NONLINEAR SEISMIC BEHAVIOUR OF CONCRETE ARCH BRIDGES

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Dedicated to my Parents
For their endless support
and encouragements
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Arch bridge structural solution has been known for centuries, in fact the simple nature of arch that require low tension and shear strength was an advantage as the simple materials like stone and brick were the only option back in ancient centuries. By the pass of time especially after industrial revolution, the new materials were adopted in construction of arch bridges to reach longer spans. Nowadays one long span arch bridge is made of steel, concrete or combination of these two as "CFST" \(^1\), as the result of using these high strength materials, very long spans can be achieved. The current record for longest arch belongs to Chaotianmen bridge over Yangtze river in China with 552 meters span made of steel and the longest reinforced concrete type is Wanxian bridge which also cross the Yangtze river through a 420 meters span (see figure 1.1). Today the designer is no longer limited by span length as long

\(^1\)CFST stands for Concrete Filled Steel Tubular
as arch bridge is the most applicable solution among other approaches, i.e. cable stayed and suspended bridges are more reasonable if very long span is desired (A comparison between largest spans of every type is presented in figure 1.3). Like any super structure, the economical and architectural aspects in construction of a bridge is extremely important, in other words, as a narrower bridge has better appearance, it also require smaller volume of material which make the design more economical. Design of such bridge, beside the high strength materials, requires precise structural analysis approaches capable of integrating the combination of material behaviour and complex geometry of structure and various types of loads which may be applied to bridge during its service life. Depend on the design strategy, analysis may only evaluates the linear elastic behaviour of structure or consider the nonlinear properties as well. Although most of structures in the past were designed to act in their elastic range, the rapid increase in computational capacity allow us to consider different sources of nonlinearities in order to achieve a more realistic evaluations where the dynamic behaviour of bridge is important especially in seismic zones where large movements may occur or structure experience $P-\Delta$ effect during the earthquake. The above mentioned type of analysis is computationally expensive and very time consuming. In recent years, several methods were proposed in order to resolve this problem. Discussion of recent developments on these methods and their application on long span concrete arch bridges is the main goal of this research. Accordingly available long span concrete arch bridges have been studied to gather the critical information about their geometrical aspects and properties of their materials. Based on concluded information, several concrete arch bridges were designed for further studies. The main span of these bridges range from 100 to 400 meters. The Structural analysis methods implemented in in this study are as following:

**Elastic Analysis:**

**Direct Response History Analysis (DRHA):** This method solves the direct equation of motion over time history of applied acceleration or imposed load in linear elastic range.

**Modal Response History Analysis (MRHA):** Similar to DRHA, this method is also based on time history, but the equation of motion is simplified to
single degree of freedom system and calculates the response of each mode independently. Performing this analysis require less time than DRHA.

**Modal Response Spectrum Analysis (MRSA):** As it is obvious from its name, this method calculates the peak response of structure for each mode and combine them using modal combination rules based on the introduced spectra of ground motion. This method is expected to be fastest among Elastic analysis.

**Inelastic Analysis:**

**Nonlinear Response History Analysis (NL-RHA):** The most accurate strategy to address significant nonlinearities in structural dynamics is undoubtedly the nonlinear response history analysis which is similar to DRHA but extended to inelastic range by updating the stiffness matrix for every iteration. This onerous task, clearly increase the computational cost especially for unsymmetrical buildings that requires to be analyzed in a full 3D model for taking the torsional effects in to consideration.

**Modal Pushover Analysis (MPA):** The Modal Pushover Analysis is basically the MRHA but extended to inelastic stage. After all, the MRHA cannot solve the system of dynamics because the resisting force $f_s(u, \dot{u})$ is unknown for inelastic stage. The solution of MPA for this obstacle is using the previously recorded $f_s$ to evaluate system of dynamics.
**Extended Modal Pushover Analysis (EMPA):** Expanded Modal pushover is one of very recent proposed methods which evaluates response of structure under multi-directional excitation using the modal pushover analysis strategy. In one specific mode, the original pushover neglect the contribution of the directions different than characteristic one, this is reasonable in regular symmetric building but a structure with complex shape like long span arch bridges may go through strong modal coupling. This method intend to consider modal coupling while it take same time of computation as MPA.

**Coupled Nonlinear Static Pushover Analysis (CNSP):** The EMPA includes the contribution of non-characteristic direction to the formal MPA procedure. However the static pushovers in EMPA are performed individually for every mode, accordingly the resulted values from different modes can be combined but this is only valid in elastic phase; as soon as any element in structure starts yielding the neutral axis of that section is no longer fixed for both response during the earthquake, meaning the longitudinal deflection unavoidably affect the transverse one or vice versa. To overcome this drawback, the CNSP suggests executing pushover analysis for governing modes of each direction at the same time. This strategy is estimated to be more accurate than MPA and EMPA, moreover the calculation time is reduced because only one pushover analysis is required.

Regardless of the strategy, the accuracy of structural analysis is highly dependent on modelling and numerical integration approaches used in evaluation of each method. Therefore the widely used Finite Element Method is implemented in process of all analysis performed in this research.

In order to address the study, chapter 2, starts with gathered information about constructed long span arch bridges, this chapter continuous with geometrical and material definition of new models. Chapter 3 provides the detailed information about structural analysis strategies; furthermore the step by step description of procedure of all methods is available in Appendix A. The document ends with the description of results and conclusion of chapter 4.
2.1 Introduction

Study of arch bridges under seismic loads requires a very precise modelling both geometrically and for material definition, the stability and efficiency of this complex structure is based on combination of these two factors; this chapter is mostly devoted to define the models based on constructed bridges or design codes. In section 2.2 the geometric shape of concrete arch bridges is studied in order to achieve reliable dimensions to define required models for various analyses of chapter 4. In section 2.3 the materials mechanical properties are developed based on Model Code 2010 [fib 2010]. Finally the finite element types are discussed in last section of this chapter.

2.2 Arch bridge description

2.2.1 Geometric aspects

Arch bridges, regardless of size all are following a similar idea: passing the load of the whole structure and what is standing on top of it to both sides of span through an arch. This design philosophy does not require materials with high tensional resistance, however the interaction of deck, piers and arch itself is a very critical point in design of efficient and narrow arch bridges. A schematic figure is presented
2.2. Arch bridge description

below (Figure 2.1) to illustrate the dominant geometry of arch bridges including several models analysed in chapter 4.

From the statistic point of view based on table 2.1 the span length to rise ratio

of long span concrete arcs is between 4 to 6 (see figure 2.2), this is also supported by other authors [Manterola 2006]. The models analysed in chapter 4 are generated accordingly.

<table>
<thead>
<tr>
<th>No</th>
<th>Name</th>
<th>Span</th>
<th>Rise</th>
<th>S/R</th>
<th>R/S</th>
<th>Year</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wanxian Bridge</td>
<td>420</td>
<td>84.0</td>
<td>5.00</td>
<td>0.20</td>
<td>1997</td>
<td>China</td>
</tr>
<tr>
<td>2</td>
<td>Qiubei Nanpanjiang Bridge</td>
<td>416</td>
<td>99.0</td>
<td>4.20</td>
<td>0.24</td>
<td>2015</td>
<td>China</td>
</tr>
<tr>
<td>3</td>
<td>Krk Bridge</td>
<td>416</td>
<td>67.0</td>
<td>6.21</td>
<td>0.16</td>
<td>1980</td>
<td>Croatia</td>
</tr>
<tr>
<td>4</td>
<td>Krk-A Bridge</td>
<td>390</td>
<td>52.0</td>
<td>7.50</td>
<td>0.13</td>
<td>2005</td>
<td>Croatia</td>
</tr>
<tr>
<td>5</td>
<td>Almonte Railway Bridge</td>
<td>384</td>
<td>81.0</td>
<td>4.74</td>
<td>0.21</td>
<td>2013</td>
<td>Spain</td>
</tr>
<tr>
<td>6</td>
<td>Zhaohua Jialing River Bridge</td>
<td>350</td>
<td>93.6</td>
<td>3.78</td>
<td>0.26</td>
<td>2012</td>
<td>China</td>
</tr>
<tr>
<td>7</td>
<td>Jiangjiehe Bridge</td>
<td>330</td>
<td>55.0</td>
<td>6.00</td>
<td>0.17</td>
<td>1993</td>
<td>China</td>
</tr>
<tr>
<td>8</td>
<td>Tajo Railway Bridge</td>
<td>324</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2013</td>
<td>Spain</td>
</tr>
<tr>
<td>9</td>
<td>Hoover bypass (Mike O’Callaghan-Pat Tillman Memorial Bridge)</td>
<td>323</td>
<td>84.4</td>
<td>3.83</td>
<td>0.26</td>
<td>2010</td>
<td>United States</td>
</tr>
<tr>
<td>10</td>
<td>Gladesville Bridge</td>
<td>305</td>
<td>40.7</td>
<td>7.49</td>
<td>0.13</td>
<td>1964</td>
<td>Australia</td>
</tr>
<tr>
<td>11</td>
<td>Friendship Bridge</td>
<td>290</td>
<td>78.0</td>
<td>3.72</td>
<td>0.27</td>
<td>1965</td>
<td>Paraguay-Brazil</td>
</tr>
<tr>
<td>12</td>
<td>Infante D. Henrique Bridge</td>
<td>280</td>
<td>25.0</td>
<td>11.20</td>
<td>0.09</td>
<td>2002</td>
<td>Portugal</td>
</tr>
<tr>
<td>13</td>
<td>Bloukrans Bridge</td>
<td>272</td>
<td>62.0</td>
<td>4.39</td>
<td>0.23</td>
<td>1984</td>
<td>South Africa</td>
</tr>
<tr>
<td>14</td>
<td>Arr bida Bridge</td>
<td>270</td>
<td>52.0</td>
<td>5.19</td>
<td>0.19</td>
<td>1963</td>
<td>Portugal</td>
</tr>
<tr>
<td>15</td>
<td>Froschgrundsee Viaduct</td>
<td>270</td>
<td>65.0</td>
<td>4.15</td>
<td>0.24</td>
<td>2010</td>
<td>Germany</td>
</tr>
<tr>
<td>16</td>
<td>Grmpen Viaduct</td>
<td>270</td>
<td>70.0</td>
<td>3.86</td>
<td>0.26</td>
<td>2011</td>
<td>Germany</td>
</tr>
<tr>
<td>17</td>
<td>Fujikawa Bridge</td>
<td>265</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2005</td>
<td>Japan</td>
</tr>
</tbody>
</table>

