An interpolation tool for aeroelastic data transfer problems

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**Problem:** Transfer deformations between meshes
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- Transfer deformations from a structural mesh (CSM grid) to an aerodynamic mesh (CFD grid)
**Motivation**

**Problem**: Transfer deformations between meshes

- Transfer deformations from an aerodynamic mesh (CFM grid) to a volumetric mesh (CFD grid)
Transfer *(using an interpolator)* deformations from a structural mesh to an aerodynamic or surface mesh.

- Low computational cost.
- Smooth representation.
- Mesh quality conservation.
- Applicability to any 3D data set (any kind of 3D meshes: structured, multiblock structured, unstructured and hybrid).
Interpolation
Given \( N_s \) centers \( \{x_1^s, \ldots, x_{N_s}^s\} \) and their displacements \( \{h_1^s, \ldots, h_{N_s}^s\} \), and \( N_a \) evaluation nodes \( \{x_1^a, \ldots, x_{N_a}^a\} \), the problem consists in obtaining the displacements \( \{h_1^a, \ldots, h_{N_a}^a\} \) via interpolation methods, in a smooth and regular way.

\[ S = \{ x_i^s, h_i^s \} \text{ Input data} \]
\[ S^o = \{ x_i^a, h_i^a \} \text{ Output data} \]
Reconstruct a continuous spatial distribution $h(\bar{x})$ using the discrete values $\bar{x}_i^s$

$$h(\bar{x}) = \sum_{i=1}^{N_s} w_i \Phi(||\bar{x} - \bar{x}_i^s||) + \Pi(\bar{x})$$

where

- $w_i$ are the coefficients.
- $\Phi$ is a basis function which is radial with respect to the Euclidean distance (Radial Basis Function)
- $\Pi$ is a $m$ degree polynomial that depends on the $\Phi$ function.
Interpolation condition

\[ h^s_i \equiv h(\bar{x}^s_i) \]

Side condition

\[
\sum_{i=1}^{N_s} w_i q(\bar{x}_i) = 0 \quad \text{deg}(q) \leq \text{deg}(\Pi)
\]

To recover translations and rotations.

To conserve forces and moments.

Zero degree polynomial

To avoid transfer of fictitious displacements

\[
\Pi(\bar{x}) = \gamma_0 \implies \sum_{i=1}^{N_s} w_i = 0
\]
Coefficients computation

\[ h^s_i = h(\bar{x}^s_i) \quad i = 1, \ldots, N_s \]

\[ \sum w_i = 0 \]

\[ \begin{pmatrix} 0 \\ h^s_1 \\ h^s_2 \\ \vdots \\ h^s_{N_s} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & \Phi_{s_1s_1} & \Phi_{s_1s_2} & \cdots & \Phi_{s_1s_{N_s}} \\ 1 & \Phi_{s_2s_1} & \Phi_{s_2s_2} & \cdots & \Phi_{s_2s_{N_s}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \Phi_{s_{N_s}s_1} & \Phi_{s_{N_s}s_2} & \cdots & \Phi_{s_{N_s}s_{N_s}} \end{pmatrix} \begin{pmatrix} \gamma_0 \\ w_1 \\ w_2 \\ \vdots \\ w_{N_s} \end{pmatrix} \]

\[ \bar{h}^s = C_{ss} \bar{\omega} \]
RBF Interpolation

Applying to evaluation nodes

\[ h_i^a = h(\bar{x}_i^a) = \sum_{k=1}^{N_s} w_k \Phi(||\bar{x}_i^a - \bar{x}_k^s||) + \gamma_0 \quad i = 1, \ldots, N_a \]

\[
\begin{pmatrix}
  h_1^a \\
  h_2^a \\
  \vdots \\
  h_{N_a}^a
\end{pmatrix} =
\begin{pmatrix}
  1 & \Phi_{a_1 s_1} & \Phi_{a_1 s_2} & \cdots & \Phi_{a_1 s_{N_s}} \\
  1 & \Phi_{a_2 s_1} & \Phi_{a_2 s_2} & \cdots & \Phi_{a_2 s_{N_s}} \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  1 & \Phi_{a_{N_a} s_1} & \Phi_{a_{N_a} s_2} & \cdots & \Phi_{a_{N_a} s_{N_s}}
\end{pmatrix}
\begin{pmatrix}
  \gamma_0 \\
  w_1 \\
  w_2 \\
  \vdots \\
  w_{N_s}
\end{pmatrix}
\]

\[ \bar{h}^a = A_{as} \bar{\omega} \]
**Strategy # 1:** $G$-matrix calculation

\[ \bar{h}^s = C_{ss} \bar{\omega} \]

\[ \bar{h}^a = A_{as} \bar{\omega} \]

\[ \Rightarrow \quad \bar{h}^a = A_{as} C_{ss}^{-1} \bar{h}^s = G \bar{h}^s \]
Applying the interpolator

