A Review of Electrodynamic Tethers for Science Applications

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Abstract. A bare electrodynamic tether (EDT) is a conductive thin wire or tape tens of kilometres long, which is kept taut in space by gravity gradient or spinning, and is left bare of insulation to collect (and carry) current as a cylindrical Langmuir probe in an ambient magnetized plasma. An EDT is a probe in mesothermal flow at highly positive (or negative) bias, with a large or extremely large 2D sheath, which may show effects from the magnetic self-field of its current and have electrons adiabatically trapped in its ram front. Beyond technical applications ranging from propellantless propulsion to power generation in orbit, EDTs allow broad scientific uses such as generating electron beams and artificial auroras; exciting Alfven waves and whistlers; modifying the radiation belts; and exploring interplanetary space and the Jovian magnetosphere. Asymptotic analysis, numerical simulations, laboratory tests, and planned missions on EDTs are reviewed.

1. Introduction: Electrodynamic tether basics

A space tether is a kilometers long wire connecting two satellites. Assuming that the tether keeps vertical in circular orbit, and its mass is small compared with end masses $m_1$ and $m_2$ at bottom and top, the common rotation velocity $\omega_{orb}$ in absence of any other force, is determined by the overall balance of gravitational and centrifugal forces in the orbiting frame,

$$2 - 2 - 2$$

$$\omega_{orb}^2 (m_2 r_2 + m_1 r_1) = GM_E \left( m_1 r_1^{-2} + m_2 r_2^{-2} \right),$$

where $M_E$ is the Earth's mass. There is a radius $r_0$ where gravitational and centrifugal forces are equal, $r_1 < (GM_E / \omega_{orb}^2)^{1/3} = r_0 < r_2$. The centrifugal force is greater for the upper mass $m_2$, which is thus subject to a net force away from the Earth. The opposite holds for the lower mass. This results in tether tension making the vertical orientation stable [1].

There is a gravity gradient force per unit mass at points away from $r_0$,

$$3 \omega_{orb}^2 (r - r_0), \quad (\text{for } r_2 - r_1 = L << r_1).$$

If the tether is conductive and carries a current as a result of interaction with the magnetized ionosphere, it will also experience a magnetic force. In addition, there is some bowing, and a Lorentz torque that may be balanced by a gravity-gradient torque at a tether tilt. All past missions involving tethers either used Low Earth Orbit (LEO) or were suborbital flights [2].

A Lorentz transformation in the non-relativistic limit relates electric fields in frames moving with spacecraft and local corotating plasma,

$$\bar{E} (\text{tether frame}) - E (\text{plasma frame}) = (\bar{\nu}_{orb} - \bar{\nu}_{pl}) \wedge \bar{B} \equiv \bar{E}_m.$$  

In the highly conductive ambient plasma outside the tether (meters away, typically) the electric field is negligible in the local plasma frame. There is then, in the tether frame, an outside (motional) field $E_m$ that drives a current $I$ inside the tether with $IE_m > 0$ in case of a passive tether system. The Lorentz force on an insulated tether of length $L$ carrying a uniform current
reads $LI \times B_0$. Since Newton’s 3rd law applies to magnetic forces between steady-current systems, a net power loss is seen to occur in the tether-plasma interaction,
\[ L \mathbf{I} \times \mathbf{B}_0 \cdot \mathbf{v}_{orb} + \left(-L \mathbf{I} \times \mathbf{B}_0\right) \cdot \mathbf{v}_{pl} = -I \frac{e}{m} E L < 0 , \]
which is (Lorentz) power naturally feeding the tether electric circuit [3].

The Lorentz force is a drag in LEO for both prograde and retrograde circular equatorial orbits, with $v_{orb}$ and $v_{op}$ in the same direction. For a vertical tether in prograde LEO, and a non-tilted, centered magnetic dipole model ($B_0$ northward, $E_0$ upward), a typical ‘motional’ field is $E_m = v_{orb} B_0 \sim 7.5 \text{ km/s} \times 0.2 \text{ Gauss} = 150 \text{ V/km}$, westward Lorentz drag and drag power reading $F_M = LI B_0$, $W_M = F_M v_{orb} = I E_m L$. Although issues of scientific interest will be here considered, the fundamental area of application of tethers is propellantless transportation [4]. The magnetic force on a tether requires no ejection of propellant, as opposite rockets or electrical thrusters. Hollow cathodes used for cathodic contact do eject (Xenon) expellant along with electrons but at extremely low rate; also, required bias is just tens of volts, leading to negligible contact impedance [5].

