ABSTRACT

As there is a need to deal with the problem of radioactive waste of nuclear plants, prototypes of subcritical reactors are being developed at the moment. They are known as ADS and they have as their main goal, the actinides transmutation. The neutron source, which keeps the reactions in a subcritical reactor, is very intense, that is why a high energy proton accelerator is required. For many applications, it is important to design methods to measure the intensity of the source. ADS are expected to work in a subcritical range between $0.92 < k_{eff} < 0.97$ and because of that, control rods have a limited use and many designers are considering not to include them. They can be substituted by reflector or combustible elements displacements. In order to calibrate in reactivity these displacements, one can either take advantage of the pulsing structure of the neutron source in microseconds time scale, or can design methods for the minute time scale wherein the source is constant, that is the objective we want to achieve here. If the reactivity control is made with the reflector, in between the extreme states of open reflector and closed reflector, both $k_{eff}$ are calculated and a reactivity ramp is estimated appropriate to the reflector movement velocity. This calibration should be made empirically by measuring the response to the ramp with neutron detectors. In the response to the ramp, the specific parameters are: the source intensity, the initial $k_{eff}$ and the ramp slope, which defines the objective to be reached in the calibration process. Measurements are available from experimental subcritical reactors in the operation range of the ADS, which allow getting ready this calibration method based on the reactivity ramps. However, in order to fit the measurements to a theoretical model, the point kinetics equations need to be solved for this case. With the Prompt-Jump approximation and the consideration of slow ramp compared to the time constants of delayed neutrons, the integration of these equations and the non-linear fit of the measurements to the resulting method are detailed. The static approximation is enough when the reactor is in a typical subcritical state. However, when close to a critical state, the reactivity is no longer a ramp but a parabola which lead us to estimate the neutron source by means of a non-linear fit of the experimental values and assuming a known reactivity initial value.

1 INTRODUCTION

For the sake of actinide transmutation, Accelerator Driven Systems (ADS) are actually under design [1],[2]. ADS are subcritical reactors operating in the approximate range $0.90 < k_{eff} < 0.97$, so they require an intense neutron source for maintaining the neutron chain. Such a neutron source is driven by a high energy proton accelerator [3]. In the microsecond time
scale, the source is pulsed; but in the minute time scale, it can be considered as a constant intensity source \[4\].

Taking advantage of both time scales, a method for reactivity calibration is developed. It is based on a quasi-static change of reactivity. Firstly, a theoretical model is built by using the prompt jump approximation of the one group point kinetic equations; later, the model is tested in a subcritical experimental reactor by changing continuously the reactivity in the range that ADS are expected to operate.

2 THEORETICAL DEVELOPMENT

The operation range of a transmutator is between -14 \( \% \) and -4 \( \% \), (referred to \(^{235}\)U); it is a very subcritical state wherein the prompt jump approximation is valid for solving the kinetics equations. The spallation neutron source comes from an accelerator pulsed at a high frequency (above 1 Kz), so that for times below 1 ms, the source is not constant, but when the characteristic unit time is the minute, the source can be considered to be constant, in the sense that a neutron source would always indicate the same counts per second if reactivity does not change.

A typical calibration process would consist in going from the initial state (for instance, open reflector) to the final state, closed reflector, at a very slow and constant velocity. If the initial and final subcritical \( k_{\text{eff}} \) are estimated through a calculation code, the velocity can be fitted to an average ramp of 1 \$/min. These ramps are slightly slower than the delayed neutrons whose characteristic time (inverse of the disintegration constant \( \lambda \)) is 12 seconds. From now on, we will name them as slow ramps.

If \( k_{\text{eff}}(0) \) is calculated in the most subcritical state and the neutron population, \( N_0 \) is measured, then \[5\]:

\[
\frac{N_0}{l} = \frac{S}{1 - k_{\text{eff}}(0)} \Rightarrow \frac{N_0}{\Lambda} = \frac{S}{-\rho} \tag{1}
\]

where \( l \) is the average neutron lifetime, \( \Lambda \) is the average neutron generation time; \( \rho \) the reactivity (negative) and \( S \) the intensity of the neutron source. The source can be obtained from the right hand side of the equation (in counts per second). The reactivity is obtained at any subcritical configuration from the source and the measurement of the neutron population. If the calculation of \( k_{\text{eff}}(0) \) is not accurate enough, then the reactivity can be measured through a pulse of the source in the microseconds time scale as it can be seen in Fig.1.
It may be observed that the response to the pulse is only required for the initial reactivity and the calculation of the source, the rest of the calibration can be achieved with the instrumentation of the Plant.

