LANDREC, A SET OF INTERACTIVE MICROCOMPUTER PROGRAMS USED AS A HELP ON LAND RECLAMATION PROJECTS.

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INTRODUCTION

LANDREC has been developed with the idea of assisting the reclamation of derelict land, especially on complex calculations requiring processes and a quick presentation of the new design. The programming of several of the used algorithms is shown in various publications, instead some others are original ones.

Although the structuring of LANDREC may suggest an application of a close methodology, it has been made according to the general process of engineering projects. Once the problem to solve is identified, the purpose which defines the on-going projects is the generation and evaluation of every proposed solution from different viewpoints. As an example, a useful methodology would be as follows:

- Design of several solutions to the reclamation problem, and their input to the computer.

- Evaluation of every cost -economic, ecologic and social ones- of each prearranged solution. Some of these costs are difficult to compute.

- Selection of the best possible solution. Among a number of possible options (the suitable generated solutions) which satisfies a set of different objectives (economical, ecological, social and so on). In this way the problem may lead to decision making under multiple-objective conditions.

To satisfy the previous needs and from a merely operative point of view, LANDREC has been arranged in three main subsets of programs:

EVALUATION programs allow to get numerical and graphic displays of every solution shaped on a map. Firstly it is necessary to input the present and the new topographic configuration in the computer storage devices, either by using a grid method or a digitizer.

INTMOREINF programs allow input of information about the influence of the reclamation on the nearby areas by a load of other programs.
developed in our department wth study landscape fragility, the vulnerability of the vegetation...

INTEGRATION programs are designed to test all the generated solutions in order to select the best one. It is possible to use algorithms developed from the utility theory and others using "soft" techniques.

An area in the valley of Jarama river has been chosen to apply these programs. Extractions of gravel have taken place in it and some of the material (200.000 m³) lies in the area and makes the recovery more difficult.

EVALUATION

These programs provide numerical and graphic displays of every proposed solution to the reclaimed area. To compute them it is necessary to input the actual state and the proposed solutions into the computer. Then it is possible to draw a conic perspective of the area from any view-point you choose, from which the designer may provide a general landscape value to the solution. It is also possible to know the economic cost of the earthworks and to minimize it using the transport algorithm. Finally, the drainage network can be studied in order to obtain the kinetic energy of runoff in every point.

The results of this approach may be "estimates" or "interpretations" rather than measurements, they will allow comparisons between solutions. This level of knowledge is certainly attainable and is sufficient in most of the reclamation processes. It goes without saying that the greater the accuracy in quantifying the values, the more precise will be the results obtained.

Introducing information into the computer storage devices

To generate and display different solutions and to display the actual topographic configuration it is necessary to shape them on topographic maps, with their vertical component (the actual or the new relief) as well as the horizontal one (the actual or the new land-use assigned to the zone). There are two ways to input both characteristics into the computer; the raster method in which topographic and land-use information can be stored as a matrix of values representing elevations or different land uses at mesh points on a grid, and the vectorial method in which the information is stored as a set of points that represent contour lines or the lines that separate different classes of land use.

Most of the applications may be done either from a raster or from a vectorial data base, but usually the vectorial way allows a bigger precision. However, because of the low speed and shortage of memory of the microcomputers, it is very frequently necessary to translate the vectorial base into a raster one. On the other hand, the graphic
outputs from a pinter of characters are more and more considered as low quality products, but plotter outputs need a vectorial base. These considerations and the comparatively high price of the digitizers allow us to suggest the use of both data base systems.

Before beginning to input information, it is necessary to make decisions about the number of points per length unit, if information is going to input from a digitizer, or about the suitable mesh size if a square grid is going to be superimposed on the map to input the information of each cell.

The first problem can be solved by studying the whole length of lines to input and the available capacity of the computer devices. The second one is a bit more complex; from a sampling point of view the ideal mesh has a spacing that ensures the existence of at least one point between each pair of contours (MacDougall-1). However, this consideration can imply a rather small grid size. To solve this problem we are developing a method which forces a grid size with a low standard error for the systematic sampling consisting in the choice of a point of each cell, and that on the other hand, it has a low processing data time cost.

There exists an automatic way to obtain a standard new topographical configuration with minimum earthwork and with a slope that allows the growth of the vegetation; this method is not however available in microcomputers.

