UNIVERSIDAD POLITÉCNICA DE MADRID
ESCUELA TÉCNICA SUPERIOR DE INGENIEROS DE MINAS

ANÁLISIS EXPERIMENTAL DE LA FRAGMENTACIÓN, VIBRACIONES Y MOVIMIENTO DE LA ROCA EN VOLADURAS A CIELO ABIERTO

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Ingeniero de Minas

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EXPERIMENTAL ANALYSIS OF FRAGMENTATION, VIBRATION AND ROCK MOVEMENT IN OPEN PIT BLASTING

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RESUMEN

La presente tesis describe y analiza el trabajo experimental realizado en la cantera El Alto (Morata de Tajuña, Madrid, España) en el marco del proyecto de investigación “Reducción de finos en el sector de minerales industriales y de áridos” financiado por la U.E. bajo contrato G1RD-CT-2000-00438. El consorcio de dicho proyecto incluye organizaciones dedicadas a la investigación –MU-Leoben (Austria), SveBeFo (Suecia), ARMINES (Francia) y UPM (España)–, fabricantes de explosivo –Dyno (Noruega) y UEE (España)– y productores de árido (Hengl-Bitustein, Austria), caliza (Nordkalk, Suecia) y cemento (Cementos Portland, España).

Se recopilan características geométricas, de carga y secuenciación en 35 voladuras de producción y tres disparos de un único barreno realizados entre 2001 y 2004. Se ha medido el movimiento del frente (velocidad inicial y tiempo de respuesta de la roca, i.e. tiempo desde la iniciación del explosivo en el barreno hasta que se tiene la primera evidencia de movimiento) a partir de blancos situados en la cara libre frente a un barreno en ocho de esas voladuras. Se proporcionan las frecuencias dominantes y velocidad de partículas de las vibraciones registradas con sismógrafos situados en la parte superior e inferior del banco, en el campo cercano (unos 50 m) y lejano (hasta 550 m) para 23 de dichas voladuras, dos barrenos sencillos confinados y un barreno sencillo sin confinamiento. Además, se da la fragmentación en el alimentador de la tolva del primario para esas 35 voladuras. Los procedimientos utilizados para realizar estas medidas han sido descritos y analizados en detalle.

Los modelos más significativos para predecir fragmentación, vibraciones y movimiento del frente se aplican a los datos disponibles. Algunos de ellos muestran diferencias significativas con los valores medidos, como el modelo de Kuz-Ram, mientras que otros como el modelo de Blair y Armstrong para predecir el espectro medio de la velocidad de partículas funcionan bastante bien.

Cuando ha sido posible, se han proporcionado fórmulas ingenieriles para predecir el movimiento del frente, fragmentación y vibraciones. Se ha obtenido: (i) una fórmula para predecir el tiempo de respuesta de la roca a una altura determinada del piso del banco en función de la densidad lineal de carga y la piedra correspondiente, (ii) una fuerte influencia en la velocidad inicial de la roca del cociente entre el retardo entre barrenos y el tiempo de respuesta, (iii) una ley de atenuación de la velocidad suma máxima de partículas, (iv) una expresión para calibrar el programa de análisis de imágenes en función de los finos naturales (cociente entre el espesor de la cobertera y la altura del banco) presentes en el banco, (v) una fórmula entre el tamaño para el que pasa el 50 %, $x_{50}$, y el valor de la pendiente de la curva granulométrica en $x_{50}$, $S_{50}$, (vi) una relación entre $x_{50}$ y el índice de uniformidad de la distribución granulométrica, $n$, (vii) sendas fórmulas para $x_{50}$ y $S_{50}$ en función del consumo específico encima del piso en la caliza, trabajo útil por unidad de masa, retardo entre barrenos y dispersión de los detonadores y (viii) el balance de energía en ocho voladuras.

A partir de los resultados obtenidos se proporcionalan las estrategias que contribuirían a mejorar la operación en El Alto y establecer las principales líneas de trabajo de futuras investigaciones en el campo de voladuras a cielo abierto.
ABSTRACT

This thesis describes and analyzes the experimental work carried out in El Alto quarry (Morata de Tajuña, Madrid, Spain) within an E.U. funded research project “Less Fines Production in Aggregate and Industrial Mineral Industry” under the contract G1RD-CT-2000-00438. The “Less Fines” consortium includes research organizations –MU-Leoben (Austria), SveBeFo (Sweden), ARMINES (France) and UPM (Spain)–, explosive manufacturers –Dyno (Norway) and UEE (Spain)– and aggregate (Hengl-Bitustein, Austria), limestone (Nordkalk, Sweden) and cement (Cementos Portland, Spain) producers.

Geometry, loading and timing characteristics are given for 35 production blasts and three single blastholes shot in El Alto quarry from 2001 to 2004. Face movement (initial velocity and response time of the rock, i.e. time from the explosive initiation in the borehole to the first evidence of rock motion) was measured in eight of those blasts from the movement of targets hanged in front of a blasthole on the highwall. Vibration data (dominant frequency and particle velocity) recorded with seismographs in the top and grade levels of the blocks at both near (about 50 m) and far field range (up to 550 m) are provided for 23 of such blasts, two confined and one unconfined single blastholes. Finally, fragmentation in the hopper of the bin from images is given for all 35 blasts. The standards used in these measurements are described and analyzed in detail.

The most outstanding models in the literature for assessing fragmentation, vibration and face movement are applied to El Alto’s data. Some of these models show poor performance, for instance Kuz-Ram model, while others like the Blair-Armstrong’s Fourier model for predicting the mean particle velocity spectra provide fairly good predictions.

Engineering formulas are derived when possible for predicting the face movement, fragmentation and vibrations. The following has been obtained: (i) formula for predicting the response time of the rock at a specific height in the free face as function of the linear density of the explosive and the burden at the target level, (ii) strong relation between initial velocity of the rock and the ratio of in-row delay to response time, (iii) an attenuation law for the peak sum particle velocity, (iv) formula for calibrating digital analysis software as function of the natural fines (ratio of the overburden thickness to the bench height) present in the block, (v) formula between median size, $x_{50}$, and the slope value of the size distribution curve at $x_{50}$, $S_{x_{50}}$, (vi) formula between $x_{50}$ and the uniformity index of the size distribution curve, $n$, (vii) formulae for $x_{50}$ and $S_{x_{50}}$ as function of powder factor above grade in limestone, explosive energy (useful work) per unit of mass, in row delay and scatter of the detonators and (viii) energy balance in eight blasts.

Finally, the strategies that may contribute to improve the operation at El Alto quarry are addressed together with the main guidelines for future research work in the blasting field at open pit mining.
Chapter 1

INTRODUCTION

The explosive energy is distributed in the rock to be mined spatially in boreholes or blastholes and temporally with delay devices. This allows the rock breakage and the movement of the rock mass already fragmented. The explosive detonation creates additionally a system of seismic waves which are transmitted into the rock mass causing the vibration phenomena.

Rock blasting is the first step in crushing, grinding and concentration processes; the loading and hauling costs depend on both fragmentation and rock movement achieved, whereas the costs in the processing plant depend only on the resulting degree of fragmentation. In aggregate operations, fragmentation of the Run of Mine (ROM) affects also to final revenues, as coarser products have higher unitary prices.

Vibrations caused by the seismic waves radiated by the blast should be controlled in order to avoid damages in structures or buildings in the surroundings of the mine. Vibrations in a particular building or structure may stop eventually the operation in specific areas of the mine or quarry. Specific blast designs are required in some cases for minimizing the vibrations effects at specific locations, although this may have a negative effect on the resulting fragmentation and rock movement.

Additionally, the increasing use of electronic programmable detonators (EPDs) emphasizes the importance of knowing its influence in fragmentation and vibrations; precisely the influence of the scatter of the detonators in fragmentation is still being investigated.

All together, outlines the importance of monitoring accurately the blasts features. This is, however, linked to the use of high-tech devices like laser profilers, seismographs, high-speed video cameras, trucks tracking systems and fragmentation monitoring systems (with digital image analysis software), that until the last decade were not developed at a commercial levels (reasonable prices and easy handing). Even now there are few references in the specialized literature about the accuracy and restrictions of some of those devices and/or systems. This applies to fragmentation measurement from images, which is the only practical tool for evaluating fragmentation of the ROM in mines and quarries, as generally the systematic sieving of blasts is not possible due to its high cost and the disruption it causes in the production cycle.

The collection of fragmentation, vibration and rock movement measurements together with the respective blasting features in a large number of blasts in which the geometry, timing (both
delay and detonators precision) and explosive type were varied is on its own of high interest, although the analysis of these data is what provides an outstanding additive value.

At present, there are still unanswered questions about the blasting process (as Chapter 2 shows) and the fact is that the knowledge of the physics behind fragmentation, face movement and vibrations is very limited for being applied in the practice. The statistical analysis of field data, either fragmentation, vibration and movement measurements, seems to be the only way for finding engineering solutions to the blasting problems. This leads, however, to prediction models that may not work in every situation and should be then considered as a rough guide of how the blasting parameters affect to fragmentation, vibration or face movement.

Literature shows a small number of fragmentation prediction models: Kuz-Ram (Cunningham, 1983 & 1987), Chung-Katsabanis (2000), SveDeFo (Ouchterlony et al., 1990), Kou-Rustan (1993), Crushed Zone Model (Kanchibotla et al, 1999; Thornton et al., 2001a & b), Kuznetsov-Cunningham-Ouchterlony (Ouchterlony, 2004a) obtained from full scale blasts in one or various production sites or from reduced-scale shots. The performance of such models is questionable due to the lack of acceptable good quality data, despite of their widespread use.

The vibration problem is mainly solved according to each country vibration standard, i.e Spanish rule, Norma UNE 22-381-93, mainly using the well-known attenuation laws in which the peak particle velocity is given as a function of the charge weight and the distance to the source. The dominant frequency of the vibrations required in the damage criteria of the vibration standards is roughly approached from measured values. Sometimes, the influence of timing in the dominant frequency is analyzed applying the linear superposition concept to seed signals coming from single productions blastholes drilled and loaded in similar conditions as in production blasts (Wheeler, 2001). This simple procedure has been improved by Blair (1999) who develops a more complex model using Monte-Carlo simulations for getting contours of vibration levels from seed signals. Additionally, Blair and Armstrong (1999) use a similar concept to estimate the mean spectra of the transversal, vertical and longitudinal components of velocity.

Face movement has been little investigated (Chiappetta et al., 1983; Chiappetta & Mammele, 1987; Chiappetta, 1998, Chiappetta et al., 2001; Mishra & Gupta, 2002 and Oñederra & Essen, 2003). Simple formulae are developed from experimental data in production blasts for predicting both the initial velocity (Chiappetta et al., 1983) and the response time of the rock, i.e. time from the explosive initiation in the borehole to the first evidence of rock motion in front of it has been determined (Oñederra & Essen, 2003).

Finally, the knowledge of the performance of any machine or industrial process is very interesting from an economic and technical point of view. Nevertheless, few data exists on how and what amount of explosive energy is delivered to the surrounding medium in the usual civil application of rock blasting, despite of the intensive use of the explosive in rock mining. Berta (1990), Hinzen (1999), Spathis (1999) and Uchterlony et al. (2003) approach the blast design from an energy point of view, calculating the energy forms –fragmentation, seismic and kinetic energy– in which the energy delivered by the explosive is transformed. The practical side of this research would only be possible if each of the energy fraction terms are sensitive to changes in the blast design.
Chapter 2

THE EXPLOSIVE-ROCK INTERACTION: GENERAL BLASTING MECHANISMS

The detonation of an elongated charge produces a shock wave, which is moving along the explosive with a certain velocity, when this velocity becomes constant (velocity of detonation), the energy added to the wave front by the chemical reaction equilibrates the work and expansion of the rear-flowing product gases. From this moment and if the conditions remain constant in the explosive column, the shock wave maintains its structure and does not suffer any attenuation process during the remaining detonation until the entire explosive has been consumed. The explosive releases a chemical energy, which may be evaluated as heat of explosion at constant volume or useful work. After the explosive detonation, the resulting products impact the rock around the blasthole, causing a rapid pressure increase on it, which results in the transmission of a pressure disturbance through the rock mass. The behaviour of the rock affected by the resulting shock wave is described by the Hugoniot-Rankine equations. Experimentally it has been found that rocks and other materials such as metals and liquids have a linear relation between the particle velocity $v$ and the shock wave front velocity $U$:

$$U = C + Sv$$

(2-1)

Where $C$ and $S$, are called shock constants and are given for some rock types in Table 2-1. The pressure $P$ induced by the shock wave in the rock mass can be estimated by applying the 1-D Equation of conservation of momentum to Equation 2-1 and assuming that $P$ is much higher than the atmospheric pressure:

$$P = \rho_r U v = \rho_r (C + Sv)v$$

(2-2)

Where $\rho_r$ is the rock density.

The stress field induced to the rock mass by the explosive detonation depends on the explosive type (density and velocity of detonation, VOD), on the coupling degree and on the way the pressure disturbance is transmitted to the rock mass. Figure 2-1 shows the induced particle velocity and the shock wave propagation velocity for different pressures for a limestone. The pressure of detonation in civil explosives ranges roughly from 3 GPa for anfo up to 15 GPa for gelatines.
Table 2-1. Shock constants for various rock types (Persson et al., 1994)

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Density, kg/m$^3$</th>
<th>C, km/s</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite, Westerley(1)</td>
<td>2630</td>
<td>2.1</td>
<td>1.63</td>
</tr>
<tr>
<td>Limestone, Solenhofen(2)</td>
<td>2600</td>
<td>3.5</td>
<td>1.43</td>
</tr>
<tr>
<td>Marble(2)</td>
<td>2700</td>
<td>4.0</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Notes: (1) For pressures smaller than 16 GPa
(2) For pressures smaller than 13 GPa

The shock wave has a conical front in the proximity of the blasthole walls, with the apex opposite to the initiation point and coaxial with the blasthole. The shock front cone angle depends directly on the relation between the VOD and the propagation velocity of the shock wave in the rock, $U$; this angle decreases with the distance from the blasthole. The Mach-cone only appears if the detonation front is moving at a supersonic velocity respect to the surrounding rock; in this case there is a good transmission of the pressure disturbance with little energy dispersion in the rock mass.

As a consequence of the simultaneous axial and radial expansion of the reaction products, the original blasthole is expanded, thereby making the pressure in blasthole wall drop quickly. This causes an additional reduction in the front pressure of the shock wave at a rate higher than that determined only by the radial expansion of the shock front, with a surface increasing with the distance. The detonation of a cylindrical charge produces, then, a continuous source of shock pulses generated during the explosive consumption, as Figure 2-2 shows. The disturbances generated in the rock are moving outwards radially, compressing the material at the wave front.

The shock wave pass originates a stress estate in the rock mass, characterised by a faster decrease of the tangential stress due to the radial outflow of the material accompanying the shock wave. The area closest to a production blasthole is affected in all the directions by strong compressive stresses higher than the compressive strength of the rock, deforming the rock mass plastically, and then crushing it completely. Berta (1990) and Johansson & Persson (1970) practically agree in the size of this area, extending from one or two blasthole radii from the hole wall. In this area the shock waves suffer a strong damping effect and as the rock immediately...
close to the hole was previously compressed, the next shock pulses generated by the gas expansion are moving faster than the previous ones. This results in the superposition and interaction of the shock waves.

![Diagram of shock wave interaction](image)

Figure 2-2. Transmitted shock wave through the rock mass (Sanchidrián & Muñiz, 2000)

As the shock wave is moving through the rock mass its energy decreases, beyond some distance away from the hole, the shock wave pressure does not exceed the compressive strength of the rock mass, although it can be above the elastic limit. Additionally, the tangential stress becomes negative (tensile), while the other components are still positive (compressive). Only if the tangential stress exceeds the tensile strength of the rock mass, it will cause tensile failures at right angles to the direction of propagation creating radial cracks that can proceed outward and inwards the blasthole. New shock waves emanate from the fracture initiation point and will continue to do so as long as the fracture propagates, resulting in the interaction between the waves systems, which will affect the fragmentation process. The radial crack propagation velocity is initially of the order of 1000 m/s, gradually decreasing (Persson et al., 1994). As the stress waves move faster than the crack extension in the rock mass, they immediately outdistance the fractures extreme. At the beginning of the fragmentation process there is a great number of radial cracks, but the grow of the longer ones produces a stress field relaxation in its surroundings, disabling the growth of the smaller ones. The result is that only few cracks reach a great length.

An area of the rock mass dominated by shear failures appears between the crushed area and the zone where the radial cracks were produced. During the whole process, the discontinuities present in the rock mass, serves of basis for the generation of new cracks or either they can be reactivated during the event. While all of this happens, some radial cracks reach the expanded blasthole walls and the gases resulting from the explosive detonation jet into the cracks. The gases act on the cracks walls with a wedge effect that will make an appreciable contribution to the tension on the crack tip, which is greater for the longer cracks. This process involves important energy consumption: friction losses as the gases are propagating at very high velocity inside the narrow crack, and fracture energy as the rock is breaking. Recent tests in small scale blasts show that an important volume of fines are generated outside the crushing area, perhaps by the friction in the radial cracks (Svahn, 2002; Moser, 2003). More controversy exists, since
for some scientists the gases play a minor role in the fragmentation while for others the fracturing process is just beginning at this stage (Chiappetta & Mammele, 1987). If some discontinuity communicates the blasthole with the atmosphere, the reaction products will eventually vent, involving a quickly decrease of the pressure in the hole, which will imply a poor fragmentation and bad muckpile shape.

The compressive waves transmitted along the rock mass reach the free face few milliseconds after the detonation, and they are reflected as tensile waves with almost the same amplitude. Normally, the incident waves are damped below the tensile strength of the rock if the burden and the explosive charge are adequate and they can not produce spalling fractures in the rock. The tensile waves created at the face of the bench are responsible of the decrease in the front pressure of the pulses that are moving towards the free face. Figure 2-3 shows the attenuation process of a square compressive wave, caused by the progression of a tensile wave in the square region.

![Figure 2-3. Attenuation of a square shock wave (Cooper & Kurowski, 1996)](image)

If the returning tensile wave is strong enough, it will create new cracks throughout the rock mass, however the decisive mechanism in the fragmentation process is its interaction with the growing radial cracks. This interaction is stronger in the cracks that are parallel to the tensile shock wave front, and as a result the crack length is increased a distance enough to reach the free face. Therefore, the more favoured cracks by the tensile waves are the ones opening in a direction of 40° to 60° to the normal of the free face, this cracks will define a wedge of angle between 80° and 120°, increasing with the burden. Figure 2-4 shows the results of a hole fired in El Alto’s quarry in 3rd October 2001, placed 22 m away from the free face; 15 to 20 cracks were visible in the surface. However, in the photo only the more important ones can be appreciated. The most opened crack is the one parallel to the free face of the bench.

Not all the scientists agree with the just explained role played by the reflected waves ("classical" theory) during the blast. For instance, Blair & Armstrong (2001) suggest that the reflected waves have negligible amplitudes. This opinion is based in the following mechanisms:

- The most dominant reflection will occur for plane waves with a normal incidence on a planar free surface. As the blasthole is close to the free face, the wave front is either conical or
hemispherical, and the resulting reflected wave will have smaller amplitude than with a planar wave: the reflected energy fraction decreases as the incident angle does.

- The free surface is ragged instead of planar, this will produce incoherent energy scatter instead of pure reflection.

![Figure 2-4. Radial cracks reinforced by tensile shock waves](image)

- The rock is more damaged in front of the hole than at the same distance behind it. This will attenuate the waves as they travel towards the free face.

- The resulting scattered waves travel back through rock already damaged during the process that follows the explosive detonation.

This reasoning seems to be logical and it is supported by data obtained in blast simulations and in the field. Consequently, the gases play a more important role in fragmentation and they are the major responsible of the extension of some cracks until the free face.

At this stage, anyway, the fragmentation has been almost completed either by the stress field created by the shock wave, the gas expansion and the combination of both. High speed cameras show that some slight fragmentation occurs during the rock fragments flight due to the collisions between them and also in their impact with the ground. The higher the bench, the greater is this type of breakage.

The remaining energy in the rock mass continues to travel through it as an elastic wave. Finally, when the fragmentation process is enough advanced and the rock wedge is almost formed if the blast is properly designed, the bench face will be bowed outwards near the bench height centre. This movement affects all the rock mass placed along the blasthole length above grade and causes the burden to break in a third dimension due to flexural rupture. In this moment the gases released in the detonation confer an initial thrust to the rock wedge, which will be enough to displace it a certain distance. The rock is still accelerated by the gases, during the first steps of the movement.

The dominant mechanisms may vary from massive rocks to jointed rocks. For the first ones, the shock wave plays an important role, while in the second, the gases effect on the natural cracks prevails.
Chapter 3

DESCRIPTION OF EL ALTO QUARRY AND ROUNDS

A total number of 35 blasts, two confined and one single production (unconfined) blastholes shot in La Concha pit of El Alto quarry were monitored from October 2001 to May 2004. The drilling and charging were carefully monitored and controlled according to the standards defined.

The quality of the blast design was increased gradually; the improvements affected mainly the subdrill length, burden and face smoothness. The conditions of El Alto quarry include certain peculiarities in the blasting features.

3.1. Description of El Alto quarry

El Alto quarry belongs to the Spanish cement group Cementos Portland Valderrivas. The quarry is located in Morata de Tajuña near Madrid. A total of 2.25 Mt/yr of limestone and clayish-marl are mined in the two pits of the quarry, La Concha and El Trasvase, by drilling and blasting.

3.1.1. Geology

The geology in El Alto is basically simple and rather constant. Three different rock types are present in the bench, see Figure 3-1. Normally, in the first four to six meters from the surface there is an overburden of clayish marl. This layer has sandy nature (its maximum size is 14 mm, as it will be shown in Chapter 6) and little cohesion. The overburden material, which mixes with the broken limestone, is considered natural fines and its relative amount is measured from the ratio of the overburden thickness to the bench height. The natural fines vary from 9 to 45 %. The particular features of the overburden affect to the vibration and fragmentation, i.e. blasts with large overburden thickness have smaller $x_{50}$ (mesh size for a passing of 50 %) and higher amount of fines. In the toe of the bench there exists a type of clay, greda, with a low content in calcium carbonate and a high content in aluminium oxide, iron oxide and silicon oxide. The limestone layer is located between both rock types; it is almost horizontal and crossed by
different sets of joints and discontinuities. A limestone ground of 1 to 1.5 m thickness, loosened by the blast, is not mucked in order to avoid the shovels getting stuck in the mud.

El Alto’s limestone has a rock density of 2560 kg/m³; the dynamic and static Young modulus are 89.1 and 64 GPa respectively; the Poisson coefficient is 0.26; the tensile strength is 64 GPa; the Rittinger coefficient obtained from the “Optimized Conminution Sequence” described by Moser et al. (2000) is 58 cm²/J; the propagation velocity of the p-waves measured in the lab is 6060 m/s (Goetz et al., 2002; Böhm & Mayerhofer, 2002), whereas the in-situ value is a 49% smaller (the methodology used to measure the p-wave velocity in the field is explained in Chapter 5). The $x_{63}$ (mesh size for a pass of 63%) and the uniformity index of the in-situ block size distributions (IBSD) are respectively 3.53 m and 2.68 for El Alto (du Mouza & Hamdi, 2002).

![Figure 3-1. Bench geology in El Alto quarry](image)

### 3.1.2. Crushing process

The crushing process in El Alto basically consists of a primary, hammer mill followed by a vertical axis roller mill that grinds and blends the limestone, the marl and the clay (plus added materials) to form the powder-like kiln feed. This is conveyed to homogenizing silos where the powder is analyzed and further treated prior to firing in the kiln.

The hammer mill is fed with the ROM, limestone and clayish-marl, coming from the quarry; the maximum size allowed is 1 m³. The primary has a bottom screen with an opening of 65 mm.

### 3.2. Description of the tests

The data monitored in each blast are given in Table 3-1 together with the procedure used to measure them. The formulae used to calculate the basic blasting parameters are also shown; the mean values in each blast of the measured parameters are used. The basic geometry of the blasting in El Alto is shown in Figure 3-2. The geotechnical features of the blocks are assumed constant from blast to blast as these were carried out in a limited area of the pit.

The “average burden” is the mean of the values given by the laser profile along the blasthole length from crest to grade. This includes the small burdens in the top part of the bench caused by the loose overburden present in the top of the block. Due to the quarry practices, sometimes the loaders leave broken material in the toe of the benches. In such cases, only the burden values...
measured between the crest and the position where the accumulation of broken rock starts are considered.

Table 3-1. Basic blasting parameters and the procedure used to obtain them.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>How it is determined</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geology</strong></td>
<td></td>
</tr>
<tr>
<td>Overburden thickness, ( h_{ob} )</td>
<td>Measured, first with measuring tape; from end of May 2002 with laser profile.</td>
</tr>
<tr>
<td>Natural fines, ( NF )</td>
<td>( NF = 100 h_{ob}/H )</td>
</tr>
<tr>
<td><strong>Drilling</strong></td>
<td></td>
</tr>
<tr>
<td>Blastholes diameter, ( d )</td>
<td>Nominal value from the size of the bit used</td>
</tr>
<tr>
<td>Blasthole inclination, ( i )</td>
<td>Measured, rig inclinometer value</td>
</tr>
<tr>
<td>Blasthole length, ( l_h )</td>
<td>Measured, tape</td>
</tr>
<tr>
<td>Bench height, ( H )</td>
<td>Measured, laser profile</td>
</tr>
<tr>
<td>Subdrill length, ( J )</td>
<td>( J = l_h - H/\cos i )</td>
</tr>
</tbody>
</table>
| Blasted height, \( h \) | If \( J \geq 0 \), \( h = H \)  
If \( J < 0 \), \( h = l_h \cdot \cos i \)                                                                                                          |
| Av. Burden, \( B \) | Measured, laser profile                                                                                                                               |
| Spacing, \( S \) | Measured, tape                                                                                                                                             |
| Spacing to burden ratio, \( S/B \) |                                                                                                                                                      |
| **Blasting**        |                                                                                                                                                      |
| Stemming length, \( l_s \) | Measured, tape                                                                                                                                           |
| Length of hole charged above grade, \( L \) | If \( J \leq 0 \), \( L = l_h - l_s \)  
If \( J > 0 \), \( L = l_h - l_s - J \)                                                                 |
| Explosives mass per hole, \( (Q_e)_r \) | Loading count, \( (Q_e)_r = Q_{cart} N_{cart} + Q_{bag} N_{bags} \)                                                                                     |
| Explosive mass above grade, cartridges, \( (Q_e)_c \) | If \( J \leq 0 \), \( (Q_e)_c = Q_{cart} N_{cart} \)  
If \( J > 0 \), \( (Q_e)_c = Q_{cart} (N_{cart} - J)/l_{cart} \) with \( J < N_{cart} l_{cart} \) |
| Explosive mass above grade, bulk, \( (Q_e)_b \) | If \( J \leq 0 \), \( (Q_e)_b = Q_{bag} N_{bags} \)  
If \( J > 0 \), \( (Q_e)_b = Q_{bag} N_{bags} - [J \cdot \pi d^2/4 - (Q_{cart} N_{cart} - (Q_e)_c)/(\rho_{e,\text{cart}})](\rho_{e,\text{bag}}) \) |
| Explosive above grade, \( Q_e \) | \( Q_e = (Q_e)_r + (Q_e)_c \)                                                                                                                           |
| Linear charge density above grade, \( \rho \) | \( \rho = Q_e/L \)                                                                                                                                          |
| Powder factor above grade, \( q \) | \( q = Q_e/(B S H/\cos i) \)                                                                                                                             |
| Powder factor, \( q_T \) | \( q = (Q_e)_r/(B S H/\cos i) \)                                                                                                                          |
| Explosive energy, \( E_Q \) (heat of explosion) and \( E_W \) (useful work) | Calculated from thermodynamic code W-Detcom (Sanchidrián, 1986 & López, 2003)                                                                      |
| VOD, \( D \) | Measured, VOD recorder: Minimate (Instantel)/ Microtrap (MREL)                                                                                          |
| In-row delay, \( t \) | Nominal value given by the manufacturer                                                                                                                   |
| Detonator scatter, \( \delta \) | Arbitrary: 10 for pyrotechnic and 1 for electronic                                                                                                      |

**Notes:**  
- \( Q_{bag} \) and \( Q_{cart} \) are the nominal masses of a bag or cartridge given by the manufacturer.  
- \( N_{bag} \) and \( N_{cart} \) are the number of bags and cartridges loaded into the blasthole.  
- \( l_{cart} \) is the nominal length of a cartridge given by the manufacturer.  
- \( (\rho_{e})_{bag} \) and \( (\rho_{e})_{cart} \) are the nominal density of the explosive in bags and in cartridges respectively.

Drilling is stopped when *grede* is reached, which sometimes leads to blastholes with negative subdrill (the bottom of the blasthole is above grade). In such situations, it is assumed that the blasthole breaks a slice of rock of height equal to the blasthole length. The blasted height, which is used for calculating the powder factor instead of the bench height, is equal to the bench height when the bottom of the blasthole is below grade.
3.1.2. Geology, drilling and charging of the rounds

The field work was organized in two blast campaigns. A total of 15 production blasts, two confined shots (named CB1 and CB2) and one single production blasthole (labelled as SB1) were monitored in the first campaign from October 2001 to May 2003. The second campaign was carried out up to May 2004; 20 more blasts were shot in this stage. A summary of the features of the single shots and production blasts is given in Tables 3-2 and 3-3 respectively. Appendix A shows the basic blasting parameters listed in Table 3-1 for the 35 production blasts and single blasthole shots.

Table 3-2. Summary of the characteristics of the single blasthole shots

<table>
<thead>
<tr>
<th></th>
<th>CB1</th>
<th>SB1</th>
<th>CB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confined</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(d, \text{mm})</td>
<td>155</td>
<td>142</td>
<td></td>
</tr>
<tr>
<td>Cart./bulk explosives</td>
<td><em>Goma 2 ECO / Alnafo</em></td>
<td><em>Goma 2 ECO / ANFO</em></td>
<td></td>
</tr>
<tr>
<td>Decking</td>
<td></td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

Blasts 103/02, 10/03, 41/02, 43/03 and 50/03 were somewhat outlying from the main area of blasts. Some geology differences, mainly related to fractures orientation with respect to the blasting face may occur in them.

One row is always drilled in El Alto; the blastholes inclination is about 6°. The mean drilling deviation in the burden and spacing directions measured with a *Boretrak MKII* in 19 holes in blasts 15/02 and 29/02 was respectively 0.1 and 0.4 m; the standard deviation was 0.2 and 0.3 m.

Commonly gelatin (*Goma 2 ECO*) cartridges and aluminized ANFO (*Alnafo*) were used as bottom and column charges respectively; the diameter of the cartridges was increased from the 65 mm used in the single shots and in blasts 15/02, 29/02, 41/02, 42/02 and 53/02 to the 85 mm
used in the rest of the blasts. In those five blasts, the gelatine was combined with Alnafo in the bottom part of the blastholes, i.e. the loading pattern was, from bottom to top: one cartridge - 12 kg of Alnafo - one cartridge - 12 kg of Alnafo - ... From blast 103/02 on, the gelatine was loaded first and next the column charge explosive.

Table 3-3. Summary of the characteristics of the 35 production blasts

<table>
<thead>
<tr>
<th>Campaign</th>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blast No.</td>
<td>15/02, 53/02 &amp; 10/03</td>
<td>29/02, 37/02 (1) &amp; 41/02, 42/02, 30/03 &amp; 32/03 &amp; 30/03, 45/03 &amp; 8/04, 13/04 &amp; 44/04 (2)</td>
</tr>
<tr>
<td>d, mm</td>
<td>142</td>
<td>155</td>
</tr>
<tr>
<td>Cart./bulk explosives</td>
<td>Goma 2 ECO / Alnafo</td>
<td>High density Alnafo Emunex 8000 Low density anfo</td>
</tr>
<tr>
<td>Cap type</td>
<td>Non-electric</td>
<td>Electronic</td>
</tr>
<tr>
<td>Decking</td>
<td>Yes (3)</td>
<td>No</td>
</tr>
<tr>
<td>t, ms</td>
<td>42</td>
<td>67</td>
</tr>
</tbody>
</table>

Notes: 1. Blast 37/02 had a variable inter-hole delay
2. Blasts with water in the blastholes are given in italics
3. In such blastholes two decks with the same delay separated with dry stemming were used.

Table 3-4. Characteristics of the explosives used.

<table>
<thead>
<tr>
<th>Explosive name</th>
<th>Explosive type</th>
<th>Packaged</th>
<th>Density, kg/m³</th>
<th>Velocity of detonation, m/s</th>
<th>Explosive energy, kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goma 2 ECO</td>
<td>Gelatin</td>
<td>Cartridges</td>
<td>1450</td>
<td>6321sd118 (6)</td>
<td>3480</td>
</tr>
<tr>
<td>Nagolita</td>
<td>Anfo</td>
<td>Bags</td>
<td>800</td>
<td>3941 (5)</td>
<td>2591</td>
</tr>
<tr>
<td>Alnafo</td>
<td>Anfo+Al.</td>
<td>Bags</td>
<td>800</td>
<td>4029sd93</td>
<td>2918</td>
</tr>
<tr>
<td>High density Alnafo</td>
<td>Anfo+Al.</td>
<td>Bags</td>
<td>950</td>
<td>3941sd93</td>
<td>3322</td>
</tr>
<tr>
<td>Emunex 8000</td>
<td>Emulsion 20AN</td>
<td>Bags</td>
<td>1260</td>
<td>3535 (6)</td>
<td>2488</td>
</tr>
<tr>
<td>Low density anfo</td>
<td>Anfo</td>
<td>Bags</td>
<td>720</td>
<td>3554sd422</td>
<td>2160</td>
</tr>
</tbody>
</table>

Notes: 1. sd. means standard deviation
2. The VOD values correspond to measurements in cartridges of 65 and 85 mm.
3. Only was measured in CB2
4. Only two measurements are available.

Normally, there was no water in the holes; except in some holes in blasts 89/03, 96/03, 21/04, 26/04, 35/04 and 44/04. In those holes, two decks with the same delay separated by dry stemming were used; the bottom deck was loaded only with gelatin. Alnafo was replaced by a “high density” Alnafo in blasts 54/03 and 58/03, Emunex 8000 in blasts 26/04 and 31/04 and a “low density” ANFO in blasts 42/04 and 49/04. Table 3-4 shows the explosive density (the actual density for the gelatine depends on the degree of packing of the cartridge in the hole), the measured VOD in the field and the explosive energy evaluated as heat of explosion at constant volume, $E_O$, and useful work to 1000 bar, $E_U$, for an ideal detonation regime; the BKW-S equation of state has been used for the energy calculations.

The current initiation and timing practices used at present in El Alto, bottom initiation with non-electric detonators, was used in the blasts of the first campaign. Electronic programmable
detonators (EPDs) were used in the next 20 blasts made in the second campaign, from May 2003.

An in-row delay of 67 ms was used in 25 blasts; 11 of these were made with non-electric caps and 14 with EPDs. Three blasts (15/02, 53/02 and 10/03) were made with non-electric detonators and in-row delay of 84 ms, combined with two decks separated by 1 m of stemming; the top deck was shot 50 ms earlier than the bottom one, leading to an average delay of 42 ms for each blast. Blast 37/02, fired with non electric caps, had a variable inter-hole delay, and some of the holes were decked and some were not; decked holes have not been considered for the statistics of this blast. In five of the electronic blasts (38/03, 45/03, 8/04, 13/04 and 44/04), a much shorter delay between holes, 17 ms, was used. In blast 50/03, also with EPDs, the delay was 30 ms.

The geometrical blasting features (burden, spacing and nominal blasthole diameter) were mainly changed in the first campaign. Twelve blasts (15/02, 29/02, 37/02, 41/02, 42/02, 53/02, 103/02, 6/03, 10/03, 15/03, 19/03, 23/03) were drilled with 142 mm holes. Blasts 15/02, 29/02, 37/02, 41/02, 42/02 and 53/02 correspond to the period from February 2002 to July 2002 in which the toe burden was measured with a pole and a plumb line. The block profiling after drilling in these blasts showed the low accuracy in the blast design, with a mean burden out of control for those blasts. It varied from 5.5 to 4.6 m, while the spacing was almost constant (5.8-6.1 m). Consequently, \( S/B \) and \( q \) (powder factor above grade) changed from 1.07 to 1.28 and from 0.31 to 0.39 kg/m³ respectively. At that stage, the subdrill length was also rather uncontrolled; the mean and standard deviations, sd., of the subdrill length of the blastholes in blasts 15/02, 29/02, 37/02 and 53/02 were 2.7sd2.9 m, 0.4sd5.3 m, 2.1sd5.6 m and 2.4sd2.0 m respectively. This would affect vibration but not fragmentation significantly, although it is certainly a waste of explosive and drilling cost. An additional blast with good layout control (number 10/03), had a similar geometrical pattern as the above mentioned six blasts.

From November 2002, a laser profiler was used prior to drilling, which enabled a better control of the subdrill and a specific blast design for a given average burden. In blasts 103/02, 6/03, 15/03, 19/03 and 23/03 the drilling pattern was opened progressively up to an \( S/B \) value of 1.49 by increasing the spacing to 6.4 m and reducing the burden to 4.3 m, which leads to a powder factor of 0.41 kg/m³. As the quarry management was concerned about a much larger spacing than usual, the \( S/B \) increase was accomplished by reducing the burden more than required for a constant powder factor, resulting in a powder factor slightly higher than usual.

Blasts 30/03, 32/03 and 35/03 were drilled with a blast-hole diameter of 155 mm. In these blasts, the burden was 5.3 m and the spacing was increased up to 7.0 m in order to get a large \( S/B \) (about 1.33) and keep the powder factor above grade as low as 0.33 kg/m³.

In the second campaign, Alnafo is used as column charge in blasts 37/03, 38/03, 43/03, 45/03, 50/03, 78/03, 89/03, 96/03, 6/04, 8/04, 13/04, 21/04, 35/04 and 44/04. In these blasts, the spacing and the average burden were changed in a similar range as in the blasts made with non electric detonators. The spacing to burden ratio, \( S/B \) and the powder factor above grade, \( q \) varied between 1.09 and 1.56 and between 0.30 and 0.39 kg/m³ respectively; the blast-hole spacing ranged from 5.8 to 6.6 m.

In the set of blasts 54/03-58/03, 26/04-31/04 and 42/04-49/04 with a different column charge explosive than Alnafo, the drilling pattern was adapted to the explosive’s features and the in-row
delay was kept at 67 ms. The powder factor above grade, $q$, for the *Emunex 8000* blasts (26/04 and 31/04) is about 0.38 kg/m$^3$; the average burden and spacing were respectively 5.3 and 6.3 m respectively. In blasts with high density *Alnafo* (54/03 and 58/03) the burden was varied between 5 and 4.4 m while the spacing was kept at 6.4 m; $q$ and $S/B$ are ranged between 0.40 and 0.43 kg/m$^3$ and 1.28 and 1.45. The smallest drilling pattern is used when low density anfo was employed; the burden was changed from 3.9 to 4.7 m and the spacing from 4.5 to 5.0 m. The $q$ and $S/B$ spans were then 0.46-0.39 kg/m$^3$ and 1.15-1.06.

The linear charge density varies somewhat from blast to blast even with the same hole diameter and the same explosive; the mean and standard deviation for the 26 blasts with 142 mm of blasthole diameter and *Alnafo* as column charge is 14.2 sd0.9 kg/m. This is probably due to some karstification in the limestone and perhaps to the effect of the bit wear, which was not controlled during the test campaigns; Ouchterlony (2004c) states that the volume difference of a bit may be as 20%. This does not affect the total powder factor, as the total amount of explosive is fixed at the design stage and is not changed when loading. A rather variable stemming length is also a consequence of this.