Continued on next page

Table 2.1: Concrete Arch Bridges-continued
2.2. Arch bridge description

<table>
<thead>
<tr>
<th>No</th>
<th>Name</th>
<th>Span</th>
<th>Rise</th>
<th>S/R</th>
<th>R/S</th>
<th>Year</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Sand Bridge</td>
<td>264</td>
<td>42.0</td>
<td>6.29</td>
<td>0.16</td>
<td>1943</td>
<td>Sweden</td>
</tr>
<tr>
<td>19</td>
<td>Contreras Railway</td>
<td>261</td>
<td>40.3</td>
<td>6.48</td>
<td>0.15</td>
<td>2009</td>
<td>Spain</td>
</tr>
<tr>
<td>20</td>
<td>Takamatsu Bridge</td>
<td>260</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2000</td>
<td>Japan</td>
</tr>
<tr>
<td>21</td>
<td>Los Tilos Arch</td>
<td>255</td>
<td>45.0</td>
<td>5.67</td>
<td>0.18</td>
<td>2004</td>
<td>Spain</td>
</tr>
<tr>
<td>22</td>
<td>Wild Gera Viaduct</td>
<td>252</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2000</td>
<td>Germany</td>
</tr>
<tr>
<td>23</td>
<td>Chateaubriand Bridge</td>
<td>250</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1991</td>
<td>France</td>
</tr>
</tbody>
</table>

Table 2.1: Concrete Arch Bridges

The cross section of arch is defined as a box girder with variable dimensions along the span, meaning the cross section is smaller in the middle and larger on foundation level. Depending on other factors such as concrete grade and reinforcement ratio, the cross section of arch at midspan is selected as 1/2 or 1/3 of larger section (see figure 2.3).

Figure 2.2: Span to rise ratio over span length.

The cross section of decks are modified according to span length for every model, the only similarity is the width which is designed to carry two lanes for two direction on highway standards, the figure 2.4 illustrates the general shape of deck section.

The first estimation of required height of deck is calculated based on equation

\[ R/S: \text{Rise to Span ratio}, \ S/R: \text{Span to Rise ratio} \]

\(^{2}\text{(-)} \text{Author was unable to access any source of information about the missed data.}\)
2.2. Arch bridge description

Figure 2.3: Cross sections of an arch with 400 meter span length with largest and smallest dimensions (all in meters)

Figure 2.4: Deck section (dimensions are in meters)

below, however its displacement is controlled during static analysis.

\[ h_{\text{deck}} = \frac{l}{20 \sim 25} \]  

where \( l \) is the distance between two consecutive column.

2.2.2 Boundary condition

The key point on stability of an arch is the constant distance between its two ends, hence the arch is always forcing its sides to move away. Therefore there must be fixities to prevent them from sliding. Defining such boundary condition is easier for the mathematical model, but on construction site building such huge foundations may be not economically acceptable and other type of bridges such as cable stayed or suspension bridges may become more applicable. However, this problem vanishes when the bridge is to be constructed in a valley or V-shaped gaps with stiff bedrock. Based on design strategy used in the project, the arch supports may be free to rotate (pin supported—see figure 2.5a) or fixed as in figure 2.5b, the fixed supports are mostly used for concrete arch bridges because of its massive arcs and low flexibility of concrete, the same type of boundary condition has been used for arch-ground connection in this work.
2.3. Materials

The material employed in different parts of bridges analysed in chapter 4 are described numerically through advanced constitutive models, the material class, especially concrete are mainly selected based on available long span bridges such as Krk and Wanxian [Bangzhu 2008]; eventually some other modifications were necessary to achieve a more numerically stable model in order to see nonlinear behaviour of structure under seismic actions. The specification of concrete, reinforcement steel and elastomer are presented below in order:
2.3. Materials

Figure 2.7: The 3D simulation of a 400m span arch bridge, designed based on data of this chapter for further analysis proposes.

Figure 2.8: The 3D simulation of a 400m span arch bridge, designed based on data of this chapter for further analysis proposes.
2.3. Materials

2.3.1 Concrete

Two normal weight (less than 2600 kg/m$^3$) class of concrete is used in every model, a C60 for arch and piers and a C40 for deck to reduce the dead load exerted form deck to columns. The materials stress-strain behaviour are formulated in equation 2.2 for compression and algorithm 1 for tension based on CIB-FIB model code 2010 [fib 2010] and a summary of the properties of C60 class concrete is presented in table 2.2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{ci}$ [Gpa]</td>
<td>40.7</td>
<td>Modulus of elasticity of concrete at age of 28 days</td>
</tr>
<tr>
<td>$\rho [kg/m^3]$</td>
<td>2600</td>
<td>Density</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.2</td>
<td>Poisson’s ratio (^1)</td>
</tr>
<tr>
<td>$f_{ck}$ [Mpa]</td>
<td>60</td>
<td>Characteristic compressive strength</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>$E_c$ [Gpa]</td>
<td>38.9</td>
<td>Reduced Modulus of elasticity</td>
</tr>
<tr>
<td>$E_{cl}$ [Gpa]</td>
<td>26.2</td>
<td>Secant modulus from the origin to the peak compressive stress</td>
</tr>
<tr>
<td>$\varepsilon_{cl}$ [%/per thousand]</td>
<td>2.6</td>
<td>Strain at maximum compressive stress</td>
</tr>
<tr>
<td>$\varepsilon_{c,lim}$ [%/per thousand]</td>
<td>3.3</td>
<td>Fracture strain</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of the properties: C60 class concrete (data are based on [fib 2010] - for more details, see figure 2.9)

$$\sigma_c = \frac{f_{cm}(\eta k - \eta^2)}{\eta(k-2)+1}$$  \hspace{1cm} (2.2)

where

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \quad \quad \quad \quad k = \frac{E_{ci}}{E_{cl}}$$

As the formulations are completed, the stress-strain values are available for every point in range; the figure 2.9 illustrates C60 concrete properties for its absolute values.

2.3.2 Reinforcement steel

As it is described in section 2.2.1, the arch’s concrete cross section is responsible to carry compressional stress. However, under some circumstances this state is not steady i.e. as a result of any applied force or displacement to arch, its symmetry

\(^1\)The exact value might be slightly different but the value 0.2 is recommended for all classes by [fib 2010]
Algorithm 1 Tension definition [fib 2010]

1. Calculate $f_{ctm}$
   
   if $f_{ck} \leq 50$ Mpa then
   
   $f_{ctm} = 0.3 \cdot (f_{ck})^{2/3}$
   
   else
   
   $f_{ctm} = 2.12 \cdot ln(1 + 0.1 \cdot (f_{ck} + \Delta f))$
   
2. Calculate $\sigma_{ct}$ based on one of following conditions:
   
   for $\varepsilon_x \leq 0.9f_{ctm}/E_{ci}$ do
   
   $\sigma_{ct}[\varepsilon_x] = \varepsilon_x \cdot E_{ci}$
   
   for $0.9f_{ctm}/E_{ci} < \varepsilon_x \leq \varepsilon_{ct}$ do
   
   $\sigma_{ct}[\varepsilon_x] = f_{ctm} \cdot \left(1 - 0.1 \left(\frac{\varepsilon_{ct} - \varepsilon_x}{\varepsilon_{ct} - 0.9f_{ctm}/E_{ci}}\right)\right)$
   
   for $\varepsilon_{ct} < \varepsilon_x \leq \varepsilon_{ct} + w_l$ do
   
   $\sigma_{ct}[\varepsilon_x] = f_{ctm} \left(1 - 0.8(\varepsilon_x - \varepsilon_{ct})\right)\frac{w_l}{w_l}$
   
   for $\varepsilon_{ct} + w_l < \varepsilon_x \leq w_c + \varepsilon_{ct}$ do
   
   $\sigma_{ct}[\varepsilon_x] = f_{ctm} \left(0.25 - 0.05(\varepsilon_x - \varepsilon_{ct})\right)\frac{w_l}{w_l}$

Figure 2.9: Compression and tension behaviour of a C60 class concrete (Absolute values).
2.3. Materials

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_s$ [Gpa]</td>
<td>210</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>$\rho$ [kg/m$^3$]</td>
<td>7850</td>
<td>Density</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$f_{0.2}$ [Mpa]</td>
<td>420</td>
<td>Steel ‘yield’ strengths are generally quoted in terms of the 0.2% proof strength</td>
</tr>
<tr>
<td>$\varepsilon_y$ [%]</td>
<td>0.2</td>
<td>Yield strain</td>
</tr>
<tr>
<td>$\varepsilon_f$ [%]</td>
<td>22</td>
<td>Maximum elongation</td>
</tr>
<tr>
<td>$f_{yk}$ [Mpa]</td>
<td>466</td>
<td>Ultimate strength</td>
</tr>
</tbody>
</table>

Table 2.3: Summary of the properties: B400 steel. [EN-1992]

and equilibrium for forces will be violated, accordingly there will be tension on some region of arch; to resist this huge source of tension over the narrow section of arch, the longitudinal reinforcements are necessary. To fulfill this requirement the reinforcement area is defined about 2% of total cross section area of arch ($\sim 160$ kg/m$^3$) which may vary depending on material class, geometry or load magnitude. This area is divided into 10-12 rebars along the arch length (see figure 2.10).

