NON-LOCAL ELECTRON HEAT-FLUX

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ABSTRACT

Electron thermal conduction in a not quite collisional unmagnetized plasma is analysed. The failure of classical results for temperature scale-length up to 100 times larger than thermal mean-free-path for electron scattering, and large ion-charge number Z_i, is discussed. Recent results from a nonlocal model of conduction at large Z_i are reviewed. Closed form expressions for Braginskii's coefficients $\alpha_0, \beta_0, \gamma_0$ for Z_i=0(1) are derived. An extension of the nonlocal model for Z_i=0(1) is discussed.

1- INTRODUCTION

Heat conduction is a crucial process for the direct-drive approach to laser fusion. On the one hand, it takes energy from the absorbing region (where density is at, or below, critical n_c) to the surface of the target that is being ablated. On the other hand, it smooths implosion asymmetries that arise from nonuniformities in either target finish or irradiation.[1]

The temperature scale-length H for the plasma lying between the critical and ablation surfaces is generally larger than the mean free path λ_T for scattering of thermal electrons. Frequently it is substantially larger: If inward conduction is mostly balanced by outward convection of energy one has

$$|\bar{u}_e . \nabla T_e| \sim \lambda_T (T_e/m_e)^{1/2} |\nabla^2 T_e|$$

where m_e, T_e and $\bar{u}_e$ are mass, temperature, and mean directed velocity of the electrons. The characteristic velocity for the quasineutral plasma expansion into vacuum is the ion-accoustic speed,

$$|\bar{u}_e| \sim |\bar{u}_i| \sim (Z_i T_e/m_i)^{1/2};$$

subscript i refers to ions, Z_i being their charge number. We then have[2]

$$H/\lambda_T \sim (m_i/Z_i m_e)^{1/2} \sim 60 \ .$$

One might thus expect classical transport results to generally hold in the plasma blowing off a laser target. A large body of experimental data has proved, however, that this is not the case.[3] Explaining the failure of classical transport at H/λ_T as high as 10^2, is clearly a problem of both practical and theoretical interest. Here we discuss some recent developments on this problem.

First, we detail the simple calculations required for the collision-dominated limit at large Z_i. We follow Braginskii's approach[4] giving electron heat flux $\bar{q}_e$ and mean ion-force on electrons $\bar{R}$ as linear expressions on both relative velocity $\bar{u} \equiv \bar{u}_e - \bar{u}_i$ and electron temperature gradient ∇T_e; $\bar{q}_e = \bar{q}_u + \bar{q}_T$, $\bar{R} = \bar{R}_u + \bar{R}_T$. Magnetic field effects are assumed small. Next, we discuss a recent (nonlocal) model for not quite collisional conditions, and $Z_i \gg 1$. Then, we use the approach of this model to derive new, approximate, analytical results for the classical limit with Z_i of order unity. Finally, we sketch recent developments on the nonlocal model at $Z_i = O(1)$.

2. CLASSICAL RESULTS FOR LARGE ION CHARGE-NUMBER

Consider the Fokker-Plank equation for the electron distribution function f_e,[4,5)]

$$\frac{\partial f_e}{\partial t} + \bar{v}.\nabla f_e - \frac{e}{m_e}\bar{E}.\frac{\partial f_e}{\partial \bar{v}} = \sum_s C_{es} . \quad (1)$$

We write the collision term C_{es} (s = e, i) in a convenient form

$$C_{es} = \frac{2\pi\Lambda e^2 e_s^2}{m_e^2}\frac{\partial}{\partial \bar{v}}\left(\frac{\partial f_e}{\partial \bar{v}} - \frac{m_e}{m_s} f_e \frac{\partial}{\partial \bar{v}}\right) : \int \bar{\bar{U}}\, f_s(\bar{v}')\, d\bar{v}', \quad (2)$$

entirely equivalent to the Landau expression;[4)] here

$$\bar{\bar{U}}\,(\bar{v}-\bar{v}') \equiv \frac{|\bar{v}-\bar{v}'|^2\, I - (\bar{v}-\bar{v}')(\bar{v}-\bar{v}')}{|\bar{v}-\bar{v}'|^3} ,$$

