Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Computer analysis and design of concrete shell roofs

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Abstract

This paper is a preliminary version of Chapter 3 of a State-of-the-Art Report by the IASS Working Group 5: Concrete Shell Roofs. The intention of this chapter is to set forth for those who intend to design concrete shell roofs information and advice about the selection, verification and utilization of commercial computer tools for analysis and design tasks. The computer analysis and design steps for a concrete shell roof are described. Advice follows on the aspects to be considered in the application of commercial finite element (FE) computer programs to concrete shell analysis, starting with recommendations on how novices can gain confidence and competence in the use of software. To establish vocabulary and provide background references, brief surveys are presented of, first, element types and formulations for shells and, second, challenges presented by advanced analyses of shells. The final section of the chapter indicates what capabilities to seek in selecting commercial FE software for the analysis and design of concrete shell roofs. Brief concluding remarks summarize advice regarding judicious use of computer analysis in design practice.

Keywords: concrete shells, shell roofs, shell design, shell analysis, finite element computer programs

1. Introduction

This paper is a preliminary version of Chapter 3 of a six-chapter State-of-the-Art Report by IASS Working Group 5: Concrete Shell Roofs. First versions of other chapters that have already been published include:

- Chapter 2 – Shape finding of concrete shell roofs [36],
- Chapter 4 – Design of reinforcement in concrete shells: A unified approach [31],
- Chapter 5 – Buckling of concrete shells: An overview [30], and
- Chapter 6 – Construction methods and quality control for concrete shell roofs [32].

The intention of this chapter is to set forth for those who intend to design concrete shell roofs information and advice about the selection, verification and utilization of commercial computer tools for analysis and design tasks. Virtually all of the relevant computer tools suitable for shell structures employ the finite element (FE) method, the only technique that is addressed in this paper. Rather than citing specific commercial software tools by name, the authors provide general guidelines and advice about what is possible, what is appropriate at different stages of design, and how to gain confidence in commercial tools that are largely "black boxes." The user of software retains professional responsibility for the design produced; therefore, an extension of this responsibility is the duty not only to develop knowledge and understanding about the computer methods used but also to develop caution and skepticism about numerical results. There are now many textbooks on the FE method, such as[5], [6], [13], [15], [26], [38], [45] and [48], plus the extensive (but now somewhat aged) state-of-the-art represented by [22], all of which include some information about shell analysis and most of which also issue cautions about informed use.

Why the focus on commercial software? Although there is ongoing research on the still-challenging aspects of the analysis and design of concrete shells, most design practitioners do not have access to research computer tools or have the in-house ability to develop them. Therefore, this chapter does not focus on current research as part of the state-of-the-art related to the typical design office. For extraordinarily challenging design projects, such as very large shell roofs or especially critical structures, designers should consider consultation with specialists and researchers who have the knowledge and tools commensurate with the best of contemporaneous research.

Much commercial FE computer software has been developed within the framework of mechanical engineering, i.e., for metallic shell structures insofar as they include capabilities for shells, and only some of them handle reinforced and/or prestressed concrete. Therefore, active and passive reinforcement design in shells is still an open problem, and a common consensus on the subject has not yet been reached (nevertheless, some recommendations are given in [31]). It should be noted that current reinforced concrete research is related mainly to one-dimensional structures, i.e., beam and column structures. Even the reinforcement design of two-dimensional structures such as simple slabs subjected to both in- and out-of-plane loadings has not been standardized. Moreover, computer simulation of three-dimensional reinforced concrete behavior is a topic still under intense research.

In the past, several recommendations, state-of-the-art reports and design guidelines on concrete shell roofs have been published, e.g., [2], [20], and some supplementary parts of the Eurocode2. Most of these sources of information have only sparse guidelines on analysis corresponding to classical linear-elastic shell procedures. Also only particular "mathematical" shell geometries such as spheres, cylinders, surfaces of revolution and hyperbolic paraboloids ("hypars") are typically considered in most of these sources whereas there has been an increasing interest in "non-mathematical" shapes obtained from shape-finding and optimization methods [36] as well as from the use of air-supported forms for construction [43].

