On the Insurmountable Size of Truss-like Structures

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1 Galileo’s problem and structural efficiency

Efficiency and Cost

Thermodynamic efficiency: \[
\frac{\text{useful quantity}}{\text{total quantity}} = \frac{\text{exergy}}{\text{energy}} < 1
\]

“Load” efficiency: \[
r = \frac{\text{useful load}}{\text{useful load + structural weight}} = \frac{Q}{Q + P} < 1
\]

Thermodynamic cost: \[
\frac{1}{\text{thermodynamic efficiency}} > 1
\]

“Load” cost: \[
k = \frac{\text{useful load + structural weight}}{\text{useful load}} = \frac{Q + P}{Q} > 1
\]
Galileo's rule:

\[ r = 1 - \frac{H}{\mathcal{L}} = \frac{1}{k} \]

\[ P = Q \cdot (k - 1) \]

\[ \mathcal{L} = \mathcal{A} = \frac{f}{\rho} \] (material scope)
Maxwell and Michell

Size measure: $L$. Useful load: $Q$

Maxwell number: $\mathcal{M} = \sum \vec{F}_i \cdot \vec{R}_i = -\frac{1}{4} Q L$

Stress volume: $\mathcal{V} = \sum |N_i| \cdot \ell_i \geq |\mathcal{M}|$

Michell number: $\nu = \mathcal{V} / Q L$

Maxwell’s problem

$\nu = \frac{3}{2}$

A Maxwell structure

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If $L = 0$ then $P = \frac{v \cdot QL}{A}$

If $L > 0$ then $P > \frac{v \cdot QL}{A}$

**Aroca rule:** If \{P\} $\approx$ \{Q\} then $P \approx \frac{v \cdot PL}{A}$, hence:

\[ \mathcal{L} \approx \frac{A}{v} \]

**GA rule:**

\[ r \approx 1 - v \frac{L}{A} \]
2 Truss-like Structures and self-weight

Model assumptions:

- bars are of constant cross-section
- identical tension-compression patterns on bars due to both useful load and self-weight (i.e., equal sign of the internal force in each bar in these two independent load conditions)
- the material has the same absolute value of allowable stress in tension than in compression
- buckling of compressed bars is not taken into consideration

Our questions:

scope $\mathcal{L}$? efficiency $r$? load cost $k$?
The model for Maxwell’s structures

\[ Q \rightarrow N_Q, \quad P \rightarrow N_P, \quad \text{subject to equilibrium condition} \]

\[ Q = HN_Q \quad P = HN_P \quad Q + P = HN_\chi \]
The model for Maxwell’s structures

\[ P = \Omega_L N_\chi \]

\[ Q = (H - \Omega_L) N_\chi \]

\[ (H - \Omega_{L=\mathcal{L}}) N_\chi = 0 \]

\[ |H - \Omega_{\mathcal{L}}| = 0 \]
The model for Maxwell’s structures

<table>
<thead>
<tr>
<th>Bar type</th>
<th>$\omega_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>straight beam</td>
<td>$\omega_{0j} = \omega_{1j} = f(h/\ell, \ell, \beta, A, \ldots)$</td>
</tr>
<tr>
<td>catenary arc</td>
<td>$\omega_{ij} = g_i(\ell, \beta, A)$</td>
</tr>
</tbody>
</table>

![Diagram](image)

- $N^i_x$: Force along the $i$th bar.
- $P_0$ = $\omega_{0i} N^i_x$: Force at the beginning of the bar.
- $P_1$ = $\omega_{1i} N^i_x$: Force at the end of the bar.
- $P_0 + P_1$ (generic bar: $\rho, A$)

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3 The scope of structural schema

Geometry Definition

Affine transformations: schema

\[ \lambda = 4 \]
\[ \lambda = 8 \]
\[ \lambda = 16 \]
Slenderness, size and structural scope

\[ N_j = -\frac{b}{h}Q \]
\[ N_t = \frac{\lambda}{8}Q \]
\[ N_{cs} = -\frac{\lambda}{4}Q \]
\[ N_{ci} = \frac{\lambda}{4}Q \]
\[ N_p = \frac{b}{h}Q \]

