A model is presented for simulation and economic evaluation of school plans within the framework of city master planning. The model has been applied to plans for a Swedish city, Västerås, and some illustrative results are reported.

1. INTRODUCTION

An important issue in city master planning in many urban areas today involves the shutdown and alternative use of existing schools (see Lerman [4]). The problem of where, when, and in what size to locate new schools is also important so as to avoid excessive busing. This problem is accentuated over time due to the fact that the aging population lives in the inner city where existing schools are located, while families of fertile age usually move to newly developed city areas. In order to determine such questions with respect to the efficiency objective, the costs and benefits of the possible alternatives have to be calculated.

The objective of this paper is to present a cost-effectiveness analysis of school plans within the framework of city master planning. The evaluation problems to be considered are discussed in Section 2. The school model, with emphasis on the cost aspects, is presented in detail in Section 3. Various limitations on the approach are also discussed. Some illustrative results from an evaluation of different school plans are presented in Section

1 We thank Professor Jan Oosterhaven, University of Groningen, Netherlands, for valuable criticism when an earlier version of this paper was presented and discussed at the Second World Regional Conference at Rotterdam in June 1984. We also thank Dr. Mats Bohman, University of Stockholm, Sweden, for his comments on this version. Financial support from the Swedish Council for Building Research is gratefully acknowledged.
In every local community there exist schools of different qualities, each with a particular location. Given the norms concerning minimum and maximum numbers of children per class and given maximum walking and busing distances, it is a simple task to calculate whether the existing schools are sufficient, too many, or too few at different points in time.

In Sweden, the number of schoolchildren will change over time in most local communities. It will therefore be necessary to increase or decrease the number of schools in these communities in the future. It is thus important to determine which new schools should be built and which existing schools should be kept or closed down, in what order and when, as well as how the community should be divided into catchment areas for each school.

The school system in Sweden is financed by taxes. This means that there are no market signals as to which new schools should be built or which existing ones should be kept or shut down. Instead, budgetary consequences exist; when a new school is built, there will be only expenditures on the budget since there are no revenues (as there are no school fees). Similarly, when a school is closed down, there are only savings in the budget since no revenues are lost. There are, however, both costs and benefits involved all the time. It is not sufficient to base a decision to build a new school or to shut down an existing one on the budgetary consequences alone. A cost–benefit analysis has to be performed to consider all the more important consequences in terms of costs and benefits. Here, a less ambitious but not quite simple cost-effectiveness analysis is presented.

The simulation and evaluation of a cost-minimizing school system for a local community with a changing number of schoolchildren over time is a complicated problem. Let us therefore first simplify the situation drastically by comparing only two particular schools in a setting where several other schools exist in the vicinity. The problem is then delimited as follows: if one of these two schools is to be shut down, which one should it be?

First, all of the cost items such as yearly maintenance must be identified and calculated. When it comes to capital costs, it is standard practice in Swedish local communities to include the historical costs of amortization of the school building. However, this is not correct; it is the actual and forecast opportunity costs for the school building and/or school site that are the real costs and that should be included.

Let us leave this simplified problem involving only two schools and look at the real situation, where several schools exist in a local community to form a system of schools. It can then be convenient to solve the following problem with the help of a cost-minimizing model: given a tentative plan
over which schools are to be kept at a particular point in time, which catchment area should "belong" to which school? The total consequences of commuting costs for the children can then be calculated in accordance with this plan.

Similarly, the cost consequences of another school plan may be simulated and calculated. These costs must then be added to all the other cost items for each plan, and the plan which minimizes total costs can be found. If the differences in benefits between the different school plans can be evaluated, such a value can be compared with the differences in costs to see if the ranking is changed.

The situation in reality is even more complicated. It may be that the number of schoolchildren decreases in one area of a city but increases in another because new housing is expected to be built in the latter and younger inhabitants of child-bearing age then move in. It is, however, possible to some extent to steer where new housing can be built—in one part of the city rather than in another—within city master planning, at least as it is carried out in Sweden. The "best" school plan found, then, is dependent on the city master plan chosen. Such a plan might involve, for instance, placing all the new housing areas outside the already built-up city (in the outskirts or satellites) or—in the opposite case—increasing the density within the already built-up city. This is why the school model was developed within a model for city master planning.