In the next section the computer analysis and design steps for a concrete shell roof are described. The following section advises on the aspects to be considered in the application of commercial FE computer programs to concrete shell analysis, starting with recommendations on how novices can gain confidence and competence in the use of software. To establish vocabulary and provide background references, this section continues with brief surveys of, first, element types and formulations for shells and, second, challenges presented by advanced analyses of shells. Then the fourth section indicates what capabilities to seek in selecting commercial FE software for the analysis and design of concrete shell roofs. Finally brief concluding remarks summarize advice regarding judicious use of computer analysis in design practice.

2. Computer analysis and design phases

For a typical concrete shell roof design project there are ordinarily a number of analysis and design phases in the iterative design process. The process can only start after finding a **conceptual design** of one or more alternatives that cover the given area and that fulfill the structural functional conditions, i.e., the boundary conditions and the number and distribution of intermediate supports. Conceptual design ideally builds upon an understanding of shell theory and behavior as well as a creative variation of existing designs. There are several textbooks on shell theory, e.g., [12], [14] and [25], but only a few that focus as well on shell design, e.g., [3], [8] and [44]. An envisioned construction process should also influence the conceptual design.

The conceptual design entails selection of the geometry of middle surface of the shell so that most of the whole shell region is under compressive stresses. In addition, although it is common to start with a uniform thickness, a possibly varying thickness over the surface is part of the conceptual design. Unless a "mathematical" shape is to be selected, a form-finding process as described in Chapter 2 [36] can be employed to find a "non-mathematical" surface. If an optimization approach is used, an appropriate variable thickness can also be obtained [36]. An alternative, less elaborate procedure is to apply analysis based on membrane shell theory [8] to a selected shell shape under a chosen primary loading system such as gravity loads and to check to see if substantially the entire shell is under compressive stresses. (It is important to realize that the membrane stress state in a shell depends not only on its middle surface geometry but also on the boundary conditions and the loading.) From such a preliminary analysis it is also possible to obtain a tentative shell thickness distribution. Typically, bending is concentrated along the shell boundaries where bending stresses appear. Some details are given in [8].

Sophisticated computer-aided shape-finding procedures described in [36] are not usually found in commercial programs, although some programs with geometrically nonlinear capabilities can be cleverly used for such processes. As is well known, an analogy in form exists between tensile and compressive structures such as cables vs. arches and membranes vs. shells. This idea has been employed in experimental form-finding for shells [21], [36]. The method can be extended to computational approaches using large-displacement FE analysis that is available in most commercial programs [41], making it possible to find a suitable shell shape and its thickness distribution when subjected to the primary loading, typically loads due to gravity. A key parameter in this case is a substitute Young's modulus for the shell [16], [41]. Caution should be taken in the use of any shape-finding technique, because sometimes optimized shapes and thickness variations can increase the imperfection sensitivity of the shell to buckling [36].

2.1. Structural modeling

The first stage of computer analysis is the creation of a structural model within – or compatible with – the software. The following entities need to be defined:

- The middle surface geometry, either by the mathematical shape or by some appropriate surface modeling such as splines or Bezier patches
- The thickness variation over the surface
- Any intersections or groins
- Any ribs or stiffeners
- All displacement boundary conditions and internal supports
- Any other constraints such as symmetry conditions or non-rigid supports
- The material properties

For reinforced concrete thin shells it is usual to model the material, at least initially, as homogeneous and isotropic, especially before the layout of reinforcement is known. At some more advanced stages of the analysis and design cycle, the structural model may need to include information about the placement, orientation and properties of the reinforcing steel and the possible influence of the concrete age differences that may occur during shell construction.