Figure 2.10: Still of rebar placement (dimensions are not realistic)

The deck and piers are also being reinforced using the same technique, although piers are containing heavier reinforcement especially at the intersection with arch to overcome the high amount shear force (up to 250 kg/m$^3$). The material used for all models is B400 grade steel which is defined in Table 2.3. for a precise material model, the line equation has been developed based on fib model code, as well as concrete (see equation 2.3, figure 2.11).

\[
\sigma_{Elastic} = \varepsilon_x \cdot E_s \quad \varepsilon_x \leq \varepsilon_y (2.3a)
\]

\[
\sigma_{Plastic} = \frac{\varepsilon_x \cdot E_s}{100} + E_s \cdot \varepsilon_y \quad \varepsilon_y < \varepsilon_x < \varepsilon_f (2.3b)
\]
2.3. Materials

Figure 2.11: Simplified stress strain behaviour of steel B400 with average 1% hardening.

| $E_1$ [Mpa] | 600 | Modulus of elasticity (Normal) |
| $E_{2,3}$ [Mpa] | Negligible | Modulus of elasticity (Transversal) |
| $G_{1,2,3}$ [Mpa] | 0.9 ~ 1.2 | Shear modulus of elasticity |
| $\rho$ [kg/m$^3$] | 3000 | Density |
| $\nu_{1,2,3}$ | 0.5 | Poisson’s ratio |

Table 2.4: summary of the properties: Elastomer bearing. [Stanton 2008, Mag 2013]

2.3.3 Elastomer

The deck and piers are connected using an elastomer type bearing, although various type of connections are suitable for this part, the elastomer type is found to be the most simple and accurate one in finite element modelling point of view. Hence the connection must be constrained in lateral direction (see figure 2.6) author decided to include this property into material definition of connection to avoid computational expenses of surface contact problem in finite element analysis $^1$. Therefore the shear modulus of elasticity in desired direction is set as a large value to make material very stiff in that transversal direction. Accordingly the elastomer bearing is introduced to finite element code as an anisotropic material defined by equation 2.4.
\[
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\gamma_{12} \\
\gamma_{13} \\
\gamma_{23}
\end{pmatrix} = 
\begin{pmatrix}
1/E_1 & -\nu_{12}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\
-\nu_{12}/E_1 & 1/E_2 & -\nu_{23}/E_3 & 0 & 0 & 0 \\
-\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{23}
\end{pmatrix}
\begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{pmatrix}
\]

(2.4)

2.4 Finite element type

The Abaqus is a finite element code used for numerical analysis of this document. In general, Abaqus provides advanced nonlinear and reinforcement concrete modeling features compared to some other FE programs. Along several element type available in Abaqus library, the beam\(^2\) element type "B31" (Timoshenko beam in space) meets the conditions of presented model. The beam cross section selected for arch and piers is "Box-section" which by default has 16 integration points (5 points in each wall) and "arbitrary-section" for deck with three point Simpson integration scheme of each section (5 section-total of 10 integration points). To define reinforcement, the command "*Rebar=Beam" must be used, this feature place the rebars parallel to beam element direction based on exact position of them. (For details on rebar positions see section 2.3.2 and figure 2.10)

To check the accuracy of abovementioned finite element code, a cylindrical reinforced specimen with dimensions of 150/300 mm is tested with four independent reinforcement area (Non-reinforced, 2.5%, 5%, 10%). The observations of this test can be summarized as following: Under compression, concrete itself works fine with both solid and beam element, however the program does not provide the rebars data individually, therefore the effect of reinforcement on specimen must be observed from the results for whole package. By reversing the elongation of specimen the way it goes under tension, concrete reacts primarily. After concrete cracks up to a certain level, (it must be defined to Abaqus using "Smeared cracking" feature), the program will partially release the strain energy to rebar until complete fracture of concrete. Eventually, the total force will be carried by reinforcements. This behaviour is illustrated in figure 2.12. This feature provided by Abaqus (smeared cracking) will

---

\(^1\)When deck moves laterally there must be some element to hold it, if the surface of deck and piers become in contact, the surface-contact calculation must be performed, which is computationally expensive. Alternatively, Abaqus provides a wide library of connections to fulfill this requirement, however by using this type of constrains, the program will not be able to produce a unique mass matrix of all element which is a necessity for external calculations of MPA and etc. (explained on chapter 3)

\(^2\)The element library in Abaqus contains several types of beam elements. A "beam" in this context is an element in which assumptions are made so that the problem is reduced to one dimension mathematically: the primary solution variables are functions of position along the beam axis only. For such assumptions to be reasonable, it is intuitively clear that a beam must be a continuum in which we can define an axis such that the shortest distance from the axis to any point in the continuum is small compared to typical lengths along the axis. [Abaqus 2013]
allow us to perform a very realistic analysis on models with composite materials under cyclic loads such as several complex bridges of chapter 4 subjected to strong ground motion.
CHAPTER 3
Seismic analysis

3.1 Introduction
Reliability of results of a complex structure such as concrete arch bridges of previous chapter, depend upon its analysis procedure, among many mathematical methods available, the designer must select the most suitable one considering the importance, environmental conditions and material behaviour expectations during the service life of structure as well as error estimation of method. The first section of this chapter describes the general system of dynamics, the chapter continues with discussion of elastic procedures and its limitations. Finally a discussion of inelastic methods and applications is presented at the end of this chapter.

3.2 The mathematical system of dynamics
The dynamic response of a N-degree of freedom structure subjected to ground motion is expressed as following system of differential equation 3.1:

\[ m\ddot{u} + c\dot{u} + f_s(u, \dot{u}) = -m\ddot{u}_g \]  \hspace{1cm} (3.1)

Where \( m \) and \( c \) are respectively the mass and damping matrices, \( u(t) \) is the relative displacement, \( f_s(u, \dot{u}) \) is the stiffness component of the force vector in the structure.
which defines the relation between force and displacement and at last, \( \iota \) represents the influence matrix connecting the degree of freedom of the structure and imposed accelerogram directions (\( \ddot{\mathbf{u}}_g(t) \)).

### 3.3 Elastic analysis

Assuming the structure remains elastic under imposed acceleration, makes the \( \mathbf{f}_s \) (Eq. 3.1) a linear relation of force displacement and can be expressed as \( \mathbf{f}_s = \mathbf{k}\mathbf{u} \) where \( \mathbf{k} \) is elastic stiffness matrix. This leads us to the traditional form of system of dynamics for linear elastic systems:

\[
\mathbf{m} \ddot{\mathbf{u}} + \mathbf{c} \dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\ddot{\mathbf{u}}_g
\]  

(3.2)

To evaluate the solution of abovementioned equation, three strategies are presented below:

#### 3.3.1 Direct Response History Analysis: DRHA

DRHA evaluates the system of equations by means of step by step numerical integration scheme, therefore DRHA produce a time history of results. If the numerical integration remains stable, this method provides accurate results, however the main drawback of this procedure is undoubtedly the large computational cost which is due to solving the system of equations for every time increment up to total duration of accelerogram. Among many other numerical integration methods available, the broadly accepted integration algorithm proposed by Hilber, Hughes and Taylor (HHT, [Hilber 1977]) is employed in this work because of its reliability to remain numerically stable in higher order vibrations. This capability becomes more important in nonlinear type of integration (NL-RHA), which will be discussed in next chapters. The Hilber-Hughes-Taylor operator is a generalization of the Newmark operator with controllable numerical damping, the damping being most valuable in the automatic time stepping scheme, because the slight high-frequency numerical noise inevitably introduced when the time step is changed is removed rapidly by a small amount of numerical damping. The operator replaces the actual equilibrium equation (Equation 3.1) with a balance of d’Alembert forces at the end of the time step and a weighted average of the static forces at the beginning and end of the time step [Abaqus 2013].

\[
\mathbf{m}\ddot{\mathbf{u}}_{t+\Delta t} + (1 + \alpha_a)(\mathbf{c}\dot{\mathbf{u}}_{t+\Delta t} + \mathbf{k}\mathbf{u}_{t+\Delta t}) - \alpha_a(\mathbf{c}\dot{\mathbf{u}}_t + \mathbf{k}\mathbf{u}_t) = \mathbf{F}_{t+\Delta t}
\]  

(3.3)

\[
\mathbf{u}_{t+\Delta t} = \mathbf{u}_t + \Delta t \mathbf{\ddot{u}}_t + \frac{\Delta t^2}{2}[(1 - 2\beta_a) \mathbf{\dddot{u}}_t + 2\beta_a \mathbf{\ddot{u}}_{t+\Delta t}]
\]  

(3.4)

\[
\mathbf{\dddot{u}}_{t+\Delta t} = \mathbf{\dddot{u}}_t + \Delta t[(1 - \gamma_a)\mathbf{\dddot{u}}_t + \gamma_a\mathbf{\dddot{u}}_{t+\Delta t}]
\]  

(3.5)
It was experienced that three dimensional nonlinear analysis of previously described models, require a higher level of numerical damping. Therefore the above mentioned parameters are selected as \(\alpha = -0.41421\), \(\beta = 0.5\), \(\gamma = 0.91421\). This configuration in Abaqus is also known as "Moderate dissipation".

