Figure 5.8: Eigen-spectrum of obtained global modes at $Re = 900$. 

5. Three-dimensional wall-bounded open cavity flow
Figure 5.9: Real part of the eigenfunction velocity field of the cuboid cavity at $Re = 900$, Eigenfunctions normalized with $\max(\hat{u})$. From upper to lower are corresponding to the eigenfunction of eigenmodes listed in table 5.5. Isosurfaces $\hat{u} = \pm 0.1$, $\hat{v}, \hat{w} = \pm 0.05$. (left): $\hat{u}$, (middle):$\hat{v}$ and (right): $\hat{w}$. 
5. Three-dimensional wall-bounded open cavity flow

5.3.2 Case 2: $Re = 950$

Base Flow

A flow over cavity of aspect ratio $L:W:D=6:2:1$ at $Re = 950$ is studied here. Figure 5.10 shows time-history of the streamwise velocity $u(3, -0.5, 1)$. Intuitively, the amplitude of the velocity oscillation decrease considerably, streamwise velocity $u(x, y, z)$ is stable with exponentially decaying amplitude, and ultimately convergences to a steady state.

A visualization of the flow structures is presented in figure 5.11. Figure 5.11(a) shows the contour of the spanwise velocity $w(x, y, z)$, the isosurface level are $\pm 0.01$. We can see the three-dimensionality associated with the rear vortex in the cavity. The spanwise velocity is antisymmetric. Figure 5.11(b) plots the negative streamwise velocity (blue color) and the streamwise vorticity (grey color). A large recirculated zone can be observed by the negative streamwise velocity inside cavity. Due to the small difference in pressure between the outside and inside of the cavity, a pair of the edge vortices are generated. The base flow structure are quite similar in the flow conditions $Re = 900$ and $Re = 950$.

Linear Instability Analysis

**Figure 5.10**: Time trace of streamwise velocity $u(3.0, 0.0, 1.0)$ and the details of the signal for the cuboid cavity at $Re = 950$. 
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.11: (a) Contour of the spanwise velocity \( w(x, y, z) \) of the cuboid cavity at \( Re = 950 \), isosurface level are \( w(x, y, z) = \pm 0.01 \). (b) Grey isosurfaces are streamwise vorticity \( \omega_x = \pm 0.15 \), the blue isosurfaces are negative streamwise velocity \( u(x, y, z) \).

Figure 5.12: Eigen-spectrum of cuboid cavity at \( Re = 950 \)
5. Three-dimensional wall-bounded open cavity flow

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \lambda_r \pm i \cdot \lambda_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>(-0.0132 \pm i \cdot 0.0000)</td>
</tr>
<tr>
<td>CT</td>
<td>(-0.0179 \pm i \cdot 0.0342)</td>
</tr>
<tr>
<td>CT1</td>
<td>(-0.0277 \pm i \cdot 0.0792)</td>
</tr>
<tr>
<td>CT2</td>
<td>(-0.0310 \pm i \cdot 0.0899)</td>
</tr>
<tr>
<td>CT3</td>
<td>(-0.0398 \pm i \cdot 0.1020)</td>
</tr>
<tr>
<td>CT4</td>
<td>(-0.0390 \pm i \cdot 0.0324)</td>
</tr>
</tbody>
</table>

Table 5.6: The first six leading eigenvalues of the cuboid cavity at \( Re = 950 \).

From the DNS simulation, we get the steady state base flow at \( Re = 950 \). TriGlobal instability analysis algorithm has been carried on this base flow. Krylov subspace dimensions \( m = 60 \) is required for convergence of the 11 eigenmodes, the time integration is \( \tau = 0.8 \). For the instability analysis, the convergence criterion for the Arnoldi iteration is based on a tolerance of \( 10^{-6} \).

In the table 5.6, the obtained leading eigenvalues are listed. We can see the least eigenmode is still the stationary eigenmode CS, it is the same steady eigenmode as the case \( Re = 900 \), with a smaller damping ratio \( \lambda_1 = -0.0132 \). Other traveling modes have the different low frequencies. The eigenvalue spectrum is shown in figure 5.12, all the modes are placed symmetrically with respect to the real axis. Figure 5.13 shows the eigenfunctions corresponding to these six least stable eigenvalues. At this Reynolds number, the traveling modes have a low frequency, all the instabilities are confined in the cavity. They are related to the centrifugal instability.
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.13: Real part of the eigenfunction velocity field of the cuboid cavity at $Re = 950$, Eigenfunctions normalized with $\max(\hat{u})$. From upper to lower are corresponding to the eigenfunction of mode 1-6. Isosurfaces $\hat{u} = \pm 0.1$, $\hat{v}, \hat{w} = \pm 0.05$. (left): $\hat{u}$, (middle): $\hat{v}$ and (right): $\hat{w}$. 


5. Three-dimensional wall-bounded open cavity flow

5.3.3 Case 3: $Re = 1000$

The surveys of the lateral walls effect on the instability properties is carried out at $Re=1000$. As interpreted in the previous chapter 4.2.4, shear layer modes behaves two-dimensional properties, only centrifugal instability has three dimensionality. Without effect of side wall, the flow attains the unstable state earlier than wall-bounded three-dimensional open cavity.

In this section, a thoroughly understanding of the cavity flow is made. The shear layer vorticity thickness are compared between two-dimensional and spanwise periodic open cavity cases. The distinguish spatial distribution of both types of eigenmodes are shown.

Base Flow

Figure 5.14 shows the temporal evolution of the kinetic energy of the cuboid cavity at $Re = 1000$. Kinetic energy converges to a steady state. Residual algorithm has been carried on the time trace of kinetic energy at time domain $[200,480]$, the damping rate of the dominant kinetic energy holder is recovered $\lambda_r = -0.025$, the frequency $St = 0.011$ is detected by the fast Fourier transformation, as shown in figure 5.16. The DNS result indicates global mode $-0.025 + i0.069$ conserved most of the kinetic energy among other perturbations.

