Systematic Method to Assess Small-Signal Stability of DC-Distributed Power-System-Architecture

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***Abstract*—The objective of this paper is to present a simplified method to analyze small-signal stability of a power system and provide performance metrics for stability assessment of a given power-system-architecture. The stability margins are stated utilizing a concept of maximum peak criteria (MPC), derived from the behavior of an impedance-based sensitivity function that provides a single number to state the robustness of the stability of a well-defined minor-loop gain. For each minor-loop gain, defined at every system interface, the robustness of the stability is provided as a maximum value of the corresponding sensitivity function. Typically power systems comprise of various interfaces and, therefore, in order to compare different architecture solutions in terms of stability, a single number providing an overall measure of the whole system stability is required. The selected figure of merit is geometric average of each maximum peak value within the system, combined with the worst case value of system interfaces.[1]**

I. INTRODUCTION

Power system architecture refers to the selection of system components and their connections in order to supply loads according to their requirements. The amount of possible architectural solutions for certain specifications can be excessive, thus the architectural selection has an important role in the overall optimization of a distributed power system. A methodology to design and optimize power distribution systems automatically is developed in [1,2]. By utilizing behavioral modeling techniques [3,4], fast simulation models are obtained, allowing the analysis of extremely large number of design options. This process results in optimized architectural solutions regarding the most fundamental system features. However, during the optimization process various features are neglected regarding the DC/DC converters: the solutions are obtained without considering stability and dynamic behavior of the converters.

It is well known that the DC/DC converters are prone to impedance-based interactions introducing destabilizing effects to the system, due to their constant-power input-terminal behavior [5]. Therefore, it is necessary to include stability assessment as a part of the optimization methodology in order to assess the feasibility of the obtained solutions. For this purpose, the DC/DC converter is represented by a two-port network, composed of a set of measurable transfer functions known as G-parameters [6-9].

Traditional stability assessment method, based on minor-loop gain [10], is widely used in various interconnected systems covering different application areas [10-13]. This method utilizes the impedance-based minor-loop gain that is a ratio of the source or upstream subsystem output impedance and the load or downstream subsystem input impedance. Stability exists if the minor-loop gain satisfies the Nyquist criterion. Typically, the impedance-ratio-based stability region is presented as certain forbidden regions in the complex plane, out of which the minor-loop gain shall stay [14-16]. It is recently stated that the forbidden regions defined by the above mentioned methods occupy unnecessary large area in the complex plane, which can be reduced to a circle around the critical point (-1,0) without compromising the robustness of stability [17]. Similar forbidden region is earlier applied in [18] without giving justification for its usability. An alternative method to assess stability, passivity-based stability criterion (PBSC), is presented in [19,20], where the passivity of a bus impedance is utilized to provide information regarding the stability thus avoiding the problem of analyzing the encirclement around the point (-1,0).

The purpose of this paper is to present a simplified method to systematically assess small-signal stability of a given power architecture as well as provide a measure of the whole system stability, based on which various architecture solutions can be compared in terms of robust stability. The applied concept of maximum peak criteria (MPC) provides the least conservative stability margins for every system interface. The selected figure of merit for stating the whole system stability is a geometric average of each maximum peak value within the system. The rest of the paper is organized as follows: the optimization methodology is briefly explained in Section II as well as the applied modeling method suitable for commercial DC/DC converters. Section III introduces the proposed MPC-concept and its application providing some practical examples. Section IV describes the proposed methodology and the

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performance metrics in detail. The conclusions are finally drawn in Section V.

II. Theoretical Background

The optimization of power architecture is a complex task and the amount of possible ways to connect various system components can be excessive. A methodology, based on simplified DC/DC converter models, to obtain optimized power system solutions is briefly described. A modeling method that enables the system small-signal stability analysis is explained, showing how to obtain a minor-loop gain to analyze the influence of the source- or load-side impedance.

A. Architecture Optimization

The power system design is typically desired to be optimized in terms of size, cost and efficiency. The main system component, DC/DC converter, is a major contributor on these features. Therefore, the optimization methodology is based on DC/DC converter models that only consider the converter size, cost and efficiency [3-4]. These simple models enable fast analysis of various architectural solutions.