It consists in using the goal programming method to minimize positive and negative deviations between the present and the new elevations of the point chosen to make the analysis. To reduce the number of the decision variables it is possible to change the size of the grid, using a higher size on the flatter areas and another smaller on those with a steep slope. In this case, it is necessary to multiply the decision variable by the area assigned to each point of the grid. As the minimum vegetation growth slope may be changed from one land use to another, it also seems suitable to construct the new grid considering the type of vegetation which will grow.

In this model the restrictions reflect the higher slope allowed in each cell. A linear way of constructing these constraints consists in obtaining the slope of a cell as the average slope of its eight nearby ones. Other constraints must reflect the incompatibility of positive and negative deviations on the same cell.

Drawing conic perspectives.
The drawing of perspectives is one of the most powerful tools in analysis and design of areas to reclaim. The conic representation
system provides a very intuitive drawing of the actual and future topographic configurations, and allows the decision maker to assign a general landscape value to every proposed solution. We have chosen the "DELPHI" method (Dalkey -2) to assign this value. The consulted experts have been selected from one multidisciplinary team that works on land reclamation. However, the advantage of this method consists in the possibility of introducing information from local authorities, ecologists and every social group interested in this decision. The average of these values was presented to the decision maker on each step of the delphi. (Fig. 1-3).

The perspective operations consists in the transformation of lines from three dimensions into two dimension coordinates on a screen between the observer and the three dimensional real space. Interchanging the focus length and the scale of representation, it is possible to obtain a drawing equivalent to a photograph (Nickerson -3). Parameters to input are the distance between the observer and the screen and the new line of vision which is obtained from two points: the observer and the viewed point. The screen will be a normal plane to the mean view line drawn at a focus distance from the observer point.

Once the screen plane is selected, the next step consists in inscribing the area to draw in a square parallelogram which must belong to the Z=0 real plane, and with two axes parallel to the screen plane. This parallelogram may be introduced into the computer by selecting three of their four vertices. Later, a set of parallel cross section diagrams has to be designed, beginning with the nearest to the observer's point. Parameters to be introduced are: distance among cross sections and the interval length along each one.

Every cross section is projected by intersecting the straight line that joins the observer and the analyzed point in the considered section with the screen plane, and then linking every projected point with a straight line. To project a point whose real coordinates are \((X,Y,Z)\) into the screen plane in which coordinates are \((X_1,Y_1,Z_1)\), it is possible to use the equation (1).

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \text{INV} \begin{bmatrix}
U_x & U_y & U_z \\
V_x & V_y & V_z \\
W_x & W_y & W_z
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

Where: \((U_x,U_y,U_z)\) Unitary vector parallel to the horizontal axes of the screen placed on the projection of the viewpoint and referred to the real coordinates.

\((V_x,V_y,V_z)\) Id., but parallel to vertical axes of the screen.

\((W_x,W_y,W_z)\) Id., but normal to the screen.
It is possible to draw the whole area with the perspective computation method that has been described. However, when you look at an area from any point, there are a lot of hidden spots which it is convenient to eliminate of the drawing. For that, we start analyzing the nearest section to the observer's point; obviously all this section will be noticed. The procedure of projecting the next nearest section may be summarized as follows:

- Analyze the section; remember that the perspective of a section is obtained by projecting each one of their lines.
- Determine if the initial and final points of the analyzed line have an "y" coordinate above or below the last section projected.
- Identify all the cutting points between the analyzed line and the last projected section.
- Decision rules to draw are:
  - If the border points are both below the last projection and there are no cutting points, then, the line has not to be drawn.
  - If the border points are both above the last projection and there are no cutting points, then, the line has to be drawn as a whole.
  - If there are cutting points, the subsegments of the line defined by the borders and the cutting points, will be drawn alternately (one is drawn and the next one is not if the first point is noticed).
- Analyze next line.
- Now the last projected section has to be completed with all the drawn subsegments of the lines, and it is saved into the computer to continue with the next nearest section.

To make prominent the perspective it is possible to draw another set of cross sections perpendicular to the explained above.