On the other hand, tether performance is ambient dependent. The bottleneck is the anodic-contact: how to efficiently collect electrons from the rarefied ionosphere. The TSS-1 and TSS-1R tethers carried a conductive sphere of radius $R = 0.8 \text{ m}$ acting as passive collector. Space charge keeps the electric field to a sheath around the sphere (ionospheric Debye length $\lambda_D$ is a fraction of cm) and the geomagnetic field guides electrons along field lines (electron gyroradius $l_e$ is a few cm); this strongly limits the current to a spherical collector [6]. The PMG tether used plasma contactors for both electron ejection and collection, which proved poor, however [7].

2 The cylindrical probe at highly positive bias

In 1991 it was proposed that [8], instead of using a large end-collector, a tether be left bare of insulation, acting as giant cylindrical Langmuir probe in the orbital-motion-limited (OML) regime [9]. Collection is efficient because the cross-section is small, while the collecting area is large because the anodic segment may be multikilometers long. Magnetic force and power now involve the length-averaged current $I_{av}$ but a length-to-radius ratio $\approx 10^6$ makes each point collect current as in a probe uniformly polarized at a local bias $AV$ [8].

The electron current to a cylindrical or spherical probe at rest in a collisionless, unmagnetized, Maxwellian plasma of density $N_0$ and temperatures $T_e$ and $T_i$ may be written as $I = I_\infty \times$ a function of $e\Delta V/kT_e$, $R/\lambda_D$, $T_e/T_i$, with $I_\infty$ the random current. Determining electron trajectories to find $I$ requires solving Poisson’s equation for the potential $\Phi(r)$, with $\Phi(0) = \Delta V > 0$ and $\Phi(\infty) = 0$. This requires solving for the density $N_e$ of attracted electrons (the density of repelled ions follows the Boltzmann law except where fully negligible anyway). Since the distribution function of electrons (originating at the ambient plasma) is conserved along trajectories, its value at given $\mathbf{r}_s, \mathbf{v}$ will be the undisturbed Maxwellian $f_M$ if their trajectory connects back to infinity, and zero otherwise.

For a cylinder, the density $N_e(r)$ will be an integral of $f_M$ over $\mathbf{v}_s$ (along the cylinder) and over all energies $E = \frac{1}{2} m (v_r^2 + v_\theta^2) - e \Phi(r) > 0$, once for radial velocity $v_r < 0$ and again for $v_r > 0$, and a $r, E$-dependent range of angular momentum $J \equiv m_r v_\theta$.

i) For a $v_r < 0$, $E$-electron at $r$ the range of integration is
\[ 0 < J < J_r^-(E) = \text{minimum} \{J_r(E); \ r \leq r' < \infty\} , \]
\[ J_r^-(E) = 2m_r^2 (E + e \Phi(r)) . \]
Condition $v_r^2 \geq 0$ reads $J \leq J_r(E)$ but $v_r^2 > 0$ is required throughout the range $r \leq r' < \infty$.

ii) For a $v_r > 0$, $E$-electron, the range of integration is $J_r^+(E) < J < J_r^+(E)$, electrons in the range $0 < J < J_r^+(E)$ having disappeared at the probe.

Collected current $I$ and density $N_e$ as function of $\Phi(r)$ are then

\[ I = \int \frac{dE}{kT_e} \exp \left(-\frac{E}{kT_e}\right) \frac{2 \left(J_r^-(E)\right)^2}{\pi m e^2 R^2 kT_e} , \]

