I. INTRODUCTION

Building a strategy to take part in external electricity markets has an additional problem: the external systems are based on marginal prices which are not time dependent. Moreover, the internal system operation is based on the minimization of the total costs with time dependence, both in costs (start up costs, for instance) as well as in the flexibility of operation (thermal units, for example). The strategy is to submit hourly buying and selling bids with the following data: maximum amount of import and export as so as block size and price of each energy block. Our methodology for building an optimal strategy consists of three steps:

1. The first step is to forecast market prices in external systems.

2. The second step is to run a Unit Commitment on the internal system taking into account the expected prices in the first step.

3. The third step, based on the results of the previous steps, consists of preparing the bids for external markets.

The computation of electricity prices is done by applying the methodology described in García-Martos et al. (2012), which consists of extracting dynamic common factors from the 24-dimensional vector of hourly electricity prices. This gives more accurate forecasts and additionally the model has a nice physical interpretation related to the load curve. The methodology applied in the third step is a methodology developed for this purpose and is a conversion of actual costs to offer prices. The methodology applied is used in a real system, the Balearic Islands, recently connected to the Iberian Peninsula with a 480 MW capacity DC submarine cable for export and import. This methodology is used not only to prepare the strategy to participate in external markets, but also for programming and planning the internal system in longer horizons (weekly, monthly and even yearly). We present the results obtained with the software tool developed for this purpose. In Section II we describe the software and methodology for price forecasting. Sections III and IV describes software and methodology applied to solve the Unit Commitment problem and Bids calculation. In Section V we demonstrate practical effects on a particular system. Section VI concludes. Finally, we list some references related to similar researches. The reference [1] is related to Price Forecasting methodology applied here. The references [2] - [4] do research into strategies for generation companies, GenCos. The reference [5] explains our methodology, using UC, about cost calculation in an electrical system. The references [6] - [10] are some additional references related to price forecasting.

II. PRICE FORECASTING BY USING A DYNAMIC FACTOR MODEL

A. Descriptive Statistics of the Hourly Series of Prices

By analyzing descriptively the hourly series of prices the methodology here applied for short and long run forecasting of electricity prices can be justified. This methodology is framed within multivariate time series methods and dimensionality reduction techniques.

In Figure 1, the 24 hourly series of electricity prices in the period January 1998 till July 2013 in the Iberian Market is shown. And a common pattern in the evolution over time of these series can be detected, not only in the conditional mean (prices) but also in the conditional variances (volatilities), i.e., that the evolution over time of prices and volatilities are common for the 24 hours.
Moreover, not only the level of the series is lower in some hours, such as those in the deep night and higher in peak hours, but also the variability depends on the hour of the day (as it is shown in Figure 2, where a boxplot of the hourly prices during the whole year 2012 is presented).

Additionally, the clearing mechanism in the Iberian Market, where every day at noon, all the prices are cleared at the same time for every hour in the forthcoming day, with the same common information, is clearly related with this multivariate modeling proposal. Thus, everyday, a 24-dimensional data including the 24 hourly prices for the following day is available. Here we deal with the 24-dimensional vector of series of prices, as follows, where \( p_{h,d} \) is the price at hour \( h \) of day \( d \):

\[
\begin{pmatrix}
  p_{1,d} & p_{2,d} & \cdots & p_{24,d} \\
  p_{1,d+1} & p_{2,d+1} & \cdots & p_{24,d+1} \\
  \vdots & \vdots & & \vdots \\
  p_{1,D} & p_{2,D} & \cdots & p_{24,D}
\end{pmatrix}
\]

B. The Seasonal Dynamic Factor Model for the Vector of Electricity Prices

A possible alternative for modeling a vector of series are Vector ARIMA models (Auto Regressive Integrated Moving Average), the so called VARIMA models (Vector ARIMA models, see [6] for a good description and examples of these models). However, each parameter to estimate is a 24 by 24 matrix, and the well known problem of “curse of dimensionality” arises.