**Earthwork calculations**

From the four techniques stated by Munson-4 to estimate the volume to be cut or filled in a construction project, we have chosen the cross sectioning. This method involves the measurement of areas on a set of parallel cross diagrams, and it is necessary to consider on each section the existing and the proposed topographic configuration, in order to pick out spots to be cut and to be filled and to measure the surface of solid material to remove. This quantity has to be multiplied by the distance between cross sections to obtain the volume. The computer processes these values to determine areas to be cut or to be filled, and provides a graphical output with these areas on a plan. (Fig.4).
Next step consists in determining the boundary of cutting and filling zones corresponding to all sections over the study area and then in calculating the volume on each enclosed zone. This may be handmade or with a computer program. JOINT program for example starts on a point in which actual and proposed solution has the same height and progresses by searching the nearest point with this feature; if in this path, the line goes only across cutting or filling areas, then this straight line belongs to the border between cutting and filling zones and then it is saved into the computer; if it is not, the search progresses towards the next nearest point and so on. When the algorithm reaches the initial point, it then progresses in the same way as the Dantzig-5 algorithm to find the shortest route on a network. Once cutting and filling areas are identified, its volume is computed by adding the volumes enclosed on each area. (Fig. 4 and Tab.1).

Table 1
Volume of material to remove on each actuation unity (see Fig 4)

<table>
<thead>
<tr>
<th>Cutting area</th>
<th>Volume (m³)</th>
<th>Filling area</th>
<th>Volume (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1558</td>
<td>1</td>
<td>375</td>
</tr>
<tr>
<td>2</td>
<td>112</td>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>12183</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>750</td>
<td>4</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>375</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>915</td>
<td>6</td>
<td>95</td>
</tr>
<tr>
<td>7</td>
<td>133</td>
<td>7</td>
<td>239</td>
</tr>
<tr>
<td>8</td>
<td>132</td>
<td>8</td>
<td>671</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>9</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>2358</td>
<td>10</td>
<td>954</td>
</tr>
<tr>
<td>11</td>
<td>233</td>
<td>11</td>
<td>21375</td>
</tr>
<tr>
<td>12</td>
<td>10860</td>
<td>12</td>
<td>4721</td>
</tr>
<tr>
<td>13</td>
<td>37</td>
<td>13</td>
<td>37</td>
</tr>
<tr>
<td>14</td>
<td>50</td>
<td>14</td>
<td>50</td>
</tr>
<tr>
<td>15</td>
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<td>878</td>
</tr>
<tr>
<td>16</td>
<td>93</td>
<td>16</td>
<td>93</td>
</tr>
</tbody>
</table>

The cross sectioning method is considerably more difficult to program than the grid method used to obtain the standard solution, but it has the advantage of accepting data from a raster or from a vectorial data base.

To consider the economic cost of earthwork for each solution let us turn to the mathematical programming, more precisely to the transportation model. In general, this model is concerned with transporting goods from multiple supply to multiple demand centers.
Such situation deals with optimization of earth work from several excavation sites to other filling or dumping areas. The algorithm employed to solve the transportation model was the Modified Distribution Method (MODI).

To apply this technique is necessary to define supply and demand centers and the distance between them. Obviously the supply centers are the areas to cut and possibly additional areas to excavate material; in the same way, demand centers are the areas to fill and possibly a waste area. The given solutions have been designed so that additional cutting or filling areas were not needed; in this way the project will be cheaper.

The distance between cutting and filling areas presents more problems. If a computer with a great storage capacity is available, then it will be possible to identify the unity of actuation as the cell and use the grid method to estimate the earthwork; in this case the distance may be defined as the length of the line that join two cells, drawing this line on the actual topographic configuration. On the other hand, if, as previously, the cutting and filling areas are the units of actuation, it may seem shocking to consider only one distance between cuts and fills, obviously a part of a cutting area may be quite near a part of a filling area, but it will be not so if we consider the area as a whole. However as we are looking for an index to compare economic cost with others, it is possible to define the distance between areas as the distance between gravity centers of cutting and filling areas. The volume of material transported inside the area may be observed on Fig.4. Once the flow of material is known, it is possible to estimate a cost as is shown on table 2.

