}), \quad v = B_2(\eta - \bar{\eta})^{-3/13}, \quad (26) \]

where \(B_1 = 1.70, B_2 = 0.78\). We notice that the expression \(v(4\eta - 3y)\) (mass flow relative to the moving plane \(\eta\)) goes to zero as \(\eta\) approaches \(\bar{\eta}\) from the right. However, since \(\eta\) must be positive everywhere, Eq. (14a) shows that \(\eta(\xi - u)\) must be monotonic throughout the plasma, increasing rightwards from a zero value at the plasma-vacuum boundary; thus, we should have \(\xi - u > 0\), that is, \(4\eta - 3y > 0\), everywhere. Clearly, then, the isentropic compression solution ceases to be valid in the neighbourhood of \(\bar{\eta}\), where the plasma becomes highly dense and cold.

IV. ISENTROPIC EXPANSION REGION

We shall now make the ansatz that the plasma expands (moving leftwards) all through the interval between \(\eta_v\) and \(\bar{\eta}\), the density being there of the order of the critical density
To match the isentropic compression solution, the density should then increase from zero at \( \eta_v \) to infinity at \( \bar{\eta} \), while the temperature should vanish at both \( \eta_v \) and \( \bar{\eta} \), having a maximum in between. As in A, a very thin transition layer of dense, cold plasma, where \( \nu \) peaks, \( z \) bottoms up and \( y \) changes sign, should exist between the expansion and compression regions; we defer to the Appendix a discussion of this layer, which affects the solution outside it negligibly.

Writing Eq. (14b) in the variables defined in (19) and integrating across the transition layer, show that in the expansion region, we have

\[
\nu \sim \frac{\bar{\eta}_e}{\bar{\eta}_f} = 0(e) \quad . \quad (27)
\]

Further, we notice that the compression solution satisfies

\[
\int_0^{1} 5\nu y d\eta = B_1 B_2 \quad .
\]

and thus Eq. (15b) leads to \( \int_{\eta_v}^{\bar{\eta}} 5\nu y d\eta = 0(1) \); since the last condition in (12) shows that \( y/\eta = 0(1) \), we arrive at \( \nu y^2 = 0(1) \) or

\[
y = 0(e^{-1/2}) \quad , \quad \eta = 0(e^{-1/2}) \quad . \quad (29)
\]
Thus, using appropriately scaled variables we find that \( \eta \) is negative throughout the expanding plasma, the interval \( 0<\eta<\bar{\eta} \) being of negligible extent. Similarly, one may set \( y(0) = y(\bar{\eta}) = 0 \), since \( y(\bar{\eta}^-) = O(1) \); moreover, \( y < 0 \) in the entire expansion, as assumed, because \( (4\eta - 3y) \) must be positive throughout the plasma (see end of last section). Finally, the energy in the compressed region comes out to be a small fraction of the total energy in the plasma; that is, \( \gamma << 1 \), Eq. (18c) leading directly to \( \gamma = \overline{\gamma} e^{1/6} \), where \( \overline{\gamma} \) is an unknown constant of order of unity.

Introducing variables scaled according to (27)-(29), into Eqs. (8)-(10), we find that the conduction and viscosity terms are \( O(\alpha^{-3/2} \varepsilon^{-2}) \) as compared with convection terms of order of unity, while the ion-electron energy relaxation term is \( O(\alpha^{3/2} \varepsilon^2) \). Thus, if \( \alpha \gg \varepsilon^{-4/3} \), the expansion region would result to be isentropic (conduction and viscosity being negligible and ion and electron temperatures being equal). However, since electron heat conduction must be important near the critical density plane, a conduction layer, necessarily thin compared with the size given in (79), should exist within the expansion region, lying between the isentropic expansion flow on the left and \( \bar{\eta} \). The structure of this layer, which may be considered as a deflagration wave because temperature and velocity increase, and density and pressure decrease across it, will be determined in the next section.

Clearly, if \( \alpha = O(\varepsilon^{-4/3}) \), the isentropic expansion and the deflagration layer merge into each other, and the transport and re-
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Relaxation terms become important throughout the expansion flow. Thus, there is no deflagration unless \( \alpha \gg \varepsilon^{-4/3} \).

Now, let \( \nu_d, z_d, \) and \( y_d \) be the unknown values of \( \nu, z \)
and \( y \) at the base of the isentropic expansion (just behind the
deflagration), assumed of order \( \varepsilon, \varepsilon^{-1} \) and \( \varepsilon^{-1/2} \) respectively; then defining

\[
\hat{\eta} = \eta / (-y_d), \quad \hat{y} = y / (-y_d), \quad \hat{\nu} = \nu / \nu_d, \quad \hat{z} = z / z_d, \quad (30)
\]