I is the unit tensor, Λ a Coulomb logarithm, $\bar{E}$ the electric field, and e_s is $-e$ for electrons and $+Z_i e$ for ions. Using the electron frame ($\bar{w} \equiv \bar{v} - \bar{u}_e$), the left-hand-side of (1) reads

$$\left(\frac{\partial}{\partial t} + \bar{u}_e.\nabla\right) f_e - \frac{\partial f_e}{\partial \bar{w}}.\left(\frac{\partial}{\partial t} + \bar{u}_e.\nabla + \bar{w}.\nabla\right)\bar{u}_e + \bar{w}.\nabla f_e - \frac{e}{m_e}\bar{E}.\frac{\partial f_e}{\partial \bar{w}} .$$

With $u_e \ll (T_e/m_e)^{1/2}$ only the last two terms remain and Eq. (1) becomes

$$\bar{w}.\nabla f_e - \frac{e}{m_e}\bar{E}.\frac{\partial f_e}{\partial \bar{w}} = C_{ei}(\bar{w}) + C_{ee}(\bar{w}) . \quad (3)$$

In the classical limit both C_{ei} and C_{ee} are dominant against the left-hand-side of (3), and therefore

$$f_e(\bar{w}) = f_M(w)\ [1+\varphi(\bar{w})], \qquad \varphi \text{ small}, \tag{4}$$

where f_M is the local maxwellian, $f_M \equiv n\ (m_e/2\pi\ T_e)^{3/2} \exp\ (-m_e w^2/2T_e)$.

We now assume Z_i large so that C_{ee} may be neglected against C_{ei}. Next we expand $\bar{\bar{U}}\ (\bar{w}-\bar{w}')$ in powers of w'/w, dropping terms of order m_e/m_i or smaller, and using (4) and $\int f_i(\bar{w})d\bar{w} = n/Z_i$, $\int f_i(\bar{w})\bar{w}d\bar{w} = -n\bar{u}/Z_i$ to get $C_{ei} \simeq C'_{ei}(f_M\varphi)-m_e\bar{u}.\bar{w}f_M/T_e\tau_{ei}$ where

$$C'_{ei}(f_e) \equiv \frac{\partial}{\partial\bar{w}}\left(\frac{\partial f_e}{\partial\bar{w}} : \frac{w^2\bar{\bar{I}}-\bar{w}\bar{w}}{2\tau_{ei}}\right), \qquad \tau_{ei} \equiv \frac{m_e^2\ w^3}{4\pi\Lambda e^4\ Z_i n}.$$

On the left of (3) we may set $f_e \simeq f_M$ and use the equation for electron momentum

$$0 = -\nabla(nT_e)\ -en\bar{E} + \bar{R}, \tag{5}$$

where the momentum itself is neglected. We finally arrive at

$$f_M\bar{w}.\bar{c} = C'_{ei}\ (f_M\varphi), \tag{6}$$

$$\bar{c}(w) \equiv \frac{\bar{R}}{nT_e} + \left(\frac{m_e w^2}{2T_e} - \frac{5}{2}\right)\nabla\ln T_e + \frac{m_e\bar{u}}{T_e\tau_{ei}(w)}, \tag{7}$$

c being the inverse of a length.

Trying a solution

$$\varphi = \bar{w}.\bar{g}(w)/f_M \tag{8}$$

in Eq.(6) one immediately finds

$$C'_{ei} = -\ \bar{w}.\bar{g}/\tau_{ei}\ , \tag{9}$$

$$\bar{g} = -\ \tau_{ei}f_M\ \bar{c}\ . \tag{10}$$

Introducing $f_e = f_M(1-\tau_{ei}\bar{w}.\bar{c})$ into

$$0 = \int f_e\bar{w}\ d\ \bar{w}, \qquad \bar{q}_e = \int f_e\ \frac{1}{2}\ m_e w^2\bar{w}d\bar{w}, \tag{11}$$

we obtain

$$\int_0^\infty \tilde{\varepsilon}^3 e^{-\tilde{\varepsilon}} \bar{c}(\tilde{\varepsilon})\, d\tilde{\varepsilon} = 0\ , \qquad \bar{q}_e = -\frac{16nT_e^2\tau_e}{9\pi m_e}\int_0 \tilde{\varepsilon}^4 e^{-\tilde{\varepsilon}}\, \bar{c}(\tilde{\varepsilon})d\tilde{\varepsilon}=0,$$

