N. A. Bezdenejnykh and J. Meseguer

Stability Limits of Minimum Volume and Breaking of Axisymmetric Liquid Bridges Between Unequal Disks

This paper deals with the stability limits of minimum volume and the breaking of axisymmetric liquid columns held by capillary forces between two concentric, circular solid disk of different radii. The problem has been analyzed both theoretically and experimentally. A theoretical analysis concerning the breaking of liquid bridges has been performed by using a one-dimensional slice model already used in liquid bridge problems. Experiments have been carried out by using millimetric liquid bridges, and minimum volume stability limits as well as the volumes of the drops resulting after breaking have been measured for a large number of liquid bridge configurations; experimental results being in agreement with theoretical predictions.

1 Introduction

The liquid bridge configuration analyzed in this paper consists, as sketched in fig. 1, of a volume of liquid held by surface tension forces between two parallel, coaxial, solid disks. Such axisymmetric fluid configuration can be identified by the following dimensionless parameters: the slenderness, $\Lambda = L/2R_0$, where L stands for the distance between the disks and $R_0 = (R_1 + R_2)/2$ is a mean radius, the ratio of the radius of the bottom disk to the radius of the top one, $K = R_1/R_2$, the dimensionless volume of liquid, $V = V/R_0^3$, V being the physical volume, and the Bond number, $B = \varrho g R_0^2/\sigma$, where ϱ is the liquid density, g the axial acceleration and σ the surface tension.

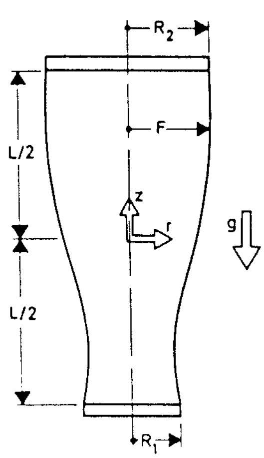

Fig. 1. Geometry and coordinate system for the liquid bridge problem

Mail address: Prof. J. Meseguer, Lamf-μg, Laboratorio de Aerodinámica, E.T.S.I. Aeronáuticos, Universidad Politécnica, E-28040 Madrid, Spain, Dr. N. A. Bezdenejnykh, Institute of Continuous Media Mechanics, Urals Scientific Center of the USSR Acad. Sci. 1, Acad. Korolev, 614061 Perm, USSR.

It is well-known that for each separation of the disks, measured by the slenderness Λ, there is a minimum volume of liquid, V, for which a stable liquid bridge can be formed. The dependence of such stability limit of minimum volume on non-symmetric effects like unequal disks, $K \neq 1$, or axial microgravity, $B \neq 0$, has been studied in refs. [1–6], among others, for the case of axisymmetric configurations. Available results show that each one of these effects separately decrease the stability of the liquid bridge (the volume of liquid must be increased or the slenderness decreased to keep a stable configuration), but both effects together can cancel or, in other words, either one of these effects can be stabilizer for the remaining one. In the particular case of cylindrical long liquid bridges ($\Lambda \sim \pi$, $V = 2\pi\Lambda$, $K \sim 1$, $B \sim 0$) it was demonstrated in ref. [2] that the maximum stable slenderness varies as

$$\Lambda = \pi\left[1 - \left(\frac{3}{2}\right)^{4/3}\left(B - \frac{1}{2\pi}\frac{1-K}{1+K}\right)^{2/3}\right] \tag{1}$$