### 3.3.2 Modal Response History Analysis: MRHA

As long as the structure remains in elastic range, equation 3.2 can be evaluated as a single degree of freedom system by decoupling equation 3.2 by means of modal transformation for the modes with frequency within the range of interest. In order to do so, it is required to expand the displacement vector \(\mathbf{u}(t)\) in terms of modal contributions:

\[
\mathbf{u}(t) = \sum_{i=1}^{N} \mathbf{\phi}_i \mathbf{q}_i(t) \tag{3.6}
\]

where \(\mathbf{\phi}_i\) is the vector of normalized displacement for degrees of freedom associated with \(i\) mode obtained from the real eigenvalue (Eq. 3.7), \(N\) is the number of modes being considered and \(\mathbf{q}_i\) is the generalized modal coordinate.

\[
\mathbf{k}\mathbf{\phi}_i = \omega^2 m \mathbf{\phi}_i \tag{3.7}
\]

By replacing equation 3.6 into equation 3.2, and pre multiplying each term by \(\mathbf{\phi}_n^T\) the general elastic equation can be rewritten as:

\[
\sum_{i=1}^{N} \phi_n^T \mathbf{m} \phi_i \ddot{q}_i + \sum_{i=1}^{N} \phi_n^T \mathbf{c} \phi_i \dot{q}_i + \sum_{i=1}^{N} \phi_n^T \mathbf{k} \phi_i q_i = -\phi_n^T \mathbf{m}\ddot{u}_g(t) \tag{3.8}
\]

Due to orthogonality relations \((\phi_i^T \mathbf{m} \phi_i = 0, \phi_i^T \mathbf{k} \phi_i = 0)\)[Chopra 2007], all terms in each part vanishes except \(i=n\):

\[
\mathbf{M}_n \ddot{q}_n(t) + \mathbf{C}_n \dot{q}_n(t) + \mathbf{K}_n q_n = -\phi_n^T \mathbf{m}\ddot{u}_g(t) \tag{3.9}
\]

Where \(\mathbf{M}_n\) is generalized mass matrix, \(\mathbf{C}_n\) is generalized damping matrix and \(\mathbf{K}_n\) is generalized Stiffness matrix, therefore:

\[
\mathbf{M}_n \ddot{q}_n(t) + \mathbf{C}_n \dot{q}_n(t) + \mathbf{K}_n q_n = -\phi_n^T \mathbf{m}\ddot{u}_g(t) \tag{3.9}
\]

On the other hand, expanding the spatial distribution of seismic excitation in base of eigenvectors which represents the vibration modes:

\[
\mathbf{s} = \mathbf{m} \mathbf{\dot{u}} = \sum_{i=1}^{N} \Gamma_i \mathbf{m}\phi_i \tag{3.10}
\]
3.3. Elastic analysis

Same as previous part, by premultiplying $\phi_n^T$ to both sides of equation 3.10 and considering the orthogonality property of mass matrix, the participation factor $\Gamma_n$ is obtained:

$$\Gamma_n = \frac{\phi_n^T m \phi_n}{\phi_n^T m \phi_n} = \frac{\phi_n^T m \phi_n}{M_n}$$ (3.11)

Dividing Eq. 3.9 by $M_n$ results:

$$\ddot{q}_n(t) + 2\xi\omega_n \dot{q}_n(t) + \omega^2 q_n(t) = -\Gamma_n \ddot{u}_g$$ (3.12)

Considering:

$$q_n(t) = \Gamma_n D_n$$ (3.13)

The Single Degree Of Freedom (SDOF) system with imposed accelerogram is ready to be solved using a numerical integration method:

$$\ddot{D}_n(t) + 2\xi\omega_n \dot{D}_n(t) + \omega^2 D_n(t) = -\ddot{u}_g(t)$$ (3.14)

Obtaining the displacement of structure can be easily done by reversing the change in variables:

$$u_n = \phi_n q_n(t) = \phi_n \Gamma_n D_n(t)$$ (3.15)

The resultant of every SDOF system belongs to one mode and as the number of included modes increase, the accuracy of results increase as well. Up to now, modal displacement of structure has been calculated and using this data, the forces in structural elements are to be determined by applying these nodal displacements into static analysis procedure (equivalent static force). The forces in structural element may be obtained as:

$$f_n(t) = k u_n(t)$$ (3.16)

Using Equations 3.10, 3.15 and 3.16 leads to:

$$f_n(t) = \Gamma_n m \phi_n \omega^2 \dot{D}_n(t) = s_n A_n$$ (3.17)

Finally, the equivalent static forces $f_n(t)$ can be expressed as product of two factors:

- The modal static response or structural response under static analysis ($r_{st}^n$) by applying spatial distribution of each mode ($s_n$).
- The pseudo-acceleration response $s_n$ of n-th mode SDOF system subjected to ground motion $\ddot{u}_g(t)$

It is concluded as:

$$r_n(t) = r_{st}^n A_n(t)$$ (3.18)
3.3. Elastic analysis

3.3.3 Modal Response Spectrum Analysis: MRSA

As long as the system of dynamics remains elastic, displacements are recovered after every cycle of motion; therefore it is not required to solve the SDOF system for every single time step as in MRHA. Simply the peak displacement is enough for calculating the maximum forces in elements and design of the sections accordingly. The peak displacement or acceleration can be obtained from earthquake response spectrum

\[ S_d = S_d(T_n, \xi_n), S_a = S_d(T_n, \xi_n) \]

which are a function of natural vibration period \( T_n \) and modal damping \( \xi_n \) then pesudo acceleration can be modified as:

\[ A_n = \omega_n^2 D_n(t) = \omega_n^2 S_d = S_a \quad (3.19) \]

Here, the previously developed expression 3.18 is modified due to new form of pesudo-acceleration Eq. 3.19 for response spectrum analysis:

\[ r_{no}(t) = r_{st} \omega_n^2 S_d = r_{st} S_a \quad (3.20) \]

Where \( r_{no} \) is the peak response of nth mode. The presented procedure is computationally less demanding in comparison to MRHA also the results of both MRHA and MRSA at peak displacement of every individual mode are identical, however after combination of modes the MRSA might slightly over estimate the forces, this is the only source of error in MRSA and the reason is the peak displacement of modes are not occurring at the same time although several Modal combination strategies are available to combine them all together.

Modal combination rules

As previously mentioned, every system of equations in MRSA calculates the response of structure and accordingly element forces for one individual mode. There are several combination rules available in order to combine the results. Some of them are represented here:

**ABSSUM** or absolute sum is a modal combination rule that assume all modal peaks occur at the same time and their algebraic sign is ignored. This rule provides an upper bond to peak value of total response.[Chopra 2007]

\[ r_o \leq \sum_{n=1}^{N} |r_{no}| \quad (3.21) \]

**SRSS** This is short term for square root of sum or squares and provides a very good estimation as long as the natural frequencies of modes are well separated. The peak modal responses are squared then summed for all number of included modes. Finally the resulting value goes under square root to complete the
3.4 Inelastic analysis

estimation [Rosenblueth 1951]:

$$r_o \simeq \sqrt{\sum_{n=1}^{N} r_{no}^2}$$ (3.22)

CQC The complete quadratic combination rule has been developed by several authors, which are mostly similar in formulation (i.e expression 3.23) but in correlation factor, in this work, the formulation developed by [Der Kiureghian 1981] has been implemented:

$$r_o \simeq \sqrt{\sum_{n=1}^{N} r_{no}^2 + \sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in} r_{io} r_{no}}$$ (3.23)

$$\rho_{in} = \frac{8 \xi^2 (1 + \beta_{in}) \beta_{in}^{3/2}}{(1 - \beta_{in}^2)^2 + 4 \xi^2 \beta_{in} (1 + \beta_{in})^2} \quad \beta = \omega_i / \omega_n$$ (3.24)

The advantage of this rule over SRSS is that CQC overcomes the limitation of SRSS in response estimation of modes with close frequencies. To use the CQC, a damping coefficient must be introduced this value is about 5% in the structures studied in this work.

3.4 Inelastic analysis

The elements of structure may be pushed above their linearly elastic strength while the structure is under extreme seismic action and deflect accordingly, which may cause other deflections in structure under secondary $P - \Delta$ effect. On the other hand traditional design strategies suggest designing the element sections by only considering the elastic strength of material, however current trends in earthquake engineering requires more rigorous and realistic procedures. By the same token, the strategies such as performance based design are including material inelasticity to achieve a more economic design. Despite the elastic analysis, the previous mode based procedures are not valid here because the system of dynamics in Eq. 3.1 cannot be directly decomposed in set of real vibration modes as $k_{inelastic} \neq k_{elastic}$ respectively $f_s(u, \dot{u}) = k_{inelastic}u \neq k_{elastic}u$. Therefore appropriate methodologies are necessary to predict these displacement demands. In following sections, the rigorous NL-RHA and approximate methods will be discussed.

\footnote{Damping within range of 1 - 5% is found to have very little effect on accuracy of modal combination. [Chopra 2007]}
3.4. Inelastic analysis

3.4.1 Nonlinear Response History Analysis: NL-RHA

The most accurate strategy to address significant nonlinearities in structural dynamics is undoubtedly the nonlinear response history analysis which is similar to DRHA (Sec.3.3.1) but extended to inelastic range by updating the stiffness matrix (k) for each iteration. This onerous task, clearly increase the computational cost especially for unsymmetrical structures that require to be analyzed in a full 3D model for taking the torsional effects in to consideration. Although approximate methods are available to calculate nonlinear response of structure, they can only be used during the design process but the final design must be based on NL-RHA.

3.4.2 Modal Pushover Analysis: MPA

The Modal Pushover Analysis is basically the MRHA but extended to inelastic stage. After all, the MRHA cannot solve the system of dynamics (Eq.3.1) because the resisting force \( f_s(u, \dot{u}) \) is unknown for inelastic stage. The solution of MPA for this obstacle is using the previously recorded \( f_s \) to evaluate system of dynamics. The force-deformation relationship (Capacity curve) can be achieved by means of a static pushover analysis when monotonically increasing forces are applied and this pushover analysis must be performed for all included modes and directions separately. The peak response results from using this method, proposed by Chopra and Goel [Chopra 2002] and improved by them [Chopra 2004a, Chopra 2004b]. Here a brief mathematical derivation of this method is available:

Repeating spatial distribution \( s_n \) from Eq.3.10:

\[
\mathbf{s} = \mathbf{m} \mathbf{u} = \sum_{i=1}^{N} \Gamma_i \mathbf{m} \phi_i \quad (3.10 \text{ revisited})
\]

And assuming that the n-th component of the expanded excitation vector in modal coordinates only affect the n-th mode,(which is not generally true in nonlinear analysis, but it is assumed to be acceptable in building structures [Chopra 2002]). In that case, the contribution of i-th mode to displacement vector when \( s_n \) is applied to the structure is null, being \( i \neq n \), therefore modal decomposition in Eq.3.17 can be represented as:

\[
\mathbf{u}(t) = \sum_{i=1}^{N} \phi_i q_i(t) \simeq \phi_n q_n(t) \quad (3.25)
\]

Then, considering the orthogonality of \( \mathbf{m} \) and \( \mathbf{c} \) (classical damping assumed here), and \( F_{sn} = \phi_n^T f_s(u, \dot{u}) \) result:

\[
M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + F_{sn} = -\Gamma_n M_n \ddot{u}_g(t) \quad (3.26)
\]

Dividing the equation by generalized mass \( M_n \) leads to:
Inelastic analysis

\[ \ddot{q}_n(t) + 2\xi\omega_n\dot{q}_n(t) + \frac{F_{sn}}{M_n} = -\Gamma_n\ddot{u}_g \]  

(3.27)

By statically imposing the specific load pattern \( s_n^* \) to push the structure to desired displacement, the capacity curve is attained.

\[ s_n^* = m\phi_n \]  

(3.28)

Integrating the Eq. 3.27 results the time history of displacement, however the maximum value is required here\(^2\), thus:

\[ u_{rn}^{\text{max}} = \phi_{rn}\max_t|q_n(t)| \]  

(3.29)

From this point forward, the MPA loses its time history, therefore a modal combination strategy is necessary to combine the results of all modes.

