DESIGN FOOTING AREA WITH BIAXIAL BENDING[a]

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The authors present a very interesting criterion for choosing a rectangular foundation. The writers should like to point out that the obtention of minimum area can be reduced to the problem of finding the minimum of $x^* + y^*$, subjected to the condition

$$x^* \cdot y^* = k^2 \quad (20)$$

whose solution is evidently

$$x^* = y^* = k \quad (21)$$

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[a]October, 1983, by Ramon Jarquio and Victor Jarquio (Paper 18267).
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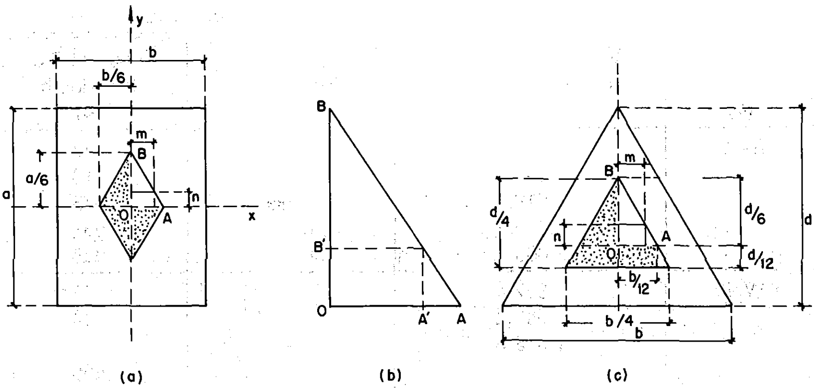

FIG. 2.—(*a–c*) Footing Area of Rectangle

As it is well-known, the kern of a rectangular section is a diamond, so that its minimum area corresponds to the minimum of a triangle, as that of Fig. 2(*b*).

It is clear that

$$\text{Area} = m \cdot n + 0.5(AA' \cdot n + BB' \cdot m) \quad (22)$$

but

$$\frac{BB'}{m} = \frac{n}{AA'} \quad (23)$$

so that

$$m \cdot n = AA' \cdot BB' \quad (24)$$

If we call

$$x^* = AA' \cdot n \quad (25)$$

$$y^* = BB' \cdot m \quad (26)$$

TABLE 1.—Computed Footing Rotations

Type of footing (1)	Area, S (2)	Maximum stress, f (3)	Rotation, B (4)
Rectangle	$144\,mn$	$2P/12^2\,mn$	$\dfrac{P\sqrt{m^2+n^2}}{\lambda 12^3 \cdot m^2 \cdot n^2}$
Square	$36(m+n)^2$	$2P/6^2\,(m+n)^2$	$\dfrac{P\sqrt{m^2+n^2}}{\lambda 3^2 \cdot 12(m+n)^4}$
Isosceles triangle	$144\,mn$	$2P/12^2\,mn$	$\dfrac{P\sqrt{n^2+9m^2}}{\lambda \cdot 2.12^3 \cdot m^2 \cdot n^2}$
Equilateral triangle	$12\sqrt{3}\,(n+\sqrt{3}m)^2$	$\dfrac{P(m+\sqrt{3}n)}{12(n+m\sqrt{3})^3}$	$\dfrac{P\sqrt{m^2+n^2}}{\lambda \cdot 24\sqrt{3}(n+m\sqrt{3})^4}$