(5)
3.43 kg/kW for \( E \) \( m^2 \) Al tether system is also characterized by a dimensional parameter to the top in a thrusting mode (a thrusting tether would be inefficient if fully bare). The overall round wires. The OML law is valid for any convex cross-section shape \[9\], with OML current-mass and the bare-tether contact impedance is negligible. This suggests moving away from power generation mode and both electric power supplied and length of insulated segment next eq tape collects the \( p/S. A \) capacitance per unit length of coaxial lines. For a thin tape, \( R \) as a classical problem in the eq electric field is about radial; this allows determining \( R \) instead of \( \pi \). Collection performance is optimum for eq \( L^* \), when current is maximum for given tether \( L \approx L^* \), and \( Z^* \) \( \approx 150 \) V/km, which corresponds to the inverse specific power of power systems. 3. Adiabatic trapping and other bare-tether issues Tether performance may cross-section area), arising from the tether resistance \( Z \) \( t \) \( c \) \( A \). \( Z^* \) \(C \) \( 3\).43 \( A \) and \( \sigma \) \( E_m A \) \( (A \) being cross-section area), arising from the tether resistance \( Z \approx L / \sigma \) \( t \) \( c \). Tether performance may depend on additional design parameters in the overall electric circuit: the load impedance in a power generation mode and both electric power supplied and length of insulated segment next to the top in a thrusting mode (a thrusting tether would be inefficient if fully bare). The overall tether system is also characterized by a dimensional parameter \( \rho \sigma E_m^2 \approx 3.43 \) kg/kW for \( A \) and \( E_m = 150 \) V/km, which corresponds to the inverse specific power of power systems. Ohmic effects are gauged by comparing the short circuit current to an average of the OML current law for a cylinder is robust; the ratio \( I_{OML}/I_{th} \) is independent of the ion distribution function, of the electron distribution if isotropic, and of \( R/\lambda_0 \) and \( T/T_e \) values over a large parameter domain. If \( R \) exceeds some radius \( R_{max} \) the ratio \( I_{OML}/I_{th} \) drops below 1 and decreases with increasing \( R/R_{max} \). The \( r^2 \Phi(r) \) minimum then lies below \( R^2 \Delta V \), trajectories that hit the probe within a range of glancing angles being unpopulated: they come from other probe points after turning back in the far field \[11\]. Recent asymptotic analyses fully agree on \( R_{max} \) current beyond \( R_{max} \) and \( \Phi \) and \( N_e \) profiles, with results from steady Vlasov calculations and particle-in-cell simulations \[12\]. [The OML current to a sphere greatly exceeds the 2D current for equal \( I_{th} \) and bias, but is never reached; faraway behavior is now \( \Phi \) \( r^2 \) \( ~ \) \( r \). Moving toward the probe, \( r^2 \Phi(r) \) decreases to a minimum (lying far from the probe for high bias and \( R \sim \lambda_0 \)): the quasineutral solution remains valid up to a sheath boundary, where \( -d\Phi/dr \) diverges. Within the sheath \( r^2 \Phi(r) \) reaches a large maximum (at minimum \( N_e \)) before again dropping to \( R^2 \Delta V \) at the probe (Fig. 2) \[10\].

The \( \Phi(r) \)-dependent structure of the \( r \)-family of straight lines \( \hat{J}^2 = J^2(r) \) in the \( \hat{J}^2-E \) plane determines the functions \( J^2_r(E) \) and \( J^2_t(E) \), which in turn determine \( N_e \) for use in Poisson's equation. Since the slope \( dE/d\Phi = 1/2m_e r^2 \) varies monotonically with \( r \), it suffices to have \( J^2_r(0) = J^2_t(0) \) for \( J^2_r(E) = J^2_t(E) \) to hold for all positive \( E \), at any particular \( r \) (Fig. 1), but \( J^2_r(0) \) varies as \( r^2 \Phi(r) \) which proves non-monotonic; this results in a complex \( r \)-family structure. The OML condition, however, requires the potential to just satisfy \( J^2_r(0) = J^2_t(0) \), i.e. \( r^2 \Phi(r) > R^2 \Delta V \) throughout the range \( R < r < \infty \). Faraway 2D quasineutrality, \( N_e \approx N_0 \), shows a behavior \( \Phi \) \( r^2 \) \( \sim \) \( r \). Moving toward the probe, \( r^2 \Phi(r) \) decreases to a minimum (lying far from the probe for high bias and \( R \sim \lambda_0 \)); the quasineutral solution remains valid up to a sheath boundary, where \( -d\Phi/dr \) diverges. Within the sheath \( R^2 \Phi(r) \) reaches a large maximum (at minimum \( N_e \)) before again dropping to \( R^2 \Delta V \) at the probe (Fig. 2) \[10\].

Ohmic effects are gauged by comparing the short circuit current to an average of the OML current law in (7) for some length \( L^* \) and bias \( E_m L^* \). This determines \( L^* \) \[8\],

\[
L^* \approx E_m^{1/3} (\sigma A_l / pN_0)^{2/3}.
\] (8)

3. Adiabatic trapping and other bare-tether issues

For a passive system, current is limited by its short circuit value, \( \sigma_e E_m A \) \( (A \) being cross-section area), arising from the tether resistance \( Z \approx L / \sigma_e A_t \). Tether performance may depend on additional design parameters in the overall electric circuit: the load impedance in a power generation mode and both electric power supplied and length of insulated segment next to the top in a thrusting mode (a thrusting tether would be inefficient if fully bare). The overall tether system is also characterized by a dimensional parameter \( \rho \sigma E_m^2 \approx 3.43 \) kg/kW for \( A \) and \( E_m = 150 \) V/km, which corresponds to the inverse specific power of power systems.