Eqs. (20)-(22) directly yield

\[
\frac{d\hat{\nu}}{d\hat{\eta}} = \frac{3\hat{\nu}}{4\hat{\eta} - 3\hat{y}} \frac{d\hat{\nu}}{d\hat{\eta}}, \quad (31)
\]

\[
\hat{\nu}[\hat{y} - (4\hat{\eta} - 3\hat{y}) \frac{d\hat{y}}{d\hat{\eta}}] = -H d(\hat{\nu}\hat{z})/d\hat{\eta}, \quad (32)
\]

\[
2\hat{z}(1 + \frac{d\hat{y}}{d\hat{\eta}}) - (4\hat{\eta} - 3\hat{y}) \frac{d\hat{\nu}}{d\hat{\eta}} = 0 \quad , (33)
\]

where \( H = z_d/y_d^2 \). As boundary conditions we have

\[
\hat{\nu} = \hat{z} = -\hat{y} = 1 \quad \text{ at } \quad \hat{\eta} = 0 \quad (34)
\]

\[
\hat{\nu} = \hat{z} = 0, \quad \hat{y}_\nu = \frac{4\hat{\nu}}{3} \quad \text{ at } \quad \hat{\eta} = \hat{\nu}_\nu.
\]

As in Sec. III, a first integral of system (31)-(33) is
It is easy to see that if \( \hat{V}(\hat{\nabla}) = 0 \), condition \( \hat{z}(\hat{\nabla}) = 0 \) follows from (35) and needs not be included as a boundary condition, that is, the isentropic expansion is indeed collision dominated [see comment preceding Eq. (6c)]. Then, we must solve Eq. (33) and a combination of (31) and (32)

\[
\hat{y} - (4\hat{n}-3\hat{y}) \frac{d\hat{y}}{d\hat{n}} = -H \left[ \frac{d\hat{z}}{d\hat{n}} + \frac{2\hat{z}}{4\hat{n}-3\hat{y}} \frac{d\hat{y}}{d\hat{n}} \right],
\]

for \( \hat{y} \) and \( \hat{z} \). Now defining

\[
\hat{y} = \frac{\hat{y}}{\hat{n}}, \quad \hat{z} = \frac{\hat{z}}{\hat{n}^2},
\]

we arrive at the same phase-space equation of the compression region (see A)

\[
\frac{d\hat{n}}{d\hat{y}} = \frac{\hat{y}-17/15}{\hat{y}+2/5} \frac{2\hat{z}}{\hat{y}-4/3} \frac{\hat{z}-\hat{z}_N(\hat{y})}{\hat{z}-\hat{z}_D(\hat{y})},
\]

together with

\[
\hat{n} \frac{d\hat{y}}{d\hat{n}} = -\left( \hat{y} + \frac{2}{5} \right) \frac{\hat{z}-\hat{z}_P(\hat{y})}{\hat{z}-\hat{z}_P(\hat{y})},
\]

where
Since $4\hat{Y} - 3\hat{Y} > 0$, the solution $\hat{Z}(\hat{Y})$ to Eq. (38) must stay to the right of $\hat{Y} = 4/3$; for $\hat{Y} > 4/3$ all three curves $\hat{Z}_P(\hat{Y})$, $\hat{Z}_N(\hat{Y})$, and $\hat{Z}_D(\hat{Y})$ cross each other at $P$ and $N$ [which are singular points of (38)] and behave as $(9/5)\hat{Y}^2$ as $\hat{Y} \to \infty$ (see Fig. 2). We also notice that in dimensional variables, $\hat{Z} = \hat{Z}(\hat{Y})$ reads

$$\left(\nu - \frac{dx}{dt}\right)^2 = \frac{5}{3} \left(\frac{Z_i+1}{m_n}\right)$$

where $x = \omega(t/T)^{4/3}$ is the moving plane corresponding to a given $\xi$; thus, the plasma velocity relative to this plane would be sonic on the curve $\hat{Z}_P(\hat{Y})$. For $\hat{Z} < \hat{Z}_P(\hat{Y})$ the velocity would be subsonic and supersonic respectively; for large $\hat{Y}$, these conditions correspond to $\hat{H} \gg 9/5$.

Equations (34), (35), (37), and (38) lead to the conditions

$$\hat{Z} \sim \hat{H}\hat{Y}^2 \quad \text{as} \quad \hat{Y} \to \infty \quad (43)$$

$$\hat{Z} = 0, \quad \hat{Y} \frac{3}{7/3} = \frac{\nu_0^3}{H^2} \frac{\hat{Z}^2}{\hat{Y} - 4/3} = 0, \quad \text{at} \quad \hat{Y} = 4/3 \quad (44)$$
Now, all solutions to Eq. (38) behave as \( \hat{Z} \sim \hat{Y}^2 \) for large \( \hat{Y} \). On the other hand, solutions to Eq. (38), crossing point P, may have any of the three following behaviours

\[
\hat{Z} = C(\hat{Y} - 4/3)^{3/13}, \quad \hat{Z} = 0, \quad \hat{Z} = \hat{Y} - 4/3
\]

\( C \) being an arbitrary constant. However, the first two behaviours may be ruled out since they lead to \( \hat{Y}^{7/3} \neq 0 \) at \( \hat{Y} = 4/3 \), and \( \hat{Z} = 0 \) everywhere, respectively. Thus, one can integrate numerically Eq. (38), starting at point P, along the curve \( \hat{Z} = \hat{Y} - 4/3 \). The solution is then found to cross the nodal point N and go to infinity as \( \hat{Z} = 1.66\hat{Y}^2 \).