<table>
<thead>
<tr>
<th>Cutting area</th>
<th>Filling area</th>
<th>Distance</th>
<th>Volume to remove from the cutting to the filling area</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>9.7</td>
<td>287</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>9</td>
<td>48</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>35</td>
<td>1223</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>12</td>
<td>88</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>24</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>27.5</td>
<td>12183</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

(As the unitary prices -A and B- are known, it is possible to obtain the total cost)

A: Pushing transport.
B: Load on truck transport
The drainage network.

This subroutine is shaped as a searching algorithm of the adjacent cell with the biggest negative slope. Once the drainage network is discovered, the flow of a cell is achieved from the amount of rain brought by the receiver watershed of the cell under consideration.

It is assumed, in a first approach, that soil nature is homogeneous, though we must consider that the soil erosion of the different soil spots may be modified.

Finally RUNOFF program fulfils the calculation gathered because of the erosive vulnerability in each cell, determining the adding of vulnerabilities of the spots that drain into the cell under consideration. This value of vulnerability may be used as an index when we liken different topographic configurations.

EVALUATION offers a subroutine in which is calculated the vulnerability to water erosion of the feasible topographical configurations. The soil erosive vulnerability is proportional to the water kinetic energy of the water that flows along the watershed given to every cell and to the soil inherent erosion because of its edaphic nature.

Kinetic energy on each cell is the result of the flow, speediness and slope of the spot, and it is analytically stated by equation (2).

\[ E = K/2.Q^{2/3} S^{2/3} \]  
(2)

\( Q \) = Cell flow  
\( S \) = Cell slope

Slope is achieved directly from topographic configuration; the unity of flow is considered in this application. The whole erosional energy on each point is obtained by adding all the individual kinetic energy of every cell belonging to its watershed. The output of these programs may be shown on fig. 5-6.

INTMOREINF

The area to be reclaimed cannot be considered as an independent spot with respect to its nearby areas. Every proposed solution will extend its influence to a superface bigger than the own actuation place. In this way INTMOREINF programs allow to call for other programming package in order to study different vulnerabilities on the surrounding spots. These programs do not belong to LANDREC package; however, they are necessary to improve a complete reclamation.
Among the available programs to study the influence of the area to be reclaimed in its nearby zone we can mention those applied to obtain: The fragility and quality of vegetation (Escribano -6, Aramburu et al -7), landscape quality and fragility (Aguiló and Ramos -8, and Aguiló, González and Ramos -9), soil erosion (González and Martínez Falero -10), the risk of watertable pollution (Sainz de Omeñaca -11) the change into the communications network (Ramos and Saenz -12, Martínez Falero -13) etc. As in EVALUATION programs, INTMOREINF ones must start from the mapping out of the basic aspects of the area. In this way it is necessary to introduce into the computer information about vegetation, geogoly, watertable,...

INTEGRATION

It does not seem necessary to emphasize the difficulty of evaluating economic, ecologic and landscape values altogether. Many times, the land reclamation process may end by giving the above mentioned values, and it corresponds to the decision maker to choose the best solution. However, there are several techniques to obtain a single value, starting from apparently not commensurable objectives. These tools avoid the hard and strict cardinal metrics to which engineers are accustomed, and take into consideration ordinal metric or mediated personal decisions which come from a knowledge of the state of the art and not from personal emotions (Ramos -14).

INTEGRATION subroutines have two available programs which may help to obtain the single value. The first is based on the utility theory and presents some difficulties for people not accustomed to such subjects. The second is based on the concordance method, a soft technique. Both start from a solutions-objectives matrix in which every solution appears as a row of the matrix; the levels of attainment of a desirable objective are represented by columns. These objectives are the above obtained values and any other than the decision maker wants to introduce.

Utility theory.

To apply these programs it is necessary to explore the decision maker preference, in order to build a real variable function representing the structure of preferences. Obviously, to shape the preference into a mathematical function it is necessary that the decisor has a criterion to establish it between each couple of solutions. To make a simplification we will suppose that the level of attainment of every objective is known. In this way it is possible to omit the utility theory under risk and uncertainty.
The first task to do is to explore the existence of such function. However theorems to verify it (Debreu -15, Fishburn -16, Luce and Suppes -17) are quite abstract and complex. For this reason we will follow the ChanKong and Haimes -18 point of view and will suppose the existence of such functions.