Based on the converter models, an architecture generation algorithm searches all suitable ways to connect these components according to the system specifications. The number of possible architectural solutions is huge and therefore, the final optimization is performed utilizing evolutionary optimization techniques [1,2]. Finally, the optimization process provides a selection of the most appropriate converters and a list of architectural solutions including the options with the smallest size, cost and losses as well as the solutions with the best trade-offs within these features. This methodology assists the design of distributed power systems as multiple architectural options can be assessed within a short time.

B. Converter Modeling Method

The DC/DC converter models utilized for the optimization process consider in this case only the static properties of the converters [2,3] in order to obtain fast simulation models for the analysis of large number of design options. Therefore, the optimized solutions are obtained without considering stability and the dynamic performance of the converters. In order to analyze it, a different modeling method that includes the effects of small-signal stability is needed. The model is required to be simple to implement as well as suitable for black-box modeling, because the utilized converters are commercial with limited available information on their internal structure.

When the converters are interconnected to a system, adverse interactions might occur due to the converter sensitivity to the external impedances. This might lead to degraded converter transient performance or even instability. The interactions can be computed based on the internal dynamic representation that can be found by performing frequency response measurements through the converter input and output terminals. The input and output sources are assumed to be ideal. The corresponding four transfer functions describing the converter dynamic behavior according to Fig. 1 and (1) are:

- Audio susceptibility $G_{io} = \frac{\hat{u}_o}{\hat{u}_{in}}$
- Input admittance $Y_{in} = \frac{\hat{i}_{in}}{\hat{u}_{in}}$
- Reverse transfer function $T_{oi} = \frac{\hat{i}_{in}}{\hat{i}_o}$
- Output impedance $Z_o = \frac{\hat{u}_o}{\hat{i}_o}$

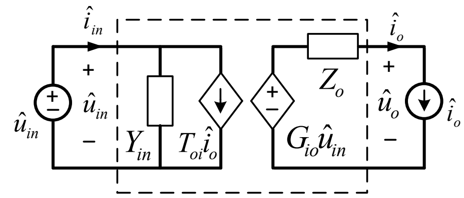

Fig. 1 Linear model of the converter with ideal source and load.

$$\begin{aligned}\hat{i}_{in} &= Y_{in}\hat{u}_{in} + T_{oi}\hat{i}_o \\ \hat{u}_o &= G_{io}\hat{u}_{in} - Z_o\hat{i}_o\end{aligned} \qquad (1)$$

This modeling method enables the small-signal stability analysis of an interconnected system and its robustness. The influence of the source or load side impedance to the internal converter transfer functions in (1) can be analyzed as described in detail in [17]. The corresponding source and load-affected transfer functions can be given according to (2) and (3), respectively.

$$\begin{aligned}\hat{i}_{in} &= \frac{Y_{in}}{1+Z_sY_{in}}\hat{u}_{in} + \frac{T_{oi}}{1+Z_sY_{in}}\hat{i}_o \\ \hat{u}_o &= \frac{G_{io}}{1+Z_sY_{in}}\hat{u}_{in} - \frac{1+Z_sY_{in-sco}}{1+Z_sY_{in}}Z_o\cdot\hat{i}_o \\ Y_{in-sco} &= Y_{in} + \frac{G_{io}T_{oi}}{Z_o}\end{aligned} \qquad (2)$$

$$\begin{aligned}\hat{i}_{in} &= \frac{1+Z_{o-oci}Y_L}{1+Z_oY_L}Y_{in}\cdot\hat{u}_{in} + \frac{T_{oi}}{1+Z_oY_L}\hat{i}_o \\ \hat{u}_o &= \frac{G_{io}}{1+Z_oY_L}\cdot\hat{u}_{in} - \frac{Z_o}{1+Z_oY_L}\cdot\hat{i}_o \\ Z_{o-oci} &= Z_o + \frac{G_{io}T_{oi}}{Y_{in}}\end{aligned} \qquad (3)$$

According to (2) and (3), the source- and load-side minor-loop gains are Z_sY_{in} and Z_oY_L, respectively. Based on these minor-loop gains, the small-signal stability and robustness can be analyzed subsequent to the system integration. Moreover, this modeling method enables more detailed assessment of the system-level interactions [9]. However, the focus of this paper is on obtaining a systematic method for the stability and robustness analysis.