One should notice, nonetheless, that this solution needs not be valid beyond point N, since there is an infinite number of solutions to Eq. (38) crossing N with the same first few derivatives. In fact, the analysis of the deflagration structure in Sec. V proves that \( H \) in (43) cannot be less than 9/5, that is, the flow velocity relative to the deflagration wave, just behind it, cannot be supersonic. In addition, it can be easily shown that solutions with \( H > 9/5 \) approach P in the form \( \hat{Z} = C(\hat{Y} - 4/3)^{3/13} \), and may thus be excluded. Consequently, we must have \( H = 9/5 \); this proves that the sonic or Chapman-Jouguet condition is satisfied here.

We find that Eq. (38) admits two solutions with \( H = 9/5 \) in Eq. (43), lying to the left and right of \( \hat{Z}_p \) respectively. The first one, however, does not reach point N, and approaches P as \( \hat{Z} \sim (\hat{Y} - 4/3)^{3/13} \); it may also be ruled out. On the other hand, the curve to the right of \( \hat{Z}_p \), first moves away from it and then turns and
reaches point \( N \); it is, then, clear that this curve, together with
the solution which was previously found to go from \( P \) to \( N \), make up
the function \( \hat{Z}(\hat{y}) \) looked for. The entire function is shown in Fig.2.
Once \( \hat{Z}(\hat{y}) \) is known, one may find immediately \( \hat{y} \), \( \hat{z} \) and \( \hat{v} \) versus \( \hat{y} \),
numerically, from Eqs. (39), (37) and (35); they are shown in Fig.3.

V. THE STRUCTURE OF THE DEFLAGRATION WAVE

The deflagration layer which separates the isentropic
expansion and compression regions is governed by dissipative terms,
the dominant one being electron heat conduction. Hence, the thick-
ers of this layer can be determined by requiring that energy con-
vection and conduction be comparable. Writing the conservation
equations (14) in variables \( \hat{n} \), \( \hat{y} \), \( \hat{v} \) and \( \hat{z} \) (allowing for different
\( 1 \)

\[ \frac{\partial}{\partial t} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] + \frac{\partial}{\partial \hat{y}} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] = 0 \]  

(45a)

\[ \frac{\partial}{\partial \hat{n}} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] - \frac{\partial}{\partial \hat{y}} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] = 0 \]  

(45b)

\[ \frac{\partial}{\partial \hat{y}} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] + \frac{\partial}{\partial \hat{y}} \left[ \hat{v}(\hat{n} - 3\hat{y}) \right] = 0 \]  

(45c)
where both $c_1$ and $c_2$

$$c_1 \equiv \left( \frac{3}{4} \right)^{5/2} \frac{Z_i H^{2-3} z_d^{3/2} \epsilon^{5/2}}{v_d (Z_1 + 1)^{7/2}} , \quad c_2 \equiv \frac{Z_i \epsilon^{-1/2}}{3 v_d (-y_d)^{3-\gamma}} ,$$

are of order of unity. Since all four $\hat{y}$, $\hat{v}$, $\hat{z}_e$ and $\hat{z}_i$ must also
be of order of unity, there then results a deflagration wave thickness
$\hat{n} = 0(\epsilon^{-2} \alpha^{-3/2})$; it also follows that such a wave (a conduction
layer, thin compared with the isentropic expansion region) will exist as long as $\alpha >> \epsilon^{-4/3}$, as found before. We notice, on the other
hand, that in the compression region $\hat{n} = 0(\epsilon^{1/2})$, and therefore the
deflagration will be thinner or thicker than this region, for
$\alpha >> \epsilon^{-5/3}$ and $\alpha << \epsilon^{-5/3}$ respectively.

We now introduce a new scaled variable

$$\hat{n} = \alpha^{3/2} \epsilon^2 \left[ \hat{n} - \left( \frac{\hat{n}}{-y_d} \right) \right] ,$$

into Eqs. (45) and integrate them once, neglecting their left-hand
sides. Making use of the conditions behind the deflagration
($\hat{n} \rightarrow -\infty$)

$$\hat{v} = \hat{z}_e = \hat{z}_i = 1 , \quad \hat{y} = -1 ,$$

and dropping $\hat{n}$ everywhere, we arrive at
\[ \hat{v}_y = -1 \quad , \quad (48a) \]

\[ 3\hat{y} + H(Z_1\hat{z}_e + \hat{z}_i)/\hat{y}(Z_1 + 1) = -3-H \quad , \quad (48b) \]

\[ \frac{3}{2}\hat{y}^2 + \frac{5}{2}H(Z_1\hat{z}_e + \hat{z}_i)/(Z_1 + 1) + \hat{z}_e \frac{5/2}{d\hat{z}_e/d\hat{\eta}} + c_2 \sigma = \frac{1}{2}(3+5H) \quad , \quad (48c) \]

\( \hat{c} \) being zero if \( \hat{\eta} < \hat{\eta}_c \), and unity if \( \hat{\eta} > \hat{\eta}_c \). A fourth equation, to determine ion temperature, is provided by (10b)

\[ \frac{\hat{z}_e}{\hat{z}_i} \frac{d\hat{y}}{d\hat{\eta}} + \frac{3}{2} \hat{y} \frac{d\hat{z}_i}{d\hat{\eta}} = \frac{3a_2Z_i^2}{4(Z_i+1)^2} H^2 \hat{y} \frac{\hat{z}_e - \hat{z}_i}{Z_e} \quad . \quad (48d) \]

