Detailed and Global Analysis of a Remedial Course’s Impact on Incoming Students’ Marks

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Abstract: - Engineering incoming students are facing great difficulties to overcome first course subjects. To tackle that situation and increase the students’ success a Remedial course in Mathematics was offered to Informatics Engineering freshmen. This study presents a statistical analysis of their results comparing the marks obtained by those joining the course (studio group) versus those who did not participate (control group). ANOVA tests are performed over the students’ marks averages as well as over each subject students marks. These tests show statistically significant differences between both groups, with the studio group consistently outperforming the control group at 99% confidence level in most cases and at more than 92% confidence level in every case.

Key-Words: - Remedial courses, Educational research, Engineering education, Mathematics learning, Informatics curricula

1 Introduction

As a matter of fact, engineering incoming students are facing great difficulties to overcome first course subjects, thus, dropping and failure ratio grow to be very high among freshmen. Those ratios are rising, which makes mandatory to find an effective way to manage the crisis.

Among the causes of these problems, we should consider the difference in didactical methodologies between University and Secondary School, but most of the mentioned difficulties come from the poor level in math knowledge and logical reasoning acquired during their secondary education.

To tackle that situation and increase the students’ success, most universities are trying diverse solutions, usually remedial or reinforcement courses, just before or during the first semester.

Among the found studies, some do not report any impact analysis. Galagedera presents a study on the performance of incoming students on first year elementary statistics. He distinguishes two groups: those who passed mathematics at matriculation level and those who did not, and took a compulsory basic mathematics course. His results suggest that the course failed, as those who did not take the course performed better, but also that performance in mathematics and statistics might be correlated. However, in [9] an online remedial course in mathematics is evaluated by measuring its impact on the outcomes of first course Statistics and Math plus a basic course on applied computer science. Their analysis compares the dropping and success rates in those subjects and it indicates that those who passed the course performed better than those who did not follow or did not pass the course. Lesik asserts that one limitation on the existing literature is determining whether participation in developmental mathematics programs has a causal impact on success in college-level mathematics and concludes that participating in the program significantly increases the odds of successfully completing a college-level mathematics course on the first try.

The study presented here goes on a deeper analysis as it performs ANOVA tests for each one of the seven compulsory subjects, not only the math related subjects, the alumni study in first course and for all the marks obtained as well as a comparison of the students’ arithmetic means and the dropping and success percentages.
In the fall of 2005, the Department of Applied Mathematics, at Informatics Engineering of Universidad Politécnica of Madrid, implemented a remedial + reinforcement course in mathematics, which was offered to incoming students.

Previous studies, analyzing this course’s impact on June’s exams marks, provided pretty encouraging records so we continued investigating its impact on the final marks of first year compulsory subjects, including not only those who passed in June’s exams but those who succeeded in September’s second opportunity as well. Data obtained are quite relevant: the means of the marks obtained by the students show a statistically significant difference between the students who joined the course matched up to those who did not participate, averaging the first ones higher than their matched counterparts. As a main effect, it is important to mention a remarkable raise of passed versus a decrease of drop out for every first course’s compulsory subject.

2 Scenario
Many incoming students on Informatics Engineering at Universidad Politécnica of Madrid are overwhelmed by first course subjects and, among them, dropping and failure ratio are getting higher every year.

As stated above, these difficulties are mainly due to the poor level in math knowledge acquired during their secondary education (pre university level). In Sept. 2005, an initial competence test, consisting of 20 questions of secondary school math contents, four options each, was taken by a 94 students group joining Informatics Engineering at our University.

![Fig. 1: Number of correct answers in the 2005 initial competence test](image)

In this test [10], as shown in Figure 1, 65.96% failed more than 10 questions while only 12.77% failed six or less. Furthermore, most of them had never used symbolic language as sets, quantifiers or propositional logics. With this lack of background, together with a significant deficiency in abstract and logical reasoning, first course subjects become an insurmountable obstacle for incoming students.

An optional curricular complement was proposed in order to increase the students’ success: a Remedial+Reinforcement course in mathematics called "Introduction to Mathematical Methodology" taught to 24 freshmen from September 2005 to January 2006.

2.1 Course’s structure
The proposed course combined remedial with reinforcement training in two differentiated blocks: First part consisting on 45 hours during September (before the regular course started). Within that period, a review of the main concepts extracted from secondary curricula was presented (with special emphasis on precalculus and basic algebra), highlighting intuition, logical reasoning and self-developed methods. A basic overview of set theory, relations and quantifiers notation was also included, since those concepts set up the basis for math language development. The course did not contain specific Formal Logics topics, as this subject starts from scratch.

The second part, which ran along with the regular first semester, was a reinforcement course. During it, they were asked to solve some exercises using Maple software in order to strengthen the concepts imparted in the following math subjects: Discrete Mathematics, Linear Algebra and Calculus.

