A Stochastic Bi-objective Location Model for Reverse Logistics

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Motivation

- Collecting, recycling, disposal is increasingly important
  - Environment aspects
  - Economic aspects

- Management Integrated Systems

- Problem studied in this work
  - Municipal
  - Regional
  - National
General Outline

A set of origins /populations ⇒ Different types of disposal are generated in each of them

- Paper
- Plastic
- Urban Solid Waste (USW)
- Glass
- ...
General Outline

- Disposal → Collection center
- Compressed disposal → Lower transportation costs to the recovery facilities
- Disposal → Recovery center
  - Residual product → Landfill
  - Valuable commodity
    - Recycling
    - Incineration
    - ...
- Origin
Technologies

Different types of technology available for each type of disposal
- Manual
- Automatic
- Semi-automatic

Different rates of conversion, costs, etc.
Assumptions

- We have a set of potential locations for installing
  - Collection centers
  - Recovery centers
  - Landfills

- Collection centers
  - Can be set-up for receiving any kind of disposal
  - There is a maximum operating capacity for each kind of disposal

- Recovery centers
  - Can receive any kind of disposal if the specific technology is installed
  - The technology is modular
  - There is a maximum number of modules of each type of technology for each type of disposal
Network Structure
Additional Assumptions

- Minimum levels are imposed on the recycling levels of some products (glass and paper)
  e.g. In most cases it is more profitable incinerating than recycling

- For a valuable commodity, there might be a maximum amount that can be sold
  e.g. Often there are limits on the amount that can be sent to the incinerators

- Each facility operates with a single technology for each product
Costs and Profits

- **Costs**
  - Set-up costs for installing the facilities (collection/recovery centers and landfills)
  - Set-up cost for preparing a collection center to process each specific disposal
  - Set-up cost for installing a specific technology in each recovery center
  - Transportation costs
  - Disposal tax associated with the residual product

- **Benefits**
  - Profit associated with valuable commodities
Environmental Impact

Obnoxious effect associated with the facilities

- Noise
- Odour
- ...

Function of the euclidian distances

We assume that incentives can be offered to the populations in order to decrease the negative feelings towards the facilities
Stochasticity

- **Sources of stochasticity**
  - Amount of each type of disposal generated
  - Transportation costs

- **Embedding stochasticity**
  - A set of scenarios
  - Probabilities associated with scenarios
In all...

Location Model
- Multiple echelons
- Multi commodity
- Stochastic
- Bi-objective
- Choice of technology

Two-stage Stochastic Mixed Integer Programming Problem
- 1st stage: strategic decisions
- 2nd stage: operational/tactical decisions
Index Sets

\( \mathcal{P}_W \) : set of disposal types
\( \mathcal{P}_F \) : set of valuable commodities
\( \mathcal{G} \) : set of different technologies available for the recovery centers
\( \mathcal{J} \) : set of origins of the disposals
\( \mathcal{I}_C \) : set of potential locations for the collection centers
\( \mathcal{I}_R \) : set of potential locations for recovery centers
\( \mathcal{I}_L \) : set of potential locations for the landfills
\( S \) : set of scenarios
Parameters

\(O_{jps}\) : amount of disposal \(p \in \mathcal{P}_W\) originated at origin \(j \in \mathcal{J}\)

\(KC_p\) : capacity of a collection center for processing product \(p \in \mathcal{P}_W\)

\(Q_{pg}\) : capacity of a module of technology \(g \in \mathcal{G}\) when processing product \(p \in \mathcal{P}_W\)

\(KR_{ipg}\) : maximum number of modules of technology \(g \in \mathcal{G}\) that can be installed at recovery center \(i \in \mathcal{I}_R\) to process product \(p \in \mathcal{P}_W\)