III. STABILITY ASSESSMENT

In order to implement the stability assessment as a part of the optimization methodology, a systematic and straightforward analysis method is preferred. For the optimized architecture, the obtained stability margins are desired to be the least conservative, i.e. optimized in terms of stability, guaranteeing the robustness. Middlebrook's criterion [10] is known to be too restrictive for general stability assessment. Therefore, applying a concept of forbidden regions, less conservative conditions for stability are obtained [14-16], where the Nyquist contour of the minor-loop gain is required to stay out of the predefined area and thus providing certain gain (GM) and phase (PM) margins for stability. A minimum forbidden region can be defined applying maximum peak criteria (MPC) [17] and this method is utilized to guarantee robust stability with the least conservative requirements.

A. Maximum Peak Criteria

A minimum forbidden region in the complex plane can be defined applying maximum peak criteria (MPC) [17]. This concept is well known in control engineering to define robust stability of a closed-loop system [21]. It is based on sensitivity function, defined in (5) where L denotes the system loop gain.

$$S = \frac{1}{1+L} \tag{5}$$

The critical area in the vicinity of the point (-1,0) determines the robustness of stability i.e. adequate gain (GM) and phase (PM) margins. Therefore, the measure of performance is assessed according to the closeness of the loop gain to the critical point (-1,0). This minimum distance is given as 1/Ms where the Ms denotes the maximum value of the sensitivity function $|S_{max}|$. Low phase or gain margins of the loop gain L would cause resonant behavior (i.e., peaking) in the converter closed-loop transfer functions. The amount of this peaking is assessed based on the maximum peak of the sensitivity function and the corresponding margins for stability are given in (6) [21].

$$PM \geq 2\arcsin\left(\frac{1}{2|S_{max}|}\right) \qquad GM \geq \frac{1}{1-1/|S_{max}|} \tag{6}$$

For instance, a maximum peak of 5dB in the sensitivity function provides minimum margins of 7dB GM and 33° PM. The impedance-based minor-loop gain forms a similar sensitivity function (7) as the loop gain L in (5)

$$S = \frac{1}{1+ML}, \tag{7}$$

where the ML can be the source or the load side minor-loop gain. Therefore, low margins (GM and PM) in the minor-loop gain would cause peaking in the corresponding sensitivity function and consequently in the internal transfer functions of the converter (2) and (3), degrading the transient behavior as shown in [22].

Based on this concept, the MPC-based forbidden region is defined as a circle having its center at (-1,0) and the radius of 1/Ms. In Fig. 4 the highlighted area illustrates the MPC-based forbidden area, having the maximum peak of 2 (6dB) compared to the regions in [14-16].

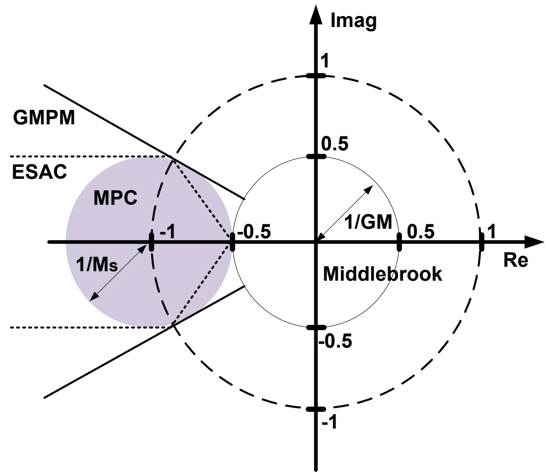

Fig. 4 The MPC-based forbidden region with the ESAC and GMPM regions.