Evaluating Eqs. (48b) and (48c) ahead of the wave (\( \hat{\eta} = 0 \)), where \( \hat{z}_e/\hat{z}_i = 1 \), \( \hat{z}_e \) and \( \hat{y} \) vanish, and \( \hat{z}_e \hat{z}_e = B_1 B_2 / \nu_d z_d \), we find the jump conditions across the wave

\[ \hat{z}_i = -Z_i \hat{z}_e - H^{-1}(Z_i+1)\hat{y}(3+H+3\hat{y}) \quad , \quad (49) \]

As in A, the continuity equation yields no condition, the mass flow at \( \hat{\eta} \) being unknown.

System (48) yields

\[ \hat{v} = -1/\hat{y} \quad , \quad \hat{z}_i = -Z_i \hat{z}_e - H^{-1}(Z_i+1)\hat{y}(3+H+3\hat{y}) \quad , \quad (50) \]

together with coupled equations for \( \hat{z}_e \) and \( \hat{y} \).
\[
\frac{d\hat{z}_e}{d\hat{y}} = \frac{6F(\hat{y})}{\hat{z}_e^{5/2}}, \quad \frac{d\hat{z}_1}{d\hat{y}} = \frac{3a_3H^2z_i}{4(z_i+1)} \left[\hat{z}_e-\hat{z}_1(\hat{y})\right]\left[\hat{z}_e-\hat{z}_2(\hat{y})\right],
\]

where

\[
F(\hat{y}) = \begin{cases} 
\hat{y} + 1 & \sigma = 0, \\
\hat{y} + 5(3+H)/12 & \sigma = 1
\end{cases}
\]

\[
\hat{z}_p(\hat{y}) = -12 \frac{Z_i+1}{Z_i} \frac{\hat{y}}{H} \left[\hat{y} + 5(3+H)/24\right]
\]

\[
\hat{z}_{1,2}(\hat{y}) = \frac{\hat{y}}{2H} \left[H + 3 + 3\hat{y} \pm \left(H + 3\hat{y}\right)^2 + \frac{48}{a_3} \frac{Z_i+1}{Z_i} F(\hat{y})\right]^{1/2}
\]

Equations (51) lead to a phase-space equation for \(\hat{z}_e\) and \(\hat{y}\)

\[
\frac{d\hat{z}_e}{d\hat{y}} = \frac{8(Z_i+1)}{3Z_iH^2} \left[\hat{z}_e-\hat{z}_1(\hat{y})\right]\left[\hat{z}_e-\hat{z}_2(\hat{y})\right]
\]

which must be solved subject to two boundary conditions

\[
\hat{z}_e(\hat{y} = -1) = 1, \quad \hat{z}_e(\hat{y} = 0) = 0
\]

We notice, however, that both \(H\) and the value of \(\hat{y}\) at the critical density plane \(\hat{y}_c\) (such \(\sigma = 0\) for \(\hat{y} < \hat{y}_c\), and \(\sigma = 1\) for \(\hat{y} > \hat{y}_c\)), must be found as part of the problem.
Clearly the solution curve $S$ will comprise two pieces: a solution to Eq. (55) with $\sigma = 1$, passing through the origin, $S_1(H)$, and a solution to Eq. (55) with $\sigma = 0$, passing through point (-1,1), $S_2(H)$, both pieces meeting at the critical point $(\hat{\gamma}_c, \hat{z}_c)$. We shall prove in what follows that $S$ does not exist for $H$ less than $9/5$, while it does for $H = 9/5$; since, on the other hand, the range $H > 9/5$ was ruled out by the analysis of Sec. IV, we would then conclude that $H = 9/5$.

We, first, find out that for $H \leq (9/5)(Z_1+1)/(1+3Z_1/5)$, point (-1,1) lies outside the parabola $\hat{z}_e = \hat{z}_p(\hat{\gamma})$, while $S_1(H)$ lies inside it for $\hat{z}_e$ small and positive, for all $H$. Thus, for $H \leq 9/5$, $\hat{z}_e - \hat{z}_p(\hat{\gamma})$ must vanish somewhere on $S$. And it also follows that either $\hat{z}_p - \hat{z}_1$ or $\hat{z}_p - \hat{z}_2$ should vanish at that point too, since otherwise we would have $d\hat{\gamma}/d\hat{\gamma} = 0$ there, and $\hat{\gamma}$ would not be a single valued function of $\hat{\eta}$.

