Assessment of irrigators’ preferences for different water supply risk management tools: option contract and insurance

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Abstract

Irrigators face the risk of not having enough water to meet their crops’ demand. There are different mechanisms to cope with this risk, including water markets (option contracts) or insurance. A farmer will purchase them when the expected utility change derived from the tool is positive. This paper presents a theoretical assessment of the farmer’s expected utility under two different option contracts, a drought insurance and a combination of an option contract and the insurance. We analyze the conditions that determine farmer’s preference for one instrument or the other and perform a numerical application that is relevant for a Spanish region.

Keywords: expected utility, crop’s insurance, option contract, risk, water.

1. Introduction

Irrigators face the risk of not having enough water to meet their crops’ demand. There are a number of strategies to cope with this risk, applying on-farm strategies to reduce vulnerability or sharing the risk with an external agent (Cummins and Thompson, 2002; FAO, 2003; Sivakumar and Motha, 2007; Garrido and Gómez-Ramos, 2009). Among all the existing tools that help irrigators to manage, or cope with this risk, this paper focuses on water option contracts and insurance.

The possibility of trading water leads to a reduction in the farmers’ risk caused by variations in the water availability (Calatrava and Garrido, 2005; Bjornlund, 2006). Options are one type of derivative contract that gives the holder the right (not the obligation) to buy or sell the underlying asset (Williamson et al., 2008; Cui and Schreider, 2009; Cheng et al., 2011). In the case of water supply risks, option contracts help the buyer (option holder) to protect

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himself from the risk of not having enough water for his activity (irrigating in the case of farmers). A farmer can pay a premium to the water seller to use an extra volume at the maturity date if needed. Option contracts may provide several benefits to water users, through a better and more efficient distribution of risk and water resources among them and a reduction of uncertainties related to water supply reliability (Brown and Carriquiry, 2007; Ranjan, 2008; Garrido and Gómez-Ramos, 2009).

Insurance provides a compensation for losses that occur with relatively low frequency and whose probability can be evaluated (Garrido et al., 2012a). It is an efficient instrument to face the losses derived from catastrophic risks in the agricultural sector (ECA, 2009) and one of the most useful risk management tools in critical situations, reducing the economic consequences derived from them (Hecht, 2008; WWAP, 2012). Here, the proposed insurance allows the farmer to receive a monetary compensation when he has not enough water for irrigating his crops.

Among the available alternatives, farmers will choose the risk management tools that are perceived to improve their benefits. The literature on farmers' water supply risks has not theoretically analyzed different mechanisms and compared to one another. Using the expected utility theory approach, we evaluate several risk management tools to obtain some conclusions about the designing parameters that make one mechanism more interesting than the other. To achieve this, we compare farmer’s expected utility in five different situations: with no contract, with an option contract (two different types are considered), with insurance, and with a combination of option contract and an insurance contract.

The paper is organized as follows: section 2 presents the theoretical framework, where the farmer’s expected utility and the risk premium for each case are calculated; section 3 contains an analysis of the farmers’ preferences for the proposed instruments; section 4 shows the results of a comparative statics analysis to determine the impact of different parameters on risk premium value; section 5 includes an application of the theoretical model to irrigators in the most arid but agricultural efficient region in Spain; in section 6 a discussion of the obtained results is provided; and finally, in section 7 some conclusions from this analysis are presented, highlighting the importance of these risk management tools for agriculture. Part of the mathematical work is presented at the end of the paper in the appendices with the aim of facilitating the reading of the rest of the document.

2. Theoretical framework

Expected Utility Theory states that a decision maker chooses between risky or uncertain prospects by comparing their expected utility values (Mongin, 1997). Between different
alternatives, a person will choose the one with higher expected utility. We are considering the Expected Utility (EU) of the farmer in relation to his profit ($\bar{\pi}$). As an irrigator, his profit is going to be a function of his water availability for each season ($w$), which follows a probability distribution function, $f(w)$.

$$EU(\bar{\pi}) = \int U(\bar{\pi}) \, f(w) \, dw$$  \[1\]

Under the expected utility hypothesis, a farmer would choose to use one instrument $i$, if:

$$EU_i(\bar{\pi}) - EU_0(\bar{\pi}) > 0$$  \[2\]

Using this theoretical framework we will make some assumptions in order to facilitate the comparison between the different cases under study, and to ease the mathematical complexity of alternative approaches.

First, acknowledging that the EU framework has been consistently discredited by empirical work, it provides a valid approach for discriminating among risk management instruments whose outcomes are not extremely different both in second and third moments.

Secondly, we assume a Constant Absolute Risk Aversion (CARA), in particular an exponential utility function ($U(\pi) = 1 - e^{-\rho \pi}$). This function assumes that the risk aversion for an individual is constant, although it can be considered decreasing with wealth level (DARA). As our aim in this analysis is to establish a comparison between different cases, in which farmers' wealth would not change significantly, we ponder the analytical convenience of CARA for range of outcomes of $\bar{\pi}$ which will not be very wide (see Calatrava and Garrido, 2005; Garrido, 2007), and farmers' wealth (land and capital values) is invariant to the choice of instrument.

Thirdly, we assume that farmer's restricted profit function dependent on $w$ is a linear function. This assumption is acceptable in cases where water is used in Leontieff production functions, where each activity enters in fixed proportion of inputs, and farmers in the short run change activities (crops) instead of searching for new production methods.

Fourth and lastly, we assume that farmer's water availability follows a gamma distribution function. This function has a considerably simple Moment Generating Function that facilitates calculation, and together with the previous assumptions provide a convenient analytical approach (Collender and Zilberman, 1985). The gamma is bounded on the left, but unbounded on the right. We assume a truncation at $\bar{w}$, leaving out the right tail of the distribution representing unlimited and unneeded water availability levels.