The applied methodology consisted in working with small groups (20 to 30 people who joined the course voluntarily) and developing together an intuitional and practical vision of mathematics. The teacher promoted direct communication within the group, trying to guide the students in such a way that they could reach the proposed problems' solutions by themselves, encouraging them to use self-developed methods, better than learned ones. In this way, the students were provided with new approaches to catch the concepts as well as intuitional approximations to the learned methods.

2.2 Students’ opinion
To get a measure of the students’ perception, they fulfilled a questionnaire at the end of the first part, rating up to 4 over 5 both contents and methodology of the course. Figures 2, 3 show contents’ and methodology’s questionnaires averages.
During one-to-one interviews, after the first semester’s examinations, the students valued the experience very positively. They considered especially beneficial the following facts: it was a small group, the work was customized to their needs, it meant a more rational and less memory based approach to mathematics and finally, they appreciated very much the possibility of using Maple software. They ended remarking an increase on self-confidence and the revision of topics facing the beginning of the course, as positive achievements.

### 2.3 Data description

We have performed a comparison between two groups of students: the studio group, which consists of 24 incoming students who joined the remedial course, and the control group, which comprises the remaining 99 students who enrolled in Informatics Engineering on June 2005. The total number of incoming students that year was 198; the remaining 75 have been excluded because they enrolled in September and did not have the opportunity to join the course. The performance of those 75 students was lower than average so if they had been computed within the control group, the results would had been more positive for the study group.

The comparison includes compulsory subjects’ marks, and dropping and success’ percentages. The students joining the remedial course were mixed up with the remaining students and distributed in groups for compulsory subjects. Thus the instructors teaching those subjects and the evaluation process have no influence on the marks obtained by both groups.

In first course, there are seven compulsory subjects, four within math fields (Calculus, Linear Algebra, Formal Logics and Discrete Mathematics), plus Programming Methodology, Foundations of Hardware and Physics Foundations of Informatics.

The marks a student can get are: P (when the students did not take the exams), S (if they took but did not pass the exam) and a numeric value from 5 to 10 according to their learning level. Since numeric values are required in order to calculate means, we have defined P=0 and S=2.5 as an average approximation.

### 3 Detailed Analysis of students outcomes by subjects

In this section we execute an ANOVA test for each one of the seven compulsory subjects the alumni study in first course.

This procedure performs a one-way analysis of variance for each subject’s marks. It constructs various tests and graphs to compare the mean values of each subject’s marks for the 2 different levels of Belonging Group. The F-test in the ANOVA table will test whether there are any significant differences amongst the means. If there are, the Multiple Range Tests will tell which means are significantly different from which others.

Tables 1 – 7 and figures 4 – 17 show the results of the ANOVA test for each subject.

### 3.1 Discrete Mathematics

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>15.6664</td>
<td>1</td>
<td>15.6664</td>
<td>3.21</td>
<td>0.0748</td>
</tr>
<tr>
<td>Within groups</td>
<td>951.962</td>
<td>195</td>
<td>4.88185</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>967.628</td>
<td>196</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 1: ANOVA test for Discrete Mathematics**

**Fig. 4: Discrete Mathematics LSD intervals**

**Fig. 5: Discrete Mathematics Box and Whisker Plot**

The ANOVA table decomposes the variance of Discrete Mathematics Marks into two components: a between-group component and a within-group component. The F-ratio, which in this case equals...
3.20911, is a ratio of the between-group estimate to the within-group estimate. Since the P-value of the F-test is lower than 0.1, there is a statistically significant difference between the mean Discrete Mathematics Marks from Studio Group to Control Group at the 90.0% confidence level.

3.2 Calculus

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>24.9038</td>
<td>1</td>
<td>24.9038</td>
<td>4.72</td>
<td>0.0311</td>
</tr>
<tr>
<td>Within groups</td>
<td>1034.9</td>
<td>196</td>
<td>5.2801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>1059.8</td>
<td>197</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: ANOVA test for Calculus

Since the P-value of the F-test is lower than 0.05, there is a statistically significant difference between the mean Calculus Marks from Studio Group to Control Group at the 95.0% confidence level.

3.3 Linear Algebra

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>83.1431</td>
<td>1</td>
<td>83.1431</td>
<td>10.25</td>
<td>0.0016</td>
</tr>
<tr>
<td>Within groups</td>
<td>1330.33</td>
<td>196</td>
<td>6.78739</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>1420.84</td>
<td>197</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: ANOVA test for Linear Algebra

Since the P-value of the F-test is lower than 0.001, there is a statistically significant difference between the mean Linear Algebra Marks from Studio Group to Control Group at the 99.9% confidence level.