For the case of a continuous reactivity insertion, expressed in dollars, the kinetic equation is:

\[
\frac{dN}{dt} = \frac{\lambda \rho(t) + \rho'}{1-\rho(t)} N + \frac{\lambda S \Lambda / \beta}{1-\rho(t)} \quad (2)
\]

where \(\beta\) is the delayed neutrons fraction and \(\lambda\) the time constant of one precursor group of delayed neutrons, and \(\rho'\) the reactivity time derivative. In the case of a ramp, this equation has an analytical solution; if the ramp is slow as well, \(\lambda \rho >> \rho'\), which simplifies the calculation considerably.

Writing the kinetics equation as:

\[
\frac{1}{\lambda} \frac{dN}{dt} = \frac{\rho N}{1-\rho} + \frac{S \Lambda / \beta}{1-\rho} \quad (3)
\]

due to the slow ramp condition, the derivative term turns out negligible; therefore, it can be derived that, as a first approximation, the solution is the static equation:

\[
N_s(t) = \frac{S \Lambda / \beta}{-\rho(t)} \quad (4)
\]

Considering that the derivative term is a perturbation, \(N(t)\) can be substituted for \(N_s(t)\) and \(N\) is obtained, leading to:

\[
N(t) = N_s(t) \left[ 1 + \frac{1-\rho}{(-\rho)\left(\frac{\rho'}{\lambda}\right)} \right] \quad (5)
\]
It may be observed that if the ramp is slow, the static approximation is correct until close to a critical state, that is, out of the operation range of transmutators.

3 EXPERIMENTAL VALIDATION

As the transmutators are still under design, the measurements were taken from the literature. In the CORAL-I reactor, with $\beta = 0.0068$, the evolution of the neutron population is measured from an initial state with reactivity -13.5 $\$ to a final state with reactivity -0.5 $. The movement lasted 15 minutes, so that the average ramp was around 0.86 $\$/min, what can be considered as “slow ramp”. The term $S / \beta = (8.6 \pm 0.2) \times 10^3 (c/s)/s$ was measured as well. The evolution of the neutron population $N(t)$ is represented in Fig.2.

![Figure 2: Example of response to the ramp of a subcritical reactor](image)

At first, it can not be stated that the inserted reactivity is exactly a ramp, to verify this is enough representing the inverse of the static approximation respect to time; if a negative slope straight line is obtained, the hypothesis of the ramp is correct. In Fig.3, $1/N$ versus time is represented:

![Figure 3: The dotted line is the experimental data, the continuous one, the fit to the static approximation.](image)
In figure 3, it can be observed that at instant $t=10$ min the ramp approximation is correct, but from 10 min up to 14.5 min there is a clear change of slope. That is why a parabolic reactivity insertion is supposed, and the static approximation has been used to fit the data. The obtained fitting parabola is:

$$\rho(t) = -13.8 + 0.288t + 0.0365t^2 \quad (6)$$

Then, Fig.4 is obtained:

![Figure 4: Inserted reactivity. Ramp between -13.4 $\text{S}$ and -8 $\text{S}$](image)

It is convenient to take into account that the static approximation needs the value of the source to obtain the reactivity, or the initial reactivity value to obtain the source and the rest of the parameters. Besides, above subcritical values of -2 $\text{S}$ (t=14 min) the static approximation must be corrected.

It is necessary to go through a non-linear fit to obtain the source from the corrected solution of the kinetics equation. Such a fit must be done assuming that the reactivity follows a parabolic equation like the following:

$$\rho(t) = a_0 + a_1t + a_2t^2 \quad (7)$$

Where initial reactivity (-13.8 $\text{S}$) is supposed to be $a_0$. In this way, the solution $N(t)$ can be written as a non linear function (rational type) with three undetermined coefficients: the source value, $a_1$ and $a_2$. By fitting the neutron population experimental data used in Figure 3, we have obtained Figure 5.

The value obtained for the neutron source is $(8.6 \pm 2.5) \times 10^3 \text{ (c/s)}$. This value is exact within an error margin of 30 %. (the exact determination of the source was made with an error of 3 %).
4 CONCLUSIONS

Data coming from a continuous reactivity insertion have been analysed in an experimental reactor at the typical subcritical conditions of the transmutators. In this range of reactivity and with a constant source, we have deduced and proved that the static approximation is enough to calibrate the movement in reactivity.

We have also derived the correction when the static approximation is not enough, as it happens in the subcritical states close to critical ones (> -2 $\delta$). The reactivity is no longer a ramp and it is necessary to approximate it to a parabola.

A known initial reactivity value, which can be obtained from the response to a pulse of the spallation source, is required. The rest of the cases need to go through a non-linear fit.

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REFERENCES


