Review and comparison of effective delayed neutron fraction calculation methods with Monte Carlo codes

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ABSTRACT

The calculation of the effective delayed neutron fraction, $\beta_{\text{eff}}$, with Monte Carlo codes is a complex task due to the requirement of properly considering the adjoint weighting of delayed neutrons. Nevertheless, several techniques have been proposed to circumvent this difficulty and obtain accurate Monte Carlo results for $\beta_{\text{eff}}$ without the need of explicitly determining the adjoint flux. In this paper, we make a review of some of these techniques; namely, we have analyzed two variants of what we call the k-eigenvalue technique and other techniques based on different interpretations of the physical meaning of the adjoint weighting. To test the validity of all these techniques, we have implemented them with the MCNPX code and we have benchmarked them against a range of critical and subcritical systems for which either experimental or deterministic values of $\beta_{\text{eff}}$ are available. Furthermore, several nuclear data libraries have been used in order to assess the impact of the uncertainty in nuclear data in the calculated value of $\beta_{\text{eff}}$.

Keywords:
Effective delayed neutron fraction
Adjunct flux
Monte Carlo
MCNPX

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1. Introduction

The effective delayed neutron fraction $\beta_{\text{eff}}$ is a crucial parameter in reactor safety since it corresponds to the maximum reactivity that can be inserted in a critical system without becoming prompt-critical. This parameter is also fundamental to describe the kinetic and dynamic response of both critical and subcritical nuclear systems to internal or external perturbations.

Calculation methodologies for $\beta_{\text{eff}}$ must take into account that it is an adjunct-weighted parameter. Since the calculation of adjoint fluxes with Monte Carlo codes is cumbersome, $\beta_{\text{eff}}$ is usually calculated with deterministic codes. Nevertheless, its calculation with Monte Carlo codes is also desirable since they allow dealing with more complex geometries, different materials, and continuous energy cross-sections. The need for accurate calculation tools for $\beta_{\text{eff}}$ is specially relevant in the case of ADS that cannot become critical, since the experimental determination of $\beta_{\text{eff}}$ is usually very difficult in a subcritical state.

For this reason, a large number of publications have appeared over the last years considering different techniques for the calculation of $\beta_{\text{eff}}$ with Monte Carlo codes. Trying to group them, we have classified them into two categories. The first one comprises techniques based on k-eigenvalue calculations; the second one comprises techniques based on different interpretations of the adjoint weighting, such as those based on interpreting the adjoint...
weighting as the next fission probability or the iterated fission probability. In addition, a third category of techniques can be considered to include those based on perturbative methods, such as those derived in Nagaya and Mori (2005, 2011), but we have not considered them in this paper.

In the first of these categories, techniques based on k-eigenvalue calculations, we include techniques based on the definition and calculation of certain parameters by analogy to the effective multiplication constant (\(k_{eq}\)), such as those of Brethes (1967) and Spriggs (2001). Techniques based on the interpretation of the next-fission probability as the adjoint weighting have been analyzed by Nauchi and Kameyama (2005), Meulekamp and Van der Mark (2008) and Nagaya (2010). Techniques based on the interpretation of the iterated fission probability as the adjoint weighting can be seen as an improvement of the previous ones, and they have been proposed by Nauchi and Kameyama (2010), Raskach et al. (2010), Chiba (2011) and Iwamoto et al. (2010). In Section 2 we will provide some discussion on the derivation and the physical meaning of all these techniques.

In Sections 3 and 4 we will present the results of the application of the above mentioned techniques against a number of critical and subcritical benchmark systems, that we consider to be representative of a wide range of nuclear systems. For this we have used the Monte Carlo code MCNPX (Pelowitz et al. 2006) with three different nuclear data libraries (ENDF/B-VII.0, JEFF-3.1 and JENDL-3.3). The use of several nuclear data libraries allows us to set a lower limit to the uncertainty on \(\beta_{eff}\) estimators, due to both the accuracy of the different techniques and the uncertainties of the basic nuclear data.

