Measurements and Prediction of Pedestrian Walking Loads on an Aluminium Catwalk

J. Fernández¹, A. Fraile², E. Alarcón¹ and L. Hermanns²
¹ Department of Structural Mechanics and Industrial Constructions
University of Technology Madrid, Spain
² Mechanical Engineering Centre CEMIM
Foundation for the Promotion of Industrial Innovations, Madrid, Spain

Abstract

This paper reviews the results obtained from measurements taken at different points of an aluminium catwalk while different persons walked back and forth along a prescribed straight path at a certain frequency. Displacements as well as accelerations were measured at several locations on the catwalk. Some mechanical properties have been estimated from the test results like the bending stiffness of the beam, the fundamental frequency as well as the damping ratio of the fundamental mode. With this information at hand, a finite element model was set up and tuned. The experimental acceleration time histories were post-processed in order to identify the ones that were thought to be best suited for comparison with predicted values resulting from applying frequently used walking load models.

Keywords: aluminium structure, human induced vibration, structural dynamics, structural damping, finite element method, walking load model, system identification.

1 Introduction

During the last two decades the topic of human induced vibration has attracted a lot of attention among civil engineering practitioners and academics alike. Usually this type of problem may be encountered in pedestrian footbridges or floors of paperless offices. Slender designs are becoming increasingly popular, and as a consequence, the importance of paying attention to vibration serviceability also increases. There are several examples like [1] that required the temporary closure of infrastructures as retrofitting of the new structure was deemed necessary. These retrofitting measures are extremely expensive and not always viable so once more prevention is much better than cure. It is thought that the lack of information regarding the best modelling practice for this type of structure under walking loads...
is responsible for this situation. Dynamic testing of existing full-scale structures is surely a good approach to increase the knowledge in this area. Apart from deterministic walking load models like that presented in [2] and [4], stochastic models for walking and jumping loads like that presented in [3] are gaining popularity. In the framework of European research projects like [4] and [5] walking load models were developed and according to the authors should accompany the Eurocodes to prevent similar situations to occur in the future. This paper resumes the results obtained from measurements taken at different points of an aluminium catwalk that was subjected to walking loads.

In section 2 some details of the structure tested are presented as well as a description of the equipment used and the types of actions the structure was subjected to. In section 3 the estimation of the mechanical properties of the structure are described. These were obtained by post-processing the test results and by means of analytical as well as numerical models. Section 4 contains the description of the Finite Element Model and the load model. A comparison between experimental results and numerical predictions is presented in section 5. Finally, section 6 contains some brief concluding comments on future research.

2 Test description

2.1 Description of the test structure

The test was performed on an aluminium catwalk which is 6 m in length by 0.6 m in width. Figure 1 shows a photograph of the catwalk and the cross section of the structure. The catwalk is composed by two aluminium sheets located on the upper and lower face respectively that are connected to two side girders which are 17 cm in height.

Figure 1: Photographs of the aluminium catwalk.

The catwalk is a modified version of a commercially available product. The modification consists in the addition of an aluminium sheet to the lower face.
2.2 Description of the performed tests

Measurements were carried out subjecting the structure to different actions:
- Static test: a steel cylinder of 35 kg was placed in the middle of the catwalk
- Dynamic test: this test consists of exciting the structure with singles impulses
- Dynamic test: people walking on the catwalk

Some of these actions are shown in Figure 2.

Figure 2: Photographs of the static test and a person walking on the catwalk

The following measurement equipment was used in the test: six piezoelectric accelerometers, two displacement laser sensors, one linear variation differential transformer (LVDT), one acquisition system and one PC.

Figure 3: Measurement equipment: accelerometer, laser and acquisition system

3 Identification of the mechanical properties

The identification of the mechanical properties of the structure was part of a student project on system identification. Therefore indirect methods were used to estimate properties like the stiffness or mass of the structure.

3.1 Support stiffness

Using the static test results it is possible to calculate the stiffness of the support springs which represent the flexibility of the auxiliary structures used as supports.
To this end, the displacement measured with the laser sensor installed at the support is used. ($\delta_{sup}=0.21$ mm). The displacement jump in figure 4 is caused by the steel cylinder that was placed in the middle of the beam.

![Figure 4: Support displacement during the static test](image)

Taking into account the support reaction and the mass of the steel block the spring stiffness can be estimated in the following way:

\[ P = mg; \quad P = 35 \times 9.8 = 343 N \]  \hspace{1cm} (1)

\[ R_{sup} = \frac{P}{2}; \quad R_{sup} = \frac{343}{2} = 171.5 N \]  \hspace{1cm} (2)

\[ k_{sup} = \frac{R_{sup}}{\delta_{sup}}; \quad k_{sup} = \frac{171.5}{0.21 \times 10^{-3}} = 816666 \frac{N}{m} \]  \hspace{1cm} (3)

### 3.2 Beam bending stiffness

The total displacement field can be considered as the sum of two contributions: the displacements corresponding to an infinitely stiff beam mounted on two spring supports and that of a simply supported beam with bending stiffness $EI$.