Unobserved component models (a good review of them can be encountered in [7]) are useful for taking into account the multivariate evolution over time of a vector of high dimension (as the vector of hourly prices is), but having a smaller number of parameters to estimate.

The intuitive idea behind Dynamic Factor Models (a particular model whose formulation can be expressed under the State-space formulation, that is a particular type of unobserved component model) is to build a small number \( r (<<24) \) of linear combinations of the observed original series. Some references in this area, not in Electrical Engineering but in Economics and Demography are [8] to [10]. The introduction of these methods and their extension to be able to deal with the particular features of the series of electricity prices are [11]-[13].

Given that the vector of prices has dimension 24, this information could be summarized by building a single linear combination of the observed series. An easy one to build would be the daily mean, which gives the same weight, 1/24, to each series (weighting them equally). However, giving the same weight to all the series does not take into account that some of the series present larger variabilities (Figure 2). The Dynamic Factor Model here considered (all the details can be encountered in Garcia-Martos et al., 2012) is the complex extension to the dynamic case of the well-known Principal Component Analysis (PCA).

The equations of the Seasonal Dynamic Factor Model are the following:

\[
y_t = \mathbf{f}_t \mathbf{P} + e_t
\]

where \( y_t \) is the 24-dimensional vector of centered hourly prices (a centered variable is built by subtracting its mean to the original one), i.e., \( y_t = p_t - \text{mean}(p_t) \), being \( p_t \) the original hourly prices. \( \mathbf{P} \) is the loading matrix, whose dimensions are 24 by \( r \), and where each column contains the weights for building the \( r (<<24) \) common factors \( \mathbf{f}_t \), which is an \( r \)-dimensional vector. Thus \( \mathbf{f}_t \) is \( r \) by \( r \), where \( r \) is the number of observations in the original vector of series. They contain the commonalities between all the original series. The weights included in \( \mathbf{P} \) are easily calculated by following the procedure that is described step-by-step in Garcia-Martos et al. (2012). Each column of \( \mathbf{P} \) is the eigenvector corresponding to the largest \( r \) eigenvalues of the autocovariance matrix of lag \( l \) of the centered prices \( y_t \). The lag \( l \) considered depends on \( y_t \) being stationary in mean or not.

\( e_t \) are the 24 specific factors. They are useful just for short-term forecasting, since they are stationary, and thus, their long-term forecasts tend to zero.

Additionally, each one of the \( r \) common factors are modeled by means of a seasonal ARIMA \((p_s, d_s, q_s) \times (P_s, D_s, Q_s)\) model as follows, \( k = 1, \ldots, r \):

\[
\phi_{p_s}(B)\Phi_{P_s}(B^S)\psi_{d_s}(B^S)\psi_q(B^S)f_{k,t} = \theta_{q_s}(B)\Theta_{Q_s}(B^S)e_{k,t}
\]

where \( B \) is the lag operator, \( s \) the order of the seasonality, \( \phi_{p_s}(B) \) is the auto-regressive polynomial of order \( p_s \), \( \Phi_{P_s}(B^S) \) is the auto-regressive polynomial corresponding to the AR seasonal part. \( \theta_{q_s}(B) \) is the polynomial for the moving average component and \( \Theta_{Q_s}(B^S) \) the polynomial that represents the...
seasonal MA component. \(d_k\) and \(D_k\) are respectively the number of regular and seasonal unit roots.

The forecasts for forecasting horizon \(h\), are calculated using the following forecasting equation that includes the \(h\)-step ahead forecasts of the common factors as well as the \(h\)-step ahead forecasts of the specific factors (which are 0 when \(h\) is long enough, since the specific factors are stationary, and their forecasts just influence in the short term). In the following equation forecasts of \(\hat{y}_{t+h}\), \(\hat{f}_{r+h}\), and \(\hat{e}_{r+h}\) appear, as well as the mean of the \(y_t\).

\[
\hat{y}_{t+h} = \hat{f}_{r+h} \mathbf{1} + \hat{e}_{r+h} + \bar{y}
\]

(3)

C. Numerical Results

When using the data from 1998 till December 2011, to compute forecasts for the whole year 2012 (forecasting horizon ranging from one day up to one year), the following forecasting errors are obtained. In Figure 3 we present the real prices as well as the forecasts, also we present percentiles 25, 50 and 75 of the Relative Forecasting Error, in percentage. For the whole year considered, the mean of the percentile 50, the so called MAPE2 (Mean Average Percentage Error 2), is 11.726%.