Capacity curve transformation and Integration of the differential equation

Previously mentioned capacity curves are made by recording the base shear forces of structure \( V_{bn} \) (sum of all boundaries) and displacement of the control point \( (u_{rn}) \) in the nth mode pushover analysis under \( s_n^* \) (Eq.3.28) and gravity load. The transformation \(^3\) of these values is required in order to be applicable in Eq. 3.27, this can be done using the equality below which is illustrated in Figure 3.1.

\[ \frac{F_{sn}}{M_n} = \frac{V_{bn}}{L_n} \]  

(3.30a)

\[ q_n = \frac{u_{rn}}{\phi_{rn}} \]  

(3.30b)

For sake of simplicity, the non-linear part of graph is idealized using equal area rule, finally the \( F_{sn}/M_n - q_n \) relationship is available to implement in system of dynamics. To evaluate \( q_n \), the differential equation 3.27 must be integrated in time domain, in this work, the central difference integration scheme is combined with non-linear cycling algorithm \(^2\) to generate an explicit incremental integration method capable of evaluating the cycling displacement of SDOF structure.

3.4.3 Extended Modal Pushover Analysis: EMPA

Expanded Modal pushover is a proposed method in Camara Ph.D. thesis [Camara 2011] to evaluate response of structure under multi-directional excitation \( \ddot{u}_g^T(t) = (\ddot{u}_g^X(t), \ddot{u}_g^Y(t), \ddot{u}_g^Z(t)) \) using the modal pushover analysis strategy. In one specific mode, the original pushover neglects the contribution of the directions different than characteristic one. This is

\(^2\) Small differences have been introduced in Eq.3.29 in comparison with the original MPA[Chopra 2002]

\(^3\) Further mathematical derivation of this transformation is available on [Chopra 2002]
3.4. Inelastic analysis

![Image of Figure 3.1: Transformation of capacity curve]

Figure 3.1: Transformation of capacity curve

![Image of Figure 3.2: The final stress is obtained by "returning" the trial stress to the yield surface through a scaling, hence the denomination return mapping.][Simo 1997]

Figure 3.2: The final stress is obtained by "returning" the trial stress to the yield surface through a scaling, hence the denomination return mapping. [Simo 1997]
Algorithm 2 Return-Mapping algorithm for one dimensional, Rate independent plasticity. Combined Isotropic/Kinematic hardening. [Simo 1997]

1. Database at $x \in B : \{\varepsilon_n^p, \alpha_n, q_n\}$

2. Given strain field at $x \in B : \varepsilon_{n+1} = \varepsilon_n + \Delta \varepsilon_n$

3. Compute elastic trial stress and test for plastic loading:
   
   $\sigma_{n+1}^{trial} := E(\varepsilon_{n+1} - \varepsilon_n)$
   
   $\xi_{n+1}^{trial} := \sigma_{n+1}^{trial} - q_n$
   
   $f_{n+1}^{trial} := \left|\xi_{n+1}^{trial}\right| - [\sigma_Y + K\alpha_n]$

   if $f_{n+1}^{trial} \leq 0$ then
      
      Elastic step: set $(\bullet)_{n+1} = (\bullet)_{n+1}^{trial}$ & Exit
   
   else
      

4. Return mapping:
   
   $\Delta \gamma := \frac{f_{n+1}^{trial}}{E + [K + H]} > 0$
   
   $\sigma_{n+1} := \sigma_{n+1}^{trial} - \Delta \gamma E \text{ sign}(\xi_{n+1}^{trial})$
   
   $\varepsilon_{n+1}^p := \varepsilon_n^p + \Delta \gamma \text{ sign}(\xi_{n+1}^{trial})$
   
   $q_{n+1} := q_n + \Delta \gamma H \text{ sign}(\xi_{n+1}^{trial})$
   
   $\alpha_{n+1} := \alpha_n + \Delta \gamma$
reasonable in a regular symmetric building but a structure with complex shape may go through strong modal coupling, meaning the modal displacement vector \( \phi_n \) will have non-zero components on non-characteristic directions although they are much smaller. Therefore the load distributions of modes are three-dimensional as it is represented in Figure 3.3.

![Figure 3.3: Left: Load distribution \( s_n^* \), Right: resulting control point displacement: \( \bar{u}_{rn} \) (Adapted from Camara Ph.D. thesis [Camara 2011])]()

Subjecting the system of dynamics to triaxial excitation, converts the right side of Eq. 3.1 to:

\[
P_{\text{effective}} = -m \ddot{u}_X^X(t) - m \ddot{u}_Y^Y(t) - m \ddot{u}_Z^Z(t)
\]  

(3.31)

Placing Eq. 3.10 into Eq. 3.31 leads to:

\[
m \ddot{u} + c \dot{u} + f_s(u, \dot{u}) = -s X \ddot{u}_g^X(t) - s Z \ddot{u}_g^Z(t) - s Z \ddot{u}_g^Z(t)
\]  

(3.32)

The Eq. 3.11 also must be redefined for triaxial form:

\[
\Gamma_j = \frac{\phi_j^T \mathbf{M}_n \phi_j}{M_n} : j = \{X, Y, Z\}
\]  

(3.33)

By neglecting the non-linear modal coupling and introducing \( \bar{q}_n \) as generalized three-dimensional coordinate, the Eq. 3.25 is reformed for J number of modes:

\[
\mathbf{u}(t) = \sum_{i=1}^{J} \phi_i \bar{q}_i(t) \approx \phi_n \bar{q}_n(t)
\]  

(3.34)

Considering \( \bar{F}_{sn} = \phi_n^T f_s(\bar{q}, \dot{\bar{q}}) \), substituting Eq. 3.34 into triaxial dynamic sys-
3.4. Inelastic analysis

tem(Eq.3.32) and finally pre multiplying it by $\phi^T_n$ leads to:

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + \dot{F}_{sn} = -M_n \left( \Gamma^X_n \ddot{u}^X_g(t) + \Gamma^Y_n \ddot{u}^Y_g(t) + \Gamma^Z_n \ddot{u}^Z_g(t) \right) \ddot{u}^*_g_n(t)$$

(3.35)

$\ddot{u}^*_g_n(t)$ being the equivalent acceleration history; dividing by $M_n$ results:

$$\ddot{q}_n(t) + 2\xi\omega_n \dot{q}_n(t) + \frac{\dot{F}_{sn}}{M_n} = \ddot{u}^*_g_n(t)$$

(3.36)

At this point, the EMPA is able to evaluate response history of structure in a three-dimensional form without neglecting the components other than dominant direction. However, the capacity curve of all directions is necessary to generate $F_{sn}/M_n - \ddot{q}_n$ relationship. to achieve that, the projection of 3D pushover on every dimension is recorded, i.e. $((V_{bn}^X - u_{rn}^X), (V_{bn}^Y - u_{rn}^Y), (V_{bn}^Z - u_{rn}^Z))$. As in MPA, the base shear-displacement records must be transformed to $F_{sn}/M_n - \ddot{q}_n$ using previously defined equations of MPA procedure, here modified for 3D format:

$$F^j_{sn} = \frac{V^j_{bn}}{L^j_n}; j = \{X,Y,Z\}$$

(3.37)

$$q^j_n = \frac{u^j_{rn}}{\phi^j_{rn}}; j = \{X,Y,Z\}$$

(3.38)

The combination of resulted vectors is necessary using following expressions for its applicability in the system of dynamics:

$$\ddot{F}^j_{sn} = \sqrt{\left(\frac{F^X_{sn}}{M_n}\right)^2 + \left(\frac{F^Y_{sn}}{M_n}\right)^2 + \left(\frac{F^Z_{sn}}{M_n}\right)^2}$$

(3.39)

$$\ddot{q}_n = \sqrt{(q^X_n)^2 + (q^Y_n)^2 + (q^Z_n)^2}$$

(3.40)

Integrating Eq. 3.36, provides the three dimensional generalized displacement of nodes of structure subjected to the equivalent accelerogram $\ddot{u}^*_g_n$, (see figure 3.3), thus the target displacement of analysis is obtained as:

$$\ddot{u}^{max}_{rn} = \ddot{\phi}_{rn} \max_t[\ddot{q}_n(t)]$$

(3.41)

Where:

$$\ddot{\phi}_{rn} = \sqrt{(\phi^X_{rn})^2 + (\phi^Y_{rn})^2 + (\phi^Z_{rn})^2}$$

(3.42)

Here, the mathematical derivation of EMPA is completed. Integrating the system of equations and modal combinations are to be done as in MPA procedure. Additionally a step by step description of procedures is available in Appendix A of this document.
3.4.4 Coupled Nonlinear Static Pushover Analysis: CNSP

The EMPA includes the contribution of non-characteristic direction to the formal MPA procedure. However, the static pushovers in EMPA are performed individually for every mode. Accordingly, the resulting values from different modes (directions) can be combined as explained previously. But this is only valid in the elastic phase; as soon as any element in structure starts yielding, the neutral axis of that section is no longer fixed for both responses during the earthquake, meaning the longitudinal deflection unavoidably affects the transverse one or vice versa. To overcome this drawback of EMPA, a new method proposed by Camara and Astiz [Camara 2012] suggests executing pushover analysis for governing modes of each direction at the same time.