Changes of blasting parameters from hole to hole may cause changes in fragmentation from some areas of the blast to others. For example, variations of 15 % in burden will change both $S/B$ and powder factor by 15 %, which, according to the fragmentation prediction models, may affect fragmentation. This is one more source of data variability. For example, the ratio of the standard deviation to the mean value is greater than 15 % in blasts 41/02, 10/03, 103/02 and 44/04 for the average burden. Variation in spacing between holes is about 2 % in all the blasts except in blast 103/02, where it is 20%.
Chapter 4

MEASUREMENT AND ANALYSIS OF THE BENCH FACE MOVEMENT

The analysis of the rock movement with video recording is a useful tool in blast design, serving as a general check of the explosive effect on the rock; inefficiencies in loading, stemming, timing, etc. become easily apparent from the inspection of the recorded images. At present, it is only possible to know the movement of the material in the face of the bench, face movement. High-speed video recording has been used for this purpose, and the movement of targets placed on the highwall has been analyzed. The raw-path coordinates of the targets are obtained from the recorded images after transformation using fixed control points of known coordinates. Profiles of the free face at several times are drawn from the target paths. This gives qualitative information of the movement and shows different possible distributions of velocities along it. The response time of the rock mass has been determined. This may be used to choose the delay for a certain degree of co-operation between adjacent holes. A trajectory model is fitted to the path of the targets, and their initial velocity is obtained so that the calculated trajectory best fits the measured one. The existing response time and initial velocity prediction formulae are analyzed and if necessary a detailed analysis is made in order to determine the main blasting parameters that affect the rock movement.

4.1. Experimental procedure and rounds description

A total of eight blasts are studied in this section: 15/02, 29/02, 37/02, 43/03, 45/03, 50/03, 54/03 and 58/03; their general basic features are given Appendix A. A Motion Meter 1000 high-speed digital camera, of Redlake Imaging with a recording velocity of 250 frames/s was used for monitoring the blasts.

The set-up described by Chiappetta et al. (1983) for recording and analysing the rock movement has been adapted to El Alto. This consists in wooden targets placed in front of the explosive column on the highwall in order to follow them easily in the image. The targets must move in
unison with the rock mass as their movement is assimilated to that of the rock fragments immediately behind them.

The information obtained from the target movement can only be extrapolated to the rest of the blast if the blasting parameters are homogeneous and similar to the ones used for the hole behind the targets. In order to avoid errors, the precise drilling and explosives data of the blastholes immediately behind the targets are used for the analysis. Table 4-1 show the main blasting features of such blastholes.

The nominal blasthole diameter is 142 mm in the eight blasts. The blasthole behind the targets have an excessive subdrill in blasts 15/02, 37/02, and 54/03 while the bottom of the hole is above grade in blast 29/02. Blasts 54/03 and 58/03 were made with high density Alnafo whereas plain Alnafo used in the others. VOD was measured along the holes behind the targets in order to check the explosive performance for the loading techniques used, see Table 3-4 above. Pyrotechnic detonators were used in blasts 15/02, 29/02 and 37/02 and EPDs in the other ones; the in-row delay was varied from 17 to 67 ms.

Table 4-1. Blasting features of the blastholes behind the targets

<table>
<thead>
<tr>
<th>Blast no.</th>
<th>15/02</th>
<th>29/02</th>
<th>37/02</th>
<th>43/03</th>
<th>45/03</th>
<th>50/03</th>
<th>54/03</th>
<th>58/03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole no.</td>
<td>9</td>
<td>11</td>
<td>15</td>
<td>2</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>(H, \text{ m})</td>
<td>20</td>
<td>17.2</td>
<td>17.2</td>
<td>16.7</td>
<td>15.7</td>
<td>17.9</td>
<td>18.5</td>
<td>16.5</td>
</tr>
<tr>
<td>(l_h, \text{ m})</td>
<td>24.9</td>
<td>15.1</td>
<td>15.1</td>
<td>24.5</td>
<td>18.1</td>
<td>17.9</td>
<td>21.9</td>
<td>19.1</td>
</tr>
<tr>
<td>(J, \text{ m})</td>
<td>4.8</td>
<td>-2.2</td>
<td>7.2</td>
<td>1.3</td>
<td>2.2</td>
<td>1.4</td>
<td>3.3</td>
<td>2.5</td>
</tr>
<tr>
<td>(h, \text{ m})</td>
<td>20</td>
<td>15.1</td>
<td>17.2</td>
<td>16.7</td>
<td>15.7</td>
<td>17.9</td>
<td>18.5</td>
<td>16.5</td>
</tr>
<tr>
<td>(S, \text{ m})</td>
<td>5.9</td>
<td>6</td>
<td>5.8</td>
<td>6.4</td>
<td>6.4</td>
<td>6.7</td>
<td>6.4</td>
<td>6.4</td>
</tr>
<tr>
<td>(B, \text{ m})</td>
<td>5.7</td>
<td>4.4</td>
<td>4.9</td>
<td>4.5</td>
<td>4</td>
<td>4.6</td>
<td>5.4</td>
<td>4.4</td>
</tr>
<tr>
<td>(S/B)</td>
<td>1.04</td>
<td>1.36</td>
<td>1.18</td>
<td>1.42</td>
<td>1.6</td>
<td>1.46</td>
<td>1.19</td>
<td>1.45</td>
</tr>
<tr>
<td>(B_{d1}, \text{ m})</td>
<td>No targ.</td>
<td>No targ.</td>
<td>No targ.</td>
<td>4.9</td>
<td>No detc</td>
<td>No detc</td>
<td>5.8</td>
<td>No detc</td>
</tr>
<tr>
<td>(B_{d2}, \text{ m})</td>
<td>No targ.</td>
<td>No targ.</td>
<td>No targ.</td>
<td>4.3</td>
<td>3.5</td>
<td>3.9</td>
<td>5.6</td>
<td>4.5</td>
</tr>
<tr>
<td>(B_{d3}, \text{ m})</td>
<td>5.4</td>
<td>4.2</td>
<td>5.3</td>
<td>3.7</td>
<td>4.2</td>
<td>4.5</td>
<td>5.2</td>
<td>4.8</td>
</tr>
<tr>
<td>(B_{d4}, \text{ m})</td>
<td>6.6</td>
<td>4.2</td>
<td>4.5</td>
<td>3.8</td>
<td>No detc</td>
<td>5.3</td>
<td>5.3</td>
<td>4.8</td>
</tr>
<tr>
<td>(H_{d1}, \text{ m})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>11.1</td>
<td>-</td>
<td>-</td>
<td>12.8</td>
<td>-</td>
</tr>
<tr>
<td>(H_{d2}, \text{ m})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>8.9</td>
<td>9.1</td>
<td>11</td>
<td>11.3</td>
<td>8</td>
</tr>
<tr>
<td>(H_{d3}, \text{ m})</td>
<td>16.1</td>
<td>10.3</td>
<td>11.6</td>
<td>6.4</td>
<td>5.5</td>
<td>6.3</td>
<td>9.6</td>
<td>5.5</td>
</tr>
<tr>
<td>(H_{d4}, \text{ m})</td>
<td>11.1</td>
<td>5.5</td>
<td>5.9</td>
<td>4.7</td>
<td>-</td>
<td>1.9</td>
<td>5.1</td>
<td>1.3</td>
</tr>
<tr>
<td>((l_s/t)_{(1)})</td>
<td>3.4</td>
<td>4.8</td>
<td>7.8</td>
<td>3.7</td>
<td>3.9</td>
<td>2.9</td>
<td>3.2</td>
<td>4.2</td>
</tr>
<tr>
<td>(\left[(Q_e)_c\right]/\left[(Q_e)_b\right])</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
<td>G2/Alf</td>
</tr>
<tr>
<td>(E_s, \text{ kJ/kg})</td>
<td>2968</td>
<td>3004</td>
<td>2969</td>
<td>2986</td>
<td>2987</td>
<td>2980</td>
<td>3335</td>
<td>3339</td>
</tr>
<tr>
<td>(q_{T}, \text{ kg/m}^3)</td>
<td>4856</td>
<td>4801</td>
<td>4854</td>
<td>4828</td>
<td>4827</td>
<td>4837</td>
<td>4901</td>
<td>4882</td>
</tr>
<tr>
<td>(q_{T}, \text{ kg/m}^3)</td>
<td>2/210</td>
<td>25/138</td>
<td>0/173</td>
<td>19/168</td>
<td>8/161</td>
<td>14/188</td>
<td>0/247</td>
<td>5/192</td>
</tr>
<tr>
<td>(q_{T}, \text{ kg/m}^3)</td>
<td>0.42</td>
<td>0.41</td>
<td>0.56</td>
<td>0.43</td>
<td>0.51</td>
<td>041</td>
<td>0.47</td>
<td>0.51</td>
</tr>
<tr>
<td>(q_{T}, \text{ kg/m}^3)</td>
<td>0.32</td>
<td>0.41</td>
<td>0.35</td>
<td>0.39</td>
<td>0.42</td>
<td>0.37</td>
<td>0.39</td>
<td>0.42</td>
</tr>
<tr>
<td>(t, \text{ ms})</td>
<td>42</td>
<td>67</td>
<td>67?</td>
<td>67</td>
<td>17</td>
<td>30</td>
<td>67</td>
<td>67</td>
</tr>
</tbody>
</table>

Notes: 1. \(\left[(Q_e)_c\right]/\left[(Q_e)_b\right]\) are the mass of cartridge and bulk explosives per hole respectively
2. G2, Alf and HdAlf respectively means Goma 2ECO, Alnafo and high density Alnafo.

One charge per hole is used in all rounds, except in blast 15/02, where two decks separated by 1 m of stemming were used; the top and bottom decks had a length of 13.8 and 6.7 m; only 1.8 m of the bottom deck was above grade. The upper deck was initiated first and the lower had a 50 ms delay. The targets used were located in front of the upper deck area.
The dimensions of the targets were 1 m x 1.8 m in blasts 15/02, 29/02 and 37/03 and 1.2 x 1.2 m in the other shots. The targets are white with a black cross along each diagonal, which provides a good contrast against the limestone colours.

The targets were placed in the free face by hanging them with a rope, whose upper extreme is tied to a stick introduced in the blasthole immediately behind the targets before loading the stemming. A detonator tied to the ropes in the top level of the blast is used to cut the rope some time after the movement has started in order to free the targets when they have certain thrust and move together with the rock, see Figure 4-1. In practice, either the detonator tied to the rope or the fastening system used allows a free movement of the targets.

![Figure 4-1. Shock tube coil and detonator lay-out to cut the rope](image)

The targets are numbered from the grade to the top of the bench face. Two wooden targets were used in blasts 15/02, 29/02 and 37/02 and two more in the others five blasts. The smallest targets were sometimes difficult to appreciate in the images as the view field must comprise all targets and control points. In blasts in which two targets were used, both targets were always visible and hence could be followed. However, when the targets were increased up to four, the targets labelled as T1 and T4 in blast 45/03 and T4 in blasts 50/03 and 58/03 could not be recognized at any time, even before the blast was initiated.

The targets were placed in front of the explosive column, except the top target in blast 37/02 that was located in the bottom part of the stemming due to vision problems in lower positions. The height and burdens at the target levels, $H_i$ and $B_i$ respectively with $i$ from 1 to 4, are given in Table 4-1. When the target is in front of the explosive column the burden is defined as the minimum distance from the face (at the height of the centre of the target) to the blasthole axis. When the target is in front of the stemming, the burden is the distance from the top of the explosive to the face (at the height of the centre of the target).

The recorded images were analysed with Motion Tracker™ 2D software in order to obtain the raw path of the targets used for calculating their initial velocity and pitch angle. The software uses the multiplier technique (Chiappetta et al., 2001), which relates the image co-ordinates to the object co-ordinates by means of two equations and eight calibration constants. These constants are calculated using four control points of known coordinates by means of a linear system of eight equations in eight unknowns. Dimensional controls, lens aberration, field set up, camera orientation and the lens characteristics are automatically calculated and adjusted by the
4 Measurement and Analysis of the Bench Face Movement

software. The angle of the optic axis and the free face is less than 40° in the blasts monitored; larger angles are inconvenient to appreciate the rock mass movement.

The four control points must form a quadrilateral, whose shape is not important, but obviously the larger the quadrilateral area in the field of view the better is for the analysis of the targets movement. The precise effect of the size of the control point’s quadrilateral on the accuracy of the measurements is unknown. Figure 4-2 shows one of the set-ups suggested for the control points (BAI, 2001). This has been used in most of the blasts monitored. The unique exception is blast 15/02, where the control point labelled as 2 in Figure 4-2 is located 12 m away from the toe of the bench, instead of being in the toe itself. A vertical pole defines the control points 3 and 4. The $L$-value in Figure 4-2 depends on the bench height, so that the angle $\theta$ is as small as possible. However, it is limited to all four points being in the field of view of the camera, which depends basically on both the bench height and the camera position. The value of $h$ in the monitored blasts is about 4.2 m and the $L$-value ranges from 20 m to 30 m.

![Figure 4-2. Control points set-up.](image)

4.1.1. Quality control

The position of the control points was obtained in all the blasts using a Quarryman ALS300 laser profiler, of MDL.

The original position of the targets has been measured in blasts 15/02, 29/02 and 37/02 in order to compare them with those given by the software. This serves for checking the goodness of the multiplier technique. In most of the cases, both co-ordinates coincide fairly well. The unique exceptions occur in the $X$-coordinate of the T1 target in blast 37/02; the actual position of the target is respectively 3.3 m and the position given by the software is 2.8 m.

The targets match well the highwall surface as given by the profile at the hole behind them in blasts 29/02, 43/03, 54/03 (only T1 and T2 targets) and 58/03. Conversely, in blasts 15/02, 37/02, 45/03 and 50/03 the targets are not well accommodated to the bench face, but only in 15/02 and 45/03 the errors are quite large. See Figure 4-9 to 4-11 below.

These discrepancies may be caused by the errors committed in the field when the position of the control points is measured, the errors due to the shape and size of the plane defined by the control points and by the errors made when the targets are pointed with the cursor in the software.
4 Measurement and Analysis of the Bench Face Movement

4.2. Response time

4.2.1. Background

With an arbitrary time reference (time zero is that at which the camera is stopped) the response time of the rock mass, $t_{\text{resp}}$, can be defined as the time after the initiation of the explosive in a blasthole, $t_{\text{init}}$, to which the rock in front of that hole starts its movement, $t_{\text{first mov}}$ (it is assumed that the rock starts to move at the same time that the targets do):

$$t_{\text{resp}} = t_{\text{first mov}} - t_{\text{init}}$$

Chiappetta & Mammele (1987) give a response time interval between 5 ms and 150 ms for a wide range of materials, burdens and explosives. This information is completed in Figure 4-3 in which the response time for different rock masses is plotted versus the burden (Chiappetta, 1998).

![Characteristic Dynamic Response Time Curves](image)

Figure 4-3. $t_{\text{resp}}$ vs. burden for different rock masses (Chiappetta, 1998).

Oñederra & Esen (2003) propose an empirical model to estimate the response time. Such model is based on 19 cases that according to the authors cover a wide range of geotechnical and blasting conditions. The response time in ms at the centre of the explosive charge is given by:

$$t_{\text{resp}} = (K_{\text{mass}} ERI) \left[ a \left( \frac{B \cdot \frac{1}{d}}{K_{\text{mass}} ERI} \right)^b \right]$$  \hspace{1cm} (4-1)

Where:

- $K_{\text{mass}}$ is the rock mass stiffness in GPa; $K_{\text{mass}} = E_d / (1 + \nu_d)$, being $E_d$ and $\nu_d$ the dynamic Young modulus and Poisson’s ratio respectively.
- **ERI** is an explosive rock interaction term function of the explosive density, \( \rho_e \) (g/cm\(^3\)); actual VOD, \( D \) (km/s); CJ detonation velocity, \( D_{CJ} \) (km/s) and P-wave propagation of the intact rock, \( c_p \) (km/s) The \( ERI \) term is given by:

\[
ERI = (0.36 + \rho_e) \rho_e \left( \frac{D}{D_{CJ}} \right) \left( \frac{D^2}{1 + D^2/c_p^2 - D/c_p} \right)
\]

- \( a \) and \( b \) are fitting constants; they are 2.408 and 1.465 respectively for the Oñederra and Esen’s data, in which \( B/d \) varies from 12 to 45 and the blastholes are fully coupled and properly stemmed.

### 4.2.2. Calculation and analysis

Chiappetta et al. (1983) describe a procedure to obtain the displacement of a target versus the associated time of each position; Motion Tracker software uses this technique. However, our experience shows that small errors tracking the targets in the first steps of their movement lead to variations up to 100% in the response time, as it is sometimes difficult to track the targets in the first steps of their movement. Therefore, the time of the first movement has been directly obtained in a frame per frame analysis of the recorded images. This involves a certain inaccuracy, but the errors committed are smaller than with the fitting method.

The initiation time of the explosive column behind the targets, \( t_{init} \), is obtained from a shock tube with one extreme coiled and hanged to a stick in the bench crest. The shock tube is introduced in the top part of the explosive column, and it is initiated by the explosive upper end. \( t_{init} \) is obtained by subtracting to the “coil” time, the time needed by the shock tube detonation to travel from the bottom part of the stemming to that point and the duration of the detonation of the explosive column (obtained from the VOD record in that hole). This set up makes it difficult to know the exact distance of shock tube burnt when the flash is noted. Besides, the shock tube flash may have taken place, in the worst case for the recording rate used, almost 4 ms earlier, and consequently it was not registered in the previous frame. Therefore, the total error in the determination of the response time may be, in the worst case, about 5 ms.

Table 4-2 shows the response times obtained in the blasts monitored. The initiation time of the explosive in blast 43/03 was not detected, which prevents to get the response time.

<table>
<thead>
<tr>
<th>Blast No.</th>
<th>15/02</th>
<th>29/02</th>
<th>37/02</th>
<th>43/03</th>
<th>45/03</th>
<th>50/03</th>
<th>54/03</th>
<th>58/03</th>
</tr>
</thead>
<tbody>
<tr>
<td>T4, ms</td>
<td>92</td>
<td>28</td>
<td>56</td>
<td>-</td>
<td></td>
<td>29</td>
<td>29</td>
<td>17</td>
</tr>
<tr>
<td>T3, ms</td>
<td>40</td>
<td>16</td>
<td>32</td>
<td>-</td>
<td>28</td>
<td>29</td>
<td>29</td>
<td>17</td>
</tr>
<tr>
<td>T2, ms</td>
<td>92</td>
<td>28</td>
<td>56</td>
<td>-</td>
<td>32</td>
<td>21</td>
<td>45</td>
<td>17</td>
</tr>
<tr>
<td>T1, ms</td>
<td>92</td>
<td>28</td>
<td>56</td>
<td>-</td>
<td>29</td>
<td>29</td>
<td>29</td>
<td>17</td>
</tr>
</tbody>
</table>

As a first approach to the response time prediction, the Oñederra & Essen’s formula is applied to El Alto’s data. The response time values shown in Table 4-2 (more than one value is available for a blasthole), the respective burdens at the targets levels given in Table 4-1 and the nominal diameter of the blastholes are introduced in Equation 4-1. A constant \( K_{mass} \) of 70.7 GPa, obtained from the data in Section 3.1.1 (\( E_d = 89.1 \) GPa and \( \nu = 0.26 \); the Poisson’s coefficient used
was obtained from the data in a static test) is assumed for all the blasts. The \( ERI \) term is 8.3 for \textit{Alnafo} and 7.8 for High density \textit{Alnafo} blasts. This is calculated from the \( \rho_e \) and \( D \) values shown in Table 3-4, the \( D_{CJ} \) velocity of detonation (4964 m/s for \textit{Alnafo} and 5532 m/s for high density \textit{Alnafo}) and the \( c_p \) measured in the field (2994 m/s). The coefficients \( a \) and \( b \) obtained from a non linear regression analysis are 217.6 and 2.95 respectively. The \( a \) value is very different to the one given by Oñederra & Esen, probably due to the wider range of the \( B/d \) ratio, \( 25 \leq B/d \leq 47 \), for El Alto’s data. The correlation factor, \( R^2 \) of the fitting is 0.55. The asymptotic standard error for \( a \) is rather large (373.9), a 172 \% of the estimated value, and its asymptotic interval includes zero at a confidence level of 95 \%. This means that \( a \) can be removed from the model without affecting significantly to the fitting.

Figure 4-4 shows the response time versus burden for each target, the data is differentiated according to the explosive type used. Most of the data is packaged between response times from 17 to 45 ms and burdens from 3.8 and 5.8 m. The response time seems to be an increasing function of the burden, especially due to the measurement made in front of the stemming with a burden of 6.6 m. However, the scatter is high. The calibrated Oñederra & Esen’s formula with \( a=217.6 \) and \( b=2.95 \) is drawn in Figure 4-4 for \textit{Alnafo} and high density \textit{Alnafo} blasts.

According to Equation 4-1, the burden at targets levels, \( B_t \), the \( ERI \) term and the measured velocity of detonation, \( D \), may affect the response time. The \( B/d \) ratio is not suitable for El Alto’s data as the actual blasthole diameter is not known exactly. However, the linear density of the explosive loaded into the hole \((\rho_l)_{t}=(Q)/((h_s-l_s))\) takes into account the diameter variations due to bit wearing. \((\rho_l)_{t}\) includes more information than \( d \), since it depends on the explosive density; the ratio \( (\rho_l)_{t}/B_t \) is then considered. Other parameters that may be included in a new \( t_{resp} \) prediction formula are:

- Height at the target level, \( H_t \).
- Blasthole length, \( h_s \).
- Blasted height, \( h \).
- Ratios \( l_b/h \), \( H_d/l_b \) and \( H_d/h \).
- Spacing between blastholes, \( S \).
- Average burden along the blasthole behind the targets, \( B \).
- Ratios \( S/B \), \( S/B_d \) and \( B_d/B \).
- Stemming length, \( l_s \).
- Total explosive mass per hole, \( (Q_e)_T \).
- Specific useful work calculated per unit of mass, \( E_u \).
- Explosive mass above grade per hole, \( Q_e \).
- Total powder factor, \( qT \), and powder factor above grade, \( q \).
- the in row delay, \( t \) and the ratios \( t/S \), \( t/B \) and \( t/B_d \).

For each blasthole, only \( B_d \) and \( H_d \) vary for the different targets from the whole set of parameters. Tables 4-3 and 4-4 show a summary of the statistics of the former parameters for 16 targets used in seven blasts, in which the entire set of parameters is known; the in-row delay for the two targets used in blast 37/02 is undetermined.

Table 4-3. Variation of the drilling parameters

<table>
<thead>
<tr>
<th>( B_d )</th>
<th>( H_d )</th>
<th>( l_b )</th>
<th>( h )</th>
<th>( l_b/h )</th>
<th>( H_d/l_b )</th>
<th>( H_d/h )</th>
<th>( S )</th>
<th>( B )</th>
<th>( S/B )</th>
<th>( S/B_d )</th>
<th>( B_d/B )</th>
<th>( l_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>m</td>
<td>m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>Av.</td>
<td>4.9</td>
<td>8.2</td>
<td>19.9</td>
<td>17.4</td>
<td>1.13</td>
<td>0.41</td>
<td>0.46</td>
<td>6.3</td>
<td>4.8</td>
<td>1.34</td>
<td>1.34</td>
<td>1.01</td>
</tr>
<tr>
<td>Sd.</td>
<td>0.8</td>
<td>4.0</td>
<td>2.9</td>
<td>1.6</td>
<td>0.07</td>
<td>0.18</td>
<td>0.21</td>
<td>0.3</td>
<td>0.6</td>
<td>0.18</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>Min.</td>
<td>3.5</td>
<td>1.3</td>
<td>15.1</td>
<td>15.1</td>
<td>1.0</td>
<td>0.07</td>
<td>0.08</td>
<td>5.9</td>
<td>4.0</td>
<td>1.04</td>
<td>0.89</td>
<td>0.85</td>
</tr>
<tr>
<td>Max.</td>
<td>6.6</td>
<td>16.1</td>
<td>24.9</td>
<td>20</td>
<td>1.25</td>
<td>0.68</td>
<td>0.81</td>
<td>6.7</td>
<td>5.7</td>
<td>1.6</td>
<td>1.83</td>
<td>1.16</td>
</tr>
<tr>
<td>Sd./Av.%</td>
<td>16.3</td>
<td>48.8</td>
<td>14.6</td>
<td>9.2</td>
<td>6.2</td>
<td>45.7</td>
<td>44.4</td>
<td>4.8</td>
<td>12.5</td>
<td>13.4</td>
<td>17.9</td>
<td>8.91</td>
</tr>
<tr>
<td>Range/Av., %</td>
<td>63.3</td>
<td>180</td>
<td>49.2</td>
<td>28.2</td>
<td>22.1</td>
<td>149</td>
<td>159</td>
<td>12.7</td>
<td>35.4</td>
<td>41.8</td>
<td>69.4</td>
<td>30.7</td>
</tr>
</tbody>
</table>

Table 4-4. Variation of the loading and timing parameters

<table>
<thead>
<tr>
<th>( (\rho)_l )</th>
<th>( E_u )</th>
<th>( (Q_e)_T )</th>
<th>( qT )</th>
<th>( q )</th>
<th>( D )</th>
<th>( t )</th>
<th>( t/S )</th>
<th>( t/B )</th>
<th>( t/B_d )</th>
<th>( ERI )</th>
</tr>
</thead>
<tbody>
<tr>
<td>kg/m²</td>
<td>kJ/kg</td>
<td>kg</td>
<td>kg</td>
<td>kg/m³</td>
<td>kg/m³</td>
<td>m/s</td>
<td>ms</td>
<td>ms/m</td>
<td>ms/m</td>
<td>m/s/m</td>
</tr>
<tr>
<td>Av.</td>
<td>3.2</td>
<td>3138</td>
<td>243</td>
<td>205</td>
<td>0.47</td>
<td>0.39</td>
<td>3764</td>
<td>51</td>
<td>8.0</td>
<td>10.5</td>
</tr>
<tr>
<td>Sd.</td>
<td>0.6</td>
<td>181</td>
<td>47</td>
<td>30</td>
<td>0.05</td>
<td>0.03</td>
<td>310</td>
<td>20</td>
<td>3.3</td>
<td>4.3</td>
</tr>
<tr>
<td>Min.</td>
<td>2.0</td>
<td>2968</td>
<td>163</td>
<td>163</td>
<td>0.41</td>
<td>0.32</td>
<td>3424</td>
<td>17</td>
<td>2.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Max.</td>
<td>4.1</td>
<td>3339</td>
<td>300</td>
<td>247</td>
<td>0.51</td>
<td>0.42</td>
<td>4029</td>
<td>67</td>
<td>11.2</td>
<td>15.2</td>
</tr>
<tr>
<td>Sd./Av.%</td>
<td>18.9</td>
<td>5.8</td>
<td>19.3</td>
<td>14.6</td>
<td>10.6</td>
<td>7.7</td>
<td>8.2</td>
<td>39.2</td>
<td>41.3</td>
<td>40.1</td>
</tr>
<tr>
<td>Range/Av., %</td>
<td>0.66</td>
<td>11.8</td>
<td>56.4</td>
<td>41.0</td>
<td>21.3</td>
<td>28.9</td>
<td>16.1</td>
<td>98.7</td>
<td>106</td>
<td>105</td>
</tr>
<tr>
<td>ERI</td>
<td>8.1</td>
<td>4.3</td>
<td>4.2</td>
<td>6.3</td>
<td>10.4</td>
<td>10.4</td>
<td>3.7</td>
<td>6.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4-5. Correlation matrix of the response time and some blast design parameters for 18 sets of data

<table>
<thead>
<tr>
<th></th>
<th>$t_{\text{resp}}$</th>
<th>$B_n$</th>
<th>$H_d$</th>
<th>$l_h$</th>
<th>$h$</th>
<th>$l_d/h$</th>
<th>$H_d/h$</th>
<th>$S$</th>
<th>$B$</th>
<th>$S/B$</th>
<th>$S/B_n$</th>
<th>$B_d/B$</th>
<th>$l_i$</th>
<th>$(Q_e)_i$</th>
<th>$(\rho_l)/B_n$</th>
<th>$E_w$</th>
<th>$Q_e$</th>
<th>$q_T$</th>
<th>$q$</th>
<th>$D$</th>
<th>$\text{ERI}$</th>
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<tbody>
<tr>
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<td></td>
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<tr>
<td>$B_n$</td>
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<td>0.38</td>
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<td>0.52</td>
<td>-0.66</td>
<td>-0.22</td>
<td>0.26</td>
<td>0.02</td>
<td>-0.67</td>
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<td>$H_d$</td>
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<td>-0.78</td>
<td>0.95</td>
<td>0.68</td>
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<td>0.26</td>
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<td>$l_h$</td>
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<td>0.43</td>
<td>0.46</td>
<td>0.25</td>
<td>0.92</td>
<td>0.98</td>
<td>-0.37</td>
<td>0.55</td>
<td>-0.56</td>
<td>-0.34</td>
<td>0.07</td>
<td>-0.04</td>
<td>0.36</td>
<td>-0.34</td>
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<td>0.24</td>
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<td>0.12</td>
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<td>0.85</td>
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<td>0.79</td>
<td>-0.78</td>
<td>-0.71</td>
<td>0.21</td>
<td>0.20</td>
<td>0.88</td>
<td>-0.65</td>
<td>0.02</td>
<td>0.46</td>
<td>0.40</td>
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<td>-0.07</td>
<td>-0.07</td>
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<td>0.15</td>
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<tr>
<td>$H_d/h$</td>
<td>1.00</td>
<td>-0.05</td>
<td>0.18</td>
<td>-0.54</td>
<td>0.43</td>
<td>-0.55</td>
<td>-0.50</td>
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<td>-0.04</td>
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<td>-0.01</td>
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<td>$S/B$</td>
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<td>0.28</td>
<td>-0.31</td>
<td>-0.06</td>
<td>-0.24</td>
<td>-0.07</td>
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<td>-0.05</td>
<td>-0.17</td>
<td>0.06</td>
<td>-0.29</td>
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<td>0.17</td>
<td>0.17</td>
<td>$H_d/h$</td>
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<tr>
<td>$S/B_n$</td>
<td>1.00</td>
<td>-0.38</td>
<td>0.39</td>
<td>-0.44</td>
<td>-0.20</td>
<td>-0.15</td>
<td>0.04</td>
<td>0.21</td>
<td>-0.16</td>
<td>-0.17</td>
<td>0.10</td>
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<td>-0.34</td>
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<td>0.16</td>
<td>0.16</td>
<td>$H_d/h$</td>
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<tr>
<td>$B_d/B$</td>
<td>1.00</td>
<td>-0.30</td>
<td>0.59</td>
<td>0.47</td>
<td>0.00</td>
<td>-0.71</td>
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<td>0.32</td>
<td>0.35</td>
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<td>0.43</td>
<td>-0.32</td>
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<td>0.11</td>
<td>0.11</td>
<td>$S/B$</td>
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<tr>
<td>$l_i$</td>
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<td>-0.17</td>
<td>0.84</td>
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<td>-0.24</td>
<td>-0.24</td>
<td>0.24</td>
<td>0.11</td>
<td>0.11</td>
<td>$S/B_n$</td>
<td></td>
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</tr>
<tr>
<td>$(Q_e)_i$</td>
<td>1.00</td>
<td>0.85</td>
<td>-0.14</td>
<td>-0.14</td>
<td>-0.73</td>
<td>0.65</td>
<td>-0.08</td>
<td>-0.50</td>
<td>0.05</td>
<td>0.72</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>$B_d/B$</td>
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</tr>
<tr>
<td>$(\rho_l)/B_n$</td>
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<td>-0.63</td>
<td>-0.09</td>
<td>-0.70</td>
<td>0.81</td>
<td>-0.22</td>
<td>-0.50</td>
<td>-0.05</td>
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<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>$B_d/B$</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>$E_w$</td>
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<td>-0.06</td>
<td>0.24</td>
<td>-0.61</td>
<td>0.24</td>
<td>0.19</td>
<td>0.13</td>
<td>-0.08</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.25</td>
<td>$E_w$</td>
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</tr>
<tr>
<td>$Q_e$</td>
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<td>0.02</td>
<td>0.69</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>$Q_e$</td>
<td></td>
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<tr>
<td>$q_T$</td>
<td>1.00</td>
<td>0.65</td>
<td>0.29</td>
<td>0.48</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-0.67</td>
<td>-0.67</td>
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<td>-0.67</td>
<td>$q_T$</td>
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<tr>
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<td>0.00</td>
<td>0.24</td>
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<td>-0.49</td>
<td>-0.49</td>
<td>-0.49</td>
<td>-0.49</td>
<td>-0.49</td>
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<td>$q$</td>
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<td></td>
</tr>
<tr>
<td>$D$</td>
<td>1.00</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>$D$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{ERI}$</td>
<td>1.00</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
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<td>0.10</td>
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<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>$\text{ERI}$</td>
<td></td>
<td></td>
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</tbody>
</table>
A correlation analysis has been done firstly between the timing parameters \((t, t/S, t/B, t/B_i)\) and the response time. The aim is to detect the blasting parameters with highest influence in the response time. The correlation coefficients (which measure the linear relationship between the variables) between \(t_{\text{resp}}\) and \(t, t/S, t/B, t/B_i\) are fairly low, -0.11, -0.08, -0.30 and -0.35 respectively. In this line, Chiappetta et al. (1983) state that the targets movement is not affected by the drilling and loading features of the adjacent hole for delays between holes greater than 25 ms. This seems to occur even in blast 45/03, in which the in-row delay was 17 ms.

A new correlation analysis is done, but this time rejecting the in-row delay; \(t_{\text{resp}}, B_{ii}, H_{ii}, l_{ii}, h, l_s/h, H_s/l_s, l_s/h, S, B, S/B, S/B_{ii}, B_{ii}/B, l_s, (Q_s)_i, (\rho)_i/B_{ii}, E_w, Q_e, q_T, q, D\) and \(\text{ERI}\) are considered for the analysis. The former set of parameters is known for 18 targets employed in eight blasts. The respective correlation coefficients are shown in Table 4-5; the p-value is below 0.05 in the relations show in bold, which indicates a statistically significant non zero correlation at the 95% confidence level.

\(B_{ii}\) and \(H_{ii}\) have the strongest relations with \(t_{\text{resp}}\), the correlation coefficients are 0.73 and 0.72 respectively. \(l_s, B, S/B, S/B_{ii}, (\rho)_i/B_{ii}\) and \(q\) have significant correlations, of about 0.66 with \(t_{\text{resp}}\). But, only \(l_s\) and \(B\) are positively correlated with \(t_{\text{resp}}\). The blasted height, \(h\) and \(H_s/h\) have a positive correlation with \(t_{\text{resp}}\) of 0.61. Conversely, \(q_T\) (only differs with \(q\) in the explosive loaded into the subdrilled part of the blasthole) and \(t_{\text{resp}}\) are weakly correlated. The explosive properties (explosive energy and velocity of detonation) and the \(\text{ERI}\) term have low influence in \(t_{\text{resp}}\), the correlation coefficients are of 0.15. Table 4-4 shows that all three, the \(\text{ERI}\) term, \(E_w\) and \(D\) vary very little in our tests.

The correlation coefficients between \(B_{ii}\) and the set of parameters, \(l_s, B, S/B, S/B_{ii}\) and \((\rho)_i\), well correlated with \(t_{\text{resp}}\) are high 0.70, 0.84, -0.78, -0.95 and -0.91. This leads to try the following simple model for predicting the response time:

\[
 t_{\text{resp}} = A \cdot B_{ii}^\alpha 
\]

The \(R^2\) of the non linear fitting is 0.48; where \(t_{\text{resp}}\) is in ms and \(B_{ii}\) in m. The estimated \(A\) and \(\alpha\) values are shown in Table 4-6. The coefficient \(A\) has a large asymptotic standard error and includes zero in the asymptotic interval at a 95 % confidence level, as occurred when Equation 4-1 was fit to El Alto data.

Table 4-6. Results of a non linear regression analysis of \(t_{\text{resp}} = A \cdot B_{ii}^\alpha\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>3.18</td>
<td>-3.8  10.1</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>1.51</td>
<td>0.2   2.8</td>
</tr>
</tbody>
</table>

Considering that \(B_{ii}\) does not vary significantly in the data, see Figure 4-4 and Table 4-3, the burden at the target level is normalized using the average of the \(B_{ii}\) values, 4.9 m. The following function is tried:

\[
 t_{\text{resp}} = A \cdot (B_{ii} / 4.9)^\alpha 
\]
A better correlation factor is obtained, 0.67. The asymptotic standard deviation for $A$, given in Table 4-7, is only a 9% of the estimated value and none of the parameters considered include zero in the asymptotic interval at the 95% confidence level. Equation 4-2 catches then better the variability of the response time and shows a similar influence of the burden in $t_{resp}$ to that shown by Oñederra & Esen (2003) and Chiappetta (1998).

Table 4-7. Results of a non linear regression analysis of $t_{resp} = A \cdot (B_{ti}/4.9)^\alpha$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95% confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>30.8</td>
<td>24.7 to 36.9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>3.2</td>
<td>2.2 to 4.3</td>
</tr>
</tbody>
</table>

Attending to the good correlation between $S/B_{ti}$ and $t_{resp}$, $B_{ti}/4.9$ is replaced by $S/B_{ti}$ in Equation 4-2. The aim is to improve $t_{resp}$ predictions. The output of the non linear regression fitting is shown in Table 4-8. The correlation factor, $R^2$ increases up to 0.78 and the asymptotic interval at the 95% confidence level for $A$ and $\alpha$ do not include zero.

Table 4-8. Results of a non linear regression analysis of $t_{resp} = A \cdot (S/B_{ti})^\alpha$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95% confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>62.9</td>
<td>54.2 to 71.6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-2.9</td>
<td>-3.6 to -2.2</td>
</tr>
</tbody>
</table>

The height at the target level is included in the model due to its strong correlation with $t_{resp}$, which results in:

$$t_{resp} = A \cdot (S / B_{ti})^\alpha \cdot H_{ti}^\beta$$

(4-3)

The estimated values for the fitting coefficients and the asymptotic intervals are listed in Table 4-9; where $t_{resp}$ is in ms and $H_{ti}$ in m. Equation 4-3 increases the correlation factor to 0.84 and shows that the response time increases potentially with $H_{ti}$. A different relation would be obtained for up-hole initiation. The response time, according to Equation 4-3, is zero at the bench grade ($H_{ti}=0$); the closest measurement to the grade lowest has an $H_{ti}$ of 1.6 m. The following formula is fitted to the data

$$t_{resp} = A \cdot (S / B_{ti})^\alpha \cdot \left( \frac{h - H_{ti}}{l_h} \right)^\beta$$

(4-4)

The correlation factor is improved up to 0.89 and again the asymptotic interval given in Table 4-10 for the three fitting constants do not include zero. The ratio of $(h-H_{ti})/l_h$ shows that the response time at the grade level decreases as the ratio of the bench height to the blasthole length increases; no target was placed at height $H_{ti}=h$. However, it must be recognised that Equations of the type of 4-3 and 4-4 are of little practical use, as the only blast design factor in then is the spacing to burden ratio. It is likely that other factors play also an important part in the response time.
Table 4-9. Results of a non linear regression analysis of \( t_{\text{resp}} = A \cdot \left( \frac{S}{B_{ti}} \right)^{\alpha} \cdot H_{ti}^{\beta} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>29.2</td>
<td>6.7 - 51.7</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-2.1</td>
<td>-3.0 - 1.3</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.3</td>
<td>0.0 - 0.6</td>
</tr>
</tbody>
</table>

Table 4-10. Results of a non linear regression analysis of \( t_{\text{resp}} = A \cdot \left( \frac{S}{B_{ti}} \right)^{\alpha} \cdot \left[ \left( \frac{h-H_{ti}}{l_{h}} \right) \right]^{\beta} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>25.9</td>
<td>13.5 - 38.3</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-1.3</td>
<td>-2.1 - 0.5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-0.6</td>
<td>-1.0 - 0.0</td>
</tr>
</tbody>
</table>

Notes: In italics the results of the fitting when the target t2 in blast 15/02 is rejected

The powder factor above grade can not be added to Equation 4-4 since the asymptotic interval for the respective exponent includes zero at the 95 % confidence level. The linear density of the explosive, included by Oñederra & Esen (2003) by means of the blasthole diameter, does not appear in the formulæ tried above. This is only possible if the spacing between blastholes is rejected and \( (\rho_{l})/B_{ni} \) is used, which leads to the following formula.

\[
t_{\text{resp}} = A \cdot \left( \frac{(\rho_{l})}{B_{ni}} \right)^{\alpha} \cdot \left( \frac{h-H_{ti}}{l_{h}} \right)^{\beta} \tag{4-5}
\]

Where \( t_{\text{resp}} \) is in ms and \( (\rho_{l})/B_{ni} \) in kg/m\(^2\). The estimated fitting coefficients and the respective asymptotic intervals at the 95 % are shown in Table 4-11. The correlation factor of the fitting, 0.88 is close to the one achieved with Equation 4-4.

