SOME MINOR PROBLEMS WITH B.E.M.

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As it is well known B.E.M. works efficiently in the treatment of a broad class of potential and elasticity problems. In this paper we present the results of several runs established with linear elements in plane potential theory and treating the importance of singularities and the pattern and size of elements used in the boundary discretization.

INTRODUCTION

Boundary elements are established through the use of partial solutions satisfying the field equations in the definition domain of the problem.

The general theory can be studied elsewhere (ref.1, 2,4) and here we shall remind only the principal facts as a means of establishing the symbols used throughout the text.

We shall concentrate in potential theory but the conclusions are equally valid for elasticity problems.

The possibility of solution can be founded on the three following items:

a) The field equation is elliptic

b) There is a reciprocity law (Green's formula)

c) There is a fundamental solution.

Using these facts and in a plane problem one can obtain a relation on the boundary domain

$$c_i \phi_i + \int_{\partial D} \phi \frac{\partial \psi}{\partial n} da = \int_{\partial D} \psi \frac{d\phi}{dn} ds \qquad ..(1)$$

where ϕ is the unknown function and ψ the fundamental solution.

If

$$q = \frac{d\phi}{dn} \quad ..(2)$$

$$c_i \phi_i + \int_{\partial D} \phi \frac{d\psi}{dn} da = \int_{\partial D} \psi q \, da \quad ..(3)$$

In the linear hypothesis, the contour is discretized with straight segments in which the function evolution is imagined as

$$\phi = \frac{1}{2}\left[(1-\xi) \; ; (\xi+1)\right] \begin{Bmatrix} \phi_i \\ \phi_{i+1} \end{Bmatrix}$$

$$..(4)$$

$$q = \frac{1}{2}\left[(1-\xi) \; ; (\xi+1)\right] \begin{Bmatrix} q_i \\ q_{i+1} \end{Bmatrix} ; -1 \leq \xi \leq 1$$

i.e : suffering a linear variation along the element betwen the nodal values (ϕ_i, ϕ_{i+1}) and (q_i , q_{i+1}).

Substitution of (4) into (3) leads to a system

$$\underset{(n\times n)}{\underline{G}} \; \underset{(n\times 1)}{\underline{\phi}} = \underset{(n\times 2n)}{\underline{H}} \; \underset{(2n\times 1)}{\underline{q}} \quad ..(5($$

in which the size of $\underline{H}$ and q reflects the fact that in every node there are three variables : the potential and the two fluxes before and after the node.

A recognition of the problem type (Dirichlet, Neumann, mixed, etc) allows the reduction of (5) to a system

$$\underset{(n\times n)}{\underline{K}} \; \underset{(n\times 1)}{\underline{X}} = \underset{(n\times 1)}{\underline{F}} \quad ..(6)$$

in which $\underline{X}$ contains the unknowns and $\underline{F}$ the weighted data of the problem.

A computer program of this procedure has been presented elsewhere.(ref. 3).

THE SIZE OF DISCRETIZATION

The possibility of using isoparametric representation opens the question of reduction of variables. In fact a better description of the contour and field variable provides better results.

It is worth noting, however, that an excessive reduction in the number of elements can produce relatively bad results inside the domain, in spite of good values in the boundary. One striking example is the simple case of thermal linear evolution shown in fig. 1.

Good results are obtained in the boundary with only four elements, but interior results suffer from the minor errors involved in the numerical values assigned to the corners.

This problem is so simple that a subsequent subdivision leads immediatly to the correct answer.

The same effects appear always, and care must be exercised in the interpretation of results. In this way it is also useful to study the evolution of values in the next paragraph examples though it is fundamentally focused to another class of problems.

THE PRESENCE OF SINGULARITIES

Singular values can affect the solution hardily. The pattern of discretization is very important, and it can be more effective to act on it than on the mesh size. The situation is very similar to that familiar to the F.E.M. users, and the classical rules of finer mesh near singular point and the regular gradation of its size, can be equally mantained.

We have run two examples previously presented by JASWON & SYMM (ref. 4).

The first one can be well represented by only 8 elements as can be seen in fig. 2, but interior points present the same feature just noted in paragraph 2. It is very interesting to observe the progressive amelioration of results with the refinement of the mesh, and more interesting, the sudden refinement introduced with intensive discretization near the singularity.

This effect is specially dramatic in the second example in which an intensive mesh produces better results than a regular one with far more elements. Several graphs on the absolute error are presented in which the amelioration of results is shown (Fig.3).

A striking fact is also the comparison with JASWON solutions obtained through constant elements and with a different approach.

