II. Optimum Load Impedance

Let us consider a bare EDT of length $L$, with a uniform cross section of area $A$, mass density $\rho$, and electrical conductivity $\sigma$. Assuming the cathodic part is insulated, a design favoring maximum power generation, and falls within the low-ohmic-effect regime, the dimensionless current, local bias, and generated power are, respectively, as follows:

$$I^* = \frac{I}{I_{sc}}, \quad \Phi^* = \frac{\Phi}{\Phi_{sc}}, \quad W = \frac{W}{W_{sc}}$$

where $I_{sc}$ is the short-circuit current, $\Phi_{sc}$ is the short-circuit power, and $W_{sc}$ is the short-circuit power. The optimum load impedance $Z_F$ is given by:

$$Z_F = \frac{1}{\rho \sigma}$$

and the maximum power density is achieved at:

$$\Phi_{max} = \frac{W_{max}}{A}$$

This expression shows that the maximum power density is proportional to the square of the current density, and inversely proportional to the mass density and electrical conductivity. This relationship is consistent with the theoretical analysis provided in Section II.
\[ i_e = i(1) = \xi_{eb}^{3/2} - \frac{2}{5} \xi_{eb}^{3} + \frac{29}{400} \xi_{eb}^{3/2} + 298500, \]
\[ \varphi_e = \varphi(1) = (1 - \xi_{eb}) (-1 + \xi_{eb}). \]
\[ \dot{w} = -i_e \varphi_e = \xi_{eb}^{3/2} (1 - \xi_{eb}) \times \left[ 1 - \frac{7}{5} \xi_{eb}^{3/2} + \frac{349}{400} \xi_{eb}^{3} \right] + \varepsilon (\xi_{eb}^{15/2}) \]

where \( \xi_{eb} \) is the nondimensional zero-bias abscissa along the tether, and \( \varepsilon \) is the ratio between the tether ohmic impedance \( Z_t \) and an equivalent tether-plasma contact impedance \( Z_c \) under OML conditions, such that

\[ \varepsilon = \frac{Z_t}{Z_c} \]

\[ Z_c = \frac{L}{\sigma A} \]

and

\[ \xi_{eb} = \frac{3 \pi}{2 p \sqrt{L}} \frac{m_e}{N_e \sqrt{2 q_e}} \]

with \( p, E, m, \) and \( q_e \) indicating, respectively, the perimeter of the tether cross section, the motional electric field projected along the tether line, the electron mass, and the electron charge [8].

It is important to highlight that, once the tether geometry, the tether conductivity, and the environmental properties are set, the parameter \( \xi_{eb} \) is completely determined by the load impedance \( Z_f \) and by the potential difference \( AV_{dc} \) between the cathodic plasma contactor and the surrounding plasma. When efficient plasma contactors (e.g., hollow cathodes) are employed and sufficiently long tethers are considered, the small effect of \( AV_{dc} \) on the bias and current profile can be safely neglected [8].

The optimum value of \( \xi_{eb} \) can be computed by solving

\[ \frac{d\dot{w}}{d\xi_{eb}} = 0 \]

which yields

\[ 8410 \xi_{eb}^{37} s^5 - 7569 \xi_{eb}^{35} s^{15} - 78880 \xi_{eb}^{14} s^4 + 69600 \xi_{eb}^{12} s^4 + 341600 \xi_{eb}^{14} s^3 - 292800 \xi_{eb}^{13} s^3 + 767800 \xi_{eb}^{12} s^3 + 628200 \xi_{eb}^{11} s^3 + 400000 s^3 + 240000 = 0 \]

\[ \xi_{eb}^{opt} = s^2 \]

Equations (6) and (7) can be solved numerically for \( 0 \leq s \leq 1 \).