Table 4-11. Results of a non linear regression analysis of \( t_{\text{resp}} = A \cdot \left( \frac{(\rho_{l})}{B_{ni}} \right)^{\alpha} \cdot \left( \frac{h-H_{ti}}{l_{h}} \right)^{\beta} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>50.2</td>
<td>6.40 - 94.0</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-0.9</td>
<td>-1.51 - 0.4</td>
</tr>
<tr>
<td>( \beta )</td>
<td>-0.7</td>
<td>-0.95 - 0.4</td>
</tr>
</tbody>
</table>

Notes: In italics the results of the fitting when the target t2 in blast 15/02 is rejected

Figure 4-5 shows the measured response times versus the predicted ones with either Equations 4-4 or 4-5. Since there is a point with a very different response time than the others, target T2 in blast 15/02 with a \( t_{\text{resp}} \) of 92 ms (see also Figure 4-4), an influential analysis is made. The influence is a statistical that measures the effect of each observation in the estimation of the coefficients of the model. The influence of an average point is 0.17 with either Equations 4-4 or 4-5. The target T2 with \( t_{\text{resp}} \) = 92 ms is an influential point, with more than three and five times, 0.79 and 0.88, the average influence in the coefficients of Equations 4.4 and 4-5 respectively. This means that more data with high response time is required, i.e. more measurements with targets in front of the stemming. For the moment, that measurement is examined carefully for determining how the model would change if it would be not present. The results of the non linear regression fitting of Equations 4-4 and 4-5 when target T2 used in blast 15/02 (\( t_{\text{resp}} \) = 92
ms) is rejected are given in italics in Tables 4-10 and 4-11. The correlation factor decreases to 0.71 for Equation 4-4 and 0.68 for Equation 4-5 respectively. The fitting coefficients are very similar than when T2 is considered in both Equations, but the asymptotic interval relative of the $A$ coefficient of Equation 4-5 includes zero.

Figure 4-5. Measured $t_{\text{resp}}$ vs. predicted ones with Equations 4-4 and 4-5

4.3. Face movement profiles

4.3.1. Background

The face movement profiles are a useful tool for understanding the rock movement. The influence of the mechanical and geotechnical properties of the rock in the rock movement are shown in Figure 4-6 for vertical benches in which the burden along the blasthole is constant (Chiapetta & Mammele, 1987). When the rock is very competent, brittle and with few joints spaced greater than the burden or the spacing between blastholes, all points of the rock mass in front of the explosive column are moving with a uniform velocity, see Figure 4-6a. Conversely, for soft and highly fissured rocks as coal or certain sedimentary rocks the bench is bowed outwards, and flexural rupture will be produced, Figure 4-6b. In this case, the greatest displacement and velocity occur near the centre of the explosive column, and for the same instant the face velocity decreases as the distance to the crest and toe decreases; the smallest displacements occur in the toe and in the crest of the bench.

In blasts made in non-competent rock with vertical faces where subdrill is used, the face movement is similar to the one depicted in Figure 4-6b. But due to the crater effect caused by the explosive of the subdrilled part of the hole, the toe of the bench is displaced upwards faster and with a greater angle with the horizontal than when no subdrill is used, see Figure 4-7 (Chiappetta & Mammele 1987).
The use of a high energy explosive in the bottom part of the explosive column in a blast with a toe burden greater than the crest burden, see Figure 4-8, implies greater displacements in the toe of the bench than in the case of not using a bottom charge (Chiappetta & Mammele 1987). High-energy content explosives will produce for the same burden higher rock displacements.

Figure 4-6. (a) Face movement for competent rocks and (b) Face movement for non-competent rocks (Chiapetta & Mammele, 1987).

Figure 4-7. Influence of the subdrilling in the rock movement. (Chiappetta & Mammele 1987)
Figure 4-8. Rock movement (a) when the same explosive is used in the entire explosive column and (b) when a bottom charge is used for a greater burden in the toe (Chiappetta & Mammele 1987).

Face profiles for blastholes with different diameters were not found in the literature. But, according to Chiappetta (1983) greater initial displacements are obtained with greater diameters for the same burden and explosive.

4.3.2. Calculation and analysis

Each target position tracked is associated to a time from the video images. The targets are tracked as far away in their trajectory as possible, before they disappear within the cloud of broken rock and dust. This takes place from 527 ms to 1540 ms after the initiation time of the explosive column. From the position of the targets along the raw path and their associated time, it is possible to know from two to four points of the face profile at several moments (only in blasts 43/03 and 54/03 four points were obtained).

The face profiles several moments after the initiation time of the explosive for the holes behind the targets are drawn in Figures 4-9, 4-10 and 4-11 for the pair of blasts 29/02-37/02, 43/03-50/03 and 54/03-58/03 respectively. In blast 43/03 the relative time of each profile is not given since the initiation time of the explosive is unknown. The profiles in blasts 15/02 and 45/03 are not included since the position of the targets with respect to the bench face is not feasible (see Section 4.1.1 above). The stemming area and the bottom explosive charge are differentiated in Figures 4-9 to 4-11; in blasts 29/02 and 37/02 there was not bottom charge properly, since the gelatine cartridges were combined with Alnafo instead of being concentrated in the bottom.

The part of the profiles between the targets is drawn with a continuous line while the upper and lower parts of the profiles are drawn with dashed line, as no information exists above or below the top and bottom targets. The continuity of the face profile is tried to keep along the highwall. Looking for better results the lower part of the profiles is completed qualitatively using the reference given by the bottom control point. Although the procedure followed is not accurate, it gives approximate information of how the face is moving.
Figure 4-9. Face profiles (a) at 184, 412, 720 and 912 ms in blast 29/02 and (b) at 167, 245 and 318 ms in blast 37/02 (the stemming is marked with light colour).

Figure 4-10. Face profiles (a) at unknown times in blast 43/03 and (b) at 167, 245 and 318 ms in blast 50/03 (the stemming is marked with light colour and the bottom charge with dark).

Figure 4-11. Face profiles (a) at 125, 360, 600, 750 and 1200 ms in blast 54/03 and (b) 150, 300, 600 and 900 ms in blast 58/03 (the stemming is marked with light colour and the bottom charge with dark).
Most of the face profiles plotted show a differential movement of the blasted rock; in blast 54/03 (Figure 4-11a) the displacement seems however, to be fairly uniform. The following conclusions can be extracted for the bench movement:

- In the stemming area (top of the bench). On blast 37/02, see Figure 4-9b, the continuity of the face profile is broken, between 712 and 1388 ms. The top target has not been displaced as much as the bottom target has, as there is not explosive behind that target, although it is in the area of influence of the explosive (vertical crater effect). The material placed in front of the stemming seems to be thrown closer; its range decreases as the distance to the top part of the explosive increases. This is also shown in blast 54/03, see Figure 4-11a, in which the target T4 filled matches T3 at 1200 ms. In none of the profiles plotted in Figure 4-10a, the rock in front of the top part of the blasthole has not moved yet. Therefore, it seems that the rock mass in front of the stemming falls down with little horizontal displacement when the rock mass that support it has been displaced enough distance.

- In the toe of the bench (bottom of the bench). There is evidence of good toe movement due to the effect of the bottom charge in the profiles of blast 58/03 drawn in Figure 5-11b. In this case, both the subdrill length and the bottom charge length are appropriate. Although no other measurements were made in this area, it is apparent from Figure 4-9b (37/02) that only a small part of the explosive placed in the subdrilled zone of the hole behind the targets contributes to the displacement of the toe of the bench. According to Langefors & Kihlström (1963) the subdrill length must be 0.3 times the burden, which for a burden of 5.7 m is only 1.7 m. The usefulness of the bottom charge in Figure 4-11a (blast 54/03) is very small due to its position respect the grade level of the profile. Care should be taken in order to have the right subdrill and bottom charge lengths; the aim is to achieve the situations in Figures 4-10 and 4-11b.

4.4. Face initial velocity

Chiappetta et al (1983) fit the initial velocity in ft/s, $V_0$, measured in dolomite, granite and iron ore production blasts with the following formula:

$$V_0 = k \left( \frac{B}{E_E} \right)^d$$

Where: $B$ is the burden in feet, $E_E$ is the explosive energy in kcal per feet of explosive column and $k$ and $d$ are fitting constants. The $k$ values obtained by Chiappetta for those blasts range between 42 and 14.5 (higher and lower bounds at a 95 % confidence level) and $d$ is 1.17.

Chiappetta & Mammele (1987) state that the initial velocity of the fragments in the face (flyrock is not considered) for a wide range of materials, burdens and explosives range between 2 m/s - 40 m/s (1987).

4.4.1. Calculation and analysis

Chiappetta et al. (1983) suggests that only displacements of less than 20 feet (6 m) the displacement can be fitted with a linear function:

$$\Delta S = Kt + \Delta S_0$$

(4-7)
Where:

- $\Delta S$ is the displacement of the target from its initial position $(x_0, y_0)$ at an instant $t$ and is given by:
  
  $$\Delta S = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2}$$

  being $x_t$ and $y_t$ are the target co-ordinates at instant $t$.

- $K$ and $\Delta S_0$ are fitting constants. The term $\Delta S_0$ appears because time zero of the measurements is not that at which movement starts and $K$ is an approximation of the initial velocity of the target. From Equation 4-7 it is possible to determine the response time, as was explained above in 4.2.1, making $\Delta S$ equal to zero.

As Equation 4-7 is physically far too simplistic, a trajectory model has been used to obtain the initial velocity. Aerodynamic drag does not play an important role in the rock flight, but it does to some extent in the targets, due to their low density (wood), large surface and flat shape. Figure 4-12 shows the forces acting on the gravity centre of the target, where: $V$ is the velocity of the target at time $t$; $x$ and $y$ are respectively the components of $V$ in the $X$ and $Y$ axis; $\theta$ is the pitch angle; $F_D$ is the aerodynamic drag; $m$ is the mass of the target and $g$ is the acceleration of gravity. The following geometrical relations apply:

$$\cos \theta = \frac{x}{V} = \frac{x}{\sqrt{x^2 + y^2}}, \quad \sin \theta = \frac{y}{V} = \frac{y}{\sqrt{x^2 + y^2}}$$

(4-8)

Aerodynamic drag is opposite to the movement of the target and is given by:

$$F_D = \frac{1}{2} \rho_a V^2 C_D S$$

Where $\rho_a$ is the air density (1.2 kg/m$^3$), $C_D$ is the drag coefficient (1.28 for a plate) and $S$ is the surface of the cross section of the object.

Figure 4-12. Forces acting on the gravity centre of the target.
Applying Newton’s second law to the system in Figure 4-12:

\[ m\ddot{x} = -\frac{1}{2} \rho V^2 C_D S \cos \theta \]  
\[ \text{ (4-9) } \]

\[ m\ddot{y} = -mg - \frac{1}{2} \rho V^2 C_D S \sin \theta \]  
\[ \text{ (4-10) } \]

Replacing Equation 4-8 in 4-9 and 4-10:

\[ x = \frac{1}{2} \frac{\rho C_D S}{m} \sqrt{\frac{2}{x+y}} \]  
\[ \text{ (4-11) } \]

\[ y = -g - \frac{1}{2} \frac{\rho C_D S}{m} \sqrt{\frac{2}{x+y}} \]  
\[ \text{ (4-12) } \]

Equations 4-11 and 4-12 form a system of second order, ordinary differential equations that can be easily solved for \( x(t) \) and \( y(t) \). The trajectory thus obtained is adjusted to the measured one by means of an iterative process in which the two components of the initial velocity of the targets are changed until a best fit is obtained. This way the whole trajectory tracked is used instead of only the initial part as it occurs with Equation 4-7, where larger errors can be made.

Figure 4-13 shows both the path given by Motion Tracker 2D and the trajectory model for the blasts analysed. The target T3 in blast 50/03 could only be tracked in two different positions than the initial, which prevents to obtain its initial velocity. The errors committed tracking the targets in the recorded images lead sometimes to paths that are not smooth with some irregularities, such as the T2 targets in blasts 29/02 and 43/03. However, in most of the cases, the simulated path matches well the measured one; only in blast 54/03 the measured trajectory of T1 seems to be linear. For T4 and T3 targets in blast 43/03, the tracked positions correspond to the raising part of the trajectory and the actual initial velocity is probably higher than the one calculated.

Table 4-12 lists the initial velocity, \( V_0 \), and the pitch angle of the targets calculated with the trajectory model.

<table>
<thead>
<tr>
<th>Blast No.</th>
<th>Target-T1</th>
<th>Target-T2</th>
<th>Target-T3</th>
<th>Target-T4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( V_0, \text{ m/s} )</td>
<td>Pitch angle</td>
<td>( V_0, \text{ m/s} )</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>15/02</td>
<td>8.3</td>
<td>12°</td>
<td>9.8</td>
<td>10°</td>
</tr>
<tr>
<td>29/02</td>
<td>15.9</td>
<td>17°</td>
<td>10.3</td>
<td>21°</td>
</tr>
<tr>
<td>37/02</td>
<td>12.4</td>
<td>56°</td>
<td>6.5</td>
<td>15°</td>
</tr>
<tr>
<td>43/03</td>
<td>5.2</td>
<td>40°</td>
<td>9.6</td>
<td>27°</td>
</tr>
<tr>
<td>45/03</td>
<td>-</td>
<td>-</td>
<td>9.2</td>
<td>27°</td>
</tr>
<tr>
<td>50/03</td>
<td>10</td>
<td>4°</td>
<td>9.8</td>
<td>7°</td>
</tr>
<tr>
<td>54/03</td>
<td>8.8</td>
<td>36°</td>
<td>8.7</td>
<td>37°</td>
</tr>
<tr>
<td>58/03</td>
<td>13</td>
<td>28°</td>
<td>16.1</td>
<td>19°</td>
</tr>
</tbody>
</table>

Notes: \( V_0 \) is the initial velocity.

A correlation analysis is made with the parameters included in Equation 4-6: \( V_0, B_t, \) and \( E_E, E_{E_0} = (Q_0), E_w(l_b-l_i). \) The correlation coefficients are shown in Table 4-13. In the 21 cases considered the influence of the burden at the target level is negligible and the correlation between \( E_E \) and \( V_0 \) is very weak. Additionally, the p-value is above 0.05 in all the relations,
which indicates a low statistical importance of the estimated correlations. All together, the results show a poor ability of Equation 4-6 for predicting the initial velocity at El Alto blasts.

The results above are striking, especially in what respects to the relation between $V_0$ and the burden. The initial velocities of 21 targets are plotted as a function of the $B$ (average burden of the blasthole behind the targets) and $B_{ti}$ in Figure 4-14; the data is differentiated according to the explosive type used. $V_0$ seems to decrease as $B$ increases, although the scatter is very large. The situation is worst as Table 4-13 shows in the $V_0$ and $B_{ti}$ plot if all the data is considered together ($Alnafo$ and high density $Alnafo$ blasts). A decrease trend can, however, be extracted if only high density $Alnafo$ blasts are considered.

Figure 4-15 shows the initial velocity of the target versus the ratio of the target height to the bench height for the blasts in which at least three targets can be tracked (43/03; 54/03 and 58/03).

Table 4-13. Correlation matrix for the parameters included in Equation 4-6

<table>
<thead>
<tr>
<th></th>
<th>$B_{ti}$</th>
<th>$E_E$</th>
<th>$V_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_0$</td>
<td>1.00</td>
<td>-0.12</td>
<td>0.27</td>
</tr>
<tr>
<td>$B_{ti}$</td>
<td>1.00</td>
<td>0.19</td>
<td>1.00</td>
</tr>
<tr>
<td>$E_E$</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the respective burden at each target, $B_i$, is given close to each point. The average burden for each blasthole is also given. Higher velocities are obtained for the blastholes with smaller average burdens and when high density Alnafo is used. Different velocities are obtained for targets with similar $B_i$, which perhaps is the source of the scatter in Figure 4-14b and the low correlation coefficient shown in Table 4-13. The initial velocity of the rock in Figure 4-15 seems to be a quadratic function of the target height; the height of the maximum velocity varies from blast to blast. The results obtained by Whitaker et al. (2001) in a Hawaiian limestone quarry for single ANFO and emulsion shots are plotted in Figure 4-16; the respective $B_i$ is given next to each point. Two quadratic functions are plotted to the data in Figure 4-16 since the number of targets per shot is enough. The results are similar to the ones obtained at El Alto.

![Figure 4-14. Plots of initial velocity, $V_0$ vs. (a) $B$ and (b) $B_i$.](image)

A new correlation analysis has been carried out. But, this time considering more blasting parameters ($B_i$, $H_i$, $l_i$, $h$, $l_i/h$, $H_i/l_i$, $B$, $S/B$, $S/B_i$, $B_i/B$, $l_i$, $(Qe)_1$, $E_n$, $Q_e$, $q_T$, $q$, $D$, $t$, $t/B$, and $t/t_{	ext{resp}}$) that may affect $V_0$, together with the response of the rock and the initial velocity. This set of parameters is known for 15 targets used in six production blasts; the response time is unknown for the 4 targets employed in blast 43/03 and $t$ for the two targets used in blast 37/02.

Table 4-14 shows the correlation coefficients and the p-values below 0.05 for the relations marked in bold. The correlation factors between $V_0$ and $B_i$, $H_i$ and $B$ are low, -0.21, -0.45 and -0.42 respectively. The ratio $t/t_{	ext{resp}}$ has the strongest influence in $V_0$; the correlation coefficient is 0.86. It is, however, difficult to explain that $V_0$ increases as the cooperation between adjacent blastholes is reduced ($t>>t_{	ext{resp}}$). A possible explanation for this may come from the fact that when the cooperation between blastholes is promoted ($t<<t_{	ext{resp}}$), the rock movement is complicated by the adjacent rock masses in motion.

The ratio $t/B_i$ has also a significant correlation (0.57) with $V_0$. The correlation analysis shows also a fairly good correlation coefficient of 0.64 between $V_0$ and the stemming length, $l_i$. This however lacks of physical sense. The correlation coefficients between the initial velocity and $l_i$, $h$, $(Qe)_1$, $Q_e$, $(Qe)_i$, and $q$ are about 0.42. Finally, it can be hardly explained that $Q_e$ and $(Qe)_i$ are negatively correlated with the initial velocity.
4 Measurement and Analysis of the Bench Face Movement

Figures 4-15. $V_0$ vs. the target height to the bench height ratio in blasts 43/03, 54/03 and 58/03 (HdAlf and Alf means high density Alnaf and Alnaf, respectively)

Figures 4-16. $V_0$ vs. the target height to the bench height ratio in a Hawaiian limestone quarry from data in Whitaker et al. (2001)
Table 4-14. Correlation matrix of initial velocity, response time and blasting parameters for 18 sets of data

<table>
<thead>
<tr>
<th></th>
<th>(Vo)</th>
<th>(t_{\text{resp}})</th>
<th>(B_{ti})</th>
<th>(H_{ti})</th>
<th>(l_{h})</th>
<th>(H)</th>
<th>(l_{h}/H)</th>
<th>(H_{l}/H)</th>
<th>(B)</th>
<th>(S/B)</th>
<th>(S/B_{ti})</th>
<th>(B_{ti}/B)</th>
<th>(l_{s})</th>
<th>((Q_{e})_{t})</th>
<th>(E_{w})</th>
<th>(Q_{e})</th>
<th>(q_{r})</th>
<th>(q)</th>
<th>(D)</th>
<th>(t)</th>
<th>(t/B_{ti})</th>
<th>(t/t_{\text{resp}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Vo)</td>
<td>1.00</td>
<td>-0.46</td>
<td>-0.21</td>
<td>-0.45</td>
<td>-0.49</td>
<td>-0.33</td>
<td>-0.36</td>
<td>-0.40</td>
<td>-0.42</td>
<td>0.29</td>
<td>0.09</td>
<td>0.21</td>
<td><strong>0.64</strong></td>
<td>-0.42</td>
<td>0.23</td>
<td>-0.41</td>
<td>0.01</td>
<td>0.41</td>
<td>-0.18</td>
<td>0.42</td>
<td><strong>0.57</strong></td>
<td><strong>0.86</strong></td>
</tr>
<tr>
<td>(t_{\text{resp}})</td>
<td>1.00</td>
<td><strong>0.73</strong></td>
<td><strong>0.71</strong></td>
<td><strong>0.69</strong></td>
<td><strong>0.66</strong></td>
<td><strong>0.57</strong></td>
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<td><strong>0.68</strong></td>
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<td>-0.35</td>
<td><strong>0.51</strong></td>
<td>-0.20</td>
<td>0.37</td>
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<td>0.14</td>
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<td><strong>0.59</strong></td>
<td>(t_{\text{resp}})</td>
</tr>
<tr>
<td>(B_{ti})</td>
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<td><strong>0.80</strong></td>
<td><strong>0.79</strong></td>
<td><strong>0.66</strong></td>
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<td>-0.82</td>
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<td><strong>0.67</strong></td>
<td>-0.42</td>
<td><strong>0.77</strong></td>
<td>0.27</td>
<td><strong>0.71</strong></td>
<td>-0.02</td>
<td>-0.62</td>
<td>-0.32</td>
<td>0.32</td>
<td>0.00</td>
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<td>(B_{ti})</td>
</tr>
<tr>
<td>(H_{ti})</td>
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<td><strong>0.89</strong></td>
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<td><strong>0.90</strong></td>
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<td><strong>0.76</strong></td>
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<td>(H_{ti})</td>
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<td>(l_{h})</td>
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<td>-0.78</td>
<td><strong>0.85</strong></td>
<td>0.04</td>
<td><strong>0.77</strong></td>
<td>-0.06</td>
<td><strong>0.85</strong></td>
<td>-0.12</td>
<td>-0.02</td>
<td>-0.31</td>
<td>-0.47</td>
<td>(l_{h})</td>
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</tr>
<tr>
<td>(H)</td>
<td>1.00</td>
<td>0.04</td>
<td>0.21</td>
<td>0.65</td>
<td>-0.56</td>
<td>-0.58</td>
<td>0.32</td>
<td>-0.42</td>
<td><strong>0.85</strong></td>
<td>0.32</td>
<td><strong>0.63</strong></td>
<td><strong>0.56</strong></td>
<td>-0.43</td>
<td>-0.42</td>
<td>0.04</td>
<td>-0.24</td>
<td>-0.30</td>
<td>(H)</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>(l_{h}/H)</td>
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<td>0.98</td>
<td>0.29</td>
<td>-0.36</td>
<td>-0.05</td>
<td>-0.35</td>
<td>0.03</td>
<td>0.06</td>
<td>-0.21</td>
<td>0.03</td>
<td>-0.30</td>
<td>-0.26</td>
<td>0.21</td>
<td>0.02</td>
<td>0.01</td>
<td>-0.32</td>
<td>(l_{h}/H)</td>
<td></td>
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</tr>
<tr>
<td>(H_{l}/H)</td>
<td>1.00</td>
<td>0.40</td>
<td>-0.46</td>
<td>-0.16</td>
<td>-0.26</td>
<td>-0.06</td>
<td>0.20</td>
<td>-0.17</td>
<td>0.14</td>
<td>-0.21</td>
<td>-0.35</td>
<td>0.15</td>
<td>0.01</td>
<td>-0.05</td>
<td>-0.38</td>
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<td>-0.83</td>
<td>0.18</td>
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<td><strong>0.85</strong></td>
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4.5. Conclusions

Quantitative data on the face rock movement under current production techniques used in El Alto are provided. Response time values are given for 18 targets used in seven blasts, whereas initial velocities of the rock are obtained for 21 targets in eight blasts. Their statistics are 34 sd. 18 ms and 10 sd. 3 m/s, which are within the range given by Chiappetta and Mammele (1987) for a variety of materials burdens and explosives. The faces profiles several moments after the initiation of the blasthole are plotted for six blasts.

The information in the literature serves as a starting point for analysing the rock face movement. The Oñederra & Esen response time prediction formula has been applied, but the results of the non-linear regression analysis show that it is not valid for El Alto’s data. A simple formula for predicting the response time of the rock in El Alto at a specific height from the grade as function of the burden and linear charge density is developed. Both burden and blasthole diameter (it is included in the linear explosive density) appear in the Oñederra & Esen formula.

Face movement profiles show a differential movement of the blasted rock along the bench height, in fact the initial velocity seems to be a quadratic function of the target height to bench height ratio. At the same time the initial velocity of the rock may be affected by blasting parameters like burden, explosive energy, in row delay, etc. A correlation analysis shows that the parameters (burden and explosive energy per unit of length of explosive column) involved in the formula proposed by Chiappetta et al. (1983) are poorly correlated (<0.4) with the initial velocity and that only the ratios that contain the in-row delay, $t/t_{\text{resp}}$ (ratio of the in row delay to the response time), and $t/B_t$ (ratio of in row delay to burden at target level) show the strongest correlations, 0.9 and 0.6. Nevertheless, the difficulty for explaining the influence of $t/t_{\text{resp}}$ in the initial velocity (it increases when cooperation between blastholes is reduced) leads to consider the results with caution and leaves aside the development of a formula for predicting the initial velocity.
Chapter 5

MEASUREMENT AND ANALYSIS OF THE SEISMIC FIELD

The blasting activity is relatively close to the cement plant in some areas of La Concha pit, about 350 m. The aim of this chapter is to understand the vibration phenomena in the particular geology of El Alto and assess the peak particle velocities and dominant frequencies.

The seismic field radiated by two confined blast holes (CB1 and CB2) and one single shot (SB1) were recorded over the clayish-marl overburden in the top of the bench; in one of these tests the propagation velocity of the P-waves was measured. Additionally, the seismic field was measured in 23 productions blasts (15/02, 29/02, 37/02, 37/03, 38/03, 43/03, 45/03, 50/03, 54/03, 58/03, 78/03, 89/03, 96/03, 06/04, 08/04, 13/04, 21/04, 26/04, 31/04, 35/04, 42/04, 44/04 and 49/04) at several distances from the blast in both the top and bottom levels of the quarry.

5.1. Vibration measurements

The ISEE recommendations (ISEE, 1998) were followed in order to assure a good coupling of the sensors with the terrain. The sensors were placed over the ground surface and spiked to the terrain whenever it was possible. The used devices were completely covered with a bag with sand when they were at distances to the blast smaller than 75 m. Despite of this cautions, eleven measurements are rejected due to the high amplitude, long period, damped oscillation apparent on the seismograms, typical of a bad coupling.
5.1.1. Features of the equipment and vibration parameters monitored

The particle oscillations were measured with digital seismographs, Multiseis Plus (Vibra-Tech, 2001). These seismographs measure the three components (vertical, $v$, transversal, $t$, and longitudinal, $l$) of the particle velocity. The peak particle velocities of each component, $ppv_i$, and the peak sum particle velocity (peak of the modulus of the velocity), $ppv_{sum}$ of the good quality seismograms are given in Appendix B.

The seismographs have a flat response curve in the range of frequencies from 2 to 300 Hz. Probably, the Multiseis-Plus incorporate a filtering system for extend somewhat the bottom range of the flat part of the response curve from the natural frequencies of electromagnetic sensors, 6 to 20 Hz. The Fast Fourier Transform is used to obtain the dominant frequencies for each component, $FFT_i$, which are shown in Appendix B.

The sample rate used in all the records was the smallest one available, 1024 samples per second. The range of the seismographs is up to 254 mm/s, their resolution is between 0.127 to 0.000625 mm/s and the accuracy of the measurements is 3% at 15 Hz.

5.1.2. Sensors location

In the recorded seismograms it is not possible to differentiate the contribution of each blasthole to the signal recorded. As an example, Figure 5-1 shows the signal obtained with the seismograph BD7101 in blast 43/03 with ten blastholes.

Figure 5-1. Seismogram recorded by BD7640 in blast 43/03
The overlapping between the signal coming from each blasthole involves that the distance from the sensor to the blast, \( r \), may be calculated as follows:

\[
 r = \frac{\sum_{j=1}^{N} (d_j^2 + z_j^2)^{\frac{1}{2}}}{N}
\]

(5-1)

Where, \( N \) are the number of blastholes; \( d_j \) is the distance between the blasthole collar \( j \) and the sensor, and \( z_j \) is depth of the gravity center of the charge in blasthole \( j \) (only one gravity center is considered in each blasthole). In the bottom level the term \( z \) is rejected.

In the single blasthole shots, the sensors were placed only in the top level of the bench at different distances to the blast.

Normally, four devices were placed around the block to be blasted in the short distance range to the blast; two sensors were located in the top and grade levels (an exception was the blast 37/03 in which all the sensors were placed in the top level). These measurements are further used for calculating the seismic energy in Chapter 7. Hence, the seismographs were placed at similar \( r \) and \( d_{\text{min}} \) (minimum distance from a sensor to a blasthole) distances in each blast; the mean and standard deviation of \( r \) and \( d_{\text{min}} \) in 70 good quality measurements made in 23 production blasts were respectively 66.0 sd. 7.9 m and 51.2 sd. 9.1 m. The sensors in the bottom level were placed slightly farther due to the breadth of the muckpile and above a thin limestone layer, which is perhaps is affected by previous blasts.

A longer distance range to the blast was covered in some blasts; a variable number of seismographs were aligned with hemispherical silo (domo) of the cement plant. A total number of 26 good quality seismograms were obtained in 15 blasts (15/02, 37/03, 38/03, 43/03, 50/03, 58/03, 78/03, 13/04, 21/04, 26/04, 31/04, 35/04, 42/04, 44/04 and 49/04) at distances to the blast from 75 to 512 m.

As an example, Figure 5-2 shows the sensor locations in blast 21/04, where four seismographs were located around the blast and two more in the direction to the domo.

---

Figure 5-2. Seismographs location in blast 21/04
5.2. Measurement of the P-wave velocity

The Uphole Surveying Technique (Telford et al., 1990) provides a background for measuring the propagation velocities of the P-waves in the overburden and in the underlying strata, limestone. Basically, the Uphole Surveying Technique requires a complete spread of sensors placed in the ground including a sensor within 3 m of the hole and a series charges shot at various depths from the bottom of the hole up to immediately below the ground surface.

The confined single blasthole shot CB2 was specifically designed for measuring P-wave velocity; the drilling and charging characteristics of this test, which are shown in Appendix A, were aiming at:

- Drill a borehole deeper than the overburden thickness.
- Minimize the effect of the greda in the transmission of the elastic waves by leaving 7.5 m between the bottom of the blasthole and the greda.
- Concentrate the explosive charge in order to assimilate the explosive to a point source and ensure that the detonation occurs entirely in the limestone. The charge length was 5.8 m; the use of smaller explosive amount was rejected in order to assure enough vibrations levels in the measurement locations. The anfo column was initiated with a cartridge of Goma 2ECO (65 mm of diameter).

The seismic waves radiated by the explosive in CB2 were recorded with a three component sensor (transversal, vertical and longitudinal) and a longitudinal sensor respectively placed at 50 m and 100 m away of the hole over the same straight line. Both sensors were connected to a MREL Microtrap VOD recorder with four channels. A sample rate of 1 MHz was used.

The velocity of the P-waves in the limestone and also in the overburden is calculated from the first arrivals to the sensors according to the Law of Snell. Probably, diffraction will occur, but even if this would happen the first arrival will correspond to the path given by the Law of Snell. Figure 5-3 shows the sensors set-up with respect to the blasthole and the paths of the direct P-waves.

The procedure for calculating the P-waves velocity in the limestone, \((c_p)_l\), and in the overburden, \((c_p)_{ob}\), is as follows. The angle with the normal to the contact limestone-overburden, \(\alpha_{ob}\) (see Figure 5-3), is:

\[
\alpha_{ob} = \sin^{-1}\left(\frac{(c_p)_{ob}}{(c_p)_l} \sin \alpha_i\right)
\]  

(5-2)

Where: \(\alpha_i\) is the angle of the incident P-waves with the normal to the contact between the limestone and the overburden.

Additionally, both the arrival time of the direct P-waves to a certain point of the ground’s surface and the horizontal distance from that point to the blasthole are:

\[
t = \frac{x_i}{(c_p)_l} + \frac{x_{ob}}{(c_p)_{ob}} = \frac{h_{l}}{(c_p)_l} \frac{1}{\cos \alpha_i} + \frac{h_{ob}}{(c_p)_{ob}} \frac{1}{\cos \alpha_{ob}}
\]  

(5-3)

\[
d = d_i + d_{ob} = h_{l} \tan \alpha_i + h_{ob} \tan \alpha_{ob}
\]  

(5-4)
Where all the geometrical parameters are shown in Figure 5-3:
- \( x_l \) and \( x_{ob} \) are the length of the direct P-waves over the ray path in the limestone and overburden respectively.
- \( h_l \) is the depth of the gravity center of the charge measured from the contact between the limestone and the overburden.
- \( h_{ob} \) is the overburden thickness.
- \( d_l \) and \( d_{ob} \) are the horizontal distances travelled by the direct waves in the limestone and in the overburden respectively.

Using Equation 5-2 for replacing \( \alpha_2 \), in Equations 5-3 and 5-4:

\[
\begin{align*}
    t &= \frac{h_l}{(c_p)_l \cos \alpha_l} + \frac{h_{ob}}{(c_p)_{ob}} \frac{1}{\cos \left( \sin^{-1} \left( \frac{(c_p)_{ob}}{(c_p)_l} \sin \alpha_l \right) \right)} \quad (5-5) \\
    d &= h_l \tan \alpha_l + h_{ob} \tan \left[ \sin^{-1} \left( \frac{(c_p)_{ob}}{(c_p)_l} \sin \alpha_l \right) \right] \quad (5-6)
\end{align*}
\]

If \( h_l, h_{ob}, (c_p)_l \) and \( (c_p)_{ob} \) are known, both the arrival time and the horizontal distance are function of the angle of incidence, \( \alpha_l \). A graph \( d-t \) can be then obtained from Equations (5-5) and (5-6).

If \( h_l \) is equated to 4 m and \( h_{ob} \) to 9.6 m (both relative to the features of CB2) and if \( (c_p)_l \) is assumed to be 2500 m/s and \( (c_p)_{ob} \) of 1000 m/s the plot \( d-t \) shown in Figure 5-4 is obtained.

Figure 5-4 shows a straight line for horizontal distances to the blasthole collar greater than 10 m (the closest sensor was located at 50 m from the blasthole collar). The slope of this line is the inverse of the phase velocity of the P-waves in the limestone layers. The time of the first arrival to the sensors placed at 50 m and 100 m is 27 ms and 43.7 ms respectively, which results in a phase velocity in limestone of 2994 m/s. The value of \( (c_p)_{ob} \) is calculated next from an iterative process in which \( h_l, h_{ob}, (c_p)_l \), the arrival time of the direct P-wave to each sensor, \( t \) (27 ms and 43.7 ms) and the horizontal distance measured from the hole to each of them, \( d \) (50 m and 100 m) are known. Equations 5-5 and 5-6 are established for both sensors (two equations for each sensor) and an the value of \( (c_p)_{ob} \) is varied until the arrival time at each sensor matches the measured one. This results in s \( (c_p)_{ob} \) of 395 m/s, which is typical of a low-velocity layer.
5.3 Analysis of the frequency of the vibrations

Blair & Armstrong (1999) use the mean spectra of the longitudinal, transversal and vertical components of acceleration, velocity and displacement for controlling ground vibration in the frequency domain. This reduces the three spectra of the recorded vibration components to one and simplifies the analysis. The dominant frequencies of the mean spectrum of the transversal, vertical and longitudinal velocity components, $FFT_{av}$, for the good quality signals (the ones from well coupled seismographs to the ground) are listed in Appendix B.

5.3.1 Mean velocity spectra radiated by single blastholes

Figure 5-5 shows the relative mean velocity spectra of the longitudinal, vertical and transversal components of the measurements made in the confined single blastholes CB1 and CB2 and in the single production shot SB1. The respective $FFT_{av}$ values are plotted in Figure 5-6 versus the distance, $r$, to the shot.

Figure 5-5 shows that the single-hole signal is not rich in high frequencies, even at 50 m to the source. It also shows that the seismic energy is more concentrated around the dominant frequency as the distance to the shot increases, which means that the energy transmitted at greater frequencies than $FFT_{av}$ becomes smaller. Nevertheless, a clear trend can not be extracted between $FFT_{av}$ and $r$ in Figure 5-6. The scatter is high and $FFT_{av}$ varies from 8 to 4.8 Hz at 50 m to the shot (the shape of the mean velocity spectra are different even for the same shot at equal distances to the charge, see Figure 5-5d). Additionally, the influence of the blasthole confinement is not apparent.
Figure 5-5. Relative mean velocity spectra in (a) CB1; (b) CB2 and (c) SB shots and (d) in CB1, CB2 and SB at about 50 m to the shot.

Figure 5-6. FFT$_{av}$ in single blastholes shots vs. the distance to the shot.
The dominant frequencies registered in CB1, CB2 and SB1 are much smaller than those reported by Bollinger (1980) in a limestone quarry at a distance range up to 250 m: 50 Hz for P-waves, 80-100 Hz for S-waves and 25 Hz for Rayleigh waves. According to Siskind (2000), the low-velocity layers, such as the clayish marl overburden, would be the cause of these discrepancies as they are strong sources of surface waves that can be developed at relatively small distances, 30 m, to the blast. Dowding (2000) reports frequency ranges between 1-40 Hz and 10-100 Hz for soils thickness greater than 2 to 3 m and rock layers respectively. The fact is that the overburden amplifies either compression or shear waves at a frequency, \( f_R \), for strong propagation velocity contrasts (Bollinger, 1989 & Dowding, 2000):

\[
f_R = \frac{c}{4h_{vl}}
\]

Where \( c \) is the phase velocity of P or S-waves and \( h_{vl} \) is the thickness of the low velocity layer. Equation 5-7 shows how variations in the overburden thickness affect the waveform shape. An \( f_R \) of 25 Hz is obtained entering with \( (c_p)_{ob} = 395 \) m/s and \( h_{vl} = 4 \) m in Equation 5-7, whereas the recorded \( FFT_{av} \) varies from 8 to 4.8 Hz at 50 m to the source.

A first conclusion comes from the predominance of low frequencies in the seed signals, which prevents a strong increase of \( FFT_{av} \) in production blasts by changing the in-row delay.

5.3.2 Mean velocity spectra radiated by production blasts

The mean amplitude velocity spectra of the seismic field recorded by the four seismographs placed at about 65 m to the production blasts 29/02 (with non-electric caps and 67 ms in row delay), 96/03 (with EPDs and 67 ms) and 8/04 (EPDs and 17 ms) are shown in Figures 5-7, 5-8 and 5-9 respectively. The position of the sensors with respect to the blasts, are given in Figure 5-10. The position seems not to affect the frequency content in the spectra, as the dominant frequencies are about the same.

The frequency spectra are similar for an in-row delay of 67 ms with either pyrotechnic or electronic caps, see Figures 5-7 and 5-8. The seismic energy is, however, more concentrated in few peaks when EPDs caps are used. This agrees with the spectral banding shown by Blair and Armstrong (1999) in blasts with electronic detonators. In blasts with an in-row delay of 67 ms, i.e. blasts 29/02 and 96/03, there is a noticeable peak, frequently the greater one, at frequencies similar to the inverse of the nominal delay, 15 Hz. Sometimes, for the records made in the top level the peak at 30 Hz (a multiple of 1/67 ms) is the dominant one; see the spectra of sensor 7640 in blast 96/03 (Figure 5-8a).

In blasts with an in-row delay of 17 ms most of the energy is concentrated in a package of frequencies between 2 and 8 Hz as occurs in the single charges, see Figure 5-5. The \( FFT_{av} \) values do not correspond with the inverse of the nominal delay and there is not spectral banding even if EPDs are used.

Very little seismic energy is transmitted to the bottom level of the quarry at frequencies greater than 20 Hz, independently of the delay.
5 Measurement and Analysis of the Seismic Field

Figure 5-7. Relative mean velocity spectra in the (a) top level and (b) grade level of blast 29/02 (non electric detonators, 67 ms).

Figure 5-8. Relative mean velocity spectra in the (a) top level and (b) grade level of blast 96/03 (electronic detonators, 67 ms).