where $\tau_e \equiv \frac{3\pi^{1/2}}{4}\tau_{ei}[(2T/m_e)^{1/2}]$ and $\tilde{\varepsilon} \equiv m_e w^2/2T_e$. We thus finally get the classsical results for $Z_i \to \infty$,[4)]

$$\bar{R} = -\beta_0 n\nabla T_e - \alpha_0 m_e n\bar{u}/\tau_e, \qquad (\beta_0 = \frac{3}{2},\ \alpha_0 = \frac{3\pi}{32}), \tag{12a}$$

$$\bar{q}_e = \beta_0 nT_e\bar{u} - \gamma_0 nT_e\tau_e\nabla T_e/m_e, \qquad (\gamma_0 = \frac{128}{3\pi}). \tag{12b}$$

Consider the range of validity of these results for $\bar{u}=0$; then

$$q_e = q_T \sim \frac{\tau_e(T_e/m_e)^{1/2}}{H}\, nT_e\left(\frac{T_e}{m_e}\right)^{1/2},$$

$$\varphi = -\tau_{ei}\bar{w}.\bar{c} \sim \lambda_{ei}(w)/H$$

where $H^{-1} \equiv |\nabla \ln T_e|$ and $\lambda_{ei} \equiv 2w\tau_{ei} \propto w^4$, is the usual mean free path for ion-electron collisions, its thermal value being λ_T. Clearly λ_T/H should be less than, or about, unity since, otherwise, q_T would exceed the so called free-streaming value $\sim nT_e(T_e/m_e)^{1/2}$, i. e. the maximum heat flux that can be carried by an electron population. Actually at $\lambda_T/H \sim 1$, we have φ(thermal) $\sim$ 1 too, and the preceding analysis should break down.

To avoid unphysically large heat fluxes in steep thermal fronts ($H \lesssim \lambda_T$), numerical codes simulating experiments have been using a crude recipe: they take q_T as either the minimum or the harmonic mean of $|\gamma_0 nT_e\tau_e\nabla T_e/m_e|$ and $fnT_e^{3/2}/m_e^{1/2}$. Clearly from the above discussion, the "flux limiter" f should be about unity, so that if $H/\lambda_T < 1$, q_T is reduced below the classical value; in a sense φ is reduced. Since about 1975, however, comparison between data from experiments, and numerical results from flux-limited codes, has indicated that the appropiate value for f should be small. This suggests that heat transport is severely inhibited, classical theory already failing at large H/λ_T. Flux limiters ranging from 0.01 to 0.2 have been proposed.[1]

3. NON-LOCAL HEAT FLUX AT LARGE ION-CHARGE NUMBER

A general failure of the classical model of Sec. 2, under conditions for which it was supposed to apply, has been traced back to the strong energy-dependence of plasma mean-free-paths.[6] Note that, for $\bar{u}=0$,

$$\varphi = -(\tilde{\varepsilon}-4)\tau_{ei}\bar{w}.\nabla\ln T_e$$

$$q_T \propto \int_0^\infty \tilde{\varepsilon}^4(\tilde{\varepsilon}-4)\, e^{-\tilde{\varepsilon}}\, d\tilde{\varepsilon}.$$

The integral above presents a sharp maximum at $\tilde{\varepsilon}=\tilde{\varepsilon}^*\simeq 6.5$. This means that electrons contributing most to the heat-flux lie in the tail of the distribution function f_e. Since $w\tau_{ei} \propto \tilde{\varepsilon}^2$, one could then have φ small at thermal energies ($\tilde{\varepsilon}\sim 1$), but $\varphi\sim 1$ at the energies of interest in transport ($\tilde{\varepsilon}\sim\tilde{\varepsilon}^*$).