Usually in the two-dimensional or two and half-dimensional open cavity cases, the shear layer vorticity thickness as a local measure to investigate the instability properties of the shear layer, it has been well documented in many cavity researches[6, 14, 21, 23, 32, 48, 55, 101, 103, 117].

It is defined as

$$\delta_\omega = \frac{U_\infty - U_c}{(\partial \bar{u}/\partial y)},$$  \hspace{1cm} (5.15)

where $U_c \approx 0$ is the flow velocity within the cavity, and $\bar{u}$ is the time-averaged streamwise velocity. Equation (5.15) assumes that the shear layer is perfectly planar and has zero transverse flow. In a strict sense, this evaluation invalidate in the three-dimensional case, we still use it as an indicator of shear layer vorticity thickness.

Most researchers report that shear layer over cavities closely resemble turbulent free shear layers, in that the spreading rate is approximately linear. It is common that the value of the spreading rate has some discrepancy in the open cavity case with laminar upstream and turbulent upstream boundary layer.

Rowley et al. [101] summary the spread study in the both experiment and numerical simulations, he documents the range of the spreading ration varied with the $L/\theta_0$ in the work of Sarohia [103] who appears to be the first to measure the spreading rate in detail: the spreading rate $d\delta_\omega/dx$ increased from 0.025 to 0.088 as $L/\theta_0$ increase from 52.5 to 105.2.

Later, Gharib and Roshko [48] supplement this range for the cavity with $L/\theta_0 > 103$, the spreading rate $d\delta_\omega/dx$ keep constant as 0.124, this is close to the turbulent free shear layer $d\delta_\omega/dx \approx 0.162$. The difference between these two kind of upstream boundary layer is the spread rate become decrease as $L/\theta_0$ increase, as shown in the experiments of Cattafesta et al. [23].

Figure 5.15 shows the shear layer vorticity thickness of the cuboid cavity at $Re = 1000$ extract from the center $X - Y$ plane at $z = W/2$, the data from Rowley et al. [101] are
5.3. Tri-Global linear instability of three-dimensional open cavity flow

also plotted because it has the same ratio of the length to depth in our case, it is clearly to observe that both case have the resemble trend of the shear layer vorticity thickness as the function of the streamwise distance over the cavity, the sharply increase in the range \([0, D]\), after that the slope become gently and grow linearly with the streamwise distance, the shear layer is suppressed near the trailing edge of the cavity, so the vorticity thickness decreases, the different values of shear layer vorticity thickness \(\delta_\omega\) is due to the flow conditions. In our result, \(\delta_\omega\) have a slight ripple in \([D, L]\) because of the three-dimensional flow.

The contour surface of the spanwise velocity \(w(x, y, z)\) at \(Re = 1000\) are plotted in figure 5.17(a), the iso-surface level is -0.01 and 0.01, it is small compared with the free-stream velocity. we can clearly observe that the three-dimensionality happened near the rear part of the cavity and downstream near to the corner of the cavity, these are never be found in the spanwise periodic three-dimensional open cavity study.

Figure 5.17(b) shows the negative streamwise velocity \(u(x, y, z)\) inside of the cavity, it corresponds to the large recirculation region as the blue contours, the flow is separated by the cavity into two regions, located inside and outside of it. The clearly symmetric streamwise vortices can be seen from the plot of streamwise vorticity \(\omega_x(x, y, z)\) which is the grey contour in the figure 5.17(b), the pair of the edge vortices are generated due to the pressure gradient between inside and outside of the cavity. In addition, the large symmetric vortices exist inside the rear part of the cavity which is related to the primary vortex of the cavity.

**Linear Instability Analysis**

Time-stepping methods have been preformed at \(Re = 1000\), the Krylov subspace dimension \(m = 60\) with integration time \(\tau = 0.8\) for recovery of the six first leading eigenvalues, the criteria tolerance is \(10^{-6}\).

Motivation on the understanding of the flow evolution promoting by the instability properties, the classification of each global mode corresponding to individual flow mechanism is included. Here the detail of the well-known cavity flow mechanism are documented.
Figure 5.15: The shear layer vorticity thickness $\delta_\omega$ along the cuboid cavity at $Re = 1000$ (○) and reference date from Rowley et al. [101](▽) at $Ma = 0.6$ of the compressible cavity flow.

Figure 5.16: (a) Time evolution of the kinetic energy residual of the cuboid cavity at $Re = 1000$, (b) Power spectral of kinetic energy at time domain [200, 480], only the most energetic frequency are captured.
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.17: (a) Contour of the spanwise velocity $w(x, y, z)$ of the cuboid cavity at $Re = 1000$, isosurface level are $w(x, y, z) = \pm 0.01$. (b) Grey isosurfaces are streamwise vorticity $\omega_x = \pm 0.15$, the blue isosurfaces are negative streamwise velocity $\omega_x(x, y, z)$.

(1) The well-know Rossiter mode (shear layer mode) can be identified by the semi-empirical formula that Rossiter developed in 1964 [100],

$$St = \frac{fL}{U} = \frac{n - \alpha}{M + 1/\kappa},$$

(5.16)

where $f$ is the frequency at a given mode number $n = 1, 2, 3, ...$, $M$ is the freestream Mach number, $\alpha$ is the average convection speed of vortices travelling over the cavity normalized by freestream speed. $\kappa$ is the phase delay of vortices against the upstream traveling acoustic waves. $\alpha = 0.25$ and $1/\kappa = 1.75$ original values used by Rossiter.

(2) Later, Sarohia [104] and Chatellier et al. [25] observed another onset of self-sustained oscillation, the vortex structures driven by convective wave and the shear layer near the trailing edge of the cavity exhibits large lateral motion, Sarohia [104] gave the wavelength $\eta$ of the disturbances bears an approximate integral relation with the length of the cavity in any particular mode of oscillation by experiments,

$$\frac{L}{\eta} = N + \frac{1}{2}$$

(5.17)

where $N = 0, 1, 2, ...$, note that the wavelength ratio for modes 0 and 1 is 3. The frequency of first self-sustained oscillation modes is $St \approx 0.67$, Zhang and Naguib [127] acquired the frequency of oscillation $St \approx 0.21$ is lower than the first Rossiter mode $St = 0.4 - 0.5$ but 1/3 times of the Sarohia’s first mode, he points out this frequency is related to the self-sustained oscillation and the mode has the 3 times wavelength ($\eta$) than the Sarohia’s. Also he speculated the large lateral oscillations of the shear layer are self-sustained through conservation of the fluid mass within the cavity.