Once an optimum design value for \( \xi_{eb} \) is established, the corresponding value of the load impedance has to be computed. For this purpose, the circuit equation at the cathodic end is considered (see Bombardelli et al. [8]). Neglecting the plasma contactor potential drop, the circuit equation provides the optimum load impedance as follows:

\[ Z_{f}^{opt} = Z_c \times \left( \frac{-\varphi_e}{\xi_{eb} - \xi_{eb}^{opt}} \right) \]

In the limit case of \( \varepsilon = 0 \), one has

\[ Z_{f}^{opt} |_{\varepsilon = 0} = \frac{2 \sqrt{15}}{9} Z_c = Z_p \]

The term \( Z_p \) can be defined as the effective plasma contact impedance of the bare EDT measured at the cathodic end. Matching the load impedance \( Z_f \) with \( Z_p \) provides maximum power transfer to the load when \( \varepsilon = 0 \).

In the other limit case of \( \varepsilon \to \infty \), it can be seen (by numerical analysis) that

\[ \lim_{\varepsilon \to \infty} Z_{f}^{opt} = \varepsilon Z_c = Z_t \]

meaning that, when the tether-plasma contact impedance is negligible, the optimum load impedance for maximum power generation has to match the ohmic impedance of the tether in agreement with the literature [6].

Figure 1 shows the variation of the optimum load impedance as a function of the parameter \( \varepsilon \). Note that, for \( \varepsilon > 5 \), the optimum load impedance is very close to the tether ohmic impedance \( Z_t \) and practically independent of local environmental conditions (\( N_e, E_t \)). Conversely, the lower the parameter \( \varepsilon \), the more \( Z_{f}^{opt} \) depart from \( Z_t \) and the more it becomes dependent on the local environment.

### III. Maximum Power Density

The dimensional power generated at the load can be obtained as

\[ \dot{W}_{t} = \dot{w} I_{ch} E_t L \]

where \( I_{ch} \) is the tether characteristic current [8]:

\[ I_{ch} = \sigma \frac{A E_t}{m} \]

When large tethers are considered, the tether mass \( m_t = \rho A L \) becomes the dominant term in the total mass of an EDT-based power plant so that the power density yields

\[ \dot{\delta} = \frac{\dot{W}_t}{m_t} = \dot{w} \frac{\sigma E_t^2}{m} \]

\[ \dot{\delta} = \frac{\dot{w}}{m} = \dot{w} \frac{\sigma E_t^2}{m} \]

The function \( \dot{w} \) under the optimum design condition (\( \xi_{eb} = \xi_{eb}^{opt} \)) grows monotonically with \( \varepsilon \), reaching the maximum asymptotic value of 0.25 (Fig. 2). The maximum power density achievable with a bare EDT is, then, as follows:

\[ \delta_{max} = \lim_{\varepsilon \to \infty} \dot{\delta}_{opt} \approx 0.25 \times \frac{\sigma E_t^2}{m} \]

which, again, in agreement with previously published results [6]. Equation (11), together with Fig. 2, highlights the fact that, to maximize power generation, the best use of tether mass is done by increasing tether length and decreasing cross-sectional area and that the improvement obtained by increasing \( \varepsilon \) beyond, say, 10 or 20, is very modest.

The power density for the case of zero ohmic effects is derived by substituting Eq. (2) into Eq. (11):

\[ \dot{\delta}_{opt} (\varepsilon = 0) = \frac{4 \sqrt{15} \rho N_e}{125 \pi A \rho} \sqrt{\frac{2 E_t^3}{m_e q_e^2 L^3}} \]
Fig. 2 Variation of the function $e \hat{w}$ under optimum design condition as a function of the parameter $e$.

<table>
<thead>
<tr>
<th>$e$</th>
<th>$0.2$</th>
<th>$0.4$</th>
<th>$0.6$</th>
<th>$0.8$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e \hat{w}$</td>
<td>$0.8$</td>
<td>$0.5$</td>
<td>$0.2$</td>
<td>$0$</td>
<td>$-0.5$</td>
</tr>
</tbody>
</table>

showing that, for the zero-ohmic-effect case, the tether length, cross section geometry, and plasma density all have a strong influence on the achievable power density.