3. THE SCHOOL MODEL

3.1. Introduction

The main objective of the school model is to compute costs of different school plans, i.e., to perform a cost-effectiveness analysis of alternative school plans. The school model is inbedded in a larger model for city master planning that was developed for the Swedish city of Västerås; see Andersson et al. [1] and Andersson and Samartin [2, 3].

The school model can be divided into three parts. First, the distribution of the schoolchildren over the city area and over time has to be forecast. Second, once this distribution is known, a simulation of the assignment of schoolchildren to the different schools has to be carried out. Third, after the assignment of schoolchildren has taken place at each intersecting time, the corresponding costs can be computed.

The three computational steps may be summarized as follows:

(a) Distribution of the total number of schoolchildren among the different residential nodes: The number of children at the initial point in time and throughout the time span of the study is given as data. The distribution of schoolchildren is also known for the initial point in time. Starting from
this distribution, specific rules are applied to determine the distribution of schoolchildren over the city area for the total time of the study.

(b) Assignment of schoolchildren to the different schools: The concept of minimum commuting costs is used to determine the choice of school and transportation mode for children residing at each residential node. The following constraints apply in this assignment process:

— Walking and busing are the only means of transportation considered for schoolchildren.

— Maximum distances for commuting are specified. (For Västerås, the maximum walking distance is set at 2 km and the maximum busing distance at 10 km.)

— The capacities of the schools are given by the number of classrooms multiplied by a mean value of schoolchildren per class as data and the number of children attending a school may not exceed the capacity of the school.

(c) Calculation of school costs such as opportunity costs for existing schools, investment costs for new schools, maintenance costs, and transportation costs.

The results can be found using these computational steps. Such results are cost-minimizing catchment areas (i.e., the residential nodes where schoolchildren live and attend a particular school), school commuting mode used from each residential node to a school, school capacity available and used at each point in time and at each school, and school costs for each time period discounted and summed to present values. The cost-minimizing school plan is the “best” plan. It should be kept in mind that possible qualitative differences among the different schools are assumed away in the model.

3.2. Forecast for the Distribution of Schoolchildren over the City Nodes

The total number of schoolchildren in the city for each point in time is assumed to be exogenously given when a mean forecast of the total population is assumed. In order to consider different levels of the total population over time, and subsequently changes in the total number of schoolchildren, the following data are assumed given for each point in time: the ratio of the total number of schoolchildren to the total population, and the ratio of the number of children in each residential zone (a set of residential nodes) to the total population of the zone.

In addition, the following information is given for the initial point in time: the ratio of the number of children at each residential node to the total population at the node.
These data permit calculation of the ratio of the number of children in each residential node to the population at the node for each point in time throughout the simulation, if the following simplifications are introduced:

(a) Nodes which belong to the same zone of the city retain the same value of this ratio throughout the time period covered by the simulation.

(b) New residential nodes, including such nodes in the inner city where old residences have been demolished and replaced by new ones, are given a value of this ratio equal to the mean value of the ratio for the entire city at the point in time that they come into being.²

(c) Nodes which exist in the initial period retain a value of this ratio that is proportional to the value in the initial period.

The proportionality factor for existing nodes is obtained from the condition that the total number of children must equal the forecasted total. This can be expressed in mathematical terms as follows:

\[ sr_n^a = \frac{SC_n^a}{P_n^a} = \frac{\text{schoolchildren living at node } n}{\text{total population living at node } n} \text{ (at time } t_a) \]  \hspace{1cm} (1)

The values of \( sr_n^0 \) (initial time) are also known as data. The hypotheses presented may then be expressed in the following way:

(a) Each zone \( z \) is homogeneous, i.e., the ratio \( sr_z^a \) for the zone is equal to \( sr_n^a \) for every node \( n \) belonging to \( z \):

\[ sr_n^a = sr_z^a \text{ for every } n \in z \]  \hspace{1cm} (2)

(b) For new residential nodes, the value of \( sr_n^a \) is equal to the mean (average) value given as data for the total city:

\[ sr_n^a = \bar{sr}^a \]  \hspace{1cm} (3)

where \( n \) is a new residential node or a residential node in the inner city where houses have been demolished and replaced by new residences.