We next find out that $S_1(H)$ should reach the critical point inside the parabola. Figure 4 shows $\hat{z}_p$, $\hat{z}_1$ and $\hat{z}_2$ in the range $-1 \leq \hat{\gamma} \leq 0$, for both $\sigma = 1$ and $\sigma = 0$, $Z_1 = 1$ and $H = 9/5$; the parabola and the loop formed by $\hat{z}_1(\sigma = 1)$ and $\hat{z}_2(\sigma = 1)$, have no common point, except the origin. As $H$ decreases to zero, both the loop and the parabola get narrower and taller, but remain separate. On the other hand, as $Z_1$ increases, the loop becomes broader while $\hat{z}_p(\hat{\gamma})$ is reduced (by as much as one half), and they finally cross each other for large enough $Z_1$. Nonetheless, $d\hat{z}_1/d\hat{\gamma}$ is positive at the cross-point; since $S_1(H)$ leaves the origin between $\hat{z}_1(\hat{\eta})$ and $\hat{z}_p(\hat{\gamma})$, it cannot, clearly, reach that point.
Now, for $H < 9/5$, point $(-1,1)$ is a saddle having a positive and a negative slope, and $\hat{z}_1(\sigma=0)$ lies in between. It is easy to show that $\hat{y}$ must be increasing at $(-1,1)$ and that the branch with positive slope, $S_{21}(H)$, cannot be part of $S$, since it is incompatible with Eqs. (51). To prove that the other branch, $S_{22}(H)$, can neither; we consider the relative positions of the line $\hat{y} = -(3 + 5H)/12$ where $F(\hat{y}, \sigma=0)$ vanishes, and point $M$ where $\hat{z}_p(\hat{y})$ and $\hat{z}_1(\hat{y}, \sigma=0)$ meet. Let $H^*(Z_1)$ be the value of $H$ for which $M$ lies on that line ($H^*$ ranges from $27/19$ at $Z_1 = 1$, to unity for $Z_1$ infinite). For $H < H^*$, $M$ lies to the left of the line, so that the slope of $S_{22}$ must remain negative until crossing either $\hat{z}_p$ or $\hat{z}_1$; since it can be shown that $\hat{z}_p > 1$ at $M$, $S_{22}(H)$ cannot be part of $S$. Actually, for $H$ small enough $\hat{z}_1(\sigma=0)$ and $\hat{z}_2(\sigma=0)$ form two loops, one starting at the origin, the other crossing both $(-1,1)$ and $(-1,0)$; then, $S_{22}(H)$ is trapped inside this second loop and finally crosses $\hat{z}_2(\sigma=0)$ toward $\hat{y}$ decreasing. For $9/5 > H > H^*$, $M$ lies to the right of $\hat{y} = -(3+5H)/12$, which meets $\hat{z}_1(\sigma=0)$ at a point which is a node of Eq. (55), while $M$ is a saddle. We find that $S_{22}(H)$ does not reach $M$. Thus, $S$ does not exist for $H < 9/5$. We notice that, actually, a negative slope at $(-1,1)$ would imply that heat would flow toward the plane where energy is absorbed (far to the left).

We find numerically that for $Z_1$ not too large, $S_{21}(H<9/5)$ crosses the node from above $\hat{z}_1(\sigma=0)$ and reaches $M$ from below $\hat{z}_1(\sigma=0)$. As $H$ goes to $9/5$, the node moves to point $(-1,1)$; then, both $S_{21}$ and $S_{22}$ leave this point below $\hat{z}_1(\sigma=0)$ and are thus com-
compatible with Eqs. (55). Since clearly $S_{21}$ still reaches $M$, $H$ must be 9/5, and the solution $S$ will be composed of $S_4(9/5)$ and $S_{21}(9/5)$, which should meet inside the parabola, determining $\hat{y}_c$. We notice, however, that as $Z_i$ increases, the critical point approaches the parabola; we let to a future note the discussion of the deflagration for large $Z_i$.

The entire solution is shown in Fig. 4 for $Z_i = 1$. Once $H$ and $\hat{y}_c$ are known, Eq. (48a) leads to $v_d$:

$$\frac{1}{\hat{y}_c} = \frac{\hat{v}}{v_d} = \frac{\hat{v}}{v_d} = \frac{\hat{v}}{v_d} = \frac{\hat{v}}{v_d}$$

Then, $z_d$ and $c_2$ can be obtained from (49). Finally, using $y_d = (5z_d/9)^{1/2}$, both $\bar{y}$ and the scale length $c_1$ are given by Eqs. (46). All $\hat{y}$, $\hat{z}_e$, $\hat{z}_i$ and $\hat{v}$ are shown in Fig. 5, as functions of $\hat{n}$, for $Z_i = 1$. To complete the solution in the isentropic expansion, we give $v_d$, $z_d$, $y_d$, $\bar{y}$ and $c_1$ in Table I for $Z_i = 1$, 2, and 4. Table I also shows the minimum of $z$ and the maximum of $\bar{y}$ and its location $\hat{n}_1$.

VI. DISCUSSION OF RESULTS

We have studied the one-dimensional self-similar motion of an initially cold, half-space plasma of electron density $n_o$, when a laser pulse of irradiation $\phi = \phi_o t/t$ ($0 < t_\tau$) is anomalously absorbed at the critical density, $n_c$ (its location being
part of the solution). We allowed electron heat conduction and
different temperatures $T_e$ and $T_i$, and showed that viscosities and
ion conduction, which could be included in the analysis, were nu-
merically negligible. Then, the solution is governed by the ion
charge number $Z_i$, the ratio $\varepsilon \equiv n_c/n_o$ (assumed small), and a pa-
rameter $\alpha \equiv (n_o^2 \tau / \phi_o)^{2/3}$. We have here considered the case $\alpha >> \varepsilon^{-4/3}$,
that is, $(n_o^2 \tau / \phi_o)^2$ large.

