# AN ANODELESS TETHER GENERATOR

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A new, simple concept for electron collection by an electrodynamic tether is presented. No anodic contactor is needed, the tether itself, left bare, drawing a current with neither shielding nor magnetic effects. Application to a generator is discussed.

There is a broad consensus that collection of electrons from the rarefied ionosphere, at the positively biased, anodic end of a conductive cable is the most critical issue facing the electrodynamics of tethers [1]. At the present time, both technology and theory of plasma contactors, intended to solve the problem, show gross uncertainties. In this work we discuss a new, simple concept, an anodeless tether, which is based on the effectiveness of elongated cylinders as collectors, for the conditions of interest, and on the use of the tether itself for that purpose[2].

The generic obstacle in the road to practical electrodynamic applications is the fact that the thermal current density, $J_t = \frac{1}{4} en_\infty \times (8kT_e/\pi m_e)^{1/2}$, is small. For typical ionospheric F-layer temperature, $T_e \simeq 10^3$K, and density, $n_\infty = 0.5\times10^{11}-10^{12}m^{-3}$, $J_t$ lies in the range 0.4–8 $mA/m^2$. To attain an useful current through a tether load, $I_L = 10A$ say, the effective collecting area, $A_{eff} \equiv I_L/J_t$, may exceed $10^4 m^2$.

Certainly, since the anode might have a positive bias $\Delta V_A$ relative to the ambient plasma, the actual area of its collecting surface need not be as large as $A_{eff}$. It is difficult, however, to obtain a high area gain,

$$G \equiv A_{eff} / \text{ Anodic area},$$

because both electron Debye length $\lambda_D$ and thermal gyroradius $l_e$ are small. If the characteristic anodic length is large compared with $\lambda_D$ ($\simeq$5mm for density at mid-range) electric shielding will severely limit the current collected; if the anodic length is also large compared with $l_e$ [$\simeq$30mm at speed $(\pi kT_e/2m_e)^{1/2}$ and magnetic field $0.3\times10^{-4}$T] magnetic guiding of electrons will have a similar effect.

To make the point definite, consider the simplest case, a passive, spherical anode of radius R. We would then have a gain

$$A_{eff}/4\pi R^2 = G_s(e\Delta V_A/kT_e\ ,\ R/\lambda_D\ ,\ R/l_e)\ .$$

In the absence of a magnetic field ($R/l_e \to 0$), and for both high $e\Delta V_A/kT_e$ and high gain, classical theory gives [3]

$$G_s \simeq 1.5\ (e\Delta V_A/kT_e)^{6/7}(\lambda_D/R)^{8/7}\ .$$

For R=3m or $4\pi R^2 \simeq 10^2 m^2$, a gain $G_s$=50 requires $e\Delta V_A/kT_e \simeq 3\times10^5$ or $\Delta V_A \simeq$30.000 volts, an absurdly high anodic bias. Clearly, taking the large value of $R/l_e$ into consideration will only compound the difficulty. In principle, plasma contactors would avoid electric shielding by ejecting ions, and magnetic guiding by having electrons scattered off plasma turbulence generated by the relative motion of attracted electrons and emitted ions.

Now, an elongated cylinder of collecting length $L_B$ and diameter d, acting as a passive anode, would avoid either effect by just introducing two disparate lengths, $d \ll L_B$. We might then have $\pi d L_B \sim 4\pi R^2$, and yet $d \ll R$. Also collection would be a two-dimensional process governed by the smallest length, d. We thus could write

$$A_{eff}/\pi d L_B = G_c(e\Delta V_A/kT_e\ ,\ d/\lambda_D\ ,\ d/l_e)\ .$$

If both $d/\lambda_D$ and $d/l_e$ are negligible there is neither guiding nor shielding, and the current is orbital-motion-limited (OML), yielding [4]

$$G_c = (4e\Delta V_A/\pi kT_e)^{1/2} \; ;$$

a gain $G_c=50$ requires $\Delta V_A \approx 200$ volts, a quite reasonable bias.