\(A_{pqg}\) : maximum proportion of valuable commodity \(p \in \mathcal{P}_F\) that can be obtained from product \(q \in \mathcal{P}_W\) when using technology \(g \in \mathcal{G}\)
Parameters II

\[ D_{ij}^O : \text{road distance between origin } j \in \mathcal{J} \text{ and facility } i \in \mathcal{I}_C \cup \mathcal{I}_R \]

\[ D_{i'i}^{NO} : \text{road distance between facility } i \in \mathcal{I}_R \text{ and facility } i' \in \mathcal{I}_C \cup \mathcal{I}_L \]

\[ MIN_{pq} : \text{the minimum rate imposed for the conversion of product } q \in \mathcal{P}_W \text{ into product } p \in \mathcal{P}_F \]

\[ MAX_p : \text{the maximum amount of valuable commodity } p \in \mathcal{P}_F \text{ that can be sold} \]
Parameters III

\(OE_{ip}^C\) : the obnoxious effect due to the collection of product \(p \in \mathcal{P}_W\) in collection center \(i \in \mathcal{I}_C\)

\(OE_{ip}^R\) : the obnoxious effect due to the processing of product \(p \in \mathcal{P}_W\) in recovery center \(i \in \mathcal{I}_R\)

\(OE_i^L\) : the obnoxious effect due to the residual product in landfill \(i \in \mathcal{I}_L\)

\(p_s\) : probability of scenario \(s \in S\)
Costs/Profits

\[ FC_i : \] fixed cost for installing a collection center at \( i \in I_C \)

\[ FR_i : \] fixed cost for installing a recovery center at \( i \in I_R \)

\[ FL_i : \] fixed cost for installing a landfill at \( i \in I_L \)

\[ FCP_p : \] fixed cost for preparing a collection center for receiving disposal
\( p \in P_W \)

\[ FRPG_{ipg} : \] fixed cost for installing a module of technology \( g \in G \) in location
\( i \in I_R \) for processing disposal \( p \in P_W \)
Costs/Profits II

\[ CTO_{ps} : \text{cost (euro per unit and per km) for shipping one unit of product} \]
\[ p \in \mathcal{P}_W \text{ from an origin to a collection center or to a recovery center under scenario } s \in S \]

\[ CTC_{ps} : \text{cost (euro per unit and per km) for shipping one unit of product} \]
\[ p \in \mathcal{P}_W \text{ from a collection center to a recovery center under scenario } s \in S \]

\[ CTR_s : \text{cost (euro per unit and per km) for shipping one unit of residual product from a recovery center to a landfill under scenario } s \in S \]

\[ B_p : \text{profit of each unit of valuable commodity } p \in \mathcal{P}_F \]

\[ DT : \text{disposal tax per unit of residual product} \]
(Strategic) Decision Variables

\[
y^C_i = \begin{cases} 
1 & \text{if collection center installed at } i \in \mathcal{I}_C \text{ is operating} \\ 
0 & \text{otherwise} 
\end{cases}
\]

\[
y^R_i = \begin{cases} 
1 & \text{if recovery center installed at } i \in \mathcal{I}_R \text{ is operating} \\ 
0 & \text{otherwise} 
\end{cases}
\]

\[
y^L_i = \begin{cases} 
1 & \text{if a landfill installed at } i \in \mathcal{I}_L \text{ is open} \\ 
0 & \text{otherwise} 
\end{cases}
\]
(Strategic) Decision Variables II

\[ w_{ip}^C = \begin{cases} 
1 & \text{if collection center } i \in \mathcal{I}_C \text{ is processing disposal } p \in \mathcal{P}_W \\
0 & \text{otherwise} 
\end{cases} \]

\[ z_{ipg}^R = \begin{cases} 
1 & \text{if recovery center } i \in \mathcal{I}_R \text{ is processing disposal } p \in \mathcal{P}_W \\
 & \text{using technology } g \in \mathcal{G} \\
0 & \text{otherwise} 
\end{cases} \]

\[ n_{ipg}^R = \text{number of modules of technology } g \in \mathcal{G} \text{ installed in recovery center } i \in \mathcal{I}_R \text{ to disposal } p \in \mathcal{P}_W \]
(Tactical/Operational) Decision Variables