Albritton et al. have given a self-consistent model for Lorentzian ($Z_i \gg 1$) plasmas based on that fact.[7,8] For Z_i large, f_e will fail to be Maxwellian while still being isotropic. For main-body (thermal) electrons Eq.(4) will hold, but for $\tilde{\varepsilon}^*$-electrons the equation should be

$$f_e = f_0(w)[1+\varphi(\bar{w})] \qquad \varphi \text{ small} \tag{4'}$$

with f_0 unknown. To dominant terms, Eq.(3), for $\bar{u}=0$, will read

$$\bar{w}.\nabla f_0 - \frac{e}{m_e}\bar{E}\,.\,\frac{\partial f_0}{\partial \bar{w}} = C'_{ei}(f_0\varphi). \tag{6'}$$

Following (8) we try $\varphi \equiv \bar{w}.\bar{g}(w)/f_0$ and obtain both (9) and

$$\bar{g} = -\,\tau_{ei}\nabla f_0(\bar{r},\varepsilon). \tag{10'}$$

For convenience we changed variables $\bar{r},w$ into $\bar{r},\varepsilon\equiv\frac{1}{2}m_e w^2 - e\psi$ ($\bar{E}\equiv-\nabla\psi$); the local Maxwellian is now $f_M \equiv n(m_e/2\pi T_e)^{3/2} \exp\,[-(\varepsilon+e\psi)/T_e]$. If $f_0 \to f_M$, one uses (5) and (7) with $\bar{u}=0$ and recovers Eq.(10).

The isotropic distribution f_0 must be determined prior to calculating $\bar{q}_e$ and $\bar{R}$. The angle-average of Eq. (3) will read

$$-\frac{1}{3}w\nabla.(\tau_{ei}w\nabla f_0) = C_{ee}(f_0, f_0); \qquad (13)$$

since $e\psi/T_e \sim 1$ [see Eq. (5)] we dropped a term $|2\tau_{ei}e\bar{E}.\nabla f_0/3m_e|$ small by a factor $2e\psi/m_e w^2$ or $1/\tilde{\varepsilon}^* \simeq 0.16$ from its left. To this same approximation, we write $\frac{1}{2}m_e w^2 \sim \varepsilon$, in the following, for power laws but not for an exponential like the Maxwellian.

The self-collision term in (2) may be written as

$$C_{ee} = \frac{2\pi\Lambda e^4}{m_e^2}\left[\frac{\partial^2 f_e}{\partial\bar{w}\partial\bar{w}} : \frac{\partial^2}{\partial\bar{w}\partial\bar{w}}\int |\bar{w}-\bar{w}'|\ f_e(\bar{w}')d\bar{w}' + 8\pi f_e^2\right]. \qquad (14)$$

For tail energies the last term is negligible. Next, for use in Eq. (13), we write $f_e = f_0$ in (14). Actually, since thermal electrons contribute most to the integral we set $f_0 = f_M$ inside it. Expanding $|\bar{w}-\bar{w}'|$ in powers of w'/w and neglecting 4^{th} and higher powers, we obtain a two-term expression, linear in f_0,

$$C_{ee} \simeq \frac{m_e w^2}{Z_i \tau_{ei}}\frac{\partial}{\partial\varepsilon}\left(f_0 + T_e\frac{\partial f_0}{\partial\varepsilon}\right). \qquad (15)$$

If f_0 followed a power law at tail energies $\varepsilon \sim \tilde{\varepsilon}^* T_e$, $T_e\partial\ln f_0/\partial\varepsilon$ would be small and the last term in (15) should be dropped for consistency. On the contrary, if we had $f_0 \simeq f_M$, $T_e\partial\ln f_0/\partial\varepsilon$ would be -1, and C_{ee} would vanish: for gradients weak enough, Eqs. (13) and (15) clearly make f_0 Maxwellian. At this point one makes an ansatz crucial to the analysis; we assume[7])

$$|T_e\partial\ln(f_0 - f_M)/\partial\varepsilon| \ll 1. \qquad (16)$$

Expression (15) for C_{ee} then becomes

$$C_{ee} \simeq \frac{m_e w^2}{Z_i \tau_{ei}}\left(\frac{\partial f_0}{\partial\varepsilon} - \frac{\partial f_M}{\partial\varepsilon}\right). \qquad (17)$$

Since we are allowing for a substantial departure from a Maxwellian,

$$|f_0 - f_M| \sim f_M, \qquad (18)$$

condition (16) may be rewritten as

$$\left| \frac{\partial(f_0 - f_M)}{\partial \varepsilon} \right| \ll \left| \frac{\partial f_M}{\partial \varepsilon} \right| . \tag{19}$$

Although conditions (18) and (19) can hold simultaneously only within a narrow energy range, $\Delta\varepsilon \sim T_e$, this range may include those electrons carrying most of the flux.