A collecting surface area of about $10^2 m^2$ might be attained with a radius of 1mm or over, and $L_B$ up to 10 Km. Then, the corresponding ratio $d/\lambda_D$ would not be small but, fortunately, in two-dimensional geometry and for a negligible magnetic field, the OML current is attained for all $d<4\lambda_D$ roughly (in the three-dimensional case, on the contrary, OML conditions are only achieved for vanishing $R/\lambda_D$)[5]. Further, magnetic effects will here be negligible, indeed, because of the cylindrical geometry too: for a cylinder with radius a few per cent of $l_e$, a current upper-bound that characterizes field-line guiding becomes much larger than the OML current when $e\Delta V_A/kT_e$ is made large; magnetic forces may thus be dropped against electric forces [6]. (Again, for spherical collection it is the other way around: the OML current greatly exceeds the bound due to field guiding).

The second piece of the new concept is that there would be no need to actually set up and connect this peculiar anode to the tether. The tether itself, if left uninsulated, totally or only in a positively biased part, could serve for the purpose. Here we apply the concept to the case of an upwards-deployed generator. A bare tether could also work, however, as a *downwards-deployed thruster* [2].

Consider first a standard (fully insulated) tether of length L, conductive cross section $A_c$, and conductivity $\sigma$. Its circuit equation is simply

$$E_m L = (R_L + L/\sigma A_c) I_L + \Delta V_A(I_L), \qquad (1)$$

where $R_L$ is the load impedance and $E_m$ the projection, along the cable, of the motional electric field (vector product of geomagnetic field and orbital velocity $v_{orb}$). We have assumed that the impedance of the anodic contactor is much larger than that of both the ionospheric closure-path and an electron-emitting hollow cathode. There are two dimensionless numbers characterizing the quality of the generator. First, the efficiency $\eta \equiv W_L/W_m$, where $W_L = R_L I_L^2$ is the useful power and $W_m = E_m I_L L$ is the mechanical power-loss due to magnetic braking. Second, a dimensionless measure of useful power per unit mass of conductive cable, $\bar{w} \equiv (\rho_M/\sigma E_m^2) W_L/M$; $\rho_M$ and $M = \rho_M L A_c$ are density and mass of cable. The range of possible values for $\rho_M/\sigma E_m^2$ is limited; for Al and $E_m = 200$ V/Km, we have $\rho_M/\sigma E_m^2 \simeq 1.9$Kg/Kw.

Using (1) one obtains parametric equations, $\eta(i_L), \bar{w}(i_L)$,

$$\eta = 1 - i_L - \Delta V_A(\sigma E_m A_c i_L)/E_m L \quad , \qquad \bar{w} = i_L \eta(i_L) \quad , \tag{2}$$

where $i_L$ is the fraction of short-circuit current. The relation $\bar{w}(\eta)$ that follows from these equations [e.g., $\bar{w} = \eta(1-\eta)$ for $\Delta V_A/E_m L$ negligible-ideal tether-] shows the trade-off common to most electrical generators: too high an efficiency gives too low a power per unit mass. Note that, accordingly, $i_L$ should be <u>moderately</u> <u>small</u>, $i_L \sim 0.1$–02. A choice of $\bar{w}$ leads to both $\eta$ and $W_L/M$; fixing $W_L$, in addition, determines $A_c L$. Since both $\Delta V_A$ and $d\Delta V_A/dI_L$ are positive, a tether with given $W_L$ and M is closer to ideal the larger is L, or the smaller is $A_c$. The most stringent limitation on length arises from the high-voltage insulation. A maximum voltage between cable and ionosphere, $\Delta V \sim 5$kV, equivalent to L~25 Km for $E_m = 200$V/Km, has been suggested [7]. This places limits on the power available.