(3) There is one type of oscillation has the low-frequency ($0.02 - 0.04$) in the three-dimensional cavity which is labeled as the centrifugal instability as interpreted by Brés and Colonius [19], this frequency at least one order of magnitude smaller than those of self-sustained oscillations, it exhibits the three-dimensionality and organizes a spanwise-distributed wavelike structures along the recirculation inside of the cavity.
5. Three-dimensional wall-bounded open cavity flow

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\lambda_r \pm i \cdot \lambda_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>$-0.0128 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CT</td>
<td>$-0.0163 \pm i \cdot 0.0345$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0251 \pm i \cdot 0.0774$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0253 \pm i \cdot 0.0878$</td>
</tr>
<tr>
<td>ST</td>
<td>$-0.0261 \pm i \cdot 1.4464$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0363 \pm i \cdot 0.1018$</td>
</tr>
</tbody>
</table>

**Table 5.7**: The first six leading eigenvalues of the cuboid cavity at $Re = 1000$.

Table 5.7 lists the obtained eigenvalues, the eigenmodes named as in the case of $Re=950$, according to the frequency of each mode. It is noteworthy that global mode CT1 gotten from linear analysis is identical to the damping stable mode extracted from residual algorithm, see in figure 5.16. Global mode CT1 contains the most kinetic energy compared with other stable modes. A new high-frequency damping mode appears in the leading global modes list. A clearly eigenspectra is plot in figure 5.18, a huge frequency contrast can be seen among global modes.

Figure 5.18 shows the eigenfunctions corresponding to the stable eigenvalues. Similar to the previous cases $Re=900$ and $Re=950$, the leading eignmode is the stationary damping mode, which sets inside of the cavity, the decay of disturbances rotating around the recirculation region. Moreover, all the spatial structure of low-frequency global modes is alike. However, a high-frequency global mode with $\lambda_{ST} = -0.0261 \pm 0.14464$ behave like a typical wake-shedding instability, the perturbation develops from the less 1/2 rear part of the cavity to the downstream. The three-dimensional perturbation has the apparent lateral behavior, which is rather distinguish from spanwise peridic open cavity case in section 4.2.4. This observation may verify the proposal in the introduction, the periodic assumption in the cavity flow may be inappropriate to understanding the flow mechanism in the realistic open cavity geometry.

Table 5.8 summarized the dominant eigenmode frequencies of two-, two and half-
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.19: Real part of the eigenfunction velocity field of the three-dimensional open cavity at $Re = 1000$. Eigenfunctions normalized with $\max(\hat{u})$. From upper to lower are corresponding to the eigenfunctions of mode 1-6. Isosurfaces $\hat{u} = \pm 0.1$, $\hat{v}, \hat{w} = \pm 0.05$. (left): $\hat{u}$, (middle): $\hat{v}$ and (right): $\hat{w}$. 
three-dimensional open cavity at the same length to depth ratio $L/D = 6$. For the two-dimensional open cavity case, the frequency obeys exactly the Rossiter prediction. The spanwise-periodic case is different because of the coexist of the shear layer mode and three-dimensional centrifugal modes. In the current flow condition, Rossiter mode II as a successful escapee from the interaction between both types of mode. The akin frequencies of numerical and experimental results in three-dimensional cavity flow indicate that ST mode has been verified.

5.3.4 Case 4: $Re = 1050$

**Base flow**

The parametric simulation is continue to preform when $Re=1050$.

In figure 5.20, time history of kinetic energy is plotted, a strong high-frequency oscillation can be observed in the detail plot at $t \in [250, 350]$. The flow is proven to be stable due to the reduced amplitude of the oscillation.

In figure 5.21(a), the dominant damping rate of the global mode is extracted from kinetic energy data using the residual algorithm (3.6). After a transition phrase, exponential decay of residual comes out, this decay purses at the same rate until convergence, the obtained damping rate is $\lambda_r = -0.011$. The discrete Fourier transform is carried out on the signal of kinetic energy at $t \in [300, 700]$ in order to extract the dominant frequency, as shown in figure 5.21(b), $St \sim 0.232$. From former assessments, the most energetic global mode has switched to the high-frequency global mode $-0.011 + i \cdot 1.457$. The comparison between both most energy global mode at $Re = 1000$ and $Re = 1050$ suggests the kinetic energy spreads from one perturbation to another global mode, the disturbance interaction seems to affect the selection of the global mode.

Similarly to the case $Re = 1000$, the spanwise velocity structure is shown in figure 5.22(a), it is antisymmetric along the center plane of the cavity, the most three-dimensionality lays in the rear part of the cavity, the represented iso-surface values correspond to 1% of the free-stream velocity. A pair of streamwise vortices are generated at the cavity edges due to a small difference in pressure between the outside and the inside of the cavity, clearly seen in figure 5.22(b) in grey, the small negative streamwise velocity $u(x, y, z)$ inside of the cavity also is plotted in figure 5.22(b) in blue. The large recirculate
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.20: Time trace of kinetic energy $E/U$ and the details of the signal for the cuboid cavity at $Re = 1050$.

Figure 5.21: (a) Time evolution of the kinetic energy $(K/U)$ residual of the cuboid cavity at $Re = 1050$, (b) Power spectral of the periodic data in figure 5.20.

region inside of the cavity.

Until $Re = 1050$, DNS results suggest three-dimensional cavity flow is still stable. The visualization of flow structure has a similar trend comparing to lower Reynolds numbers. A large flow separation exist inside of the cavity and edge vortices generated due to pressure differences along the side edges of the cavity. However, perturbation selection have been conducted as Reynolds number increases. The global mode which conserved the most of the kinetic energy switches from steady to traveling, from low-frequency to high-frequency. So, linear instability analysis is performed to discover the perturbation evolution along $Re$ changes.