2.2. Loading hypotheses

Computer modeling of loads and load combinations demands special attention. As has already been emphasized in the discussion of the conceptual design, it is important for the shell designer to identify the so-called primary loading, i.e., the one producing the most important stresses in the shell roof, typically consisting of gravity loads that include the dead loading and perhaps a uniform live load such as snow. But other distributions of loads over the shell surface also need to be modeled, including non-uniform and non-symmetric live loads such as wind and drifting snow. Because shells are inherently statically indeterminate, it is possible that thermal loading may need to be considered. Quasi-static wind loading is generally normal to the shell surface and should include possible uplift pressure. Potential lateral loads such as seismic forces may need to be considered. However, dynamic loads seldom need to be considered for the design of concrete shell roofs unless the shells are very large or very flexible.

Shells do not perform well under concentrated loads but usually they must be subject to some such loadings due to fixtures that may need to be suspended from the roof. In addition, it may be appropriate to model any prestressing effects by point and/or line loads on the shell structure. In case of prestress, cable deviation loads due to the prestressing forces should be included in the analysis.

2.3. Element selection, meshing and mesh refinement

Implicit in this description of the typical phases of the FE analysis process is that the structural and load modeling is entirely independent of the meshing of the shell. The meshing is an artifact of the analysis process, and the elements and edges of any mesh should properly inherit the geometry, properties and loads of the model that have been described in the previous two sections.

Virtually all modern commercial programs have algorithmic capabilities for generating meshes over surfaces and for refining these meshes, either uniformly or selectively. Meshing tessellates the surface into either triangular or quadrilateral elements, and the choice relates to the selection of an element shape from the program's element library. If there are multiple shell elements in the library, the selection of the most appropriate is the starting point of the discretization process (see Section 5.1 for more information about shell elements). Generally triangular meshing is the most flexible for surface shapes that have acute corners because quadrilaterals may need to be unduly distorted to fit such zones.

The mesh density has a strong effect on the accuracy of the numerical results because the FE method is an approximate numerical procedure. The analyst typically has some control over the density and gradations of the mesh. Finer gradations in zones of anticipated high gradients of behavior are desirable. Moreover, any FE analysis should employ two or more meshes, increasingly refined, to see whether the meshing is adequate, that is, whether the numerical results for displacements and stresses seem to be converging to stable values. Some commercial computer programs have the capability to perform **a posteriori error analysis** [46], [48], and with this information mesh refinement may be carried automatically until a uniform estimated numerical error can be reached throughout the shell surface.

2.4. Preliminary analyses

In a well designed concrete shell, the membrane stresses are typically small, and it is the deflections and bending moments that will affect the design. Therefore, it is always appropriate to begin with linear elastic analysis to evaluate the stress resultants and deflections. To interpret the performance of a design one wishes to visualize the principal stresses trajectories that indicate the how the external loads are transferred to the reactions at the supports or foundations. In addition, one wishes to ensure that substantially the entire shell is under compressive stresses, including in areas where there may be bending moments. Finally, one wishes to see that the elastic deflections are only a very small fraction of the thickness of the shell while recognizing that these deflections will increase due to creep.

2.5. Results evaluation

Errors or inadequacies in a FE element shell analysis can arise due to numerical errors, modeling errors, poor FE technology in the software (such as less-than-adequate shell element formulations), software bugs, or human errors in the use of the software. There are some experts who say that one should not perform a finite element analysis unless one already knows the "answer"; and then the analysis is to be used only to gain greater richness of detail if one can gain confidence in the solution. Therefore, it is essential that the results of any FE analysis be examined critically.

One should not only be certain that the numerical analysis appears to converge with mesh refinement to stable values of both deflections and stress resultants but also that the results "make sense." The best approach is to assume that the FE element results are wrong until comparison with alternative solutions, such as point wise membrane analysis for stresses, indicates otherwise. Other comparative solutions might include FE analysis with different software, hand solutions based on similar geometries, and published results for similar shells. It is also important to perform overall and local equilibrium checks, for example to ensure that all applied loads are equilibrated by the calculated reactions. Aspects of the FE solution that do not make apparent sense must be resolved before using the analysis results for further phases in the process.