so that, for a given value of K the critical slenderness increases if $B > 0$ (gravity points to the smaller disk) whereas the contrary occurs if $B < 0$ (gravity points to the larger disk). This behaviour is qualitatively the same regardless of the values of K and B, and seems to be in agreement with experimental evidence [6]. Those results are summarized in fig. 2, where the stability limits of minimum volume of liquid bridges between unequal disks ($K = 0.6$) have been plotted for two different values of the Bond number (details on the method used to calculate each minimum volume stability limits can be found in [6]). In this plot it can be observed that in the $B < 0$ case the slope of the stability limit curve is continuous in the range of values of Λ of interest, whereas in the case of $B > 0$ there is a discontinuity in the slope at point A. Numerical results published in ref. [3] show that, in the latter case, when the stability limit is reached and breakage of the long liquid bridge takes place, there is a sudden jump in the values of the volume of the drops resulting after breaking, in such a way that if $\Lambda < \Lambda_A$ the final configuration consists of a large drop at the top disk and a small one at the bottom disk, but when $\Lambda > \Lambda_A$ the large drop appears at the bottom disk whereas the small one is formed at the top.

This behaviour at breaking seems to be in connection with the equilibrium interface shape of the long liquid bridge at the corresponding minimum volume stability limit. If $K < 1$ and $B < 0$ equilibrium interface shapes have

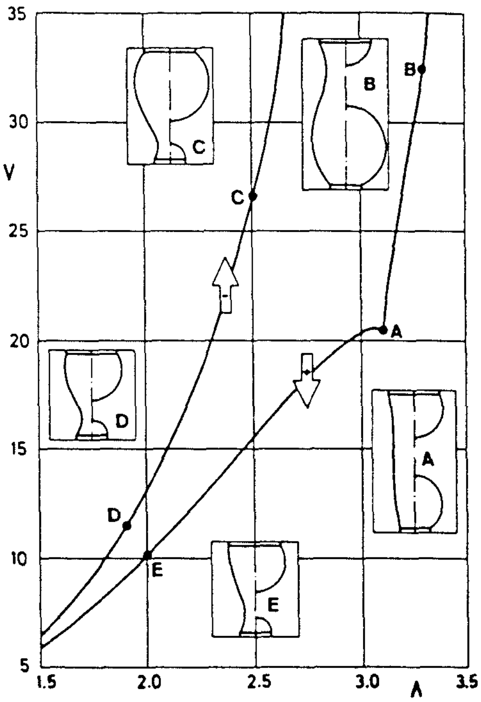

Fig. 2. Typical stability diagrams (minimum volume V versus slenderness Λ) of liquid bridges between unequal disks subjected to an axial microgravity field whose direction is indicated by arrows on the curves. The different sketches show the liquid bridge interface at selected points of stability limits as well as the drops resulting after the liquid bridge breakage

always a neck close to the smaller disk, and this position of the neck roughly determines the volume of the two drops resulting after breaking, in such a way that the final configuration consists, leaving apart satellite droplets, of a large volume drop of liquid attached to the larger disk and a small drop attached to the smaller one.

The behaviour is rather different when $K < 1$ and $B > 0$, where the interface shapes at the stability limit have the neck close to the smaller disk when $\Lambda < \Lambda_A$ and close to the larger one when $\Lambda > \Lambda_A$ (the variation of the equilibrium interface shape with Λ, K and B has been experimentally analyzed in [4]). Also in this case the position of the neck of the equilibrium interface shapes at minimum volume stability limit seems to determine the volume of the drops resulting after breaking.

These two characteristics associated to point A (discontinuity of slope of the curves of stability limit of minimum volume and a sudden change in the final drop configuration resulting after breaking) become more and more smooth as Λ_A decreases, in such a way that if $\Lambda_A < 1.5$ the discontinuity practically disappears and there is a continuous transition in the value of the volume of the drops resulting after breaking.

In this paper the stability limits of minimum volume of axisymmetric liquid bridges, as well as the breaking of liquid bridges at such stability limits, are analyzed both theoretically an experimentally. Concerning the liquid bridge breakage the physical magnitude measured has been the ratio of the volume of the drop attached to the larger disk to the whole liquid bridge volume. In the following this ratio is defined as the partial volume and denoted by V_p. Theoretical results have been obtained by using a one-dimensional slice model already used in liquid bridge problems [3, 7], whereas experiments have been performed in a millimetric liquid bridge facility using water as working fluid; experimental results being in agreement with theoretical predictions.