One of the bigger problems in defining the utility function lies in the multidisciplinarity of the problem. A logical way to remove it consists in reducing the dimension of the problem as far as possible. To attain the reduction it is possible to group the objectives in independent sets. Once the global utility has been disarranged, the simplest resulting function may be easily obtained, and then, combined to get the total one. There are many kinds of decompositions. Expression (3) shows some of them.

\[ U(\vec{x}) = k_1 U_1(x_1) + k_2 U_2(x_2) + \ldots + k_n U_n(x_n) \]  
Additive function (3.1)

\[ U(\vec{x}) = k U_1(x_1) U_2(x_2) \ldots U_n(x_n) \]  
Multiplicative (3.2)

\[ U(\vec{x}) = k U_1(x_1) U_2(x_2) \ldots U_n(x_n) \]  
Polynomial (3.3)

The ideal case would consist in that every objective was independent. If there are two or more objectives which are not reciprocally independent and, however, they are independent if considered as a whole, it is possible to reduce these objectives into only one by applying the cross-matricial way. This method transforms two objectives into only one by asking the decision maker for the combined value of every couple of numbers, the first representing the value of an objective and the second representing the value of the other.

Additive function.

Once the decision matrix has been introduced into the computer, the first thing to test consists in studying the possibility of disarranging the objectives into a complete set of independent groups. In this way the computer program asks for the "mutual and preferential" independence relation among the elements of the \( \sigma \)-field generated by the set of all the objectives. Every time the decision maker has to perform an opinion about independence, the computer displays the expression (4).

\[ \theta = (\ldots) \]  
objectives that belong to the considered \( \sigma \)-field element \( \theta \) is mutual and preferential independent \( \equiv \) given a determinated value in \( \theta \) \( \vec{x}_\theta \), \( \forall x_\theta, \vec{x}_\theta, \forall \vec{x}_\theta \) :

\[ (\vec{x}_\theta \vec{x}_\theta) \geq (\vec{x}_\theta \vec{x}_\theta) \Rightarrow (\vec{x}_\theta \vec{\vec{x}}_\theta) \geq (\vec{x}_\theta \vec{x}_\theta) \]  
(4)

and this happens to every chosen fixed value.
Then, the computer displays all the above inequations in a combinatorial way, in order to test the decision maker opinion. If the level of attainment of objectives are not integer numbers, it is possible to build up an integer scale and to apply the same process.

When it was possible to obtain the set of all objectives as the union of independent and disjointed -field elements, then an additive function may be defined on these elements.

Once the complete set of elements has been chosen, it is necessary to get the utility function of every one. In this way, the computer provides information and allows to make use of the "direct" and the "half point" methods (Fishburn -19) which are combined with the Kirwood and Sarin -20 theorem. We will not describe them to simplify this description. However, it is important to remark that this process is based on questions put to the decision maker in order to reflect his preference as accurately as possible.

Next step consists in obtaining the value of the $k_i$ parameters ($i=1,\ldots,n$) of the (3.1) expression. There are two ways to do it: the handmade method developed by Ramos and Otero -21, and the computer one. Parameters to input in this last one are $n$ couples of solutions with the same utility. Then, it is possible to establish a $n$-linear equation system and to solve it in order to obtain the unknown parameters.

Finally the global additive utility may be obtained by multiplying the $k_i$ value of every independent element of the -field by its unitary utility, and adding these values all over the complete set.

Other utility functions.

The remaining functions exposed on (3) expression may be developed in a similar way. However, the independence concept becomes more complex (multiplicative decomposition may be seen on Keeney and Raiffa -22). For this reason we postulate the use of soft models when additive decomposition is not possible.

Soft models.

These techniques have been developed to be used when the decision maker may not establish a preferential order among all the suitable solutions. We all know how difficult it is to compare every couple of solutions in order to decide which is "at least as preferred" as the other. This point may suggest the construction of a weaker "relation of order". To solve this problem, Roy -23,24, stated a very intuitive method which was applied by Nijkamp -25. In the case of "n" different solutions, it may be summarised as follows:
- Ask the decision maker the "meaning" of performing each objective, in order to assign a fixed value corresponding to its importance.