Using (17) in (13), taking gradients along the x-axis, and defining ξ (the square of an energy) by $d\xi/dx \equiv (3/8Z_i)^{1/2}\ m_e^2\ w^3/\tau_{ei}$ one obtains a parabolic equation for f_0,

$$\frac{\partial f_0}{\partial(-\varepsilon^4/4)} - \frac{\partial^2 f_0}{\partial \xi^2} = -\frac{1}{\varepsilon^3}\ \frac{\partial f_M}{\partial \varepsilon} \tag{20}$$

with $-\varepsilon^4/4$ as "time" variable and a "time" dependent source-term. Since we must have $f_0 \to 0$ as $-\varepsilon^4/4 \to -\infty$, the solution to (20) is[7]

$$f_0(\xi,\varepsilon) = \int \frac{d\xi'}{\pi^{1/2}} \int_{\varepsilon}^{\infty} \frac{f_M(\xi',\varepsilon')d\varepsilon'}{T_e'(\varepsilon'^4 - \varepsilon^4)^{1/2}} \exp\left[\frac{-(\xi-\xi)^2}{\varepsilon'^4 - \varepsilon^4} \right] \tag{21}$$

with $T_e' \equiv T_e(\xi')$. Note that the non-Maxwellian population for suprathermal energy ε, at "point" ξ, arises from a Maxwellian source of electrons at $\varepsilon' > \varepsilon$, which lost energy while making a random walk from a neighbor "point" ξ'!

Solution (21) for f_0 satisfies (16). We now introduce a second ansatz, verified a posteriori,

$$H d\xi/dx \ll (\varepsilon^5/T_e)^{1/2}, \qquad \text{at} \quad \varepsilon \sim \bar{\varepsilon}^* T_e ;$$

one can then show that, in general, only values $\varepsilon' \approx \varepsilon$ contribute to f_0, which simplifies to[8]

$$f_0(\xi,\varepsilon) = \int \frac{f_M(\xi',\varepsilon)d\xi'}{2(\varepsilon^3 T'_e)^{1/2}} \exp\left[\frac{-|\xi-\xi')}{(\varepsilon^3 T')^{1/2}}\right] \quad (22)$$

Note that the characteristic scale-length for the model is $H \sim (\varepsilon^3 T_e)^{1/2}\, dx/d\xi$ at $\varepsilon \sim \tilde{\varepsilon}^* T_e$, or

$$H \sim \lambda_T Z_i^{1/2} (\tilde{\varepsilon}^*)^{3/2} , \quad (23)$$

which numerically agrees with the length $\lambda_T (m_i/Z_i m_e)^{1/2}$ shown in the Introduction to characterize the overdense region of a laser target. For such H , (22) gives $f_0 \simeq f_M$ at thermal energies.[8)]

If H is well above the value in (23) one finds $f_0 \simeq f_M$ at energies $\varepsilon \sim \tilde{\varepsilon}^* T_e$, recovering the classical result for q_T. On the other hand at H well below (23), the model will ultimately fail: If f_M changes in a distance $\Delta\xi \ll (\varepsilon^3 T_e)^{1/2}$, f_0 will lag behind it and condition (19) will not hold. Prasad and Kershaw have illustrated thisfailure for some extreme profiles.[9)] Ramirez has shown that the nonlocal model extends the validity of classical results for laser fusion by over one order of magnitude in laser intensity.[10)]

Using $f_e = (1 - \tau_{ei} \bar{w}.\nabla) f_0(\bar{r},\varepsilon)$, with f_0 given by (22), in (11), one obtains coupled equations

$$\left\{0, q_T\right\} = \int \frac{\{1, T'_e\} n' dx'}{4\pi (3 m_e Z_i T'_e)^{1/2}} \left[\left\{I^*, K^*\right\} \frac{dT'_e}{dx'} + \left\{J^*, L^*\right\} eE'_{ne} \right] \quad (24)$$

where the kernels or propagators are functions of $\theta \equiv |\xi-\xi'|/T'^2_e$:[8)]

$$J^* \equiv 8\pi^{1/2} \int_0^\infty s^{3/2} \exp\left(-s - \frac{\theta}{s^{3/2}}\right) ds ,$$

$$I^* = 3J^* - 2\theta dJ^*/d\theta, \qquad L^* = \frac{1}{4}(3I^* + J^*), \qquad K^* = 4L^* - 2\theta dL^*/d\theta.$$

From (24) and given profiles one may obtain both q_T and the auxiliary field E_{nl} (also linear in the temperature gradient); this field has been defined by $eE_{nl} \equiv eE + T_e d\ln n/dx - \frac{5}{2}\, dT_e/dx$.