\begin{align*}
x_{ijps}^{OC} & = \text{amount of disposal } p \in \mathcal{P}_W \text{ sent from origin } j \in \mathcal{J} \text{ to collection center } i \in \mathcal{I}_C \text{ under scenario } s \in S \\
x_{ijps}^{OR} & = \text{amount of disposal } p \in \mathcal{P}_W \text{ sent from origin } j \in \mathcal{J} \text{ to recovery center } i \in \mathcal{I}_R \text{ under scenario } s \in S \\
x_{ii'ps}^{CR} & = \text{amount of disposal } p \in \mathcal{P}_W \text{ sent from collection center } i \in \mathcal{I}_C \text{ to recovery center } i' \in \mathcal{I}_R \text{ under scenario } s \in S \\
x_{ii's}^{RL} & = \text{amount of residual disposal obtained in recovery center } i \in \mathcal{I}_R \text{ sent to landfill } i' \in \mathcal{I}_L \text{ under scenario } s \in S \\
x_{ipqgs}^{RV} & = \text{amount of valuable commodity } p \in \mathcal{P}_F \text{ obtained in recovery center } i \in \mathcal{I}_R \text{ from product } q \in \mathcal{P}_W \text{ using technology } g \in \mathcal{G} \text{ under scenario } s \in S
\end{align*}
Flow Variables and Unitary Transportation Costs
Objective Function 1. Costs

$$\text{MIN } \sum_{i \in I_C} FC_i y_i^C + \sum_{i \in I_R} FR_i y_i^R + \sum_{i \in I_L} FL_i y_i^L$$
$$+ \sum_{p \in P_W} \left( FCP_p \sum_{i \in I_C} w_{ip}^C \right) + \sum_{i \in I_R} \sum_{p \in P_W} \sum_{g \in G} FRPG_{ipg} n_{ipg}^R$$
$$+ \sum_{s \in S} \rho_s \left[ \sum_{j \in J} \sum_{i \in I_C} \sum_{p \in P_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j \in J} \sum_{i \in I_R} \sum_{p \in P_W} CTO_p D_{ij}^O x_{ijps}^{OR} \right]$$
$$+ \sum_{i \in I_C} \sum_{i' \in I_R} \sum_{p \in P_W} CTC_p D_{ii'}^NO x_{ii'ps}^{CR} + \sum_{i \in I_R} \sum_{i' \in I_L} CTR \times D_{ii'}^NO x_{ii's}^{RL}$$
$$+ DT \sum_{i \in I_R} \sum_{i' \in I_L} x_{ii's}^{RL} - \sum_{i \in I_R} \sum_{p \in P_F} \sum_{q \in P_W} B_p \sum_{g \in G} x_{ipqgs}^{RV}$$
Objective Function 1. Costs

\[
\text{MIN} \sum_{i \in \mathcal{I}_C} FC_i y_i^C + \sum_{i \in \mathcal{I}_R} FR_i y_i^R + \sum_{i \in \mathcal{I}_L} FL_i y_i^L \\
+ \sum_{p \in \mathcal{P}_W} \left( FC_{p} \sum_{i \in \mathcal{I}_C} \beta_{ip}^C \right) + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} FR_{pg} n_{pg}^R \\
+ \sum_{s \in \mathcal{S}} \rho_s \left[ \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \right. \\
+ \sum_{i \in \mathcal{I}_C} \sum_{i' \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
\left. + DT \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} x_{ii's}^{RL} - \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_F} \sum_{q \in \mathcal{P}_W} B_p \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \right]
\]
Objective Function 1. Costs

\[
\begin{align*}
\text{MIN } & \sum_{i \in \mathcal{I}_C} FC_i y_i^C + \sum_{i \in \mathcal{I}_R} FR_i y_i^R + \sum_{i \in \mathcal{I}_L} FL_i y_i^L \\
& + \sum_{p \in \mathcal{P}_W} \left( FCP_p \sum_{i \in \mathcal{I}_C} w_{ip}^C \right) + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} \text{FRPG}_{ipg} n_{ipg}^R \\
& + \sum_{s \in \mathcal{S}} \rho_s \left[ \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
& + \sum_{i \in \mathcal{I}_C} \sum_{i' \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTC_{p} D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii's}^{RL} \\
& + DT \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} x_{ii's}^{RL} - \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_F} \sum_{q \in \mathcal{P}_W} B_p \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \right]
\end{align*}
\]
Objective Function 1. Costs