(c) For each zone \( z \) where no new residences will be built, the value of \( sr_z^a \) is scaled by a factor \( \lambda_z^a \) that is the same for all zones. The value of \( sr_z^a \) is obtained from the condition that the number of schoolchildren in the city

²These assumptions may appear to be rigid because they do not represent the full complexity of the real world. It is, however, possible to relax some of these assumptions without too much effort. The use of the mean value of the ratio for the entire city as a representation of the proportion of schoolchildren to the total population could, for instance, be changed by introducing a factor that considers the higher fertility of the usually young families who are moving to live there. These assumptions are under revision and can be improved in a future version of the model.
at each point in time must equal the forecasted number; thus

$$\lambda_a \sum_{z \in Z} sr_z^a p_z + \overline{sr}_z^a \sum_{z \in Z'} p_z^a = \overline{sr}_z^a \sum_{z \in Z + Z'} p_z^a$$  \hspace{1cm} (4)$$

where $Z$ is the set of city zones in which no new residences are built at time $t_a$ and $Z'$ is the set of remaining zones. The total city population is

$$P_a = \sum_{z \in Z + Z'} p_z^a$$

and $p_z^a$ is the total population living at zone $z$.

Thus, the following equations are obtained. If $p_n^a$ is the population at node $n$ at time $t_a$, the number of schoolchildren in zone $z$ is

$$SC_z^a = sr_z^a \sum_{n \in z} p_n^a$$  \hspace{1cm} (5)$$

and at node $n$ is

$$SC_n^a = SC_z^a \frac{SC_n^0}{\sum_{n \in z} SC_n^0} = sr_n^a SC_n^0 \frac{\sum_{n \in z} p_n^a}{\sum_{n \in z} SC_n^0}$$  \hspace{1cm} (6)$$

where $SC_n^0$ corresponds to the number of schoolchildren at node $n$ at initial time $t_0$.

Equation (6) applies to all existing nodes. For new nodes, the number of schoolchildren is

$$SC_n^a = \overline{sr}_n^a p_n^a.$$  \hspace{1cm} (7)$$

3.3. School Assignment

The main objective of this computational step is to assign the children living at each residential node to the existing or planned school centers in an efficient way. This part of the school model is developed on the basis of these two models, with some strong simplifications.

The schoolchildren choose their commuting mode (busing or walking) and school according to the following procedure, to be applied to every city node:

(a) The commuting costs $c_b$ from a particular residential node to each of the school centers are calculated for the two modes of transportation allowed (walking and busing), i.e., the costs $c_{bW}$ and $c_{bB}$ from node $n$ to school centers $M$. The minimum of these two values of $c_b$ is chosen for each school and the mode of transportation is also determined implicitly.
This minimum is

$$c_{b}^{n,M} = \min_{W, B} \left( c_{b}^{n,M}, c_{b}^{n,M'} \right).$$  (8)

(b) Maximum walking and busing distances are specified. If the distance to the school exceeds the maximum walking distance, then busing is compulsory. However, if the distance also exceeds the maximum busing distance, then the school is not a feasible one for the particular city node.

(c) If there is a feasible school relative to a given residential node, then the schoolchildren living at the node will be sent to the school with the least commuting costs if there is sufficient capacity at that school. If not, then the next cheapest school is selected, if it has enough capacity. Otherwise, the process is continued until a school with sufficient capacity is obtained.

In other words, if the minimum value of $c_{b}^{n,M'}$ over all school centers $M'$ is $c_{b}^{n}$ and it corresponds to a particular school center $M$, this indicates that the schoolchildren living at node $n$ will attend school center $M$, providing there exists enough free capacity at this school. Otherwise, the next cheapest school center $M$ (producing the possible minimum value of $c_{b}^{n,M'}$) with sufficient capacity for the schoolchildren living at node $n$ has to be found.