2.1. Expected utility function with no contract (case 0)
Farmers’ water availability would depend only on his water allotment ($\tilde{w}$). A very simplified farmer’s linear restricted profit function is used. We are not taking into account the costs associated with the farming activity neither the income\(^2\).

$$\tilde{\pi}_0 = a + b\tilde{w}$$

where $a$ is net benefit of agriculture, independent of water availability; $b$ is water productivity\(^3\). The CARA exponential utility function for this case, is:

$$U(\tilde{\pi}_0) = 1 - e^{-r\tilde{\pi}_0} = 1 - e^{-r(a+b\tilde{w})}$$

$r$ is farmer’s risk aversion coefficient. The farmer’s expected utility can be expressed as (see Appendix 1 for the entire calculation):

$$EU_o(\tilde{\pi}) = \int_0^{\bar{w}} U(\tilde{\pi}) f(w) dw = \int_0^{\bar{w}} [1 - e^{-r(a+bw)}] f(w) dw = 1 - e^{-ra}MGF_w(-rb)$$

$\bar{w}$ is the maximum water availability for the farmer, being zero the minimum. $MGF_w(-rb)$ is the Moment Generating Function\(^4\) of the variable $w$ of order $(-rb)$. Not all distribution functions have their own MGF. As explained before, we assume that our variable $\tilde{w}$ follows a gamma distribution, which has a considerably simple MGF:

$$MGF_w(-rb) = \left(\frac{1 + \frac{rb}{\lambda}}{\lambda}\right)^{-a}$$

$\lambda$ and $\alpha$ are parameters of the gamma function, $w \sim \text{Gamma} (\alpha, \lambda)$; with mean $(\alpha/\lambda)$ and variance $(\alpha/\lambda^2)$.

2.2. Expected utility function with the option contract (cases 1 and 2)

A farmer that decides to sign an option contract with a water seller has to pay a premium ($P$) to have the right to purchase the optioned water volume at the maturity date if needed. The premium represents the value of the flexibility gained by the buyer from postponing the decision to purchase water (Hansen et al., 2006). Also, this premium must compensate the seller for giving away a part of his water allotment.

For our analysis, two option contracts with different exercise conditions have been defined. The first one (1) allows the option holder to exercise the option whenever his water allotment is below a pre-established guaranteed level. Under the second type (2), the option can be exercised if another external condition (trigger) is also satisfied, lowering the probability of

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\(^2\) Our assessment only takes into account the changes in the farmer’s expected utility caused by different water availability (due to an option contract or an insurance).

\(^3\) $b$ is net of the water price; $b = c - P_w \cdot c$ is the marginal profit of water use and $P_w$ the water tariff.

\(^4\) A MGF of a random variable is a specification of its probability distribution, which give us a convenient way of collecting together all the moments of a random variable into a single power series.
exercising the option in comparison to contract 1. In the following sections, each case is presented in further detail.

2.2.1. Option contract (1)

This option contract establishes a minimum water volume, \( w_g \). When the farmer’s yearly water allotment (\( \bar{w} \)) is below this threshold (which occurs with a probability \( \gamma \)), he can exercise the option and obtain (\( w_g - \bar{w} \)), paying an exercise price to the seller. This guarantees that every year the option holder is going to have, at least, a water volume equivalent to \( w_g \). If we assume that the irrigator will always exercise the option at the maturity date when his water allotment is below \( w_g \), his profit function is:

\[
\pi_1(w) = a + bw - P_1 \quad \text{if} \quad \bar{w} \geq w_g
\]

\[
\bar{\pi}_1(w) = a + bw - P_1 - P_e(w_g - \bar{w}) \quad \text{if} \quad \bar{w} < w_g
\]

\[5\]

\( P_1 \) is the contract premium; \( \bar{w} \) is the guaranteed volume accessible through the option contract; and \( P_e \) is the exercise price\(^5\). When his water allotment is higher than \( w_g \), he cannot exercise the option and the only water that he would have is his water allotment \( \bar{w} \).

The farmer’s expected utility if he decides to sign an option contract with these characteristics would have the following expression (see Appendix 2.a.):

\[
EU_1(\bar{w}) = 1 - e^{-r(\alpha + bw_0 - P_1 - P_e w_0)}LIUMGF_w(-rP_e) - e^{-r(\alpha + P_e)}UIUMGF_w(-rb)
\]

\[8\]

LIUMGF is the Lower Incomplete Moment Generating Function of \( \bar{w} \) of order \((-rP_e)\); and UIUMGF is the Upper Incomplete Moment Generating Function\(^6\) of order \((-rb)\).

Now, the expression of the premium that equals farmer’s both expected utilities (without and with this option contract) can be analytically obtained. This expression determines the value of the maximum premium that makes the contract attractive for the farmer, i.e., the risk premium \( (R_1) \) (See Appendix 2.b. for the entire mathematical calculation).

\(^5\) A farmer exercising the water supply option contract will pay \( P_w \) plus \( P_e \) for the optioned volume. \( P_e \) is defined as a price additionally paid, besides the price paid for the normal source of water supply (\( P_w \)). If the exercise price agreed in the option contract were lower than the price paid for the normal source of water supply, \( P_e \) would then be negative. This situation is not very common, but it can be possible when the contract is established between water users who have very different water productivities. In order to simplify the presentation of this approach, only positive \( P_e \) values are considered in the analysis, unless stated otherwise. In section 5, an example of an option contract with a lower exercise price than \( P_w \) is presented, with the aim of addressing this issue.

\(^6\) The UIMGF and the LIMGF are calculated in the same way, the only thing that changes is the value of the integral's limits (In Appendix 3, the expression of UIUMGF \((-rb)\) is obtained).
The risk premium value depends on several parameters, including the risk aversion coefficient \((r)\), the water productivity \((b)\), the exercise price \((P_e)\), the guaranteed water volume \((w_g)\), and the parameters of the gamma function that represents farmer’s water availability \((\alpha\) and \(\lambda)\). Changes in these parameters make the value of the risk premium increase or decrease. The effect of each parameter on the risk premium is analyzed in section 4, pointing out which factors are more determinant when designing this type of risk management tool.

It is important to distinguish between the risk premium and the premium actually paid by the farmer. The risk premium represents the value that the option contract has for the farmer, i.e., his willingness to pay (WTP). The premium actually paid is the amount of money that the farmer has to pay to the seller at the beginning of the year, to have access to the optioned water volume if needed. Obviously, as the risk premium represents the WTP for the option contract, the farmer is not going to pay a premium higher than \(R_i\).

2.2.2. Option contract (2)

This case is more restrictive than option contract (1), as the condition for exercising the option (trigger) is stiffer. The farmer would only be able to exercise the option when two different conditions are met: his water availability is lower than \(w_g\); and the water stock \((S)\) in the reservoir which stores the seller’s water allotment is higher than a pre-established limit \(k\). The probability of exercising the option has been obtained applying the Bayes Theorem. The above expression is the probability of having less water than \(w_g\) when the stock in the reservoir is higher than the limit, \(k\). For the rest of the paper, this probability is going to be denoted by \(Z\). This is the probability of meeting the conditions needed to exercise the option. When one of these conditions is not met, the option contract cannot be exercised (prob = 1 – \(Z\)), despite \(\bar{w} < w_g\). The farmer’s profit function in this case is:

\[
\frac{1}{\tau} \ln \left( \frac{MGF_w(-rb)}{e^{-r(P_e-P_c)W}} LMGF_w(-rP_e) + UMGF_w(-rb) \right)
\]

\[EU_0(\bar{\pi}) = EU_1(\bar{\pi})\]

\[R_i = \frac{1}{\tau} \ln \left( \frac{MGF_w(-rb)}{e^{-r(P_e-P_c)W}} LMGF_w(-rP_e) + UMGF_w(-rb) \right)\]

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7 This is the case of farmers relying on inter-basin transfers where, because of area-of-origin preferences no volume is transferred unless minimum water volumes are stored in the region where the transfer is derived from.