Consider now a tether with no anodic contactor, deployed upwards in a <u>normal</u> west-east orbit. The local bias $\Delta V(y)$ will be positive over some upper segment AB, with a peak value $\Delta V_A$ (Fig. 1). We shall first assume that only a lower, larger portion of the cable, from C to around, or below, B, is insulated. We may then neglect ion collection, if any, and write $I = I_L$ over the segment BC; the circuit equation is here

$$E_m L = E_m L_B + [R_L + (L - L_B)/\sigma A_c] \; I_L \, . \tag{1'}$$

From A to B the tether will collect electrons in the OML regime [4]

$$dI/dy = e n_\infty d (2e\Delta V/m_e)^{1/2} \tag{3}$$

with $d\equiv(4A_c/\pi)^{1/2}$. The local bias is $\Delta V=V_t-V_p$, where the tether potential $V_t$ satisfies Ohm's law, $I(y)=\sigma A_c dV_t/dy$, and the faraway plasma potential (in the tether frame) is given by $dV_p/dy=E_m$. Then we have

$$d\Delta V/dy = -E_m + I/\sigma A_c \ . \qquad (4)$$

Using (4) we get a power loss, $W_m\equiv\int_0^L E_m I(y)dy=\sigma E_m A_c(E_m L_B-\Delta V_A)+E_m I_L(L-L_B)$.

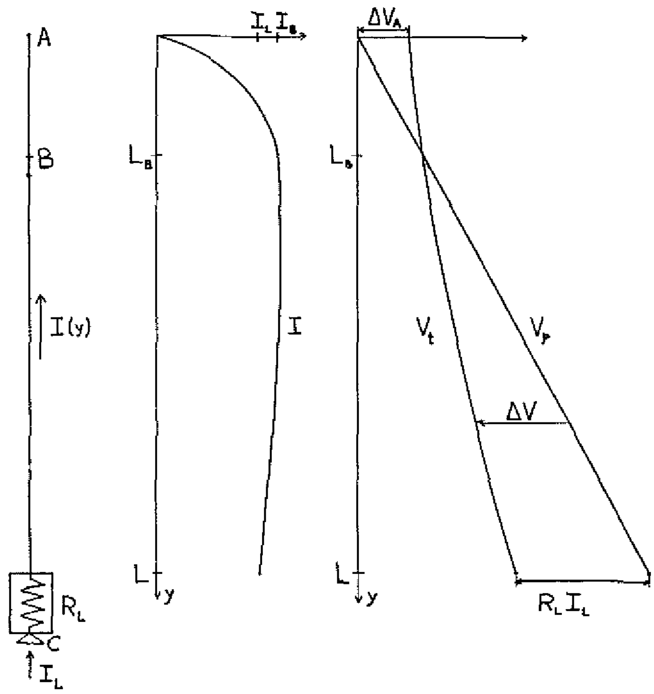


Figure 1.- Upwards-deployed generator with cathodic contactor C and load impedance $R_L$; $V_p(y)$ and $V_t(y)$ are the faraway plasma and tether potentials. Segment AB collects electrons ($\Delta V\equiv V_t-V_p>0$). The load current equals $I_B$ if segment BC is insulated; it is $I_L<I_B$, if BC is left bare.

The boundary conditions for equations (3),(4) are I=0 at y=0, and $I=I_L$ where $\Delta V=0$. In addition, however, we have $\Delta V=\Delta V_A$ at point A and $y=L_B$ at B. This yields two relations,

$$\Delta V_A= E_m L_*(2i_L-i_L^2)^{2/3} \quad , \quad L_B= L_*\xi_B(i_L) \quad ,$$

with

$$L_* = (9\pi m_e\sigma^2 E_m A_c/128\ e^3 n_\infty^2)^{1/3} \quad ;$$

for $E_m$ = 200V/Km and an aluminum cable one has $L_*\approx 6.2$Km at a mid-range density and $A_c$=1mm$^2$. For $i_L$ small, $\xi_B$ takes the form

$$\xi_B \simeq (2i_L)^{2/3}\ (1+i_L/4)\ .$$

Finally, $\eta$ and $\bar{w}$ are then given by

$$\eta = (1-i_L)\left[1+\frac{3}{5}\,\frac{L_*\xi_B(i_L)}{L-L_*\xi_B(i_L)}\right]^{-1}\ , \qquad (2'a)$$

$$\bar{w} = i_L(1-i_L)\ [1-L_*\ \xi_B(i_L)/L]\ . \qquad (2'b)$$

Figure 2 shows $\bar{w}(\eta)$ from (2') for several values of $L_*/L$, together with the ideal law, $\bar{w}=\eta(1-\eta)$. Again, $i_L$ must be moderately small; also, for given $W_L$, the larger L (or smaller $A_c$), the closer to ideal. Setting $L-L_B$=25 Km from insulation restrictions, and using $W_L \propto A_c L\bar{w}$, $A_c \propto n_\infty^2 L^3$ we get an estimate of power limitation:

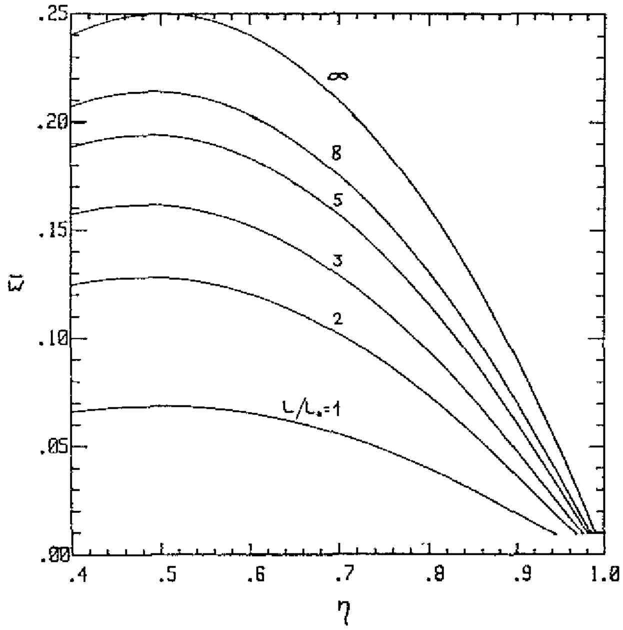


Figure 2.- Partially bare generator. Dimensionless load power $\bar{w}$ versus efficiency $\eta$, for several values of length parameter $L_*/L$; the ideal tether corresponds to $L_*/L$=0.

$$W_L^{max}(kW) \simeq \frac{459(L_*/L)^3}{(1-\xi_B L_*/L)^4}\left(\frac{n_\infty}{10^{11}m^{-3}}\right)^2 \bar{w} \quad .$$

A low value of $L_*/L$, or equivalently, $L_B/L$, though convenient in (2'), would produce too low a power. A reasonable compromise could be $\xi_B L_*/L=1/5$. At $\bar{w}=0.1$ the efficiency is then $\eta\simeq0.74$ ($i_L\simeq0.15, \xi_B\simeq0.46$) and $W_L^{max} \simeq 2.5$–$1000$Kw for $n_\infty \simeq 0.5\times10^{11}$–$10^{12}m^{-3}$.

We can get rid of insulation considerations by simply letting the entire tether go bare. Ion collection along the segment BC will somewhat decrease both $\bar{w}$ and $\eta$ by reducing the current through the load, $I_L<I(y=L_B)$. Over most of BC, $e|\Delta V|$ is large compared with $\frac{1}{2}m_i v_{orb}^2$ ($\simeq$10eV for $O^+$ ions), and we again have OML current,

$$dI/dy = - end_\infty(2e|\Delta V|/m_i)^{1/2} \quad , \qquad L_B<y<L \quad .$$

For $i_L$ small, the bias is roughly proportional to $y-L_B$ (Fig.1); we find

$$\text{Ion current/Electron current} \sim (m_e/m_i)^{1/2}\,(L-L_B)^{3/2}/L_B^{3/2} \; .$$

Clearly, a high efficiency requires a ratio $L_B/L$ moderately small: this ratio, if too high, would reduce the voltage available at the load but, if too low, would produce a large ionic current.

When the ion current into BC is taken into account, the envelope of the family $\bar{w}(\eta,L_*/L)$, $L_*/L$ parameter, gives an upper bound $\bar{w}_M(\eta)$ [less than the ideal value $\eta(1-\eta)$], which may be said to correspond to an optimal design for a fully bare tether. A trade-off choice of $\eta$ determines $\bar{w}=\bar{w}_M(\eta)$, as well as $i_L$, $L_*/L$, $L_B/L$, and $(I_B-I_L)/I_B$. In Fig. 3 we took $\bar{w}_M$ as abscissa to represent all design dimensionless relations. Note that the optimal ion current is small, so that secondary emission due to ion bombardment of segment BC should hardly affect the results.