2 Experimental Apparatus and Procedures

The experimental set-up, as sketched in fig. 3, consists of a microzone (liquid bridge) facility in which a small liquid bridge (1) can be formed between two coaxial, solid disks. Working fluid (distilled water) is injected or removed through the bottom disk by using a callibrated syringe (3). The bottom disk is fixed to the supporting frame, whereas the upper one can be rotated and displaced up and down. Uniform background illumination (2) is achieved with the help of a translucent plate and a lamp. To avoid excessive heating of the liquid column the lamp is placed some distance apart of the liquid bridge, the light being conducted to the translucent plate through an optic-fiber cable. Opposite to the translucent grid there is a microscope (4) to which a CCD camera (5) is connected. Output from the CCD camera is stored in a video recorder (6) for further analysis in an image analyzer (7) which in turn is driven by a desktop computer (8).

A typical experiment (an experimental point in Λ-V stability diagrams) is performed as follows: first of all the upper disk is placed close enough to the bottom one and working fluid is injected through the lower disk until a short liquid bridge is formed. Then, the upper disk is moved upwards slowly until the desired slenderness is reached, and, during this process, working fluid is injected from time to time if necessary. After this process, the fluid configuration is a liquid with the selected slenderness and a volume of liquid greater than that of the corresponding minimum

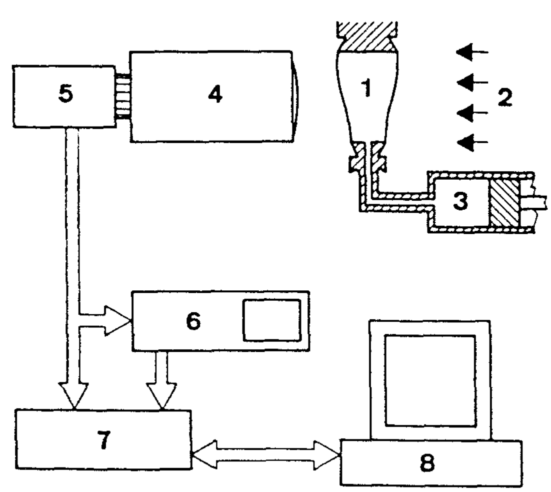

Fig. 3. Experimental setup: 1) liquid bridge, 2) syringe, 3) illumination, 4) microscope, 5) CCD camera, 6) video recorder, 7) image analyzer, 8) computer

volume stability limit. From this point on the experiment runs alone. Due to water evaporation the volume of liquid in the fluid bridge continuously decreases and when the minimum volume stability limit is reached the breaking of the liquid column takes place. The volume of the liquid bridge just before breaking (the minimum volume stability limit), as well as the volume of the two drops resulting after breaking, are further obtained from recorded images. Both the volume of the liquid bridge and the volume of the drops are measured with an error less than 1%.

3 Experimental Results

A problem arising when water is used as working fluid is that the value of surface tension can be, due to interface contamination, rather different from the nominal value. However, since surface tension, σ, appears in the expression of Bond number, $B = \varrho g R_0^2/\sigma$, its value can be determined from the values of B resulting from experimental data, by fitting theoretical curves of minimum volume (which once the geometry parameter K is fixed depend only on the Bond number[1]) to available experimental data.

In our case three different series of experiments were performed, varying the values of K and R_0, and therefore the value of the Bond number, from one to another series. The results concerning minimum volume stability limits are shown in figs. 4, 6, and 8. In these plots experimental data as well as theoretical minimum volume stability limits have been represented (as already stated, minimum volume stability limits have been calculated by using the method described in [6]). In each one of the plots the stability limits corresponding to three different values of B have been drawn, to get an idea of the influence of this parameter on the results.