3.4 Formal Logics

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>90.5092</td>
<td>1</td>
<td>90.5092</td>
<td>13.33</td>
<td>0.0003</td>
</tr>
<tr>
<td>Within groups</td>
<td>1330.33</td>
<td>196</td>
<td>6.78738</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>1420.84</td>
<td>197</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: ANOVA test for Formal Logics

Since the P-value of the F-test is lower than 0.001, there is a statistically significant difference between the mean Formal Logics Marks from Studio Group to Control Group at the 99.9% confidence level.
### 3.5 Programming methodology

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>57.6658</td>
<td>1</td>
<td>57.6658</td>
<td>9.74</td>
<td>0.0021</td>
</tr>
<tr>
<td>Within groups</td>
<td>1154.85</td>
<td>195</td>
<td>5.9223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>1212.51</td>
<td>196</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: ANOVA test for Programming Methodology

Since the P-value of the F-test is lower than 0.01, there is a statistically significant difference between the mean Programming Methodology Marks from Studio Group to Control Group at the 99.0% confidence level.

### 3.6 Foundations of Physics

Since the P-value of the F-test is lower than 0.001, there is a statistically significant difference between the mean Foundations of Physics Marks from Studio Group to Control Group at the 99.9% confidence level.

### 3.7 Foundations of hardware

Since the P-value of the F-test is lower than 0.001, there is a statistically significant difference between the mean Foundations of hardware Marks from Studio Group to Control Group at the 99.9% confidence level.
4 General Analysis of students outcomes

This section analyzes the impact of the remedial course by studying three types of comparisons:

- For each student the arithmetic mean of the obtained marks has been calculated and the two groups’ data have been compared.
- For each compulsory subject the dropping and success percentages of both groups have been compared.
- A multifactor analysis of variance for marks has been performed to determine which factors have a statistically significant effect on marks. Apart from this, it also allows to examine for significant interactions amongst the factors.

4.1 Comparison of arithmetic means

Data compared here are, for each student, the arithmetic mean of the marks obtained in compulsory subjects.

<table>
<thead>
<tr>
<th></th>
<th>Control G.</th>
<th>Studio G.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count</td>
<td>99</td>
<td>24</td>
</tr>
<tr>
<td>Average</td>
<td>2.902</td>
<td>3.69333</td>
</tr>
<tr>
<td>Variance</td>
<td>4.93926</td>
<td>4.89898</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2.22245</td>
<td>2.21336</td>
</tr>
<tr>
<td>Range</td>
<td>8.22857</td>
<td>7.57286</td>
</tr>
<tr>
<td>Std. skewness</td>
<td>2.57047</td>
<td>0.120151</td>
</tr>
<tr>
<td>Std. kurtosis</td>
<td>-0.965559</td>
<td>-0.925003</td>
</tr>
</tbody>
</table>

Table 8: Summary Statistics for arithmetic means

According to 2.3 we are studying the whole population enrolled in Informatics Engineering on June 2005. The standardized skewness value outside the normal range in Control Group is due to the huge dropping and failure ratios.

Figure 18 compares the means obtained by the components of both groups.

4.1.1 Comparison of means for students’ arithmetic means

95% confidence interval for mean of Control G.: 2.902 +/- 0.44326 [2.45874, 3.34526]
95% confidence interval for mean of Studio G.: 3.69333 +/- 0.934623 [2.75871, 4.62796]
95.0% confidence interval for the difference between the means assuming equal variances: -0.791337 +/- 1.00032 [-1.79165, 0.20898]

T-test to compare means

Null hypothesis: mean Control G = mean Studio G
Alt. hypothesis: mean Control G < mean Studio G

Assuming equal variances: t = -1.67077 and P-Value = 0.0486756

The T-test has been constructed to determine whether the difference between the two means equals 0 versus the alternative hypothesis that the difference is below 0. Since the computed P-value is less than 0.05, we can reject the null hypothesis in favor of the alternative, what means that there is a statistically significant difference between the means of the two groups, with the mean of the control group lower than the mean of the studio group at the 95.0% confidence level.
F-test to compare the standard deviations that gives a P-value of 0.808785.

4.2 Drop out and success

Tables 9–10 and figures 20–21 represent the comparison between the dropping rates of incoming students who did not join the course matched up with those who joined the course in the compulsory subjects taught in the first year: Linear Algebra (LAlg), Discrete Mathematics (Disc), Calculus (Calc), Programming Methodology (Prog), Formal Logics (Log), Foundations of Hardware (FHw) and Foundations of Physics (FPh). Data are expressed in percentage on the group totals.