\[
\begin{align*}
s_{nX}^* &= m\phi_X^n \\
s_{nY}^* &= m\phi_Y^n
\end{align*}
\]

Governing transverse mode \(nY\) Governing longitudinal mode \(nX\) Coupled pushover: \(nX + nY\)

![Figure 3.4: Schematic three-dimensional coupled load distribution \(s_C^*\) in CNSP. \(\bar{V}_{bC}\) is the magnitude of the base shear in 3D coupled pushover. (Adapted from Camara and Astiz article. [Camara 2012])](image)

The final coupled load pattern \(s_C^*\) results from the algebraic weighted addition of both modal excitation vectors in order to obtain a coupled response in longitudinal and transverse directions during the non-linear static analysis. These components are multiplied by factor \(\Lambda\), which takes into account the difference in spectral accelerations associated with each governing mode [Huang 2007]:

\[
s_C^* = \Lambda_X s_n^* X + \Lambda_Y s_n^* Y
\]
3.4. Inelastic analysis

\[ \Lambda_X = \frac{S_{aX}}{\max(S_{aX}, S_{aY})} \]
\[ \Lambda_Y = \frac{S_{aY}}{\max(S_{aX}, S_{aY})} \]

(3.45)

Where \( S_{aX} \) and \( S_{aY} \) are the spectral accelerations associated with the transversal directions. It is worth mentioning that both \( s^*_{nX} \) and \( s^*_{nY} \) are three dimensional by their very nature hence there are non-zero values in directions other than dominant. Therefore \( s^*_C \) is resulting from by combination of two sets of three vectors; this concept is illustrated in figure 3.4.

Performing pushover under \( s^*_C \) load pattern, provides the base shear-displacement behaviour(\( \bar{V}_{bC} - \bar{u}_{rC} \)) and its projection on every dimension must be simultaneously recorded to be applicable in the system of dynamics(see figure 3.5).

Figure 3.5: Summary of CNSP philosophy. \( \bar{V}_{bC} \) and \( \bar{u}_{rC} \) respectively magnitude of the base shear and control point displacement in the 3D coupled pushover.\[Camara 2012\]

The achieved \( \bar{F}_{snX,Y}/M_n - \bar{q}_{nX,Y} \) relationship now can be placed in previously derived formulation of EMPA as the rest of procedure is similar. Furthermore a step by step description of CNSP is available at Appendix A of this document.
Chapter 4

The ground motion and discussion of results

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4.1 Ground motion

All bridges in following analysis are subjected to same type of ground motion, the accelerograms are generated using SismoArtif program [SeismoArtif-2.1.0], a simulation of a far field inter-plate regime with 6.5 moment magnitude and linear site effect by selecting a generic rock (V30=630 m/s). The total of 8 synthetic accelerograms are generated based on EC8(type1-A) [EN-1998] elastic spectrum with peak ground acceleration of 0.5 and 5 % damping. An example of aforementioned accelerograms and spectra are presented in figure 4.1 and the complete data can be found in appendix B.

4.2 Discussion of results

4.2.1 Result of Elastic procedures

The elastic analyses are completed using procedures described in section 3.3. The direct integration method takes longer, as it needs to regenerate the stiffness matrix and solve the equation of dynamics for every time step. Second is Modal Response History Analysis which uses accelerogram as base motion to calculate the resultant of each mode and combine them all using CQC (for further details on CQC see section 3.3.3) The time consuming part of this procedure is the fact that the method needs to go through the time history for all included modes. Finally the fastest elastic procedure is Modal Response Spectrum Analysis, similar to MRHA it is a mode based method but despite MRHA, it is using spectrum as it evaluates the applied
loads to structure. In this method the time history is not available therefore only the extreme deflected shape of structure is resulted.

The following graphics (Figures 4.2 and 4.3) are presented to illustrate the difference in results of elastic procedures in same model:

![Graph](image)

**Figure 4.2:** The normal forces over arch section of a 400m span arch bridge in elastic analysis

A very critical factor in mode base methods is the number of modes that are to be included, this matter depends on participation factor which means how effective one mode can be on general behaviour of whole structure the decision on number of included modes plays a very important role in mode base procedures and their results. Figures 4.4, 4.5 and 4.6 are representing the result of MRSA of several independent modes.

The number of included modes is even more important in inelastic mode base methods especially when governing modes are to be selected in CNSP.
4.2. Discussion of results

Figure 4.3: The shear force along arch height of a 400m span arch bridge in elastic analysis

Figure 4.4: The effect of individual modes on structure- Normal section forces over half arch of a 400m span (SF1).
4.2. Discussion of results

Figure 4.5: The effect of individual modes on structure- Longitudinal section forces over half arch of a 400m span (SF2).

Figure 4.6: The effect of individual modes on structure- Transversal section forces over half arch of a 400m span (SF3).
4.2 Discussion of results

4.2.2 Results of Inelastic procedures

When structure is subjected to extreme seismic actions, it is expected to behave above its linear range, accordingly the nonlinear procedures must be applied. Designing a structure to perform beyond its elastic range will reduce the dimensions or materials required strength, but as it is economically efficient, it is computationally expensive when the only rigorous available method is NLRHA which is a time history procedure, the provided results by this method is considered as exact values. However, the NLRHA is extremely time consuming and implementing it in a design procedure is very frustrating although the final calculations must be based on this rigorous method. An alternative solution is using the mode based estimations, which are previously explained in section 3.4. Application of either MPA, EMPA or CNSP requires the integration of the nonlinear equivalent SDOF system (For Example, see expression 3.27) in order to obtain the target displacement, which has been performed by employing a code developed by author in Mathematica [Mathematica-10.0 ], the equation of dynamics is solved while the algorithm 2 proposed by Simo and Hughes [Simo 1997] is performed in each time step to consider the nonlinear behavior and dependency with the history of the response $F_{sn} = (q_n, \dot{q}_n)$. An example of nonlinear SDOF cyclic loading is plotted in figure 4.7.

![Figure 4.7: An example of force-deformation relation generated during evaluation of a SDOF system in MPA procedure.](image-url)
4.2. Discussion of results

Figure 4.8: a 400m span showing slight yielding during seismic action.

Figure 4.9: a 300m span acting in its Inelastic range. (Normal Forces)

Figure 4.10: a 300m span acting in its Inelastic range. (Shear Forces)
4.2. Discussion of results

Figure 4.11: a 300m span acting in its Inelastic range. (Shear Forces)

Figure 4.12: a 300m span acting in its Inelastic range. (Section Moment-Normal axis)

Figure 4.13: a 300m span acting in its Inelastic range. (Section Moment-Longitudinal)
4.2. Discussion of results

Figure 4.14: a 300m span acting in its Inelastic range. (Section Moment-Transversal)

Figure 4.15: Percentage error of first 40 elements of arch (2 only modes included in MPA to compare with the coupling condition of CNSP)
4.3 Conclusion

The general conclusion about the seismic analysis of arch bridge resulted from this chapter are summarized as following:

- The forces obtained by modal dynamics are generally larger than spectrum analysis. The reason is the inability of MRSA to combine peak response in the exact time of occurrence, although modal combination rules are implemented to overcome this drawback, the estimation is not accurate as modal dynamic procedure which is resulted by exact integration. Therefore the modal dynamic method is assumed to provide exact results in elastic range analysis.

- The MRSA found to be the fastest elastic approach. This advantage makes it very appealing for designer to use it in early design phase; however, it may fall on unsafe side and should not be referenced for final analysis.

- Both SRSS and CQC rules are providing almost equal and reliable results while combining extreme seismic response. Therefore SRSS is selected to be used in calculation of MRSA as it has a simpler formulation.

- When the bridge is expected to act in its inelastic range, the nonlinear response history analysis is the only method which designer can rely on. However, execution of numerical analysis on a complex model is extremely time consuming.

---

1The maximum observed calculation time among all models was 11 seconds including data export.
and requires large computational power; this problem can be omitted during design phase by using MPA, EMPA or CNSP to reduce the calculation time to minutes. It is experienced that in long span arch bridge, the CNSP provide the results within range of less than 20% error (almost similar to MPA and EMPA) with 10% of time required for MPA or EMPA.
Appendix A

Description of pushover analyses

This appendix illustrates the step by step procedure of MPA, EMPA and CNSP respectively, the following is adopted from Camara Ph.D thesis [Camara 2011] since it is applicable here as well. although a step by step procedure of MPA is primarily published by Chopra [Chopra 2007].The procedures are presented in a general form and could be applied to any structure.

A.1 Modal Pushover Analysis: MPA

1. Apply first the gravity loads considering geometric nonlinearities (P-\(\Delta\) effects).

2. Perform modal analysis from the deformed configuration of the structure, computing frequencies \(f_n\), participation factors \(\Gamma_n^j\ (j = X, Y, Z)\) and mode shapes \(\phi_n\) up to the maximum frequency of interests; \(f_{\text{max}} = 25\) Hz. After the modal analysis, a study about the participation of each mode below \(f_{\text{max}}\) in the global response in terms of forces or displacements should be conducted, identifying the governing longitudinal \(f_{nX}\) and transverse modes \(f_{nY}\). MPA should involve both dominant modes. the limit frequency which marks the end of the range where pushover is to be conducted is established as \(f_{\text{gov}} = \max(f_{nX}, f_{nY})\).