8 \(\beta\) is the probability of having a stock in the reservoir higher than \(k\), when farmer’s water availability is lower than \(w_g\). \(\bar{\beta}\) is the probability of having a stock in the reservoir higher than \(k\), when farmer’s water availability is higher than \(w_g\).
being ϕ a binomial variable (0,1), with a probability Z of being 1, so the option can be exercised. The farmer’s expected utility function with this option contract is (Appendix 4.a.):

\[ EU_2(\bar{\pi}) = 1 - e^{-\gamma(a-p_2-MGF_w(-rb))} - Z e^{-\gamma(a+bw_g-p_2-p_3w_g)}limGF_w(-rP_2) + Z e^{-\gamma(a-p_2)}limGF_w(-rb) \]

The premium that makes equal the expected utility with no contract \((EU_0(\pi))\) with the one for this option contract is obtained (See Appendix 4.b. for the entire mathematical calculation):

\[ R_2 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{(1-Z)MGF_w(-rb) + Z e^{-\gamma(a-bw_g-p_2-p_3w_g)}limGF_w(-rP_2) + UIMGF_w(-rb)} \right) \]

Considering the farmer as a risk-averse agent, his WTP for option contract 1, which offers him more guarantees, would be higher than his WTP for contract 2.

2.3. Insurance (3)

Another risk management tool available for farmers to cope with water supply risks is water supply insurance. A farmer can contract an insurance that offers a financial compensation for the lost profit due to water shortage. When the water volume received by the farmer is below \(w_g\), he will be compensated with the forgone profit that he would have obtained if he had also \((w_g - \bar{w})\). The profit function of the irrigator in this case is:

\[ \bar{\Pi}_3(w) = a + b\bar{w} - P_3 \quad \text{if} \quad \bar{w} \geq w_g \]

\[ \bar{\Pi}_3(w) = a + bw_g - P_3 \quad \text{if} \quad \bar{w} < w_g \]

The farmer’s expected utility is (see Appendix 5.a.):

\[ EU_3(\bar{\pi}) = 1 - e^{-\gamma(a+bw_g-p_3)} - e^{-\gamma(a-p_3)}UIMGF_w(-rb) \]

Following the same methodology as the two previous cases, the risk premium is obtained (See Appendix 5.b.):

\[ R_3 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{\gamma e^{-\gamma(bw_g-p_3)} + UIMGF_w(-rb)} \right) \]

The value of the premium for purchasing an insurance policy providing coverage against water shortages \((P_3)\) would be, at least, the value of the expected compensation \((C^e)\) that the farmer would receive through the insurance, i.e., the equivalent amount of money that he would obtain from having an extra water volume \((w_g - w)\) (See Appendix 6).
\[ C^e = bw_g \gamma + \frac{b^2}{\alpha \Gamma(a)} \left[ \Gamma(a + 1, \lambda w_g) - \alpha \Gamma(a) \right] \]  

\( \Gamma(.) \) is a gamma function.

2.4. **Combination of an option contract and an insurance (case 4)**

This case analyzes the farmer’s expected utility under a combination of the option contract (case 2) and the insurance (case 3). The option contract would allow the farmer to purchase the optioned volume when the two conditions are met (water allotment lower than \( w_g \), and stock in the reservoir higher than \( k \)). When these two conditions are not met, the option cannot be exercised. The insurance contract would protect the farmer in this situation, paying him a compensation for having less water than the guaranteed level (\( w_g \)) when the other condition for exercising the option (\( S > k \)) is not met. The profit function in this case would be:

\[
\begin{align*}
\pi_4(w) &= a + bw - P_4 & \text{if } w \geq w_g \\
\pi_4(w) &= \varphi \left( a + bw_g - P_4 - P_2 (w_g - w) \right) + (1 - \varphi) \left( a + bw_g - P_4 \right) & \text{if } w < w_g
\end{align*}
\]

\( Z \) is the same probability as the one considered in case 2. The farmer’s expected utility for this case is (see Appendix 7.a.):

\[ EU_4(\pi) = 1 - e^{-(a-P_4)} \left[ U1MGF_w(-rb) +Ze^{-r(b-P_2)}w_gLIMGF_w(-P_2) + (1-Z)Ye^{-rbw_g} \right] \]

The premium for this risk management tool is the sum of the premium of the option contract (2) plus the insurance premium (\( P_4 = P_2 + P_3 \)). Comparing this expected utility with the one of not contracting this tool, the risk premium can be obtained (see Appendix 7.b.):

\[ R_4 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{U1MGF_w(-rb)+Ze^{-r(b-P_2)}w_gLIMGF_w(-rP_2)+ (1-Z)Ye^{-rbw_g}} \right) \]

3. **Comparison of instruments**

The risk premium represents the willingness of the farmer to purchase insurance or to sign an option contract. Thus, the higher the risk premium of the instrument is, the higher the farmer’s WTP for it will be. A comparison of the cases under study is presented below.

3.1. **Comparison between option contract (1) and insurance (3)**

The risk premium for both cases (\( R_1 \) and \( R_3 \)) has been obtained in the previous section ([9] and [16]). As it can be seen, the difference between them is the denominator of the logarithm. In order to compare the value of the risk premium for these cases, we are going to compare the value of their denominators.
If \( e^{rP_{w}g}LIMGF_{w}(-rP_{e}) > \gamma \), then \( R_3 > R_1 \). Further algebra allows us to conclude that, if \( P_{e} > 0 \), \( R_3 \) is going to be always higher than \( R_1 \).

\[
e^{rP_{w}g}LIMGF_{w}(-rP_{e}) - \gamma > 0
\]

\[
e^{rP_{w}g} \int_{0}^{w_{g}} e^{-rP_{e}w}f(w)dw - \int_{0}^{w_{g}} f(w) dw > 0
\]

\[
\int_{0}^{w_{g}} (e^{-rP_{e}(w-w_{g})} - 1)f(w)dw > 0
\]

[22]

As the upper limit of the integral is \( w_{g} \), \( w \) is going to be always smaller than \( w_{g} \). Thus, \( e^{-rP_{e}(w-w_{g})} > 1 \); and this expression would be positive for \( P_{e} > 0 \).