Figure 5-9. Relative mean velocity spectra recorded in the (a) top level and (b) grade level of blast 08/04 (electronic detonators, 17 ms)
The dominant frequencies, $FFT_{av}$, are plotted in Figure 5-11 versus the distance $r$ to the blast for the available measurements made in the top and bottom levels respectively; the data is differentiated according to the explosive, detonators and in-row delays used. Blast 37/02 is excluded since its delay is undetermined.

In the top level the $FFT_{av}$ values are packaged between 16 and 2 Hz (exception is one of the records made in blast 96/03 that has a $FFT_{av}$ of 30 Hz). With respect, to the grade level, the upper bound of the $FFT_{av}$ values is also 16 Hz, but the bottom range is slightly higher, about 5 Hz. The low $FFT_{av}$ values for the grade level measurements show that low frequencies waves, surfaces waves, are also predominant at relatively small distances to the blast. Perhaps, they are generated at the contact between limestone and *greda*.

Figure 5-11 shows that the dominant frequency level is constant at any distance to the blast when the same in-row delay is used. This occurs in both quarry levels for the distance range covered, 512 and 100 m in the top and bottom levels respectively (most of the data correspond to distances smaller than 200 m and 65 m in top and bottom levels respectively). The dominant frequency is about

- 14.6 Hz (14.7sd. 3.1 and 14.6sd. 1.1 Hz in the top and bottom levels respectively; the mean of the distances to the blast is 103 and 70.8 m respectively) for in row-delays of 67 ms.
- 4.9 Hz ( mean of the one and two measurements in the top and bottom levels respectively; the mean of the distances is 70 m ) for in row delays of 30 ms.
- 4.6 Hz (3.9sd.1.4 and 5.8sd 0.8 Hz in the top and bottom levels respectively; the mean of the distances to the blast is 113 and 67 m respectively) for in row delays of 17 ms.

In blast 15/02 with and in row delay of 84 ms and two decks delayed 50 ms, $FFT_{av}$ is 11 Hz (mean of 4 measurements made in the top level and one more in the grade level; the mean of the distances to the blast is 188 m).

From the data available (only for in-row delay of 67 ms), the explosive type does not affect to the measured $FFT_{av}$.
Figure 5-11. $FFT_{av}$ recorded in the (a) top and (b) bottom levels vs. $r$ in 22 several production blasts
5.3.3. Influence of the in-row delay in the mean velocity spectra of production blasts.

In production blasts, there is room for changing the respective $FFT_{av}$ when the in-row delay is larger than one fourth of the inverse of the dominant frequency of a single blasthole. However, this simple rule of thumb is valid for ground vibrations of the type of harmonic functions and when the commonality between the waveforms radiated by each blasthole is large.

The statistics of the dominant frequencies registered in CB1, CB2 and SB1 at 247 sd. 213 m are 5.8 sd. 1.5 Hz. The critical value is, then, 43 ms ($0.25/FFT_{av}$). Consequently, the 30 and 17 ms delays are in the constructive time area of El Alto’s long period waves. The use of decking in blast 15/02, leads to a combined delay between charges of 50 and 34 ms, the former is below one fourth of the dominant period.

Blair & Armstrong (1999) propose a Fourier model for predicting the frequency spectrum at a specific location. This model considers the delay sequence, the scatter in the planned sequence and the random fluctuations between the signatures radiated by each blasthole. The latter is a big novelty and for Blair & Armstrong (2001) it is mainly caused by the alteration of ground conditions during the blast rather than other factors such as the burden. It seems, however, that random fluctuations in vibrations between similarly charged blastholes are not significant in small blasts (Blair & Armstrong, 1999). In that work the blasts considered are considerably bigger than the ones in El Alto; the smallest one consists of two rows with 18 holes per row.

If random fluctuations are neglected, the mean amplitude spectrum of many Monte-Carlo simulations is given by the called pooled firing function, $P(f)$, (Blair & Armstrong, 1999):

$$P^2(f) = A_B^2(f) \left( \sum_{j=1}^{N} w_j e^{-2(\alpha \sigma_j f)^2} \cos 2\pi T_j f \right)^2 + \left( \sum_{j=1}^{N} w_j e^{-2(\alpha \sigma_j f)^2} \sin 2\pi T_j f \right)^2 \quad (5-8)$$

Where:

- $A_B(f)$ is the seed amplitude function radiated by a single blasthole in the frequency domain. It is not possible to create a frequency energy that does not exist in $A_B(f)$; obviously this is not a matter of delay. $A_B(f)$ is not available for the bottom level of the quarry.

- $w_j$ is a weighting factor for the blasthole $j$ ($j=1,.., N$) that accounts for the total charge mass used in the blasthole $j$, $[(Qe)_j]$, and the distance from the $j$-blasthole to the monitoring station, $d_j$, as follows:

$$w_j = k(Q_j^{0.5} / d_j)^\alpha S_j$$

Being:

- $k$ and $\alpha$ the site constants of the attenuation law of the peak vibration of single blastholes. For the case study $k$ and $\alpha$ are obtained from the data of the three single blastholes shots made and are respectively taken as 337.5 and -1.338, see Figure 5-16 below.

- $S_j$ the screening effect that accounts for the influence in vibrations of the holes fired previously that lie along the wave travel path between the current blasthole and the monitoring location. This is not the case of El Alto blasts, hence $S_j=1$.

- $\mu_j$ and $\sigma_j$ are the mean and standard deviation of the delay used in blasthole $j$. They depend on the features of the detonator and are unknown for either EPDs or non-electric caps.
-\( T_j \) is given by:
\[
T_j = \mu_j + d_j / c_p
\]

If EPDs caps are used, \( \sigma_j \approx 0 \) and Equation 5-8 becomes, then, in a Fourier equation:
\[
P^2(f) = A_0^2(f) \left[ \sum_{j=1}^{N} w_j \cos 2\pi T_j f \right]^2 + \left[ \sum_{j=1}^{N} w_j \sin 2\pi T_j f \right]^2 \quad (5-9)
\]

![Graphs showing modeled and measured spectra](image)

Figure. 5-12. Modeled with different \( A_d(f) \) functions and observed amplitude spectra for the monitoring station (a) 6783 and (b) 7640 used in blast 96/03.

Figures 5-12 and 5-13 show the measured and modeled spectra of frequencies in the monitoring stations placed in the top level in blasts 96/03 (seismographs 6783 and 7640) and 8/04 (sensors 7640 and 7101) respectively. The values of the parameters, \( d_j, Q_j, w_j, \mu_j \) and \( T_j \), introduced in Equation 5-9 for each case of study are shown in Appendix C. The five \( A_d(f) \) functions recorded at about 50 m of the single shots CB1, CB2 and SB1 are used since the sensors in blasts 96/03 and 8/04 were placed at a distance \( r \) to the blast ranged from 54 to 64 m. The sensors were
placed, however, at different locations in the single shots than in production blasts. For this reason some differences between the predicted and measured signals are expectable.

The peak of the inverse of the nominal delay, 67 ms, and the multiple ones are predicted for the sensors 6783 and 7640 used in the blast 96/03. However, the peak at 30 Hz has a relevant amplitude in the modeled signal, when the seed function labeled as 7102-CB2 is used (this is the unique function with a significant amount of energy at frequencies of 30 Hz). Unfortunately, the peak at 4 Hz is not modeled. The banding in the spectra is more apparent in the modeled spectra since the scatter of EPDs caps is considered as zero.

![Graph](image1)

**Figure. 5-13.** Modeled with different $A_0(f)$ functions and observed amplitude spectra for the monitoring station (a) 7640 and (b) 7101 used in blast 08/04.

In blast 08/04, there is not good agreement between the measured and the modeled signals for the sensor 7640, see Figure 5-13. Only the $FFT_{av}$ peak, which is close to the lower bound of the frequency response, 2 Hz, of the Vibra-Tech seismographs is roughly caught (except when the seed signal recorded by 7101 in CB2 is used). The situation is better for the other sensor, but again large discrepancies are obtained when the seed signal 7101-CB2 is used.
Figures 5-12 and 5-13 show that the reconciliation between the measured and modeled velocity spectra depends on the seed function considered. If the amount of energy transmitted at a specific frequency is negligible in the seed function, the predictions may not be improved by considering random fluctuations between the waves radiated by each blasthole.

The good performance of Equation 5-9 allows its use for analyzing the influence in $FFT_{av}$ of different in row delays than the 67, 30 and 17 ms used in the practice. The mean amplitude spectra recorded by sensor 6783 in blast 96/03 is modeled considering three different in-row delays, 30, 50 and 80 ms; delays larger than 80 ms are unpractical. The modeled spectra are plotted in Figure 5-14. The predicted $FFT_{av}$ values are 4.3 Hz for an in-row delay of 30 ms, 19.5 Hz for 50 ms and 12.3 Hz for 80 ms; they are similar to the inverse of the nominal delay except when the 30 ms delay is used. $FFT_{av}$ increases from the one obtained with 67 ms, 14.6 Hz, if the delay is decreased to 50 ms. The practical implications of this analysis are, for instance that the peak particle velocity allowed by the Spanish damage criteria shown in Figure 5-15 would increase from 20 to 26 mm/s for structures of the Group I (Industrial buildings) and from 9 to 11.6 mm/s for buildings of Group II (Offices, dwellings and commercial grounds).

![Graph showing modeled amplitude spectra](image-url)

Figure. 5-14. Modeled amplitude spectra for the monitoring station 6783 in blast 96/03 using the average seed amplitude function shown in Figure 5-5d
5.4. Analysis of the peak particle velocity

The peak sum particle velocity, $ppv_{sum}$, is used for analyzing the vibration level, instead of using the velocity component with a higher peak as the Spanish vibration standard (Norma UNE 22-381-93) suggests. This conservative approach agrees with the German vibration standard, DIN 4150 (Dowding, 2000).

The $ppv_{sum}$ recorded in the top level on the overburden in confined and unconfined single shots is plotted in Figure 5-16 versus the scaled distance, $r/(Q_{max})^{0.5}$ (ratio of the distance to the blast to the square root of the maximum charge mass per delay detonated within 8 ms). It shows that the particle velocity in the unconfined shots is lower at any scaled distance, although more measurements are required to draw a definitive conclusion.

Figure 5-16 shows the attenuation law for single charges over the clayish-marl overburden; the data for confined and unconfined shots is considered together due to the lack of data especially in unconfined shots.

The $ppv_{sum}$ recorded in the top and grade levels of production blasts is also plotted versus the scaled distance in Figure 5-16. In the bottom level, the data is packaged within a narrow scaled distance range due to the lack of data farther away than 100 m, since the main goal of all the seismic measurements at the bottom level was the calculation of the seismic energy at close distances. This prevents to obtain the respective attenuation law. Conversely, a wider range of scaled distances is covered in the top level. The $ppv_{sum}$ values in production blasts are similar to the ones for single shots. The attenuation law of $ppv_{sum}$ in the top level for production blasts is given in Figure 5-16. The respective correlation factor, 0.85 is slightly better than relative to the attenuation law of single shots. Therefore, both parameters $r$ and $Q_{max}$ explain well the variations of the peak sum particle velocity in the top level. However, if more data is available at large scaled distances, $R^2$ would be smaller (Dowding, 2000). For instance, the scatter in $ppv_{sum}$ at scaled distances of 4±1 m/kg$^{0.5}$, where large amount of data exist, is significant at
either top or bottom levels of the quarry. This is reasonable, since the data in Figure 5-16 include a large variety of differing circumstances that affect the peak particle velocity, such as the not accounted overburden thickness beneath the seismograph in the top level. The statistics for the $ppv_{sum}$ values at a scaled distance between 3 and 5 m/kg$^{0.5}$, are 94.7 sd 42.1 mm/s on the top level and 42.4 sd 17.4 on the bottom level. Probably, the different features of the clayish-marl overburden, a low velocity layer of variable thickness, and the narrow broken limestone layer over the greda are responsible of these differences.

![Figure 5-16. $ppv_{sum}$ from single shots and production blasts vs. scaled distance](image)

The $ppv_{sum}$ measurements made in the top level are plotted again versus the scaled distance in Figure 5-17, but this time the data is differentiated according to the explosive, detonator type and in-row delay used. Blasts with different column charge than Alnaflo lie about the attenuation law for the top level. The use of either EPDs or non electric detonators for the same in row delay (67 ms) does not affect to the $ppv_{sum}$ values. However, few data was registered for blasts delayed with non electric caps.
The use of 17 ms as in-row delay instead of 67 ms increases the peak particle velocity from 91.5 sd. 31.8 mm/s (from 22 values) to 136.9 sd. 22.7 mm/s (from 7 values) in a scaled distance between 3 and 5 m/kg^{0.5}. This occurs as a result of the constructive interference between the blasthole signatures radiated by each blasthole; the same conclusion was drawn in Section 5.3.3. On the other hand, no differences in the peak particle velocities can be seen at larger scale distances, 10-20 m/kg^{0.5}. The attenuation laws for the ppv_{sum} in blasts with in row delays of 67 and 17 ms are drawn in Figure 5-17. The attenuation law for the blasts delayed 67 ms is very close to the general attenuation law for the data recorded in the top level (also plotted in Figure 5-17); the correlation factor does not change. The attenuation law for blasts with a delay of 17
ms is above the general attenuation law at small scaled distances, while the opposite occurs at large scaled distances.

It seems then, that the in-row delay affects to the $ppv_{sum}$ only at close distances to the blast, let’s say 100 m. This is supported by the mean amplitude spectra for single shots plotted in Figure 5-5; in the near distance range, $r=50$ m, some of the seismic energy is transmitted to the field at greater frequencies (>12 Hz) than $FFT_{av}$, while for large distances to the blast the energy transmitted at those frequencies is negligible. The physical background for this is that P and S waves are still present in the seismogram in the near distance range, although they are not the predominant ones. Consequently, the time window for calculating the operating charge, $Q'_{max}$, at $r$ distances smaller than 100 m is increased to 43 ms (=0.25/$FFT_{av}$, where $FFT_{av}$ is the dominant frequency of the seismogram radiated by a single shot). The resulting $ppv_{sum} - r/Q'_{max}$ plot is shown in Figure 5-18. The attenuation law for all the data recorded in the top level is also given; the correlation factor increases to 0.89.

Finally, more data is required in order to determine the influence of the use of decking, although the references about its use confirm its positive effect for reducing vibrations.

![Figure 5-18. $ppv_{sum}$ from production blasts in the top level vs. scaled distance](image)

(The explosive mass considered is the operating charge in a 43 ms time window for $r<100$ m)

### 5.5. Conclusions

Vibration data (dominant frequencies for the longitudinal, vertical and transversal components, dominant frequencies of the mean velocity spectrum of the three recorded components, $FFT_{av}$, peak particle velocities for the three components and peak sum particle velocities, $ppv_{sum}$) are
provided for 108 good measurements; 12 correspond to three single-blasthole shots monitored in the top level of the quarry and the remaining 96 are relative to 23 production blasts monitored in the near distance range, at 66.0 sd. 7.9 m (31 records were obtained in the top level and 39 in the bottom level; these measurements are used to calculate the seismic energy) and at distances from 75 m to 512 m (26 measurements were made in the top level).

A P-wave velocity of 2994 m/s is measured in the field for limestone and 395 m/s (typical of low velocity layers) for clayish-marl overburden.

The $FFT_{av}$ values do not decrease as the distance to the source increases, although the energy transmitted at higher frequencies than $FFT_{av}$ becomes smaller. Other factors like the sensor position with respect to the initiation sense of the blast, the explosive type (only investigated in blasts delayed 67 ms) and the scatter of detonators seems to not affect $FFT_{av}$ (also analyzed for blasts with an in-row delay, $t$, of 67 ms).

The $FFT_{av}$ values from single-hole shots are 5.8 sd.1.5 Hz at 247 sd.213 m, which shows the predominance of surface waves in the seismogram at even close distances to the source, about 50 m. Production blasts measurements show that in-row delas of 17 and 30 ms leads to $FFT_{av}$ of about 4.5 Hz, whereas in row delays of 67 ms leads to greater $FFT_{av}$ than the natural frequency of the terrain, about 15 Hz (1000/67). These values are valid for both quarry levels at any distance to the blast (the distance range considered is up to 512 m).

The dominant frequency of the vibrations radiated by production blasts would be higher than the natural frequency of the terrain (5.8 Hz), when the in-row delay is larger than $\frac{1}{4}$ of the inverse of the dominant frequency of a single blasthole, 43 ms $(=250/5.8)$. The Blair & Armstrong (1999) model provides acceptable predictions of the $FFT_{av}$ values in the two blasts delayed with EPDs in which it has been applied; negligible random fluctuations between the signatures radiated by each blasthole were assumed. A further application of this model shows that the use of 50 ms (slightly above 50 ms) would lead to an $FFT_{av}$ of 19.5 Hz, which according to the Spanish damage criteria allows an increase of 30 % of the peak particle velocity.

In a conservative approach the vibration level is analyzed from $ppv_{sum}$. The influence of other blasting parameters different than the explosive mass may affect to the vibration level in the near distance range, i.e. let’s say 100 m, where there are still internal waves, although they are not predominant. Higher $ppv_{sum}$ are obtained in single confined shots than in unconfined, but more data is required to draw a definitive conclusion. The use of either EPDs or non electric caps does not affect to $ppv_{sum}$ for delays of 67 ms, but the use of 17 ms of in row delay instead of 67 ms increases 1.5 times $ppv_{sum}$ values. This seems however, to apply in the near field.$ppv_{sum}$ can be reasonably predicted in the top level from attenuation laws; since the correlation factors are at least 0.83. Attenuation laws are provided for single-blasthole shots and production blasts (for overall blasts, for blasts delayed 67 and 17 ms). An attenuation law for the bottom level is not defined.
Chapter 6

MEASUREMENT AND ANALYSIS OF FRAGMENTATION

Fragmentation is a key factor in order to control and minimize the loading, hauling, crushing, classification and processing costs in aggregate and industrial minerals operations. In El Alto, similarly than in other quarries, the fragment size of the blasted material affects to the consumables of loaders, shovels and trucks, and the loading cycle. Both $x_{80}$ and $x_{50}$ have an effect on energy consumption (according to Bond’s formula) and hammer mill throughput. The last is also affected by $P_{63 \text{ mm}}$ (fraction passing at 63 mm) since the fines material tend to block the screen, especially in rain conditions, due to caking; the overburden of clayish marl contributes to this blocking. This could be easily solved by removing the overburden; however this is not economically interesting, as the clayish marl is also used for the cement manufacture.

The aim of this chapter is to improve the overall operation at El Alto quarry. Basically, this requires: i) a definition of the problem, (ii) measuring fragmentation, (iii) analysis of the actual fragmentation data, (iv) decision about the main blasting parameters affecting fragmentation and (v) ability to implement the blasting changes decided.

A continuous fragmentation monitoring system was installed in El Alto quarry (Spain) as preliminary task for a long-term goal of developing a fragmentation prediction model for El Alto. The operation of the digital image analysis system is described. The calibration of the system, required in order to obtain moderately reliable fragmentation values, is done from muckpile sieving data by tuning the image analysis software settings so that the fragmentation curve measured matches as close as possible the sieving. The sieving data have also been used to extend the fragment size distribution curves measured to sizes below the system’s optical resolution and to process the results in terms of fragmented rock, discounting the material coming from the loose overburden ($natural \ fines$) that is blasted together with the fragmented rock. Despite of the unavoidable errors due to sampling, image processing and fines corrections, the system is sensitive to relative changes in fragmentation.

Fragmentation data for the 35 production blasts described in Chapter 3 are given. Since there is not a strong physical background behind any of the existing fragmentation prediction models, the performance of such models has been assessed from the blasting and fragmentation data of
35 production blasts. A detailed statistical analysis of the data has been made from which some correlations are derived.

6.1. Fragmentation measurement by image analysis

The evaluation of the fragmentation of the ROM by sieving is generally not possible due to its high cost and the disruption it causes in the production cycle. Several alternative procedures can be used to measure fragmentation, such as belt scale readings in selected positions of the processing plant, production data of the various size fractions or digital image software. The selection of the fragmentation monitoring system is to a large extent site-dependent and the features of the crushing circuit must be considered.

Digital image software was developed through the 1990s and at present it is a worldwide accepted tool in the mining and mineral processing industries; *Fragscan* (Schleifer & Tessier, 2000), *Split*® (Split Engineering, 2001), *PowersSieve*®, and *Wipfrag* (Maerz & Palangio, 1999) are the most significant. Several references of them can be found in the literature. Their main advantage is that they can be used on a continuous basis without affecting the production cycle, which makes them the only practical tool for evaluating fragmentation of the run of mine, despite of their inherent limitations (thoroughly discussed by Maerz & Zhou 1998). Some of the errors of image analysis system can be overcome to some extent by means of an on-site-calibration from sieving data, as Latham et al. (2003) suggest for industrial applications. This is not a difficult task for material in a conveyor belt but is problematic for the run of mine in a muckpile. Ouchterlony (2003) makes an excellent revision of digital image analysis methods.

6.2. Description of the measurement system

Fragmentation measurements may be done in a variety of locations depending on the intended use of the fragmentation control. If the purpose is blasting control, measurements in the processing plant, after the primary crusher, make it difficult to relate with the fragmentation in the muckpile and cannot be used to assess the influence of blast design on fragmentation. Given this, a suitable option is to measure fragmentation in the hopper of the primary crusher. Other convenient measuring position before the primary crusher is on the top of the trucks, but this was thought to be less appropriate than the hopper, since the fines are more segregated during the truck run from the pit to the mill. In both cases, fragments larger than about 1 m³ are excluded from the fragmentation measurements, as they are not hauled to the crusher until they have been secondarily fragmented. Direct measurements on the muckpile were found to be impossible to carry out on a systematic basis.

The monitoring system installed at El Alto is formed by a video camera (able to take an image of moving material), a trigger for the camera, a set of lights, a PC installed in the control room of the crushing station and a connection between the camera and the PC. The camera is above the primary crusher’s hopper, 6.5 m from the surface of the material to be analyzed, when the bin is full; the set-up is shown in Figure 6-1. The resolution of the camera for this set-up is 50 mm (Tessier et al., 2002). The images are processed with digital image analysis software, *Split Desktop*® hereafter *Split* (Split Engineering, 2001 & Kemeny et al., 1999).
6 Measurement and Analysis of Fragmentation

Figure 6-1. Camera set-up installed in the primary crusher building in El Alto (Tessier et al., 2002).

Figure 6-2. Working procedure for photographs assignment to blasts in El Alto.

Normally, more than one blast is loaded at the same time. In order to assign the photographs to the respective blast, a color is associated to every blast. Software installed in the computer of the control room shows the trucks that are currently working and the blasts that are being loaded—represented by their color and identification number—. The working procedure is shown graphically in Figure 6-2. A color is assigned to each blast from the time the block is drilled until the muckpile is completely loaded; a flag of this color is allocated in the highwall crest. At the beginning of the shift, the crusher operator enters in the computer the identification numbers of the blasts being mucked and their colors, and the trucks are assigned to the different blasts (trucks load from a unique muckpile during the whole shift and they carry in the front a flag of the color of the blast they are currently hauling). The screen in the computer looks then like that shown above in Figure 6-2, with the truck identification and the color of the blast it is currently...
hauling. When a truck dumps, the mill operator presses the button that refers to the truck; once
the conditions are favorable for a good quality image (settlement of the dust), the operator
triggers the camera. The resulting photo is stored in the PC. A database links the computer time
at which each photo was made with the dumping time—which differ by a lapse of some
seconds—and the color, so that the origin of the material is known: the blast of that color being
loaded at that time.

6.3. Calibration of the digital image analysis software

This section describes the tests made in order to optimize the performance of the digital image
analysis software. These tests were relevant in developing the methodology described below.

6.3.1. Performance of the image analysis software in automatic mode

Split can function in automatic or manual modes, or a combination of both called auto &
manual. In the latter, the operator uses the automatic delineation done by the code and corrects
the errors visually. The correction process can be very time-consuming. A test to check the
performance in automatic mode (obviously advantageous from an economic standpoint),
without corrections, versus the auto & manual mode was done on a conveyor belt. The belt
conveying material from the outlet of the hammer mill was used for this purpose; the belt was
stopped and a photo was made of the material on it (Figure 6-3). A sample of about 60 cm in
length was taken from the photographed area of the belt, for further laboratory standard
screening. This sampling procedure was considered enough for our purposes, despite of its
simplicity—a more involved method for making belt-cuts, sieving and calibrating the system is
given by Kemeny et al. (1999).

The photograph in Figure 6-3 was analyzed with Split in both automatic and auto & manual
modes; the fines adjustment factor—a calibration parameter that, roughly, accounts for the
material smaller than the resolution of the image, “fines” (Split Engineering, 2001 & Kemeny et
al., 1999)—was given the value 100 % in both analyses (the fines factor can be less than 100 %,
if the fines are overrated, or greater, if the fines are hidden). The “autofines” option—that
enables the automatic detection of the fines patches—was activated in the automatic mode.
The actual fragmentation of the material on the belt, obtained by sieving, and the size distribution curves from *Split*-automatic and *Split*-auto & manual are plotted in Figure 6-4. The size distribution curve given by *Split* in auto & manual mode is quite close to the one obtained by sieving, while the one given by *Split* in automatic mode is well below. Maerz and Zhou (1998), working with *Wipfrag*, mention that particles with different textures and grey densities—clayish-marl’s fines patches appear between the limestone fragments—tend to confuse the delineation. In fact, the automatic delineation incurs in certain errors, sometimes quite significant, especially in the detection of the fines patches that are often confused with rock fragments. This leads to an overestimation of the sizes when the automatic delineation is not corrected, resulting in larger median size and more uniform distribution, and fewer fines than in reality.

![Figure 6-4. Size distribution curves for material in the belt obtained by sieving and with *Split*](image)

The errors made in the automatic mode can thus be attributed to the system’s ability for delineating the particles, allocating particles to size ranges, and recognizing and treating the fines.

### 6.3.2. Muckpile sieving. Calibration of the system

As shown in the previous section, the manual correction of the automatic delineation is required in order to have reliable fragmentation results. Besides this, the fines factor must be calibrated from sieving data. This can be easily done for the material on a belt, but such calibration would be useless for the material in the hopper of the crusher, as its topology is very different. For the calibration of the fines factor in photographs at the hopper, a sample of a blast was sieved, the resulting fractions were homogenized and then loaded into the primary crusher, and photographs were taken.

Production blast no. 10/03 was selected for this purpose. A cut perpendicular to the face was opened in the muckpile with a Cat 992 front-end loader in order to collect material ranging from
the free face to the opposite end of the muckpile—about 30 m away from the wall. The loader moved its bucket from bottom to top, taking material across the vertical of the pile. This sampling procedure allowed taking a representative sample, with material coming from both the heart of the blast and the overburden. The sampled material was stocked in a new pile from which an excavator fed it to a mobile Powerscreen Turbo Chieftain 1400, see Figure 6-5.

The mobile screen had three screens of sizes 90/110 mm, 46/48 mm and 20 mm. The first screen was formed by non-parallel rods separated 90 mm in one side and 110 mm in the opposite side. The second screen had rectangular openings of 48×46 mm. Screen openings were measured on-site. Fragments exceeding the excavator’s capacity (larger than about 800 mm) were not fed onto the screen. Four fractions: 800-110, 110-48, 48-20 and <20 mm were thus obtained.

The piles relative to each fraction were loaded into trucks with on-board scales. A negligible amount of material was lost in this operation. The mass of each fraction and the cumulative pass percentages are given in Table 6-1. A sample of the fine fraction was screened in the laboratory, with the aim to extend the size distribution down to 0.063 mm. This is shown in Table 6-2. The total fragment size distribution from the sieving is shown in Figure 6-6. A pile was then rebuilt from the individual screening piles, homogenized, loaded into trucks and dumped to the primary crusher. A total of four photos of this material were taken in the hopper and the fragmentation determined with Split. These photos are shown in Figure 6-7.

Table 6-1. Fragmentation data of the run of mine obtained by sieving.

<table>
<thead>
<tr>
<th>Size class, mm</th>
<th>Mass, t</th>
<th>x, mm</th>
<th>Pass, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>800–110</td>
<td>205</td>
<td>800</td>
<td>100</td>
</tr>
<tr>
<td>110–48</td>
<td>103</td>
<td>110</td>
<td>57.4</td>
</tr>
<tr>
<td>48–20</td>
<td>47</td>
<td>48</td>
<td>36.0</td>
</tr>
<tr>
<td>&lt;20</td>
<td>126</td>
<td>20</td>
<td>26.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>481</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 6-2. Size distribution of a sample of the muckpile fraction <20 mm.

<table>
<thead>
<tr>
<th>Sieve size, mm</th>
<th>20</th>
<th>14</th>
<th>12.5</th>
<th>10</th>
<th>6.3</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
<th>0.125</th>
<th>0.1</th>
<th>0.063</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass, %</td>
<td>100</td>
<td>90.9</td>
<td>85.5</td>
<td>76.6</td>
<td>62.4</td>
<td>50.8</td>
<td>34.0</td>
<td>21.4</td>
<td>12.1</td>
<td>6.5</td>
<td>3.2</td>
<td>2.49</td>
<td>1.36</td>
</tr>
</tbody>
</table>

A constant density for all size fractions has been assumed, so that the size distribution curves given by *Split* (which are volume-based) and the one obtained by sieving (mass-based) are directly compared. The four binary images were analyzed in auto & manual mode. The “best fit” option was selected for the fines distribution, which means that the software uses the distribution, Schuhmann or Rosin-Rammler, which works better (Split Engineering, 2001). The manual analysis was done by two different persons and the average curves from both were calculated. Different fines factors were used.

The *Split* data points and the sieving points have different abscissa values; in order to assess the differences of both size distributions, the Swefrec function (Ouchterlony, 2004a & 2004b) was fitted to the points from sieving down to 4 mm, the minimum size given by *Split*; this fitting provides a very good description of the existing data points ($R^2=99.8$), as can be seen in Figure 6-6. The root mean squared relative differences between the sieved-Swefrec values and the *Split* values from 750 mm down to 4 mm (14 points) were 0.128, 0.075, 0.05 and 0.083 for fines correction factors of 130, 120, 110 and 100 %, respectively. A factor of 110 % was thus chosen. Figure 6-6 shows the *Split* curves for fines factors of 100, 110 and 120 %. It should be noted that the *Split* curves separate significantly from the Swefrec fit in the coarse region; this is due to the fact that the zone between 110 and 800 mm is unresolved in the muckpile screening. If a Swefrec function is newly fitted to the best *Split* curve (fines factor 110) down to 4 mm, the
goodness of the fit is again remarkable ($R^2=99.8$). Figure 6-6 also shows the Split curve for FF=110 % in automatic mode, illustrating the bad results with this option, as discussed in Section 6.3.1.

Figure 6-7. Photographs analyzed from the material sieved in the calibration test

6.4. Practical procedure for the determination of fragmentation

The basics steps, once the photographs from a blast are available, are: sampling of the photos, processing the images with Split, obtaining the size distribution curve of the blast and subtracting the contribution of the overburden material.

6.4.1. Photo sampling

The photos are randomly sampled from the whole set of images assigned to a blast. Photos with bad quality, poor or uneven lighting and partially empty bin (this has an influence on the distance between the camera and the material, hence on the scale of the image), are rejected manually. In some blasts, more than half of the photographs are eliminated with this criterion. At least twenty photos per blast are selected for the analysis. However, if the images sampled are not representative of the fragmentation of the blast (details on this are given in Section 6.4.3), more images will be required until a minimum of 20 representative images are obtained. The size of the sample used is a compromise between the available resources (time spent in the manual edition of the images) and the requirement for the set of images to be representative of the whole blast. Assuming a standard deviation for a parameter (e.g. $x_{50}$) 25% of the mean value, twenty images result in a confidence interval at a significance level of 95 % of 16% of the mean. The confidence interval would be smaller if more photos per blast were be analyzed.
6.4.2. Scaling, automatic delineation and manual edition

The scale of the photograph is known from the camera opening and the distance to the rock surface (see Figure 6-1), which is approximately perpendicular to the camera’s optical axis. The resolution of the camera in the present case is 5.4 mm/pixel and the height of the photo is 576 pixels.

The “autofines” option is switched off in the automatic delineation menu, as large errors are committed using it. According to Norton (2003), a proper delineation should be made in this stage in order to cut the required amount of editing. For this purpose, the three delineation parameters –noise size, watershed ratio and gradient ratio (Split Engineering, 2001)– are modified conveniently according to the topology of the material in the photo. As an example, Figure 6-8 shows two automatic delineations from the same photo with different sets of values of the delineation parameters.

The manual edition is focused in the main errors of the automatic delineation. This consists of correcting the over-divided large rocks and marking the fines areas, which according to Ouchterlony (2003) are the main causes of the tendency of digital image analysis software to give more uniform distributions. Many of the smaller particles, whose delineated edges are not modified, are under the maximum size of fines –called fines cut-off and obtained from the peak or peaks of the histogram of volume of particles (Kemeny et al., 1999 & Split Engineering, 2001)– and they will be included in the fines estimation. Split calculates the total area of fines from the sum of the particle areas smaller than the fines cut-off and the product of the fines patches’ area times the fines adjustment factor.

This quick manual edition leads to a slightly different size distribution curve than if a very detailed edition, particle by particle, of the automatic delineation was made (the differences between each type of manual edition are shown in Figure 6-13). However, such very detailed edition was rejected because it is highly time-consuming.

![Figure 6-8. Auto-delineations made with the Split delineations parameters set to (a) minimum and (b) maximum](image-url)
6.4.3. Calculation of the size distribution curves of each image and of the blast

The size distribution for each single image is calculated using the fines adjustment factor determined in the calibration test, 110 %, and the “best fit” option activated. The time spent per photo in the analysis is about 20 minutes, most of the time being dedicated to correct the automatic delineation.

Most of the size distribution curves of the twenty photos of a blast form a fairly tight package. However, sometimes few curves have a different slope and lie outside that package. This is illustrated in Figure 6-9, where one curve of the twenty photos analyzed is very different to the others, with a very small median size. This curve relates to a photograph with few fragments and with most of the surface covered with fines patches. The opposite case (photos with very large fragments and few fines) also happens sometimes. Other cases of bad photographs, such as hopper half-empty, bad illumination, shadows, etc., are directly rejected before doing the digital analysis, as commented previously.

![Figure 6-9. Influence of outlier curves on the overall fragmentation](image)

The identification of outlying curves is done assuming that the distribution of medians \(x_{50}\) is log-normal, which can be confirmed by a goodness-of-fit test to the log-normal fit of the set of medians of many blasts (this could be expected, as the individual size distributions from each photograph can themselves be acceptably described as log-normal). The median absolute deviation about the median \(MAD\) has been used as robust variance estimator (Rousseeuw & Leroy, 1987):

\[
MAD = \text{median}\left\{ \left| \ln x_{50i} - \text{median}(\ln x_{50}) \right| \right\}
\]

And the deviations relative to the \(MAD\) are calculated as:

\[
Z = \left| \ln x_{50i} - \text{median}(\ln x_{50}) \right| / MAD
\]

The individual curves are rejected when \(Z \geq 5\) (Miller & Miller, 2000).
A new photograph is analyzed whenever one photograph has been taken out as outlier, so that 20 valid photographs are always used for making up the fragmentation curve of the blast (unless the total number of valid photographs be less than 20, a very rare case).

### 6.4.4. Calculation of the size distribution curve of the fragmented limestone

The overburden contributes to the fines in the muckpile and its thickness varies from blast to blast. In order to get the actual fragmentation produced by blasting, the loose overburden has to be discounted from the raw data given by Split. For this purpose, a representative sample of the overburden was collected and the size distribution obtained by standard sieving; it is given in Table 6-3.

<table>
<thead>
<tr>
<th>Sieve size mm</th>
<th>14</th>
<th>12.5</th>
<th>10</th>
<th>6.3</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
<th>0.125</th>
<th>0.1</th>
<th>0.063</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass, overburden (%)</td>
<td>100.0</td>
<td>96.8</td>
<td>93.3</td>
<td>79.3</td>
<td>71.4</td>
<td>55.8</td>
<td>42.1</td>
<td>24.1</td>
<td>14.0</td>
<td>7.7</td>
<td>5.8</td>
<td>3.3</td>
</tr>
<tr>
<td>Pass, muckpile (%)</td>
<td>100.0</td>
<td>94.0</td>
<td>84.3</td>
<td>68.6</td>
<td>55.9</td>
<td>37.4</td>
<td>23.5</td>
<td>13.3</td>
<td>7.2</td>
<td>3.5</td>
<td>2.7</td>
<td>1.5</td>
</tr>
</tbody>
</table>

For sizes larger than or equal to 14 mm (maximum size of the overburden), the size distribution \( P'(x) \) of the fragmented limestone is as follows:

\[
P'(x) = 100 \cdot \frac{P(x) - NF}{100 - NF}
\]

where \( P(x) \) is the cumulative pass given by Split for a size \( x \) and \( NF \) is the natural fines fraction present in the block (in the blast sieved, \( NF = 10.8 \% \)).

For sizes smaller than 14 mm, the size distribution of the fragmented rock is obtained from the muckpile and the overburden sieving data. The cumulative size distribution of the muckpile below 14 mm for the blast sieved can be obtained from Table 6-2 by renormalizing with respect to 90.9 \% passing; it is given in Table 6-3. The classes for each size interval are formed from these; let \( f_{Mi} \) and \( f_{OBi} \) be the percentages of each class in the muckpile and in the overburden, respectively (from the total material below 14 mm); they are shown in Figure 6-10.

If the total mass of material in the muckpile is \( M \), that of a class \( i \) is:

\[
m_{Mi} = M \cdot \frac{P_{14mm} \cdot f_{Mi}}{100 \cdot 100}
\]

\( P_{14mm} \) being the fraction of material below 14 mm in the muckpile, from the Split analysis. The mass of the same class originally in the overburden is:

\[
m_{OBi} = M \cdot \frac{NF \cdot f_{OBi}}{100 \cdot 100}
\]

And the mass of class \( i \) in the fragmented limestone is:

\[
m_{Li} = m_{Mi} - m_{OBi}
\]
The frequency of the classes in the fragmented limestone relative to the material below 14 mm are:

\[
f_{Li} = 100 \frac{m_{Li}}{\sum_{i=1} m_{Li}} = 100 \left( \frac{P_{14 \text{mm}}}{100} \right) \frac{f_{Mli}}{100 - NF} \frac{f_{OBl}}{100 - NF}
\]

These are shown in Figure 6-10. The cumulative size distribution below 14mm (the limestone fines), \( P_L(x) \), formed from \( f_{Li} \) is shown in Table 6-4; \( P_L(x=14 \text{ mm}) \) is obviously 100 %.

Table 6-4. Size distribution below 14 mm, \( P_L(x) \), of the fragmented rock.

<table>
<thead>
<tr>
<th>Sieve size, mm</th>
<th>14</th>
<th>12.5</th>
<th>10</th>
<th>6.3</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>0.5</th>
<th>0.25</th>
<th>0.125</th>
<th>0.1</th>
<th>0.063</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass, %</td>
<td>100.0</td>
<td>91.8</td>
<td>76.8</td>
<td>59.8</td>
<td>43.1</td>
<td>22.3</td>
<td>8.3</td>
<td>4.5</td>
<td>1.6</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Moser et al. (2000) show a strong similarity between the shape of the finer part of the size distribution curves of a given rock type irrespective of the fragmentation process applied, either comminution or blasting, for sizes up to about 10 to 50 mm (rock dependent). Assuming that this holds true for the fragmented rock in El Alto, the size distribution curve for the fragmented limestone below 14 mm obtained for the blast that was sieved, shown in Table 4, is used for the other blasts. This fines tail is linked at 14 mm:

\[
P'(x) = P'(x = 14) \frac{P_L(x)}{100}
\]

Figure 6-11 shows the various size distribution curves of the correction process.
6 Measurement and Analysis of Fragmentation

6.5. Sources of error

The limitations of the fragmentation monitoring system encompass different errors that may affect its reliability and hinder its use for fragmentation control. Errors can be grouped into three categories, namely:

- from the sampling method,
- from the image analysis process, and
- from the method of discounting the natural fines.