2. Calculation methodologies

The usual definition of \(\beta_{eff}\) is:

\[
\beta_{eff} = \frac{\langle \phi_{d+}, F_{d+}\phi_{d} \rangle}{\langle \phi_{d+}^{2}, F_{d+} \rangle} \quad (1)
\]

where \(\tilde{F}\) is the creation operator, that takes into account all neutrons (prompt and delayed) created in the phase space by fission, and \(F_{d}\) is the delayed neutron creation operator, that takes into account only delayed neutrons. The brackets indicate integration over the whole phase space. More specifically, the expressions in the numerator and in the denominator of Eq. (1) can be expanded as:

\[
\langle \phi_{d+}^{2}, F_{d+} \rangle = \int \phi_{d+}^{2} \{\tilde{F}, \Delta E, \tilde{\Omega}\} \Sigma_{d} \langle \tilde{F}, \tilde{E}, \tilde{\Omega} \rangle \nu_{d}(E) \times \chi_{d} \{E, \Omega, \tilde{E}, \tilde{\Omega}\} \phi_{d}(E) \, d\tilde{E} \, d\tilde{\Omega} \, dE \, d\tilde{\Omega} \, d\tilde{E} \quad (2)
\]

\[
\langle \phi_{d+}, F_{d+}\phi_{d} \rangle = \int \phi_{d+}^{2} \{\tilde{F}, \Delta E, \tilde{\Omega}\} \Sigma_{d} \langle \tilde{F}, \tilde{E}, \tilde{\Omega} \rangle \nu(E) \times \chi \{E, \Omega, \tilde{E}, \tilde{\Omega}\} \phi_{d}(E) \, d\tilde{E} \, d\tilde{\Omega} \, dE \, d\tilde{\Omega} \, d\tilde{E} \quad (3)
\]

\(\nu(E)\) and \(\nu_{d}(E)\) denote, respectively, the average number of total (both prompt and delayed) and delayed neutrons at energy \(E\) produced per fission. \(\chi_{d}\{E, \Omega, \tilde{E}, \tilde{\Omega}\}\) and \(\chi \{E, \Omega, \tilde{E}, \tilde{\Omega}\}\) represent, respectively, the spectrum of energy and angular distribution \(\{E, \tilde{E}, \tilde{\Omega}\}\) of the total and delayed neutrons produced by an incoming neutron with \((E', \tilde{\Omega'})\). \(\Sigma_{d}\) is the macroscopic fission cross section.

Finally, \(\phi_{d}\) and \(\phi_{d}'\) are respectively the \(\tilde{E}\)-mode direct and adjoint neutron fluxes, that is, the fundamental mode solutions of the eigenvalue equations:

\[
M\phi_{d} = \frac{1}{k_{eq}} F_{d+}\phi_{d} \quad (4)
\]

\[
M^{+}\phi_{d}' = \frac{1}{k_{eq}} F_{d\prime+}\phi_{d}' \quad (5)
\]

being \(M\) the migration and losses operator, that takes into account the net number of neutrons leaving the phase space element by capture, out-scattering or streaming, and \(\tilde{F}\) is the creation operator, already defined. \(M^{+}\) and \(F_{d\prime+}\) are their corresponding adjoints.

In this work, we consider only "effective" the delayed neutron fraction defined in Eq. (1) with the fluxes \(\phi_{d}\) and \(\phi_{d}'\). Several other delayed neutron fractions \(\beta\) can be defined considering fluxes other than \(\phi_{d}\) or \(\phi_{d}'\) but they will not be the "effective" values anymore. See, e.g., Bell and Glashorne (1970), Henry (1975) or Ott and Neufhold (1985) for further discussions on this topic. For instance, considering the adjoint flux to be constant over the whole phase space, we can define a non-adjoint weighted delayed neutron fraction, \(\beta_{0}\), that can be expressed as:

\[
\beta_{0} = \frac{\langle \phi_{d}, F_{d}\phi_{d} \rangle}{\langle \phi_{d} \rangle} \quad (6)
\]

The determination of \(\beta_{0}\) with Monte Carlo codes poses no major difficulty and can be performed by simply counting the number of total and delayed neutrons produced in fission processes. On the contrary, the determination of adjoint-weighted parameters requires the development of specific methodologies.