Both instruments offer the farmer the same level of guarantee, but here we are taking into account the exercise price of the option contract. That is why the insurance risk premium is higher when the exercise price of the option contract is positive. The farmer would have to pay all the costs of the insurance at the beginning of the year, and the risk premium of the option contract is only a part of the total payment. However, if the price of the water acquired through the option contract were lower than the usual source of water (\( P_{e} < 0 \)), then the risk premium of the option contract could be greater than that of insurance\(^9\).

The final decision to purchase one instrument or the other would depend on the effect that each tool has on farmer’s welfare. If \( P_{1} \) and \( P_{3} \) are the premiums paid by the farmer for each instrument, he is going to purchase the one that provides him higher welfare; i.e., the difference between the risk premium and the premium paid is higher. If \( R_1 - P_{1} > R_3 - P_{3} \), the farmer would purchase the option contract, whereas if \( R_1 - P_{1} < R_3 - P_{3} \), the farmer would purchase the insurance. When \( R_1 - P_{1} = R_3 - P_{3} \), the farmer would be indifferent between the option contract and the insurance.

\[
R_1 = \frac{1}{r} \ln \left( \frac{N}{N_{1}} \right) \quad \text{and} \quad R_3 = \frac{1}{r} \ln \left( \frac{N}{N_{3}} \right)
\]

\[
R_1 - P_{1} > R_3 - P_{3}
\]

\(^9\) Expression [22] can be rewritten as \( \int_{0}^{w_{g}} (1 - e^{-rP_{e}(w-w_{g})})f(w)dw < 0 \).

\( \int_{0}^{w_{g}} (1 - e^{-rP_{e}(w-w_{g})})f(w)dw \) is the expected utility of \(-P_{e}(w-w_{g})\), i.e., the expected utility of the increase in the cost of water due to obtaining it through the option contract instead of the usual water source. If \( P_{e} < 0 \), such expected utility would be positive and thus \( R_3 < R_1 \).
Therefore, if \( P_1 < P_3 + \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \), the farmer would prefer the option contract, whereas if \( P_1 > P_3 + \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \), he would prefer the insurance. The value of the premium paid that makes the farmer indifferent between both alternatives is: \( P_1 = P_3 + \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \). For a risk-averse farmer and \( p_e > 0 \), \( D_1 \) is greater than \( D_3 \) (and \( R_3 \) greater than \( R_4 \)), and therefore \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) < 0 \). Expression [23] would be:

\[
P_3 - P_1 > -\frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) > 0
\]

This result implies that the farmer will choose the insurance even if the premium to be paid for it is greater than the one to be paid for the option contract, as long as the former does not overpass the latter in more than \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \). The difference between \( P_3 \) and \( P_1 \) must be greater than \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \) for the farmer to choose the option contract.

On the other hand, if \( p_e < 0 \) (the optioned water is cheaper than the price the farmer pays for his water allotment), and for a risk-averse farmer, \( D_3 \) is greater than \( D_1 \) (and \( R_1 \) greater than \( R_3 \)), and therefore \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) > 0 \). Expression [23] would then be:

\[
P_1 - P_3 < \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) > 0
\]

In this case, the farmer will purchase the option contract even for a greater premium to be paid for it, as long as this premium does not overpass the premium to be paid for the insurance in more than \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \). Thus, if the difference between \( P_1 \) and \( P_3 \) is greater than \( \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \) the farmer would purchase the insurance rather than the option contract, even if the exercise price is lower than the usual price of water.

3.2. Comparison between option contract (2) and insurance (3)

The same comparison is made between the other type of option contract (case 2) and the insurance. The probability of getting the compensation through the insurance is higher than the probability of getting the optioned volume through this option contract. Thus, \( R_3 > R_2 \).

\[
(1 - Z) \text{MGF}_w(-rb) + Z \left[ e^{-r(b-p_e)w} \text{MGF}_w(-rP_e) + U \text{MGF}_w(-rb) \right] > ye^{-rbw} + U \text{MGF}_w(-rb)
\]

(1 - Z) \text{MGF}_w(-rb) + Z \left[ e^{-rwb_p(b-p_e)} \right] \text{MGF}_w(-rP_e) \right] > ye^{-rbw} \quad \text{[26]}

This condition is always met for a risk averse farmer, so \( R_3 > R_2 \). Below is the assessment that determines which instrument is going to be purchased by the farmer. If \( R_2 - P_2 > R_3 - P_3 \), then the farmer would purchase the option contract.

\[
P_2 - P_3 < \frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) \tag{27}
\]

So, if \( P_2 < P_3 + \frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) \) he would prefer the option contract; and if \( P_2 > P_3 + \frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) \), he would prefer the insurance. \( D_2 \) is higher than \( D_3 \) when considering a risk-averse farmer. Therefore, \( \frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) < 0 \).

\[
P_3 - P_2 > -\frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) > 0 \tag{28}
\]

The farmer will choose the insurance even if \( P_3 \) is greater than the premium to be paid for this option contract, as long as the former does not overpass the latter in more than \( \frac{1}{r} \ln \left( \frac{D_3}{D_2} \right) \).

### 3.3. Comparison between the two option contracts (1) and (2)

The third comparison has been made between the two different proposed option contracts. First the comparison of the risk premiums is presented; and then the assessment of the conditions that make one instrument more attractive to the farmer than the other. \( R_1 \) is going to be higher than \( R_2 \) for all cases. Intuitively, the same conclusion can be obtained, as the option contract (1) offers higher guarantees than contract (2), allowing the farmer to purchase the optioned volume at the maturity date with higher probability. If \( R_1 > R_2 \), then:

\[
e^{-r(b-P_e)w}LIMGF_w(-rP_e) + UIMGF_w(-rb) < (1 - Z) MGF_w(-rb) + Z \left[ e^{-r(b-P_e)w}LIMGF_w(-rP_e) + UIMGF_w(-rb) \right] \]

\[
e^{-r(b-P_e)w}LIMGF_w(-rP_e) + UIMGF_w(-rb) < MGF_w(-rb) \tag{29}
\]

For \( R_1 \) to be positive, the above expression has to be always met (as the numerator, on the right side of expression [29] has to be higher than the denominator, on the left side).