6.5.1. Errors due to sampling

A first limitation comes from the requirement of the photographs to be representative of all the rock fragments. In most blasts, the 20 photos analyzed represent about a 12 % of the muckpile material in mass. Forty photographs from one blast were chosen and processed; groups of 20, 10, 5, 2 and 1 photos were randomly formed from those. This arrangement was done twice (samplings a and b). The respective size distribution curves of the ROM are plotted in Figure 6-12. The Swebrec size distribution function (Ouchterlony, 2004a & 2004b) was fitted to them; the characteristic Swebrec parameters: maximum size, $x_{\text{max}}$, mean size, $x_{50}$ and undulation parameter, $b$ together with the correlation factor are listed in Table 6-5. The correlation factor is quite good in all the cases; it is at least 99.8 %. The errors in $x_{50}$ and $b$ with respect to the fragmentation data obtained with the complete set of 40 photos –considered to be the best.
approach to the actual fragmentation of the muckpile— are shown in Table 6-5; $x_{\text{max}}$ is about the nominal maximum size loaded into the primary and is not considered in the analysis. If 20 photos are considered, the agreement with the 40 photos’ size distribution curve is good (the errors are within 7%). If less than 10 photos are considered, the errors may increase strongly, especially in $x_{50}$.

Table 6-5. Swebrec parameters, correlation factor and errors committed for different samples of photos.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Sample</th>
<th>$x_{\text{max}}$, mm</th>
<th>$x_{50}$, mm</th>
<th>$b$</th>
<th>$R^2$, %</th>
<th>Errors, (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>-</td>
<td>1000</td>
<td>87</td>
<td>1.62</td>
<td>99.9</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>(a)</td>
<td>1253</td>
<td>83</td>
<td>1.71</td>
<td>99.9</td>
<td>4, -6</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>1127</td>
<td>85</td>
<td>1.73</td>
<td>99.9</td>
<td>3, -7</td>
</tr>
<tr>
<td>10</td>
<td>(a)</td>
<td>1116</td>
<td>85</td>
<td>1.67</td>
<td>99.9</td>
<td>2, -3</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>1137</td>
<td>79</td>
<td>1.43</td>
<td>99.9</td>
<td>9, 12</td>
</tr>
<tr>
<td>5</td>
<td>(a)</td>
<td>1000</td>
<td>62</td>
<td>1.66</td>
<td>99.8</td>
<td>28, -2</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>1000</td>
<td>89</td>
<td>1.50</td>
<td>99.8</td>
<td>-2, 8</td>
</tr>
<tr>
<td>2</td>
<td>(a)</td>
<td>1070</td>
<td>118</td>
<td>1.53</td>
<td>100.0</td>
<td>-36, 5</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>1000</td>
<td>83</td>
<td>1.55</td>
<td>99.8</td>
<td>5, 4</td>
</tr>
<tr>
<td>1</td>
<td>(a)</td>
<td>1006</td>
<td>189</td>
<td>1.11</td>
<td>99.6</td>
<td>-118, 32</td>
</tr>
<tr>
<td></td>
<td>(b)</td>
<td>1000</td>
<td>131</td>
<td>2.21</td>
<td>99.9</td>
<td>-51, -36</td>
</tr>
</tbody>
</table>

Figure 6-12. Size distribution curves of ROM for two analyses (a) and (b) for different number of photographs

Secondly, the muckpile in El Alto is formed by fragmented limestone and natural fines, so that two different size distributions are superimposed. When the amount of natural fines is high, the coarse particles may be hidden by the fines spread all over the surface, causing an overrepresentation of fines, whereas if the amount of natural fines is small, the surface view may represent fairly well the material below. As Latham et al (2003) state, the fines adjustment factor is not expected to account for the unpredictable effects of gravity segregation. As both of the above situations (large and small amount of fines) may take place in photographs of the same blast, several calibration tests with material coming from different types of size distributions would be required. A variable fines adjustment factor would then be used...
depending on the topology of the photograph. This however would complicate the process and would probably bring about more subjectivity into it.

### 6.5.2. Errors due to the image processing

The calibration of the system was done from the comparison of optic and sieving measurements, each with its own limitations. For instance, optical systems measure the long and intermediate diameters of the fragments while the intermediate and small diameters are measured when the material is sieved (Maerz & Zhou, 1998); additionally, the errors inherent to a 2D analysis, like the fragments overlapping, are not present when a sample is sieved.

The required manual edition is a new source of errors. A test was carried out in which a very detailed edition particle by particle of about two hours, and a quick one—as described in Section 6.4.2—were done on two photos (A and B). The size distribution curves for both the detailed and the “time optimized” analyses are shown for each photo in Figure 6-13; the differences in pass increase as the size decreases; in $P_{63\text{mm}}$ they are about 3 %, while the differences in $P_{4\text{mm}}$ are 10 and 27 %. The latter is rather large, but 4 mm is anyway far below the optical resolution of the system, 50 mm.

![Photo A](image1.png)
![Photo B](image2.png)

Figure 6-13. Analysis of two photos; results with detailed and quick analyses

The unavoidable manual edition of the photographs encompasses some subjectivity. Figure 6-14 shows the size distribution curves from the analysis of the same photograph made by two operators. The resulting fragmentation is very similar for sizes larger than 88 mm. The fines cut-off of the photographs in Figure 6-14 is 100 and 78 mm. So, the manual corrections seem to be relatively independent of the user at sizes larger than the fines cut-off. Below that size, the curves clearly diverge; the errors at 63 mm (the last point above the camera resolution limit) are still low, 4 %, while they become important, 45 %, at 4 mm. It is clear that the precise identification and delineation of patches of fines and fine particles is difficult not only for the image analysis software but also for the human eye.
6.5.3. Errors due to subtracting the natural fines

The correction for natural fines is probably a new source of error. Firstly, the estimation of the amount of natural fines present in the block is based on a plane section of the block blasted (the highwall face). Secondly, the maximum size of the natural fines, 14 mm, obtained from only one sample, may be subject to changes across the quarry. Finally, the Split passing at 14 mm is a weak point, as it is well in the range where errors of the measurements are relatively important, as Figures 6-13 and 6-14 show.

6.6. Consistency of the fragmentation monitoring system

The sieving of blast 10/03 shows that the muckpile size distribution curve given by Split from photographs in the primary crusher feed can acceptably match the measured one (see Figure 6-6). It is not possible to check the accuracy of the system in each particular blast. For most applications related with fragmentation by blasting, however, it may be enough if the system is able to respond to relative changes consistently.

A way to check the consistency of the measurements is to correlate the size distributions with the amount of natural fines: the passing at 14 mm ($P_{14\text{ mm}}$), which is the maximum size of the overburden, should at least be equal to the natural fines fractions; $P_{14\text{ mm}}$ should then increase as the amount of natural fines does. The fraction passing at 14 mm obtained with the procedure described in Section 6.4 is plotted versus the natural fines fractions in Figure 6-15 for the 35 blasts described in Chapter 3. The trend line that fits the $P_{14\text{ mm}}$ data, named FF-110, and the line that shows that no fines are created in the blast, i.e. $P_{14\text{ mm}}=NF$, i.e., are also drawn. In those blasts the blasting parameters –drilling pattern, explosives and timing– were varied, so that fragmentation cannot be expected to be constant. Nevertheless, this does not hide the influence of the natural fines: $P_{14\text{ mm}}$ (obtained from fitting a Rosin-Rammler distribution, see Equation 6-2, to the passing at 16 and 11 mm given by Split) increases with the natural fines; although 26% of the blasts are below the line that represents a pass equal to the natural fines, which forces to reject them.
6.6.1. Recalibration of the fines adjustment factor

The natural fines fraction present in the block can be much larger than the 10.8 % present in blast 10/03, which was used for calibrating the fines factor, FF. It is clear from the results in Figure 6-15 that the FF of 110 % does not account for all the fines present in the bin when NF is much larger than 10.8 %. This may be solved by using a variable FF function of NF that increases as NF does.

At least two production blasts with Median and large amounts of natural fines, about 25 and 40 % respectively, should be sieved in order to draw a relation between the fines adjustment factor and the natural fines. From that and according to the NF present in the block, the size distribution, $P^*(x)$, could be easily recalculated using Split from the resulting FF value. Since only one calibration tests are available, the fines adjustment factor is back-calculated; so that the new $P^*_{14\text{mm}}$ values [$P^*(x=14\text{mm})$] follow the trend line determined from two known $P^*_{14\text{mm}}$:

1) In a hypothetical situation, if there is no limestone in the block, NF=100 %, hence $P^*_{14\text{mm}}$ should be 100 %.

2) In blasts with natural fines content similar to the sieved blast, around 10.8 %, the FF of 110 % works well and the new fines adjustment factor should be very similar. Since the sieved blast has a NF fraction close to the blast with the smallest NF, 9.7 %, the FF-110 trend line must tilt up around this point; for a NF $=9.7 \%$, $P^*_{14\text{mm}}$ is 23.5 % from the Equation of the FF-110 trend line.

The points (100 %, 100 %) and (9.7 %, 23.5 %) define the ideal trend line of $P^*_{14\text{mm}}$:

$$P^*_{14\text{mm}} = 0.85 \cdot NF + 15.3$$

which is drawn in Figure 6-15. This trend line is used to calculate $P^*_{14\text{mm}}$ from $P_{14\text{mm}}$ for each blast assuming that the $P^*_{14\text{mm}}$ values must be in the same relative position, $\Delta P_{14\text{mm}}$ about their trend line than $P_{14\text{mm}}$ are with respect to FF-110 trend line; the latter avoids biasing the measurements. Figure 6-15 shows as an example $\Delta P_{14\text{mm}}$ for a blast with NF of 40 %. The points $P^*_{14\text{mm}}$ obtained with this operation are plotted for the 35 blasts in Figure 6-15. Once $P^*_{14\text{mm}}$ is known, it can be approached from an iterative process in which the set of already delineated images of each blast are analyzed with different FF that are increased or reduced conveniently in multiples of 10 % (i.e. 120, 130, 140,...) until the target $P^*_{14\text{mm}}$ is included between the two values, $(P^*_{14\text{mm}})_{FF+}$ and $(P^*_{14\text{mm}})_{FF-}$. Both $(P^*_{14\text{mm}})_{FF+}$ and $(P^*_{14\text{mm}})_{FF-}$ are plotted for each blast in Figure 6-15. The resulting upper and lower bounds, FF+ and FF, respectively, for the 35 blasts are plotted in Figure 6-16 versus the natural NF. If a linear function is fitted to the overall data, the following expression is defined:

$$FF = 83 + 2.8 \cdot NF \quad (6-1)$$

which is used to obtain $P^*(x)$ in the 35 blasts by recalculating the fragmentation with Split with the resulting fines factor. The final $P^*_{14\text{mm}}$ values are plotted in Figure 6-15; $P^*_{14\text{mm}}$ with Equation 6-1 are very similar to $P^*_{14\text{mm}}$ obtained without Split from $\Delta P_{14\text{mm}}$. Notice, however, that there are still two blasts, 96/03 and 08/04 (they can be easily identified with their abscissas 35.4 and 24.8 % respectively) whose $P_{14\text{mm}}$ is still below the line that represents a pass equal to the natural fines. They are definitively excluded for further analysis as outliers.
6.7. Fragmentation data. Analytical description

Fragmentation of the 35 blasts described in Chapter 3 was determined following the practical procedure described in the Section 6.4 and using the FF in Equation 6-1. Figure 6-17 shows the resulting size distribution curves of the ROM (limestone plus overburden) for those blasts and the quantitative data are given in Appendix D (passing for each fraction, maximum fragment...
size and the fines cut-off, together with the fines adjustment factor used in the calculation). The statistics of the maximum fragment size given by Split, 1098 ±108 mm, agree with the maximum size loaded into the primary crusher.

The procedure described in Section 6.4.4 for calculating the distribution curves of the fragmented limestone is applied to the ROM data. The resulting size distribution curves of fragmented limestone for 33 blasts are also shown in Appendix D and plotted in Figure 6-18; blasts 96/03 and 8/04 are excluded. The size distribution curves of the fragmented limestone cover a wide fragmentation range from very flat curves to rather steep ones. In all the curves, there is a kink at 14 mm (junction point between the variable and constant parts of the size distribution curves), which is more marked for the very flat and steep curves.
The set of passing fractions for a finite number of mesh sizes is frequently represented analytically with continuous distribution functions. This reduces the amount of data to work with. The Rosin-Rammler distribution (R-R) is probably the most common one; it is a function of the type:

$$
P(x) = 1 - e^{-\ln\left(\frac{x}{x_{50}}\right)^n}$$

Where $P(x)$ is the cumulative fraction of rock passing a sieve of size $x$; $x_{50}$ is the 50% passing mesh size and $n$ is the uniformity index.

The Rosin-Rammler function represents well the size distribution of fragmented rock for size ranges of two or even up to three orders of magnitude, for instance between 10 and 1000 mm. This means that usually R-R does not describe well both the coarse and fines part of the fragmentation curve. This limitation has leads to the recently developed Swebrec function (Ouchterlony, 2004a & 2004b):

$$
P(x) = 1/\{1+[\ln(x_{max}/x)/\ln(x_{max}/x_{50})]^b\}$$

Where $x_{max}$ is the upper limit of the fragment size distribution and $b$ is a curve undulation parameter (it may be understood as a uniformity index).
Ouchterlony (2004a & 2004b) shows that the Swebrec function (S) usually fits the size distribution curve obtained by blasting better than the Rosin-Rammler does. Frequently, this avoids the use of bimodal distributions with up to five parameters that consider two sets of fragments, each of them for the coarse and fines particles (Djordjevic, 1999; Kanchibotla et al., 1999; Thornton et al., 2001a & b).

The actual fragmentation values of fragmented limestone for each blast are fitted with the Rosin-Rammler and Swebrec functions, Equations 6-2 and 6-3 respectively. The size range considered is 1000 to 14 mm, almost two orders of magnitude, with a total of 12 points. The data below 14 mm is rejected as the fines tail obtained from the sieved blast is assumed to be a feature of the rock type and kept constant in all the blasts. Ouchterlony (2004a & 2004b) states that $x_{\text{max}}$ should be related to the in-situ block size. However, in this particular case the measurements are made at the hopper of the primary and the boulders from the blast do not appear in the photographs; the number of boulders, 1.5% of the mass of the blasted block, was not quantified. For this reason, the Swebrec’s $x_{\text{max}}$ parameter is fixed to the value given by Split. The resulting R-R and S parameters ($x_{50}$ and $n$, and $x_{\text{max}}$, $x_{50}$ and $b$ respectively) and the correlation factors are listed in Appendix D together with the absolute percentage errors in pass for varying sizes (for 1000, 750, 500, 250, 125, 88, 63, 44, 31, 22, 16 and 14 mm) and the absolute percentage errors in size for varying passes ($x_{30}$, $x_{50}$ and $x_{90}$) for each blast.

As an example, Figure 6-19, shows the fits made in blasts 41/02, 103/02 and 58/03; linear scale was used for the cumulative pass axis in order to outline better the differences between each fitting. The percentage errors for each size committed with the S and R-R fittings are also plotted. Blasts 103/02 and 58/03 are representative of blasts that are well fitted with the S and R-R distributions respectively, whereas fragmentation in blast 41/02 is described similarly with either R-R or S functions, the mean of the 12 absolute percentage errors in pass is 3.3% for R-R and 3.9% for S. The Rosin-Rammler fit is always below the actual curve in the vicinity of $x_{\text{max}}$, whereas the Swebrec distribution tends to lift from the actual curve for sizes below 22 mm.

The mean and standard deviation of the absolute percentage errors in pass and sizes for 33 blasts with the S and R-R fittings are given in Tables 6-6 and 6-7 respectively and plotted in Figure 6-20.

### Table 6-6. Mean and standard deviation of the absolute percentage error in pass for all 33 blasts

<table>
<thead>
<tr>
<th>$x$, mm</th>
<th>1000</th>
<th>750</th>
<th>500</th>
<th>250</th>
<th>125</th>
<th>88</th>
<th>44</th>
<th>31</th>
<th>22</th>
<th>16</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-R</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Av.</td>
<td>4.0</td>
<td>3.7</td>
<td>1.6</td>
<td>6.7</td>
<td>6.4</td>
<td>4.3</td>
<td>4.2</td>
<td>4.9</td>
<td>5.6</td>
<td>5.9</td>
<td>11.2</td>
</tr>
<tr>
<td>Sd.</td>
<td>1.4</td>
<td>1.5</td>
<td>1.1</td>
<td>3.9</td>
<td>3.4</td>
<td>2.4</td>
<td>2.5</td>
<td>1.7</td>
<td>4.0</td>
<td>10.0</td>
<td>16.6</td>
</tr>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Av.</td>
<td>1.0</td>
<td>1.8</td>
<td>2.9</td>
<td>2.4</td>
<td>3.3</td>
<td>5.9</td>
<td>7.1</td>
<td>5.6</td>
<td>2.2</td>
<td>8.3</td>
<td>23.4</td>
</tr>
<tr>
<td>Sd.</td>
<td>0.5</td>
<td>1.0</td>
<td>1.8</td>
<td>1.3</td>
<td>2.1</td>
<td>3.7</td>
<td>3.6</td>
<td>2.1</td>
<td>1.3</td>
<td>7.6</td>
<td>20.8</td>
</tr>
</tbody>
</table>

*Note: Av and sd. means average and standard deviation respectively*

### Table 6-7. Mean and standard deviation of the absolute percentage error in size for all 33 blasts

<table>
<thead>
<tr>
<th>$P(x)$</th>
<th>$x_{30}$</th>
<th>$x_{50}$</th>
<th>$x_{90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-R</td>
<td>14.7sd6.9</td>
<td>12.8sd8.1</td>
<td>16.2sd11.5</td>
</tr>
<tr>
<td>S-S</td>
<td>14.6sd8.7</td>
<td>7.1sd5.2</td>
<td>8.8sd4.9</td>
</tr>
</tbody>
</table>
The statistics of the correlation factor, $R^2$, obtained with the Rosin-Rammler and Swebrec functions in the 33 blasts are 0.988 sd. 0.009 % and 0.992 sd. 0.06 % respectively. None of the functions tried fit properly all the size range down to 14 mm, as seen in Figure 6-19; the R-R fails in the coarse part while the S is unsuccessful in the fines part. The statistics of the mean of the percentage absolute errors in pass shown in Table 6-6 are 8.2sd.6.5 % for S and 6.2sd.4.2 % for R-R fittings. The mean absolute errors in size for $x_{50}$ and $x_{90}$ are larger with the R-R distribution, as $x_{50}$ and $x_{90}$ correspond to the coarse part of the size distribution curve; the average values for the 33 blasts are 130 mm for $x_{50}$ and 600 mm for $x_{90}$. Conversely $x_{30}$, whose mean value of the 33 blasts is about 40 mm, is similarly fitted with either R-R or S.

Ouchterlony (2004a) gives a range of variation for the undulation parameter, $b$, between 1 and 4. In the case of study, the mean and standard deviation for the $b$ values in the 33 blasts are 1.907 and 0.268 respectively. The statistics in 33 blasts for the $x_{50}$ Swebrec parameter are 129 sd. 29 mm. The ratio of the standard deviation of $x_{50}$ to its mean is 22 %, while for $b$ is 14 %.

The R-R uniformity index, $n$, from the 33 fitted blasts is 0.659 sd. 0.094; non $n$-value is above unity. Cunningham (1987) states that the “normal range of $n$ for blasting fragmentation in reasonably competent ground is between 0.75 and 1.5, […] more competent rocks have higher values”. He adds that “values of $n$ below 0.75 represent a situation of dust and boulders which if occurs on a wide scale in practice indicates that the rock conditions are not conducive to control of fragmentation through changes in blasting”, which occurs typically when overburden is stripped in weathered ground. In our, where the natural fines have been discounted the $n$-values are still rather low, according to Cunningham’s statement. It will be shown in Section 6.8.7, that Cunningham’s Kuz-Ram model predicts very high $n$-values. This could lead to quote then-values below unity as extreme. The statistics of the $x_{30}$ for the R-R fitting are 110 sd. 25 mm; the ratios of the standard deviation of $x_{50}$ and $n$ to their mean values, 23 % and 14 %, are similar to those obtained for the Swebrec’s $x_{50}$ and $b$ parameters.

### 6.8. A survey of fragmentation prediction models

The knowledge of the different models and recommendations existent in the literature is a first step towards the control of the fragmentation through the blast design. Fragmentation models define empirical relations between a set of parameters that include rock, drilling and blasting features, and the characteristic parameters of the fragmentation distribution. Those relations may be inferred either from full scale blasting in one or various production sites, or from reduced-scale shots. The models may not work in every situation and they should be considered as a rough guide of how the blasting parameters affect fragmentation. As the aim is to develop engineering models as general as possible, the models should be generalized to different rock types than the ones form which they are drawn by introducing a rock parameter that can be understood as a fitting factor to the actual fragmentation.

Ouchterlony (2002) makes an excellent compilation of the available models on fragmentation by blasting. Of them, the Kuz-Ram (Cunningham, 1983 &1987), Chung-Katsabanis (Chung & Katsabanis, 2000), SveDeFo (Ouchterlony et al., 1990), Kou-Rustan (Kou & Rustan, 1993) and Kuznetsov-Cunningham-Ouchterlony, KCO (Ouchterlony, 2004b), formulae are described, analyzed and applied in the following to the data of 33 production blasts.
Figure 6-19. Size distribution curve of fragmented limestone, fitted S and R-R functions and the respective percentage errors in pass for blasts (a) 41/02; (b) 103/02 and (c) 58/03
Figure 6.20. Mean absolute percentage errors in (a) pass and (b) size of the data of 33 blasts

6.8.1 Kuz-Ram model

The Kuz-Ram model (K-R) is widely used, despite the data on which it is based are not reported in the original work (Cunningham, 1983). Until recently, it was the only model available for predicting fragmentation. The K-R model makes use of the Rosin-Rammler distribution, Equation 6.2, for describing the fragmentation of the blasted rock. Fragmentation was evaluated using a set of photographs of artificial muckpiles of known \( x_{50} \) and \( n \); these standard photographs were compared with the ones relative to the actual muckpile to find the one that matched both in texture and size.

The mean size in centimeters is obtained from:

\[
x_{50} = A Q_e^{1/6} q^{-0.8} \left[ \frac{RBS}{115} \right]^{-19/30}
\]  

(6-4)
Where:

- \( A \), is a rock factor, which is 7 for Median rocks, 10 for hard and highly fissured rocks, and 13 for hard and weakly fissured rocks; Table 6-8 shows the rock factors given by AECI (1986) for different rock types. Cunningham (1987) includes in the revision of the model an equation for calculating the rock factor from the geotechnical (massive/powdery character, spacing between joints and orientation of the joints) and mechanical features (rock density, elastic modulus and U.C.S). Cunningham (1983) cautions that the drilling pattern must be adapted to the separation between joints.

<table>
<thead>
<tr>
<th>Blasting behavior</th>
<th>Rock type</th>
<th>Rock factor</th>
<th>Technical powder factor (kg/m³)</th>
<th>Rock constant (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bad</td>
<td>Andesite, Dolerite, Granite Ironstone, Silcrete</td>
<td>12-14</td>
<td>0.70</td>
<td>0.62</td>
</tr>
<tr>
<td>Fair</td>
<td>Dolomite, Quartzite, Serpentine, Schist</td>
<td>10-11</td>
<td>0.45</td>
<td>0.40</td>
</tr>
<tr>
<td>Good</td>
<td>Sandstone, Calcrete Limestone, Shale</td>
<td>8-9</td>
<td>0.30</td>
<td>0.27</td>
</tr>
<tr>
<td>Very good</td>
<td>Coal</td>
<td>6</td>
<td>0.15-0.25</td>
<td>0.14– 0.22</td>
</tr>
</tbody>
</table>

- \( Q_e \), is the mass of explosive per hole in kg. Cunningham (1983) states that they “normally exclude the explosive in the subdrill section, as this seldom contributes significantly to fragmentation in the column area”. This agrees with the analysis made by Daniel (1996).

- \( q \), is the powder factor in kg/m³. The powder factor above grade is normally used, according to Cunningham (1983).

- \( RBS \), is the explosive relative weight strength based on the heat of explosion (ANFO=100). The energy delivered by the explosive must be close to the relative weight strength of the explosive, which for Cunningham (1987) is more probable if:
  i) The explosive composition is homogenous.
  ii) The blasthole diameter is at least three times greater than the critical diameter.
  iii) The confinement (elastic modulus above 50 GPa) is strong.
  iv) Point initiation and correct boostering are used.
  v) The explosive has appropriate water resistance.

All the above are conditions that favors an ideal detonation. Emulsions are close to the ideal behavior in large diameters but not heterogeneous explosives like ANFO. For the latter, the model would predict a finer fragmentation than actual, as energy delivered would be smaller than what the RWS may indicate.

The initial formula for the uniformity index is:

\[
n = \left( 2.2 - 0.014 \frac{B}{d} \right) \left( 1 - \frac{W}{B} \right) \left( \frac{1 + S/B}{2} \right) \frac{L}{H} \quad (6-5)
\]

where \( B \) is the burden, \( d \) is the blasthole diameter, \( W \) is the standard deviation of the drilling errors, \( S/B \) is the spacing to burden ratio, \( L \) is the charge length above grade and \( H \) is the bench
height in meters. All dimensions are in meters or consistent units. The $S/B$ used is that of the drilling layout, not the effective one resulting from the initiation sequence, and should not exceed 2 (Cunningham 1983).

All the terms that make up the powder factor except the explosive density are included in Equation 6-5; the term affecting $B/d$ is probably obtained from a linear fitting in a plot of $n$ versus $B/d$. If a staggered drilling pattern is used, the value of $n$ given by Equation 6-5 is increased by 10% (Cunningham 1983). According to Equation 6-5, a more uniform distribution can be achieved by reducing the burden to blasthole diameter ratio and the drilling deviations, and by increasing the charge length to the bench height ratio and the spacing to burden ratio. The influence of $S/B$ was afterwards limited (Cunningham 1987) from the original model by raising the term comprising it to the power of 0.5. Additionally, a new term was incorporated to Equation 6-5 for considering the use of different bottom and column charges, of lengths $L_B$ (above grade) and $L_C$. Thornton et al. (2001a) reject that term, as it is raised to 0.1. The uniformity index becomes finally:

$$n = \left(2.2 - 0.014 \frac{B}{d} \left(1 - \frac{W}{B} \left(\frac{1 + S/B}{2}\right)^{0.5} \left(\frac{L_B - L_C}{L} + 0.1\right)^{0.1} \frac{L}{H}\right)\right)$$

(6-6)

The mean size depends on the drilling pattern (burden and spacing) and bench height by means of the powder factor. Then, changes in the powder factor caused by variations in the drilling pattern affect to the mean size and uniformity index. But if $S/B$ is changed and the burden varied so that the powder factor is kept constant, the distribution tilts around the mean and gets steeper as the $S/B$ increases.

The explosive mass per borehole is directly proportional to the explosive density, the square of the blasthole diameter and the length of the explosive charge. The last two parameters also affect the uniformity index, which increases as the hole diameter and charge length do, although the effect of the diameter is smaller than that of the charge length. Therefore, the mean size could be changed without affecting the uniformity index by changing the explosive properties (density and energy).

Timing is not present in the K-R model but it is assumed to be within normal millisecond limits (Cunningham 1987).

### 6.8.2. Chung and Katsabanis model

The K-R model was tested using the results of 29 small-scale blasts made in dolomite with well coupled charges of dynamite. These tests, in which fragmentation was measured by sieving, were reported by Otterness et al (1991), who used a Weibull and Normal distributions to fit respectively the fines and coarse material (the boundary between coarse and fine were 40 mm for charge diameter 16 mm and 76 mm for charge diameter 25 mm). However, Chung and Katsabanis (C-K) found that excellent fitting was obtained using a Rosin-Rammler distribution, which greatly simplifies the model. They also showed that the uniformity index predicted by the K-R model (packaged mainly between 2 and 1.5) was somewhat higher and more variable than the measured ones, which were smaller than 1 and with a tight range of variation, between 1.0 and 0.7. Such large differences were not appreciated in the mean size, but the existing discrepancies were associated to the fact that the Kuznetsoy equation does not consider the
timing and that the explosive distribution is only accounted by the powder factor. They found that the uniformity index could be decreased from the one calculated with the K-R model by considering the coarse part of the size distribution:

\[ n = \frac{0.842}{\ln x_{80} - \ln x_{50}} \]  

(6-7)

Additionally, the data of scale blasts were used to find a general relation for \( x_{50} \):

\[ x_{50} = A' Q_e^\alpha B^\beta (S / B)^f H^\delta t^\epsilon \]  

(6-8)

Where \( A' \) is a rock factor, \( Q_e \) is the explosive mass per hole (kg), \( t \) is the in-row delay and \( \alpha, \beta, \gamma, \delta \) and \( \epsilon \) are constants. The in-row delay was dropped from Equation 6-8 since there were very few observations on the effect of the delay, and the following was then obtained from non-linear regression analysis:

\[ x_{50} = A' Q_e^{-1.193} B^{2.461} (S / B)^{1.254} H^{1.266} \]  

(6-9)

being \( x_{50} \) in centimeters.

If the powder factor, \( q \equiv Q_e / (B S H) \), is introduced in 6-9, this can be compared with the K-R formula 6-4:

\[ x_{50} = A' B^{0.014} S^{0.061} H^{0.073} q^{-1.193} \]  

and, letting the explosive mass per hole appear and minimizing the exponents of \( B, S, H \) and \( Q_e \):

\[ x_{50} = A' B^{-0.023} S^{0.024} H^{0.036} Q_e^{0.037} q^{-1.230} \]  

(6-10)

Equation 6-10 shows that the influence of \( q \) is higher than in K-R, and that the effect of explosive mass per hole, burden, spacing and bench height in \( x_{50} \), other than that borne in the powder factor, is negligible, as the exponents affecting those parameters are very low compared with the dominant one, 1.230.

Chung and Katsabanis start from the Bond equation to predict the necessary energy per ton of rock for a required degree of fragmentation \( x_{80} \). With such relation, it rapidly comes out the dependence of \( x_{80} \) on the explosive energy and the powder factor, as the product of the powder factor times the explosive energy is the energy input per unit mass of rock. Of course, the Bond equation cannot possibly be used directly, as the fraction of explosive energy that goes into the rock for fragmentation in blasting is largely unknown. Chung and Katsabanis then conclude by performing a non linear regression analysis, which results in the following equation for \( x_{80} \):

\[ x_{80} = 3 A' Q_e^{-1.07} B^{2.43} (S / B)^{1.013} H^{1.111} \]  

(6-11)

which, in terms of \( q \), can be written:

\[ x_{80} = 3 A' B^{0.347} S^{-0.057} H^{0.041} q^{-1.07} \]  

The C-K uniformity index increases as the mean size does if \( x_{80} \) is kept approximately constant. However, as both \( x_{80} \) and \( x_{50} \) are function of the same parameters powered to different exponents, \( x_{80} \) is likely to change if \( x_{50} \) does. If equations 6-11 and 6-9 are replaced in (6-7):
Or, using the powder factor-explicit forms:

\[
\frac{n}{\ln(3B^{0.333}S^{-0.118}H^{-0.032}q^{0.123})} = 0.842
\]

The small values of the exponents show that the influence of all variables in the uniformity index is very weak. The uniformity index in the C-K and K-R formulae have different forms, being it difficult to compare them directly. The uniformity index is in both models independent of the rock factor. The burden and \(S/B\) have a similar effect than in the K-R model, but the blasthole diameter, charge length (both included in the explosive mass), and bench height have an opposite effect in each model; in C-K an increase of the mass implies a decrease of \(n\).

At present there is no information about the performance of this model in full-scale blasts. Explosive energy is not included in the model, as all the tests were made with the same explosive.

### 6.8.3. SveDeFo fragmentation formula

The Swedish Detonics Research Foundation (Ouchterlony et al., 1990) model comprises only an estimation of the mean size:

\[
x_{50} = \frac{1}{6.99} \left( B^2 \left( \frac{1.25}{S/B} \right)^{0.29} \left( \frac{c}{q_T} \right)^{1.35} \right)
\]

Where:

- \(c\), the rock constant (Langefors & Kihlström, 1963), a blastability index. It is usually made equal to 0.4 kg/m\(^3\) as a default value. Sanchidrián et al. (2002) calibrated it for different rock types and found a linear relation with the powder factor above grade (AECI, 1986) required, see Table 6-8.
- \(q_T\), is the specific charge including the explosive in the subdrill zone.
- \(s\), is the strength relative to a Swedish dynamite (LFB), which combines both the heat of explosion and the volume of gases (weighed, respectively, 5 to 1); strength of ANFO is 0.84.

Equation 6-12 shows a different approach to the previous formulae, as the drilling parameters affect the mean size not only through the powder factor. The exponents affecting \(B\) and \(S\) in the SveDeFo formula are one order of magnitude higher than the ones shown in C-K’s Equation 6-10. Hence, in the SveDeFo formula, conversely to K-R and C-K, \(x_{50}\) changes when \(S/B\) or the burden vary, even if the powder factor is kept constant. Additionally, the influence of the explosive energy in Equation 6-12 is greater than the one shown in K-R.

The SveDeFo’s size distribution function is a Rosin-Rammler with a constant uniformity index:

\[
P(x) = 1 - e^{-\left( \frac{0.76x}{x_{50}} \right)^{1.35}}
\]

Note that \(0.76^{1.35} \approx \ln 2\).
6.8.4. Kou-Rustan formula

Kou and Rustan (1993) present a $x_{50}$ fragmentation prediction formula based on the analysis of the literature and the data of small-scale blasts. The purpose is to predict $x_{50}$ with a precision of $\pm 15\%$ for every rock types:

$$x_{50} = \frac{0.01 \left( \rho_r c_p \right)^{0.6} \left( BS \right)^{0.5}}{B^{0.8} \left( L_{tot} / H \right)^{0.7} D^{0.4} \rho_r}$$

(6-13)

Where $x_{50}$ is in centimetres, $c_p$ is the propagation velocity of the P-waves, $\rho_r$ is the density of the rock, $L_{tot}$ is the total length of the explosive charge and $D$ is the velocity of detonation.

In this model, the rock factor is expressed in a different way than in the previous models; the rock features are described by means of the acoustic impedance of the Median $\rho_r c_p$. With respect to the explosive strength, the heat of explosion was replaced in the original formula by the velocity of detonation since this seems to have more influence in fragmentation. The inclusion of VOD is of particular interest as it is easily measured and is a good indicator of the explosive performance.

The geometrical parameters $B$ and $S$ appear explicitly in Equation 6-14 as occurs with the SveDeFo formula. Operating in 6-13:

$$x_{50} = \frac{0.01 \left( \rho_r c_p \right)^{0.6} \left( S / B \right)^{0.5} B^{0.2}}{(L_{tot} / H)^{0.7} D^{0.4} \rho_T}$$

The $S/B$ term has more weight and affects inversely to $x_{50}$ than in SveDeFo’s Equation 6-12. This contradiction is due to the different source of the data used for developing each formula. The inaccuracy of the data may also have some influence on this. This model outlines that the actual effect of $S/B$ on fragmentation is unknown. The other parameters included in Equation 6-13 affect $x_{50}$ in a similar way as occurs in the SveDeFo formula, although the exponents are different.

6.8.5. Kuznetsov-Cunningham-Ouchterlony model

The Kuznetsov-Cunningham-Ouchterlony model (KCO) updates the Kuz-Ram model for using the Swebrec distribution; the Rosin-Rammler distribution is replaced by the Swebrec function. Ouchterlony (2004b) states that the KCO model overcomes two important drawbacks of K-R, the poor predictive capacity in the fines range and the infinite upper limit to block sizes. The KCO model is based on the following formulas:

- Prediction of the mean size, $x_{50}$:

$$x_{50} = g(n) A Q_e^{1/6} q^{-0.8} \left[ \frac{RBS}{115} \right]^{-19/30}$$

with $g(n) = (\ln 2)^{1/n} / \Gamma(1+1/n)$

(6-14)

The mean size depends on the uniformity index thorough $g(n)$; Ouchterlony (2004b) uses Equation 6-6 for $n$ predictions. $g(n)$ is plotted versus $n$ in Figure 6-21. The factor $g(n)$ shifts the fragment size distribution to smaller $x_{50}$, i.e. more fines are predicted, but its effect becomes negligible for rather uniform distributions, $n > 1.7$. Ouchterlony (2004b) advises that it must be analysed if the factor $g(n)$ is really needed as the Swebrec function has a
built-in fines bias. If \( g(n) \) is made equal to unity, Equation 6-14 is exactly that of the Kuz-Ram, Equation 6-4.

![Graph of g(n) vs. n](image)

Figure 6-21. \( g(n) \) vs. \( n \)

- Prediction of the undulation parameter, \( b \):

\[
b = 2 \ln 2 \ln \left( \frac{x_{\text{max}}}{x_{50}} \right) n
\]  \hspace{1cm} (6-15)

- Prediction of \( x_{\text{max}} \):

\[x_{\text{max}} = \min(\text{in-situ block size, } S \text{ or } B)\]

Ouchterlony (2004b) suggests that the expression for \( x_{\text{max}} \) is tentative and would be replaced when a better description of blasting in a fractured rock mass becomes available.

### 6.8.6. Summary of the fragmentation prediction models

There is more information available about how to predict the mean size in fragmentation by blasting, than what is available for predicting the shape of the size distribution curve. For the latter, it is required a second parameter that gives an idea of the uniformity, which is an indication, for instance of the fines and oversizes fractions. The mean size of the size distribution is an easy concept and relatively easy to measure, even with the actual limitations of the digital image analysis softwares. On the other hand, it is more difficult to estimate the fines at present, as their sizes are usually below the resolution of the images used. Despite of this, there are models like the CZM (Kanchibotla et al., 1999; Thornton et al., 2001a & b) that assume a special law for the fines generated around the blasthole. These models are wrong in their basic assumption that all the fine material comes from the surroundings of the blasthole (Svhan, 2002; Moser, 2003). Hence, the fact that the bimodal distributions used in these models fit well the size distribution, better than the R-R, it is not an evidence of two independent fracture mechanisms (Ouchterlony, 2003).

None of the existent models consider the influence of the timing in fragmentation.

Segarra & Sanchidrian (2004) made a parametric analysis of the Kuz-Ram, Chung-Katsabanis and SveDeFo formulae. Typical values at El Alto quarry were given to the variables involved.
Table 6-9 shows the different situations considered and the respective predicted variations in $x_{50}$ and $n$.

Table 6-9. Predicted variations in $x_{50}$ and $n$ by changes in blasting (Segarra & Sanchidrián, 2004).

<table>
<thead>
<tr>
<th>Changes</th>
<th>Constraints</th>
<th>Variations in $x_{50}$ %</th>
<th>Variations in $n$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q=0.37 \text{ kg/m}^3; \phi=142 \text{ mm};\rho_e=800 \text{ kg/m}^3$</td>
<td>K-R</td>
<td>C-K</td>
</tr>
<tr>
<td>$1.0 \leq S/B \leq 1.5$</td>
<td>$5.3 \leq B \leq 4.5 \text{ m}$</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>$120.7 \leq \phi \leq 152.4 \text{ mm}$</td>
<td>$4.0 \leq B \leq 5.1 \text{ m}$</td>
<td>+9</td>
<td>+3</td>
</tr>
<tr>
<td>$0.3 \leq q \leq 0.4 \text{ kg/m}^3$</td>
<td>$5.3 \leq B \leq 4.6 \text{ m}$</td>
<td>−23</td>
<td>−35</td>
</tr>
<tr>
<td>$700 \leq \rho_e \leq 950 \text{ kg/m}^3$</td>
<td>$4.5 \leq B \leq 5.2 \text{ m}$</td>
<td>+5</td>
<td>+1</td>
</tr>
<tr>
<td>$0.32 \leq q \leq 0.44 \text{ kg/m}^3$</td>
<td>$\phi=142 \text{ mm}; S=6 \text{ m}; \rho_e=800 \text{ kg/m}^3$</td>
<td>−25</td>
<td>−38</td>
</tr>
</tbody>
</table>

The main hints extracted in order to analyze the influence of blasting parameters in fragmentation and particularly on the fines production are the following:

a) Moderate changes in blasthole diameter (from 121 to 152 mm) and explosive density (from 700 to 950 kg/m$^3$) do not affect significantly the fragmentation; Figure 6-22 show the $x_{50}$ and $n$ plot versus the blasthole diameter and the burden, whereas Figure 6-23 show the variations of fragmentation parameters versus the explosive density and the burden.

b) Variations in $S/B$ (from 1 to 1.5) keeping constant the powder factor only lead to large changes in fragmentation if SveDeFo and/or K-R apply; $x_{50}$ and $n$ are plotted versus $S/B$ and the respective burden in Figure 6-24.

c) An increase of powder factor above grade (from 0.3 to 0.4 kg/m$^3$) keeping constant $S/B$ moves the size distribution curve almost parallel to itself onto smaller sizes, as only the mean size is affected and not the uniformity index; $x_{50}$ and $n$ are plotted versus the powder factor and the burden in Figure 6-25.

d) An increase of powder factor keeping constant the spacing (and not $S/B$) moves the curve toward smaller sizes in C-K (more fines and less oversizes are obtained); the changes in $x_{50}$ control variations in fines and oversizes. Similar behavior is obtained for the SveDeFo formula (which also predicts a decrease of $x_{50}$ when the powder factor increases), as the uniformity index is a constant in this case. In the K-R model both $x_{50}$ (decreases) and $n$ (increases) when the powder factor is increased, which leads to less oversizes and less fines. $x_{50}$ and $n$ are plotted versus the explosive density and the respective burden in Figure 6-26 and the size distribution curves predicted by K-R and C-K models for the extremes of the ranges defined are shown in Figure 6-27.
6 Measurement and Analysis of Fragmentation

Figure 6-22. Effect of blasthole diameter variation in (a) $x_{50}$ and (b) $n$; $q=0.37 \text{ kg/m}^3$; $\rho_e = 800 \text{ kg/m}^3$; $S/B=1.25$.