It can be seen that JASWON's results and ours bound the exact solution from different sides. We are studying the generalizations of this result because it could be a useful tool for error bounding.

THE NEUMANN PROBLEM

A pure Neumann problem can disturb the results of the inexperienced analyst. The obvious solutions can be easily implemented using a very well known tool in matrix structural analysis i.e : zeroing row and column corresponding to a chosen value of Φ.

Fig. 4 presents an example taken, once again, from JASWON's book.

CONCLUSIONS

The size of elements in B.I.E.M. can seriously affect the precision of results. This is especially true for interior points and singular points situations in which a gradually refined mesh near the point of interest must be used.

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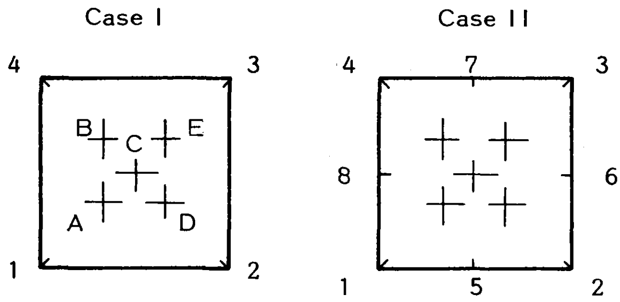


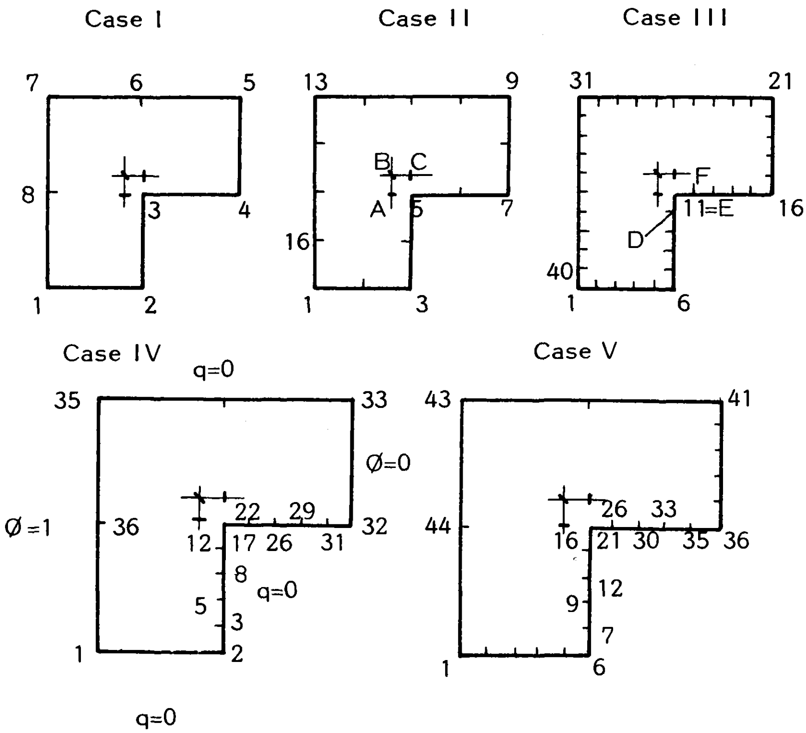
Node	Theory	Case I	Case II
1	50.0 (f.b.)	50.0004	49.9999
2	-50.0 (f.a.)	-50.0003	-49.9998
3	-50.0 (f.b.)	-50.0004	-50.0003
4	50.0 (f.a.)	50.0003	49.9999
A	200.00(Ø)	201.376	200.000
B	200.00 (Ø)	201.376	200.001
C	150.000 (Ø)	149.793	150.001
D	100.00 (Ø)	100.014	100.005
C	100.00 (Ø)	100.014	100.005