![Figure 5: Structural model used for hand calculations](image)
The deflection in the middle of the beam that was measured with the second laser sensor \((\delta_{\text{mid}}=2.948 \text{ mm})\) has been used to estimate the bending stiffness of the beam.

![Figure 6: Deflection at midspan of the beam](image)

The measured displacement at midspan contains both contributions i.e. the spring displacement as well as the deflection resulting from bending moments:

\[
\delta = \delta_{\text{mid}} - \delta_{\text{sup}}; \quad \delta = 2.948 - 0.21 = 2.738 \text{mm} \tag{4}
\]

\[
\delta = \frac{PL^3}{48EI}; \quad EI = \frac{P \times L^3}{48 \delta}; \quad EI = \frac{313 \times 6^3}{48 \times 2.738 \times 10^{-3}} = 514427 \text{Nm}^2 \tag{5}
\]

Obviously the obtained value depends on the chosen structural model i.e. in this first approach the stiffness of rotational springs at the supports has been considered negligible.

### 3.3 Natural frequency

The fundamental natural frequency of the catwalk has been estimated using two methods that have been applied to the acceleration time history of an accelerometer that has been attached to the midspan of the beam:

#### 3.3.1 Time domain analysis

This simple method consists of counting the cycles per second in the acceleration time history. As can be seen in figure 7 there are approximately 8.5 cycles per second corresponding to a natural frequency of 8.5 Hz.
3.3.2 Frequency domain analysis

This method consists in estimating the Power Spectral Density (PSD) by means of Welch’s method and then identifying the frequency corresponding to the peak value of the PSD. Figure 8 displays the PSD obtained for the free vibration time history. The peak value is located at 8.5 Hz thus confirming the result obtained by the first method.

\[ \zeta = \frac{1}{2\pi} \ln \left( \frac{u(t)}{u(t+T_d)} \right) \]

(6)

3.4 Damping ratio

The damping ratio corresponding to the fundamental mode of vibration has been estimated by calculating the logarithmic decrement. The damping ratio can be determined by the comparison of the structural response at two instants in time separated by \( T_d \) or its integer multiples i.e. \( j \cdot T_d \). For \( j=1 \) the following expression applies

\[ \frac{u(t)}{u(t+T_d)} = e^{\zeta \omega T_d} \]
Approximating the damped period $T_d$ by the one corresponding to the undamped system $T_n$ the following expression is obtained:

$$\delta = \frac{1}{j} \ln \left( \frac{u(t)}{u(t + jT_d)} \right) \approx 2\pi \zeta$$  \hspace{1cm} (7)$$

The damping ratio calculated by this way depends on the number of cycles $j$ chosen and the quality of the experimental data. The free vibration record has been subdivided into three parts and the number of cycles $j$ has been varied between 5 and 25. The corresponding results are resumed in table 1.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Amplitude (m/s²)</th>
<th>Logarithmic decrement</th>
<th>Damping ratio $\zeta$ (%)</th>
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<tr>
<td></td>
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<td>Initial peak</td>
<td>Final peak</td>
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<tr>
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<td>1</td>
<td>5</td>
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</table>

Table 1. Estimated damping ratio of the fundamental mode

The results suggest that the damping ratio depends on the maximum amplitude of vibration. This is confirmed by estimating the damping ratio using a separation in time of $20 T_d$ all along the free vibration signal. The result is displayed in figure 9 where the damping ratio is plotted vs. time. It seems that the damping ratio converges to 0.23% for low amplitudes of vibration.

### 3.5 Mass of the structures

From the properties calculated previously the mass of the structure has been estimated by two methods:

#### 3.5.1 Rayleigh method

The harmonic response of a system may be described by the following equation:

$$u(x,t') = z_0 \sin \omega_n t' \psi(x)$$ \hspace{1cm} (8)
Where $\psi(x)$ is an assumed deformation shape of the system, $z_0$ is the maximum amplitude and $\omega_n$ is the natural vibration frequency. For the velocity the following expression is obtained:

$$\dot{u}(x,t') = \omega_n z_0 \cos \omega_n t' \psi(x)$$  \hspace{1cm} (9)

The maximum potential energy of the system over a vibration cycle is equal to its strain energy associated with the maximum displacement $u_0(x)$. According to Bernoulli's beam theory the following expression results:
The maximum kinetic energy of the system over a vibration cycle is associated with the maximum velocity \( u_0(x) \):

\[
E_{k_0} = \int_0^L \frac{1}{2} m(x)[\dot{u}_0(x)]^2 dx
\]

If there are no dissipative mechanisms in the system both energies, \( E_{k_0} \) and \( E_{s_0} \), are equal. Substituting \( u_0(x) = z_0 \psi(x) \) and \( \dot{u}_0(x) = \omega_n z_0 \psi(x) \) in equations (10) and (11) and taking into account the potential energy of the support springs, the following expression is obtained:

\[
\omega_n^2 = \frac{\int_0^L (EI(x)[\psi''(x)]^2 + 2K_{sup}\delta_{sup}^2) dx}{\int_0^L m(x)[\psi''(x)]^2 dx}
\]

This is the Rayleigh quotient for a system with distributed mass and elasticity [6]. Using the following deformation shape

\[
\psi(x) = (\delta_{mid} - \delta_{sup}) \times \sin \left( \frac{\pi x}{L} \right) + \delta_{sup}
\]

the total mass of the structure is estimated as 80 kg.