This MPC-based forbidden region occupies the minimum area in the complex plane and guarantees robust stability: the minor-loop gain is required to comply with the Nyquist criterion as well as to stay away from the circular forbidden area. The MPC-based forbidden region is determined by the maximum allowed peak of the sensitivity function and the area is, therefore, definable according to the robustness requirements of a particular application.

B. Application of the MPC-concept

The robustness of the stability can be most reliably determined at the interface closest to the direct input or output of the converter power stage as explicitly demonstrated in [17]. In addition, the operation point where the frequency response measurements are performed influences on the stability margins. Few practical examples based on measurement data and simulations illustrate the application of this concept.

The peak sensitivity function provides information of the stability margins: the lower the peak value, the better in the sense of robust stability. The MPC- based forbidden region utilized in the following examples is obtained selecting the maximum peak 2 (6dB), corresponding to minimum margins of GM =6dB and PM ≈ 29°. Fig. 5 shows the Nyquist contour of two minor-loop gains. Minor-loop gain 1 is formed between measured output impedance of the source converter, voltage-mode-controlled synchronous buck (Uin 12V;Uo = 5V; Io = 2A; fsw = 200kHz) and simulated input impedance of the load converter that operates as a constant power load (R = -10Ω; Cin = 17μF). The second plot presents the minor-loop gain formed between a measured input impedance of Ericsson Power Module, PMB 8518TP (12V, 3.3V, 10A) and an input filter that is designed to comply with the Middlebrook's criterion with a GM of 6dB.

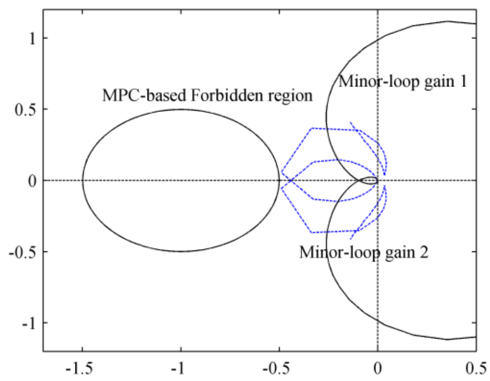

Fig. 5 Minor-loop gains 1(black line) and 2 (blue line), formed based on measured Zo of a buck and simulated Zin of a cascaded converter and measured Zin of a buck and Zo of a filter, respectively with the MPC-based forbidden region.

Based on visual observation, neither of the plots encircles the point (-1,0) nor intersects the forbidden region thus guaranteeing the minimum margins of 6dB GM and 29deg PM. For systematic stability analysis, the same information is easily obtained computationally. The peaking of both minor-loop gains can be computed: 1.25 (1.92dB, minor-loop gain 1) and 1.9 (5.8dB, minor-loop gain 2) at the frequency of 3.46 kHz and 447 Hz, respectively. These values correspond to the minimum distance between the Nyquist contour and the point (-1,0): 0.8 for minor-loop 1 and 0.52 for the second plot as can be observed from the figure. The computed peaks of the sensitivity functions are lower than the predefined value for the MPC-based circle, guaranteeing the robustness.

The following example, described in detail in [17] demonstrates how the excessive peaking of the sensitivity function influences on the converter performance. The system consists of a bus converter, and two identical point-of-load converters, POL1 and POL2 as shown in Fig. 6 with the system specifications. The source-side minor-loop gain of the POL2 is measured at two operating conditions: firstly POL1 is operating at full load (4A) and POL2 at 1A and secondly POL2 is operating at full load (4A) and POL1 at 1A. These minor-loop gains are shown in Fig. 7 together with the MPC-based forbidden region.

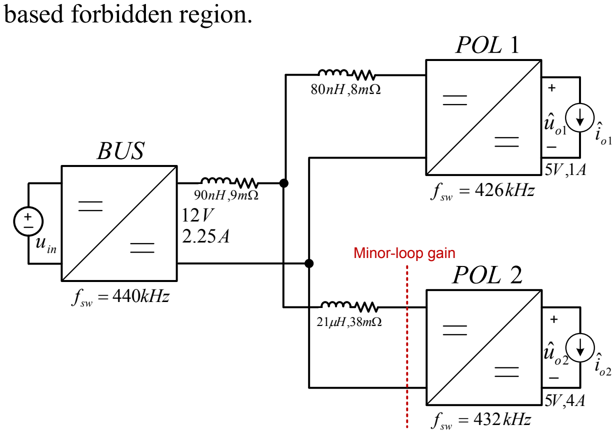

Fig. 6 Distributed power system consisting of a bus converter and two POL converters.