The formalism, to be used in conjunction with the conservation equations, can be greatly, and approximately, simplified. At large H , only the complete integrals of the kernels, e.g. $\int_0^\infty I^*(\theta)d\theta$, are involved in the results. For H small, only the values at $\theta=0$ affect the results. In addition, the kernels satisfy the property[8)]

$$\frac{\int_0^\infty K^* d\theta}{\int_0^\infty L^* d\theta} - \frac{\int_0^\infty I^* d\theta}{\int_0^\infty J^* d\theta} = \frac{K^*(0)}{L^*(0)} - \frac{I^*(0)}{J^*(0)} = 1 \ .$$

One then finds that a simple formula

$$q_T = \int \frac{-T'_e n' dx'}{4\pi(3m_e Z_i T'_e)^{1/2}} L^* \frac{dT'_e}{dx'} \tag{25}$$

agrees exactly with (24) for both large and small H; in particular, it exactly recovers the result (12b) at large H. Equation (25) is a convenient approximation for arbitrary H.[8)]

Using (5) in the definition of E_{nl} one gets

$$R_T = ne\, E_{nl} + \frac{7}{2}\, ndT_e/dx \ .$$

R_T is found to vary from its classical value $-\frac{3}{2}ndT_e/dx$ at large H to $+\frac{1}{2}ndT_e/dx$ at small H. To determine q_u and R_u we take $\bar{u}\neq 0$ The kinetic equation is now best solved in the ion frame. Since only the isotropic part of C_{ee} is needed, solution (22) is still valid with $\bar{w}\to\bar{w}^* \equiv \bar{v}-\bar{u}_i=\bar{w}+\bar{u}$. In (24) one just makes the change

$$\left\{0, q_T\right\} \to \left\{ nu,\ q+ \frac{5}{2}\, nTu \right\} ,$$

and use $eE_{nl} = R/n - \frac{7}{2}\, dT_e/dx$, so that q and R are then determined in terms of relative velocity and temperature gradient. At short H, Onsagers's principle $(q_u d\ln T_e/dx+R_T u=0)$ is not satisfied.[8)]

4. CLASSICAL RESULTS FOR ION-CHARGE NUMBER OF ORDER UNITY

For Z_i of order unity, electron-electron collisions contribute to the scattering. Results for $\bar{q}_e$ and $\bar{R}$ in the classical limit are still given by Eqs. (12a,b), but $\alpha_0, \beta_0, \gamma_0$ are now functions of Z_i that can not be obtained in closed form; some numerical approach, usually based on a broken Laguerre expansion, is required. Here, using one basic idea of the nonlocal model of Sec.3 (the departure of a Maxwellian, $f_e - f_M$, is needed only at energies well above thermal), we derive approximate, explicit values for $\alpha_0(Z_i), \beta_0(Z_i)$ and $\gamma_0(Z_i)$.

From Eqs. (3) and (4) we now get

$$f_M \bar{w}.\bar{c} = C'_{ei}(f_M \varphi) + C_{ee}. \qquad (6")$$

In expression (14) for C_{ee} one may still neglect the last term. Expanding $|\bar{w}-\bar{w}'|$ inside the integral in powers of w'/w, neglecting 3[rd] and higher powers, and using $\int f_e \bar{w} d\bar{w}=0$, only f_M in Eq. (4) enters the integral of (14). Trying solution (8), C_{ee} takes the form

$$C_{ee} = \frac{-\bar{w}}{Z_i \tau_{ei}} \cdot \left[\bar{g}\left(1-\frac{1}{2\tilde{\varepsilon}}\right) - \frac{\partial}{\partial w}\left(w\bar{g} + \frac{\partial}{\partial \tilde{\varepsilon}} w\bar{g}\right)\right] \qquad (26)$$

while Eq.(6") gives

$$f_M \bar{c} = -\frac{Z_i+1}{Z_i \tau_{ei}} \bar{g} - \frac{1}{Z_i \tau_{ei}} \frac{\partial}{\partial w}\left(w\bar{g} + \frac{\partial}{\partial \tilde{\varepsilon}} w\bar{g}\right) \qquad (27)$$

where we dropped a term $1/2\tilde{\varepsilon}(1+Z_i)$ agains unity.