2.6. Design modifications

Only after confidence has been gained in the results of the structural analysis is it appropriate to use the insights gained from these elastic analysis results to modify the configurations, support conditions, prestressing, surface shape and thickness variations of the shell. This modification step is inherently part of the usual cyclical design process in structural engineering; if changes to the design are deemed necessary, a return to the first phase, structural modeling (Section 2.1), is the next step in this cycle.

Some FE software packages may have re-design capabilities for suggesting modifications to surface shape and/or thickness variation, but few if any such semi-automated design capabilities are able to suggest more sweeping changes to the design such as configuration and support changes. It is advisable for the engineer to exercise the same degree of skepticism about programmatic design suggestions as for analysis results, that is, these re-designs should pass the test of whether they "make sense" in comparison to similar existing designs that are performing well.

When and if design modifications are deemed no longer necessary, one typically proceeds to the reinforcement design and possibly thereafter to more refined analyses as described in the following sections.

2.7. Reinforcement design

The layout of reinforcement is significant for control of cracking, shrinkage and creep under service loads and also affects the ductility and behavior of the shell at ultimate loads. Typically reinforcement design is not accomplished within FE software but is a separate process that uses the results of FE analysis. Although there is still debate about what is a comprehensive approach to the design of reinforcement, there is advice in Chapter 4 [31], and possible constitutive equations to be used to consider different reinforcing layouts have been proposed [31], [40].

If FE collapse analysis is to be performed, the reinforcing layout and properties must be added to the structural model, Section 2.1.

2.8. Advanced analyses

Depending upon the overall design requirements, upon the size and importance of the structure and upon comparisons to similar existing shells that may have exhibited sensitive behavior, more advanced FE analyses may be required to evaluate various design conditions and limit states. Usually such analyses should be performed only after a thorough design has been posed based on linear elastic analysis. Moreover, such advanced FE analyses should always build upon the modeling, meshing and evaluative experience gained from linear elastic analysis. Although advanced analysis can certainly provide insights that motivate design modifications, for concrete shell roofs the advanced analysis phase can be viewed as a means of confirming or checking the adequacy of the linear elastic design. This is in contrast with design practice of large industrial concrete shells such as cooling towers for which design practice now conventionally includes explicit modeling of nonlinear limit states and dynamic behavior.

Among the advanced FE analyses that may be appropriate are:

- Classical linear buckling analysis of the perfect shell as the starting point of the traditional buckling design ideas described in Chapter 5
- Linear buckling analyses of hypothesized nonsymmetrical imperfect surface shape to gain some insight about imperfection sensitivity of the shell
- Geometrically nonlinear analysis of perfect and imperfect surface shapes to estimate buckling limit loads, possible snap-through failure and/or post-buckling behavior
- Elastic free vibration analysis to check for possible resonance
- Response spectrum analysis for seismic performance
- Transient dynamic analysis, either linear or fully nonlinear, to estimate extreme responses to earthquake and/or wind loadings
- Creep analysis under gravity loads
- Fully nonlinear analysis to explore the development of cracking and crushing as selected loading condition(s) are increased and to estimate the ultimate (collapse) load

Not all of these capabilities are present in commercial FE programs as some capabilities are still under advanced research and development. Some further comments regarding these analysis types are given in Section 3.3.

If any of these analyses undertaken indicate that design refinements or changes are necessary, much of the analysis and design process may need to be repeated, beginning with the first phase (Section 2.1).

3. Advice about commercial FE computer programs

An instance of commercial FE software may or may not be well suited for concrete shell roof analysis and design. This section and the next deal with general advice for designers that is intended to provide some background to facilitate the use and selection of commercial software. In addition, it is noteworthy that there exists NAFEMS, "an independent not-for-profit body with the sole aim of promoting the effective use of engineering simulation such as finite element analysis…" [34], which provides or sells training, books, benchmark problems and other aids. This is another source of information and advice that can be useful for designers of shells [17], [23].