### IV. Load Power Efficiency

In general, a passive EDT generator can extract power from its own orbital energy (and the one of the spacecraft to which it is attached) and from the energy of the plasma sphere with which it interacts. In this regard, it becomes important to establish what fraction of the available power is actually converted into useful power for onboard use. The load power efficiency, introduced by Sanmartin et al. [4], can be written as

$$\eta = \frac{W_L}{W_{mag}}$$

where $W_L$ is the power generated at the load, and $W_{mag}$ is the power associated with the Lorentz force $F$, acting both on the tether and on the ambient plasma

$$W_{mag} = \mathbf{F} \cdot (v_w - v_{pl})$$

with $v_w$ and $v_{pl}$ indicating the spacecraft and plasma velocity vectors with respect to an inertial reference system.

The Lorentz force can be written as

$$\mathbf{F} = i x B L (\mathbf{u}_t \times \mathbf{u}_p)$$

where $B$ and $\mathbf{u}_p$ are the magnetic field intensity and unit vector, $\mathbf{u}_t$ is the tether unit vector, and $i_x$ is the dimensionless current averaged along the tether line [8], which can be determined as follows:

$$i_x = \frac{\xi - 1}{2} + \frac{\xi^2}{40} (7 \xi - 16) + \frac{\xi^3}{4400} (319 - 130 \xi)$$

From Eqs. (15) and (16) one has

$$W_{mag} = -Li_{ix}B \mathbf{L} (\mathbf{u}_t \times \mathbf{u}_p)$$

which substituted into Eq. (14), together with Eq. (9), yields

$$\xi = \frac{\dot{\gamma}}{i_x}$$

Finally, the power load efficiency $\eta_{opt}$ for the maximum power density can be derived by substituting Eqs. (1) and (17) into Eq. (19) and evaluating for $\xi_{opt} = \xi_{opt}$. For the limit cases of $e = 0$ and $e \rightarrow \infty$, one has

$$\eta_{opt}(e = 0) = \frac{10}{9} = 0.556, \quad \eta_{opt}(e \rightarrow \infty) = 0.5$$

The function $\eta_{opt}(e)$ is plotted in Fig. 3, showing that the efficiency is close to 50% throughout all the range of $e$ in line with results published in Sanmartin et al. [4].

### V. Applications

#### A. Earth Orbit

Assuming the tether is in a circular equatorial orbit, perfectly aligned with the local vertical and neglecting the small magnetic field variations along the orbit due to tilt, offset and higher harmonics $E_t$ become solely a function of the orbit radius $r$ so that, for a prograde (−) and retrograde (+) orbit, one has

$$E_t \approx \left( \frac{\mu_p}{r} \pm \omega_p r \right) \times \frac{B_s r_p^3}{r^3}$$

where $\mu_p$, $r_p$, $\omega_p$, and $B_s$ are, respectively, the planet gravitational constant, the planet radius, the angular velocity of the planet-corotating plasmasphere, and the planet surface magnetic field measured at the magnetic equator.

After taking $B_s = 30 \mu T$, $r_p = 6378 km$, $\omega_p = 7.2722 \times 10^{-5}$ rad/s, $\mu_p = 3.986 \times 10^{24} m^3 s^{-2}$, and substituting Eq. (20) into Eq. (12), one obtains the maximum power density in circular orbit, which is plotted in Fig. 4.

The obtained power density values can be higher than current flexible foldout solar arrays (~40-100 W/kg [9]), which, unlike EDTs, require continuous sun tracking to avoid losing performance. On the other hand, when comparing EDTs with conventional power-generation systems, one has to consider that the generated power comes at the expense of orbital energy, which may be an important drawback of this approach, unless the deorbiting operation is included in the mission objectives. Should this not be the case, the EDT orbit would need to constantly reboosted if continuous power is desired. Because the load power efficiency is about 50% for an optimized system, the amount of orbital energy loss per unit time will be about twice the generated power.