Defining

$$h \equiv \frac{3\pi^2 T_e w}{Z_i m_e n \tau_e} g \quad , \qquad N(Z_i) \equiv \frac{1+Z_i}{2}$$

the equation for h becomes

$$\frac{d^2h}{d\tilde{\varepsilon}^2} + \frac{dh}{d\tilde{\varepsilon}} - \frac{N}{\tilde{\varepsilon}} h = \tilde{\varepsilon}\ e^{-\tilde{\varepsilon}} c(\tilde{\varepsilon})\ . \qquad (28)$$

The homogeneous part of (28) is the equation for an associated Laguerre polynomial, $L_N^{(-1)}(-\tilde{\varepsilon})$, which we just write $L_N(\tilde{\varepsilon})$

$$L_N(\tilde{\varepsilon}) \equiv \frac{\tilde{\varepsilon}}{N!} e^{-\tilde{\varepsilon}} \frac{d^N}{d\tilde{\varepsilon}^N} \left(e^{\tilde{\varepsilon}}\ \tilde{\varepsilon}^{N-1} \right),$$

normalized so that $L_N/\tilde{\varepsilon} \to 1$ as $\tilde{\varepsilon} \to 0$ ($L_1 \equiv \tilde{\varepsilon}, L_2 \equiv \tilde{\varepsilon} + \tilde{\varepsilon}^2/2, \ldots$). A convenient second solution of the homogeneous equation is $L_N \int_0^\infty e^{-\tilde{\varepsilon}'} d\tilde{\varepsilon}'/L_N^2(\tilde{\varepsilon}')$.

The full solution of (28) with boundary condition $h \to 0$ as $\tilde{\varepsilon} \to \infty$ is

$$h = L_N \int_{\tilde{\varepsilon}}^{\infty} \frac{e^{-\tilde{\varepsilon}'} d\tilde{\varepsilon}'}{L_N^2(\tilde{\varepsilon}')} \left[A - \int_0^{\tilde{\varepsilon}'} \tilde{\varepsilon}''\ L_N(\tilde{\varepsilon}'')\ c(\tilde{\varepsilon}'')\ d\tilde{\varepsilon}'' \right].$$

The constant A is determined by requiring that the solution h makes our simplified form for C_{ee} to satisfy $\int C_{ee}\ m_e \bar{w}\ d\ \bar{w} = 0$, as it should. Note that our approximation for the self-colision term had lost this as a general property (no such condition was needed in the nonlocal model at large Z_i because the effects of the non-isotropic part of C_{ee} could be neglected in the analysis).

Using h now to write f_e in (11), with $\bar{c}$, $\bar{R}$ and $\bar{q}_e$ as given in Eqs. (7) and (12), we get 4 equations for the three coefficients $\alpha_0, \beta_0, \gamma_0$: Our approximation violates Onsager's principle, because we obtain two different values for β_0. In order to enforce the principle we just use some mean value for β_0. To chek the accuracy of the entire procedure, note that errors should be larger at low Z_i, since the relative effect of self-collisions decreases as Z_i increases. For $Z_i = 1$ ($N = 1$), which is also the simplest case for carring out a full calculation, we find

$\frac{3\pi}{4}\gamma_0(1) = 10 - 4\beta_0(1)$, $\quad\frac{3\pi}{4}\beta_0(1) = 4\alpha_0(1) - \frac{15\pi}{64}$,

$\alpha_0(1) = \frac{15\pi}{154} + \frac{2}{11}$, $\quad\beta_0(1) = \frac{185}{154} - \frac{15}{88}$,

or

$\alpha_0(1) \simeq 0.49$, (exact numerical value $\simeq 0.51$);

$\beta_0(1) \simeq 0.52$ and 1.03, (exact value $\simeq 0.71$),

harmonic and geometric means $\simeq 0.77$ and 0.73;

$\gamma_0(1) \simeq 2.93$ and 3.01 (exact value $\simeq 3.16$).