Linear Instability Analysis

The instability of the flow over the open cavity at $Re = 1050$ is analyzed through time-stepping method. Time integration $\tau = 0.8$ and krylov subspace dimensional $m = 60$
5. Three-dimensional wall-bounded open cavity flow

Figure 5.22: (a) Contour of the spanwise velocity $w(x, y, z)$ of the cuboid cavity at $Re = 1050$, isosurface level are $w(x, y, z) = \pm 0.01$. (b) Grey isosurfaces are streamwise vorticity $\omega_x = \pm 0.15$, the blue isosurfaces are negative streamwise velocity $u(x, y, z)$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>$\lambda_r \pm i \cdot \lambda_i$</td>
</tr>
<tr>
<td>ST</td>
<td>$-0.0100 \pm i \cdot 1.4555$</td>
</tr>
<tr>
<td>CS</td>
<td>$-0.0124 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CT</td>
<td>$-0.0149 \pm i \cdot 0.0347$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0201 \pm i \cdot 0.0863$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0229 \pm i \cdot 0.0754$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0331 \pm i \cdot 0.1015$</td>
</tr>
</tbody>
</table>

Table 5.9: The first six leading eigenvalues of the cuboid cavity at $Re = 1050$.

for the purpose of calculating six global modes. The analysis is carried on until reaches the criteria tolerance $10^{-6}$.

The eigenvalues of these global modes are listed in table 5.9. Distinguish from lower Re cases, the high-frequency ST becomes the leading eigenmode. This high-frequency mode may be responsible for the lose of stability because of the largest increment over $Re$.

Figure 5.24 shows spatial structure of global modes at $Re = 1050$. The ST modes become the first leading stable mode in this $Re$, exhibiting symmetry structure. The modes resemble shear-layer instability oscillations but with a more complex structure. The oscillatory structure in the wake presents a nearly constant wavelength, which can be extracted visually from the figures.

CS mode becomes the secondary leading stable eigenmode with a lower damping rate than ST mode. From the spatial structure, it is clearly suggested this global mode is related to the recirculation region of the cavity, with a tail sweeping downstream. This perturbations property is anti-symmetry.

Figure 5.23 shows eigenspectrum of global mode at $Re = 1050$.

As $Re$ increases, damping rate of the global modes grows, the leading global mode shifts from CS mode to ST mode. Moreover, ST mode has an apparently increase and becomes leading eigenmode at $Re = 1050$. The frequency of mode ST and CT increases
5.3. Tri-Global linear instability of three-dimensional open cavity flow

with $Re$ and relatively increment $\Delta \lambda_i$ of ST mode is $\sim 10$ times of CT mode. Take an example between $Re = 1000$ and $Re = 1050$, $\Delta \lambda_i,ST = 7.4\%$, $\Delta \lambda_i,CT = 0.6\%$. This reveals further unsteadiness is responsible to ST mode.

In order to validation the ST mode as the least stable mode in this cavity flow, we turn our attention to nonlinear runs, as preformed in the reference of Theofilis et al. [115], initialized at linearly low levels of the amplitude of ST mode imposed as initial condition in the DNS. Obviously, this initial condition is an exact solution of the Navier-Stokes equations, which manifests in the absence of transient behavior in the early stages of the temporal evolution. The exponential decay of the kinetic energy are plotted in figure 5.25(a), linear decay starts at the early time without transient behavior as expected. The temporal evolution of kinetic energy is shown in figure 5.25(b), the flow remains oscillation with a travelling damping mode. $St \sim 0.232$ captured via FFT in figure 5.25(b) indicates ST mode frequency.

Figure 5.23: Spectrum of open cavity cavity at $Re = 1000$
Figure 5.24: Real part of the eigenfunction velocity field of the cuboid cavity at $Re = 1050$, Eigenfunctions normalized with $\max(\hat{u})$. From upper to lower are corresponding to the eigenmodes listed in table 5.9. Isosurfaces $\hat{u} = \pm 0.1$, $\hat{v}, \hat{w} = \pm 0.05$. (left): $\hat{u}$, (middle): $\hat{v}$ and (right): $\hat{w}$. 
5.3. Tri-Global linear instability of three-dimensional open cavity flow

Figure 5.25: (a) Nonlinear simulation initialized using a single ST mode at initial amplitude $\epsilon = 10^{-4}$ and $Re = 1050$. (b) Upper: Temporal evolution of kinetic energy $K$. Lower: Power density spectrum of kinetic energy at $Re = 1050$. 

$Re = 1050$ 
$\lambda_r = -0.01$
5. Three-dimensional wall-bounded open cavity flow

Figure 5.26: The damping rates of the first three leading modes and the linear interpolation of ST mode.

<table>
<thead>
<tr>
<th>Re</th>
<th>1000</th>
<th>1050</th>
<th>$Re_{cr} \sim 1080$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_r$</td>
<td>-0.0260</td>
<td>-0.010</td>
<td>$\lambda_{r, ne} = 0$</td>
</tr>
</tbody>
</table>

Table 5.10: The damping rate and the corresponding $Re$ are used for the linear interpolation.

5.4 Critical Reynolds number evaluation

The critical Reynolds number at which the steady base flow first becomes unstable was determined by preforming a parametric study of the eigenvalue problem. According to the linear instability analysis results, when $Re$ increases, the spectrum of the eigenmodes reach $\lambda_{r, ne} = 0$, the first instability comes when the damping rate of the least eigenmode greater than 0. so the neutrally stable happens, it corresponds to the critical Reynolds number $Re_{cr}$. $Re_{cr}$, as the key parameter, evaluates the transition of flow from laminar to turbulent.

The damping rate of the first three leading modes at $Re = 1050$ are illustrated in figure 5.26. As document before, the increment $Re$ has a large influence on ST mode. It has the largest growth ratio among the global modes and becomes the suspect of the first instability in this kind of flow.