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\] (8)

Collection performance is optimum for \( L >> L^* \), when current is maximum for given tether mass and the bare-tether contact impedance is negligible. This suggests moving away from round wires. The OML law is valid for any convex cross-section shape \[9\], with OML current-density uniform over the probe surface; it suffices to write \( p/\pi \) instead of \( 2R \) in Eq. (7).

As regards non-OML results, there is an equivalent radius \( R_{eq} \neq p/2 \pi \) for any cross section. Because of the high bias, Laplace’s equation holds in a large probe vicinity, reaching where the electric field is about radial; this allows determining \( R_{eq} \) as a classical problem in the capacitance per unit length of coaxial lines. For a thin tape, \( R_{eq} \) is \( p/8 \). A tape collects the same OML-current as a round wire of equal perimeter, while being much lighter, the optimal
tether thus presenting three disparate dimensions, \( L \gg \text{tape width } w \gg \text{thickness } h \). This reflects on length \( L^* \) varying as \( R^{2/3} \) for round-wires and as \( h^{2/3} \) for tapes [13].

The resilience of the OML law further shows in its being reasonably applicable to non-convex cross sections, for which it breaks independently of size due to behavior of the potential near the probe; the law may still be used if \( p \) is replaced by the perimeter of the minimum-perimeter envelope of the cross section [13]. All above results apply, however, to unmagnetized plasmas at rest. Geomagnetic effects might in principle break the 2D-OML law because of 3D effects. There is, however, an upper (Parker-Murphy) bound to current to a cylinder in a magnetized plasma, reading at high bias [14]

\[
I_{\text{PM}} \approx \frac{1}{\text{OML}} \sqrt{\frac{2}{\pi}} \times \frac{1}{e} R.
\]

(9)

The geomagnetic field is thus expected to hardly affect the current for \( R \ll l_e \).

The field due to the tether own current might reduce current collection. Strong reduction would roughly occur at tether points where some average radius \( r^* \) of a magnetic separatrix modifying field topology in the cross-section plane exceeds the sheath radius \( r_{sh} \) [15],

\[
r_{sh} < r^* = \frac{1}{2} \pi r_0^2 e B_0
\]

(10)

Self-field effects, however, are typically negligible for thin tapes because ohmic effects severely limit current and separatrix radius. They are also negligible for round tethers at the Van Allen Belts, at Jupiter, and in interplanetary space, where the plasma density is very low.

As regards \( v_{\text{orb}} \) effects, the 2D-OML law might hold in principle because the plasma flow in LEO barely breaks ambient electron isotropy. However, the mesothermal character of that flow raises a paradox for the high-bias tethers. With the ambient electron population (nearly) isotropic, a fundamental result [9] shows \( N_e < N_0 \) in any 2D potential field \( \Phi(r, \theta) \). On the other hand, the high bias would ram back the 5 eV hypersonic ions, yielding \( N_i(r, \theta) > N_0 \) and thus breaking quasineutrality over front distances much larger than \( \lambda_D \).

The way out of this quandary seems to hinge on \( E < 0 \) electrons trapped in bound trajectories not accounted for in Ref. [9]. The key process is collisionless (adiabatic) trapping. As troughs in electron potential energy develop when quasineutrality is first broken, electrons are trapped in fast bound orbits slowly changing in an unsteady potential controlled by ion motion, finally leading to steady quasineutrality [16].

Laboratory tests involving the combined plasma-flow and magnetic effects have proved inconclusive. Bare-tether current collection by the long lattice of tensioning rods for each mast in the International Space Station solar array did explain, however, current balance in the Station south of Australia in early 2001. Also, bare-tether collection will be tested in August 2009 by a dedicated sounding-rocket mission (S-520-25) of the Japanese Space Agency [17]. Note that, to ensure 2D, cylindrical conditions in testing high-bias OML collection, length \( L \) must be much larger than sheath radius, not just much larger than \( R \),

\[
r_{sh} / \lambda_D e \approx 2 \sqrt{e \Delta V / kT_e} \gg 1, \quad \text{for } R \sim R_{\max} \sim \lambda_D.
\]

(11)

4. Determining E-layer neutrals density profiles

If the cathodic contactor is switched off, a tether \textit{floats} electrically, i.e., current vanishes at both ends. Because of the large ion-to-electron mass ratio, the floating tether is biased negative except over a \((m_e/m_i)^{1/3} \approx 0.03\) fraction of its length at the top. The low ion current makes for no ohmic effects, bias increasing linearly with distance \( d \) from the top at the \( E_m \) rate. Ambient ions impacting the tape both leave as neutrals and liberate additional secondary electrons, which race down the magnetic field and excite neutral molecules in the E-layer, resulting in auroral emissions [18].