f.b. flux before

f.a. flux after

Ø potential

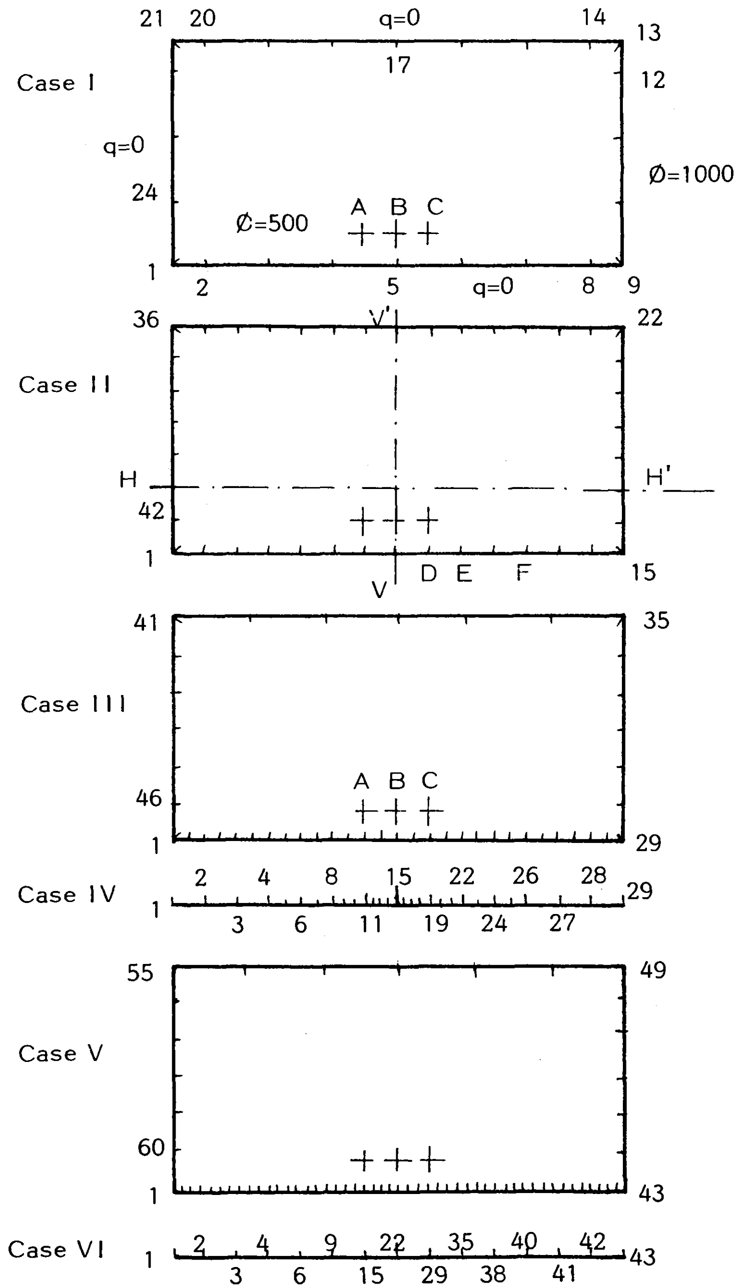
Figure 1



Prob.	Points Numb.	Internal Points			Boundary Points		
		A	B	C	D	E	F
I	8	0.746	0.705	0.611	---	0.6674	----
II	16	0.749	0.706	0.607	---	0.6668	---
III	40	0.754	0.706	0.604	0.7911	0.6668	0.4972
IV	36	0.756	0.706	0.601	0.7966	0.6664	0.4867
V	44	0.756	0.705	0.601	0.7953	0.6659	0.4867
Jasw.	80	0.756	0.706	0.6019	0.7961	0.6667	0.4869
(1)		0.756	0.706	0.6026	0.7948	0.6663	0.4884