In addition, for those with little experience in both shell design and FE analysis, it is advisable to initially use simpler means to develop a sense of understanding of the general structural behavior of a contemplated design prior to beginning FE shell analysis. These simpler means may be manual computations, such as by membrane-theory analysis recommended earlier in Section 2, or they may employ capabilities of the selected commercial FE software that differ from shell analysis. One possibility is to consider a three-dimensional skeletal model of the shell to obtain a rough idea of the overall behavior while recognizing that the virtue of a doubly curved shell is the tendency to convert bending behavior into membrane actions. Another possibility, especially for shells of positive Gaussian curvature, is to perform an initial FE analysis of an equivalent surface using membrane elements rather than shell elements.

3.1. Gaining competence with a commercial program

Section 4 provides hints about selecting a commercial FE program to be employed for concrete shell roof design. Once a program is selected, there are a number of steps that can be taken to gain confidence and capability in the software package. These are to be undertaken in the same sense of responsibility alluded to in Section 1 – that the user of software is responsible for the design, and therefore, the user should develop confidence (or skepticism) about the application of any software for analysis and design.

Recommended steps to gain competence with a commercial FE program include:

- Study background material as well as the technical documentation and sample problems provided with the program
- Use the program to replicate the sample problems provided
- Test the software against various standardized benchmark problems, e.g., [23], and other published results, including some already designed and well functioning shells
- Always compare results with alternative methods of analysis such as membrane theory [8] to see if the numerical results "make sense"
- Understand that it is more difficult for FE analysis to predict stresses and stress resultants accurately than deflections so that testing should examine both types of results
- Always start testing with simple models and linear elastic analysis
- Use mesh refinement in testing to observe different levels of FE approximation
- Try automated error estimation and mesh refinement, if available
- Be cautious about testing the more advanced types of analysis outlined in Section 2.8 – see Section 3.3

3.2. Some background on shell finite elements

As an extension of the advice being offered to designers of concrete shell roofs, this section provides an overview of shell element formulations to establish vocabulary and to indicate some potential difficulties exhibited by some types of formulations. Many of the general FE textbooks have information on shell elements, e.g., [13] and [48], and should also be consulted. In addition, there are advisory publications that specifically address shell elements, for example [17].

There are multiple alternative formulations for shell finite elements, and research to find the best formulations, modeling and analysis techniques continues – as is evidenced by papers presented at recent conferences on computational methods for shells such as [1] and [37]. As of this writing, it is unlikely that the best of the most recent developments have been incorporated into commercial FE programs, but this is an evolving situation.

The challenge in formulating satisfactory shell finite elements is associated with the difficulty of posing the shell theories upon which they are based. Development of such theories has been called "attempting the impossible" because such theories have tried to provide a two-dimensional representation of an intrinsically three-dimensional phenomenon [24]. Therefore, much of recent research has focused on how 3D mechanics can inform shell FE analysis. However, the results of this most recent research have generally not yet been implemented in commercial FE programs, although a variety of longstanding element formulations based on "degeneration" of 3D mechanics [4] are available in many commercial programs.

The "thinness" of shells is based on the ratio of the radii of curvature to the shell thickness. Metallic shells and some industrial concrete shells (such as cooling towers) often have high ratios and may be termed "thin." However, concrete shell roofs may in some cases have a sufficiently small radius-to-thickness that transverse shearing deformations become significant in static or dynamic behavior, and such shells may be called "moderately thick." (Thick concrete shells such as arch dams can usually be analyzed as 3D solids rather than shells; and thick shells are not given much attention here.)

Finite elements for shells may have the following characteristics:

- They may be flat or curved surfaces
- They may be triangular or quadrilateral in shape
- The are based on one of the following formulative principles:
  - Simple superposition of membrane and plate bending behaviors (flat elements only)
  - Thin shell theory according to Kirchhoff-Love (KL) in which straight normals to the undeformed middle surface remain straight and normal after deformation
  - Moderately thick shell theory according to Reissner-Mindlin (RM) in which straight normals to the undeformed middle surface remain straight but no longer normal after deformation
  - Three-dimensional mechanics of solids, usually degenerated to a surface with either KL or RM kinematics

In addition, elements for the nonlinear analysis of reinforced concrete divide the elements into layers through the thickness representing concrete or steel reinforcing. Examples of this type of element are given in [11], [27], [28] and [33].