From these data it is clear that:

- Studio group’s success ratio is higher in every compulsory subject, with the exception of Discrete Mathematics, reaching the difference of nearly 16% in Foundations of Physics.
- Drop out percentages diminish in every subject.
- Dropping ratios difference rises to more than 21% in Programming Methodology and Foundations of Hardware.
- The Studio group clearly outperforms the Control group

Among the problems the incoming students have to face, one of the most important is that they must pass a minimum number of credits for staying at the University. One of the chosen subjects for fulfilling this obligation is Discrete Mathematics. The better success ratio in this subject could be addressed to this fact.

4.3 Multifactor analysis

A multifactor analysis of variance (ANOVA) was selected to investigate the effects of different factors (in this case, the subject and the belonging group) and their interactions on the students’ marks. Table 11 summarizes the results. Though the subject is introduced as a factor, it is not relevant to our analysis, since is well-know that students behave differently in front of diverse subjects.

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<tr>
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<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F-Ratio</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Subject</td>
<td>236.488</td>
<td>5</td>
<td>47.297</td>
<td>5.65</td>
<td>0.0000</td>
</tr>
<tr>
<td>B: Belonging group</td>
<td>100.546</td>
<td>1</td>
<td>100.546</td>
<td>14.42</td>
<td>0.0001</td>
</tr>
<tr>
<td>INTERACTIONS</td>
<td>12.655</td>
<td>6</td>
<td>2.109</td>
<td>0.30</td>
<td>0.9357</td>
</tr>
<tr>
<td>RESIDUAL</td>
<td>5907.09</td>
<td>847</td>
<td>6.974</td>
<td>0.6741</td>
<td></td>
</tr>
<tr>
<td>TOTAL (CORRECTED)</td>
<td>6403.6</td>
<td>860</td>
<td>7.386</td>
<td>0.9357</td>
<td></td>
</tr>
</tbody>
</table>

| Table 11: Analysis of Variance for Marks - Type III Sums of Squares |

Interactions & 90.0 % LSD intervals

Fig. 22: Comparison by subject
The ANOVA table decomposes the variability of the marks into contributions due to each of the factors. Since P-value for belonging group is less than 0.001, this factor has a statistically significant effect on marks at 99% confidence level.

Figure 22 shows the differences on marks’ averages by subjects and the Least Significant Differences intervals at 90% confidence level. It is clear that considering the subjects separately, in most cases there is a statistically significant difference among both groups, with the studio group surpassing the control group.

| Method: 99 percent LSD |
|-------------------------|-----------------|
| Belonging group         | Count | LS Mean | LS Sigma | Homogeneous Groups |
| Control Group           | 693   | 2.87388 | 0.100318 | X                |
| Studio Group            | 168   | 3.73619 | 0.203747 | X                |
| Contrast                | Difference | +/- Limits |
| Control Group - Studio Group | *-0.862309 | 0.373554 |

* denotes a statistically significant difference.

Table 12: Multiple Range Tests for Marks by Belonging group

Table 12 and Figure 23 show the least squares mean of each group marks. It also shows the standard error of each mean, which is a measure of its sampling variability.

Means and 99,0 Percent LSD Intervals

Fig. 23: Comparison of belonging group factor

They evidence a statistically significant difference between both groups at 99% confidence level and prove the impact of the presented course on students’ results.

5 Conclusions
From the above exposed, the following conclusions may be obtained:

- There is a statistically significant difference between the means of the two groups, with the mean of the control group lower than the mean of the studio group at the 99.0% confidence level for every compulsory subject except for Discrete Mathematics, with a 92% confidence level and Calculus where the confidence level is higher than 96%.
- There is a statistically significant difference between the means of the marks averages of the two groups, with the mean of the control group lower than the mean of the studio group at the 95.0% confidence level.
- Studio group’s success ratios are higher, except for Discrete Mathematics, reaching a difference of nearly 16%, while drop out ratios are, except for one case, visibly lower with a difference rising up to more than 21%.
- Once having removed the effect of other factors, the fact of belonging to the studio group has a statistically significant effect on the marks at the 99% confidence level.
- The lack of mathematical basis and reasoning ability results in high dropping and failure ratios.
- Both enhanced reasoning and analyzing ability must get the credit for outstanding results in math as well as non math subjects.

The results clearly demonstrate that there are significant differences between both groups, with the studio group consistently outperforming the control group, which proves the effectiveness of the experience. Consequently, the convenience of complementing Engineering Curricula by means of a Remedial/Reinforcement course like the presented one is inferred. Thus, incoming students’ negative results might be amended.

Additionally, the development of mathematical reasoning entails an enhancement in logical and abstract reasoning, needed in other first course subjects. Therefore, as we had suspected the course’s positive impact has spread to every subject’s outcomes.

After this analysis, the requirement of improving the alumni’s mathematical basis is clear. Math constitutes a foundation for every science or engineering topic, as it is an essential tool for modeling, as well as a main language. Apart from this fact, but not less important, there is an increase in logical reasoning capacity as well as scientific method provided by math.

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