Obviously the mean value of gravity, g, does not change in an earth laboratory, and one could expect that the value of surface tension, σ, is kept constant unless the liquid bridge interface becomes contaminated, so that it seems that there is no reason to consider any uncertainity in the value of the Bond number. However, in an earth laboratory there is some level of noise that could be of importance when checking stability limits, mainly if millimetric liquid bridges are used. This effect could be avoided by using some isolation device, or it can be accounted for as an uncertainty in the value of the actual acceleration acting on the liquid bridge or as an uncertainty in the value of surface tension, and therefore as an uncertainty in the value of the Bond number.

From the values of the Bond number resulting from the experiments, the mean value $\sigma = 0.059\ \mathrm{Nm}^{-1}$ is deduced. Observe that the values of σ derived from the different series of experiments, table 1, are almost the same, so that it can be concluded that there has not been degradation of the surface properties of the working fluid (water) during the experiments.

Table 1. Geometry parameters, R_0 and K, and measured values of Bond number, B, and surface tension, σ, during the different series of experiments performed

K	R_0[mm]	B	σ[Nm^{-1}]
0.61	0.70	−0.08	0.060
0.60	0.69	0.08	0.058
0.78	2.20	0.8	0.059

The results corresponding to the configuration $K = 0.61$ and $B = -0.080$ are shown in figs. 4 and 5. Experimental values of the minimum volume stability limit as well as theoretical ones corresponding to four different values of the Bond number have been plotted in fig. 4, whereas those related to the dynamics of breaking, partial volume V_p (the ratio of the volume of the drop attached to the larger disk, the top one, to the whole liquid bridge volume) versus the

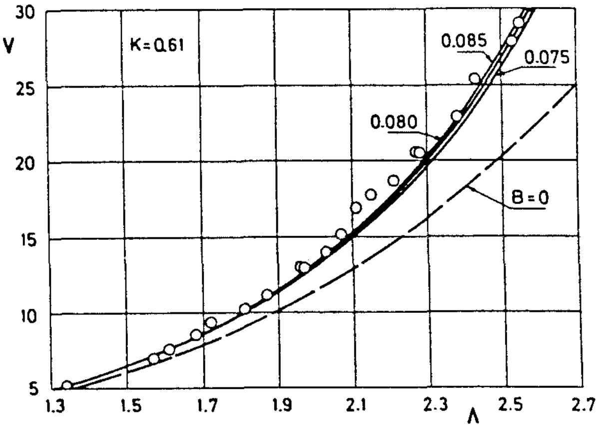

Fig. 4. Variation with the slenderness, Λ, of the minimum stable volume, V, of liquid bridges between unequal disks ($K = 0.61$). The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

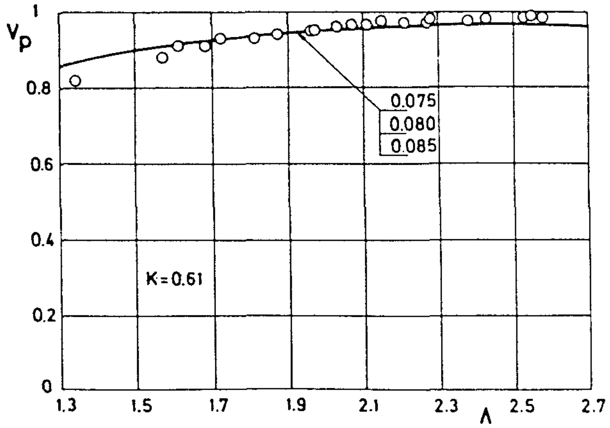

Fig. 5. Variation with the slenderness, Λ, of the partial volume at minimum volume stability limit, V_p, of liquid bridges between unequal disks ($K = 0.61$). The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

[1] Obviously in an earth laboratory gravity always acts in the same direction. However, results here presented are referred to the reference system sketched in fig. 1, which is the one normally used in the literature. Therefore, a negative value of Bond number means that during experiments the larger disk was on the bottom and the small disk on the top of the liquid bridge.

slenderness Λ are shown in fig. 5. As it can be observed in fig. 4, experimental values of minimum volume are in agreement with theoretical ones except for a small interval of values of the slenderness ranging from $\Lambda = 2.1$ to $\Lambda = 2.3$, where measured values of minimum volume stability limits are some 10 % higher than those corresponding to the theoretical limit for $B = -0.080$. These differences are explained because of the perturbation existing in the laboratory during these experiments. In fig. 4 also the minimum volume corresponding to gravitationless conditions ($B = 0$) is shown. Note that a negative value of the Bond number (this means that gravity is pointed to the larger disk) decreases the stability of liquid bridges between unequal disks, this effect being more and more important as the slenderness of the liquid bridge increases.