2.1. k-eigenvalue methods

Some of these methodologies can be classified as k-eigenvalue methods because they are based in defining and solving eigenvalue equations similar to (4) and (5). A first method is applied by Bretschner (1997) and it has been named the prompt method by Meulekamp and Van der Mark (2006) and the prompt k-ratio method by Nagaya and Mori (2011). It is obtained by defining the following eigenvalue equations:

\[
M\phi_{p} = \frac{1}{k_{p}} F_{p}\phi_{p} \quad (7)
\]

\[
M^{+}\phi_{p}' = \frac{1}{k_{p}} F_{p\prime+}\phi_{p}' \quad (8)
\]

where \(F_{p}\) is the prompt neutron creation operator. Assuming that \(\phi_{0} \simeq \phi_{d}\), in which principle seems a great approximation since over 99% of the neutrons produced in fission are prompt neutrons, we can obtain that:

\[
\beta_{eff} = \frac{\langle \phi_{d}, F_{d}\phi_{d} \rangle}{\phi_{d}, F_{d}\phi_{d}} = 1 - \frac{\langle \phi_{d}, F_{d}\phi_{d} \rangle}{\phi_{d}, F_{d}\phi_{d}} \simeq -\frac{\langle \phi_{d}', k_{eff} M\phi_{d} \rangle}{\phi_{d}', k_{eff} M\phi_{d} \rangle} = 1 - \frac{k_{p}}{k_{eq}} \left( \frac{\langle \phi_{d}', M\phi_{d} \rangle}{\phi_{d}', M\phi_{d} \rangle} = 1 - \frac{k_{p}}{k_{eq}} \right) \quad (9)
\]

Notice that the k-mode flux, \(\phi_{k}\), obtained as solution of Eq. (4) only corresponds to the physical flux for a critical system. As the system departs from criticality, the physical flux also begins to differentiate from \(\phi_{k}\). Hence, the concept of effective delayed neutron fraction loses significance for systems far away from critical.
The eigenvalue $k_2$ can be easily evaluated in MCNPX without requiring any modification of the code, making the most of the ability of this code to “switch off” the delayed neutron transport (this is a standard feature of MCNPX and MCNP since version 4C, and it is performed by overriding the KCODE card with a TOTNUO card). To perform this calculation it is obviously required that nuclear data libraries contain information about delayed neutron spectra, which is included in the latest versions of the most common ones.

A problem of the prompt method lies, in our opinion, in understanding how it takes into account adjoint weighting. In fact, the calculation of an adjoint-weighted parameter like $\rho_{ad}$ should not be possible using expressions that only contain k-eigenvalues, i.e., that are not adjoint-weighted parameters. Notice that both eigenvalues $k_n$ and $k_p$ can be determined (Eqs. (4) and (7)) with no need to define any adjoint flux. For instance, Bretscher (1997) considers no adjoint fluxes for deriving the method.

Notice as well that, in principle, any weighting function (e.g. $\Phi_p$ as defined by Eq. (8)) can be used instead of $\Phi_1$ in Eq. (9), leading to the same numerical value for $\rho_{ad}$. In particular, we could consider a constant weighting function over the whole phase space and therefore the method must provide values for $\rho_{ad}$ instead of $\rho_{ad}$. In our opinion, it is not fully understood how the appropriate adjoint weighting is taken into account when applying the prompt method, as defined by Eq. (9), and, in particular, when it is applied with a Monte Carlo code. These issues have not been addressed when the prompt method is discussed by authors like Meulekamp and Van der Marck (2005) or Nagaya and Mori (2011).