### 3.5.2 Iterative matrix method

The mass can also be estimated using matrix structural analysis. To this end, beam elements with four degrees of freedom and simple spring elements have been used. The corresponding stiffness and consistent mass matrix of the beam element are as follows:

\[
K_{elem} = \begin{bmatrix}
12 & 6 & -12 & 12 \\
6 & 4 & -6 & 2 \\
-12 & -6 & 12 & -6 \\
6 & 2 & -6 & 4 \\
\end{bmatrix}
\]

\[
M_{elem} = \begin{bmatrix}
\frac{13}{35} \rho L^A & \frac{11}{210} \rho L^A & \frac{9}{70} \rho L^A & \frac{-13}{420} \rho L^A \\
\frac{11}{210} \rho L^A & \frac{1}{105} \rho L^A & \frac{13}{420} \rho L^A & \frac{-1}{140} \rho L^A \\
\frac{9}{70} \rho L^A & \frac{13}{420} \rho L^A & \frac{35}{140} \rho L^A & \frac{-11}{210} \rho L^A \\
\frac{-13}{420} \rho L^A & \frac{-1}{140} \rho L^A & \frac{-11}{210} \rho L^A & \frac{1}{105} \rho L^A \\
\end{bmatrix}
\]

For the estimation of the mass, the catwalk has been discretized with four elements. Varying the mass of the beam elements and comparing the numerical and experimental natural frequencies permits to estimate the mass of the catwalk using the iterative procedure depicted in figure 10.
Using this method the total mass of the structure is estimated as 79.2 kg. The same result is obtained if the number of elements is increased i.e. a discretization with 4 elements is sufficient.

The small difference between the values obtained by both methods indicates that the assumed deformation shape used for Rayleigh’s method is very similar to the one obtained using matrix structural analysis.

4 Numerical prediction of the structural response due to walking loads

For the numerical prediction of the structural response a walking load model developed in the framework of European research projects [4] and [5] in combination with matrix structural analysis has been used.

4.1 Walking load model

According to the load model developed under the JRC – ECCS cooperation agreement for the evolution of Eurocode 3 [5], the normalized contact force during a step can be described by a polynomial function that depends on the step frequency. In figure 11 the resulting polynomial functions for 3 different step frequencies are displayed.

The contact force is obtained multiplying the normalized contact force by the body weight of the person considered. During the test a metronome was used to help the students maintain a constant step frequency \( f_s \). The frequency chosen was 1.5 Hz. In order to define the contact points they were marked on the catwalk with red insulating tape. The step length has been estimated using the following expression:

\[
d = 1.6389 - 1.5556 \cdot f_s + 0.5556 \cdot f_s^2 \quad [\text{m}]
\]  

(15)
4.2 Structural model

A structural analysis model has been set up using the same beam elements and simple springs described in section 3. The discretization was chosen to facilitate the definition of the loads. To this end, nodes have been placed at the contact points. Thus 12 beam elements have been used to discretize the structure.

Figure 12: Structural model used for transient dynamic analysis

The Newmark-beta method has been used for direct integration of the system of structural dynamics equations. All calculations have been performed in Matlab.
5 Comparison of numerical predictions and measurements

In order to facilitate the comparison of the experimental results with the predicted acceleration values the transient vibration value (TTV) is used.

\[
TTV(t) = \frac{1}{\tau} \int_0^t \phi(t) \, dt
\]

The value used for the time constant \( \tau \) is 1s. A total of 19 students participated in the test but only few of them managed to maintain the step frequency and to hit the previously marked contact points.

In general the numerically predicted values are always higher than the measured ones. One reason for this is that a constant damping ratio of 0.28 % has been used for the simulations. Using a higher damping ratio leads to lower values however, the difference between measured and predicted values is so big that other causes must exist that justify these differences.

Commonly it is thought that walking people are adequately modelled by means of a suitable load model like the one used in this study whereas sitting or standing people are simulated by means of mass, spring and damper systems. More information on
this type of model may be found in [7]. In the light of the results obtained the authors think that the dynamic interaction between the pedestrian and the catwalk has to be accounted for in the model. It seems that a person in contact with the catwalk changes significantly the dynamic behaviour of the system. At this point, it is worth to remember that the estimated mass of the structure is around 80 kg i.e. the mass of a pedestrian is, in general, not very different from this value.

6 Conclusions

In the present study experimental results and numerical predictions for the response of an aluminium catwalk subjected to walking loads have been compared. The damping of this light weight structure depends on the amplitude of vibration which complicates the tuning of a structural model. In the light of the results obtained it seems that the used walking load model is not appropriate as the predicted TTV values are much higher than the measured ones. A simulation model that takes into account the interaction between the structure and the pedestrian seems to be necessary in order to reduce the difference between measured and predicted vibration values.

In the future, the structural model will be improved with the consideration of rotational springs at the supports and using beam elements with torsional degrees of freedom.

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References