In respect to the dynamics of breaking, partial volume V_p, versus slenderness Λ (fig. 5), the experimental results show the same trends as theoretical predictions: the final configuration consists of a very large drop attached to the larger disk and a small drop attached to the smaller one. This behaviour can be explained taking into account the antisymmetric character of both stimuli (unequal disks and axial gravity) and that in this case both stimuli act in the same sense: each one of them separately cause the liquid bridge interface to have a neck close to the small disk regardless of the value of Λ. Note also that V_p grows as Λ increases and that, for the range of values of Bond number considered ($B = -0.075, -0.080, -0.085$) the theoretical values of V_p are practically independent of the value of B. It should be pointed out that the differences between theoretical and experimental results increase as Λ decreases, the reason for this discrepancy being that the accuracy of the slice model used to calculate theoretical values of V_p decreases as the slenderness decreases: this model neglects radial inertial effects, which is only justified if the slenderness of the liquid bridge is large enough, say $\Lambda > 1.5$.

The most interesting results are probably those corresponding to the case $K = 0.60$ and $B = 0.080$, which are shown in fig. 6 (minimum volume V versus slenderness Λ) and in fig. 7 (partial volume V_p versus slenderness Λ). As already stated, if Λ is large enough and the values of the parameters K and B are appropriate, an axial gravity pointing to the small disk is a stabilizing effect for a liquid bridge between unequal disks, as it is clearly shown in fig. 6. Note, for instance, that in liquid bridges with a slenderness $\Lambda = 2.9$, if $B \sim 0.08$, the volume of liquid can be reduced an amount of the order of 30 % of the stable minimum volume corresponding to $B = 0$, and the configuration be still stable. A characteristic of the experimental results shown in fig. 6 is that they become more and more dispersed as one approaches the value Λ where the discontinuity in minimum volume curves appears ($\Lambda_A \approx 3.1$), which is in agreement with theoretical predictions. Compare the theoretical curves of fig. 6 with those shown in fig. 4 (in both figures the same scale has been used), a variation of the same magnitude in the value of the Bond number causes a variation in the minimum volume stability limit which is greater if $B > 0$ than in the contrary case, $B < 0$.

Concerning the partial volume, fig. 7, the theoretical results show a good agreement with experimental ones. In this case ($K = 0.60$, $B \approx 0.080$) both stimuli are in opposition, and the relative importance of each one of them depends on the value of the slenderness Λ. Bond number becomes more and more important as the slenderness increases, in such a way that this effect is dominant if $\Lambda > 3.1$; in that case equilibrium shapes show a neck close to the larger disk, and when the liquid bridge disruption takes place the final configuration consists of a large drop of liquid attached to the bottom disk and a small drop attached to the top one. The contrary occurs when $\Lambda < 3.1$: the interface shape is mainly driven by the fact that the disks are unequal in diameter, liquid bridge interfaces have a neck close to the smaller disk and this position of the neck determines the volume of the drops resulting after the breaking of the liquid column.