In Meulekamp and Van der Marck (2006), it is also remarked that the prompt method is always an approximate method, because of the approximation $\Phi_0 \approx \Phi_1$. For this reason, we have considered the possibility of removing the approximation by defining a new parameter $k'_p$ as:

$$k'_p = \frac{\frac{\Phi_1}{M\Phi_1}}{\frac{\Phi_0}{M\Phi_0}}$$

Notice that Eq. (10) is not an eigenvalue equation. With this parameter, a new delayed neutron fraction $\beta'$ can be defined in an analogous manner to Eq. (9):

$$\beta' = 1 - \frac{k'_p}{k_{ad}}$$

Although with this definition of $k'_p$, it is expected that $\beta'$ will be equivalent to the non-adjoint weighted neutron fraction $\rho_{ad}$, as defined by Eq. (6), we have also included the results obtained with this parameter in Section 4, to help clarifying how the adjoint weighting influences the prompt method. We will denote the prompt method using $k'_p$ as prompt method with the total eigenfunction, as opposed to the previous method, which we will denote as prompt method with the prompt eigenfunction.

$k'_p$ can be calculated with MCNP performing a single KCODE cycle switching off the delayed neutron transport and taking as initial fission source distribution a previously calculated one with a flux that includes both prompt and delayed neutrons. An alternative to improve statistics is to perform many of these KCODE switching off delayed neutrons and upgrading the initial fission source in another KCODE cycle run in parallel, this one considering both prompt and delayed neutrons.

2.2. Methods based on different interpretations of the adjoint flux

A second class of methods for the calculation of $\rho_{ad}$ is based on the interpretation of the physical meaning of the adjoint weighting. To derive this interpretation, let us consider the case of a subcritical reactor maintained in a steady state by an external neutron source. The neutron transport equation applied for this case takes the form:

$$\nabla \Phi - M \Phi = -S_{ni}$$

As the adjoint equation, let us consider:

$$\nabla \Psi^T - M \Psi^T = -\Sigma_j$$

where we have considered the macroscopic fission cross section as the adjoint source. Then, subtracting Eq. (12) multiplied by $\Psi^T$ and Eq. (13) multiplied by $\Phi$ and integrating the resulting expression over the entire range of the variables taking into account the properties of the adjoint operators, we end up with the following expression:

$$\int S_{ni}(\vec{r}, \vec{G}, E) \Psi^T(\vec{r}, \vec{G}, E) \, d\vec{G} \, dE$$

$$= \int \Sigma_j(\vec{r}, \vec{G}, E) \Phi(\vec{r}, \vec{G}, E) \, d\vec{G} \, dE$$

The right-hand side of the last equation is just the response of the system to the external source, that is to say, the number of fissions per unit time that takes place in the whole system in the presence of the external source. We can further clarify this if we consider the external source to have the shape of:

$$S_{ni}(\vec{r}, \vec{G}, E) = \delta(\vec{r}_0, \vec{G}_0, E_0)$$

Then we find out that:

$$\Psi^T(\vec{r}_0, \vec{G}_0, E_0) = \int \Sigma_j(\vec{r}, \vec{G}, E) \Phi(\vec{r}, \vec{G}, E) \, d\vec{G} \, dE$$

where $\Phi(\vec{r}, \vec{G}, E)$ is the solution of the equation $\nabla \Phi(\vec{r}, \vec{G}, E) = -\Sigma_j(\vec{r}, \vec{G}, E) \delta(\vec{r}_0, \vec{G}_0, E_0)$. That is to say, the value of the adjoint flux in a point of the phase space is the response of the system to a unit source introduced in this point. Hence, adjoint functions can be interpreted as the source importance, meaning the total number of fission reactions caused by the introduction of a source neutron in the point of the phase space $\left(\vec{r}, \vec{G}, E\right)$.

Alternative ways to derive the interpretation of the adjoint weighting as the neutron importance or the iterated fission probability are presented in Henry (1975) and Hurwitz (1964). Be aware that in the previous derivation, the macroscopic fission cross section has been considered as the adjoint source. Bell and Glassstone (1970), for instance, consider a macroscopic detector cross section instead, and interpret the adjoint flux as the importance of a neutron to the detector response.