(d) If there is no school within the maximum busing distance from the residential node, then a new school center near the node is required. This situation is revealed by the model.

3.4. Calculation of School Costs

School costs are calculated within the model for economic evaluation in the model for city master planning. The details of this calculation procedure may be summarized as follows. The costs of schools are calculated according to the type of school, i.e., existing or new. The following cost items are considered for each case.

**Existing schools:** opportunity costs for the schools ($T_{C_{alt}}$), maintenance costs ($T_{C_{m}}$), and transportation costs ($T_{C_{t}}$).

**New schools:** investment costs ($T_{C_{inv}}$), maintenance costs ($T_{C_{m}}$), and transportation costs ($T_{C_{t}}$).

Teachers’ wages are excluded from the cost calculation. The rationale for this is that teachers’ wage costs will be approximately the same, regardless of which new schools are opened or old one closed when the forecasted number of students is the same in the different alternatives. The total school costs, $T_{C}$, are computed by adding the costs occurring at each intersection time in present values throughout all the periods of study. The costs $T_{C}(a)$ at the intersection time are given by the expression

$$T_{C}(a) = \sum_{m \in M_{1}} T_{C_{alt}}^{M} + \sum_{M \in M_{2}} T_{C_{inv}}^{M} + \sum_{m \in M_{1} + M_{2}} (T_{C_{m}} + T_{C_{t}})$$  (9)

where $M_{1}$ is the set of the existing schools and $M_{2}$ is the set of the schools.
planned to be built. The computation of these cost items may be commented on as follows.

**Opportunity costs for existing schools.** The approach used to calculate capital costs for school buildings and school sites involves basing amortization on historical costs, i.e., on the nominal value of the costs at the time the school was built. This approach is standard in Swedish local communities, but the evaluation principle is erroneous. Instead, we base our cost evaluation on the opportunity cost approach. This means that the estimated actual and forecasted value of the school building and/or school site for its best alternative use should be included as a cost.

Let us justify our choice of approach. Assume that an old school is situated in a very attractive area in the middle of the city. The historical cost of purchasing the site and of building the school has already been amortized a long time ago. According to this cost concept, the cost for the school building is zero. (As a matter fact, we found a couple of such schools in the books of the local community under study.) But if the building is of high quality, is well-constructed, etc., it may still be used not only as a school but also for alternative purposes, such as offices or residences, after some remodeling. Therefore, the school building and its site can be sold at a price determined by its value for its best alternative use. This value is determined by both the location of the school site in the city (the land-rent distribution) and the present status of the building. In order to calculate the last part of this value, we used estimations of the actual selling value for each school. Assuming an estimated economic lifetime for each school, the costs could be calculated in terms of annual rents for the school buildings and summed to present values for the given time horizon, also assuming a given real interest rate (6%). The land rents for the school sites were calculated by means of a model for land-rent distribution.

**Investment costs for new schools.** Investment costs, i.e., construction costs, for new schools are computed in a similar way to the above costs, in the sense that the estimated value of the existing school for an alternative use is replaced by the costs for new schools on sites outside the built-up city. These costs are expressed as costs per annum for the remaining economic lifetime and are summed to a present value. The land rents for the different school sites were calculated using a model for land-rent distribution.

**Maintenance costs.** The given data for maintenance costs correspond to the annual maintenance costs per student such as for cleaning, water and

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3 Due to lack of space, this model cannot be presented here. It is described in detail in Andersson and Samartin [3].

4 Maintenance costs can in principle be divided into two parts: one depending on the actual number of schoolchildren attending the school and the other as a function of the size (capacity) of the school. The model could easily be extended using this more realistic computational procedure if data of this kind were available.
sewage, and electricity. Once the number of students attending the school is known, these maintenance costs are obtained by multiplication and are discounted and summed to present values.

Transportation costs. Transportation costs are the minimum of the costs for the two possible commuting modes between residence and school. The individuals' walking costs are simply

\[ c_{r1} = \frac{d_1}{v_1} p_{r1} \]  

where \( d_1 \) is the straight-line distance between the residence and school. If this distance is greater than some given limit (2 km), then walking will be excluded (i.e., these costs are assumed to be very large relative to the costs for busing); \( v_1 \) is the walking speed; and \( p_{r1} \) is the value of time spent walking to school. It is assumed to be half the value of time spent (by workers) walking to work.