As in the two previous cases, the conditions that determine the participation of the farmer in the option contract or the insurance are obtained:

\[
P_1 - P_2 < \frac{1}{r} \ln \left( \frac{D_3}{D_1} \right) \tag{30}
\]
So, if \( P_1 < P_2 + \frac{1}{r} \ln \left( \frac{D_2}{D_1} \right) \); he would choose the option contract 1. And if \( P_1 > P_2 + \frac{1}{r} \ln \left( \frac{D_2}{D_1} \right) \); he would purchase the option contract 2 (\( D_2 \) is always higher than \( D_1 \)). The farmer would be indifferent between them if \( P_1 = P_2 + \frac{1}{r} \ln \left( \frac{D_2}{D_1} \right) \).

From all these comparisons\(^{10}\), the final conclusion that can be obtained is the order of preferences for these instruments considering a risk-averse farmer (Figure 1).

Figure 1. Farmer’s willingness to pay for the risk management tools (according to the obtained risk premium for each case).

Blue: \( P_e > 0 \); Yellow: \( P_e < 0 \).

A low exercise price in comparison to the price normally paid for the water allotment can change farmer’s preferences for the different risk management tools considered in this study. This scenario occurs in the Inter-basin water markets in Spain (see Garrido et al., 2012b).

When comparing two of these risk management tools (\( i \) and \( j \)), the decision rule that is going to determine which instrument is going to be purchased by the farmer is:

\[
P_i < P_j + \frac{1}{r} \ln \left( \frac{D_i}{D_j} \right) \quad \Rightarrow \quad i
\]

\[
P_i > P_j + \frac{1}{r} \ln \left( \frac{D_i}{D_j} \right) \quad \Rightarrow \quad j
\]

4. Factors affecting the value of the risk premium

A comparative statics analysis has been carried out in order to determine the influence of the main parameters on the value of the risk premium for each instrument. The parameters included in this analysis are: the exercise price (\( P_e \)), the guaranteed level (\( w_g \)), and the parameters of the water availability function (Gamma function, with parameters \( \alpha \) and \( \lambda \)). Each of them is going to affect the farmer’s WTP for a risk management tool, increasing or decreasing its value in a different magnitude.

Table 1. Sign of the impact of each parameter on the risk premium for each case.

\(^{10}\) See Appendix 8, where the remaining comparisons between the proposed tools are shown.
Risk premium

<table>
<thead>
<tr>
<th></th>
<th>$w_g$</th>
<th>$a$</th>
<th>$\lambda$</th>
<th>$P_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R_2$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R_3^*$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R_4$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*R_3^*, as it is the risk premium for the insurance, is not affected by the exercise price.

The results shown in this table were obtained from the partial derivatives of the risk premium for each parameter. As expected, an increase of $P_e$ would lead to a decrease in the risk premium value; and an increase of $w_g$ would have the opposite effect. For all cases, the most affecting factor is the guaranteed level, then $\alpha$, and the less affecting parameter is the exercise price. For $R_4$, the impact of $P_e$ on the risk premium value is considerably low if we compare it with the other cases. The difference between $b$ and $P_e$ would be crucial for the value of the risk premium. The higher this difference is, the higher the value of the risk premium. Higher values of $\alpha$ increase the guaranteed volume (which would increase the WTP for an instrument, i.e., the risk premium), whereas higher values of $\lambda$ have the opposite effect.

The combined effects of the above parameters affect farmers’ WTP for each instrument and their preferences for one risk management tool of the other. For example, a risk-averse farmer would be willing to pay more money if the included guarantees in the option contract or the insurance are high.

5. Application to the Spanish case

In Spain, as in Mediterranean countries, farmers experience water scarcity problems. When there is not enough water for all uses, normally irrigators suffer cuts in their water allotments to secure urban water supply. Currently, some water trading mechanisms and crop insurance help farmers to protect themselves against the risk of not having enough water to irrigate their crops.

The Spanish Water Law allows the establishment of bilateral agreements between water users to exchange water rights (temporary or permanent), under several conditions and restrictions (Garrido et al., 2012b). During drought periods, some exchanges through the spot water market have taken place in order to obtain enough water for irrigation. However, in these situations it may be very difficult to find a water seller and prices are normally high. Besides, the needed administrative procedures for the approval of the exchange could take

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\[11\] This material can be provided by the authors upon request.

Spanish insurance system is one of the most developed worldwide (Antón and Kimura, 2011). Several studies show the potential of drought insurance for the Spanish agriculture. Quiroga et al. (2011) highlight the importance of reliable drought information to help farmers to avoid negative impacts and also to develop hydrological risk insurance schemes for them. Pérez and Gómez (2012, 2013) focus on the potential of drought insurance to reduce aquifers’ overexploitation during water scarcity periods.

Due to the existence of wide differences in climatic and edaphic conditions, irrigation water productivity in Spain vary among regions, ranging from 0.3 to 3.4 €/m$^3$ (Gil et al., 2009). Also, the price that irrigators have to pay for water is not the same in all areas (Calatrava and Garrido, 2010). But, in general, the price for irrigation water in Spain is considerably low, covering only the operation and management costs. This heterogeneity in water productivity levels and water prices could lead to differences in farmer's preferences for the risk management mechanisms proposed here. Also, this heterogeneity favors the establishment of water exchanges between users with different productivity levels.

The theoretical framework presented in the previous sections is applied to farmers in the Campo de Cartagena irrigation district in the Segura Basin (Southeast Spain), to obtain some numerical results. This irrigation district receives water from the Tagus Basin through the Tagus-Segura Aqueduct (from its 141 hm$^3$ of water allotment, 122 hm$^3$ come from the Aqueduct). So, these tools are meant to protect irrigators in this community from the risk of not having enough water resources from the Aqueduct.

From the annual water allotment data for this irrigation district (1979-2012)$^{12}$, a mean water allotment per hectare has been obtained, and then a gamma distribution function has been fitted to these data ($\chi^2 = 1.8235$; $p$ value = 0.9352). The risk premium values for each instrument under different scenarios is obtained, changing some of the parameters’ values affecting the risk premium (Table 3).

Table 3. Parameters’ values for the Campo de Cartagena irrigation district.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercised price ($P_x$)</td>
<td>0.120</td>
<td>0.120</td>
<td>-0.020</td>
<td>0.120</td>
</tr>
<tr>
<td>Probability of having a water allotment lower than $w^*_y$ ($y$)</td>
<td>0.200</td>
<td>0.200</td>
<td>0.200</td>
<td>0.250</td>
</tr>
</tbody>
</table>

$$\alpha$$ and $$\lambda$$ values were obtained from the fitted gamma distribution of the water allotment. $$w_g$$ is set as the p20 value of farmer’s water allotment (649 m$^3$; p25 for scenario D, 746 m$^3$), as the probability of exercising the option is 20% (25% for scenario D). $$b$$ and $$P_e$$ are normal values for the region under study. Below, a figure for each scenario (A, B, C and D) is shown, being A the baseline scenario.