The electron beam from a tether is free from effects marring the “standard” beams introduced in 1969 for producing artificial auroras. Tether emission takes place far from any instrument, and has no effect on S/C potential. Beam density and flux are low, thus avoiding nonlinear plasma-interaction problems that arise with standard beams, which require high flux.
to make thin-beam ground observation possible [19]. Beam is about twice the electron gyroradius (\(\propto \text{energy}^{1/2}\)) thick, flux being less than \(10^5\) times the ambient plasma flux at emission, and the beam-to-ambient electron density ratio being less than \(10^5\),

\[
\Phi_\infty (h) \approx N_0 \Omega e _w \frac{m_e \gamma(eE_m d)}{m_i 2\pi \cos I(dip)},
\]

\[
\frac{N_b}{N_0} \approx \frac{m_e \omega}{m_i 4\sqrt{2} \cos I(dip)} \frac{\gamma(eE_m d)}{eE_m d},
\]

where \(\gamma\) is the energy-dependent secondary yield and \(I(dip)\) the magnetic dip angle.

Each point in the tether emits monoenergetic secondary electrons with a definite pitch-angle distribution, and energy and flux increasing with distance \(d\). As beam electrons move in helical paths down magnetic lines, they find a density of neutral molecules increasing with decreasing altitude \(z\). Beam electrons lose energy in inelastic ionization and excitation collisions, followed by photon emission; they are also scattered in elastic collisions with air molecules, which both affect pitch distribution and beam flux through diffusion across magnetic lines. The beam dwell-time at any particular point does permit excited states with prompt emission through \textit{allowed} transitions (lifetimes \(\sim 10^7\) s) to reach a steady-state, emission rates then being proportional to excitation rates. Since cross sections have similar energy dependence for all collisional interactions, there exist simple approximate relations between emission and ionization rates for prominent spectral bands and lines.

The low-flux, thin beam exhibits brightness for ground observation as low as 1 Rayleigh, light sources in the night sky masking such signals. On the other hand, brightness is much greater for observation from the spacecraft, over a hundred Rayleigh for prominent bands and lines, say 427.8 or 391.4 nm for \(N_2\), and 777.4 and 844.6 for \(O\) and \(O_2\), allowing continuous measurements (impractical for standard-beam cross sections). They involve ‘column’-integrated emission rates along straight lines extending over the ionization region and determining a relation \(z(d, \psi)\); brightness, at each small angle \(\psi\) from the magnetic field, mix \(z/d\) effects.

As a result, the narrow emission footprint of the beam, which is tens of kilometers long and covers a line-of-sight range of about 6°, shows a peak in brightness. Tomographic inverse to determine vertical profiles of densities involves density values at a number of altitudes equal to the number of pixels along one side of an imaging camera, each pixel corresponding to a line-of-sight. An iterative solution scheme uses density values at a step in evaluating a \(10^3 \times 10^3\) linearized kernel matrix, to determine densities at the next step. Proceeding with inversion requires a regularization technique; a direct approximation to the actual density profile used as good initial guess to start the iteration, which would not converge otherwise, is first obtained by fitting parameters in a model and using a Direction Set (Powell) technique [18].

5. Tether wave radiation

A tether carrying a steady current in the orbital frame \((\omega = v_{ref}k, \ k \equiv \text{wave vector})\), radiates waves with refraction index \(n = c k/\omega \gg 1\), just allowing Slow Extraordinary (SE), Fast Magnetosonic (FM), and Alfven (A) wave emission into the ionospheric cold-plasma. The radiation impedance is weak for FM and A emissions (and extraordinary weak for SE)

\[
Z_A \approx (2V_A/c^2) \ln(Qe^{1-1} \Omega L/v_{ref}), \quad Z_{FM} \propto 1/\omega_{pe} \omega_{pi},
\]

where \(V_A\) is the Alfven velocity and \(\omega_{pe}\) (\(\omega_{pi}\)) the electron (ion) plasma frequency [20].