To implement the interpretation of the adjoint weighting as neutron source importance to calculate $\rho_{ad}$ it is necessary to determine the different importances of prompt and delayed neutrons in generating new fissions. For this, first notice that, as it is well known, the total progeny of a given neutron is given by $k + k^2 + k^3 + \ldots$ in case the multiplication constant $k$ remains constant between fission generations; otherwise it will be given by $k_0 + k_0k_1 + k_0k_1k_2 + \ldots$. Therefore, it is necessary to determine the multiplication constant of delayed neutrons.

To obtain these values, a modification of the MCNPX code has been implemented to track the delayed neutron creation. In this way, if we start from an already converged fission source distribution, we can now obtain the subset of the fission source corresponding to delayed neutrons. Once this delayed neutron source

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4 Take into account that this is equivalent to the track length estimator of $k$. Hence, we will use this estimator for all calculations.
is known, a value of $k_{on}$ can be calculated (in MCNPX this is performed with a single KCODE cycle, taking this delayed neutron source as initial fission source distribution). Notice that, with this definition, $k_{on}$ is not an eigenvalue, as it is calculated from the delayed component of a converged source.

This first KCODE cycle will also provide a second fission source distribution that can be used as the initial fission source distribution for another KCODE cycle, in order to obtain a second multiplicative constant $k_{on}$ and a third fission source distribution. In turn, this third source distribution can be used to obtain a third multiplicative constant, $k_{on}$, and so on. Notice that after several cycles the value of $k_{on}$ will tend to the value of $k_{on}$, as the neutron sources used to calculate them tend to reach the initial fission source distribution (considering both prompt and delayed neutrons), and therefore the results obtained with this technique will converge.

Once the values $k_{on}, k_{on}$, etc., have been determined, the value of $\beta_{on}$ can be calculated as:

$$\beta_{on} = \frac{\text{Equilibrium number of delayed neutrons}}{\text{Equilibrium number of all neutrons}} \times \frac{\text{Average multiplication of delayed neutrons}}{\text{Average multiplication of all neutrons}}$$

$$= \beta_{on} \lim_{k_{on} = k_{on} \cdots k_{on} = k_{on}} \times \frac{\langle 1 \rangle \cdots \langle 1 \rangle}{k_{on} \cdots k_{on}} \cdots k_{on}$$

$$= \beta_{on} \lim_{k_{on} \to 0} k_{on} \cdots k_{on} = k_{on} \cdots k_{on} = k_{on}$$
generations. Therefore, in this case we consider that a better estimate of the neutron importance is the total (or integrated) number of fissions produced by a given neutron, i.e., $k_{inf} + k_{inf}k_{inf} + k_{inf}k_{inf}k_{inf} + \ldots$, which tends to converge to a finite value. Thus, this is the interpretation we have adopted in this paper. It must be remarked that this same interpretation has already been applied by Feghali (2007, 2008) to calculate neutron importance functions and importance-weighted neutron generation times.\footnote{In fact, we consider that for a critical system both interpretations are actually equivalent. In a critical system, the expression $k_{inf} + k_{inf}k_{inf} + k_{inf}k_{inf}k_{inf} + \ldots$ in the numerator and $k_{inf} + k_{inf} + k_{inf} + \ldots$ in the denominator both diverge. Therefore, we can neglect the first $m$ terms (which take a finite value) in both the numerator and the denominator, and take out the common factors, to be left with:}

In practice, we only need to calculate a limited number of values of $k_{inf}$ providing that this number is enough to reach convergence. If we are left with a single cycle we will be considering only the multiplicity of the delayed neutrons but not of their progeny, which is defined as the next fission probability in Meulekamp and Van der Marck (2006):

$$\beta_{inf} = \beta_0 \frac{k_{inf} k_{inf}}{k_{inf}}$$

In Meulekamp and Van der Marck (2006) it is also discussed the validity of this approximation, arguing that the exact knowledge of the adjoint flux is not critical since the value of $\beta_{inf}$ is largely determined by the value of $\beta_0$, and hence the accuracy in the definition of the adjoint weighting function has little impact. However, the validity of this approach has been questioned by other authors, for instance Irwanto et al. (2010) and Chiha (2009). We discuss the validity of this approximation in Section 4.