Strategy # 1: $G$-matrix calculation

\[ \bar{h}^s = C_{ss} \bar{\omega} \]
\[ \bar{h}^a = A_{as} \bar{\omega} \rightarrow \bar{h}^a = A_{as} C_{ss}^{-1} \bar{h}^s = G \bar{h}^s \]

Strategy # 2: Solving linear algebraic system

- Calculate the $\bar{\omega}$ vector of coefficients
- Construct matrix $A_{as}$
- Calculate the new values $\bar{h}^a = A_{as} \bar{\omega}$
<table>
<thead>
<tr>
<th><strong>Function</strong></th>
<th><strong>Definition</strong> $\Phi(\bar{x})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume Spline</td>
<td>$</td>
</tr>
<tr>
<td>Wendland $C^0$</td>
<td>$(1 -</td>
</tr>
<tr>
<td>Wendland $C^2$</td>
<td>$(1 -</td>
</tr>
</tbody>
</table>
Test cases
Test cases

**FE model**

- Transfer deformations from the structural mesh to the aerodynamical mesh.
Test cases

- **FE model**
  - Transfer deformations from the structural mesh to the aerodynamical mesh.

- **Stick model**
  - Generate a virtual FE structural mesh
    - According to stick nodes
    - Near of the aerodynamic surface
  - Transfer deformations from the stick to the virtual structural mesh.
  - Transfer deformations from the structural mesh to the aerodynamical mesh.
Stick model strategy
Stick model: Strategy

Structural Nodes
Stick model: Strategy

Normal Planes
Stick model: Strategy
Stick model: Strategy

Virtual Structural Mesh
Stick model: NACA0012 wing

- 21 stick-nodes
- 34007 aerodynamic-nodes and 67918 mesh-elements

An interpolation tool for aeroelastic data transfer problems—p. 16
Stick model: NACA0012 wing

bullet Work structure (2666 nodes)
Stick model: NACA0012 wing

Deformation

\[ \eta(y) = \frac{y^2(6L^2 - 4Ly + y^2)}{3L^4} \]
\[ \eta_{\text{max}} = 10\%L \]
Error contour maps

one step computation  multiple steps computation

\[ Error_k = \| x_{k,\text{exact}} - x_{k,\text{calc}} \| \]

An interpolation tool for aeroelastic data transfer problems—p. 19
Maximum error estimation

\[ Error = \max \left| \mathbf{x}_{k,\text{exact}} - \mathbf{x}_{k,\text{calc}} \right| \]
Full configuration aircraft
Decomposition in domains or blocks

Block 1 (fuselage)
Block 2 (wing)
Block 3 (horizontal)
Block 4 (fin)
Deformation by blocks
Correction based on deformed junction nodes
Junctions

Block 1 (fuselage)

Block 2 (wing)

Junction node

\( d_1 \)

\( d_2 \)
Junctions strategy
\[ h_J = \alpha h_J^{(1)} + (1 - \alpha) h_J^{(2)} \quad 0 \leq \alpha \leq 1 \]
Junctions strategy

Structure: \( \{ J_1, \Delta h_{J_1} \} \cup \{ J_2, \Delta h_{J_2} \} \) \rightarrow \Delta h_{i}^{(1)}
Aerodynamic: \( B_1 \)

Structure: \( \{ J_1, \Delta h_{J_1} \} \) \rightarrow \Delta h_{i}^{(2)}
Aerodynamic: \( B_2 \)

Structure: \( \{ J_2, \Delta h_{J_2} \} \) \rightarrow \Delta h_{i}^{(3)}
Aerodynamic: \( B_3 \)
68 stick-nodes
253 FE-nodes
67040 aerodynamic-nodes

Structured mesh
Stick – FE model: aircraft

Work structural mesh (8286 nodes)
Deformed work structural mesh
Interpolated aerodynamic mesh
\[ \text{Error}_k = \| \mathbf{x}_{k,\text{exact}} - \mathbf{x}_{k,\text{calc}} \| \]
An interpolation tool based on radial basis functions has been developed.

Useful to transfer forces from a structural mesh to an aerodynamic mesh or loads from an aerodynamic mesh to a structural mesh.

Direct application to any dimension problems, both structured and non structured meshes.
Thank you