It is important to remark that all the forecasting errors presented in this paper correspond to out-of-sample forecasts. Thus, the data used to estimate the model does not coincide with the period being forecasted.

![Graph](image)

Fig. 3. The real and forecasts prices. Percentiles 25, 50 (median) and 75 of the Relative Forecasting Errors for the whole year 2012 using the data till the end of December 2011, by hour of the day. Y-axis: Percentiles of the Relative Forecasting Error.

III. UNIT COMMITMENT

A. Introduction

A Unit Commitment (UC) is an operation scheduling software which from a given forecasted condition for the electrical power system, develops an optimal feasible resource schedule in order to minimize the system operating costs, meeting system requirements for a given period (Figure 4).

Significant resources and system constraints are considered within the scheduling process.

![Graph](image)

Fig. 4. Example - daily operation plan for each unit.

B. System description and model

1) System Resources:

These are the thermal unit, hydro basin, pump units, independent units, import and export across an interconnector, RES (Renewable Energy Sources), among others.

2) System Constraints

System load, system reserve, fuel limitations, transmission constraints, etc.

C. Methodology

The methodology to solve the UC problem is based on Lagrange Relaxation & Decomposition technique, Mixed Integer Programming and Decommitment techniques.

IV. BID CALCULATION

A. Hourly variable cost of thermal generating units

The variable cost of the thermal generating units is formed by the following terms:

- The fuel cost is a quadratic function of power.

\[
c_{\text{fu}}(i, h) = a(i) + b(i) \cdot e(i, h) + c(i) \cdot e^2(i, h) \]

(4)

- The start-up cost is represented by an exponential curve and depends on the time the unit has been off prior to start-up

\[
c_{\text{sr}}(i, h) = a'(i) + b'(i) \cdot (1 - \exp(-t / te)) \]

(5)

- Operation and maintenance cost is a linear function of the power.

\[
c_{\text{om}}(i, h) = a''(i) + b''(i) \cdot e(i, h) \]

(6)

where:

- \(i\) - generating unit; \(h\) - hour period scheduling interval; \(a,b,c\) - quadratic curve coefficients; \(e\) - scheduled energy for unit \(i\) in period \(h\); \(a',b',te\) - exponential curve coefficients for unit \(i\) in period \(h\); \(a'',b''\) - linear curve coefficients. In the optimization process \(c_{\text{om}}\) and \(c_{\text{sr}}\) are approximated by piecewise linear curves.

Hourly costs are obtained as a result of the total costs (except start up cost) for each unit and for each time period. Figure 5 presents an example:
B. Hourly redistribution of the variable costs for Ordinary Regime units

Once the hourly variable cost of each generating unit is calculated, part of this cost should be redistributed among the hours. Due to possible existence of temporal constraints (minimum run times, startup costs, ramps, etc…), the cost incurred at certain times is caused by the need of the generating unit in other times. The purpose is to reflect each hour the cost of generation for each unit where by the unit is responsible for. Failure to do so may result in greater average costs in valley hours than average costs in peak hours, and that would lead to bid prices that would not reflect the real structure of prices in the internal electric system and an import/export operation that would contravene the expected and reasonable technical and economic operation of the internal system.

For this, we shall compare the actual dispatch with a dispatch in which the generating units will have no temporal constraints. This will assess, for each generating unit, the shift induced cost at certain hours to the hours that this cost should be imputed, and the total cost for each generating unit already calculated above is maintained.