3. Benchmark systems

We have implemented the techniques described in the previous section with a modified version of the Monte Carlo code MCNPX 2.7 and validated them against a set of benchmark experiments for which measured or deterministic data for $\beta_{inf}$ are available. These benchmarks have been chosen to be representative of wide ranges of systems. Hence, they include two homogeneous bare fast systems (Godiva and Jezelbel), four reflected fast systems (Topsy, Popsy, Big-Ten and CORAL-1), a thermal system (TCA), two configurations of a subcritical system (Yalina-Booster) and three large fast systems (MUSE-4, ESFR and MYRRHA).

MCNP inputs have been taken from the JHECSBE (2008) (the reference number is given), unless indicated otherwise. A brief description of these system follows. The same $\beta_{inf}$ reference values considered by Meulekamp and Van der Marck (2006) have been used when available; otherwise, the source is referenced.

- Godiva (HEU-MET-FAST-001). A bare, highly enriched (94 w/o) uranium spherical core containing 52.42 kg of uranium. The proposed MCNP model consists of five shells with slightly varying composition and an additional sixth shell added to compensate for reflections in the supporting elements and other factors.
- Jezelbel (PU-MET-FAST-001). A bare plutonium sphere (4.5 w/o of Pu-240) containing 17.020 kg of a plutonium and gallium alloy (1.02 w/o of gallium).
- Topsy (HEU-MET-FAST-028). A sphere of 93 w/o enriched...
uranium (17.84 kg) surrounded by a thick (19 in.) U-238 reflector.
- **Popsy (PU-MET-FAST-006).** A plutonium/gallium sphere (6.06 kg) surrounded by a uranium reflector.
- **Big Ten (HEU-MET-FAST-007).** The Big Ten core contained uranium with three different enrichments (93%, 10% and natural). It was surrounded by a depleted uranium reflector. Total uranium mass in the reactor was 10 metric tons.
- **Coral-I.** A reflected, highly enriched uranium cylinder (containing 26 kg of 90% enriched uranium) surrounded by a reflector of natural uranium (Hernández et al., 1972). It was in operation at CIEMAT (formerly JEN) between 1968 and 1988. MCNP models for CORAL-I are available (Villamarin et al., 1999). Experimental $\beta_{ef}$ data were measured and are presented in De Francisco et al. (1973).
- **TCA (LEU-COMP-TERM-006).** The TCA (Tank Critical Assembly) consists of a tank containing an array of fuel assemblies of 2.5 w/o UO$_2$ and water that acts as moderator. The number of fuel rods and the lattice geometry and pitch can be changed. The reactor is made critical by adjusting the water level.
- **Yalina-Booster.** The Yalina-Booster subcritical facility is located at the JIPNR-Sosny of the National Academy of Sciences of Belarus. The facility consists of a fast and a thermal zones partially decoupled by a thermal neutron absorbing layer between them. The fast zone is formed by HEU (90% and 36% enrichment) in a lead matrix and the thermal zone is formed by LEU (10% enrichment). The whole assembly is surrounded by a graphite reflector. Experimental $\beta_{ef}$ data are not available, but deterministic calculations with the ERANOS code have been reported by Aliberti et al. (2009) for two different configurations of the system with a different number of fuel rods in the thermal zone (902 and 1141, respectively). The reference value we consider here is the average of several libraries. No estimation of the error is given. MCNP inputs were supplied by JIPNR-Sosny and have been modified to fit these two configurations.
- **MUSE-4.** The MUSE-4 experiment was carried out at the MASURCA reactor at the CEA-Cadarache facility (France) between 2001 and 2004 and comprised several critical and subcritical configurations. The reference critical configuration consisted of a core formed by a mixture of MOX fuel and sodium rodslets surrounded by a reflector made up of stainless steel and sodium rodslets. The MCNP input has been made using the specifications given in Soule et al. (2001). $\beta_{ef}$ data have been taken from Lebrat (2008). It must be remarked that the values of $\beta_{ef}$ were measured in a later configuration than those of the MCNP input file used, with 1115 fuel elements instead of 1112. These three elements were added to compensate for the decay of some of the Pu-241 in the MOX fuel. Nevertheless the difference in the measured value of $\beta_{ef}$ is not expected to be significant.
- **ESFR (European Sodium Fast Reactor).** The 3600 MWth core under consideration is divided in two zones with different Pu content, which is surrounded by a stainless steel reflector. Two different cores are studied. The first core is loaded with (U,Pu)O$_2$ MOX fuel and the second has (U,Pu,MA)O$_2$ fuel. The two zones averaged Pu content is 15 w/o and in
the second case the MA content is 4 w/o. In both cases, the results for beginning of life (BOL) and after 1230 equivalent full power days (representative of end of Equilibrium Cycle) are presented. The reference results for $\beta_{ef}$ were calculated by PSI with the deterministic code ERANOS (Martín-Fuertes et al., 2011). The MCNP input was developed within the participation of CIEMAT in the CP-ESFR project (Pérez-Martin et al., 2011).