Figure 6-23. Effect of explosive density variation in (a) $x_{50}$ and (b) $n$; $q=0.37 \text{ kg/m}^3$; $d=142 \text{ mm}$; $S/B=1.25$.

Figure 6-24. Effect of variations of $S/B$ in (a) $x_{50}$ and (b) $n$. $q=0.37 \text{ kg/m}^3$; $\rho_e = 800 \text{ kg/m}^3$; $d=142 \text{ mm}$.
### Measurement and Analysis of Fragmentation

Table 6-1: Effect of powder factor variation in $X_{50}$ and $n$; $d=142$ mm; $\rho_e = 800$ kg/m$^3$; $S/B=1.25$.

<table>
<thead>
<tr>
<th>$q$ (kg/m$^3$)</th>
<th>$X_{50}$ (mm)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>0.32</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>0.34</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>0.36</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>0.38</td>
<td>210</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>220</td>
<td></td>
</tr>
</tbody>
</table>

- **Svedefo ($\Delta = 46\%$)**
- **K-R ($\Delta = 23\%$)**
- **C-K ($\Delta = -35\%$)**

### Figures

**Figure 6-25.** Effect of powder factor variation in (a) $X_{50}$ and (b) $n$; $d=142$ mm; $\rho_e = 800$ kg/m$^3$; $S/B=1.25$.

**Figure 6-26.** Effect of variations of powder factor and burden in (a) $X_{50}$ and (b) $n$; $d=142$ mm; $S=6$ m; $\rho_e = 800$ kg/m$^3$.

**Figure 6-27.** Size distribution curves obtained for $0.32 \leq q \leq 0.44$ kg/m$^3$ & $5.5 \leq B \leq 4.0$ m ($\phi = 142$ mm; $S=6$ m; $\rho_e = 800$ kg/m$^3$).
Medium size

In all the models reviewed the Equation that predicts \( x_{50} \) is the product of rock, geometry-charging and explosives parameters, all powered to different exponents. The KCO includes, additionally, the term \( g(n) \) that makes \( x_{50} \) smaller as the distribution is less uniform. The following comments can be made for each group of parameters:

- Rock parameters: they are lumped in the rock factor/rock constant, which must be understood as a blastability index that describes the response of the rock mass to the blast. The rock factor/rock constant must consider in some way the strength of the rock and the in situ geotechnical features of the rock mass.

  Each of the models reviewed defined the rock differently. The C-K model does not give any formula for calculating the rock factor, and it is then inferred that \( A' \) is a fitting constant to the actual \( x_{50} \). The Kuz-Ram and the models derived from it use the more complete rock factor. It is however worth noting, that the tensile strength is not accounted, although the tensile failure of the rock is one the predominant mechanisms in the breakage process. The mechanical features of the rock are neither referred from a dynamic point of view, despite of being the blast a dynamic process. Finally, the exponents or weights affecting each of the parameters included in the reviewed models are independent of the rock type.

- Geometrical and charging parameters: they are mainly described with the burden \( B \), the ratio \( S/B \) (both variables only have an explicit influence in the SveDeFo and Kou-Rustan formulae, the explosive mass (only in K-R) and the charge length (only in Kou-Rustan). The exponents that affect to the former parameters are different in each models and even contradictory. The powder factor (it lumps all the previous parameters) is the predominant parameter, with power dependence of 0.8 in Kuz-Ram and KCO, 1 in Kou-Rustan, 1.230 in Chung-Katsabanis and 1.35 in SveDeFo. Other related published works, such as that by Otterness (1991) speak in the same line, with an exponent 1.17.

- Explosive parameters: all the models except Chung-Katsabanis consider some characteristic of the explosive (besides the density, which is implicitly involved in the explosive mass per blasthole). Each model defines the performance of the explosive in a different way: Swedish strength in SveDeFo; heat of explosion at constant volume in Kuz-Ram and velocity of detonation in Kou-Rustan. The last one should perhaps be more widely used in the fragmentation models, since it shows how far the explosive is from its ideal detonation regime. The VOD is also determinant for the pressure in the blasthole. However, the velocity of detonation must be accompanied with some index of the explosive energy –this may explain why explosives with similar VODs like ANFO and aluminized ANFO deliver different amounts of energy to the rock, as Table 3-4 shows. Additionally, it would be advisable a normalization with respect to the explosive strength; in this line it is more common nowadays to define the strength of the explosive from the useful work.

Uniformity parameters

The model of Chung-Katsabanis gives a very narrow range of variations, while Kuz-Ram predicts large variations as Table 6-9 and Figures 6-22 to 6-26. Nevertheless, both models are based in the Rosin-Rammler, whose accuracy in the fines part of the size distribution curve is
frequently limited. The formulae that predict the slope of the size distribution curves do not include neither rock parameters nor explosive ones. It could be argued that, having a strong influence on the oversize fraction, and being this fraction largely dependent on the in-situ fractures of the rock mass, the uniformity index should bear some dependence on the rock properties, specially the in situ fractures of the rock mass. The KCO model takes this into account, as it includes in the $b$-formula, the maximum size of the size distribution curve (somewhat related with the geotechnical features of the rock mass) and $x_{50}$ (which includes the rock factor). The C-K and K-R models only consider geometrical and charging parameters; the influence of the powder factor it is not apparent in Kuz-Ram and very small in Chung-Katsabanis.

It seems difficult to give strategies about how to decrease the fines from variations in the uniformity index; the KCO model has been only recently developed and its $b$-prediction ability should be checked. Therefore, it is more convenient for controlling fragmentation to act upon the median size, as the influence of the blasting characteristics on $x_{50}$ is more contrasted. Therefore, it is probably safer to act upon the variables with a greater weight in $x_{50}$ in order to outline the changes in fragmentation over the inherent scatter in production blasts; for instance, explosive energy and powder factor.

6.8.7. Application of the fragmentation prediction models

The data of the fragmented limestone in 33 blasts are compared with the predictions of the models described in Sections 6.8.1 to 6.8.5. The required drilling and loading parameters given in Appendix A are introduced in Equations 6-4 and 6-6 (K-R); 6-7, 6-9 and 6-11 (C-K); 6-12 (SveDeFo); 6-13 (Kou-Rustan); and 6-14 and 6-15 (KCO). In the calculations, $h$ is used instead of $H$, see Table 3-1. $B$ is taken as the average burden. The explosive mass in the subdrilled zone is not included in K-R; additionally, in Equation 6-6 the term that considers $L_B$ and $L_C$ is rejected as $W/B$ term; the last one due the negligible drilling deviations. The Swedish strength (heat of explosion and volume of gasses calculated with W-Detcom code) for Goma 2 ECO, Alnafo, High density Alnafo, Emunex 8000 and Low density ANFO are 0.85, 0.84, 1.00, 0.67 and 0.78 respectively. In the KCO model, $g(n)$ is not taken as one and the Swebrec parameter $x_{max}$ is fixed to 1000 mm (maximum size loaded in the hammer mill).

**Median size**

Figure 6-22 shows the predicted $x_{50}$ with K-R, C-K, KCO, SveDeFo and Kou-Rustan models versus the actual one for 33 blasts; the actual $x_{50}$ is obtained from the Rosin–Rammler fitting except in KCO model, in which the data from the Swebrec’s function used.

It is assumed that the rock behaves uniformly in all blasts; according to Table 6-8, $A$ (K-R) and $c$ (SveDeFo) are taken as 7 and 0.27 kg/m$^3$. For consistency $A'$ (C-K) is also taken as 7. The data given in Section 3.1.1 is used to calculate the term $\rho_r c_p$ in the Kou-Rustan formula. The predicted $x_{50}$ is always above the observed ones, as Figure 6-28 shows. This, however, is not a restriction for analyzing a possible relation between the model predictions and the reality. The rock factors $A'$ and $A$, as expected, are not equivalent since the mean size predicted by K-R and C-K for each blast are different.
Figure 6-28. Predicted $x_{50}$ with K-R, KCO, C-K, SveDeFo and Kou-Rustan vs. the actual $x_{50}$ for 33 blasts.

Figure 6-29. Predicted $x_{50}$ with K-R, KCO, SveDeFo and Kou-Rustan when $h$ and $q$ are refereed to limestone vs. the actual $x_{50}$ for 33 blasts.
In K-R, SveDeFo and CKO, the predicted $x_{50}$ increases as the experimental one does, although less than expected. In KCO, the factor $g(n)$ reduces $x_{50}$ and leads to a flatter trend line than K-R. The slopes of the trend lines to SveDeFo’s and Kuz-Ram’s data are similar. The C-K and Kou-Rustan predictions decrease as $x_{50}$ increases, which has no sense.

Although some models comply with the general trend, the slope of the trend lines in K-R and SveDeFo’s is far from unity. Perhaps, the blasting parameters $L_{tot}$, $h$ and $q$, should be referred to the limestone, as our data is relative to fragmented rock. The parameter $L_{tot}$ includes the explosive in the top marl, however usually there is not much explosive there, except when the overburden is really thick. $h$ and $q$ (or $q_T$) change strongly if the overburden is neglected; if $h$ and $q$ are replaced by $h' = h - h_{ob}$ and $q' = q [h/(h-h_{ob})]$ respectively, Figure 6-29 shows the $x_{50}$ predicted with K-R, C-K, KCO, SveDeFo and Kou-Rustan models versus the actual one for 33 blasts. The respective exponents in each formula are kept, although the overburden thickness, $h_{ob}$, was not considered in any of the original formulae.

Figure 6-29 shows that the slope of the trend lines is positive in all the series, even in Kou-Rustan, but in this case the predicted $x_{50}$ varies very little as the actual one does. The trend lines for K-R, KCO and SvedeFo models are steeper than in Figure 6-28. However, although the data is more packaged about the trend line the scatter is still large in all the series resulting in a $R^2$ extremely low. This may be caused by:

(i) A variable behavior of the rock from blast to blast due to some differences in geotechnical features like orientation of discontinuities with respect to the face. Rock constants $A$ and $c$ are not rock properties but blastability indexes and they might conceptually change from blast to blast depending on features like those. Indeed, this cannot possibly be done in practical terms

(ii) The influence of timing in fragmentation is not included in any model.

(iii) Bad characterization of the explosive energy (in C-K the explosive energy is not considered) or different relative importance of the parameters included in the formulae.

**Uniformity parameters**

The K-R and C-K formulae for predicting $n$ and the KCO prediction formula for $b$ are compared with the results of the 33 blasts in Figure 6-30. No relation between experimental and predicted uniformity values can be drawn from it. The $n$-values given by the K-R model are clearly above the actual ones. In the best case (achieved with Equation 6-6) the K-R’s $n$ varies from 1.09 to 1.62, which is twice or more the measured values. The predicted $n$ values with the Chung-Katsabanis model are close to the actual ones, but the values for the 33 blasts with very different blast layouts are in a tight range, from 0.68 to 0.73. This makes the model too stiff, unable to detect changes and leads in practice to a constant uniformity index as the SveDeFo formula does.

The predicted undulation factor, $b$ with the KCO model varies in a similar range, from 1.6 to 2.7, as the actual data, which changes between 1.4 and 2.5 and in some blasts the predicted values matches the actual ones. However, the trend line (not shown) is almost horizontal.

The situation, see Figure 6-31, does not improve when $h$ is replaced by $h'$; the exponents and coefficients in Equations 6-6 (K-R), 6-9 and 6-11 (C-K) are not varied. A negative correlation is
obtained between the predicted and the actual uniformity parameters for K-R and KCO. C-K remain unchanged.

Figure 6-30. Predicted uniformity parameters with K-R, KCO and C-K vs. the actual $x_{50}$ for 33 blasts

Figure 6-31. Predicted uniformity parameters with K-R, KCO and C-K when $h$ is refereed to limestone vs. the actual $x_{50}$ for 33 blasts

6.9. An approach to the factors governing fragmentation

The fragmentation prediction models reviewed above (Kuz-Ram, Chung-Katsabanis, SveDeFo, Kou-Rustan and Kuznetsov-Cunningham-Ouchterlony) show a first set of blasting parameters that may affect to the resulting fragmentation. They are blasthole diameter, $d$, average burden, $B$, spacing to burden ratio, $S/B$, blasted height, $h$ (note that when the hole bottom is above grade, the bench height is taken equal to the vertical projection of the hole length), subdrill length, $J$, 

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explosive mass per hole, \((Q_e)\), explosive mass per hole above grade, \(Q_e\), total length of the hole charged, \(L_{rot}\), length of hole charged above grade, \(L\), total powder factor, \(q_T\), powder factor above grade, \(q\), explosive energy per unit mass, heat of explosion, \(E_Q\), and useful work, \(E_w\), velocity of detonation, \(D\), and timing, by means of \(t\) (in-row delay) and \(\delta t\) (scatter of the in-row delay).

Other blasting parameters to consider in our case are the overburden thickness, \(h_{ob}\), rock height, \(h'\), total powder factor in limestone, \(q_T\), linear charge density, \((Q_e)\)/\(L_{rot}\), linear charge density above grade, \(Q_e/L\), linear charge density per bench height, \(Q_e/h\), linear charge density in the limestone, \(Q_e/h'\), explosive density, \(\rho_e\), energy (useful work) above grade per hole, \(Q_e/E_w\), energy (heat of explosion) above grade, \(Q_e/E_Q\), linear energy (useful work) density, \((Q_e/L)E_w\), linear energy (heat of explosion) density, \((Q_e/L)E_Q\), energy (useful work powder factor), \(q' E_w\), energy (heat of explosion) powder factor, \(q' E_Q\), energy (useful work) powder factor in limestone, \(q' E_w\), and in-row delay to burden ratio, \(t/B\).

Non-dimensional groups of blasting parameters may also have some influence in the resulting fragmentation. Of them, the most popular is the spacing to burden ration. Besides this, the following relations are selected: blasted block aspect ratio, \(h/B\) and relative to limestone \(h'/B\), \(L/B\), \(L/h\), \(h/(B\cdot S)^{0.5}\), and for limestone, \(h'/(B\cdot S)^{0.5}\), \(B/d\), \((t/B)\cdot D\), ratios of explosive energies per unit mass to VOD, \(E_w/D^2\) and \(E_Q/D^2\) and ratio of detonator scatter to in-row delay, \(\delta t/t\).

Since there were not significant differences in the quality of the fitting between the Rosin-Rammler and the Swebrec distributions, both the \(R-R\) and \(S\) parameters are considered, \((x_{50})_{R-R}, n\), \((x_{50})_{S}, b\) and \(x_{max}\). Additionally, Ouchterlony (2004c) suggests the use of the slope value at \(x_{50}\) of either Rosin-Rammler or Swebrec distributions, \(S_{R-R}\) and \(S_{S}\):

\[
S_{R-R} = 0.5 \ln \left( \frac{n}{x_{50}} \right) \tag{6-16}
\]

\[
S_{S} = 0.25 \frac{b}{x_{50} \ln(x_{max}/x_{50})} \tag{6-17}
\]

Appendix E gives a summary of the statistics of those parameters/non dimensional relations for the 32 blasts in which all the blasting parameters are known (blast 37/02 is not considered since the timing is unknown) and the fragmentation data has an acceptable quality (blasts 96/03 and 8/04 are not included as \(P_{14\,mm}\) is smaller than the \textit{natural fines}, see Section 6.6.1). The mean and standard deviation for \(d\) and \(\delta t\) are not given. Blasting data in Appendix A and fragmentation data in Appendix D. The mean values for the whole blast are used, when blasting parameters are measured hole per hole (e.g. explosive mass, burden, etc). The overburden thickness, \(h_{ob}\), subdrill length, \(J\), blasted block aspect ratio in limestone \(h'/B\), heat of explosion above grade, \(Q_eE_Q\), \((t/B)\cdot D\), \(E_w/D^2\), in-row delay, \(t\), \(t/B\), and \(\delta t/t\) are the blasting parameters with a ratio of the standard deviation to the mean value greater than 20\%, i.e. parameters with largest variations.

A correlation analysis has been done with each of the sets of blasting parameters described above (parameters from fragmentation prediction models \((d, B, S/B, h, J, (Q_e)\), \(Q_e, L_{rot}, L, q_T, q, E_Q, E_w, D, t\) and \(\delta t\)) other parameters \((h_{ob}, h', q_T', q', (Q_e)/L_{rot}, Q_e/L, Q_e/h, Q_e/h', \rho_e, Q_eE_Q, Q_eE_Q, (Q_e/L)E_w, (Q_e/L)E_Q, q' E_w, q' E_Q, q' E_w\) and \(t/B\)) and non dimensional groups \((h/B, h'/B, \ldots)\).
6 Measurement and Analysis of Fragmentation

\[ \frac{L}{B}, \frac{L}{h}, \frac{h'(B-S)^{0.5}}{B/d}, \frac{t(ByD)}{E_w/D^2}, \frac{E_P/D^2}{\delta/t} \]

and the fragmentation parameters \((x_{50})_{R-R}, n, S_{R-R}, (x_{50})_S, b, x_{\text{max}}\) and \(S_\delta\). The respective correlation coefficients, which measure the linear relationship between variables, are given in Tables 6-10, 6-11 and 6-12 for the set of 32 blasts; the p-value is below 0.05 in the relations shown in bold, which indicates a statistically significant non-zero correlation at the 95% confidence level.

The correlation analysis shows numerous relations, but only the most outstanding for our interests are commented. The maximum size, \(x_{\text{max}}\), has very low correlation coefficients with either \(b\) or \((x_{50})_S\). There are strong relations between \((x_{50})_{R-R}\) and the other two R-R parameters, \(n\) and \(S_{R-R}\), of 0.9 and -0.8 respectively. The relations between the respective Swebrec parameters, \((x_{50})_R-b\) and \((x_{50})_S-S_\delta\) are weaker; their correlation coefficients are 0.5 and -0.4. The correlations between \(S_{R-R}-n\) and \(S_\delta-b\) , -0.5 and -0.4 respectively, are opposite what Equations 6-16 and 6-17 suggest. In general, \(x_{50}\) controls changes in fragmentation; less fines and oversizes are obtained as \(x_{50}\) increases. This leads to smaller \(S_{R-R}\) and \(S_\delta\).

The R-R and S mean sizes are equivalent, their correlation coefficient is one. Additionally, the uniformity index, \(n\), is strongly related with the Swebrec distribution parameters, \(b\), and \((x_{50})_S\), with correlations of 0.9 and 0.8 respectively. This shows the equivalency between the R-R and S fittings.

The correlations within group of parameters, either blasting or fragmentation, are stronger than between them. The inherent scatter within each blast, i.e. variations in the burden from hole to hole, is probably the source of the low correlation coefficients obtained between blasting and fragmentation parameters, up to 0.4.

The in-row delay, \(t\) (only for Swebrec fitting) and the scatter \(\delta t\) affect \(x_{50}\) with a correlation coefficient of -0.4; the influence of the powder factor above grade, \(q\), is very low, -0.1, whereas the powder factor in limestone, \(q'\) and the energy powder factor have a higher influence of -0.3. The parameters \(t\), \(\delta t\), \(t/B\), linear charge density in the limestone \(Q_e/h'\), \(q'\) and overburden thickness, \(h_{ob}\), affect \(S_{R-R}\) positively with a correlation coefficient of 0.4. From those, only \(t\) and \(t/B\) have the strongest correlation coefficient with \(S_\delta\), 0.4. The highest correlation factor between \(b\) and \(n\) and some of the blasting parameters is -0.3; they are \(b-h_{ob}\), \(b-q'\), \(n-h_{ob}\), \(n-\delta t\), \(n-Q_e/h'\), \(n-Q_e-E_Q\) and \(n-(Q_e/L)E_Q\). Again more parameters affect to the R-R’s \(n\) than to the Swebrec’s parameter. The powder factor above grade in limestone is the parameter with highest influence in \(x_{\text{max}}\), the correlation factor is -0.4. This verifies that the maximum size in the bin of the crusher is not related with either of the parameters shown in the KCO model, in situ block size, burden and spacing.
Table 6-10. Correlation matrix of blasting parameters from fragmentation prediction models and fragmentation parameters for 32 blasts

| $(x_{50})_R$ | $n$ | $S_{R,R}$ | $(x_{50})_S$ | $b$ | $x_{max}$ | $S_S$ | $d$ | $B$ | $S/B$ | $h$ | $(Q_e)_T$ | $Q_e$ | $L_{tot}$ | $q_r$ | $q$ | $E_Q$ | $E_W$ | $D$ | $t$ | $\delta t$ |
|-------------|-----|----------|---------------|----|-----------|------|----|-----|------|----|----------|------|--------|------|----|--------|------|----|-------|------|-----|
| 1.0         | 0.9 | -0.8     | 1.0           | 0.7| 0.1       | -0.3 | -0.3| 0.1  | -0.2 | -0.2| -0.1     | -0.2 | -0.1   | 0.0  | -0.1| -0.2   | -0.3 | 0.2 | -0.3   | -0.4 | 0.4 |
| 1.0         | -0.5| 0.8      | 0.9           | -0.3| -0.2     | 0.2  | 0.1  | -0.2| -0.2 | -0.1| -0.2     | 0.0  | -0.1   | 0.0  | -0.1| -0.2   | -0.2 | 0.2 | -0.1   | -0.3 | 0.1 |
| 1.0         | -0.9| -0.3     | 0.7           | -0.3| -0.1     | 0.2  | 0.3  | 0.0  | 0.2  | 0.3  | 0.2     | 0.2  | 0.0    | 0.1  | 0.2 | 0.3    | -0.2 | 0.4 | 0.4    | 0.4  | 0.3 |
| 1.0         | 0.5 | -0.4     | 0.5           | 0.1 | 0.1      | -0.3 | 0.1  | -0.2| -0.3 | -0.1| -0.2     | 0.0  | -0.1   | 0.0  | 0.1 | 0.2    | -0.2 | 0.4 | 0.3    | 0.4  | 0.3 |
| 1.0         | 0.1 | 0.5      | 0.0           | 0.1 | 0.0      | 0.2  | 0.1  | 0.2  | 0.2  | 0.3  | 0.2     | 0.0  | -0.1   | 0.1  | 0.0 | 0.2    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.2 | 0.0      | 0.1           | 0.1 | 0.0      | 0.1  | 0.2  | 0.2  | 0.2  | 0.0  | 0.1     | 0.1  | 0.2    | 0.0  | 0.4 | 0.2    | -0.2 | 0.4 | 0.1    | 0.6  | 0.1 |
| 1.0         | 0.4 | 0.1      | 0.2           | 0.1 | 0.0      | 0.3  | 0.3  | 0.0  | 0.0  | 0.0  | 0.0     | 0.0  | 0.3    | 0.0  | 0.1 | 0.2    | 0.6  | 0.1 | 0.2    | 0.8  | 0.1 |
| 1.0         | -0.6| 0.1      | 0.0           | 0.3 | 0.3      | 0.0  | 0.0  | 0.0  | 0.0  | -0.6 | 0.7     | -0.1 | 0.0    | 0.3  | 0.1 | 0.2    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.4 | 0.8      | 0.8           | 0.8 | 0.9      | 0.2  | 0.0  | 0.4  | 0.2  | -0.3| 0.1     | 0.6  | 0.1    | 0.3  | -0.1| 0.4    | 0.6  | 0.1 | 0.2    | 0.9  | 0.8 |
| 1.0         | 0.6 | 0.4      | 0.6           | 0.4 | 0.4      | 0.2  | 0.1  | 0.4  | 0.2  | -0.3| 0.1     | 0.4  | 0.4    | 0.2  | 0.5 | 0.4    | 0.5  | 0.4 | 0.6    | 0.9  | 0.4 |
| 1.0         | 0.9 | 0.8      | 0.8           | 0.4 | 0.1      | 0.2  | 0.3  | 0.2  | -0.2| 0.0  | 0.4     | 0.4  | 0.4    | 0.2  | 0.5 | 0.4    | 0.5  | 0.5 | 0.0    | 0.9  | 0.8 |
| 1.0         | 0.4 | 0.0      | 0.5           | 0.4 | 0.0      | 0.5  | 0.4  | 0.0  | -0.5| 0.5  | 0.0     | 0.5  | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.9 | -0.1     | 0.0           | 0.5 | -0.4     | -0.1 | -0.1| -0.1| 0.0  | 0.1    | 0.0     | 0.1  | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.8 | -0.6     | 0.0           | 0.8 | -0.6     | -0.2| 0.3  | 0.2  | -0.4| -0.1| 0.2     | 0.4  | 0.1    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | -0.4| 0.0      | 0.2           | 0.0 | 0.1      | -0.3| 0.2  | 0.2  | -0.2| 0.1  | 0.0     | 0.1  | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.1 | 0.0      | 0.0           | 0.2 | 0.0      | 0.2  | 0.2  | 0.0  | 0.2  | 0.0  | 0.0     | 0.0  | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |
| 1.0         | 0.2 | 0.0      | 0.0           | 0.1 | 0.0      | 0.2  | 0.2  | 0.0  | 0.2  | 0.0  | 0.0     | 0.0  | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 | 0.0    | 0.0  | 0.0 |

Notes: Values in bold are those correlations with a p-value below 0.05.
Table 6-11. Correlation matrix of other blasting parameters and fragmentation parameters for 32 blasts

| $(x_{so})_{R,R}$ | n | $(x_{so})_{S}$ | b | $x_{max}$ | $S_{R}$ | $h_{ob}$ | $h'$ | $q_{T}'$ | $q'$ | $(Q_{e})/L_{tot}$ | $Q_{e}/L$ | $Q_{e}/h$ | $Q_{e}/h'$ | $\rho_{e}$ | $Q_{e}/E_{W}$ | $Q_{e}/E_{Q}$ | $(Q_{e}/L)/E_{W}$ | $(Q_{e}/L)/E_{Q}$ | $q'E_{W}$ | $q'E_{Q}$ | $q'E_{W}$ | $t/B$ |
|----------------|---|----------------|---|----------|--------|---------|------|--------|-----|----------------|--------|---------|----------|--------|-------------|-------------|----------------|----------------|-------------|-----------|-----------|-------|--------|
| 1.0            | 0.9 | -0.8          | 1.0 | 0.7      | 0.1    | -0.3    | -0.3  | 0.1    | -0.3 | 0.0            | -0.1   | -0.1    | -0.1    | -0.3 | -0.3       | -0.1       | -0.3           | -0.1           | -0.1       | -0.3       | -0.3   |
| 1.0            | -0.5 | 0.8           | 1.0 | 0.9      | -0.3   | -0.3    | 0.1   | -0.2   | -0.2  | -0.1          | -0.1   | -0.1    | -0.1    | -0.3 | -0.3       | -0.1       | -0.1           | -0.2           | -0.1       | -0.3       | -0.3   |
| 1.0            | -0.9 | -0.3          | 1.0 | 0.2      | 0.7    | 0.4     | -0.1  | 0.3    | 0.4   | 0.0           | 0.0    | 0.2     | 0.4     | -0.1 | 0.3        | 0.3        | 0.2            | 0.3            | 0.1        | 0.3        | 0.1    |
| 1.0            | 0.5  | 0.1           | 1.0 | 0.0      | 0.6    | 0.7     | 0.1   | -0.2   | -0.2  | 0.0           | -0.1   | -0.3    | -0.3    | -0.2 | -0.3       | -0.2       | -0.3           | -0.3           | -0.2       | -0.3       | -0.2   |
| 1.0            | 0.1  | 0.5           | 1.0 | 0.0      | 0.4    | 0.5     | 0.1   | -0.3   | -0.3  | 0.0           | 0.0    | 0.1     | 0.1     | -0.1 | 0.1        | 0.0        | 0.1            | 0.0            | 0.1        | 0.0        | 0.0    |
| 1.0            | 0.2  | 0.0           | 1.0 | 0.1      | 0.7    | 0.1     | -0.2  | -0.2   | 0.0   | 0.0           | 0.0    | 0.1     | 0.1     | 0.1  | 0.2        | 0.0        | 0.0            | 0.0            | 0.2        | 0.0        | 0.0    |
| 1.0            | -0.7 | 0.6           | 1.0 | 0.1      | 0.0    | 0.4     | 0.2   | -0.1   | -0.2  | 0.0           | 0.0    | 0.1     | 0.0     | -0.1 | 0.1        | 0.0        | 0.0            | 0.0            | 0.0        | 0.0        | 0.0    |
| 1.0            | -0.4 | -0.6          | 1.0 | 0.1      | 0.0    | 0.5     | 0.1   | -0.3   | -0.3  | 0.0           | 0.0    | 0.1     | 0.1     | -0.1 | 0.0        | 0.0        | 0.0            | 0.0            | 0.0        | 0.0        | 0.0    |
| Notes: $h'$, $q_T'$ and $q'$ are \((h-h_{ob})\), \(q_T' \cdot [h/(h-h_{ob})]\) and \(q' \cdot [h/(h-h_{ob})]\) respectively.
Table 6-12. Correlation matrix of non-dimensional groups of blasting parameters and fragmentation parameters for 32 blasts

<table>
<thead>
<tr>
<th>((x_{50})_{R,R})</th>
<th>n</th>
<th>(S_{R,R})</th>
<th>((x_{50})_S)</th>
<th>(b)</th>
<th>(x_{max})</th>
<th>(S_S)</th>
<th>(h/B)</th>
<th>(h'/B)</th>
<th>(L/B)</th>
<th>(L/h)</th>
<th>(h'(B·S)^{0.5})</th>
<th>(h'(B·S)^{0.5})</th>
<th>(B/d)</th>
<th>((t/B)·D)</th>
<th>(E_{ql}/D^2)</th>
<th>(E_{ql}'/D^2)</th>
<th>(\delta/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.9</td>
<td>-0.8</td>
<td>1.0</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.3</td>
<td>-0.2</td>
<td>0.0</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.3</td>
<td>0.1</td>
<td>-0.2</td>
<td>-0.2</td>
<td>-0.2</td>
<td>((x_{50})_{R,R})</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.5</td>
<td>0.8</td>
<td>1.0</td>
<td>0.9</td>
<td>-0.1</td>
<td>0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>-0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.1</td>
<td>0.2</td>
<td>-0.3</td>
<td>-0.1</td>
<td>-0.2</td>
<td>(S_{R,R})</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.9</td>
<td>-0.3</td>
<td>1.0</td>
<td>0.7</td>
<td>0.2</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>-0.2</td>
<td>-0.1</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.2</td>
<td>(x_{50N})</td>
<td>(n)</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>(0.4)</td>
<td>1.0</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.3</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>0.2</td>
<td>-0.3</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.2</td>
<td>(h_{max})</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
<td>0.7</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>(S_S)</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>-0.1</td>
<td>1.0</td>
<td>0.8</td>
<td>0.9</td>
<td>0.4</td>
<td>0.9</td>
<td>0.8</td>
<td>-0.7</td>
<td>0.1</td>
<td>(h/B)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>(h'/B)</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
<td>0.7</td>
<td>0.0</td>
<td>-0.5</td>
<td>0.0</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
<td>(h'/B)</td>
<td>0.1</td>
<td>0.3</td>
<td>(L/B)</td>
<td>0.3</td>
<td>(h'(B·S)^{0.5})</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.2</td>
<td>0.4</td>
<td>1.0</td>
<td>0.7</td>
<td>(0.4)</td>
<td>-0.6</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.5</td>
<td>0.3</td>
<td>(h'(B·S)^{0.5})</td>
<td>0.1</td>
<td>0.3</td>
<td>(L/h)</td>
<td>0.1</td>
<td>(h'(B·S)^{0.5})</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.8</td>
<td>-0.4</td>
<td>1.0</td>
<td>0.7</td>
<td>0.0</td>
<td>0.2</td>
<td>0.4</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.3</td>
<td>(h'(B·S)^{0.5})</td>
<td>0.4</td>
<td>0.0</td>
<td>(B/d)</td>
<td>0.1</td>
<td>(h'(B·S)^{0.5})</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>-0.3</td>
<td>0.0</td>
<td>1.0</td>
<td>0.8</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.2</td>
<td>-0.1</td>
<td>0.1</td>
<td>-0.1</td>
<td>0.1</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>(E_{ql}/D^2)</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.1</td>
<td>-0.2</td>
<td>1.0</td>
<td>0.7</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>-0.1</td>
<td>(E_{ql}'/D^2)</td>
<td>0.0</td>
<td>0.0</td>
<td>(\delta/t)</td>
<td>1.0</td>
<td>(\delta/t)</td>
<td></td>
</tr>
</tbody>
</table>
A principal component analysis (PCA) has been made. The blasting parameters \((h_{ob}, h, h', d, Q, l_{tot}, E_w, D, t, \delta_t, q', Q_e/h', Q_eE_Q, Q_eE_{w}, (Q_e/L)E_Q, q'E_Q, q'E_{w}, t/B, h/B, (t/B)D, E_Q/D^2\) and \(h'/(B'S)^{0.5}\) with a correlation coefficient greater than 0.3 with either of the fragmentation parameters, are introduced in the analysis together with \((x_{50})_{R-R}, n, S_{R-R}, (x_{50})_S, b, X_{max}\) and \(S_S\). The aim is to obtain a small set of independent linear combinations (the components) of the fragmentation and blasting parameters. The parameters considered are standardized by subtracting their mean and dividing by their standard deviation. A total of four components are extracted, \(C_1, C_2, C_3\) and \(C_4\). The percent of variance accounted in each component are 30, 20, 11 and 11%; the four components explain just a 72% of the variability in the data. Figure 6-32 shows the weight of each parameter in each component.

In this particular case, poor results were expectable from the PCA as the correlation matrix does not show strong correlations between the blasting and fragmentation parameters. An interpretation of the components is not possible, as the weights of the variables in the components are rather low—in the best case they are 0.5—and the same variable has significant weights in various components—e.g. explosive mass above grade in \(C_1\) and \(C_4\) and \(Q_e/h'\) in \(C_2\) and \(C_4\).

![Figure 6-32. Weight of the blasting and R-R’s parameters in the four principal components](image)

(the parameters with higher weights in each component are marked with light color)
Consequently, the analysis in Figure 6-32 does not add relevant information. \( C_2 \) has very low weights in all variables, and it is difficult to extract a significant group out of them. This component seems to be a mix of the parameters considered, mainly fragmentation \([ (x_{50})_{R,R}, S_{R,R}, (x_{50})_S]\) plus overburden thickness, charge and geometry of the block-\( q' \) and \( Q_e/h' \). Note for instance that \((x_{50})_{R,R}, S_{R,R}, (x_{50})_S\) have high mutual correlation factors. It is worth mentioning, however, that the explosive energy per mass, which is relevant on fragmentation, is not in this component. The fragmentation parameters have very low coefficients in the other three components; \( C_3 \) is mainly explained by the in row delay, \( t \), and the groups in which \( t \) is involved, \( t/B \) and \((t/B)D \). The loading parameters \( h, \ h', Q_e, l_{tot} \) and either \( Q_e E_Q \) or \( Q_e E_w \) are the dominant parameters in \( C_1 \) (the correlation coefficients between \( h, Q_e \) and \( l_{tot} \) are 0.8), whereas the nominal blasthole diameter, \( d \), \( Q_e \) (and \( Q_e/h' \)), VOD, and energy powder factor are significant in \( C_4 \) (the explosive mass above grade is directly related with the square of the blasthole diameter, however they have a correlation coefficient of 0.5).

There are stronger mutual correlations between the Rosin-Rammler fragmentation parameters (also shown in \( C_2 \)) than within the Swecrec ones. The first ones are then more convenient for defining fragmentation prediction formulae for El Alto quarry in order to have greater correlation coefficients.

A potential function is fit between \((x_{50})_{R,R} \) and \( n \), with a \( R^2 \) of 0.78:

\[
n = 0.055 \cdot (x_{50})_{R,R}^{0.531} \tag{6-18}
\]

where \( x_{50} \) is given in mm. Figure 6-33 shows the \((x_{50})_{R,R} \) vs \( n \) plot, the resulting trend line and the 95 % prediction limits built from the standard error of the estimate, 0.07 (the formula for the upper limit is \( n=0.064 \cdot (x_{50})_{R,R}^{0.531} \), and for the lower limit is \( n=0.047 \cdot (x_{50})_{R,R}^{0.531} \)). The \( n \) values may vary around the trend line given by Equation 6-18, but it is clear that as \((x_{50})_{R,R} \) increases is more probable to have a more uniform size distribution curve. The KCO model shows that \((x_{50})_{R,R} \) is related with the uniformity index, together with some blasting parameters. Some blasting parameters, i.e. \( \delta t, Q_e/h' \), \( Q_e E_Q \), were added to Equation 6-18, with the aim of increasing \( R^2 \). They were dropped since their exponents (one order of magnitude smaller than that of \( x_{50} \)) include zero in the asymptotic interval at a 95 % confidence level.

\( S_{R,R} \) and \((x_{50})_{R,R} \) can also fitted with a potential function:

\[
S_{R,R} = 18.97 \cdot (x_{50})_{R,R}^{-0.47} \tag{6-19}
\]

where \( S_{R,R} \) is \( m^{-1} \) and \((x_{50})_{R,R} \) in mm. The correlation factor is again good, 0.74. \( S_{R,R} \) is plotted versus \((x_{50})_{R,R} \) in Figure 6-33; the trend line is also drawn together with the 95 % prediction levels (whose formulae are \( S_{R,R}=22.3 \cdot (x_{50})_{R,R}^{-0.47} \) and \( S_{R,R}=16.3 \cdot (x_{50})_{R,R}^{-0.47} \)).

If \((x_{50})_{R,R} \) increases, \( n \) also increases and \( S_{R,R} \) decreases. This leads to a negative correlation between \( n \) and \( S_{R,R} \) (-0.5), although Equation 6-16 shows the opposite; if 6-18 is replaced in 6-16 a similar Equation to 6-19 is obtained. Figure 6-34 shows the Rosin -Rammiller distribution curves for two assumed \( x_{50} \) values, 80 and 140 mm; the respective uniformity indexes are obtained applying Equation 6-18; linear scale has been used in both axes in order to outline the decrease in the slope value at \( x_{50} \) as the median size increases.

In fact, as \( x_{max} \) has a moderate variation over the nominal value of the maximum sizes loaded into the primary crusher, the size distribution curve can only be steeper if the mean size
increases. Hence, the direct relations between \((x_{50})_{R-R} - n\) and \((x_{50})_{R-R} - S_{R-R}\). Those relations may not necessarily stand for the real muckpile distribution.