Curved elements, for which bending and membrane behavior are coupled, are generally based on shell theories, but curved shell-theory elements, either triangular or quadrilateral, with sufficient inter-element continuity of displacements and rotations (so-called C^1

continuity or conformity, theoretically required for convergence to shell theory) are scarce in the element libraries of commercial computer programs. Nevertheless there have been relatively successful non-conforming formulations, and these have appeared early in some commercial programs. Planar elements are simpler than the curved ones, because they can be constructed as a combination of membrane and plate elements. However, they have the same difficulties in meeting conformability requirements when their faceted mesh represents a smooth surface. Most shell elements based on plate+membrane or shell theories lack any stiffness against rotational deformation about the normal to the surface (the so-called "drilling degree of freedom"), and this can give rise to numerical singularities unless properly accounted for in the element formulation or the software platform [48].

Planar or curved triangular elements can always be meshed to approximate a curved shell surface but they can produce results dependent on the arrangement and orientation of the mesh to fit the shell geometry. On the other hand, planar quadrilateral elements must be distorted (twisted) to lie on a doubly curved shell surface and can therefore be inadequate. Both triangular and quadrilateral elements can exhibit poor behavior if their geometry is distorted, e.g., elements with large length:width aspect ratios, with overly acute or obtuse corner angles or with side nodes located with unequal spacing along the side.

The formulation of curved elements degenerated from 3D isoparametric brick elements is well known [4], and these elements appear in several commercial computer programs and can be applied to either thin or thick shells. But unless proper corrective measures are taken, a numerical anomaly known as "locking" can occur in situations, such as very thin shells or shells with high membrane stresses, where the assumed displacement interpolations inherent in the element formulation overconstrain the deformations and the results become useless [42]. Following the lead of [47], reduced-order and selectively reduced-order quadrature techniques have been applied to eliminate locking difficulties in various manifestations of the degenerated solid elements, e.g., [7]. An anomaly converse to locking that must also be avoided arises from underconstraining of some element formulations through reduced-order quadrature that leads to numerical instabilities, sometimes known as "hourglassing." An alternative formulation approach to circumvent both of these anomalies uses assumed strain distributions rather than assumed displacement fields within elements [10]. Shell designers need to be aware of these potential anomalies because some commercial programs offer the user choices in the order of quadrature for some elements in the software library.

3.3. Analysis challenges

In addition to potential difficulties presented by the available element formulations such as described in the previous section, the nonlinear analysis process can be fraught with challenges that should be faced with extreme caution. Some of these difficulties are directly affected by possible locking phenomena of the elements and others are inherent in the nonlinear analysis procedures. The shell designer intending to undertake advanced analyses such as outlined in Section 2.8 must be aware of these potential challenges.

Especially for shells that may be sensitive to instabilities, the tracing of the geometrically nonlinear equilibrium path, the detection of limit or snap-through points and the estimation of post-buckling behavior are particularly difficult and precarious. These problems naturally also can carry over to fully nonlinear analysis of shells, although for concrete shells the ultimate load often is dominated by material collapse rather than instability.

Another area of potential difficulty is nonlinear transient dynamic analysis where the choice of algorithms (explicit or implicit) and associated time steps may strongly affect the quality of the predicted results.

Therefore, an informed and responsible user of shell analysis software requires a sound understanding the theoretical background of the selected computer program. There are a number of textbooks that specifically address the challenges of nonlinear analysis and structural behavior, and it is recommended that one or more of these be consulted for use with the technical documentation of a commercial program: [6], [9], [26], [29], [35] and [38]. Relevant journal papers include [18] and [19], while general FE textbooks that include some consideration of these challenges include [5], [13] and [48].