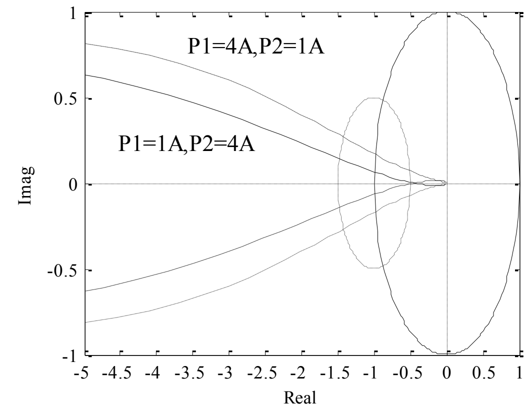

Fig. 7 The Nyquist plots of the same minor-loop gain at both operating conditions (solid line: POL2 4A, dashed line POL2 1A).

Both minor-loop gains intersect the MPC-based forbidden region. However, depending on the operational conditions, the stability margins vary. The peaking during the first operating condition is computed to be 13.7dB and in the second condition, 23.7dB. In both cases, the predefined peaking value 6dB is exceeded. When the POL2 is operating at full load the worst case stability margins are 0.6dB of GM and 4° of PM, respectively. The influence of this peaking can be observed from the measured output impedance of the POL2 in both operating conditions as shown in Fig. 8.

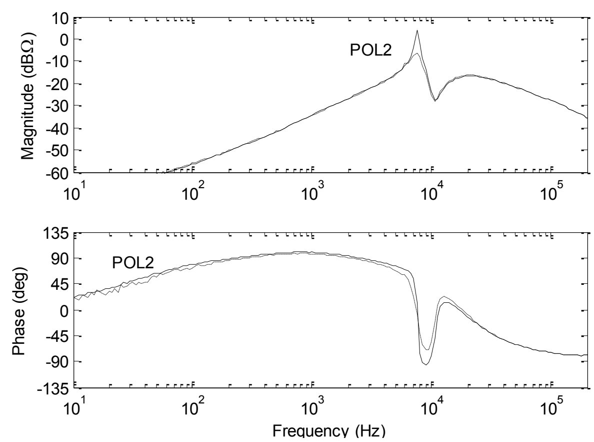

Fig.8 Measured source-affected output impedance of the POL2 at both operating conditions.

The converter output impedance is affected due to the source impedance (system interconnection) implying that its performance is deteriorated. This can be observed in time-domain, when a load step is applied at the output of the POL2 as shown in Fig. 9. The damped oscillation in the output voltage response is due to the resonance in the source-affected output impedance of the POL2.

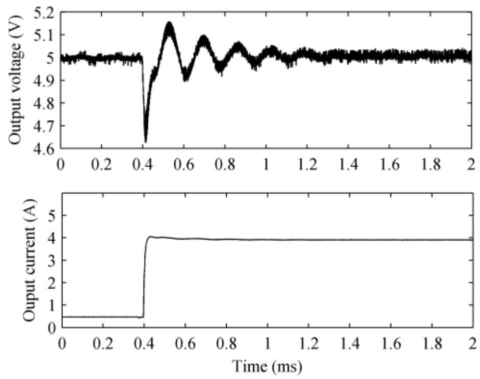

Fig. 9 Measured time-domain behavior of POL2 during a load step from 0.5 to 4.0A (250mA/µs) at the output of POL2.

IV. PROPOSED METHODOLOGY

The optimization process provides a set of architectural solutions that are optimized regarding the system size, cost and efficiency. In order to obtain information of the system stability and robustness, the presented concepts are applied. The system stability is analyzed based on the encircling of the minor-loop gain around the point (-1,0). By utilizing the MPC-concept, for each minor-loop gain, a single value that combines the effects of both margins, is obtained thus enabling systematic stability assessment. In addition, as the architectures are desired to be comparable, a figure of merit to provide a measure of the whole system stability is selected.