According to the correlation matrix and the PCA, the blasting parameters can be packaged in the following groups: loading \((h, h', d, L_{tot}, Q_e, \text{ and } Q_e/h')\) whose mutual correlation coefficients are at least 0.6, except for \(d\) and \(Q_e/h'\), strength of the explosives \((E_{bl}, D, \text{ ratio of the explosive energy to the square of the velocity of detonation and powder factor energy})\), loading-geometry \((q')\) and timing \((t, \delta t, (t/B)D\) and \(t/B)\). The parameters with little influence in fragmentation have been rejected.
Formulae for \((x_{50})_{R-R}\) as a function of the blasting parameters are tried. The function,

\[
(x_{50})_{R-R} = A \left( q \cdot \frac{h}{h - h_{ob}} \right)^{\alpha} \left( \frac{E_W}{2918} \right)^{\beta}
\]

leads to a low correlation factor, \(R^2\) of 0.28; where \((x_{50})_{R-R}\) is in mm, \(q\) in kg/m\(^3\) and \(E_W\) in kJ/kg. The results of the non linear regression analysis are shown in Table 6-13. The powder factor above grade can only be kept in the model when the term \(h/(h-h_{ob})\) is included; \(h/(h-h_{ob})\) is equal to one when there is no overburden. If the total powder factor is considered, \(R^2\) decreases to 0.19. The explosive energy is normalized by the useful work of \(Alnaf\) given in Table 3-4. The use of the other explosive energy parameters leads in some cases to lower \(R^2\) and in others to exponents that includes zero in the asymptotic interval at a 95 % confidence level. For instance, if the useful work is replaced by the heat of explosion at constant volume and normalized by the respective value for \(Alnaf\), the correlation factor decreases to 0.23, and if \(E_W/D^2\) replaces \(E_W/2918\) the respective exponent includes zero in the asymptotic level.

Table 6-13. Results of a non linear regression analysis of \(x_{50}=A \cdot [q \cdot h/(h-h_{ob})]^{\alpha} \cdot (E_W/2918)^{\beta}\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>58.7</td>
<td>31.3</td>
<td>86.1</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>-0.83</td>
<td>-1.44</td>
<td>-0.23</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-1.47</td>
<td>-2.53</td>
<td>-0.42</td>
</tr>
</tbody>
</table>

The correlation factor is better when the in-row delay and the scatter (both in ms) are introduced in Equation 6-20. The best situation is achieved with:

\[
(x_{50})_{R-R} = A \left( q \cdot \frac{h}{h - h_{ob}} \right)^{\alpha} \left( \frac{E_W}{2918} \right)^{\beta} \cdot t^\varepsilon \cdot \delta^\lambda
\]

in which \(R^2\) increases up to 0.45. The output of the fitting is shown in Table 6-14. The exponents of \(t\) and \(\delta\), \(\varepsilon = -0.12\) and \(\lambda = -0.06\) respectively, have a positive –though very small– value for the upper bound of its asymptotic interval at confidence level of 95 %. If the confidence level is decreased to 89 %, zero is left out of the interval for both \(\varepsilon\) and \(\lambda\). Similar results are obtained if the in-row delay is divided by the burden (in m), see Table 6-15.

Table 6-14. Results of a non linear regression analysis of \(x_{50}=A \cdot [q \cdot h/(h-h_{ob})]^{\alpha} \cdot (E_W/2918)^{\beta} \cdot t^\varepsilon \cdot \delta^\lambda\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>93.4</td>
<td>30.7</td>
<td>156.0</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>-0.91</td>
<td>-1.50</td>
<td>-0.33</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-1.46</td>
<td>-2.46</td>
<td>-0.46</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>-0.12</td>
<td>-0.26</td>
<td>0.02</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>-0.06</td>
<td>-0.12</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The loading parameters mainly \(d\), \(L_{oa}\), \(Q_e\) and \(Q_e/h^*\) can not be added to the model in Equation 6-21 since the exponent affecting them includes zero at a 95 % confidence level, conversely than in other models, like Kuz-Ram and Kou-Rustan formulae.
Table 6-15. Results of a non linear regression analysis of $x_{50} = A \cdot \left[ q \cdot h/(h-h_{ob}) \right]^{\alpha} \cdot \left( E_w/2918 \right)^{\beta} \cdot (t/B)^{\gamma} \cdot \delta t^{\lambda}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>80.0</td>
<td>36.2 124.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.88</td>
<td>-1.46 -0.30</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.45</td>
<td>-2.44 -0.46</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>-0.12</td>
<td>-0.26 0.02</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-0.06</td>
<td>-0.12 0.01</td>
</tr>
</tbody>
</table>

In $(x_{50})_{R-R}$’s prediction formula, Equations 6-21, the fitting constant $A$ may be understood as a blastability index of El Alto’s limestone for this particular fit. Most of the independent blasting parameters with higher significance in the principal components $C_1$, $C_2$, $C_3$ and $C_4$ are included in 6-21. Ideally, both $E_w$ and powder factor in limestone should have the same exponents, since $E_w \cdot q \cdot [h/(h-h_{ob})]$ is the effective specific energy per unit of volume supplied to fracture the rock. However, $\beta$ ($E_w$ exponent) in 6-21 is 1.5 times larger than $\alpha$ ($q \cdot [h/(h-h_{ob})]$ exponent), which according to Ouchterlony (2004c) means that the useful work is not a good measure of the fragmentation capacity of the explosive or that something relevant is missing in 6-21. If the energy powder factor in limestone, $E_w \cdot q \cdot [h/(h-h_{ob})]$, is introduced in Equation 6-21,

$$(x_{50})_{R-R} = 10.69 \cdot 10^{h} \left[ E_w \cdot q \left( \frac{h}{h-h_{ob}} \right) \right]^{-1.46} \left( q \frac{h}{h-h_{ob}} \right)^{0.55} t^{-0.12} \delta t^{-0.06}$$

(6-22)

the powder factor in limestone is raised to a positive power that penalises the specific energy per unit of volume supplied to the rock mass. From an energy standpoint, this means that the fragmentation performance decreases as the powder factor in limestone increases. As an example two powder factors above grade, $q$, are assumed, 0.32 and 0.40 kg/m³. For each of them, the explosive energy is varied as required in order to have the same energy powder factor values, whereas $t$ is kept in 67 ms, $\delta t$ in 10 ms, $h$ in 18 m and $h_{ob}$ in 4 m. The resulting $(x_{50})_{R-R}$ are plotted in Figure 6-35 versus the energy powder factor in limestone; it shows greater median sizes for the same $E_w \cdot q \cdot [h/(h-h_{ob})]$ for blasts with $q=0.40$ kg/m³. Hence, better results are obtained acting upon the explosive energy than upon the powder factor.

![Figure 6-35. $(x_{50})_{R-R}$ versus the energy powder factor in limestone for blasts with $q$ of 0.32 and 0.40 kg/m³](image.png)
Equation 6-22 shows an opposite situation than Kuz-Ram’s $x_{50}$ formula, in which the explosive distribution has a larger influence in fragmentation than the explosive energy.

As a distinctive feature, Equations 6-21 includes the influence of timing. The median size is inversely related to the in-row delay and its respective scatter; although both parameters have a weak influence in the mean size, as their exponents are low. The trend shown in (6-21) agrees with the fact that the coarsest fragmentation is obtained for instantaneous blasts. The fitting made by Chung & Katsabanis (2000) for $x_{50}$ predictions is also in this line, although they finally drop the timing from the model arguing that few observations exist. This result seems to be contradictory with the statement given by Oñederra & Essen (2004): “In-row delays ≤ response time promote interaction between blastholes and fragmentation”. The influence of the scatter of the in-row delay is opposite to that shown in the literature. In general, recent works (Bartley & McClure 2001; McKinstry et al. 2002; Cunningham 2003), show that the uniformity index increases as the mean size decreases, even if the powder factor is slightly decreased with respect to the used in blasts with pyrotechnic detonators. For instance, Bartley & McClure (2001) show a decrease in $x_{50}$ of 42 %, whereas $n$ is practically constant (it is reduced about 1%) for the same drilling pattern. However, these works should be considered with caution as few blasts are reported and frequently not all the parameters are shown.

With respect to $S_{R-R}$, the parameters in Equation 6-21 are kept with the purpose of fitting the data of 32 blasts:

$$S_{R-R} = A \left( q \frac{h}{h - h_{ob}} \right)^{\alpha} \left( \frac{E_W}{2918} \right)^{\beta} t^\epsilon \delta \xi$$

(6-23)

The correlation factor increases to 0.56. The results of the non linear regression analysis are shown in Table 6-16. All the exponents are positive, opposite to those in Equation 6-21, since $(x_{50})_{R-R}$ and $S_{R-R}$ are negatively correlated. The lower bound of the asymptotic interval at the 95 % confidence for the $\lambda$ exponent that affects $\delta \xi$ is zero.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated</th>
<th>Asymptotic interval at a 95 % confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower bound</td>
</tr>
<tr>
<td>$A$</td>
<td>2.09</td>
<td>1.24</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.54</td>
<td>0.26</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.80</td>
<td>0.26</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.04</td>
<td>0</td>
</tr>
</tbody>
</table>

There are no references in the literature about the influence of blasting parameters in the value of the slope of the size distribution curve at $x_{50}$. 

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6 Measurement and Analysis of Fragmentation

Figure 6-35. Predicted \((x_{50})_{R-R}\) with Equation 6-21 vs. the actual one for 32 blasts (LdAnf: low density anfo; HdAlf: high density Alnafo; Alf: Alnafo; E8000: Emunex 8000; non-el: non-electric detonator; EPDs: electronic detonators).

The \((x_{50})_{R-R}\) predicted with Equation 6-21 is plotted in Figure 6-35 versus the actual size; the blasts are differentiated according to the explosive, detonator type and in-row delay used. Equation 6-21 grasps the influence of the blasting parameters and the predictions are quite acceptable. The median size is however overestimated for small \((x_{50})_{R-R}\) (i.e.<100 mm) and underestimated for large ones. Precisely, there is few data in those areas, especially for \((x_{50})_{R-R}\) about 170 mm. The greatest and smallest actual \(x_{50}\) correspond to blasts 38/03 (\(x_{50}=54\) mm) and 30/03 (\(x_{50}=165\) mm) respectively, whose size distribution curves are the upper and lower bounds of the curves shown in Figure 6-18.

6.10. Conclusions

The use of a digital image analysis system applied to photographs taken over the hopper of the bin in the primary crusher has been thoroughly studied.

Manual edition of the photographs has proved to be required in order to obtain acceptable results, although it does not prevent large errors at small particles sizes. Twenty images per blast are used to obtain fragmentation. This amount is a compromise between sample representativity and cost of the analysis in person-hours; the manual edition is carried out for about twenty minutes per photograph; a longer time spent may not pay in terms of added accuracy of the size distribution curves.
The system operation aims at controlling the fragment size of the limestone from blasting. This requires the subtraction of the loose overburden material (natural fines) from the raw fragmentation curves of the feed of the primary crusher. This has been done from laboratory screening data of the overburden material and the muckpile’s fine fraction, together with the estimation of the natural fines fraction as the ratio of overburden thickness to bench height ratio.

The errors, either potential or real have been discussed; the following must be highlighted:

- Sample size. Twenty photographs per blast have been found to be an acceptable sample size when manual editing must be done. Ten photographs may still do, though results are less reliable and more sample-dependent.

- Manual editing of the images and correction of the particles delineation cause errors due to the operator’s limited visual sharpness and his personal skills; they tend to be larger at smaller particle sizes, but are limited at sizes larger than the optical resolution of the system.

- Errors in the fines range, far below the fines cut-off or the resolution of the image, should be expected to be high.

For the system to be effective, it was first calibrated using screened data from a full scaled blast. The calibration was done setting the fines correction factor to a level where the image analysis output was as coincident as possible with the screened data. Although the absolute precision of the system on a blast per blast basis is impossible to state, the correlation of the passing at 14 mm (maximum size of overburden) with the amount of natural fines, NF, makes apparent that the fines correction factor, FF, must be variable and function of NF present in the bench. The system is recalibrated with an expression between FF and NF, which is used to get the fragmentation of each blast with the corresponding FF.

The image analysis system and the working method described have been found useful for assessing fragmentation for blast control purposes. Size distribution curves of the feed of the primary crusher (limestone and clayish-marl overburden) for 35 blasts and fragmented limestone (natural fines subtracted) for 33 blasts (two blasts have less fines than the natural fines) are provided.

The size distribution of the fragmented limestone in 33 blasts is fitted with either Rosin-Rammler (R-R) or Swebrec (S) functions in the size range 1000 to 14 mm (variable part of the size distribution that may change from blast to blast); in the Swebrec fitting, $x_{max}$ was fixed to the maximum fragment size given by digital image analysis software. There are no significant differences in the goodness of both fittings; the statistics of $R^2$ for the 33 blasts in the R-R and S fittings are very similar, 0.988sd.0.009 % for R-R and 0.992sd. 0.06 for the S. None of the functions tried fit properly all the size range down to 14 mm (the statistics of the mean of the percentage absolute errors in pass for the 12 mesh sizes fitted are 8.2sd.6.5 % with S and 6.2sd.4.2 % with R-R distributions) and in general, the R-R fails in the coarse part while the S is unsuccessful in the fines part. Although, Ouchterlony (2004c) thinks that the “authentic” or actual sieved fragments size distributions are basically not R-R, our results do not mean that El Alto’s fragmentation data are of bad quality.

An insight of the fragmentation prediction models provides a sound background for understanding the role of the blasting parameters in fragmentation. The performance of the Kuz-
Ram, Chung-Katsabanis (C-K), SveDeFo, Kou-Rustan and KCO formulae has been checked with the data of 33 blasts. With respect to the $x_{50}$ of the size distribution, extremely low correlation factor within all the series is obtained between the predicted and measured $x_{50}$ and a negative trend is obtained in C-K and Kou-Rustan. The results are improved if the powder factor, $q$, and the blasted height, $h$, are referred to limestone $- h$ and $q$ are replaced by $h-h_{ob}$ and $q \cdot [h/(h-h_{ob})]$; the predicted $x_{50}$ with all the models increases as the actual does, but the scatter is still high. The analysis also shows a bad behavior of K-R, C-K and KCO models in estimating the uniformity parameter ($n$ or $b$); the range given by C-K is very narrow to be practical, from 0.68 to 0.73, and K-R predictions for $n$ (1.09 to 1.62) are about twice the measured ones (0.441 to 0.823). The predicted $b$ values with KCO (1.6 to 2.7) are similar to the actual ones (1.4 to 2.5) and in some cases the predictions match the actual values, although no relation between the experimental and predicted undulation parameters can be drawn.

The available data are not broad enough to establish a conclusive fragmentation prediction model, but they allow detecting the most decisive parameters in fragmentation. A correlation analysis between fragmentation parameters from Rosin-Rammler and Swebrec fittings (they include the slope value at $x_{50}$) and 16 blasting parameters derived from the fragmentation prediction models, 17 other blasting parameters/groups, and 11 non-dimensional groups of blasting parameters has been made. The correlations factors are greater within each set of parameters than between them; the highest correlation factor between blasting and fragmentation parameters is 0.4.

The mutual correlations between the Rosin-Rammler parameters are stronger than within Swebrec, up to 0.8 and 0.5 respectively. The relations between the R-R parameters are then further investigated because higher correlation factors would be possible within the Rosin-Rammler parameters; the uniformity index, $n$, is found to be proportional to $(x_{50})_{R,R}^{0.5}$, whereas the slope value of the R-R distribution at the median size, $S_{R,R}$, is related with $(x_{50})_{R,R}^{0.5}$; the correlations factors for the respective formulae are about 0.75. These relations may not stand for real muckpile distributions, where the maximum size is not fixed.

The formula between $(x_{50})_{R,R}$ and $n$ is outstanding from a practical standpoint, as it allows calculating $n$ from $(x_{50})_{R,R}$ and hence building the respective size distribution curve. From the correlations between the parameters, an expression for predicting $x_{50}$ as function of the powder factor (referred to limestone with a ratio equal to one when there is not overburden), explosive energy (normalized by the Alnafó energy), in-row delay and detonator scatter is obtained. Those parameters are powered to negative exponents. The correlation factor is 0.45 %, which is acceptable given the noise due to unpredictable variations for the typical production conditions, where variations of the data from hole to hole within a blast are always present and the geology from one blast to another may vary.

The $(x_{50})_{R,R}$ formula does not include any geometrical parameter different to the powder factor and are basically of the type of the Kuz-Ram formula with some differences, e.g. the influence of the explosive mass (or either parameter related) is not enough strong to be included in the model, whereas the influence of timing is included. Both, the explosive energy and the powder factor are the predominant parameters. If the energy powder factor is considered in $(x_{50})_{R,R}$ formula, the powder factor appears as a penalty parameter of $(x_{50})_{R,R}$; fragmentation performance decreases as the powder factor increases, i.e. greater median sizes are obtained in
blasts with greater powder factor but with the same specific energy per unit of volume (the timing features are not varied).

The exponents that affect the timing parameters (in-row delays and detonators scatter) are about one order of magnitude smaller. However, if they both are rejected the correlation factor decreases to 0.28.

A similar formula, but this time for the slope at $x_{50}$ has been derived. The same blasting parameters are involved and the correlation factor is 0.56. All the exponents are opposite to those in the median size prediction.
Chapter 7

A VIEW OF QUARRY BLASTING FROM AN ENERGY STANDPOINT

Explosives are used all over the world in mining and quarrying. They deliver energy at a fast rate, this being their basic usefulness. Explosives energy is rated in a variety of ways, either obtained by calculation, such as the heat of explosion or the useful expansion work, or by experimental tests, such as the underwater or the cylinder tests. However, few data exist on how and what amount of that energy is transferred to the surrounding medium in the usual civil application of rock blasting (Berta, 1990; Spathis, 1999 and Ouchterlony et al., 2003). This matter is addressed in this Chapter by setting up an energy balance in blasting, where most of the energy forms in which the energy delivered by the explosive is transformed are measured by different means. Additionally, practical applications, i.e. blast guiding for better the results, are looked for.

The theoretical and experimental background for the determination of the energy balance in eight fully monitored production blasts (FMBs) and in a single confined blast-hole (CB) is addressed. Seismographs, high-speed video camera and a fragmentation monitoring system were used in each FMB, in order to measure the seismic field, the initial velocity of the blasted rock face and the fragment size distribution curve from which the various energy terms are calculated, while only vibrations were monitored in the confined single blasthole shot.

7.1. The components of the energy balance

During the blast, the chemical energy of the explosives is used in a useful form for breaking the rock, fragmentation energy and in throwing the broken rock mass, kinetic energy. An important part of energy is lost as seismic wave to the rock, causing the vibration phenomenon, seismic energy. Other energy losses take place, such as the heat transferred to the rock, the energy
contained in the hot detonation products vented to the atmosphere, airblast, etc. These forms of energy are difficult to evaluate and they are considered “other losses” of the process. The energy balance of the blast can be expressed by:

\[ E = E_F + E_K + E_S + E_{OL} \]  

(7-1)

Where: \( E \) is the explosive energy; \( E_F \) is the fragmentation energy; \( E_K \) is the kinetic energy; \( E_S \) is the seismic energy and \( E_{OL} \) are the energy losses different than seismic energy of the process. The terms fragmentation, kinetic and seismic efficiency are used hereafter for the ratios of the respective energies to the explosive energy.

The explosive energy can be rated in different ways from thermodynamic codes like W-Detcom (Sanchidrián, 1986; López, 2003) or from tests like the cylinder test (Nyberg et al., 2003), which leads to the Gurney energy. Sanchidrián & López (2003) show an agreement of the Gurney energy with the useful work values at a non-ideal detonation regime of the order of experimental errors.

Spathis (1999) outlines as a recommendation for future works, the practical side of Equation 7-1 with the aim of guiding the blast design for affecting the fragmentation and kinetic energy terms. At the same time, the seismic energy could be minimized, all resulting in lower vibration levels and a more efficient use of the explosive energy.

### 7.1.1. Fragmentation energy

The fragmentation energy, \( E_F \) is obtained as the product of the new surface area generated by the blast, \( A \) and the specific surface energy, \( \gamma_f \):

\[ E_F = A \gamma_f \]  

(7-2)

The specific surface energy \( \gamma_f \) is of the order of 50 J/m² (Spathis, 1999) and it can be calculated either from the Rittinger coefficient \( R \) (m²/J) or the fracture toughness \( K_{IC} \) (Pa·m⁰.⁵). The first one is derived from millions of fractures in the rock, while the fracture toughness is obtained from tests in which only one fracture is formed. The last one is affected strongly by the material structure and additionally it does not take into account the aspects like the crack roughness or the side cracking. For the estimation of the fragmentation efficiency by blasting, where a great amount of fines is produced, we shall use the inverse of the Rittinger coefficient.

\[ A = 6V \sum_{k=1}^{z} \frac{P_k}{d_k} \]  

(7-3)

Where:
- \( V \) is the volume of the fragmented rock in the blast. If it is assumed that there is no back-break, \( V \) may be calculated as:

\[ V = \frac{B}{\cos i} S N (h - h_{ob}) \]  

(7-4)

Where \( B \) is the average burden (mean of all the burden values given by the laser profile in front of the blast-holes); \( i \) is the blasthole inclination from the vertical; \( S \) is the blast-hole spacing; \( N \) is the number of blastholes; \( h \) is the average blasted length and \( h_{ob} \) is the overburden thickness.
- \( P_k \) is the percentage of pass relative to the bin \( k \).
– $d_k$ is the average size for bin $k$
– $z$ is the number of bins.

The surface area for cubic particles is the same that for spherical particles as the ratio of the surface area to the volume is identical. Nevertheless if parallelogram particles whose sides are in the ratio $1:a:b$ (with $a \geq 1$ and $b \geq 1$) are assumed, $A$ is smaller.

Ouchterlony et al. (2003) based on the work of Hamdi & du Mouza (2000) calculates the surface area generated in the blast by an integral instead of the summation in 7-3 using the Rosin-Rammler distribution. In such formula the mean size and the uniformity parameter of the size distribution curve affect inversely to the specific surface area ($m^2/m^3$) generated by the blast.

Some authors like Hamdi et al. (2001) and Ouchterlony et al. (2003) subtract the area of the in situ block size distribution to the area of broken rock in the muckpile.

7.1.2. Seismic Energy

A simple calculation of the seismic energy, $E_s$, radiated by a production blast using the velocity records is as follows:

$$ E_s = 4\pi R^2 \rho_c c_p \int_0^R v_r^2 \, dt $$

Where:
- $R$ is the distance to the source;
- $\rho_c$ is the rock density;
- $c_p$ is the propagation velocity of the compressional waves;
- $t$ is the integration time and $v_r$ is the modulus of the sum particle velocity at a distance $r$.

Equation 7-6 is a simplification of the general expression (Achenbach, 1975):

$$ E_s = \int_0^R \int_S \mathbf{t} \cdot \mathbf{v} \, dS \, dt + \int_0^R \int_V \rho \mathbf{f} \cdot \mathbf{v} \, dV \, dt $$

Where $S$ and $V$ are respectively the surface and volume of the control domain; $\mathbf{t}$ is the normal stress vector field on the surface, $\mathbf{v}$ the velocity field and $\mathbf{f}$ the body forces field. The use of Equation 7-6 would require a large number of velocity and stress measurements within the rock, which is far beyond of the existent capabilities. A large number of assumptions and simplifications are required to obtain 7-5 from 7-6, namely:

i. The vectors $v$ is unknown (only the modulus of the radial, transversal and vertical components of the rock particles velocity are obtained) and it is assumed to have radial direction.

ii. The stress is not measured and it should be related in some way with the velocity given by the sensor. It is assumed that the relation, $t_r = c_p \rho v_r$ (Rinehart, 1975), between the radial stress, $t_r$, and radial velocity, $v_r$, at the front of a spherical wave is valid everywhere for the stress and velocity modules.

iii. According to (ii), the seismic energy is transmitted at the velocity of the P-waves, although wave types with different propagation features than P-waves are present in the recorded signal. Ouchterlony et al (2003) made their measurements at depth in order to avoid Rayleigh waves, which according to Graff (1975) may carry a 67% of the total energy.
iv. The seismic energy is radiated spherically (a spherical domain is considered); the ground surface and the free face interrupt the spherical wave front and there is no spherical spreading from the gravity centre of the cylinder charge; different waves paths result for each wave front system

v. Reflected waves go back into the spherical control surface

vi. No measurements are made within the volume and the function to be integrated is only known in the boundary.

vii. The body forces, \( f \) (like gravity, which is partially responsible for the state of velocities measured) are assumed to be insignificant.

viii. The rock mass is anisotropic and in-homogenous; azimuth variation of vibrations may occur.

All together, the application of Equation 7-5 leads to several uncertainties.

### 7.1.3. Kinetic energy

The kinetic energy, \( E_k \), may be calculated from the initial velocity of the rock face and from the displacement of the gravity centre from the position in the block to that in the muckpile.

**Calculation based on the measured face velocity**

The initial velocity of the blasted rock face is known from measurements in the rock face; it may vary along the burden and bench height directions, \( x \) and \( y \) directions, see Figure 7-1. If \( v_0 \), as discussed by Segarra et al (2003), is assumed constant for all \( x \), \( v_0 = V_0(y) \) and the kinetic energy of the rock blasted by a hole is:

\[
E_k = \frac{1}{2} \int_0^H \rho(y)B(y)V_0^2(y) dy 
\]

Where \( \rho(y) \) and \( B(y) \) account for lithology and burden variations along the height respectively.

Depending on the degree of knowledge of the velocity, i.e. number of points or targets used in the measurements, step or quadratic distributions of velocities, \( V_0(y) \), along the bench height may be introduced in Equation 7, see Figure 7-1. If a quadratic distribution of velocities is assumed, the resulting kinetic energy is about 80% that when a step distribution is considered (maximum kinetic energy). The step distribution of velocities is more accurate as the number of targets increase. However, when few targets were used, the step distribution of velocities overestimates the velocities in the proximity of the crest and toe of the bench.

The high-speed film and radar measurements in the work of Sheahan and Beattie (1990) showed that i) the face velocity distributions for many blasts were relatively narrow and ii) the rocks behind the face generally move in unison with the face. This behaviour is according to Chiappetta & Mammele (1987) typical of competent and brittle rocks with few joints that are spaced greater than the burden or the spacing between blastholes.
The rock behind the face probably moves slower due to the collisions of the fragments. Some of the kinetic energy is then lost during the rock flight into fragmentation and elastic-plastic energy in the fragments and a velocity gradient within the projected rock could be assumed:

$$V_0(x, y) = \frac{V_0(y)}{B} x$$

The kinetic energy is from Equation 7-7:

$$E_k = \frac{1}{2} S \int_0^H \int_0^B \rho(y) \frac{x^2}{B^2} V_0^2(y) \, dx \, dy = \frac{1}{6} SB \int_0^H \rho(y) V_0^2(y) \, dy$$

Equation (7-8) leads to a kinetic energy value one third of that obtained with constant velocity within the burden. Either a step or quadratic distributions of velocities may be used for $V_0(y)$.

**Calculation based on the displacement of the centre of gravity**

An alternative method to estimate the kinetic energy is to determine the initial velocity that would bring a displacement of the gravity centre from its initial position, gravity centre of the block, to the final one, gravity centre of the muckpile. They are respectively obtained from the face and muckpile profiles in the blasthole located just behind the targets. The material coming from the backbreak is now, forcefully, considered in the calculation of the rock mass. The new profile for the bench after the blast is assumed to be equal to the initial one with an overburden drag as measured, see Figure 7-2. This method implicitly considers that the resultant fragments
travel different distances at different velocities. The resulting energy does not include the energy lost in the collisions during the flight.

![Figure 7-2. Displacement of the gravity centre for the blasthole No. 11 in the blast 29/02.](image)

7.2. Brief review of the FMBs

The energy balance is calculated in eight production blasts: 15/02, 29/02, 37/02, 43/03, 45/03, 50/03, 54/03 and 58/03, in which face movement, vibrations and fragmentation were all measured. Additionally, the seismic energy term is calculated from the seismic measurements made in the confined single blasthole CB2; fragmentation and rock movement can not be measured as it was a confined deep shot.

Blasts 29/02, 37/02, 45/03, 54/03 and 58/03 are in the same area of the pit, although the last three were made one year later. Blasts 15/02, 43/03 and 50/03 were in different areas. The features of those blasts and of CB2 were reviewed in Chapter 3, and their features compiled in Appendix A. Summarizing: ANFO was used as column charge in CB2, Al-nafo in the production blasts 15/02, 29/02, 37/02, 43/03, 45/03, 50/03 and high density Al-nafo in the pair of FMBs 54/03 and 58/03; gelatine cartridges were used as bottom charge. Non-electric detonators were used in blasts 15/02, 29/02 and 37/02, while EPDs were employed in the other FMBs. The timing was also varied; it was 84 ms in 15/02, 67 ms in 29/02 and 43/03, 30 ms in 50/03 and 17 ms in 45/03; the in-row delay in blast 37/02 at El Alto quarry was mainly 67 ms, although this was variable along the blast since some of the holes were decked and some not. One charge per hole is blasted in all rounds, except in blast 15/02 shot in El Alto, where two decks separated by 1 m of stemming and a 50 ms delay top-bottom were used.

The face movement, vibrations and fragmentations measurements were made as was described in Chapters 4, 5 and 6 respectively.

7.3. Calculation of the energy balance

The explosive energy is evaluated from the thermodynamic code W-Detcom (Lopez, 2003) as heat of explosion at constant volume, $E_Q$, and useful work to a cut-off pressure of 1000 bar, $E_W$. The explosive energy per hole, $(E_Q)_h$ or $(E_W)_h$, is used in the calculations; product of either $E_Q$ or $E_W$ by the explosive mass per hole, $(Q_e)_h$. 

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7.3.1. Calculation of fragmentation energy

The size distribution curves of broken limestone (natural fines discounted) of the FMBs are drawn in Figure 7-3; the quantitative data is given in Appendix D. The average fragmentation curve of the five blasts reported by Ouchterlony et al. (2003) in a limestone quarry is also plotted.

![Figure 7-3. Size distributions curves of the 8 FMBs blasts monitored and average fragmentation curve of the blasts monitored in Klinthagen (Ouchterlony et al., 2003)](image)

Equations 7-2 (with $\gamma_f = 172.4$ J/m$^2$), 7-3 and 7-4 are used for the calculation of $E_F$. The fines tail is considered in the calculations due to its large specific surface, although the Split’s fines cut-off is significantly higher (the smallest fines cut-off is 82 mm). A bin from zero to 0.25 mm has been considered for including the very fine material roughly in the calculation of the new surface area generated by the blast.

Table 7-1 shows for each blast the volume of broken rock mass per blast-hole, the surface area generated in the blast per unit of volume, $A_s$, the fragmentation energy per hole, $E_F$, the explosive energy per hole, $(E_W)_h$ and $(E_Q)_h$, obtained from the average values for the whole blast (see Appendix A) and the fragmentation efficiencies. The specific surface area of the ISBD in El Alto, 2.4 m$^2$/m$^3$ (obtained from the $x_{63}$ and $n$ values given in Section 3.1.1) is not considered in the calculations, since it is up to a 0.8% of $A_s$. 

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Table 7-1. Fragmentation energy and efficiency and steps to final results

<table>
<thead>
<tr>
<th>Blast No.</th>
<th>Rock volume, m³/hole</th>
<th>$A_S^{(1)}$, m²/m³</th>
<th>$E_F$, MJ/hole</th>
<th>$E_F/E_r$, %</th>
<th>Statistics $E_F/E_r$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>15/02</td>
<td>539</td>
<td>349</td>
<td>32.4</td>
<td>700</td>
<td>1156</td>
</tr>
<tr>
<td>29/02</td>
<td>485</td>
<td>403</td>
<td>33.7</td>
<td>690</td>
<td>1125</td>
</tr>
<tr>
<td>37/02</td>
<td>496</td>
<td>441</td>
<td>37.7</td>
<td>733</td>
<td>1209</td>
</tr>
<tr>
<td>43/03</td>
<td>471</td>
<td>436</td>
<td>35.4</td>
<td>665</td>
<td>1097</td>
</tr>
<tr>
<td>45/03</td>
<td>371</td>
<td>518</td>
<td>33.1</td>
<td>615</td>
<td>1014</td>
</tr>
<tr>
<td>50/03</td>
<td>486</td>
<td>247</td>
<td>20.7</td>
<td>702</td>
<td>1163</td>
</tr>
<tr>
<td>54/03</td>
<td>518</td>
<td>858</td>
<td>76.6</td>
<td>944</td>
<td>1409</td>
</tr>
<tr>
<td>58/03</td>
<td>386</td>
<td>490</td>
<td>32.6</td>
<td>783</td>
<td>1156</td>
</tr>
</tbody>
</table>

The greatest fragmentation efficiency, 8.1 and 5.4 % of $E_W$ and $E_Q$ respectively, is obtained in blast 54/03, while the smallest one, 2.9 % of $W_a$, corresponds to blast 50/03. In the other six blasts, the fragmentation efficiency range is tighter, between 4.2 and 5.4 %. The average fragmentation efficiency for El Alto is 3.2 % of the heat of explosion.

The reliability of the results depends on the difference between the actual and the obtained size distribution curves; in El Alto the digital analysis software has been calibrated. It should be remarked that, if the internal microcracking and fracture surface roughness were accounted in any way, the fragmentation energy would increase.

### 7.3.2. Calculation of seismic energy

The seismographs placed around the block at a distance to the source of about 65 m in the top and bottom quarry levels are used for calculating $E_s$. As an example, Figure 7-4 shows the seismographs lay-out used in blasts 29/02 and 37/02 for measuring seismic energy.

![Figure 7-4. Seismographs disposition for measuring $E_s$ in blasts 29/02 and 37/02](image-url)

A unique value of the seismic energy is obtained for each sensor as it is not possible to discriminate the vibrations coming from each blast-hole, see Figure 5-1.

In Equation (7-5), the integration time, $t$, includes the whole signal; the distance $r$ is taken as the mean of the distances from the respective sensor to the blast-holes of the blast (Equation 5-1 is
used); according to the measured values in the limestone, \( \rho_r \) and \( c_p \) are taken as 2560 kg/m\(^3\) and 2994 m/s respectively (see Section 3.1.1) and \( v_R \) is obtained from the recorded transversal, longitudinal and vertical components of the particle velocity.

The total seismic efficiency for each blast is obtained as the average of the seismic energy given by each sensor. This force to have similar \( r \) distances between the sensors and the blast as no energy dissipation terms like the one considered by Hinzen (1998) are included in Equation 7-5; in the eight FMBs the statistics for \( r \) in the 26 good quality measurements are 65.5sd.4.4 m

Table 7-2 show the average of the \( r \) distances from each sensor to the blast, the seismic energies recorded by each sensor (the sensor position in the quarry – top or bottom level – is given), the total seismic energy and efficiency for each shot (calculated from the explosive energies per hole given in Table 7-1), and the overall statistics (mean and standard deviation of the seismic and efficiency values) of the FMBs.

Table 7-2. Seismic energies and efficiencies

<table>
<thead>
<tr>
<th>Blast No.</th>
<th>CB</th>
<th>15/02</th>
<th>29/02</th>
<th>37/02</th>
<th>43/03</th>
<th>45/03</th>
<th>50/03</th>
<th>54/03</th>
<th>58/03</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average dist. to blast, m</td>
<td>51.8</td>
<td>75.7</td>
<td>63.5</td>
<td>64.3</td>
<td>66.7</td>
<td>63.4</td>
<td>68.9</td>
<td>64.6</td>
<td>67.6</td>
</tr>
<tr>
<td>Sensor 7101(^{(1)})</td>
<td>35.9</td>
<td>6.4B</td>
<td>37.9T</td>
<td>29.3T</td>
<td>8.2T</td>
<td>25.8T</td>
<td>7.6B</td>
<td>16.9B</td>
<td>-</td>
</tr>
<tr>
<td>Sensor 6783</td>
<td>-</td>
<td>-</td>
<td>27.9T</td>
<td>12.2B</td>
<td>44.9B</td>
<td>23.1B</td>
<td>-</td>
<td>10.9T</td>
<td>35.9T</td>
</tr>
<tr>
<td>Sensor 7102(^{(2)})</td>
<td>25.9</td>
<td>26.7T</td>
<td>21.1B</td>
<td>6.6B</td>
<td>44.2B</td>
<td>43.0T</td>
<td>4.4B</td>
<td>7.4B</td>
<td>18.9B</td>
</tr>
<tr>
<td>Sensor 7640</td>
<td>34.8</td>
<td>-</td>
<td>17.3B</td>
<td>-</td>
<td>49.8T</td>
<td>14.7B</td>
<td>-</td>
<td>110.3T</td>
<td>33.5B</td>
</tr>
<tr>
<td>Blast</td>
<td>32.2</td>
<td>16.6</td>
<td>26.1</td>
<td>16.0</td>
<td>36.8</td>
<td>26.7</td>
<td>6.0</td>
<td>34.6</td>
<td>29.5</td>
</tr>
<tr>
<td>Statistics</td>
<td>-</td>
<td>-</td>
<td>24.0sd.10.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( E_p/(E_w)_{bh} \)

| | 15.7 | 2.4 | 3.8 | 2.2 | 5.5 | 4.3 | 1.0 | 3.7 | 3.8 |
| | 10.7 | 1.4 | 2.3 | 1.3 | 3.4 | 2.6 | 0.5 | 2.5 | 2.6 |
| Statistics | - | - | 3.3sd1.4 |
| \( E_p/(E_w)_{bh} \) | - | - | 2.1sd0.9 |

Notes: (1) T and B means that the sensor is placed in the top and bottom levels of the block respectively. 
(2) In blasts 45/03 and 50/03 the seismograph 4862 was used instead of the 7102.

Table 7-2 shows a large scatter in the seismic energies values from one sensor to another within the same blast. The largest seismic efficiencies correspond to the confined shot, CB2 which is about five times the mean efficiency of El Alto FMBs. This large difference may be caused by the absence of overlapping of the signal, the confinement of the charge or/and the undamaged nature of the rock mass around the shot. According to Blair & Armstrong (2001), the last one seems to have a larger influence in vibration than the burden, while according to Blair & Armstrong (1999) the destructive or constructive interference between the blasthole signals seems to not affect as random fluctuation between the seed waves is expected to occur in the time domain.

The seismic efficiency monitored at El Alto at about 65 m is compared with the measured one with the same methodology at 37 m in Eibenstein, an amphibolite quarry (Sanchidrián et al., 2003); the statistics of the FMBs monitored at El Alto are 2.1 sd 0.9 % of the heat of explosion, whereas for Eisenstein the average value of two blasts is 1.2 % (the value given by Sanchidrián et al.(2003) has been corrected by a factor of two since they use in Equation 7-5 a factor of \( 2\pi \) instead of \( 4\pi \)). The physical background for this could be the amplification effect caused by the low-velocity clayish-marl overburden, of compression or shear waves, when a strong propagation velocity contrast occurs (Dowding, 2000).
7.3.3. Calculation of kinetic energy

The kinetic efficiency has been calculated using a step velocity distribution along the bench height in Equation 7-7. A linear variation of velocity within the burden has been also considered together with the step velocity distribution along the bench height; Equation 7-8 is used for that purpose.

The data in Table 4-1 is used in the calculation of the kinetic energy; \( \rho \) is taken as 2560 and 1600 kg/m³ for limestone and clayish-marly overburden; \( B(y) \) is known from face profile measurements and \( V_0(y) \) from the face movement measurements given in Chapter 4. The kinetic energy is then calculated for the blastholes behind the targets hanged at the free face.

The kinetic energy has been also calculated from the displacement of the centre of gravity in the blasts 29/02 and 37/02; the profiles of the block and the muckpile were used for that purpose.

Table 7-3 shows the kinetic energies and efficiencies (calculated from the explosive energy values of the holes behind the targets, see Table 4-1). There were some measurement problems when performing the muckpile profile from round 37/02, so the kinetic efficiency obtained from the gravity centre of the muckpile in this blast was rejected.