Here, we use the one-dimensional linear interpolation method to determine the critical Reynolds number of this flow, the expression is defined as follows,

$$Re_{cr} = Re_0 + (Re_1 - Re_0) \frac{\lambda_{r, ne} - \lambda_{r, 0}}{\lambda_{r, 1} - \lambda_{r, 0}},$$

(5.18)

where two flow conditions are necessary to obtain the interpolation value $Re_{cr}$ in (5.18).
We employ $Re = 1000$ and $Re = 1050$ to acquire $Re_{cr}$, so $Re_1 = 1050$, $Re_0 = 1000$, $\lambda_1 = -0.0100$ and $\lambda_0 = -0.0260$ are introduced in (5.18), the result is $Re_{cr} \sim 1080$, as listed in table 5.10. The Hopf bifurcation can be associated with the shear layer instability prevailing at $Re > 1080$.

5.5 Topological changes exerted by the global modes

Motivation on the onset of instability existed in the three-dimensional open cavity flow. Only the global modes ST, CT, CS at $Re = 1050$ are discussed in detail.

The nature and topological bifurcations exerted by ST mode on the steady three-dimensional flow will be the subject of the analysis presented herein. The methodology follows the previous work of Rodríguez and Theofilis [97, 98], who were the first to relate critical points formation with linear amplification of global mods of instability in separated flow. The amplitude functions of the ST modes are normalized to have maximum streamwise velocity component equal to unity, and is then added linearly to the base flow by multiple a given small amplitude $\epsilon$, the composite flow field $u_{new}$ is obtained as follows,

$$u_{new} = \bar{u} + \epsilon \cdot \hat{u} e^{\lambda t},$$

(5.19)

The instantaneous composite flow field, considered as the topological bifurcations induced by ST mode, with an amplitude $\epsilon = 0.1$ is calculated at $t = 0$. The powerful influence of three-dimensional shear layer mode over the base flow can be observed in the downstream of the cavity flow, the surface streamlines become bent, the flow waves along the downstream and the structures keep symmetric. The comparison of experiment and numerical results are emphasized on the downstream flow structures, as shown in the figure 5.27. The surface streamlines pattern generated by the ST mode is strongly reminiscent of observations in the experiments [31], indicating the existence interaction of ST mode.
The streamlines corresponding to a representative base flow are depicted in figure 5.28(a). The flow structures inside of the cavity are symmetric along the center plane of the cavity. The rear vortices and front vortex are evident. The streamlines over the downstream of the cavity are parallel.

Figure 5.28 shows the wall streamlines for a flow reconstruction using the centrifugal modes CS and CT, with an amplitude $\epsilon = 0.01$. The amplitude $\epsilon$ used for the reconstruction allows a better visualization of the topological changes exerted by the linear mode while remaining topologically equivalent to a reconstruction with infinitesimal amplitude. Compare to the flow pattern of the base flow in figure 5.28(a). The huge impact of these two eigenmodes is confined in the cavity. The downstream flow pattern keep the same as base flow state. As the consequence of mode CS, the flow topology becomes unsymmetrical and the front and rear vortices squeeze to one side of the cavity, which can be seen in figure 5.28(c). Whereas in the effect of CT mode, the large unsymetrical flow feature occurs on the front vortex, the rear vortex has a slight influence and almost is symmetric. See in figure 5.28(d).
5.6 Sensitivity sensitivity

The adjoint analysis yields a very efficient way to determine the receptivity properties of the flow, which reveals the initial disturbance amplitude and the influence of external excitation on this flow. Thus, inspecting the adjoint eigenfunctions, assessing the receptivity regions of the flow, provide the precious information to the structural sensitivity.

The adjoint perturbations are convected upstream, a proper upstream length is required. Here the base flow of the adjoint problem (A.6) is using base flow from original geometry with an 3D longitude extension of beginning of computational domain. Regarding the boundary conditions, \( v = 0 \) is using for the outlet \( D_{out} \) \([10, 50]\), other boundary conditions are keep constant as in the direct simulation in section 5.2.

The spatial distribution of the direct, adjoint modules and the product between both of them \( \delta \lambda \) are displayed in figure 5.29. The disturbance (direct eigenfunction) and the receptivity (adjoint eigenfunction) of the open cavity flow are distributed at the trailing and leading edge of the cavity, respectively. The spatial region where a modification of the structure of the problem produces the greatest drift of the eigenvalues, locates above the cavity, with related to the shear layer region.

The contour plot in figure 5.30 represents the area of maximum sensitivity of eigenvalue,

Figure 5.29: (a)Normalized module of the ST mode, iso-surface value is 0.2 (grey) and 0.4 (blue). (b)Normalized module of the adjoint ST mode, iso-surfaces represent a value of 0.2 (grey) and 0.4 (blue). (c) Sensitivity function \( \delta \lambda \) of the three-dimensional cavity at \( Re = 1050 \). Iso-surfaces represent a normalized value of 0.3 (grey) and 0.7 (blue).
occurs near the leading edge of the cavity, also the local Blasius thickness \( \delta = \sqrt{\nu x/U_\infty} \) of the base flow is plotted in order to quantify the location of structure sensitivity. This numerical conclusion accords to the previous experiments in the cavity flow control. As summarized in the reference [24], either the active flow control or passive flow control, are confined near the upstream lip in order to interact with the shear layer as strong as possible. From recently experiment results of Dudley and Ukeiley [36], the passive control comes from placing a spanwise aligned cylinder in the boundary layer near the leading edge of the cavity. From the experiment observation, the presence of the rod is shown to decrease the mean shear gradient. This is in good agreement with the sensitivity structure prediction here. At the same time, the experiment show the flow control becomes more effectively for the large rod placed at the top of the boundary layer. This discrepancy due to the different flow conditions, the experiment carried on the supersonic flow \( M_\infty = 1.4 \), which interacted with acoustic waves.