On the other hand, switching the cathodic contactor on from the \textit{off} (floating) condition of Sec. 4 would produce a large surge in both current and radiated power, accompanied by bias/current pulses along the tether that can be modelled as a transmission line

\[
\frac{\partial I}{\partial s} = G_1 \Delta V + C_1 \frac{\partial \Delta V}{\partial t}, \quad \frac{\partial \Delta V}{\partial s} = -E_m + \frac{Z_I}{L} I + L_1 \frac{\partial I}{\partial t},
\]

where \(Z_I\) is the operational impedance of the contactor.
allowing signal emission. Dropping time derivatives recovers the equations describing profiles $\Delta F(s), h(s)$ along distance $s$ from the bare-tether anodic end. The conductance per unit length follows the OML collection law, $G_t \propto 1/\sqrt{|\Delta V|}$. Capacitance $C_t$ and inductance $L_t$ involve sheath radius and some radius characterizing current closure in the ambient plasma.

It was recently suggested that current modulation in tethers could generate non-linear, low frequency wave structures attached to the spacecraft. A magnetic pumping process, through magnetic oscillations in the near field of the radiated wave, would result in a parametric instability [21]. Pumping Fast Magnetosonic waves would involve a cylindrical array of parallel tethers flying vertical in the equatorial plane and stabilized by the gravity-gradient, which is perpendicular to the geomagnetic field when ignoring its tilt. A following nonlinear stage in the wavefront moving with the orbiting array might be represented by an equation of Korteweg de Vries type, and serve to study wave interactions in space plasmas.

Whistlers could be excited by a square array of electrodynamic tethers, made of two perpendicular rows of tethers that carry equal time-modulated currents with a $90^\circ$ phase shift. The array would fly vertical in the orbital equatorial plane, perpendicular to the geomagnetic field. Pumping by the whistler wave radiated along the field gives rise to coupled whistler perturbations with growth rate maxima at angles $38.36^\circ$ and $75.93^\circ$ away from it (Fig. 3).

6. Radiation Belt Modifications

There is recent interest in artificially modifying the high-energy particle populations trapped in the Earth Radiation Belts. Their densities are small (typically 10 m$^{-3}$) and natural replenishment rates are slow enough that mechanisms to scatter particles into their loss cone, over significant space volumes, might require reasonable power. Actually, calculations of electron loss rates due to several natural mechanisms (Whistler waves or Coulomb scattering), and due to a few high power VLF ground antennas suggest that man-made wave injections can be a dominant depletion channel. Recent observational confirmation was obtained by the Demeter Satellite, which measured energetic populations at 720 km altitude. Using ground stations for intentional Belt clean-up is inefficient, however, because only a fraction of order of 1% of kHz power is coupled to whistler radiation through “plasma ducts” in the Ionosphere. On the other hand, in-situ emission by an orbiting spacecraft carrying a very long antenna might be practical; the USAF DSX spacecraft will test this idea in 2010 [22].

Weak wave fields need act repeatedly over a particle to significantly modify its motion. This implies a resonance condition, which, for electrons, corresponds to the Whistler dispersion branch, with frequencies approaching values between $\sqrt{\Omega_e \Omega_e}$ (Lower Hybrid frequency) and $\Omega_e$, at wavelengths smaller than the skin depth $c/\omega_{pe}$,

$$1 \ll (2\pi)^2 \ll c^2k^2/\omega_{pe}^2.$$  

For waves that are nearly but not quite perpendicular to the magnetic field, the group velocity is nearly parallel

$$\sqrt{\frac{m_e}{m_i}} \ll \frac{\omega}{\Omega_e} \approx k_{par}/k \ll 1$$  \hspace{1cm} (16)

Wavelengths range from fraction of kilometer to several kilometers, impractical for a rigid boom but possible with a flexible tether. The index of refraction $n$ is high,

$$\frac{1}{n} = \frac{\Omega_e}{\omega_{pe}} \frac{\omega_{pe}}{c} \Omega_e \ll 1,$$  \hspace{1cm} (17)

making relativistic particles interact dominantly with the magnetic field of the wave. This results in just deflections with no energy change, thus describing a pitch-angle random walk that gradually diffuses particles towards the loss-cone boundary.