Useful Load

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Slenderness, size and structural scope

\[ N_j = -\frac{b}{h} Q \]
\[ Q/2 \]
\[ \frac{Q}{2} \]
\[ N_t = \frac{\lambda}{8} Q \]
\[ N_{ci} = \frac{\lambda}{4} Q \]
\[ N_{cs} = -\frac{\lambda}{4} Q \]

Useful Load

\[ N_j = -\frac{b}{h} (P_1 + P_2) \]
\[ N_{cs} = -\frac{\lambda}{8} (P_1 + 2P_2) \]
\[ N_{p} = \frac{b}{h} P_2 \]

Self Weight

\[ N_t = \frac{\lambda}{8} (P_1 + P_2) \]
\[ N_{ci} = \frac{\lambda}{8} (P_1 + 2P_2) \]
Slenderness, size and structural scope

\[ N_j = -\frac{b}{h}Q \]
\[ N_t = \frac{\lambda}{8}Q \]
\[ N_{ci} = \frac{\lambda}{4}Q \]
\[ N_{cs} = -\frac{\lambda}{4}Q \]
\[ N_p = \frac{b}{h}Q \]

Useful Load

Layout with Catenary Bars for the Insurmountable Size

Case \( \lambda = 8 \), \( \mathcal{L}(\lambda = 8) = 0.66A \)
Direct application of GA rule \((L \rightarrow 0)\)

\[ \nu(\lambda) = \frac{5\lambda}{27} + \frac{12}{9\lambda} \]

\[ \lambda_{\text{opt}} = \min_{\lambda} \nu(\lambda) = \frac{6}{\sqrt{5}} = 2.6833 \]

<table>
<thead>
<tr>
<th>(\lambda_{\text{opt}})</th>
<th>GA rule</th>
<th>Present Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>scope (L/A)</td>
<td>2.6833</td>
<td>2.455</td>
</tr>
<tr>
<td></td>
<td>1.006</td>
<td>1.1851</td>
</tr>
</tbody>
</table>
Slenderness, size and structural scope

\[ \lambda = 2.455 \]

- Numerical solution
- GA rule

\[ \frac{L}{A} \]

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Catenary Arcs

- Numerical Results
- GA Rule

Rectangular Cross-Section Beams, Numerical Results
- \( k_1 = 0.1, \ell/h = 10 \)
- \( k_1 = 0.2, \ell/h = 5 \)
## 4 Use of Galileo’s rule in practise

**Design examples**

\[ A = 2300 \text{ m}, \text{ rectangular cross-section, } \lambda = 8 \]

<table>
<thead>
<tr>
<th>( h/\ell )</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{L} (\text{m}) )</td>
<td>407</td>
<td>688</td>
</tr>
<tr>
<td>size (m)</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>relative size</td>
<td>2.5%</td>
<td>12%</td>
</tr>
<tr>
<td>( r )</td>
<td>98%</td>
<td>88%</td>
</tr>
<tr>
<td>( k )</td>
<td>1.02</td>
<td>1.14</td>
</tr>
<tr>
<td>( P/Q )</td>
<td>2%</td>
<td>14%</td>
</tr>
<tr>
<td>quality</td>
<td><strong>OK</strong></td>
<td><strong>Ok</strong></td>
</tr>
</tbody>
</table>
5 Conclusion: future researches

The proposed method when applied to well definite Maxwell problems can be used to resolve concrete structural form determining the thickness of its bars, its total self-weight and its structural efficiency. Besides, the method can be used to explore the space of solutions associated to the Maxwell problem and to a definite structural schema, determining the relationships between size, slenderness, insurmountable size, and efficiency of each solution in the search space.

It could be improved in several ways:

- incorporating a more realistic model of the stress distribution for beams, e.g., Timoshenko beam theory
- managing different values for tension and compression allowable stress
- considering the local buckling of beams and compressed catenary arcs.
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