5.7 Eigenvalue sensitivity to the size of computational domain

As mentioned by Giannetti and Luchini [50], the global mode is dictated mainly by the structural sensitivity region where values of \( \delta \lambda \) substantially different from zero. To a certain extent, resize the computational domain is equivalent to changing the boundary condition of discretized Navier-Stokes equations. The eigenvalues change substantially only when the domain is smaller than the region of \( \delta \lambda \) is significantly different from zero. In order to validate this hypothesis in three-dimensional open cavity flow, the instability analysis is repeated on short height domain \( L_y = 3D \) with four different Reynolds numbers.

Table 5.11 gives the obtained eigenvalues of global modes by performing the instability analysis on two different height domains \( L_y = 3D \) and \( L_y = 5D \). The damping ratio of mode ST, CS, and CT4 have a slightly decrease as the computational domain height increase, while the damping rates of mode CT1, CT2 and CT3 decline as the height decreases. Overall, the eigenvalues are in good qualitative agreement in both computational domains. ST mode has been identified as the least stable mode in this flow.
5.7. Eigenvalue sensitivity to the size of computational domain

<table>
<thead>
<tr>
<th>Mode</th>
<th>$L_y = 5D$</th>
<th>$L_y = 3D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>$-0.0136 \pm i \cdot 0.0000$</td>
<td>$-0.0130 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CS</td>
<td>$-0.0198 \pm i \cdot 0.0340$</td>
<td>$-0.0199 \pm i \cdot 0.0342$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0310 \pm i \cdot 0.0810$</td>
<td>$-0.0317 \pm i \cdot 0.0814$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0380 \pm i \cdot 0.0924$</td>
<td>$-0.0397 \pm i \cdot 0.0934$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0424 \pm i \cdot 0.1016$</td>
<td>$-0.0405 \pm i \cdot 0.1024$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>$L_y = 5D$</th>
<th>$L_y = 3D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>$-0.0132 \pm i \cdot 0.0000$</td>
<td>$-0.0126 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CS</td>
<td>$-0.0179 \pm i \cdot 0.0342$</td>
<td>$-0.0181 \pm i \cdot 0.0344$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0277 \pm i \cdot 0.0792$</td>
<td>$-0.0282 \pm i \cdot 0.0795$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0310 \pm i \cdot 0.0899$</td>
<td>$-0.0317 \pm i \cdot 0.0912$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0398 \pm i \cdot 0.1020$</td>
<td>$-0.0395 \pm i \cdot 0.1024$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>$L_y = 5D$</th>
<th>$L_y = 3D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>$-0.0261 \pm i \cdot 1.4464$</td>
<td>$-0.0226 \pm i \cdot 1.4453$</td>
</tr>
<tr>
<td>CS</td>
<td>$-0.0128 \pm i \cdot 0.0000$</td>
<td>$-0.0123 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CT</td>
<td>$-0.0163 \pm i \cdot 0.0345$</td>
<td>$-0.0165 \pm i \cdot 0.0346$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0251 \pm i \cdot 0.0774$</td>
<td>$-0.0253 \pm i \cdot 0.0776$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0253 \pm i \cdot 0.0878$</td>
<td>$-0.0257 \pm i \cdot 0.0890$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0363 \pm i \cdot 0.1018$</td>
<td>$-0.0360 \pm i \cdot 0.1019$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>$L_y = 5D$</th>
<th>$L_y = 3D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>$-0.0100 \pm i \cdot 1.4555$</td>
<td>$-0.0066 \pm i \cdot 1.4543$</td>
</tr>
<tr>
<td>CS</td>
<td>$-0.0124 \pm i \cdot 0.0000$</td>
<td>$-0.0119 \pm i \cdot 0.0000$</td>
</tr>
<tr>
<td>CT</td>
<td>$-0.0149 \pm i \cdot 0.0347$</td>
<td>$-0.0151 \pm i \cdot 0.0349$</td>
</tr>
<tr>
<td>CT2</td>
<td>$-0.0201 \pm i \cdot 0.0863$</td>
<td>$-0.0203 \pm i \cdot 0.0874$</td>
</tr>
<tr>
<td>CT1</td>
<td>$-0.0229 \pm i \cdot 0.0754$</td>
<td>$-0.0228 \pm i \cdot 0.0755$</td>
</tr>
<tr>
<td>CT3</td>
<td>$-0.0331 \pm i \cdot 0.1015$</td>
<td>$-0.0328 \pm i \cdot 0.1017$</td>
</tr>
</tbody>
</table>

**Table 5.11:** Eigenvalue sensitivity to the size of the computational domain at $Re=900,950,100,1050$. 

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5. Three-dimensional wall-bounded open cavity flow

5.8 Nonlinear evolution of supercritical flow

Regarding to the previous investigation, critical Reynolds number is revealed at this condition, $Re_{cr} \sim 1080$, flow condition of $Re=1100$ is considered as a supercritical flow condition. For further understanding the flow unsteadiness in this type of flow, the direct numerical simulation continues to carry out at $Re = 1100$.

In figure 5.31, the strong oscillations of streamwise velocity at probe point $(3,0,1)$ have been detected, two dominant frequencies have been observed clearly. Further assessment on these two frequencies is preformed using discrete Fourier transform, two probes are considered: inside cavity $(3,-0.5,1.25)$ and downstream of the cavity $(7,0.1,1.25)$. Figure 5.32(a) shows the PSD of a probe placed inside of the cavity, two strong peaks appear, the lower frequency mode $St \sim 0.004$ is related to the centrifugal mode inside of the cavity, which preserves a little bit more energy than the high frequency mode $St \sim 0.230$. In the figure 5.32(b), PSD for a probe positioned downstream of the cavity is shown, a clear peak round $St \sim 0.230$ are observed, with three orders of magnitude higher of amplitude than the modes detected in figure 5.32(a). The results manifest the governing role of high-frequency mode.