\[
\text{MIN } \sum_{i \in \mathcal{I}_C} FC_i y_i^C + \sum_{i \in \mathcal{I}_R} FR_i y_i^R + \sum_{i \in \mathcal{I}_L} FL_i y_i^L \\
+ \sum_{p \in \mathcal{P}_W} \left( FCP_p \sum_{i \in \mathcal{I}_C} w_{ip}^C \right) + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} FRPG_{ipg} n_{ipg}^R \\
+ \sum_{s \in \mathcal{S}} \rho_s \left[ \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OC} + \sum_{j \in \mathcal{J}} \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTO_p D_{ij}^O x_{ijps}^{OR} \\
+ \sum_{i \in \mathcal{I}_C} \sum_{i' \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} CTC_p D_{ii'}^{NO} x_{ii'ps}^{CR} + \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} CTR \times D_{ii'}^{NO} x_{ii'ps}^{RL} \\
+ DT \sum_{i \in \mathcal{I}_R} \sum_{i' \in \mathcal{I}_L} x_{ii'ps}^{RL} - \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_F} \sum_{q \in \mathcal{P}_W} B_p \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \right]
\]
Objective Function 2. Obnoxious Effect

\[
\text{MIN} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} OE_{ip}^C w_{ip}^C + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} OE_{ip}^R \sum_{g \in \mathcal{G}} z_{ipg}^R + \sum_{i \in \mathcal{I}_L} OE_{i}^L y_{i}^L
\]
Objective Function 2. Obnoxious Effect

\[
\text{MIN} \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} O E_{i,p}^C w_{i,p}^C + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} O E_{i,p}^R \sum_{g \in \mathcal{G}} z_{ipg}^R + \sum_{i \in \mathcal{I}_L} O E_i^L y_i^L
\]
Objective Function 2. Obnoxious Effect

\[
\text{MIN } \sum_{i \in \mathcal{I}_C} \sum_{p \in \mathcal{P}_W} O\!E_{ip}^C w_{ip}^C + \sum_{i \in \mathcal{I}_R} \sum_{p \in \mathcal{P}_W} O\!E_{ip}^R \sum_{g \in \mathcal{G}} z_{ipg}^R + \sum_{i \in \mathcal{I}_L} O\!E_i^L y_i^L
\]
Objective Function 2. Obnoxious Effect

\[
\text{MIN } \sum_{i \in I_c} \sum_{p \in P_w} OE_{ip}^C w_{ip}^C + \sum_{i \in I_R} \sum_{p \in P_w} OE_{ip}^R \sum_{g \in G} z_{ipg}^R + \sum_{i \in I_L} OE_i^L y_i^L
\]
Constraints I

All waste generated must be sent to a facility

\[
\sum_{i \in I_C} x_{ijps}^{OC} + \sum_{i \in I_R} x_{ijps}^{OR} = O_{jps} \quad p \in P_W, j \in J, s \in S
\]

Flow conservation in collection centers

\[
\sum_{j \in J} x_{ijps}^{OC} = \sum_{i' \in I_R} x_{ii'ps}^{CR} \quad i \in I_C, p \in P_W, s \in S
\]

Balance of valuable products obtained from waste products

\[
A_{pqg} \left( \sum_{j \in J} x_{ijqs}^{OR} + \sum_{i' \in I_C} x_{ii'qs}^{CR} \right) \geq x_{ipqgs}^{RV} \quad i \in I_R, q \in P_W, p \in P_F, g \in G
\]
Constraints II

Residual product to be sent to landfills

$$\sum_{i' \in \mathcal{I}_L} x_{ii'}^{RL} = \sum_{q \in \mathcal{P}_W} \left( \sum_{j \in \mathcal{J}} x_{ijqs}^{OR} + \sum_{i' \in \mathcal{I}_C} x_{i'i'qs}^{CR} \right) - \sum_{q \in \mathcal{P}_W} \sum_{p \in \mathcal{P}_F} \sum_{g \in \mathcal{G}} x_{ipqg}^{RV} \quad i \in \mathcal{I}_R, s$$