Considering different scenarios allow us to assess the values of the risk premium for the four proposed risk management tools under different circumstances, obtaining the farmers’ WTP for them in this irrigation district.

Under scenario B, a higher Z value is considered in comparison to scenario A. This change has a pronounced effect on R2, which is very sensitive to changes in Z values. R4 value is very similar to the one for scenario A, so the impact of this change is imperceptible.

Figure 2. Risk premium value (in €) for each proposed instrument under a different scenario.
R1: risk premium for the option contract (1); R2: option contract (2); R3: insurance; R4: combination of option contract (2) plus the insurance.

A reduction in the risk aversion coefficient (scenario C) would cause a decrease of the WTP for the risk management tools, except for R2, causing an increase in the farmer’s risk premium for this option contract. From all the scenarios, we can conclude that the WTP for the option contract 2 (R2) is considerably lower than for the rest of the instruments, even when $p_e < 0$ (scenario C). In this scenario, the order of farmers’ preference for these instruments changes, and the highest WTP is for option contract 1 and for the combination of the option contract and the insurance (case 4). When the price of the water from the option contract is lower than the price that farmers normally pay for the water, the WTP for these instruments increases. A reduction of the exercise price has a very low effect on R2, but a lower risk aversion level has an important impact on it.

If the risk management tool offers the farmer a higher guaranteed water volume (scenario D), the WTP for the instrument will increase for all instruments. The option contract 2 is the less sensitive, remaining almost invariable.

6. Discussion

The application of this theoretical framework to an irrigation district in Spain shows the potential of this approach to evaluate the farmers’ WTP for different risk management tools.

The differences among farmers’ WTP for the option contract (1), the insurance (case 3) or the combination of the option contract and the insurance (case 4) is not very significant. At the end, the final decision to purchase one instrument or the other would be based on:

- The price actually paid for each instrument (as previously explained in section 3).

- The design of the instrument. Some of the features of the risk management tool (maturity date, process to get the indemnization/optioned water, time needed to arrange the contract, etc.) would make the instrument more or less attractive to the farmer.

13 During the 2005-2008 drought period, the Spanish Government allowed for inter-basin market exchanges to alleviate the conditions of the most affected river basins (Garrido et al., 2012b). That was the case of the agreement between the irrigation district Canal de Estremera (Tagus Basin) and the SCRATS (Sindicato Central de Regantes del Acueducto Tajo-Segura, Segura Basin). The signed contract between them worked as an option contract, and the exercise price was 0.06 €/m³ rather than 0.12 €/m³ (the water tariff paid by irrigators in the Segura basin). This case illustrates the possibility of the exercise price being lower than the normal water tariff ($p_w$). In Spain, with considerable differences in water productivity among regions, as mentioned before, this situation would be possible if water is sold from a low-productive region, to a very high-productive one.
- Farmer's reliability on the other agent involved in the contract (the water seller in the case of the option contract, and the insurance company when he purchases the insurance).

- The exercise price. If it is considerably lower than the price that the farmer has to pay for his water allotment, the WTP for the option contract could increase and become significantly higher than the WTP for the insurance.

The obtained results of the farmers' WTP from the application of the theoretical model to the Campo de Cartagena irrigation district (Segura Basin) are similar to the results from previous works on the same region. For example, Rigby et al. (2010) assert that farmers in this same irrigation district would be prepared to pay a premium near to €330 for an increase in their water supply reliability.

Irrigators in this region have bought water from other water users (some of them in a different river basin) during the drought period 2005-2008; so they are interested in protecting their activity against the risk of not having enough water. Because the Segura basin is a very productive region, irrigators might be willing to pay more for securing a minimum water volume every year than in other areas of Spain.

7. Conclusions

Water supply uncertainty is one of the main risks faced by water users. The existence of risk management tools can protect them, guaranteeing a minimum water volume each season to cover, at least, their basic water needs.

The farmer's decision to sign an option contract or an insurance policy depends on his utility function. The associated expected utility determines the preferences of the farmer for one instrument or the other. There are several factors that affect farmer's utility: risk aversion, profit function, risk premium for each instrument, and the exercise price in the case of the option contract.

Knowing in which cases the farmer is going to prefer one risk management instrument to the other help us to understand the potential demand of these tools, and to design the most appropriate mechanism for a certain region or agent. All the comparisons collected in this work can be applied to a certain case, given values to the different parameters, and finding the best option for a farmer based on his risk preferences.

The potential of this type of mechanisms for the Spanish agriculture is very high, as drought episodes in this country are a recurrent phenomenon. Differences in water productivity among different water users facilitate the arrangement of this type of contracts between them. Though in this study we are considering the case of a farmer as a water option holder
or as an insured, this same mechanism can be used by cities as well, increasing cities’ water supply during drought periods; or by regional governments to protect the environment.

8. References


**Appendix 1: Expected utility for case 0 (with no risk management tool).**

\[
EU_0(\bar{\pi}) = \int_0^{\bar{w}} U(\pi_0) f(w) dw = \int_0^{\bar{w}} \left[ 1 - e^{-r(a+bw)} \right] f(w) dw = \int_0^{\bar{w}} f(w) dw - \int_0^{\bar{w}} e^{-ra} e^{-rbw} f(w) dw = 1 - e^{-ra} \int_0^{\bar{w}} e^{-rbw} f(w) dw = 1 - e^{-ra} MGF_w(-rb)
\]

**Appendix 2: Expected utility and risk premium calculation for case 1 (with option contract 1).**

a) \[
EU_1(\bar{\pi}) = \int_0^{w_g} \left[ 1 - e^{-r(a+bw_g-P_1-P_e(w_g-w))} \right] f(w) dw + \int_{w_g}^{\bar{w}} \left[ 1 - e^{-r(a+bw-w)} \right] f(w) dw = \gamma - e^{-r(a+bw_g-P_1-P_ew_g)} \int_0^{w_g} e^{-rp_{1w}} f(w) dw + (1-\gamma) - e^{-r(a-P_1)} \int_{w_g}^{\bar{w}} e^{-rbw} f(w) dw = 1 - e^{-r(a+bw_g-P_1-P_ew_g)} LMGF_w(-rP_e) - e^{-r(a-P_1)} UIMGF_w(-rb)
\]

b) \[
EU_0(\bar{\pi}) = EU_1(\bar{\pi})
1 - e^{-ra} MGF_w(-rb) = 1 - e^{-r(a+bw_g-R_1-P_ew_g)} LMGF_w(-rP_e) - e^{-r(a-R_1)} UIMGF_w(-rb)
-e^{-ra} MGF_w(-rb) = -e^{-r(a+bw_g-R_1-P_ew_g)} LMGF_w(-rP_e) - e^{-r(a-R_1)} UIMGF_w(-rb)
MGF_w(-rb) = e^{rR_1} \left[ e^{-r(b-P_e)w_g} LMGF_w(-rP_e) + UIMGF_w(-rb) \right]
R_1 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{e^{-r(b-P_e)w_g} LMGF_w(-rP_e) + UIMGF_w(-rb)} \right)
\]