5. NON-LOCAL MODEL FOR ION-CHARGE NUMBER OF ORDER UNITY

An extension of the non-local model of Sec. 3 to include self-scattering faces two difficulties. First when $Z_i = O(1)$, there is no large parameter allowing for an asymptotic expansion, with f_e not nearly Maxwellian and yet near-isotropic; this had made possible to handle the kinetic equation by first taking its dominant terms, and then its angle-average. To the order considered, however, that approach is equivalent to expanding f_e in Legendre polynomials, P_n, and neglecting terms involving P_2 against P_0-terms. There is substantial evidence suggesting that this last approximation has a range of validity reaching down to values $Z_i = O(1)$;[11] in a sense, this indicates that f_e becomes isotropic sensibly faster than Maxwellian.

Thus, the non-isotropic part of the kinetic equation for $f_e = f_0(1+\varphi)$ will read

$$\bar{w}.\nabla f_0 = C'_{ei}(f_0\varphi) + C_{ee} \qquad (6'')$$

where, on the left, we already changed to variables $\bar{r}$, $\varepsilon \equiv \frac{1}{2}m_e w^2 - e\psi$. Making again $\varphi = \bar{w}.\bar{g}(w)/f_0$ we have C_{ee} as given by (26). Instead of Eq. (27) we now get

$$\frac{\partial f_0}{\partial x} = -\frac{Z_i+1}{Z_i\tau_{ei}} + \frac{m_e w}{Z_i\tau_{ei}}\frac{\partial}{\partial\varepsilon}\left(wg + T_e\frac{\partial}{\partial\varepsilon}wg\right) . \qquad (29)$$

The second difficulty arises from the last term above. On the one hand, if retained, it is hard to proceed and get closed-form results. On the other hand, the evidence from Sec. 4 is against dropping it: If we had dropped the equivalent term in Eq. (27), assuming $|\partial \ln wg/\partial\tilde{\varepsilon}| \ll 1$ [in a way similar to ansatz (16)], the resulting solution would had been found not to verify the assumption; moreover, (28) would then be a first order equation, making impossible for the solution to satisfy $\int C_{ee} m_e \bar{w} d\bar{w} = 0$.

Recently Minotti and Ferro-Fontan did drop that last term in (29).[12] Using the isotropic part of the kinetic equation

$$\frac{w}{3}\frac{\partial}{\partial x} wg = C_{ee}(f_0, f_0)$$

to eliminate wg, the equation for f_0 becomes

$$\frac{\partial f_0}{\partial \varepsilon} + \frac{Z_i}{1+Z_i}\varepsilon^3 \frac{\partial^2 f_0}{\partial \xi^2} = \frac{\partial f_M}{\partial \varepsilon} - \frac{2\varepsilon^2}{1+Z_i}\frac{\partial}{\partial \varepsilon}\left[\frac{1}{\varepsilon}\frac{\partial}{\partial \varepsilon}(f_0 - f_M)\right] \qquad (30)$$

which recovers (20) as $Z_i \to \infty$. Equation (29) is parabolic, however. Introducing

$$F \equiv wg/\varepsilon^N \quad , \qquad \zeta \equiv \xi\,(2/Z_i)^{1/2}$$

with ξ and N as defined in Secs. 3, 4, the equation for F, from which the heat flux is directly determined, reads

$$\frac{\partial^2 F}{\partial \zeta^2} - \frac{1}{\varepsilon^{N+1}}\frac{\partial}{\partial \varepsilon}\left(\varepsilon^{N-1}\frac{\partial F}{\partial \varepsilon}\right) = \frac{-3^{1/2}}{\varepsilon^{N+1}}\frac{\partial^2 f_M}{\partial \varepsilon \partial \zeta} . \qquad (31)$$

The solution for $F(\zeta, \varepsilon)$ involves Hankel functions of order $|N-2|/4$ and large argument. Using asymptotic expansions for fixed order $[Z_i = O(1)]$ Minotti and Ferro-Fontan found very good agreement with detailed experimental data for which $Z_i = 1$. They could not, however, recover the large Z_i formulae.[12] Recently, it has been shown that complete agreement and generality can be obtained by using the asymptotic expansions of Hankel functions for large argument and order.[13]

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