- To every couple of solutions \((x, y)\), the following sets must be obtained:
  
  \[
  I^+(x, y) = \{i / \exists \ i \in n : x_i \geq y_i\}
  
  I^-(x, y) = \{i / \exists \ i \in n : x_i = y_i\}
  
  I^-(x, y) = \{i / \exists \ i \in n : x_i < y_i\}
  
  \]
  
  Where \(x\) is the level of attainment of the \(k\) objective.

- Built the \(I^x\) and \(I^y\) concordance index.

  \[
  I^x = \frac{\left(\sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{n} \lambda_i^*\right)}{\left(\sum_{i=1}^{n} \lambda_i + \sum_{i=1}^{n} \lambda_i^*\right)}
  
  \]
  
  \(\lambda_i\) is the value assigned to the "i" objective.

  \(-x \gg y \in I^x\) and \(I^y \geq 1\). A value is a decision maker's choice.

The above algorithm is easy to compute.

In this way it is possible to reduce the dimension of every suitable solution, from the "n" original objectives, to two components. The first one reflecting the number of preferred solutions, and the second the number of inferior ones. It is possible to increase the number of solutions affected by this "relation of order" by reducing the parameter, although this reduction implies an increase of the risk and must be handled carefully.

REFERENCES


PRESENT TOPOGRAPHIC CONFIGURATION

VIEW FROM THE SOUTH

VIEW FROM THE EAST

ESTIMATED LANDSCAPE VALUE: ?

Fig. n° 1.
VIEW FROM THE SOUTH

VIEW FROM THE EAST

ESTIMATED LANDSCAPE VALUE: ?

Fig. n° 2.
SOLUTION II

ESTIMATED LANDSCAPE VALUE: ?

VIEW FROM THE SOUTH

VIEW FROM THE EAST

REMOTION OF WASTE MATERIAL

Fig. n° 3.
THE CROSS SECTIONING METHOD IN ORDER TO SHAPE CUTTING AND FILLING AREAS AND THE FLOW OF MATERIAL (m³)

OPTION I

Economic cost of material removing: 14.500.000 PTA

OPTION II

Economic cost of material removing: 5.150.000 PTA
DRAINAGE NETWORK AND EROSION VULNERABILITY

PRESENT TOPOGRAPHIC CONFIGURATION

SOLUTION I
DRAINAGE NETWORK AND EROSION VULNERABILITY

PRESENT TOPOGRAPHIC CONFIGURATION  SOLUTION II
STANDARD SOLUTION

Determine homogeneous units based on:
- Proposed land use
- Homogeneous slope
And estimate a maximum admisible slope compatible with the land use proposed.

If slope (i) > maximum admisible slope \( i \) is a supply spot
\[ s_K = (z_K - z'_K)^2 \]
\( K = 1, 2, \ldots, K_1 \)

If slope (i) < maximum admisible slope \( i \) is a possible demand spot
\[ d_K = (z'_K - z_K)^2 \]
\( z_K = \text{Height of the K-spot} \)
\( z'_K = \text{Maximum height of the k-spot with an admisible slope} \)

Minimize
\[ \sum_{r=1}^{K_1} \sum_{s=1}^{K_2} d_{rs} s_{rs} \]
s.t.
\[ \sum_{s=1}^{K_2} s_{rs} = S_r (r = 1, 2, \ldots, K_1) \]
\[ \sum_{r=1}^{K_1} s_{rs} \leq d_s (s = 1, 2, \ldots, K_2) \]

\( D_{rs} = \text{Distance between r-supply spot and k-demand spot} \)
\( S_{rs} = \text{Material to remove from r-spot to K-spot} \)

If \( K_1 = n^2 \) of \( \leq \) constraints \( \Rightarrow K_1 \) slack variables
\( K_2 = n^2 \) of \( = \) constraints \( \Rightarrow K_2 \) artificial variables
\( N = K_1 + K_2 \Rightarrow N \) decision variables \( (s_{rs}) \)

For Phase I \( \Rightarrow 3(2+N)(2+2N) \) Simplex table:

<table>
<thead>
<tr>
<th>N^2 of d.v</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>170</td>
<td>500K</td>
</tr>
<tr>
<td>200</td>
<td>750K</td>
</tr>
<tr>
<td>240</td>
<td>1000K</td>
</tr>
<tr>
<td>355</td>
<td>2000K</td>
</tr>
</tbody>
</table>