3. For the modes below the limit \(f_{\text{gov}}\) their dominant direction \((DR_n)\) is selected because MPA does not consider the three-directional contribution of modes, and subsequently the analysis continues in 2D (either longitudinally, transversely or vertically depending on the governing direction of the specific mode). The control point is selected for each vibration mode as the node with maximum modal displacement in the dominant direction. A proposed method to obtain the characteristic direction of the mode in presented in section 6.4.4.1, summarized in the expression below.

\[
\Gamma_n^{DR} = \max_j(\Gamma_n^j)\ ; \text{with } j = X, Y, Z \rightarrow DR_n \tag{A.1}
\]

4. For the \(n\) – th mode within the range studied by pushover analysis \((f_n \leq f_{\text{gov}})\), develop the base shear versus control point displacement curve in the dominant direction, \(V_{bn} - u_{rn}\), by means of nonlinear static analysis (pushover) of the structure when the load distribution \(s_n^* - m\phi_n\) is incrementally applied (self-weight included), ignoring the components in the direction different than the dominating one. The base shear \((V_{bn})\) is the sum of the shear recorded in
A.1. Modal Pushover Analysis: MPA

each foundation (towers, intermediate piers and abutments) in the dominant direction of the mode, and the control point displacement $u_{rn}$ is consequently the displacement of this point along the dominant direction in the n-th mode (it position depends on the vibration mode).

5. Transform $V_{bn} - u_{rn}$ pushover curve to $F_{sn}/M_n q_n$ coordinates using expressions 3.30a and 3.30b, repeated here for convenience:

$$\frac{F_{sn}}{M_n} = \frac{V_{bn}}{L_n}$$  \hspace{1cm} (A.2)

$$q_n = \frac{u_{rn}}{\phi_{rn}}$$ \hspace{1cm} (A.3)

6. Idealize the real $F_{sn}/M_n q_n$ curve for the equivalent SDOF system associated with the $n-th$ mode. Here, a bi-linear curve considering a modified 'Equal Area' rule presented in figure 3.1 is proposed because of its simplicity, but more realistic curves may be considered. The kinematic properties of the nonlinear spring behavior in the SDOF system need to be addressed.

7. Compute the peak generalized displacement $q_n$ of the n-th mode inelastic SDOF system. Here, it is proposed to integrate the SDOF differential equation of the corresponding mode in time domain (repeated below), by implementing the algorithm 2.

$$\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \frac{F_{sn}}{M_n} = \Gamma_n \ddot{g}$$ \hspace{1cm} (A.4)

8. Obtain the peak control point displacement $u_{rn}$ with expression: $u_{rn} = q_n \phi_{rn}$, where $\phi_{rn}$ is the modal displacement in the dominant direction $DR_n$ of n-mode at the selected control point.

9. Interpolate with $u_{rn} + u_G$ (being $u_G$ the control point displacement in the dominant direction due to gravity loads) from the database of pushover analysis, to obtain the combined effects of lateral loads and gravity due to n-mode contribution ($r_n + G$).

10. Obtain the contribution of the n-mode to the seismic response exclusively, by extracting the effect due to the self-weight: $r_n = r_n + G - r_G$, where $r_G$ is the contribution considering the self-weight acting alone.

11. Repeat steps 4 to 10 for all modes with frequencies below or equal $f_{gov}$.

12. Obtain the total nonlinear seismic response by combining the contribution of each mode with frequency below $f_{gov}$, using an appropriate combination rule, here CQC rule is selected for this purpose: $r_{nl}$.

13. Compute the higher mode seismic response ($r_{el}$) by means of spectrum analysis (MRSA) including modes with frequencies higher than $f_{gov}$ and lower or equal than $f_{max}=25$ Hz.
A.2. Extended Modal Pushover Analysis: EMPA

14. Combine the participation of first modes with the higher mode effect (between \( f_{gov} \) and \( f_{max} \)), employing SRSS combination rule to obtain the dynamic response of the structure: 
\[
{r_d} \approx \sqrt{{r^2_{nl}} + {r^2_{el}}}.
\]
Frequencies higher than \( f_{max} = 25 \) Hz are ignored.

15. Calculate the total demand \( (r) \) by combining the self-weight effect \( (r_G) \) with the dynamic contribution exclusively due to the earthquake \( (r_d) \). Since the sign of seismic forces is lost in pushover procedures\(^1\), two hypotheses are made, considering both positive and negative signs in the earthquake responses to take into account that the seismic input, and the consequent structural behavior, have alternating sign reversals.

\[
r \approx \max(r_G \pm r_d)
\]  
(A.5)

A.2 Extended Modal Pushover Analysis: EMPA

EMPA proposed procedure is summarized next, repeating for completeness some steps introduced in MPA:

1. Apply first gravity loads considering geometric nonlinearities (P-\( \Delta \) effects).

2. Perform modal analysis from the deformed configuration of the structure computing frequencies \( f_n \), participation factors \( \Gamma_j^n \) \((j=X,Y,Z)\) and mode shapes \( \phi_n \) up to the maximum limiting frequency; \( f_{max} = 25 \) Hz. Next, a study about the participation of each mode below \( f_{max} \) in the global response in terms of forces or displacements is conducted, identifying the governing longitudinal \((f_{nX})\) and transverse modes \((f_{nY})\), EMPA covers both dominant modes. The limit frequency which marks the end of the range where pushover is to be conducted is established as \((f_{gov} = \max(f_{nX}, f_{nY}))\).

3. For the \( n\)th mode, included in the range studied by pushover analysis \((f_n \leq f_{gov})\) develop the base shear-control point displacement curve, \( V_{bn}^X - u_{rn}^X \) \(^2\), by means of three-dimensional nonlinear static analysis of the structure when the load distribution \( S_n^x = m\phi_n \) is incrementally applied (self-weight included), now fully considering its three-dimensional characteristics. Three pushover curves are obtained while the structure is being pushed beyond the linear range, each one associated with the longitudinal, transverse and vertical directions: \((V_{bn}^X - u_{rn}^X), (V_{bn}^Y - u_{rn}^Y), (V_{bn}^Z - u_{rn}^Z)\) (see figure 3.3) The base shear \( V_{jn}^j \) is the sum of the shear recorded in each connection of the model to the ground in \( j \)-direction during the pushover analysis of the \( n\)th mode, whilst the control point displacement \( u_{rn}^j \) is consequently the displacement of the control point in \( j \)-direction; this point, is selected as the node with maximum modal displacement regardless of the direction where it is recorded.

\(^1\)Pushover is equivalent to spectrum analysis in the elastic range
\(^2\)the bar symbol means ‘magnitude’ of the three directional displacement or shear components.
A.2. Extended Modal Pushover Analysis: EMPA

4. Transform each $V_{bn}^j - u_{rn}^j$ pushover curves to $F_{sn}^j/M_n$ coordinates using expressions 3.38 and 3.39, repeated below:

$$F_{sn}^j = \frac{V_{bn}^j}{L_n^j}; \text{with } j = X, Y, Z$$  \hspace{1cm} (A.6)

$$q_{sn}^j = \frac{u_{rn}^j}{q_{rn}^j}; \text{with } j = X, Y, Z$$  \hspace{1cm} (A.7)

5. Idealize the real $F_{sn}^j/M_n$-$q_{sn}^j$ curve for equivalent SDOF system associated with $n$-mode in $j$-direction. Here, a bi-linear curve considering a modified ‘Equal Area’ rule presented in figure 3.1 is proposed because of its simplicity (more realistic curves could be considered.) The kinematic properties of the non-linear spring behaviour in the SDOF system need to be addressed, defining the loading and unloading branches appropriate for the structural system and material.

6. Obtain a modular $\bar{F}_{sn}/M_n$-$\bar{q}_n$ from the three-directional results using expressions 3.39 and 3.40, repeated next:

$$\bar{F}_{sn} = \sqrt{(F_{sn}^X)^2 + (F_{sn}^Y)^2 + (F_{sn}^Z)^2}$$  \hspace{1cm} (A.8)

$$\bar{q}_n = \sqrt{(q_{sn}^X)^2 + (q_{sn}^Y)^2 + (q_{sn}^Z)^2}$$  \hspace{1cm} (A.9)

7. Compute the peak generalized modular displacement $\bar{q}_n$ of the $n$-mode inelastic SDOF system. Here, the integration of the SDOF differential equation 3.36 of the corresponding mode in time domain (repeated below) is proposed.

$$\ddot{\bar{q}}_n + 2\zeta_n\omega_n\dot{\bar{q}}_n + \frac{\bar{F}_{sn}}{M_n} = -\ddot{\bar{u}}_{g,n}(t)$$  \hspace{1cm} (A.10)

where $\ddot{\bar{u}}_{g,n}(t)=\Gamma_n^X \ddot{u}_g^X(t) + \Gamma_n^Y \ddot{u}_g^Y(t) + \Gamma_n^Z \ddot{u}_g^Z(t)$, and $\ddot{u}_g$ the accelogram 3D components.

8. Obtain the peak modular control point displacement $\bar{u}_{rn}$ with:

$$\bar{u}_{rn}^{max} = \phi_{rn} \max_t[\bar{q}_n(t)]$$  \hspace{1cm} (A.11)

9. Interpolate with $\bar{u}_{rn} + \bar{u}_G$ (being $\bar{u}_G$ the modular displacement of control point due to gravity loads; $\bar{u}_G = \sqrt{(u_{G}^X)^2 + (u_{G}^Y)^2 + (u_{G}^Z)^2}$ from the database of the three-dimensional pushover analysis, in order to obtain the combined effects of lateral loads and gravity due to $n$-mode contribution $r_n+G$.

10. Obtain the contribution of $n$-mode to the seismic response exclusively, by extracting the effect due to the self-weight: $r_n=r_{n+G}-r_G$, where $r_G$ is the
A.3. Coupled Nonlinear Pushover Analysis: CNSP

contribution of gravity loads acting alone.

11. Repeat steps 3 to 10 for all modes with frequencies below or equal to \( f_{gov} \).

12. Obtain the total nonlinear seismic response \( (r_d) \) by combining the contribution of each studied mode using an appropriate combination rule, here CQC rule is selected for this purpose.

13. Compute the higher mode seismic response \( (r_{el}) \) by means of spectrum analysis (MRSA), including modes with frequencies higher than \( f_{gov} \) and lower or equal than \( f_{max}=25 \) Hz.