Capacity constraints for each technology at each recovery centers

$$\sum_{j \in \mathcal{J}} x_{ijps}^{OR} + \sum_{i' \in \mathcal{I}_C} x_{i'i'ps}^{CR} \leq \sum_{g \in \mathcal{G}} Q_{pg} n_{ipg}^{R} \quad i \in \mathcal{I}_R, p \in \mathcal{P}_W, s \in S$$
Constraints III

Minimum rates of conversion (law)

$$\sum_{i \in \mathcal{I}_R} \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \geq MIN_{pq} \left( \sum_{j \in \mathcal{J}} O_{jqs} \right) \quad q \in \mathcal{P}_W, \; p \in \mathcal{P}_F, \; s \in S$$

Market constraints

$$\sum_{i \in \mathcal{I}_R} \sum_{q \in \mathcal{P}_W} \sum_{g \in \mathcal{G}} x_{ipqgs}^{RV} \leq MAX_p \quad p \in \mathcal{P}_F, \; s \in S$$

Capacity constraints for the collection centers

$$\sum_{j \in \mathcal{J}} x_{ijps}^{OC} \leq KC_p \omega_{ip}^C \quad i \in \mathcal{I}_C, \; p \in \mathcal{P}_W, \; s \in S$$
Constraints IV

Consistency for product conversion and the available technology

\[ x^{RV}_{ipqgs} \leq \left( \sum_{j \in J} O_{jq} \right) z_{ipg}^{R} \quad i \in \mathcal{I}_{R}, \ q \in \mathcal{P}_{W}, \ p \in \mathcal{P}_{F}, \ g \in \mathcal{G}, \ s \in \mathcal{S} \]

A collection center can only be used if it was installed

\[ \omega_{ip}^{C} \leq y_{i}^{C} \quad i \in \mathcal{I}_{C}, \ p \in \mathcal{P}_{W} \]

A landfill can only be used if it was installed

\[ x_{ii'}^{RL} \leq \left( \sum_{j \in J} \sum_{p \in \mathcal{P}_{W}} O_{jp} \right) y_{i}^{L} \quad i \in \mathcal{I}_{R}, \ i' \in \mathcal{I}_{L} \]
Constraints V

Unicity of technology

$$\sum_{g \in G} z_{ipg}^R \leq 1 \quad i \in I_R, \ p \in P_W$$

Consistency in recovery centers (technology)

$$\sum_{g \in G} z_{ipg}^R \leq y_i^R \quad i \in I_R, \ p \in P_W$$

Constraint for the number of modules of each technology

$$n_{ipg}^R \leq K R_{ipg} z_{ipg}^R \quad i \in I_R, \ p \in P_W, \ g \in G$$
Province of Córdoba

- 78 towns
- 8 eligible locations: Collection, recovery and landfills
- 4 waste products: Glass, paper, USW, yellow
- 5 valuable products: Glass, paper, plastic, compost, incineration
- 2 technologies: Manual/automatic
- 9 scenarios

- 10144 constraints
- 48571 variables (64 integer, 120 binary)
Goals and Methodology

Goals:
- To obtain non-dominated solutions
- To check if these solutions can be found in reasonable computational times

Methodology:
- Solving the problem with objective function 1 (cost)
- Solving the problem with objective function 2 (obnoxious effect)
- Solving a problem
  - Objective function: convex combination of the two objective functions
  - Original constraints
  - Two additional constraints (area to explore)
Optimal Solution (Cost). Flows for Scenario 1. USW
Optimal Solution (Obnoxious Effect). Flows for Scenario 1. USW
Non-dominated Solutions

- 275 sec.
- 493 sec.
- 8739 sec.
- 65441 sec.
- 3359 sec.
Conclusions and Further Work

- **Conclusions**
  - Multi echelon, multi commodity, stochastic, bi-objective facility location problem with choice of technology
  - Reasonable computational times
  - Non dominated solutions can be obtained

- **Further work**
  - More comprehensive models (e.g. obnoxious effect dependent on flows/capacities)
  - Efficient solution techniques for large-scale instances
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Thank you all

Comments, suggestions, questions are more than welcome