We found that a deflagration wave (inside which energy
absorption occurs, ion and electron temperatures are unequal, and
conduction is important) separates an isentropic compression flow
on the right, bounded by a shock from the undisturbed plasma, and
an isentropic expansion flow on the left, bounded by the vacuum.
We determined the motion entirely throughout these three regions.
A very thin transition layer, lying between the deflagration and
the isentropic compression is studied in the Appendix. The order
of magnitude, throughout the plasma, of the self-similar variables
defined in Sec. II, is

Isentropic Compression

\[
\bar{n} = 0(1), \, u = 0(\alpha^{1/3} \varepsilon^{1/6}), \, \Delta \varepsilon = 0(\alpha^{1/3} \varepsilon^{1/6}), \, \theta_j = 0(\alpha^{-1/3} \varepsilon^{1/3})
\]

(55)

Transition Layer

\[
\bar{n} = 0(\varepsilon^{-3/20}), \, u = 0(\alpha^{1/3} \varepsilon^{1/6}), \, \Delta \varepsilon = 0(\alpha^{-7/6} \varepsilon^{13/24}), \, \theta_j = 0(\alpha^{-1/3} \varepsilon^{-2/3})
\]

(57)

Deflagration

\[
\bar{n} = 0(\varepsilon), \, u = 0(\alpha^{1/3} \varepsilon^{-1/3}), \, \Delta \varepsilon = 0(\alpha^{-7/6} \varepsilon^{-7/3}), \, \theta_j = 0(\alpha^{-1/3} \varepsilon^{-2/3})
\]

(58)
Isentropic Expansion

\[ \bar{n} = 0(\epsilon), \ u = 0(\alpha^{1/3} \epsilon^{-1/3}), \ \Delta \xi = 0(\alpha^{1/3} \epsilon^{-1/3}), \ \theta_j = 0(\alpha^{-1/3} \epsilon^{-2/3}). \quad (59) \]

Also there is a thin precursor ahead of the shock, of width \( \Delta \xi = 0(\alpha^{-7/6} \epsilon^{2/3}) \).

We notice the following results: 1) A deflagration exists if, and only if, \( \alpha \gg \epsilon^{-4/3} \), 2) When a deflagration exists, the expansion flow is isentropic (not isothermal), 3) The deflagration layer is thin compared to the expansion flow; however, it will be thinner (thicker) than the compression flow for \( \alpha \gg \epsilon^{-5/3} \) (\( \alpha \ll \epsilon^{-3/5} \)). 4) We completely determined the structure of the deflagration layer; in particular, we proved the Chapman-Jouguet condition and found the value of the density behind the deflagration (not \( n_c \)). 5) There is a density maximum and a temperature minimum in a thin transition layer ahead of the deflagration.

In the analysis, we assumed near-Maxwellian distributions functions; this implies \( \lambda_j \ll \Delta x \) and \( t_j \ll t \), where \( \lambda_j \) and \( t_j \) are the mean free path and collision time for species \( j \). In self-similar variables, the first condition reads

\[ \theta_j^2 / \bar{n} \Delta \xi \ll \alpha^{1/2} (m_j / m_e)^{1/2} \quad ; \quad (60) \]

using Eqs. (56)-(59), the left-hand side is \( 0(\alpha^{1/2}) \) in the deflagration and much less elsewhere, so that (60) is well satisfied. Condition \( t_j \ll t \) is more restrictive for ions, and reads
for the expansion and the deflagration (where temperature is larger and density smaller) condition (61) becomes $(\alpha \varepsilon^{4/3})^{3/2}(m_i/m_e)^{1/2} \gg 1$, which is clearly satisfied.

Quasineutrality has been also assumed throughout the analysis. This condition requires a Debye length small compared with $\Delta x$, and in self-similar variables reads

$$\omega \tau(t/\tau) \gg \left( A_i \varepsilon_\theta / n \right)^{1/2} \left( 10 / \Delta \xi \right),$$

(62)

($\omega$ and $A_i$ being laser frequency and ion mass number). Clearly, charge separation is not self-similar; we shall consider $0.1t < t < \tau$ in discussing (62). This condition is less restrictive for the isentropic regions than for the deflagration layer, for which it becomes

$$\omega \tau \gg 10^2 A_i^{1/2} \left( \alpha \varepsilon^{4/3} \right)^{3/2},$$

for $\omega = 1.8 \times 10^{15}$ sec$^{-1}$ (Nd laser), $\tau = 30$ nsec, $A_i = 2.5$ (DT) and $\alpha \varepsilon^{4/3} = 10^{3/2}$, the condition leads to $10^3 \gg 1$. We notice that according to Eqs. (15), $\alpha \varepsilon^{4/3} \alpha(2_i \tau / \phi_o)^{2/3} \omega^{8/3} A_i^{-1}$ cannot be large (unless $\phi_o$ is very much less than $10^{14}$ W/cm$^2$) except for the largest $\omega$, $A_i^{-1}$ and $\tau$; for the values just taken, $\phi_o = 10^{11}$ W/cm$^2$.