The individuals' busing costs are

\[ C_{r2} = t_4 p_{s4} + \frac{N_{BUS}}{SC} \left( d_4 \cdot p_{s4}^{bus} + t_4 \cdot p_{driv} + \frac{P'_4}{2N_{DAY}} \right) \]  

where \( t_4 = d_4/v_{40} \) is the time required for traveling by bus from the residential node to school; \( d_4 \) is the busing distance between the residential node and school (not the straight-line distance); \( v_{40} \) is the bus velocity without congestion; \( p_{s4} \) is the value of time spent traveling to school by bus; \( p_{bus} \) is the costs for fuel, oil, and tires, and depreciation and maintenance per kilometer; \( N_{BUS} \) is the number of bus trips from the residential node to school and is equal to the number of students living at the residential node divided by the capacity of a bus (\( q_4 \)); \( p_{driv} \) is the cost of manning the bus; \( P'_4 \) is the yearly maintenance costs; \( N_{DAY} \) is the number of school days, assumed to be the same as the number of working days; and \( SC_n \) is the number of school children living at residential node \( n \).

These costs are discounted and summed to a present value in a way similar to that for other cost items in the master city planning model.

3.5. Limitations in the Approach

The model presented above has some limitations that we want to point out. First, it is a cost-effectiveness analysis, i.e., the items on the benefit side are not dealt with. For a decision maker, it is necessary to analyze the benefit side as well as the cost side. Then any existing qualitative differences between two or more alternatives must be identified and in some way
evaluated. Such items include whether one of the schools has better premises specially designed for music, gymnastics, etc., or has better-coordinated teams of teachers, or better facilities on the school site such as green areas for playing. Evaluation of such items is a rather complicated task.

Second, the assignment problem of the school model can alternatively be handled using standard procedures developed in the framework of the mathematical theory of transportation such as by linear programming.

The use of linear programming to solve the assignment problem presented here has the following advantages. First, it is based on a recognized mathematical method and many standard algorithms for its solution are available. Second, it represents an extension of the assignment problem discussed in Section 3.2, because children living at a particular node can attend different schools instead of only one. Third, it produces an answer that minimizes the total costs for society. In the previous model, this fact was not explicitly considered.

On the other hand, the disadvantages may be summed up as follows. First, the solution of the problem in practical cases demands quite large computational resources and rules out the possibility of including the school model as part of a model for city master planning. Second, the answers obtained by the linear programming method are continuous numbers for the number of schoolchildren at a residential node that attend a particular school, and so can produce noninteger numbers that can represent unrealistic situations and therefore demand further treatment. Third, the linear programming method does not take into complete consideration the economic behavior of the individual, e.g., that schoolchildren want to attend the nearest school independently of the resulting net benefits to society. Apart from this, the concept of the catchment area is diffuse or lost entirely in this approach, because children living at the same node will attend different schools. The implementation of this model in practical situations is therefore probably very difficult.

4. RESULTS

We return to the main results from the evaluation of two city master plans for Västerås. First, the calculated school costs are presented and interpreted. A tentative sensitivity analysis is also shown. Some complementary results for the assignment of schoolchildren to different schools are given in the form of a computer-drawn map.

It should be emphasized that these results are preliminary. Therefore, only the results corresponding to one path for the working population are

5 It is then possible to use integer linear programming, but in this case, the complexity of the mathematical solution increases tremendously.
reported. The same simulated path is used for all the plans to facilitate comparison.

4.1. School Costs

Two different master city plans for Västerås are studied. Alternative B is a master city plan with an emphasis on building in the outskirts and satellites of Västerås. Alternative D is an urban renewal plan with demolition and concentration of new residences in the inner city.

The distribution of schools among different cost items is shown in Table 1 for alternatives B and D. The difference in school costs between the two alternatives is due to the fact that six new schools are planned to be built in the satellite alternative B and only two new ones in the urban renewal alternative D.