The contour of spanwise velocity $w(x,y,z)$ at $Re = 1100$ is displayed in Figure 5.33(a), the spatial structure is distinct from the case at $Re = 1050$, three dimensionality no longer confine inside the cavity, The structures of the spanwise velocity observed are resemble to the three dimensional shear layer mode in the lower Reynolds number cases. A typical wake-shedding pattern is visualized. The spanwise velocity is antisymmetric and develops along the downstream of the cavity. From this results, numerical simulation at supercritical case confirms the presence of the ST mode. Figure 5.33(b) shows the streamwise vortices, a three-dimensional wave shedding appears downstream of the cavity. The edge vortices in the previous cases has been submerged.

As comparing the energy spectra of centrifugal mode and shear layer mode in figure 5.32, it is clear to see the shear layer mode are dominant energy keeper of the unsteady cavity flow at $Re = 1100$. Fluctuation velocity are analyzed in order to she the light to the complex flow structure. As shown in the figure 5.34, three different times of the spanwise velocity fluctuation exhibit the flow topology which reminiscent of shear layer mode at
5.8. Nonlinear evolution of supercritical flow

![Graph showing power density spectrum (PSD) of streamwise velocity](a) St ∼ 0.004

![Graph showing power density spectrum (PSD) of streamwise velocity](b) St ∼ 0.230

**Figure 5.32:** Power density spectrum of the streamwise velocity $u$ of probes (a) inside of the cavity $(3, -0.5, 1.25)$ and (b) downstream of the cavity $(7, 0.1, 1.25)$.

![Contour of spanwise velocity](a) $w(x, y, z)$

![Grey isosurfaces are streamwise vorticity $\omega_x = \pm 1$, the blue isosurfaces are negative streamwise velocity $u(x, y, z)$](b)

**Figure 5.33:** (a) Contour of the spanwise velocity $w(x, y, z)$ of the cuboid cavity at $Re = 1100$, isosurface level are $w(x, y, z) = \pm 0.01$. (b) Grey isosurfaces are streamwise vorticity $\omega_x = \pm 1$, the blue isosurfaces are negative streamwise velocity $u(x, y, z)$. 

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sub-critical flow condition \( Re = 1050 \), this observation is consistent to the PSD results.

Later, \( Q \) criterion \cite{63} is introduced to identify the vortex which triggered by fluctuation velocity in the three-dimensional open cavity.

\( Q \) criterion is a vortex detection technique based on the second invariant of velocity gradient. It is based on the capability of decomposition of the velocity gradient into two parts: a symmetric part, the strain rate tensor \( S \), that represents the flow strain effects; and an asymmetric part, the vorticity tensor \( \Omega \), that represent the flow rotational effects.

\[
\nabla U = S + \Omega \tag{5.20}
\]

Following the latter decomposition, \( Q \) is defined as the difference of the Euclidean norm of the vorticity tensor and the strain tensor:

\[
Q = \frac{1}{2} (u^2_{i,i} - u_{i,j}u_{j,i}) = \frac{1}{2} u_{i,j}u_{j,i} = \frac{1}{2} (||\Omega||^2 - ||S||^2) 
\tag{5.21}
\]

When rotational effects are dominant over strain effects, \( Q > 0 \). \( Q \) criterion considers that the regions of positive \( Q \) are coherent structures.

In figure 5.35, the instantaneous isosurface of fluctuation vortices are illustrated. Around the trailing edge of the cavity, a sequence of interlacing hairpin vortices form and grow. Further downstream, the hairpin vortices slightly break down, shrink laterally for decreasing spanwise distance because of the viscous effect.
5.8. Nonlinear evolution of supercritical flow

The instantaneous surface streamlines of three-dimensional open cavity flow at $Re = 1100$ are shown in figure 5.36. The main oscillations occur in the rear part of the cavity and the downstream of the cavity, this flow pattern is in good agreement with the experiment observation from [31]. The velocity fluctuating resulting from shear layer instability alters the streamlines in the downstream. In the rear part of the cavity bottom, two symmetric vortices merged into one vortex occupied the whole spanwise space.

Regarding to the symmetry of flow feature, the instability results from sub-critical simulation manifest the symmetric shear layer mode and anti-symmetric centrifugal mode. In order to understand more deeply the second bifurcation of the supercritical flow, anti-symmetric flow structure are extracted from DNS result, as shown in figure 5.37. The anti-symmetric structure of $u$ and $v$ are similar to the centrifugal stationary mode at $Re = 1050$, while the third component of anti-symmetric structure pertains to the shear layer mode. The quantities of $u$ and $v$ are six orders magnitude lower than the third component $w$, which proves the shear layer mode is the dominant unstable mode, contains the majority energy of the flow, while the centrifugal mode might be another bifurcation following the first shear layer bifurcation.
Figure 5.37: Anti-symmetric structure of three components (a) \( u \), (b) \( v \) and (c) \( w \). The three components are normalized with \( \max(u) \), \( \max(v) \) and \( \max(w) \), respectively, iso-surface is \( \pm 0.01 \).
Chapter 6

Attempts for the cavity flow control

Theoretical flow control indicated by adjoint-based sensitivity analysis has been widely applied in the two-dimensional wall-bounded or open flow, such as the flat-plate boundary layer and wake past a circular cylinder [79, 80, 86, 91]. Structure sensitivity analysis is verified to be able to provide correct indications on where to position passive control devices, e.g. small base flow spatial modification, in order to suppress the unstable eigenmode.

Here we can use the map of the eigenvalues drift $\delta \lambda$ in previous chapter as a guideline to inset the small square cylinder inside flow in the attempt to suppress the three-dimensional shear layer instability in three-dimensional cavity flow. Since it is the firstly three-dimensional cavity flow control conducted by structural sensitivity analysis, previous investigations are all carried on two-dimensional flow, spanwise base flow modifications are considered.

A quantitative evaluation for the structural sensitivity results is necessary to provide informations to the specifically control strategy. In figure 6.1, spatial distributions of largest eigenvalue draft are shown again. The location of the eigenvalue draft on the level of $\delta \lambda = 0.7$ (green) are denoted by the following parameters: the width of eigenvalue draft $w_s$, the length $l_s$, the height $h_s$ and the distance from wall of the leading edge to the eigenvalue draft $h_{0s}$. The detail of the parameters are listed in table 6.1.