A second concept would use a pair of conductive bare tethers biased at potentials of the order of 1MV with respect to each other [23]. Since the ion current to the negative tether must equal the electron current collected by the positive tether, this one plays the role of the 3% segment in the electrically floating condition. The negative tether bias relative to the ambient
plasma will then be close to the full MV value. Its sheath radius would both be a fraction of 
kilometer and depend only weakly on the tether radius \( R \ll \lambda_D \) [12],

\[
1.53 \left[ 1 - 2.56 \left( \frac{\lambda_D}{r_{sh}} \right) \right]^{4/5} \left( \frac{r_{sh}}{\lambda_D} \right)^{4/3} \ln \left( \frac{r_{sh}}{R} \right) \approx \frac{e\Delta V}{kT}.
\]  

(18)

A fraction of all high-energy electrons and ions passing through the sheath would be scattered 
into their respective loss cones, while the wire radius, if small enough, could make both 
collected current and required power small, too.

7. Bare tethers at Jupiter

Because of both rapid rotation (about 10-hour period) and low mean density (1.32 
g/cm\(^3\)), the stationary orbit in Jupiter, which lies at radius

\[ a_s \sim R_J \left( \frac{\rho_J^{1/3}}{\Omega_J^{2/3}} \right) \]

(19)
is one third the relative distance for Earth, or \( a_s = 2.24 \, R_J \). In turn \( B_0 \) at its surface is greater 
than at Earth’s by one order of magnitude (with the motional field near Jupiter more than one 
order of magnitude greater). As a result, there is magnetospheric plasma co-rotating beyond \( a_s \),
allowing for Lorentz thrust on tethers in prograde Jovian orbit beyond \( a_s \). Also, maximum \( N_0 \)
is typically \( 10^2 \) times smaller than at the F-layer daytime maximum, and the Alfvén velocity, \( V_A \)
\( \propto B_0/\sqrt{\rho_0} \), may be up to \( 10^3 \) times greater than in LEO. This makes impedances for Fast 
Magnetosonic and Alfvén radiations greater for Jupiter by 2 and up to 3 orders of magnitude in 
(14a, b), both affecting the tether-current circuit and making significant signals possible [24].

Insertion in orbit and touring the Jovian moons afterwards, which are transport 
applications of interest, prove possible, a tens-of-km long tape with mass a sensible fraction of 
the full spacecraft mass being required. Radiation dose accumulated at repeated passes through 
the Jovian radiation belts appears as the limiting factor for such missions. This makes missions 
that avoid the belts, such as NASA’s Juno mission, particularly interesting. Typical power needs 
may be generated with tethers of moderate size and little effect on orbital dynamics because of 
the giant gravitational well of Jupiter [25].

This would also apply to a mission final-stage, with a spacecraft starting in circular, 
equatorial orbit, safe below the Radiation Belts, at radius \( 1.3 / 1.4 \, R_J \). A light, few kilometers 
long, thin tape bare-tether could make the spacecraft spiral in a controlled manner, over several 
months, while generating power onboard. A number of scientific goals might be attained. From 
its slowly decaying orbit the spacecraft could carry out spatially resolved observations as 
required for understanding transport in the atmosphere, and broad studies on its variability over 
different time scales. The proximity to Jupiter would allow highly accurate determination of 
magnetic and gravity fields and water content [26].

The initial location would be at the inner region of the Halo ring and the 2:1 Lorentz 
resonance, allowing for in situ measurements on charged grains. Lorentz resonances occur at 
circular equatorial orbits commensurate with periodicities of the magnetic field, \( B \propto \nabla \Psi \),

\[
\Psi = -R \sum_{l=1}^{l+1} \left( \frac{R}{r} \right)^l \sum_{k=0}^{l} \frac{k\cos \theta}{g_{l}^k \cos k\phi + h_{l}^k \sin k\phi} P_l^k(\cos \theta)
\]

(20)

with \( g \) and \( h \) Schmidt coefficients determined from observations, and \( P_l^k \) Schmidt-normalized 
associate Legendre functions. The 2-1 Lorentz resonance is the strongest one and arises from 
the \( g_2^2 \) term, which is about 0.4-0.5 Gauss in inner magnetosphere models.

The gravity gradient force is characteristically weak at Jupiter because of both low density 
and orbit radius well above \( R_J \) in most missions. The tether is kept taut by a spin in the orbital 
plane. With the gravity-gradient torque averaging to zero, the average Lorentz torque must 
vanish too. Keeping a hollow cathode at just one end, end masses appropriately different make 
the torque vanish over the active HC spin half-period. Over the other half-period the tether is
electrically floating. The emitted e-beam could result in multiple magnetic mirroring in the field of Eq. (20), before eventually excite auroral emissions, throughout the spacecraft orbital decay.