14. Combine the participation of first mode with the higher mode effect by means of SRSS combination rule to obtain the dynamic response of the structure:

\[
r_d \approx \sqrt{r_{nl}^2 + r_{el}^2}.
\]

15. Calculate the total demand by combining the effect due to self-weight \( (r_G) \) with the dynamic contribution due to the earthquake exclusively \( (r_d) \). Since the sign of earthquake forces is lost in pushover procedure, two hypotheses are made, considering both positive and negative signs in the earthquake response, which takes into account the alternating nature of the seismic response.

\[
r \approx \max(r_G \pm r_d)
\]

A.3 Coupled Nonlinear Pushover Analysis: CNSP

The steps involved in the proposed coupled pushover (CNSP) are detailed next; repeating for convenience several features introduced in MPA and EMPA:

1. Apply first the gravity loads considering geometric nonlinearities (P-\( \Delta \) effects).

2. Perform modal analysis from the deformed configuration of the structure, computing frequencies \( f_n \), participation factors \( \Gamma_n^j (j=X,Y,Z) \) and mode shapes \( \phi_n \) up to the limit maximum frequency \( f_{max}=25 \) Hz. Next, a study about the participation of each mode below \( f_{max} \) in the global response in terms of forces or displacements is conducted, identifying the governing longitudinal \( (f_{nX}) \) and transverse \( (f_{nY}) \) frequencies.

3. Expand the excitation vector of the dominant longitudinal \( (nX) \) and transverse \( (nY) \) modes, and compute the weight of dominant \( nk \)-mode in direction \( j(\alpha_{nk}^j) \) by adding its expanded nodal forces \( (s_{nk,i}^j \text{ in node } i) \) along the whole

\(^3\text{CNSP, unlike MPA and EMPA, only conducts one pushover analysis using a combination of loads from the transverse and longitudinal governing modes, instead of performing nonlinear static analysis for all the modes with frequencies below or equal to } f_{gov}=\max(f_{nX},f_{nY})\)
structure \((k=X,Y, j=X,Y,Z)\). Alternatively, the ratio could be modified substituting the summations by the corresponding participation factors.

\[
\alpha_{nk}^j = \frac{\sum_{i=1}^{N_{node}} s_{nk,i}^j}{\sum_{i=1}^{N_{node}} s_{nX,i}^j + \sum_{i=1}^{N_{node}} s_{nY,i}^j}
\]  

(A.13)

4. Considering the two dominant longitudinal \((nX)\) and transverse \((nY)\) modes, develop the base shear versus control point displacement curve, \(\bar{V}_{bc}-\bar{u}_C\), by means of 3D nonlinear static analysis of the structure (self weight included), incrementally increasing the coupled load pattern \(s^*_{C}\) of the expression 3.44 (considering the fully 3D characteristic of the excitation vector \(s^*_{n}\)):

\[
s^*_C = \Lambda Y s^*_nY + \Lambda X s^*_nX
\]  

(A.14)

\[\Lambda_j = S_{aj}/\max(S_{aY}, S_{aX}), \text{ where } S_{aj} \text{ is the spectral acceleration associated with the governing mode in } j\text{-direction, } k=X,Y.\]

This displacement is measured simultaneously in two points during the coupled pushover, corresponding to the control points of the longitudinal and transverse governing modes (selected according to EMPA described above).

5. The contributions of each dominant vibration mode to the longitudinal and transverse projections of the coupled capacity pushover curve, are computed by multiplying the projection in \(j\)-direction \((V_{bnX}^j-u_{rnX}^j)\) by the weight factor \(\alpha_{nk}^j\) or \(\alpha_{nk}^j\) expressed above. Three pushover curves are obtained per both two dominant modes; \((V_{bnX}^j-u_{rnX}^j)\) and \((V_{bnY}^j-u_{rnY}^j)\) with \(j=X,Y,Z\). The base shear \(V_{bnX}^j\) and \(V_{bnY}^j\) is respectively the sum of the shear recorded in each foundation of the model in the \(j\)-direction of each dominant mode, whilst the control point displacement \(u_{rnX}^j\) and \(u_{rnY}^j\) is, consequently, the displacement in the \(j\)-direction for each governing longitudinal and transverse mode at the corresponding control point in a respective way.

6. Transform each \(V_{bnX}^j-u_{rnX}^j\) and \(V_{bnY}^j-u_{rnY}^j\) pushover curves respectively into \(F_{snX}/M_n-q_{nX}^j\) and \(F_{snY}/M_n-q_{nY}^j\) coordinates by means of:

\[
\frac{F_{snk}^j}{M_{nk}} = \frac{V_{bnk}^j}{L_{nk}^j}; \text{with } j = X,Y,Z \text{ and } k = X,Y
\]  

(A.15)

\[
q_{nk}^j = \frac{u_{rnk}^j}{\theta_{rnk}^j}; \text{with } j = X,Y,Z \text{ and } k = X,Y
\]  

(A.16)

7. Idealize the real \(F_{snX}/M_n-q_{nX}^j\) and \(F_{snY}/M_n-q_{nY}^j\) curves for the equivalent SDOF systems associated with each governing mode in \(j\)-direction. Here, it is proposed to use a bi-linear curve considering a modified ‘Equal Area’
rule presented in figure 3.1 (more realistic curves maybe considered). The
kinematic properties of the nonlinear spring behavior in the SDOF system
need to be simulated, defining the loading and unloading branches appropriate
for the structural system and material.

8. Obtain modular $\bar{F}_{snX}/M_nX, \bar{F}_{snY}/M_nY, \bar{q}_{nX}$ and $\bar{q}_{nY}$ relations from the di-
rectional results using expressions ($k = X, Y$):

$$\frac{\bar{F}_{snX}}{M_{nk}} = \sqrt{\left(\frac{F_{Xnk}}{M_{nk}}\right)^2 + \left(\frac{F_{Ynk}}{M_{nk}}\right)^2 + \left(\frac{F_{Znk}}{M_{nk}}\right)^2}$$ (A.17)

$$\bar{q}_{nk} = \sqrt{(q_{Xnk})^2 + (q_{Ynk})^2 + (q_{Znk})^2}$$ (A.18)

9. Compute the peak generalized modular displacement $\bar{q}_{nX}$ of the governing lon-\ngitudinal $nX$-mode inelastic SDOF system, and analogously for the dominant
transverse $nY$-mode the peak $\bar{q}_{nY}$. Here, integrating the differential equation
3.36 of the corresponding governing mode in time domain (repeated below for
$nk$-mode) is proposed, following the algorithm 2.

$$\ddot{\bar{q}}_{nk} + 2\xi_{nk}\omega_{nk}\dot{\bar{q}}_{nk} + \frac{\bar{F}_{snk}}{M_{nk}} = -\ddot{u}^*_g,nk(t)$$ (A.19)

10. Obtain the peak modular control point displacement of the governing modes;
$u_{rnX}$ and $u_{rnY}$ respectively, with expressions $\bar{u}_{rnX} = \bar{\phi}_{rnX} \max_t [\bar{q}_{nX}(t)]$ and
$\bar{u}_{rnY} = \bar{\phi}_{rnY} \max_t [\bar{q}_{nY}(t)]$, where $\bar{\phi}_{rnX}$ and $\bar{\phi}_{rnY}$ are the modular modal dis-
placements at control point in longitudinal and transverse dominating modes
respectively.

11. Interpolate with $\bar{u}_{rnX} + \bar{u}_G$ (being $\bar{u}_G$ the modular displacement of control
point due to gravity loads: $\bar{u}_G = \sqrt{(u_{GX})^2 + (u_{GY})^2 + (u_{GZ})^2}$ from the database
of three-dimensional pushover analysis particularized for the longitudinal gov-
erning mode ($V_{bnX}$-$\bar{u}_{rnX}$), to obtain the combine effects of lateral loads and
gravity due to $nX$-mode contribution; $r_{nX+G}$. Analogously, the contribu-
tion of the governing transverse mode $nY$, $r_{nY+G}$, is obtained by interpolating
$\bar{u}_{rnY} + \bar{u}_G$ in the pushover curve ($V_{bnY}$-$\bar{u}_{rnY}$).

12. Obtain the contribution of $nX$-mode and $nY$-mode to seismic response exclu-
sively by extracting the effect due to self-weight: $r_{nX} = r_{nX+G} - r_G$ and $r_{nY} = r_{nY+G} - r_G$, where $r_G$ is the contribution of gravity loads alone.

13. Combine the three-dimensional contribution of governing $nX$-mode and $nY$-
mode through the CQC combination rule, obtaining $r_{nl}$.

14. Compute the contribution of vibration mods different than the governing ones
and below $f_{max} = 25$ Hz, assuming their response elastic ($r_{ul}$). This elastic
effect is obtained by means of spectrum analysis (MRSA). Vibration modes above 25 Hz are neglected.

15. Combine through SRSS rule the contribution of governing modes with the contribution of modes below 25 Hz and different than the governing ones, in order to obtain the dynamic response of the structure: 
   \[ r_d \approx \sqrt{r_{nl}^2 + r_{el}^2}. \]

16. Calculate the total demand by combining the self-weight effect \( r_G \) with the dynamic contribution due to the earthquake exclusively \( r_d \). Since the sign of earthquake forces is lost in pushover procedure, two hypotheses are made (taking into account the alternating nature of the seismic input), considering both positive and negative signs in the earthquake response.

   \[ r \approx max(r_G \pm r_d) \quad (A.20) \]
The section 4.1 includes a brief description of the ground motion implemented in this analysis. Every bridge is subjected to two accelerogram at the same time in transversal and longitudinal directions in order to simulate the three directional nature of earthquake. The time history approaches are using the accelerogram as the source of motion directly, but in response spectrum analysis, the combination of spectrums is achieved using SRSS (discussed in section 3.3.3). However, it is required to analyze every model with several number of accelerogram, therefore the total of 8 synthetic accelerogram are generated based on EC8 (type1-A) [EN-1998] Elastic spectrum with peak ground acceleration of 0.5 and 5 % damping. These synthetic ground motions are plotted in following figures of this appendix.
Figure B.1: Plot of 8 artificial Spectrums
Figure B.2: Plot of 8 artificial accelerograms
Bibliography


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