- **MYRRHA.** This facility is a Lead–Bismuth cooled research reactor planned to be built at SCK–CEN site in Mol (Belgium) (Sarotto et al., 2012). It is conceived to operate in both critical and subcritical modes, in this case acting as a prototype of accelerator driven system (ADS). Both configurations have been considered in our study: a 100 MW (th) critical core at beginning-of-cycle (fresh fuel Pu content 34% wt.) and a 94 MW subcritical core at end-of-cycle (Pu content 30% wt. in the fresh fuel). Both MCNP reference inputs were developed by SCK–CEN within the CDT FASTEF project. The reference results for $\beta_{ef}$ were calculated by ENEA using ERANOS code. Fuel depletion calculations for the end-of-cycle core have been performed with the EVOLCODE system (Álvarez-Velarde et al., 2006).

In all the cases except the ESFR and MYRRHA, the $\beta_{ef}$ results with three different nuclear data libraries (ENDF-VII.0, JEFF-3.1 and JENDL-3.3) processed at room temperature are presented. For the ESFR and MYRRHA, we only present results for the JEFF-3.1.1 library processed at operating temperatures.

4. Validation results

The results of the validation are presented in Tables 1–3. They include results of $\beta_{ef}$ with the techniques described in Section 2 as well as results for $\beta_{b}$. The ratios between the values of $\beta_{ef}$ calculated with the different techniques and the reference values are presented graphically in Fig. 1.

The prompt method (both with the prompt and the total eigenfunction) has been applied considering the simple average of the three $k_{ef}$ estimators provided by MCNPX (collisions, absorptions and track length estimators). In the case of the I.F.P. and N.F.P. methods only the track length estimator has been considered.

Results show that the prompt method, with the prompt eigenfunction, fits the reference experimental or deterministic results with good accuracy in all the cases. These results are consistent with the conclusions of previous benchmarks (e.g. Meulekamp and Van der Marck, 2006), in spite that, as it was commented in Section 2, the question of how adjoint weighting is taken into account with this method, is not, to our understanding, fully explained.

On the other hand, the prompt method with the total eigenfunction has been found to provide values of $\beta_{b}$ rather than values of $\beta_{ef}$, as expected. Although in homogeneous systems (Godiva, Jezebel) both values are similar, in heterogeneous (reflected) systems (Topsy, Popsy, Big Ten, CORAL), where $\beta_{b}$ and $\beta_{ef}$ are very

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6 It has to be noticed that the $\beta_{b}$ estimator must keep the correlation among the two calculations (with and without delayed neutrons), which, in general, is lost using the combined average $k_{ef}$ estimator.
different, the prompt method with the total eigenfunction overestimates the value of $\beta_{gf}$ by a factor of up to two. For thermal systems (TCA and Yalina-B), however, it has been found to underestimate the values by about a 10%.