**Appendix 3: Upper Incomplete Moment Generation Function (UIMGF)**

We consider that variable \( \bar{w} \) follows a Gamma distribution \( f(w) \):
$$f(w) = \frac{\lambda^a}{\Gamma(a)} w^{a-1} e^{-\lambda w}$$

$$UIMGF_w(-rb) = \int_{w_g}^{\infty} e^{-rbw} \frac{\lambda^a}{\Gamma(a)} w^{a-1} e^{-\lambda w} dw = \left[ -\frac{\lambda^a w^a E_1(a(\lambda + rb)w)}{\Gamma(a)} \right]_{w_g}^{\infty}$$

\(E\) is an exponential integral function.

$$E_n(x) = z^{n-1} \Gamma(1-n, z)$$

$$E_{1-a}(\lambda + rb)w = (\lambda + rb)w)^{-a} \Gamma(\lambda + rb)w)$$

So, the expression of the \(UIMGF_w(-rb)\) is:

$$UIMGF_w(-rb) = -\frac{\lambda^a}{\Gamma(a)} \left[ w^a (w(\lambda + rb))^{-a} \Gamma(\lambda + rb)w) \right]_{w_g}^{\infty} = -\frac{\lambda^a}{\Gamma(a)} \left[ \Gamma(\lambda + rb)w) \right]_{w_g}^{\infty}$$

$$= \frac{\lambda^a}{(\lambda + rb)^a} \left[ -\Gamma(\lambda + rb)w) \right]_{w_g}^{\infty} = MGF_w(-rb)([Q(\lambda + rb)w)] - [Q(\lambda + rb)w)]$$

\(Q(\cdot)\) is a regularized gamma function, whose domain is [0,1].

**Appendix 4: Expected utility and risk premium calculation for case 2 (with option contract 2).**

a)

$$EU_2(\bar{r}) = Z \int_{w_g}^{\infty} \left[ 1 - e^{-r(a+bw-r_2-P_r w)} \right] f(w) dw + (1-Z) \int_{w_g}^{\infty} \left[ 1 - e^{-r(a+bw-P_2)} \right] f(w) dw +$$

$$\int_{w_g}^{\infty} \left[ 1 - e^{-r(a+bw-P_2)} \right] f(w) dw =$$

$$Z\gamma - Z e^{-r(a+bw-r_2-P_r w)} \text{LIMGF}_w(-rP_e) + \gamma - e^{-r(a-P_2)} \text{LIMGF}_w(-rb) - Z\gamma +$$

$$Z e^{-r(a-P_2)} \text{LIMGF}_w(-rb) + (1-\gamma) - e^{-r(a-P_2)} \text{UIMGF}_w(-rb) = 1 - e^{-r(a-P_2)} \left( \text{UIMGF}_w(-rb) + \right.$$

$$\text{LIMGF}_w(-rb) - Z e^{-r(a+bw-r_2-P_r w)} \text{LIMGF}_w(-rP_e) + Z e^{-r(a-P_2)} \text{LIMGF}_w(-rb) =$$

$$1 - e^{-r(a-P_2)} \text{MGF}_w(-rb) - Z e^{-r(a+bw-r_2-P_r w)} \text{MGF}_w(-rP_e) + Z e^{-r(a-P_2)} \text{LIMGF}_w(-rb)$$

b)

$$EU_0(\bar{r}) = EU_2(\bar{r})$$

$$= 1 - e^{-r\alpha \text{MGF}_w(-rb)}$$

$$= 1 - e^{-r(a-R_2)} \text{MGF}_w(-rb) - Z e^{-r(a+bw-r_2-P_r w)} \text{LIMGF}_w(-rP_e)$$

$$+ Z e^{-r(a-R_2)} \text{LIMGF}_w(-rb)$$

$$-e^{-r\alpha \text{MGF}_w(-rb)} = -e^{-r\alpha} e^{R_2} [\text{MGF}_w(-rb) + Z e^{-r(b-P_2)w} \text{LIMGF}_w(-rP_e) - Z \text{LIMGF}_w(-rb)]$$

22
\[ R_2 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{(1-Z) MGF_w(-rb) + Z \left[ e^{-r(b-P_e)w_g} LIMGF_w(-rP_e) + UIMGF_w(-rb) \right]} \right) \]

Appendix 5: Expected utility and risk premium calculation for case 3 (with insurance).

a) 

\[
EU_3(\bar{\alpha}) = \int_0^{w_g} 1 - e^{-r(a+bw_g-P_3)} f(w)dw + \int_{w_g}^{w} 1 - e^{-r(a+bw-P_3)} f(w)dw
\]

\[= \gamma - \gamma e^{-r(a+bw_g-P_3)} + (1 - \gamma) - e^{-r(a-P_3)}UIMGF_w(-rb)\]

\[= 1 - \gamma e^{-r(a+bw_g-P_3)} - e^{-r(a-P_3)}UIMGF_w(-rb)\]

b) 

\[EU_3(\bar{\alpha}) = EU_3(\bar{\alpha})\]

\[1 - e^{-r}\Delta MGF_w(-rb) = 1 - \gamma e^{-r(a+bw_g-P_3)} - e^{-r(a-\bar{\alpha})}UIMGF_w(-rb)\]

\[R_3 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{\gamma e^{-rbw_g} + UIMGF_w(-rb)} \right)\]

Appendix 6: Expected compensation

\[
C^e = \int_0^{w_g} b(w_g - w)f(w)dw = bw_g \int_0^{w_g} f(w)dw - \int_0^{w_g} bw f(w)dw
\]

\[= bw_g \gamma - \int_0^{w_g} bw \frac{\lambda^\alpha}{\Gamma(\alpha)} w^{\alpha-1} e^{-\lambda w} dw = bw_g \gamma - \frac{\lambda^\alpha}{\Gamma(\alpha)} \int_0^{w_g} bw^\alpha e^{-\lambda w} dw\]

\[= bw_g \gamma - \frac{\lambda^\alpha}{\Gamma(\alpha)} \left[ -bw^\alpha + 1F_{\alpha}(\lambda w) \right]_0^{w_g} = bw_g \gamma + \frac{b}{\lambda \Gamma(\alpha)} \Gamma(\alpha + 1, \lambda w_g) - \alpha \Gamma(\alpha)\]