For the transition layer, quasineutrality would require

$$\omega \tau \gg 10^2 A_i^{1/2} \left( \alpha \varepsilon^{4/3} \right)^{3/2} e^{-69/40},$$
and it may thus be easily violated. However, a review of the analysis in the Appendix shows that the only modification in the results given there, is the change $\bar{n} + (\bar{n}_e \bar{Z}_i + \bar{n}_i)/(Z_i + 1)$. Moreover, we stress the point that the structure of the transition layer bears no dependence upon the results outside it.

While we assumed initially a half-space of plasma at zero temperature, this is not essential to the analysis; we could have as well a half-space of solid matter, as long as its initial specific energy could be later neglected. Letting aside the transition layer, the assumption that the perturbed matter is a plasma is most restrictive for the dense, relatively cold compression region; clearly, however, the results would not be modified if we had there a monatomic ideal gas instead of a plasma.

The assumption of constant Coulomb logarithms only affects the deflagration layer (where heat conduction and ion electron energy relaxation was used), but certainly, it can be only considered as an approximation. We notice, however, that in the structure of the layer itself, Coulomb logarithms enter only as a ratio, through the coefficient $a_3$ defined in Sec. II; furthermore, the expressions for such parameters as $a$, $T_0$, and $v_0$ (see Sec. II) depend only weakly on a Coulomb logarithm.

We finally point out that the results of the present analysis should remain qualitatively valid for spherical geometries and other pulse shapes. Furthermore, our results for thin layers (such as deflagration) should in fact remain quantitatively valid.
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APPENDIX

To analyse the deflagration, we assumed $y(\tilde{\eta} = 0) = 0$. Actually, for matching with the isentropic compression solution, we should have $y(\eta = \tilde{\eta}) = 4\tilde{\eta}/3$, that is, $y(\tilde{\eta} = 0) = 4\tilde{\eta}/3(-y_d) = 0(\varepsilon^{1/2})$. Then, for $\tilde{\eta} \to 0$, we obtain

$$z_i = z_e = z_d(45/4)^{2/5}(-\tilde{\eta})^{2/5}, \quad (A1)$$

$$v = B_1B_2/z_e, \quad y = 4\tilde{\eta}/3 + (3y_d/8)z_e/z_d,$$

which are just the deflagration results, except for the shift in $y$.

The analysis of the transition layer around $\tilde{\eta}$ is now similar to that carried out in A. Defining $J = v(4\eta - 3y)$ and using Eqs. (A1), we have

$$J(\eta = \tilde{\eta}) + 9B_1B_2y_d/8z_d = 0(\varepsilon^{1/2}) \quad (A2)$$

On the other hand, the value of $J$ in the isentropic compression is, from Eqs. (26), $(26B_2/5)(\eta - \tilde{\eta})^{10/13}$, which is $0(\varepsilon^{1/2})$ at a point

$$\eta_1 = \tilde{\eta} + b \varepsilon^{13/20}, \quad b = 0(1) \quad .$$

Equations (26) also give $z(\eta = \eta_1) = 0(\varepsilon^{3/20})$. Then, we introduce a stretched variable
to appropriately retain electron heat conduction near \( \eta_1 \); we verify that electron and ion temperatures are equal (and \( v_2 = \text{constant} = B_1 B_2 \)) throughout the layer. The equation for \( J \) is

\[
\frac{dJ}{d\eta} = \frac{\varepsilon^{3/8} B_1 B_2}{\alpha^{3/2} \eta^{3/2}}.
\] (A3)

Since \( z = O(\varepsilon^{3/20}) \), the change in \( J \) across the layer is small compared to \( \varepsilon^{1/2} \); we shall try as solution

\[
J = (26 B_2^2/5)b^{10/13} \varepsilon^{1/2} + J_1, \quad J_1 = O(\varepsilon^{9/40} \alpha^{-3/2}).
\] (A4)

The equation for \( z \) then reads

\[
B_1 B_2 = \frac{13}{3} B_2 b^{10/13} \alpha^{3/2} \varepsilon^{1/8} \frac{dz}{d\eta} = (\frac{3}{4})^{5/2} Z_i \Gamma \frac{\alpha^{3/2}}{\beta Z_i + 1} \frac{\varepsilon^{1/4}}{\eta} \frac{d}{d\eta} \frac{\varepsilon^{5/2}}{\eta} \frac{dz}{d\eta}.
\] (A5)

Once \( z \) is known, \( J_1 \) follows from (A3).