The calculation procedure consists of the following steps:

1. Calculation of the average operation variable cost \(c'_{\text{om}}(i,h) = c_{\text{om}}(i)/\text{Sum}_h(i,h)\) of the daily net energy production result of the unit commitment execution:

\[
cgm_{\text{fun+c\text{om}}} (i) = \text{Sum}_h \left( c_{\text{fun}}(i,h) + c_{\text{om}}(i,h) \right) / \text{Sum}_h(e(i,h)) \quad (7)
\]

2. Execution of an economic dispatch, hourly independent, in merit-order according to the previous \(cgm_{\text{fun+c\text{om}}}(i)\) values . This process gives the hours in which the unit is necessary \(h'\), those when is not necessary \(h''\), and the highest average cost \(c_{\text{maxfun+c\text{om}}}(h)\) (cost of the last necessary unit).

3. The hours when the generating unit is necessary absorb:

- the redistributed start-up cost \(c'_{\text{om}}(i,h) = c_{\text{om}}(i)/\text{Sum}_h(i,h)\)
- the excess of the average cost of the hours when is not necessary with respect to the highest average cost \(c_{\text{maxfun+c\text{om}}}(h)\) of the units needed in each hour (passing costs from valley hours to peak and plain hours)

\[
c_{\text{pm}}(i,h) = \left( (cgm_{\text{fun+c\text{om}}}(i) - c_{\text{maxfun+c\text{om}}}(h'))/\text{Sum}_h(i,h) \right) \quad (8)
\]

4. The resulting variable costs are obtained:

- hours in which the group is necessary to cover demand

\[
c_{\text{var}}(i,h) = cgm_{\text{fun+c\text{om}}}(i) + c'_{\text{om}}(i,h) + c_{\text{pm}}(i,h') \quad (9)
\]

- no generating hours

\[
c_{\text{var}}(i,h) = 0 \quad (10)
\]

In Figure 6 the final result is shown:

Once these new hourly costs are calculated for each generating unit and taking into account the import/export UC results, the bids for the external markets can be prepared. The process is as follows:

- The hourly bids’ blocks, prices against power output, are associated to the unit results, cost and energy
- The maximum quantity to be presented is limited by the UC optimal results for import/export taking into account the forecasting prices in the external market.

The figure 7 shows an example results’ of this process:
V. PRACTICAL DEMONSTRATION

As a practical demonstration, it is shown the Balcanic system that is connected to the Iberian Peninsula (system based in an electricity market) with a submarine cable. The Figure 8 shows its geographical situation.

A. Initial conditions
- Market prices forecast (Figure 9)

B. Results

TABLE I shows summary of UC results for two different cases and TABLE II shows comparison results.

<table>
<thead>
<tr>
<th>TABLE I. SUMMARY OF UC RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>Total Costs</td>
</tr>
<tr>
<td>Import Price</td>
</tr>
<tr>
<td>Average Price</td>
</tr>
<tr>
<td>Difference “A” - “B”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II. MANUAL VS MARKET STRATEGY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 (discount of difference in imported energy)</td>
</tr>
<tr>
<td>Case 2 (market strategy)</td>
</tr>
</tbody>
</table>

As it can be seen in TABLE II the reduction in costs in the internal system is significant.

VI. CONCLUSIONS

Nowadays there are many small electrical systems with limited connection capacities with major electrical systems based on markets. The problem to be solved is not only how much to import/export but also at what price.

To solve this problem two things are necessary, first, to have a good forecast prices of the external markets and second, a good estimation (calculation) on the internal costs.

As prices on external markets and the internal costs are not directly comparable, it is necessary to relocate the costs, both in time as dividing them into related production blocks.
As it is demonstrated, by applying the methodology described here the costs of the internal system can be significantly reduced.

The methodology developed in this paper not only serves to apply it in the daily and intraday markets, also serves to the negotiations of bilateral long-term contracts (the process would be the same, only substituting forecasting prices in external market by offered prices from client would be needed).

In the near future, the idea is to develop Risk-Constrained Bidding Strategy with the same methodology but using a probabilistic forecasting and a probabilistic Unit Commitment.