The assignment of children to the existing 43 schools at the initial point in time is given. The capacities available are also given. Ten of these schools are closed during the period covered by the simulation due to assumed expiring lifetimes. Figure 1 shows the assignment of schoolchildren to the remaining schools and to newly built schools for alternative D in the final period (1996–2000). Such maps can illustrate the consequences of the different plans and may provide suggestions for iterative changes in the plans which could be worthwhile.

The results should not be interpreted to imply that the schools which are scheduled to be closed down should actually be closed. The value of a particular school does not necessarily depend only on the age of the school building. It might also depend on its location relative to other schools. Also, the quality of a team of teachers at a school and the ability of the principal to stimulate the teachers to engage in fruitful educational activities might differ substantially from one school to the next. Such differences in the quality of school services might be difficult to transfer from a school that is closed down to another that still exists. Therefore, some of the schools assumed to be closed in accordance with an expiring lifetime might be remodeled at some cost and be allowed to continue, while others on the list might be closed on schedule.

<table>
<thead>
<tr>
<th>Cost items</th>
<th>Alternative B</th>
<th>Alternative D</th>
<th>Alternative B−Alternative D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost for busing</td>
<td>12</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Cost for school buildings</td>
<td>67</td>
<td>44</td>
<td>23</td>
</tr>
<tr>
<td>Maintenance costs</td>
<td>158</td>
<td>158</td>
<td>0</td>
</tr>
<tr>
<td>Total costs</td>
<td>237</td>
<td>214</td>
<td>23</td>
</tr>
</tbody>
</table>
4.2. A Sensitivity Analysis

A sensitivity analysis for alternative B with respect to some assumed values is shown in Table 2. The following variations are studied: a reduced maximum busing distance (from 10 to 5 km); a reduced remaining economic lifetime for already existing schools (minus 8 years); a decrease in
Sensitivity Analysis for Alternative B with Respect to School Costs (Millions of SEK)

<table>
<thead>
<tr>
<th>Cost items</th>
<th>Reduced busing distance (from 10 to 5 km)</th>
<th>Reduced economic lifetime for the school buildings (minus 8 years)</th>
<th>Reduced number of schoolchildren per class (from 25 to 20)</th>
<th>Increased number of schoolchildren per class (from 25 to 30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs for busing</td>
<td>9</td>
<td>17</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>Costs for school buildings</td>
<td>66</td>
<td>68</td>
<td>69</td>
<td>64</td>
</tr>
<tr>
<td>Maintenance costs</td>
<td>151</td>
<td>152</td>
<td>186</td>
<td>135</td>
</tr>
<tr>
<td>Total costs</td>
<td>227</td>
<td>237</td>
<td>270</td>
<td>208</td>
</tr>
</tbody>
</table>

The following observations can be made concerning the results for the main case of alternative B:

— A reduced busing distance will decrease the costs for busing and maintenance.

— Busing costs will increase when the assumed economic lifetime of the school building is reduced, as the average commuting distance to the schools increases. The capacity of student places then will not be sufficient at the end of the time horizon under study.

— Maintenance and busing costs will increase when the number of schoolchildren per class is decreased and vice versa. The capacity of student places will not be sufficient at the beginning and at the end of the time horizon. On the other hand, there will be a considerable overcapacity during the whole period when the number of students per class is increased.

5. CONCLUSIONS

If the objective of efficiency is of interest for the decision maker, it is necessary to at least calculate total costs of alternative school plans. The model presented in this paper may therefore be helpful in an iterative procedure, where school plans are successively revised in light of information obtained from previous evaluations. The model can provide planners with information such as how great additional benefits of a more costly alternative must be as a minimum for that alternative to be preferred.

In view of the strong interdependence among various sectors, a local government cannot achieve efficient solutions to its planning problems unless it has access to a tool for drafting school plans within the framework of city master planning. The school model presented here has the advantage of being developed within a city master planning model.
The school model has been applied in a pilot study in the Swedish city of Västerås and some illustrative results are presented. The computer program for the school model is now operating in the environment of the work of practical planners (the Västerås City Hall). In future research, this school model can be improved, for instance, by attempts to introduce benefit items.