Let us now assume

\[
z = B_1 b^{3/13} \varepsilon^{3/20} + z_1, \quad |z_1| \ll \varepsilon^{3/20}.
\] (A6)

Then, linearizing Eq. (A5), and integrating twice we get

\[
z_1 = a^{-3/2} \varepsilon^{-1/8} \left[ \frac{3B_1 n}{13} \right]^{5/2} b^{5/26} (Z_i + 1)^{7/2} \frac{\varepsilon^{5/2}}{Z_i B^{5/2} - \frac{5}{3}} \right],
\] (A7)
where \( F \) and \( G \) are unknown constants. For \( \eta^* \to \infty \), (A7) gives

\[
z = B_1 b^{3/13} \varepsilon^{3/20} + \frac{3 B_1}{13 b^{10/13}} \varepsilon^{-1/2}(\eta - \eta_1),
\]

(A8)

while from (A3) and (A4) we get

\[
J = (26 B_2/5) b^{10/13} \varepsilon^{1/2} + \frac{4 B_2}{b^{3/13}} \varepsilon^{-3/20}(\eta - \eta_1);
\]

(A9)

Eqs. (A8) and (A9) match to the behaviors of \( z \) and \( J \) in the isentropic compression, given in (26), near \( \eta_1 \).

For \( \eta^* \) negative, Eq. (A7) shows that (A6) ceases to be valid when \(|\eta^*| = O(\ln \varepsilon^{-1})\). Defining then \( z = B_1 b^{3/13} \varepsilon^{3/20} z^* \), dropping \( B_1 B_2 \) in (A5), and integrating twice we arrive at

\[
-\eta^* = \left(\frac{3}{4}\right)^{5/2} \frac{3 B_1^{5/2}}{52 B_2 b^{5/26}} \frac{2}{(Z_1 + 1)^{7/2}} \left[ \frac{2}{3} z^*^{5/2} + \frac{2}{3} z^*^{3/2} + \frac{2}{3} z^*^{1/2} + 1 \right] + \ln \frac{z^*^{1/2} - 1}{z^*^{1/2} + 1};
\]

for \( \eta^* \to -\infty \),

\[
z^* = \frac{4 b^{1/13}}{3 B_1} \left[ \frac{130(Z_1 + 1)^{7/2}}{3 \varepsilon^{3} Z_1} \right] \left( -\eta^* \right)^{2/5}.
\]

Then, matching to Eqs. (A1) and (A2), we find \( b, z_{\text{min}} \) and \( v_{\text{max}} \); they are given in Table I.
REFERENCES AND FOOTNOTES

14 In general, heat flux includes both thermal conduction and a electric current term; this term, obviously, vanishes here.
18 In (A this width was $O(\alpha^{-7/6})$ [erroneously given as $O(\alpha^{-5/6})$].
<table>
<thead>
<tr>
<th>( Z_i )</th>
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<tr>
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<td>0.165</td>
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<td>-1.22</td>
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<td>( \bar{\gamma} )</td>
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<td>0.677</td>
<td>0.869</td>
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<td>( c_i )</td>
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<td>0.166</td>
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<td>0.975</td>
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<td>( \epsilon^{3/20} \nu_{\text{max}} )</td>
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<td>1.36</td>
<td>1.34</td>
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<tr>
<td>( \epsilon^{-13/20} (\eta_{1\text{-}} - \bar{\eta}) )</td>
<td>0.086</td>
<td>0.090</td>
<td>0.097</td>
</tr>
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</table>

TABLE I. Values of normalized density, \( \nu_d \), temperature, \( \zeta_d \), and velocity, \( y_d \), behind the deflagration; shock location parameter, \( \bar{\gamma} \) (Sec. IV), and deflagration scale length, \( c_i \) [Eq. (46)]; minimum temperature, \( \zeta_{\text{min}} \); maximum density, \( \nu_{\text{max}} \), and their location, \( \eta_{1\text{-}} \) (see the Appendix).
LIST OF FIGURES

Fig. 1. Density, $\nu$, velocity, $y$, and ion and electron temperatures, $z$, vs. distance, $\eta$, within the isentropic compression region; the self-similar variables have been normalized with the values behind the shock.

Fig. 2. Integral solution $\mathcal{V}(\mathcal{V})$ of Eq. (38) in the phase-plane $(\mathcal{V},\mathcal{Z})$. The inserts represent the neighborhood of the singular points N (schematic) and P.

Fig. 3. Density, $\hat{\nu} = \nu/\nu_d$, velocity, $\hat{y} = y/(-y_d)$, and ion and electron temperatures $\hat{z} = z/z_d$, vs. distance, $\hat{\eta} = \eta/(-y_d)$, within the isentropic expansion region; $\nu_d = O(\varepsilon)$, $z_d = O(\varepsilon^{-1})$, and $y_d = O(\varepsilon^{-1/2})$ are the values behind the deflagration, given in Sec. V for $Z_i = 1$.

Fig. 4. Integral solution $\mathcal{V}_e(\mathcal{Y})$ of Eq. (55) in the phase-plane $(\mathcal{Y},\mathcal{Z}_e)$ for $Z_i = 1$ ($H = 9/5$). The insert represents schematically the neighborhood of point $(-1,1)$.

Fig. 5. Density, $\hat{\nu} = \nu/\nu_d$, velocity, $\hat{y} = y/(-y_d)$, and electron and ion temperatures, $\hat{z}_j = z_j/z_d$, vs. distance, $\hat{\eta} = (\frac{3/2}{c_1^2}(\mathcal{E}-\mathcal{H})(\pi-\eta))/(-y_d)$, within the deflagration layer, for $Z_i = 1$; $c_1 = O(1)$ is given in Eq. (46).
Fig. 1
Fig. 3
Fig. 4