(1) Papamichael solution

Figure 2

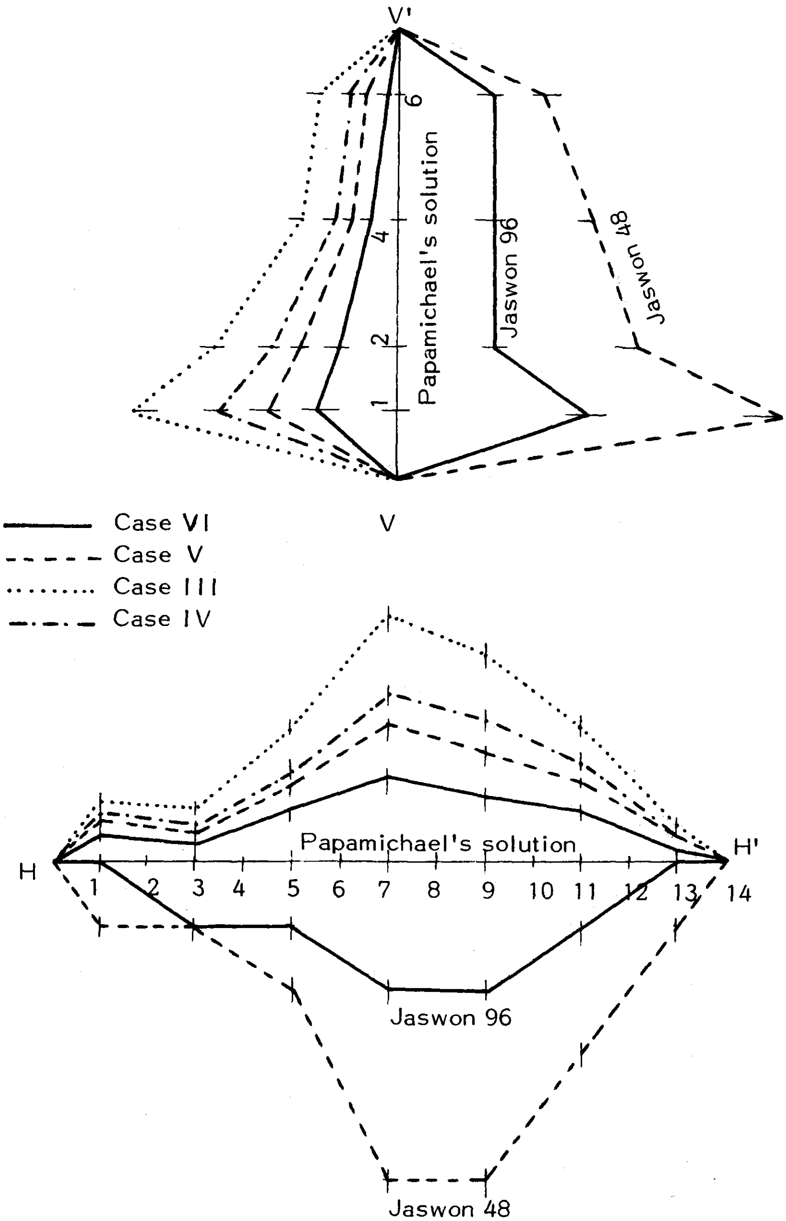


21
20
q=0
14
13
Case I
17
12
q=0
Ø=1000
24
Ø=500
A B C
1
2
5
q=0
8
9
36
V'
22
Case II
H
H'
42
1
D E F
15
V
41
35
Case III
A B C
46
1
29
Case IV
2 4 8 15 22 26 28
1
29
3 6 11 19 24 27
55
49
Case V
60
1
43
Case VI
1
2 4 9 22 35 40 42
43
3 6 15 29 38 41

Figure 3a

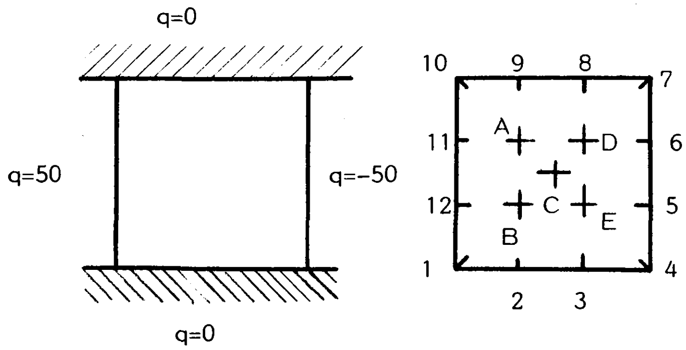
Figure 3b

Problem	Points Number	Internal Points			Boundary Points		
		A	B	C	D	E	F
I	24	546	584	649	---	715	837
II	42	556	593	659	645	720	840
III	46	559	598	664	650	724	842
IV	46	561	601	667	653	726	843
V	60	560	600	666	652.3	725.8	843
VI	60	561	602	668	654.6	727.3	843.8
Jasw.	24	570	623	686	677	743	853
Jasw.	48	566	612	678	667	736	848
Jasw.	96	564	608	670	662	732	846
Papam.	--	562	604	670	656	728	844



V'
6
4
2
1
Papamichael's solution
Jaswon 96
Jaswon 48
V
Case VI
Case V
Case III
Case IV
H
H'
Papamichael's solution
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Jaswon 96
Jaswon 48

Figure 3c



NODE	POTENTIAL (1)	POTENTIAL (2)	POTENTIAL THEORY
1	28.941	299.9996	300.000
2	-71.058	199.9999	200.000
3	-171.058	99.9995	100.000
4	-271.058	0.0000	0.000
5	-271.058	-0.0002	0.000
6	-271.058	-0.0002	0.000
7	-271.058	-0.0004	0.000
8	-171.058	99.9995	100.000
9	-71.058	199.9999	200.000
10	28.941	299.9997	300.000
11	28.941	299.9996	300.000
12	28.941	299.9996	300.000
A	-71.059	200.001	200.000
B	-71.059	200.001	200.000
C	-121.059	150.000	150.000
D	-171.059	100.000	100.000
E	-171.059	100.000	100.000

(1) DIRECT SOLUTION

(2) SOLUTION FIXING A POINT

Figure 4