4. What to seek in selecting commercial FE software

In selecting a commercial FE computer program for analysis and design of concrete shells one should seek to satisfy several requirements that reflect what has been presented in Sections 2 and 3 above. These necessary or desirable features and capabilities include:

- Necessary: Good documentation, including technical information and solved sample problems for shell analysis
- Necessary: The availability of shell elements, preferably several types, that are able to model arbitrarily shaped shells, both thin and moderately thick shells, and general boundary conditions
- Desirable: Shell elements that can model reinforced concrete, i.e., layered elements and nonlinear steel and concrete material modeling
- Necessary: Capabilities for graphical pre- and post-processing, the latter including interpolation or smoothing of stresses, stress resultants and stress trajectories for shells
- Necessary: Capabilities for structural geometric modeling – mathematical and free-form surfaces, groins, ribs and edge beams, supports and boundary conditions and eventual shell imperfections
- Necessary: Option to select different element formulations and to combine them (e.g., shells with beams to represent stiffeners)
- Necessary: Choices of material modeling – elastic and various types of inelastic
- Desirable: Nonlinear material modeling of reinforced concrete including yielding, crushing, cracking and tension stiffening
- Necessary: The possibility to simulate various models of loadings – dead, wind, snow, thermal, seismic, dynamic loads
- Desirable: The possibility to model prestressing
- Necessary: Mesh generation and mesh refinement capabilities
- Desirable: Capability – applicable to shell FE models – for *a posteriori* error analysis that leads to automatic mesh enrichment or refinement

- Necessary: Inheritance of attributes by any mesh, i.e., the structural geometry, properties, support conditions and loadings are automatically associated with the FE mesh upon generation or refinement
- Necessary: Choices of analysis types including most or all of: static, classical buckling (elastic stability), free vibrations, dynamic, geometrically nonlinear, materially nonlinear and fully nonlinear
- Desirable: User choice and control of advanced analysis capabilities such as implicit vs. explicit formulations for transient dynamic analysis, selection of an incremental-iterative procedure for nonlinear analysis [39]
- Necessary: Capability to produce analysis results directly usable for the designer of shell structures, i.e., stresses in local (surface) coordinates at middle surface and extreme fibers plus stress resultants (normal, bending and shear forces per unit length)
- Desirable: Possibility of computer-assisted design and/or re-design capabilities for shape, thickness variation, and/or configuration, possibly by optimization
- Desirable: Possibility of computer-assisted design and/or re-design capabilities for concrete reinforcement
- Desirable: Capabilities for the advanced user to program and integrate modules such as specialized material models, element stiffnesses or reinforcement design

5. Concluding remarks

The insights obtained by the proper use of modern computer analysis and design strongly benefit the designer of concrete shell roofs by helping ensure safe designs that minimize use of materials. Nevertheless, the conceptual design phase remains central to the creativity of a design process that seeks to achieve innovation and elegance as well as efficiency and economy. Judicious use of computational tools to evaluate conceptual designs may enhance, but cannot replace the creativity and the attention to details of the engineer.

The tone of this chapter is intended to make clear that FE analysis of concrete shell roofs is a powerful tool but one that must be used with knowledge, judgment and caution. There is a risk of over-trusting and over-using computational power, especially when the focus is on what is still as challenging a computational problem as the simulation of the full-range behavior of concrete shells. There may be a strong temptation to immediately bring to bear on a shell design problem all the tools of advanced analysis as outlined in Section 2.8, but this is almost always a mistake. Not only are numerical models of structures inherently incomplete representations of the real objects, but FE analysis is also always approximate – especially for nonlinear analysis.

The overall advice of this chapter can be summarized by a few aphorisms phrased in the terms of a builder: Build knowledge to understand shell behavior and numerical structural analysis. Build confidence and competence by careful testing of software and by comparisons to other solutions and examples. Build insights into structural behavior and performance by starting with the simple and progressing cautiously to the more advanced.

And build experience by continually re-evaluating earlier designs by physical monitoring and/or numerical re-analysis.

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