A. Implementation

Each converter within the power architecture is represented as its two-port model that contains the information from the internal converter dynamics. This modeling structure allows the interconnection of the DC/DC converters to form a system according to the given architectural structure.

The stability analysis is based on the minor-loop gain at each interface and divided into two parts:

- Stability analysis according to the Nyquist criterion
- Robustness analysis based on the MPC concept

Unstable system is detected by assessing whether the Nyquist contour of the minor-loop gain encircles the point (-1,0) or not. For a stable system, the stability margins are assessed. In order to correctly predict the robustness within the system, the source and load side minor-loop gains are analyzed for each converter, as illustrated in Fig. 10. The robustness of the stability is stated by computing the maximum value of the minor-loop-gain-based sensitivity function in (7). This number is utilized to evaluate whether the system interconnection might deteriorate the converter performance.

B. Performance metrics

For each minor-loop gain, defined at every system interface, the robustness of the stability is provided as a maximum value of the corresponding sensitivity function. Typically power systems comprise of various interfaces and, therefore, in order to compare different architecture solutions in terms of stability, a single number providing an overall measure of the whole system stability is required. Geometric average of each $S_{\max}$ value within the system is selected as a figure of merit regarding the robustness of the stability as given in (5).

$$S_{syst} = \sqrt[n]{S_1 \cdot \ldots \cdot S_n} \tag{5}$$

It provides a meaningful metrics for system comparisons: the best system in terms of robust stability is the one that minimizes this index. The maximum allowed peak of the sensitivity function is definable according to the robustness requirements of a particular application. For the performance metrics presented in this paper, the maximum value is selected as $S_{\max} = 2$, corresponding to a peaking of 6dB and stability margins PM ≈ 29º and GM = 6dB while the minimum value is 1 (0dB). In order to provide more insight to the usage of the selected figure of merit, Table I presents different values for the system interfaces in Fig. 10. By applying the proposed performance metrics, a value for the whole system stability is obtained.

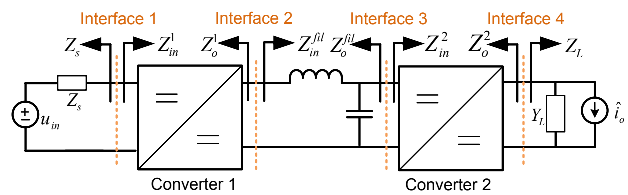

Fig. 10 Source-and load- side minor-loop gains for cascaded converters.

Table I Maximum peak values for Case 1 and 2.

Interface	Case 1 $S_{\max}$	Case 2 $S_{\max}$
Int. 1	1.1	1.4
Int. 2	1.2	1.3
Int. 3	2	1.4
Int. 4	1.2	1.3
S_{syst}	**1.33**	**1.35**

The obtained figure of merit in both cases is approximately the same. However, in the case 1, the interface 3 is close to deteriorate the predefined stability margins ($S_{\max}$ = 2), whereas all the interfaces of the case 2 consist of good average values. Therefore, it is of interest to provide additional information regarding the system robustness to facilitate the architecture comparisons. Thus, in addition to geometric average, the weakest point of the system in terms of robustness is detected by providing the worst case $S_{\max}$ value.

V. CONCLUSIONS

This paper presented a methodology for small-signal stability analysis of a given power system. The applied MPC-concept, based on the minor-loop-gain-based sensitivity function provides the least conservative method to obtain stability margins at each interface. Typically, power systems consist of various interfaces and, therefore, a geometric average of the peak values in each interface was selected as a performance metrics for the small-signal stability. This figure of merit provides a meaningful metrics of the overall system small-signal stability: the best system in terms of robust stability is the one that minimizes this index. Moreover, the largest $S_{\max}$ value within the system is given thus providing additional information regarding the robustness and facilitating the system comparison by detecting the weakest point of the system in terms of robustness.

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