It has been already stated that the value of Λ_A depends on the values of the parameters K and B. From results published in [6] one can conclude that the discontinuity in the minimum volume stability limit vanishes if Λ_A is small

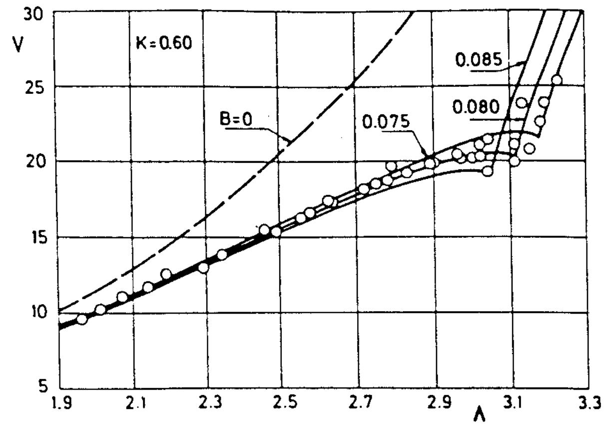

Fig. 6. Variation with the slenderness, Λ, of the minimum stable volume, V, of liquid bridges between unequal disks ($K = 0.60$). The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

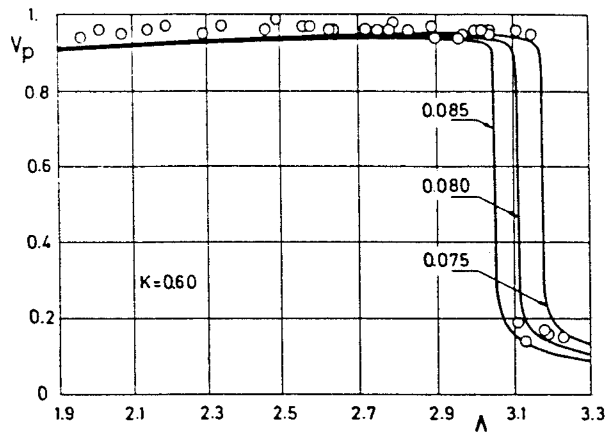

Fig. 7. Variation with the slenderness, Λ, of the partial volume at minimum volume stability limit, V_p, of liquid bridges between unequal disks ($K = 0.60$). The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

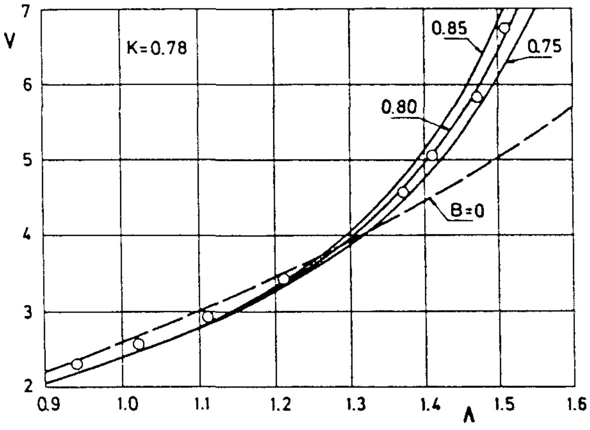

Fig. 8. Variation with the slenderness, Λ, of the minimum stable volume, V, of the liquid bridges between unequal disks (K = 0.78), The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

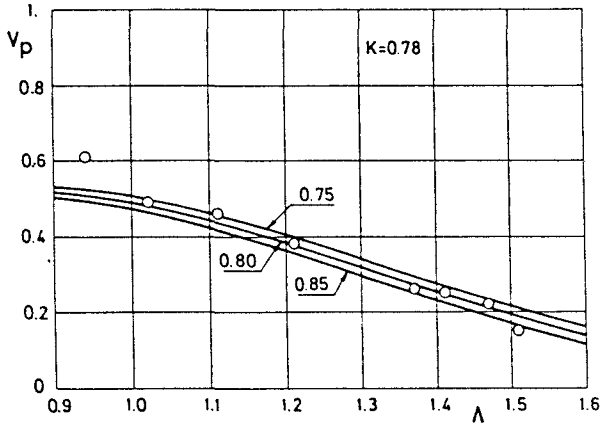

Fig. 9. Variation with the slenderness, Λ, of the partial volume at minimum volume stability limit, V_p, of liquid bridges between unequal disks (K = 0.78). The symbols indicate experimental results whereas the curves correspond to theoretical ones. Numbers on the curves indicate the value of the Bond number, B