It is important to underline that the power-density values plotted in Fig. 4 refer to the ideal case, in which $e \rightarrow \infty$ and $Z_{opt} \rightarrow Z_t$. In real applications, the parameter $e$ is limited by the maximum tether length
that can be deployed and the minimum tether cross section $A$, providing reasonable strength, as well as by the available environmental plasma density $N_e$. The latter can vary along the orbit by one to two orders of magnitude, primarily due to solar illumination conditions (a drop in plasma density is experienced during orbital eclipse transition).

Figure 5 plots the instantaneous optimum load impedance and power density achievable with a 20-km aluminum tape tether of 0.05 mm thickness flying on a 600 km altitude equatorial orbit around the Earth during maximum and minimum solar activity. (The reference dates for the simulations are 1 January 2000 and 1 January 1996 for maximum and minimum solar activity, respectively). The IRI2007 ionospheric model and the IGRF95 Earth magnetic field models were employed. During maximum solar activity, the optimum load impedance is almost constantly equal to the tether ohmic impedance, meaning that the parameter epsilon [Eq. (2)] for this particular tether design and environmental conditions is practically always greater than $\approx 5$ (see Fig. 2). Conversely, during minimum solar activity, the same tether has a much more variable optimum power-generation impedance. More specifically, the peaks in Fig. 5a correspond to eclipse transition periods, during which a decrease in plasma density (corresponding to a decrease in $\epsilon$) is observed. By looking at Fig. 5b, one can notice that power-density values close to the maximum possible are obtained during high solar activity, whereas a relatively small decrease in the average value is experienced during low solar activity. Note that the oscillatory behavior of the power density is due to the different Earth magnetic field harmonics accounted for by the IGRF95 model. Nevertheless, the maximum average power density remains close to the one plotted in Fig. 4.

B. Jupiter Orbit

The lower ionospheric density and stronger magnetic field of Jupiter, compared with the Earth case, make ohmic effects negligible for EDT of reasonable size (say less than 20 km) [8], so that Eq. (13), rather than (12), must be used for computing the achievable power density.

The motional electric field for circular equatorial orbits is computed using Eq. (20), as in the Earth case, and taking $B_0 = 420 \mu T$, $r_{eq} = 71492$ km, $\omega_p = 1.7585 \times 10^{-4}$ rad/s, and $\mu_r = 1.267 \times 10^{13}$ m$^3$ s$^{-2}$. By substituting Eq. (20) into (13) and employing a Divine-Garrett ionospheric density model [10], one finally obtains the zero-ohmic-effect power density, which is plotted in Fig. 6, considering an aluminum tape tether of 0.05 mm thickness and 20 km length.

The plot highlights the advantage of using EDTs as power plants (as opposed to low-power-density radioisotope thermoelectric generator (RTG)-based power sources (typical power density levels for RTGs do not exceed 8 W/kg) in the lower part of Jupiter’s plasmasphere, as already pointed out in the literature [1,2,11–13].

VI. Conclusions

The performance of bare EDTs as power-generating systems under OML-theory conditions has been analyzed based on a previously published asymptotic model for the tether current and bias. The model allows for the easy determination of the optimum load impedance for maximum power generation at the cathodic end of the tether, and, in turn, the computation of the corresponding optimum power, power density, and load power efficiency. Limit cases of zero and dominant tether ohmic impedance provide closed-form analytical formulas that match previously published results.

The maximum power density theoretically achievable by an EDT is seen to be proportional to the square of the local motional electric field along the tether and to the conductivity/density ratio of the tether material. A numerical evaluation of the maximum performance in circular equatorial Earth and Jupiter orbits show that power density levels comparable to the ones of solar arrays (for the Earth) and superior to RTGs (for Jupiter) are achievable. Simulations with a 20-km-long 0.05-mm-thick tape EDT in low circular equatorial Earth orbit at 600 km altitude show that nearly optimum power-generation conditions are reached during maximum solar activity, with optimum load impedance always very close to tether ohmic impedance and, therefore, virtually insensitive to local plasma density variations. A power density decrease is experienced by the same system during eclipse transitions under minimum solar activity conditions, in which a tuning of the load impedance based on the local ionospheric density becomes necessary.

References