8 Electric Sails in Interplanetary Space

Hans Alfven first considered bare wires for interplanetary transportation but the solar-wind motional field, assumed to power an electric thruster, is so weak that superconductors were required [27]. Actually, using the Lorentz force on the bare tether would be more efficient. Recently, an array of very thin (R ~ 10 μm) bare tethers was proposed as a new type of solar sail, using the dynamic pressure of the solar wind for propulsion [28]. Because the sheath of each tether will be much larger than its radius, electric solar sailing may be efficient; the tether array, constituting a virtual sail, could be comparatively light. The Coulomb thrust is here dominant against the Lorentz thrust because of extreme values $B_{sw} \sim 0.00003$ Gauss, $v_{sw} \sim 400$ km/s, and $\lambda_0 \sim 10$ m, at 1 AU, say.

Early estimates of Coulomb drag on LEO satellites involved a high ion Mach number $M$, complicated 3D geometries with radius $R \gg \lambda_0$, and a floating probe condition ($-eAV$ a few times $kT$). In Middle Earth Orbit, satellites such as LAGEOS I and II can float positive because of dominant photoelectron-emission; tin/copper (few cm long) “dipoles” in orbit since the 60’s involve 2D geometries, like tethers. Coulomb forces have lately acquired relevance in Formation-flying satellites and in Dusty plasmas, where drag on charged (3D) grains involves a range of $R/\lambda_0$ and $M$ values.

Since the motional field is negligible for the e-sail, bias from a power supply will be uniform throughout the wire. The Coulomb force is thrust because the solar wind overtakes the sail, whether wire bias is positive or negative. Collecting electrons at positive bias is simpler because it requires ejecting electrons at a plasma contactor past the power supply. For a single wire, the Coulomb-to-Lorentz thrust ratio can be estimated as

$$\frac{F_{\text{Coulomb}}}{F_{\text{Lorentz}}} = \frac{2r_{sh} L \times N \cdot m_{w} \cdot v_{sw}^{2} / m_{i} \cdot e^{AV / m_{e}}}{L \Omega_{i} \cdot R \cdot e^{AV / m_{e}} \cdot \sqrt{m_{i} \cdot m_{e} / 2}} = \left(\frac{m_{i}}{m_{e}}\right) \left(\frac{r_{sh}}{R}\right)$$

where a very large $r_{sh}/R$ is given in Eq.(18) in terms of $R/\lambda_0$ and $eAV/kT$. Note the ordering $eAV \sim 10$ keV $\gg \frac{1}{2}m_{i}v_{sw}^{2} \sim 1$ keV $\gg kT \sim 6$ eV. A negatively biased, ion-attracting wire requires an ion gun but its Coulomb-to-Lorentz thrust ratio is larger by a factor $\sqrt{m_{i}/m_{e}} \sim 43$.

The ratio of $L\Omega_{i}$ to a relative velocity of interest also appears in comparing the Lorentz drag $L_{\lambda_{0}}OML$ on a tether in LEO to the standard aerodynamic drag by hypothetical neutrals, having the ion mass and density, on a body of equal frontal area $2RL$,

$$\frac{F_{\text{Lorentz}}}{F_{\text{neutrals}}} = \frac{L_{\lambda_{0}}OML}{2RLc_{D}m_{i}N_{0}v_{\text{orb}}^{2}} = \frac{4\sqrt{2}}{5c_{D}}\sqrt{\frac{m_{i}}{m_{e}}\left(\frac{\Omega_{i}L}{v_{\text{orb}}}\right)^{3/2}}$$

with $c_{D}$ typically about 2. The ratio above is usually extremely large. For oxygen, $\sqrt{m_{i}/m_{e}}$ is about 171. For $L = 10$ km, and typical LEO values (ion-gyrofrequency $\Omega_{i} \sim 200$ /s and $v_{\text{orb}} \sim 7.5$ km/s), Eq. (7) gives a ratio of order of $10^6$. This means that Lorentz drag from weakly ionized plasma can be effective where neutrals drag is negligible.

9. Conclusions

Issues related to applications of conductive tethers away from transportation in LEO have been briefly reviewed. The validity of the 2D OML collection regime at highly positive bias, and the adiabatic trapping of electrons by bare tethers orbiting in LEO were considered. Science applications of tethers such as generation of electron beams for atmospheric research, generation of nonlinear wave structures, and Modification of the Radiation Belts, as well as tether use at Jupiter and in the solar wind, were discussed.
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Figure 1 A $\mathbf{J}^2 = 2m_e r^2[E + e\Phi(r)]$ line

Figure 2 Potential vs $1/r^2$ in OML regime

Figure 3 Tether array to excite whistlers