INITIAL PORE PRESSURE FROM VERTICAL SURFACE LOADS[a]

Discussion by Enrique Alarcón[2] and Pascual Martí[3]

The author presents a very interesting application of the ideas developed by Scott (3) to determine the initial pore pressure in excess of the hydrostatic pore pressure in linear, elastic, homogeneous and isotropic soil-skeleton. Scott demonstrates that under vertical surface loads the problem is governed by Laplace's equation. Nevertheless the writers think that it could be interesting to state clearly the conditions under which this analogy can be applied.

In terms of total stresses and in the absence of body forces the Beltrami-Mitchell equations lead to the condition

$$\nabla^2 l_1 = 0 \quad (11)$$

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in which l_1 = the first stress invariant.

Following the classical line of reasoning of Skempton, it is possible to establish the equality of volume change between the fluid and the soil skeleton as

$$\frac{nu}{K_f} = \frac{l_1'}{K_s'} \qquad (12)$$

in which l_1' = the first invariant of effective stress; K_s' = the bulk modulus of the soil skeleton; and $1/K_f = n/K_w + (1 - n)/K_s$, in which K_w = the bulk modulus of the fluid and K_s that of the soil particles.

Using Terzaghi's principle and rearranging the equation:

$$u = \frac{1}{1 + \dfrac{nK_s'}{3K_f}} \frac{l_1}{3} \qquad (13)$$

that, in conjunction with Eq. 11 yields

$$\nabla^2 u = 0 \qquad (14)$$

Nevertheless, it is a fundamental fact that the boundary conditions for the potential problem of Eq. 14 are governed by Eq. 13.

In order to use Eq. 14 to determine the pore pressures under undrained conditions, you have to solve a problem where the boundary conditions are specified by Eq. 13.

Of course, if it is possible to assume total incompressibility of the fluid and soil particles, Eq. 13 reduces to

$$u = \frac{l_1}{3} \qquad (15)$$

$$l_1 = 3q \qquad (16)$$

in which q = the pressure applied on the surface.

What the author, as well as Sundaram (Ref. 5), have done is to solve only the absolutely incompressible case ($\nu = 1/2$). But in this case it seems that it would have been better to refer directly to the enormous amount of information collected in the classical "Elasticity" manuals, and to apply Eq. 15. Taking into account, for instance, the book by Poulos and Davis (Ref. 6, p. 43), one finds the following expressions for stresses in points below center of uniformly loaded circular areas:

$$\sigma_z = q\left[1 - \left(\frac{1}{1 + \left(\dfrac{a}{z}\right)^2}\right)^{3/2}\right] \qquad (17)$$

$$\sigma_r = \sigma_\theta = \frac{q}{2}\left[(1 + 2\nu) - \frac{2(1 + \nu)z}{\sqrt{a^2 + z^2}} + \frac{z^3}{(a^2 + z^2)^{3/2}}\right] \qquad (18)$$

$$\text{i.e.} \quad l_1 = 2(1 + \nu)q\left[1 - \frac{z}{\sqrt{a^2 + z^2}}\right] \qquad (19)$$

i.e., for $\nu = 1/2 \quad l_1 = 3q\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right)$ .... (20)

and $\quad u = \frac{l_1}{3} = q\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right)$ .... (21)

which is, of course, the same equation as presented by Sundaram (Ref. 5).

On page 54 (Ref. 6) for the uniform loading on a rectangular area, Poulos and Davis give the following expressions for stresses beneath the corner:

$$\sigma_z = \frac{q}{2\pi}\left[\tan^{-1}\frac{ab}{zR_3} + \frac{abz}{R_3}\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\right] \qquad (22)$$

$$\sigma_x = \frac{q}{2\pi}\left[\tan^{-1}\frac{ab}{zR_3} - \frac{abz}{R_1^2 R_3}\right] \qquad (23)$$

$$\sigma_y = \frac{q}{2\pi}\left[\tan^{-1}\frac{ab}{zR_3} - \frac{abz}{R_2^2 R_3}\right] \qquad (24)$$

$$R_1 = \sqrt{a^2 + z^2} \quad R_2 = \sqrt{b^2 + z^2} \quad R_3 = \sqrt{a^2 + b^2 + z^2} \qquad (25)$$

i.e. $\quad l_1 = \frac{3q}{2\pi}\left[\tan^{-1}\frac{ab}{zR_3}\right]$ .... (26)

and $\quad u = \frac{q}{2\pi}\tan^{-1}\left[\frac{ab}{z\sqrt{a^2 + b^2 + z^2}}\right]$ .... (27)

which is the same Eq. 3.

The same scheme could be applied to every case listed in the Poulos and Davis book, for linear, elastic, homogeneous, isotropic bodies, conditions under which Eq. 13 is valid.

For instance, for uniform vertical loading on an infinite strip we have (Ref. 6 p. 36)

$$\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\delta)] \qquad (28)$$

$$\sigma_x = \frac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\delta)] \qquad (29)$$

$$\alpha = \tan^{-1}\frac{x + b}{z} - \tan^{-1}\frac{x - b}{z} \quad \delta = \tan^{-1}\frac{x - b}{z} \qquad (30)$$

i.e. $\quad l_1 = \frac{3q\alpha}{\pi}$ .... (31)

and $\quad u = \frac{q\alpha}{\pi}$ .... (32)

Values of σ_z, u and $\bar{\sigma}_z = \sigma_z - u$ have been presented in the form of

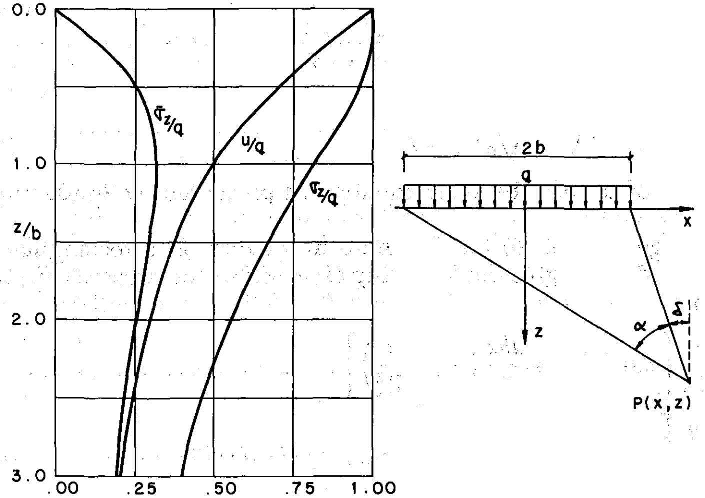

FIG. 3.—Distribution of Initial Excess Pore Pressure (*u*), Total Vertical Stress (σ_z) and Initial Effective Stress ($\bar{\sigma}_z$) Below Center of Uniformly Loaded Infinite Strip

a normalized graph in Fig. 3, for points below the center of the loaded infinite strip.

Appendix.—Reference

6. Poulos, H. G., and Davis, E. H., *Elastic Solutions for Soil and Rock Mechanics*, John Wiley and Sons, Inc., New York, N.Y., 1974.