The integrated fission probability methodology has also been found to provide accurate results for $\beta_{gf}$ in all cases for the number of KCODE cycles considered (50). If only a single KCODE cycle is performed (next fission probability approximation) the values of $\beta_{gf}$ obtained can be up to about 10% lower than the reference values for the case of reflected systems (Popsy, Topsy, Big-Ten, CORAL).

In these cases, we have analyzed the number of KCODE cycles required in order to reach convergence in the value of $\beta_{gf}$. The results of this analysis are presented in Fig. 2. Notice how the value of $\beta_{gf}$ increases rapidly with the first cycles, and after about 15 cycles, a relatively constant level is reached. A very accurate determination of $k_{gf}$ is required for this level to remain constant with the number of cycles; otherwise, the bias in $k_{gf}$ will accumulate with the increasing number of cycles and it will result in an increasing systematic deviation in the value of $\beta_{gf}$ with the number of cycles. Therefore, a larger number of KCODE cycles require a higher precision in the determination of $k_{gf}$ (more statistics).

To better understand the evolution of the $\beta_{gf}$ results with the number of KCODE cycles, we have obtained the radial distributions of the initial delayed neutron fission source, and the fission sources after a different number of cycles for the cases of the four spherical reactors (Fig. 3). It can be observed how, for the cases of the non-reflected systems (Godiva and Jezebel), the fission source distribution does not experience major changes with the number of cycles. For these systems, there was little difference between the $\beta_{gf}$ results considering the next fission and the integrated fission probability interpretations of the adjoint weighting. In the reflected systems (Popsy and Topsy), however, there is a considerable difference among the initial delayed neutron fission source, the neutron source obtained from the first KCODE cycle, and subsequent fission sources (from the second one, it is apparent that the fission source has reached a distribution that is stable with the number of cycles). These noticeable differences among fission sources can explain the differences in the $\beta_{gf}$ results obtained with the N.F.P. and the I.F.P. techniques.

Finally, concerning the uncertainty in the results due to the nuclear data library used, we would like to stress that it is of the same magnitude as the difference between the prompt method (with the prompt eigenfunction) and the integrated fission probability method. With both methods, changing the nuclear data library can drive to changes in the $\beta_{gf}$ results of up to the order of 10 pcm. Therefore, we cannot conclude whether any library is better suited than the others for $\beta_{gf}$ calculations for a particular type of systems.

5. Conclusions

We have analyzed and benchmarked the performance of several proposed Monte Carlo techniques for the calculation of the effective delayed neutron fraction, $\beta_{gf}$. They are two versions of the so-called prompt method, both with the prompt eigenfunction and the total eigenfunction, and the techniques based on the interpretation of the next fission probability (N.F.P.) and the integrated fission probability (I.F.P.) as the adjoint weighting.

Results show that the prompt method with the prompt eigenfunction and the technique based on I.F.P. provide the most accurate values for $\beta_{gf}$. The first is the simplest to be applied with MCNPX and does not require any modification of the code. However, it has a weak theoretical justification, as it is necessary to explain how adjoint weighting is approximated with it. The usage of the total eigenfunction instead of the prompt eigenfunction has also been investigated and has been found to provide values for $\beta_{gf}$ instead of $\beta_{gf}$.

The technique based on I.F.P. provides similar values for $\beta_{gf}$. It has been noticed that as the number of KCODE cycles becomes larger, the uncertainties in the determination of $k_{gf}$ and $\beta_{gf}$ cause the uncertainty in the $\beta_{gf}$ results obtained with this method to increase. The technique based on N.F.P. can be considered the first cycle of the technique based on I.F.P. and has been found to work reasonably well for most of the systems considered but it causes noticeable systematic errors in reflected heterogeneous systems.

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