For an Al tether the optimal dimensions are

$$\frac{d}{1mm} \simeq \frac{0.75}{\bar{w}_M^{\;3/8}}\left(\frac{L_*}{L}\right)^{3/8} \times \left(\frac{W_L}{1Kw}\right)^{3/8}\left(\frac{n_\infty}{10^{11}m^{-3}}\right)^{1/4}\left(\frac{100V/Km}{E_m}\right)^{7/8} ,$$

$$\frac{L}{1Km} \simeq \frac{6.4}{\bar{w}_M^{\,1/4}} \left(\frac{L}{L_*}\right)^{3/4} \times \left(\frac{W_L}{1Kw}\right)^{1/4} \left(\frac{10^{11}m^{-3}}{n_\infty}\right)^{1/2} \left(\frac{100V/Km}{E_m}\right)^{1/4} .$$

For a choice $\eta$=0.75 ($\bar{w}_M \simeq 0.09, i_L \simeq 0.119, L_*/L \simeq 0.34, L_B/L \simeq 0.137, I_B - I_L \simeq 0.090 I_B$), a 1 Kw generator, at $n_\infty = 10^{12} m^{-3}$ and $E_m$=200V/Km, would have a 1.15 mm diameter and a 7.5 Km length, with a mass M=21Kg. The load current and impedance would be $I_L \simeq 0.87A$, $R_L = 1.31K\Omega$.

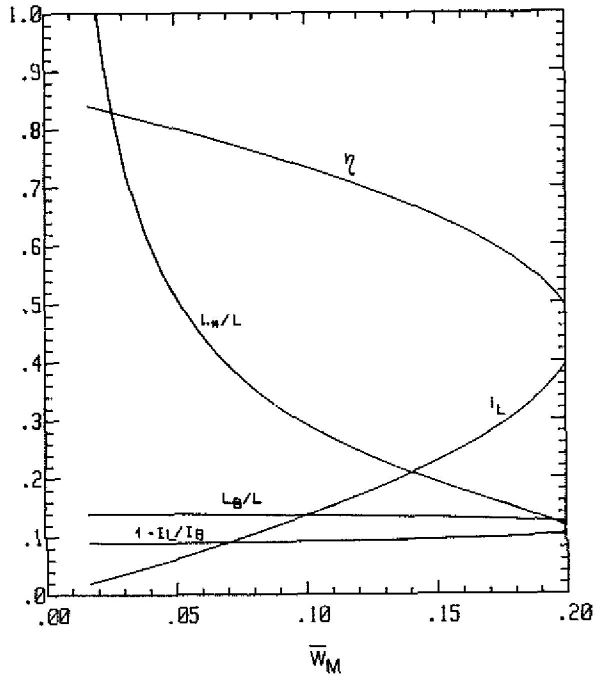


Figure 3.- Optimal fully-bare generator: $\eta$, $L_*/L$, fraction of short-circuit current $i_L$, and fractional anodic length $L_B/L$ and ion-current loss $1-I_L/I_B$, versus dimensionless load power.

## References


1. "Tethers in Space Handbook", 2nd ed., NASA, Washington(1988), pp. 7, 53, 166.
2. J.R.Sanmartín, E.Ahedo, and M.Martínez-Sánchez, submitted to AIAA Journal.
3. S.H.Lam, Phys.Fluids 8, 73(1965).
4. P.M.Chung, L.Talbot, and K.J.Touryan, "Electric probes in stationary and flowing plasmas", Srpinger-Verlag, New York(1975).
5. J.Laframboise, Report 100, UTIAS, Toronto(1966).
6. J.Rubinstein and J.Laframboise, Phys.Fluids 21, 1655(1978); 25, 1174(1982).
7. M.Martínez-Sanchez and D.E.Hastings, J.Astronautical Sci. 35, 75(1987).