enough, let us say $\Lambda_A < 1.5$. In the third series of experiments, aiming at checking this behaviour, the values of the parameters $K = 0.78$, $B = 0.8$ were chosen, and the results obtained are shown in figs. 8 and 9. As it can be observed in fig. 8, in spite of the value of Bond number ($B > 0$) there is not any discontinuity in the minimum volumes curves. Note that, depending on the value of the slenderness, an axial microgravity increases or decreases the stability margin in respect to the minimum volume stability limit corresponding to $B = 0$: the curves for $B > 0$ cross the one for $B = 0$ (or course the same happens with the results shown in fig. 6, although the different stability limit curves corresponding to $B \sim 0.08$ cross that for $B = 0$ at points in the Λ-V stability diagram which are out of the range of values of interest).

In respect to the partial volume of this series of experiments ($K = 0.78$) it is difficult to compare with theoretical results, the problem coming from the fact that the slice model fails if the slenderness is not high enough. However, within this range of slenderness, liquid bridge interfaces have a well pronounced neck which, according to experimental evidence and theoretical results corresponding to liquid bridges with slenderness higher than the ones under consideration, strongly determines the volume of the resulting drops. In consequence, theoretical values of V_p plotted in fig. 9 have been calculated assuming that the volume of each one of the two final drops is the volume of liquid existing between the neck of the liquid column at stability limit and the corresponding disk. Observe that this estimation of V_p agrees with experimental values, which supports the hypothesis that the position of the neck determines the volume of the drops resulting after liquid bridge disruption if the slenderness of the liquid bridge is small enough.

4 Conclusions

In this paper the influence of Bond number on the minimum volume stability limits of axisymmetric liquid bridges betweeen unequal disks has been studied experimentally using millimetric liquid bridges. In addition, the volumes of the drops resulting after liquid bridge breaking have been measured for a large number of liquid bridge configurations and compared with the results predicted by a one-dimensional slice model already used in liquid bridge dynamics.

In respect to minimum volume stability limits, experimental results corroborate the theoretical predictions published in [6] concerning the combined effects of Bond number and supporting disks of different diameters. On the other hand, experimetnal results related to partial volume are in agreement with theoretical ones predicted by the one-dimensional slice model, provided that the liquid bridge slenderness is high enough, within the range of validity of such slice model ($\Lambda > 1.5$).

Acknowledgements

This work has been performed during a stay of Dr. Bezdenejnykh at Lamf-μg (Madrid) sponsored by the European Space Agency and Universidad Politécnica de Madrid. This work has been supported also by the Spanish Comisión Interministerial de Ciencia y Technologia (CICYT), Project No. ESP88-0359. The authors wish to thank Dr. J. M. Perales for his helpful advice.

References

1 *Martinez, I.:* Stability of axisymmetric liquid bridges. ESA SP-191, 267–273 (1983)
2 *Meseguer, J.:* Stability of slender, axiysmmetric liquid bridges between unequal disks, J. Crystal Growth, 67, 141–143 (1984)
3 *Meseguer, J.:* The dynamics of axisymmetric slender liquid bridges between unequal disks, J. Crystal Growth, 73, 599–608 (1985)
4 *Meseguer, J., Mayo, L. A., Llorente, J. C., Fernández, A.:* Experiments with liquid bridges in simulated microgravity, J. Crystal Growth, 73, 609–621 (1985)
5 *Martinez, I., Perales, J. M.:* Liquid bridge stability data, J. Crystal Growth, 78, 369–378 (1986)
6 *Perales, J. M., Meseguer, J., Martinez, I.:* Minimum volume stability limits for axisymmetric liquid bridges subject to steady axial acceleration J. Crystal Growth, 110, 855–861 (1991)
7 *Meseguer, J.:* The breaking of axisymmetric slender liquid bridges, J. Fluid Mech., 130, 123–151 (1983)