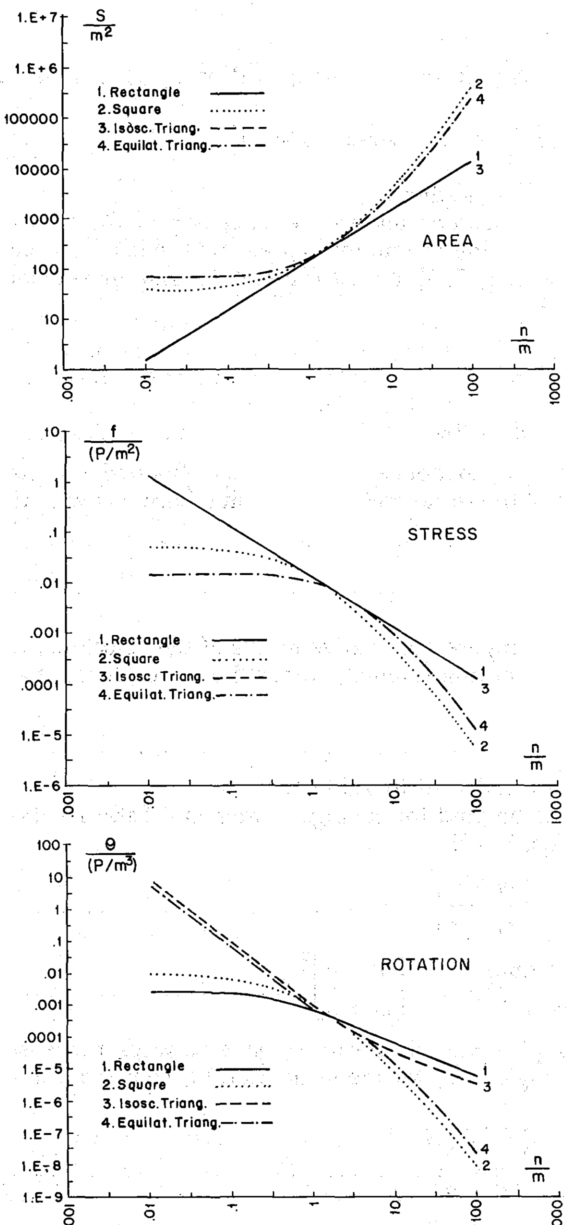

FIG. 3.—Area, Stress, and Rotation for Rectangle, Square, and Isosceles and Equilateral Triangles

then $x^* \cdot y^* = (m \cdot n)^2$ .... (27)

and the area will be minimum for $x^* + y^*$.

The solution is then

$$AA' \cdot n = BB' \cdot m = m \cdot n; \quad AA' = m; \quad BB' = n \quad (28)$$

$$\text{and} \quad \frac{b}{6} = 2\,m; \quad \frac{a}{6} = 2\,n; \quad \text{Area} = 144\,mn \quad (29)$$

as obtained by the authors.

That line of reasoning allows the extension of the previous result to the triangular footing of minimum area. As the kern is homotetic to the external shape, [Fig. 2(*c*)], it is clear that the minimum is obtained through Eq. 28. That is

$$\frac{b}{12} = 2m; \quad \frac{d}{6} = 2n \quad (30)$$

$$\text{or} \quad b = 24m; \quad d = 12n \quad (31)$$

It is interesting to notice that the area obtained for that triangle is equal to that of the rectangle, and the same happens with the maximum stress

$$f = \frac{2P}{12^2 mn} \quad (32)$$

In order to compare the relative merits of the solutions, we have computed also the footing rotation, under the assumption of a ballast coefficient λ;

$$\delta = \lambda^{-1} \cdot f \quad (33)$$

The results are collected in Table 1.

The price to be paid for savings in area is greater rotation, as can be seen from Eqs. 34–35:

$$\frac{S(\text{rect})}{S(\text{squa})} = 1 - \left[\frac{m-n}{m+n}\right]^2 < 1 \quad (34)$$

$$\text{but, also} \quad \frac{B(\text{squa})}{B(\text{rect})} = 1 - \left[\frac{m-n}{m+n}\right]^2 < 1 \quad (35)$$

Also, a simple comparison can be made between the isosceles triangle and the rectangle, i.e., for the same S and f, compare their rotations as follows:

$$\frac{B(\text{isos})}{B(\text{rect})} = \frac{1}{2}\sqrt{\frac{n^2 + 9m^2}{m^2 + n^2}} \quad (36)$$

which establishes the limit

$$\left(\frac{m}{n}\right)^2 > \frac{3}{5} \quad \text{if} \quad \frac{B(\text{isos})}{B(\text{rect})} > 1 \quad (37)$$

Other interesting relationships are shown in Fig. 3.