<table>
<thead>
<tr>
<th>Blast No.</th>
<th>15/02</th>
<th>29/02</th>
<th>37/02</th>
<th>43/03</th>
<th>45/03</th>
<th>50/03</th>
<th>54/03</th>
<th>58/03</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step E_k</td>
<td>68.5</td>
<td>91.4</td>
<td>61.8</td>
<td>47.3</td>
<td>33.3</td>
<td>66.7</td>
<td>55</td>
<td>122.1</td>
<td>-</td>
</tr>
<tr>
<td>Step &amp; grad. E_k</td>
<td>22.8</td>
<td>30.5</td>
<td>20.6</td>
<td>15.8</td>
<td>11.1</td>
<td>22.2</td>
<td>18.3</td>
<td>40.1</td>
<td>-</td>
</tr>
<tr>
<td>G.C. displac. E_k</td>
<td>-</td>
<td>30.5</td>
<td>45.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>E_k / E_E, %</td>
<td>8.2</td>
<td>18.7</td>
<td>7.6</td>
<td>7.7</td>
<td>5.5</td>
<td>9.9</td>
<td>5.5</td>
<td>15.5</td>
<td>9.9sd4.8</td>
</tr>
</tbody>
</table>

**Notes:** In italics kinetics efficiencies rejected

The different assumptions on the distribution of the velocity result in large differences in the kinetic efficiency. It seems unlikely that the velocity varies so much along the burden as Equation (7-8) assumes, although it looks apparent that the rock behind the face moves somewhat slower than the rock fragments at the free face, due to fragments collisions. However, the kinetic efficiency calculated from the displacement of the gravity centre for blast 29/02 at El Alto, matches the one calculated assuming a step distribution of velocities and a linear velocity variation within the burden. Both methods leave aside energy losses, although it could be argued that the energy was initially transferred to the rock in the form of kinetic energy.

The kinetic efficiency for the eight rounds monitored in El Alto is 6.2sd3.1 % of the heat of explosion (9.9sd4.8 % of \( W_u \)) assuming a step distribution along the bench face and constant velocity within the burden. The largest kinetic efficiency is obtained for blast 29/02, where the bottom of the blasthole does not reach the grade.
7.4. Discussion of the results

Table 7-4 summarises the fragmentation, seismic and kinetic efficiencies obtained in El Alto for useful work and heat of explosion energies; the upper and lower bounds of the range are obtained by adding and subtracting the standard deviation to the average of the efficiencies of the eight FMBs respectively. The highest and smallest values of the kinetic efficiency range are obtained from the statistics of the values calculated with constant and linear velocity within the burden respectively.

The unaccounted losses make up more than a half of the explosive energy released by the explosive; both ways for rating the explosive consider an ideal detonation regime. The term $E_{DL}$ in Equation (7-1) ranges for production blasts from 92 to 73 % (useful work) and from 96 to 84 % (heat of explosion). The situation is non-better for a confined blast-hole, in which the losses are 84 and 89 % of $E_W$ and $E_Q$ respectively.

The results have been compared with available data in the literature. The conclusions that can be drawn from them are limited, as the blasts are made in different conditions (geology, drilling and blasting parameters) and so are probably the blasts results (fragmentation, throw and vibrations). Table 7-4 also shows the energy balances from Berta (1990), Spathis (1999) and Ouchterlony et al (2003). Berta’s energy balance (1990) is for bench blasting, but the data in which it is based is not reported nor referred. Conversely Spathis’s work (1999) is more rigorous and it is based on the data given by Sheahan & Beattie (1990) for ten small-scale blasts in granite. The energy balance from Ouchterlony et al (2003) is obtained in five production blasts in a limestone quarry (Klinthagen; Gotland, Sweden) with the aim of investigating if the energy balance could be used to get less fines. Hinzen’s results (1998) are added to Table 7-4; he compiled seismic information for five blasts made during the excavation of a drift through a gneiss formation.

The partitioning of the explosive energy in seismic, fragmentation and kinetic is very different for Berta and all other authors, although the losses given by Berta (1990) are 37 % (including flyrock, airblast and other losses), which is within the wide window, 22 % to 84 %, given by Spathis.

Table 7-4. Energy terms from different authors.

<table>
<thead>
<tr>
<th>Energy efficiencies</th>
<th>Berta</th>
<th>Spathis</th>
<th>Hinzen</th>
<th>Ouchterlony et al</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_Q$</td>
<td>$E_W$</td>
<td>$E_Q$</td>
<td>$E_Q^{(1)}$</td>
<td>$E_Q$</td>
</tr>
<tr>
<td>Fragmentation, %</td>
<td>17</td>
<td>0.2 – 1</td>
<td>–</td>
<td>0.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Seismic, %</td>
<td>40</td>
<td>3 – 14</td>
<td>&lt;1–25</td>
<td>6-19</td>
<td>16(2)</td>
</tr>
<tr>
<td>Kinetic, %</td>
<td>6</td>
<td>13 – 63</td>
<td>–</td>
<td>6-20</td>
<td>11</td>
</tr>
</tbody>
</table>

Notes: (1)$E_Q$ means Gurney energy (obtained from the cylinder expansion test). (2) In italics the value obtained for the CB shot at El Alto.

Spathis evaluated the explosive energy using the useful work down to 100 MPa as it was done in this work. He uses different explosives, ignition systems and ways for charge decoupling in a total of ten reduced scale blasts. Hinzen (1998) calculated the seismic efficiency for each detonator time step in five blasts in which three explosive types were used; one being an emulsion with energy of 3.21 MJ/kg, which probably corresponds to the heat of explosion. The seismic efficiencies are ranged between 25 % for the cut holes – 11 % of $E_Q$ was obtained in the
single confined blasthole CB2− and <1 % for some of the perimeter holes. The seismic efficiency calculated by Spathis (1990) and Ouchterlony et al. (2003) for surface blasts and the range given by Hinzen (1998) cannot be directly compared as the explosives are evaluated in different ways. Additionally, the blast mechanics are different for underground and surface blasts, and the confinement for the cut holes is higher than in a surface blast. The seismic records in all those works were obtained with accelerometers, which makes possible to calculate the seismic energy radiated by each blast-hole; no overlapping occurs and the integration time in (7-5) covers the complete signal from a single blast-hole. This, together with the fact that Spathis’ and Ouchterlony’s data correspond to the upper level of the bench, may be the reason why the seismic efficiencies from both authors are very similar, whereas their high end figures are larger than the seismic efficiencies obtained in El Alto. Only the seismic efficiency for the CB is close to the upper bound of the intervals given in those publications. Ouchterlony et al. (2003) outline the difficulty in analysing the seismic data since the standard deviation in seismic energy is about half or more of the mean, even for the same gauge in a particular blast.

In what respects to the fragmentation energy, the fines tail of the size distribution curve has a great influence in the resulting value, as small sizes have a large specific surface and consequently a great amount of the fracture energy is required for their generation. For the calculation of the fragmentation energy, Spathis (1999) used the size distribution curves obtained from screening the muckpile material down to 16 mm where the cumulative pass ranged from 5 % to 2 %; the steeper curve of El Alto’s FMBs corresponds to blast 50/02, which has a pass at 16 mm of 8%, see Figure 7-2. Ouchterlony et al.’s fragmentation efficiency is very small, like the lower figure given by Spathis (1999). Fragmentation was measured in Klinthagen with Fragscan (Schleifer & Tessier, 2000) which according to Ouchterlony et al. (2003) lead to a non credible fraction at 90 mm, of about 0.8 %. Latham et al. (2003), whom use the same version in their blind comparison between digital image analysis software, show that Fragscan systematically predicts a more uniform distribution than the other tested systems. El Alto’s efficiencies are about 5 times the values given by Spathis, because the fragmentations curves at El Alto are flatter and material below 16 mm has been considered.

Spathis (1999) and Ouchterlony et al. (2003) calculate the kinetic energy from the mean features of all the blast. The kinetic efficiency may change by a factor of 3, depending on the distribution of velocities considered; the lowest value is obtained assuming a linear velocity within the burden. If a constant velocity within the burden is assumed, the kinetic energy term is the one with a largest efficiency. Spathis considers a constant velocity along the bench height and within the burden. The upper range of the kinetic efficiencies given here and by Ouchterlony et al. is close to the lower range given by Spathis.
Chapter 8
GENERAL CONCLUSIONS

This thesis compiles blasting data – geometry, loading and timing – for a total of 35 production blasts and three single shots made at a limestone quarry in the period from October 2001 to May 2004. Aluminized ANFO was used as column charge in 29 blasts and in two of the single shots; non electric and electronic detonators were used in 15 and 14 of these blasts respectively. The in-row delay was varied in 10 of those blasts (the timing in blast 37/02 is unknown); in-row delays of 84 ms (combined with two decks per hole delayed 50 ms), 30 ms and 17 ms were used. The column explosive was varied in six blasts and in one of the single shots; three pairs of those blasts were made with high density aluminized ANFO, low density ANFO and emulsion respectively, whereas plain ANFO was used in the single shot.

The blasting parameters have been changed according to production requirements – mainly sufficient rock heave, small number of boulders and good toe breakage. The majority of the blasts had 142 mm blasthole diameter. The burden and spacing were varied from 3.9 to 5.5 m and from 5.8 to 6.4 m (only increased up to 7 m in three blasts). The powder factor above grade was changed between 0.30 and 0.46 kg/m$^3$.

The geotechnical features of the blocks were not differentiated from blast to blast, although they were all carried out in the same bench and in a relatively limited area of the quarry. The quality of the blasting data was controlled in all the tests. It was increased gradually from February 2002 to July 2002 by replacing the old quarry practices with a rigorous procedure based on the use of monitoring devices. The improvements affected mainly to the subdrill length, burden and face smoothness.

Small variations of the data from hole to hole within a blast were always present and difficult to avoid; they mainly affect to the burden (and hence to the powder factor) and the linear charge density. Mean values of the variables that were measured from hole to hole have been used. The conditions of El Alto quarry include certain particularities in the blasting features, e.g. negative subdrill exists in some blastholes.

Quantitative data of bench face movement, vibrations and fragmentation is provided under current production techniques used in quarry blasting.

Rock face movement was measured in eight of these 35 blasts from wooden targets. Rock initial velocity and response time in blasting are provided for a total of 21 and 18 targets respectively. Response times, $t_{\text{resp}}$, range from 16 to 92 ms and initial velocities, $V_0$, from 5 to 16 m/s; their
statistics are 34 sd. 18 ms and 10 sd. 3 m/s respectively. Burdens between 3.5 and 6.6 were involved. Such wide range is also reported (from 2 to 40 m/s and from 5 to 150 ms) by Chiappetta & Mammele (1987) for a variety of materials, burdens and explosives. The Oñederra & Esen response time prediction formula has been applied, but the results of the non-linear regression analysis show that it is not valid for El Alto’s data since one of the fitting constant in the formula includes zero in the asymptotic interval at a 95 % confidence level. A correlation analysis shows that the in-row delay, \( t \), is weakly correlated with \( t_{\text{resp}} \). Conversely, the burden at the target level, \( B_t \), and the height from the grade at the target level, \( H_t \), have the strongest influence in \( t_{\text{resp}} \). A formula for predicting the response time (in ms) at specific height from the grade, \( H_t \), is developed:

\[
 t_{\text{resp}} = 50.2 \cdot \left( \frac{(\rho_l)_t}{B_t} \right)^{-0.9} \cdot \left( \frac{h - H_t}{l_h} \right)^{-0.7}
\]

where: the ratio of the linear density of the explosive loaded into the hole to the burden, \((\rho_l)_t/B_t\), is in kg/m\(^2\) and the blasted height, \( h \), \( H_t \) and the blasthole length, \( l_h \), are in consistent units. The correlation factor of the fitting is high, 0.88 and the fitting constant, 50.2, is related in some way with the features of El Alto’s rock mass. Both burden and blasthole diameter (it is included in the linear explosive density) appear in the Oñederra & Esen formula.

More measurements in front of the stemming are required, since the unique observation available at that position appears to have large influence in both non-linear regression analyses; if this measurement is rejected, the fitting constants are very similar to that shown above, but \( R^2 \) decreases to 0.68.

Profiles of the free face at different times are given for six of the blasts. These profiles often show a non-uniform displacement along the bench height, which advise the use of more than two target points in the measurements. The initial velocity of the rock, \( V_0 \), in front of a blasthole seems to be a quadratic function of \( H_t \). But weak correlations between \( B_t - V_0 \) and \( H_t - V_0 \) are obtained when data from different blasts is considered together. Burden and explosive energy involved in the formula proposed by Chiappetta et al. (1983) are poorly correlated with the initial velocity (<0.4), according to our results. Conversely, the ratio of the in row delay to the response time, \( t/t_{\text{resp}} \), and the ratio of in row delay to burden at target level, \( t/B_t \), show the strongest correlations, 0.9 and 0.6. Nevertheless, the difficulty for explaining the influence of \( t/t_{\text{resp}} \) in the initial velocity (if the cooperation between blastholes is promoted (\( t<<t_{\text{resp}} \)) the rock movement is complicated by the adjacent rock masses in movement) leads to consider the results with caution and leaves aside the development of a formula for predicting the initial velocity.

A total of 108 good vibration quality measurements were obtained with 3D-digital seismographs; 12 seismograms were monitored in the top level in two confined and one unconfined single-hole shots, whereas 96 correspond to 23 of the monitored production blasts. From those, 31 measurements were made in the top level and 39 in the bottom level at an average distance from the sensor to the blast of 66.0sd. 7.9 m (these seismograms have used to calculate the seismic energy in eight blasts) and the remaining 26 measurements were made in the top level at a distance range from 75 to 512 m. These data have been used to investigate the transmission phenomena of seismic waves, and to assess the resulting vibrations.
A P-wave velocity of 2994 m/s is measured in the field for limestone and 395 m/s (typical of low velocity layers) for clayish-marl overburden.

The dominant frequency, $FFT_{av}$, of the mean velocity spectra of the longitudinal, transversal and vertical components (Blair & Armstrong, 1999) has been used for investigating the frequency of the vibrations. The influence of blasthole confinement, sensor position with respect to the initiation sense of the blast, the explosive type (only investigated in blasts delayed 67 ms) and the scatter of the detonators (also analysed for 67 ms delayed blasts) seems to not affect to $FFT_{av}$ values. However, the frequency spectrum is clearer and the energy is concentrated in few peaks when EPDs are used.

The $FFT_{av}$ values do not decrease as the distance to the source increases, although the energy transmitted at higher frequencies than $FFT_{av}$ becomes smaller. The $FFT_{av}$ values from single-hole shots are not rich in high frequencies, 5.8 sd.1.5 Hz at 247 sd.213 m. This shows the predominance of surface waves in the seismogram at even close distances to the source, about 50 m. Production blasts measurements in the top and bottom levels show at any distance to the source (up to 512 m in the top level and 100 m in the bottom level) that in-row delays of 17 and 30 ms leads to $FFT_{av}$ of about 4.5 Hz, whereas $FFT_{av}$ is about 15 Hz (1000/67) for in-row delays of 67 ms.

$FFT_{av}$ of the vibrations radiated by production blasts would be higher than the natural frequency of the terrain (5.8 Hz), when the in-row delay is larger than 1/4 of the inverse of the dominant frequency of a single blasthole, 43 ms (=250/5.8). The Blair & Armstrong (1999) model provides acceptable predictions of the $FFT_{av}$ values in the two blasts delayed with EPDs in which it has been applied; negligible random fluctuations between the signatures radiated by each blasthole were assumed. A further application of this model shows that the use of 50 ms (slightly above 50 ms) would lead to an $FFT_{av}$ of 19.5 Hz, which according to the Spanish damage criteria allows an increase of 30 % of the peak particle velocity.

In a conservative approach the vibration level is analyzed from $ppv_{sum}$. The influence of other blasting parameters different than the explosive mass may affect to the vibration level in the near distance range, i.e. let’s say 100 m, where there are still internal waves, although they are not predominant. Higher $ppv_{sum}$ are obtained in single confined shots than in unconfined, but more data is required to draw a definitive conclusion. The use of either EPDs or non electric caps does not affect to $ppv_{sum}$ for delays of 67 ms, but the use of 17 ms of in row delay instead of 67 ms increases 1.5 times $ppv_{sum}$ values. This seems however, to apply in the near field.

$ppv_{sum}$ can be reasonably predicted in the top level from attenuation laws; since the correlation factors are high. Attenuation laws of $ppv_{sum}$ (in mm/s) are provided for:

- Single-blasthole shots monitored in the top level of the quarry (12 points available from unconfined and confined tests),

$$ppv_{sum} = 337.5 \left( \frac{r}{Q_{max}} \right)^{0.5}$$

with $R^2=0.83$

where $r$ (in m) is the average distance from the sensor to the blast and $Q_{max}$ (in kg) is the maximum charge mass per delay detonated within 8 ms.

- Production blasts monitored in the top level of the quarry (57 points available),

$$ppv_{sum} = 1234.7 \left( \frac{r}{Q_{max}} \right)^{1.829}$$

with $R^2=0.85$
- Production blasts monitored in the top level of the quarry (57 points available),

\[
ppv_{\text{sum}} = 904.1 \left( \frac{r}{Q'_{\text{max}}} \right)^{0.5} \right) \right)^{-1.7092} \text{ with } R^2 = 0.89
\]

For \( r < 100 \text{ m} \), \( Q'_{\text{max}} \) is the maximum charge detonated per delay within 43 ms.

For \( r > 100 \text{ m} \), \( Q'_{\text{max}} \) is the maximum charge detonated per delay within 8 ms.

In the bottom level the scaled distance is little variable, between 3 and 8 m/kg^{0.5} and an attenuation law can not be then obtained with moderate reliability; the mean of the \( ppv_{\text{sum}} \) values are about twice as large in the top level than in the bottom for scaled distances between 3 and 5 m/kg^{0.5}.

With respect to fragmentation, the use of a digital image analysis system applied to photographs taken over the hopper of the bin in the primary crusher has been thoroughly studied. Manual edition of the photographs has proved to be required in order to obtain acceptable results, although it does not prevent large errors at small particles sizes. Twenty images per blast are used to obtain fragmentation. This amount is a compromise between sample representativity and cost of the analysis in person-hours; the manual edition is carried out for about twenty minutes per photograph; a longer time spent may not pay in terms of added accuracy of the size distribution curves.

The system operation aims at controlling the fragment size of the limestone from blasting. This requires the subtraction of the loose overburden material (natural fines) from the raw fragmentation curves of the feed of the primary crusher. This has been done from laboratory screening data of the overburden material and the muckpile’s fine fraction, together with the estimation of the natural fines fraction as the ratio of over burden thickness to bench height ratio.

The errors, either potential or real have been discussed; the following must be highlighted:

- Sample size. Twenty photographs per blast have been found to be an acceptable sample size when manual editing must be done. Ten photographs may still do, though results are less reliable and more sample-dependent.

- Manual editing of the images and correction of the particles delineation cause errors due to the operator’s limited visual sharpness and his personal skills; they tend to be larger at smaller particle sizes, but are limited at sizes larger than the optical resolution of the system.

- Errors in the fines range, far below the fines cut-off or the resolution of the image, should be expected to be high.

For the system to be effective, it was first calibrated using screened data from a full scaled blast. The calibration was done setting the fines correction factor to a level where the image analysis output was as coincident as possible with the screened data. Although the absolute precision of the system on a blast per blast basis is impossible to state, the correlation of the passing at 14 mm (maximum size of overburden) with the amount of natural fines, \( NF \), makes apparent that the fines correction factor, \( FF \), must be variable and function of \( NF \) present in the bench. The system is recalibrated with an expression between \( FF \) (in %) and \( NF \) (in %):

\[
FF = 2.8 + 83NF
\]

This is used to get the fragmentation of each blast with the corresponding \( FF \).
The image analysis system and the working method described have been found useful for assessing fragmentation for blast control purposes. Size distribution curves of the feed of the primary crusher (limestone and clayish-marl overburden) for 35 blasts and fragmented limestone (natural fines subtracted) for 33 blasts (two blasts have less fines than the natural fines) are provided.

The size distribution of the fragmented limestone in 33 blasts is fitted with either Rosin-Rammler (R-R) or Swebrec (S) functions in the size range 1000 to 14 mm (variable part of the size distribution that may change from blast to blast); in the Swebrec fitting, \( x_{\text{max}} \) was fixed to the maximum fragment size given by digital image analysis software. There are not significant differences in the goodness of both fittings; the statistics of \( R^2 \) for the 33 blasts in the R-R and S fittings are very similar, 0.988sd.0.009 % for R-R and 0.992sd. 0.06 for the S. None of the functions tried fit properly all the size range down to 14 mm and in general, the R-R fails in the coarse part while the S is unsuccessful in the fines part. Although, Ouchterlony (2004c) thinks that the “authentic” or actual sieved fragments size distributions are basically not R-R, our results do not mean that El Alto’s fragmentation data are of bad quality.

An insight of the fragmentation prediction models provides a sound background for understanding the role of the blasting parameters in fragmentation. The performance of the Kuz-Ram, Chung-Katsabanis (C-K), SveDeFo, Kou-Rustan and KCM formulae has been checked with the data of 33 blasts. With respect to the \( x_{50} \) of the size distribution, extremely low correlation factor within all the series is obtained between the predicted and measured \( x_{50} \) even if the powder factor, \( q \), and the blasted height, \( h \), are refereed to limestone (\( h \) is replaced by \( h-h_{\text{ob}} \) and \( q \) by \( q[h/(h-h_{\text{ob}})] \)). The analysis also shows a bad behavior of K-R, C-K and KCO models in estimating the uniformity parameter (\( n \) or \( b \)); the range given by C-K is very narrow to be practical, from 0.68 to 0.73, and K-R predictions for \( n \) (1.09 to 1.62) are about twice the measured ones (0.441 to 0.823). The predicted \( b \) values with KCO (1.6 to 2.7) are similar to the actual ones (1.4 to 2.5) and in some cases the predictions match the actual values, although no relation between the experimental and predicted undulation parameters can be drawn.

The available data are not broad enough to establish a conclusive fragmentation prediction model, but they allow detecting the most decisive parameters in fragmentation. A correlation analysis between fragmentation parameters from Rosin-Rammler and Swebrec fittings (they include the slope value at \( x_{50} \), \( S_{x_{50}} \)) and 16 blasting parameters derived from the fragmentation prediction models, 17 other blasting parameters/groups, and 11 non-dimensional groups of blasting parameters has been made. The correlations factors are greater within each set of parameters than between them; the highest correlation factor between blasting and fragmentation parameters is 0.4.

The mutual correlations between the Rosin-Rammler parameters are stronger than within Swebrec, up to 0.8 and 0.5 respectively. This may not stand for other quarries. The relations between the R-R parameters are then further investigated because higher correlation factors would be possible within the Rosin-Rammler parameters; the uniformity index, \( n \), and the slope value of the R-R distribution at the median size, \( S_{R-R} \) (in m\(^{-1}\)) are found to be related with the median size, \( (x_{50})_{R-R} \) (in mm),

\[
n = 0.055 \cdot (x_{50})_{R-R}^{0.531} \quad \text{with} \quad R^2=0.78
\]

and
The median size is then a key parameter as $x_{\text{max}}$ has a moderate variation over the nominal value of the maximum sizes loaded into the primary crusher; if the median size increases the size distribution curve is steeper, whereas the slope value at the median size decreases. Those relations may not necessarily stand for the real muckpile distribution.

The formula between $(x_{50})_{R-R}$ and $n$ allows calculating $n$ from $(x_{50})_{R-R}$ and hence building the respective size distribution curve (the value of $S_{R-R}$ is not required). An expression for predicting $x_{50}$ is defined as a function of the powder factor above grade (in kg/m$^3$) referred to limestone with a ratio of the blasted height, $h$, to the difference between the blasted height and the overburden thickness, $h_{\text{ob}}$ (it is equal to one when there is not overburden), explosive energy normalized by the Alnafo energy (in kJ/kg) and in-row delay (in ms) and detonator scatter (arbitrary 10ms for pyrotechnic and 1 ms for electronic):

$$x_{50} = 93.4 \left(\frac{q}{h - h_{\text{ob}}}ight)^{-0.91} \left(\frac{E_{\text{W}}}{2918}\right)^{-1.46} \cdot t^{-0.12} \cdot \delta^{-0.06}$$

The correlation factor is 0.45 %, which is acceptable given the noise due to unpredictable variations for the typical production conditions, where variations of the data from hole to hole within a blast are always present and the geology from one blast to another may vary.

The $(x_{50})_{R-R}$ formula does not include any geometrical parameter different to the powder factor and are basically of the type of the Kuz-Ram formula with some differences, e.g. the influence of the explosive mass (or either parameter related) is not enough strong to be included in the model, whereas the influence of timing is included. Both, the explosive energy and the powder factor are the predominant parameters. If the energy powder factor is considered in $(x_{50})_{R-R}$ formula: the powder factor appears as a penalty parameter and the fragmentation performance decreases as the powder factor increases, i.e. greater median sizes are obtained in blasts with greater powder factor but with the same specific energy per unit of volume (the timing features are not varied). The exponents that affect the timing parameters (in-row delays and detonators scatter) are about one order of magnitude smaller. However, if they both are rejected the correlation factor decreases to 0.28.

An energetic approach to the blasting process has been made by calculating the basic energy components of the blasting process from measurements in eight production blasts. The energy spent in useful effects (fragmentation and throw) accounts for less than 20% of the chemical energy (heat of explosion). Seismic energy is, at most, about 3 % of heat of explosion in production blasts (and up to 11 % in a confined single blasthole). This means that a large part of the chemical energy is not recovered in the experimental measurements. Part of that energy may have been used in the generation of very fine material (<0.25 mm), with large specific surface, not recovered in the calibration test from which the fines tail has been derived, and in the creation of cracks within the fragments, without surface separation. The former can not be accounted, and may be the cause of a worst performance of a single blast-hole versus a production blast.

Seismic energy determinations are done generally in an oversimplified manner, as a precise calculation would require large number of records and measurements done at depth and the use of gauges for measuring the stress. Measurements closer to the hole usually lead to higher
seismic energy values, which means that energy is lost to the ground as the seismic wave proceeds, and therefore is not detected in the gauges. Additionally, the use of accelerometers is advisable, since there is no overlapping between the waveform signature coming from each blasthole.

In this work, the kinetic energy is calculated for one of the blastholes of the blasts. This energy fraction seems to have less variability from different sources (except the large range by Spathis), although it is also subjected to changes from different assumptions in the calculation.

All together, the results obtained and their comparison with data from the literature, give an idea of the large undetermination that presently exists in the energy balance of rock blasting; such variations may arise from the blast features, explosive and rock types, but also from the difficulties existing in the experimental measurements and further elaboration of the data and assumptions in the calculations thereof.

It is apparent that a very large amount of the explosive energy may go with the high pressure gases to the atmosphere. The energy measured is also a small fraction (hardly above 40%) of the Gurney energy so there must be a large amount of irreversible losses in the process of energy transfer to the rock. The explosive process in rock blasting, though excellent for a fast rate delivery of energy, seem to be, from a thermodynamic point of view, a rather little efficient work machine. A deeper understanding of the losses associated with the energy transfer from the charge to the surrounding rock, including the near-hole region of the rock mass still has some potential for improving the blasting process.

The ambition to use the energy balance to guide the blast design to a specific goal became impractical and according to Ouchterlony et al (2003) more is to be gained by a deeper understanding of the fragmentation process or by relying on systematically gathered experience. However, the determination of the energy balance provides a better understanding of the whole process and serves for controlling and analysing the blasts results.

8.1. Recommendations for El Alto quarry

This section summarizes the strategies that must be considered for improving the operation at El Alto quarry:

1) Laser profiling prior to drilling for blasts design and data gathering must continue.

2) The variations of explosive mass and burden from hole to hole must be taken in account and minimized (similar powder factors).

3) A formula for estimating the response time of the rock is provided. Basically at a specific height, \( t_{\text{resp}} \) decreases as \( (\rho_e)/B \) increases and/or \( h/l_h \) increases.

4) The initial velocity, \( V_0 \), increases as \( t/t_{\text{resp}} \) does. Flatter muckpiles would be then obtained if long in-row delays are used, 67 ms, the average burden is decreased and if short subdrill as possible to assure good toe breakage are used.

5) Looking to \( FFT_{\text{av}} \), the higher frequencies are obtained with 67 ms in-row delays. However, the effect of a 50 ms delay must be checked according to the predictions of the pooled function given by Blair & Armstrong (1999).
6) An attenuation law of the \( ppv_{sum} \) is given for the top level; the maximum charge detonated in a 43 ms window, \( Q'_{max} \), is considered for distances to the blast smaller than 100 m. Hence short in-row delays must be avoided (<43ms).

7) Two decks must be used in the areas of the pit closer to cement plant in order to reduce \( Q'_{max} \) (in those blasts the use of a delay between charges of 50 ms will increase the resulting dominant frequency).

8) In the near field (not at the moment), high burden values are not advised, although the influence of the confinement in vibrations has not been proved.

9) The boulder size can be calculated in the future from the formula provided in the KCO model; the spacing must be kept under control.

10) A decrease of powder factor from 0.41 to 0.32 kg/m\(^3\) (a very large change) leads to an increase in both \( n \) and \( x_{50} \) of the broken rock of 15 % and 30 % respectively, which would probably be a significant reduction of fines in general terms. But, considering that the amount of natural fines (the overburden) ranges from 14 to 40 %, the overall effect with respect to the screen blocking may be meager. External uncontrollable factors such as rain (humid fines cake and block the screen more easily) doubtlessly have a stronger effect in the mill’s outlet screen throughput.

11) From an energy standpoint, it is advisable (higher fragmentation performance) to act upon the explosive energy instead than upon the powder factor. However, since Alnafo is generally used there is little room for this.

12) Considering a suitable powder factor range in El Alto from 0.34 to 0.42 kg/m\(^3\) and the use of ANFO or Aluminized ANFO with the same density but different explosive energy (\( E_W \) is 2591 and 2918 kJ/kg respectively) a high value of powder factor and the use of Aluminized ANFO (smaller mean size and more fines from blasting) would result in:

- Reduced energy consumption and maintenance cost in the mill.
- Increased mill throughput in dry conditions.
- Reduced mucking cost by more efficient loading and less consumables of loaders and shovels.
- Reduced mill throughput due to screen blocking in rain conditions.
- Higher drilling and explosives cost.

Conversely, a low powder factor and ANFO (larger mean size and fewer fines from blasting) will result in:

- Increased energy consumption and maintenance costs in the mill.
- Reduced mill throughput in dry conditions.
- Increased mucking cost due to less efficient loading and more consumables of loaders and shovels.
- Increased mill throughput due to reduction of screen blocking in rain conditions.
- Reduced drilling and explosives costs.

The strategy to follow would probably be in the line of a powder factor in the high range in dry conditions and a lower one under humid weather. The definition of an optimum value
can only be given as the result a detailed economic analysis of the overall operation, which is beyond the scope of this work.

13) The high cost and the training required for using EPDs does not justify their use in terms of resulting vibration and fragmentation (given the small effect of the detonator scatter on $x_{50}$).
Chapter 9

GUIDELINES FOR FUTURE WORK

Future work should focus in:

1. Rigorous data gathering with systematic recording in order to build large databases.
2. Replacing old quarry practices for blasts design by using advanced techniques, mainly block profiling and control of blasthole deviations.
3. Reduced scale-blasts in order to have less scatter in blasting parameters and better understand the physics of rock blasting.
4. Today there is still little quantitative data about the face movement. Simpler ways for measuring it are required; the procedure used in this work is highly time consuming. Radar devices for measuring the face movement provide information in many points of the free face and not only in front of some blastholes. The resulting muckpile profile is also interesting.
5. Determination of velocities from rock fragments behind the free face in order to determine the variation of the initial velocity within the blasted rock; the accurate kinetic energy value could be then calculated.
6. Developing an engineering model for predicting the distribution of initial velocity along the bench height.
7. The potential for the influence of the response time in rock movement and fragmentation must be exploited. The response time seems to be an important factor to choose the in-row delay in quarry blasting.
8. Understanding the physics behind the parameters affecting the response time and initial velocity. In this sense numerical code may be helpful in order to simulate the rock movement.
9. The use of the \( FFT_{av} \) reduces the amount of data to work with and simplifies the frequency analysis of the vibrations.
10. The random fluctuations between signatures radiated by each blasthole should be further investigated, especially for the sizes of blasts in many quarries.
11. Using different time windows for calculating the maximum charge detonated per delay depending on the dominant frequencies of seed signals.
12. Analysing the influence of confinement and explosive type in the particle velocity at the near distance range.
13 Applying statistical models for ground vibration particle velocity like the proposed by Blair (1999). The Fourier model proposed by Blair & Armstrong (1999) for modelling the frequency spectra provides interesting results.

14 Stress measurements should be made in order to estimate better seismic energy than only from particle velocities.

15 Use of the Swebrec function for the description of the fragment size distribution.

16 The use of Swebrec distribution may also be implemented in digital image analysis software in order to improve the extrapolation of the distribution for smaller sizes from the delineated particles.

17 Better delineation algorithms in order to reduce the amount of manual edition; the number of photos analysed per blast could be then increased.

18 Use of digital image analysis software in other quarries where the amount of fines is smaller; building large fragmentation databases.

19 Defining a method for accounting the boulders production, when fragmentation is measured in the primary crusher.

20 Defining the parameters affecting the maximum size (muckpile fragmentation measurements required).

21 Checking if the relation between the median size and the uniformity index of the size distribution curve is valid everywhere for the feed of crushers and if there should be some blasting parameters involved (at present the uniformity index is not well described by any model).

22 Defining an $x_{50}$ prediction formula with a higher correlation coefficient than the 0.5 obtained in this work.

23 Developing a simple procedure for the estimation of the rock factor. This is a basic requirement to establish an engineering model and analyzing the influence of the rock mass in the blasting parameters affecting $x_{50}$.

24 Additional testing of different explosives in order to verify the influence of the explosive properties and to check the best way of rating the explosive energy. Of the explosive properties, the velocity of detonation is one that requires particular attention in order to determine whether or not it has an influence in the mean size.

25 The cylinder expansion test and developments of it will be a key factor for understanding the losses in the energy transfer from the charge to the surrounding rock, e.g. checking the influence of the powder factor in fragmentation performance.

26 Checking the influence of timing and detonator scatter in the mean size. At the moment there are not conclusive data in the literature on this matter, nor have they been obtained in this work.
Chapter 10

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Notation

$A$  Rock factor
$A'$  Rock factor
$A_B(f)$  Seed amplitude function of a single blasthole in the frequency domain
$B$  Av. Burden
$B_{Ti}$  Burdens at the target levels, with $i$ from 1 to 4
$b$  Undulation parameter
cart.  Cartridge
c  Propagation velocity of either P or S-waves
c  Rock constant
$c_p$  Propagation velocity of the p-waves
$(c_p)_l$  P-waves velocity in the limestone
$(c_p)_{ob}$  P-waves velocity in the overburden
$D$  Velocity of detonation
$D_CJ$  CJ velocity of detonation
d  Blasthole diameter
d  Horizontal distance between the blasthole collar and the seismograph
dl  Horizontal distances travelled by the direct waves in the limestone
d_{min}  Minimum distance between a seismograph and the blasthole collar
d_{ob}  Horizontal distances travelled by the direct waves in the overburden.
$E_d$  Dynamic Young modulus
$E_E$  Explosive energy per length of explosive column
$E_Q$  Heat of explosion at constant volume
$E_W$  Useful work
EPDs  Electronic programmble detonators
$f$  Frecuency
$F_D$  Aerodynamic drag
$FFT$  Fast Fourier Transform
$FFT_{av}$  Dominant frequency of the mean velocity spectra of the transversal, vertical and longitudinal components
$f_R$  Frequency of the compression or shear waves amplified by low velocity layers
g  Acceleration of gravity
$H$  Bench height
$H_{Ti}$  Height at the target levels, with $i$ from 1 to 4
$h$  Blasted height
$h'$  Rock height (blasted height substracted to overburden thickness)
h_{l}  Depth of the gravity center of the explosive charge measured from the contact between the limestone and the overburden
$H_{lvl}$  Thickness of a low velocity layer
$h_{ob}$  Overburden thickness
$i$  Blasthole inclination
ISBSD  In situ block size dsitribution
$J$  Subdrill length
$K_{mass}$  Rock mass stiffness
$L$  Length of hole charged above grade
$L$  Longitudinal component of a seismic signal
$L_B$  Length of the bottom charge above grade
$L_C$  Length of the column charge above grade
$L_{tot}$  Total length of the explosive charge
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{\text{cart}}$</td>
<td>Nominal length of a cartridge given by the manufacturer.</td>
</tr>
<tr>
<td>$l_b$</td>
<td>Blasthole length</td>
</tr>
<tr>
<td>$(l_{s})_T$</td>
<td>Top stemming length</td>
</tr>
<tr>
<td>$(l_{s})_I$</td>
<td>Intermediate stemming length</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of a target</td>
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<tr>
<td>$N$</td>
<td>Number of blastholes in a blast</td>
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<tr>
<td>$N_{\text{bag}}$</td>
<td>Number of bags loaded into the blasthole</td>
</tr>
<tr>
<td>$N_{\text{cart}}$</td>
<td>Number of Cartridges loaded into the blasthole</td>
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<tr>
<td>$N_{\text{F}}$</td>
<td>Natural fines</td>
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<tr>
<td>$n$</td>
<td>Uniformity index</td>
</tr>
<tr>
<td>$P(f)$</td>
<td>Pooled firing function</td>
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<tr>
<td>$P(x)$</td>
<td>Cumulative fraction of rock passing a sieve of size $x$</td>
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<td>$Q_{\text{bag}}$</td>
<td>Nominal mass of a bag given by the manufacturer</td>
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<td>$Q_{\text{cart}}$</td>
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<td>$Q_e$</td>
<td>Explosive above grade</td>
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<td>Explosive mass above grade, bulk</td>
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<tr>
<td>$(Q_e)_c$</td>
<td>Explosive mass above grade, cartridges</td>
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<td>$Q_{\text{max}}$</td>
<td>Maximum charge mass per delay detonated within 8 ms</td>
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<tr>
<td>$ppv_i$</td>
<td>Peak particle velocity of each component ($t, v$ and $l$ means transversal, vertical and longitudinal components)</td>
</tr>
<tr>
<td>$ppv_{\text{sum}}$</td>
<td>Peak sum particle velocity</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance from a seismograph to the blast</td>
</tr>
<tr>
<td>RBS</td>
<td>Relative bulk strength</td>
</tr>
<tr>
<td>ROM</td>
<td>Run of mine</td>
</tr>
<tr>
<td>R-R</td>
<td>Rosin-Rammler</td>
</tr>
<tr>
<td>$S$</td>
<td>Spacing</td>
</tr>
<tr>
<td>$S_{\text{Swbrec}}$</td>
<td>Swebrec distribution</td>
</tr>
<tr>
<td>$S_{B-R}$</td>
<td>Slope of Rosin-Rammler distributions at $x_{50}$</td>
</tr>
<tr>
<td>$S_{S}$</td>
<td>Slope of Swebrec distributions</td>
</tr>
<tr>
<td>$s$</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$s$</td>
<td>Strength relative to a Swedish dynamite (LFB)</td>
</tr>
<tr>
<td>$T$</td>
<td>Transversal component of a seismic signal</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Target, with $i=1$ to 4</td>
</tr>
<tr>
<td>$t$</td>
<td>In-row delay</td>
</tr>
<tr>
<td>$t_{\text{init}}$</td>
<td>Initiation time of the explosive</td>
</tr>
<tr>
<td>$t_{\text{first mov}}$</td>
<td>Time at which the rock starts its movement</td>
</tr>
<tr>
<td>$t_j$</td>
<td>Time required by the seismic waves to reach the monitoring station</td>
</tr>
<tr>
<td>$t_{\text{resp}}$</td>
<td>Response time of the rock</td>
</tr>
<tr>
<td>$V$</td>
<td>Transversal component of a seismic signal</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Initial velocity.</td>
</tr>
<tr>
<td>VOD</td>
<td>Velocity of detonation</td>
</tr>
<tr>
<td>$W$</td>
<td>Standard deviation of the drilling</td>
</tr>
</tbody>
</table>
Mesh size

Components of \( V \) in the X axis

Length of the direct P-waves over the ray path in the limestone

Length of the direct P-waves over the ray path in the overburden

Upper limit of the fragment size distribution

50 % passing mesh size

50 % passing mesh size for the Rosin-Rammler fitting

50 % passing mesh size for the Swebrec fitting

Components of \( V \) in the Y axis

Depth of the gravity center of the charge in blasthole \( j \)

Displacement of the target from its initial position

Detonator scatter

Gamma function of \( x \)

Poisson’s coefficient

Mean of the delay used in the blasthole \( j \)

Pitch angle

Explosive density

Nominal density of the explosive in bags

Nominal density of the explosive in cartridges

Linear charge density of the explosive loaded into the blasthole

Rock density

Standard deviation of the delay used in the blasthole \( j \)

\( x_{50} \) is the 50 % passing mesh size and \( n \) is the uniformity index