Appendix 7: Expected utility and risk premium calculation for case 4 (with option contract 2 + insurance).

a) 

\[
EU_4(\bar{\alpha}) = Z \int_0^{w_g} \left[ 1 - e^{-r(a+bw_g-P_4-P_e(w_g-w))} \right] f(w)dw + (1-Z) \int_0^{w_g} \left[ 1 - e^{-r(a+bw_g-P_3)} \right] f(w)dw
\]

\[+ \int_{w_g}^{w} \left[ 1 - e^{-r(a+bw-P_3)} \right] f(w)dw\]

\[= Z\gamma - Z e^{-r(a+bw_g-P_4-P_e w_g)} LIMGF_w(-rP_4) + \gamma - \gamma e^{-r(a+bw_g-P_4)} - Z\gamma + Z \gamma e^{-r(a+bw_g-P_4)}\]

\[+ (1 - \gamma) - e^{-r(a-P_3)}UIMGF_w(-rb)\]

\[= 1 - e^{-r(a-P_4)} \left[ UIMGF_w(-rb) + Z e^{-r(b-P_e)w_g} LIMGF_w(-rP_e) + (1-Z)\gamma e^{-rbw_g} \right]\]
b)  

\[ EU_4(\pi) = EU_4(\pi) \]

\[ 1 - e^{-rU}MGF_w(-rb) = 1 - e^{-r(a-R_4)}[U1MGF_w(-rb) + Ze^{-r(b-P_e)w}LIMGF_w(-rP_e) + (1 - Z)Ye^{-rbw}] \]

\[ MGF_w(-rb) = e^{rU}[U1MGF_w(-rb) + Ze^{-r(b-P_e)w}LIMGF_w(-rP_e) + (1 - Z)Ye^{-rbw}] \]

\[ R_4 = \frac{1}{r} \ln \left( \frac{MGF_w(-rb)}{U1MGF_w(-rb) + Ze^{-r(b-P_e)w}LIMGF_w(-rP_e) + (1 - Z)Ye^{-rbw}} \right) \]

**Appendix 8: Comparison of instruments**

a) **Comparison between cases (1) and (4)**

\[ R_4 > R_1 \text{ if} \]

\[ e^{-r(b-P_e)w}LIMGF_w(-rP_e) + U1MGF_w(-rb) > U1MGF_w(-rb) + Ze^{-r(b-P_e)w}LIMGF_w(-rP_e) + (1 - Z)Ye^{-rbw} \]

\[ (1 - Z)e^{-r(b-P_e)w}LIMGF_w(-rP_e) > (1 - Z)Ye^{-rbw} \]

\[ e^{rP_e}wLIMGF_w(-rP_e) > Y \]

This condition is always met for \( P_e > 0 \) (see the explanation related to equation [22] and footnote 9 for the case of \( P_e < 0 \)).

If \( R_1 - P_1 > R_4 - P_4 \), then the farmer would purchase the option contract 1. If \( P_1 - P_4 < \frac{1}{r} \ln \left( \frac{D_4}{D_1} \right) \), the farmer would sign this option contract rather than the combination of the insurance plus the option contract (case 4). \( D_4 \) is always smaller than \( D_1 \). So, \( \frac{1}{r} \ln \left( \frac{D_4}{D_1} \right) < 0 \).

\[ P_4 - P_1 < -\frac{1}{r} \ln \left( \frac{D_4}{D_1} \right) > 0 \]

The farmer would choose the combination of instruments rather than the option contract when \( P_4 > P_1 \) as long as the difference between them does not exceed \( \frac{1}{r} \ln \left( \frac{D_4}{D_1} \right) \).

b) **Comparison between cases (2) and (4)**

\[ R_2 > R_4 \text{ if} \]

\[ (1 - Z)MGF_w(-rb) + Z[e^{-r(b-P_e)w}LIMGF_w(-rP_e) + U1MGF_w(-rb)] < U1MGF_w(-rb) + Ze^{-r(b-P_e)w}LIMGF_w(-rP_e) + (1 - Z)Ye^{-rbw} \]
The right side of this expression is the denominator of $R_3$, and the left side the numerator. So, for $R_3$ to be positive, this condition should be the opposite. That is why $R_4 > R_2$. Intuitively, as this management tool (4) offers more guarantees to the farmer than the option contract (2), the WTP for it would be always higher for a risk averse agent. For $R_4$ to be higher than $R_2$, the denominator of the former should be smaller than the denominator of the latter.

If $P_2 - P_4 < \frac{1}{r} \ln \left( \frac{D_4}{D_2} \right)$, the farmer would sign the option contract (2) rather than the combination of this option contract and the insurance.

$$\frac{1}{r} \ln \left( \frac{D_4}{D_2} \right) < 0$$

$$P_4 - P_2 < -\frac{1}{r} \ln \left( \frac{D_4}{D_2} \right) > 0$$

**c) Comparison between cases (3) and (4)**

$R_3 > R_4$ if

$$\beta e^{-rbw} + UIMGF_w(-rb) < UIMGF_w(-rb) + Ze^{-r(b-p_e)w} LIMGF_w(-rP_e) + (1 - Z) \beta e^{-rbw}$$

$$Z \beta e^{-rbw} < Ze^{-r(b-p_e)w} LIMGF_w(-rP_e)$$

$$\beta < e^{rPew} LIMGF_w(-rP_e)$$

Note that the above expression is exactly the opposite to the one shown in the previous case. So, this condition is going to be always met, and as a conclusion $R_3 > R_4$ (so $D_4 > D_3$).

If $P_3 - P_4 < \frac{1}{r} \ln \left( \frac{D_4}{D_3} \right)$ the farmer would purchase the insurance. As $\frac{1}{r} \ln \left( \frac{D_4}{D_3} \right) > 0$; he is going to purchase the combination of instruments even if the premium is higher, whenever the difference between them is not higher than $\frac{1}{r} \ln \left( \frac{D_4}{D_3} \right)$. 

\[(1 - Z) MGF_w(-rb) < (1 - Z)UIMGF_w(-rb) + (1 - Z)\beta e^{-rbw}\]

$$MGF_w(-rb) < UIMGF_w(-rb) + \beta e^{-rbw}$$