From previous evaluation, we may judge that the zone for the largest eigenvalue draft is too small to make a control. So the passive control strategy resorts to low iso-surfaces region, here we choose the sensitivity region with iso-surface $\delta \lambda = 0.4$. The parameters are combined in table 6.1.

6.1 Passive cavity flow control via square cylinder

As mentioned in the study of Pralits et al. [91]. In order to suppress the instability, we need to produce a enough variation for the eigenvalue. The success of such operation depends on how large of the damping/growth rate of the base flow is, on its sensitivity to structure perturbation and on the size of spatial modification.

In order to show the control impact by positioned a small square cylinder in regions, where expected a substantial decrease of the damping/growth rate according to current analysis. Two sets of control are performed based on the variation of span length of control cylinder.

<table>
<thead>
<tr>
<th>$\delta \lambda$</th>
<th>$w_s$</th>
<th>$l_s$</th>
<th>$h_s$</th>
<th>$h_{0s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.86</td>
<td>0.045</td>
<td>0.03</td>
<td>0.025</td>
</tr>
<tr>
<td>0.4</td>
<td>2.00</td>
<td>4.2</td>
<td>0.14</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1: Details of the parameters of $\delta \lambda$. 
Figure 6.1: Quantitative evaluation of the structural sensitivity for three-dimensional cavity at $Re = 1050$. (a) Isosurface of $\delta \lambda = 0.7$ (front view). (b) Isosurface of $\delta \lambda = 0.7$ (side view). (c) Isosurfaces of $\delta \lambda = 0.4$ (grey) and 0.7 (green).

Figure 6.2 presents the sketch of small square cylinder positioned on the three-dimensional open cavity, notated as Case 1. Three views are shown. The small square cylinder is located with $x$, $y$, $z$ axis ranges of $[3, 3.02] \times [0.1, 0.12] \times [0.8, 1.2]$. Figure 6.2 (c) shows the spatial location of control cylinder with eigenvalue draft at iso-surface level $\delta \lambda = 0.4$.

Figure 6.3 presents the sketch of small square cylinder positioned on the three-dimensional open cavity, notated as Case 2. Three views are shown. The small square cylinder are located with $x$, $y$, $z$ axis ranges $[3, 3.02] \times [0.1, 0.12] \times [0.97, 1.03]$. Figure 6.3 (c) shows the spatial location of control cylinder with eigenvalue draft at iso-surface level $\delta \lambda = 0.4$, which is smaller than the previous control.

DNS is preformed on this two sets of control system at $Re=1050$, until either a steady-state or a time-periodic solution is obtained. Depress or active the three-dimensional shear layer instability is considered as the evaluation for the control strategy.
6.1. Passive cavity flow control via square cylinder

Figure 6.2: Computational configuration of Case 1. (a) Front view of small square cylinder. (The ratio is not to scale.) (b) Side view of small square cylinder. (The ratio is not to scale.) (c) Three-dimensional view of small square cylinder positioning over cavity.

Figure 6.3: Computational configuration of Case 2. (a) Front view of small square cylinder. (The ratio is not to scale.) (b) Side view of small square cylinder. (The ratio is not to scale.) (c) Three-dimensional view of small square cylinder positioning over cavity.
Figure 6.4 shows the numerical mesh employed in the Case 1. A refined mesh is defined near the small square cylinder. The simulation is carried on the total discretized 23594 elements with polynomial order $n=7$. In the Case 2, a small mesh adjusting is executed around the small cylinder, the number of discretized elements is 22382 with the same polynomial order as previous case.

The boundary conditions of small square cylinder are the same as wall boundary in three-dimensional open cavity, $u=(0, 0, 0)$. DNS starts from an incompressible Blasius velocity profile above the cavity, the same initial condition as no control case.

The time histories of streamwise velocities of these two control systems at probe inside cavity $(4.0, -0.5, 0.75)$, as well as DNS result of three-dimensional cavity case are compared in figure 6.5 (a). A clearly attenuation of high-frequency oscillation can be observed. Both control strategies can modify hydrodynamic stability characteristics through manipulating the shear layer oscillation. It is noteworthy that Case 2 is more powerful to suppress the shear layer instability, the low-frequency of the centrifugal instability seems to be detected in the time range $[225, 300]$.

Figure 6.5 (b) present the time histories of streamwise velocities in the downstream of the cavity $(7.0, 0.1, 0.75)$, where is the most active zone of shear layer mode in the no control cavity flow. The visualization confirms the aforementioned observation. The amplitude of shedding waves is decreased sharply in Case 1. Shear layer oscillation dies out at the transient phrase in Case 1.

In conclusion it may be postulated that the spanwise length of the small spatial modification is an important factor in the three-dimensional flow control, which makes a more complex subject than two-dimensional flow control.
6.1. Passive cavity flow control via square cylinder

![Graph showing time traces of streamwise velocity](image)

**Figure 6.5:** The comparison of time traces of streamwise velocity at (a) inside cavity (4.0, -0.5, 0.75) and (b) downstream of cavity (7.0, 0.1, 0.75).
6. Attempts for the cavity flow control
Chapter 7

Final Remarks

The three-dimensional open cavity with lateral walls is a geometry representative of aircraft devices of engineering significance. Despite of the geometrical simplicity, flow structures and instability properties are still under discussion, specially for realistic three-dimensional open cavity with lateral walls effect. The study performed in this thesis addresses the complex flow structures and instability properties appearing as $Re$ increases, as well as the adjoint-based sensitivity analysis associated to the structural sensitivity of this particular flow. The finding of this thesis at incompressible and low $Re$ flow condition strives to investigate the onset of instability in three-dimensional wall-bounded open cavity flow. Concerning the topology of the three-dimensional flow evoked by the global mode, the flow patterns encountered on the cavity at very high $Re$ experiments [31] are qualitatively identical to these found for low $Re$ in incompressible flow.

The incompressible flow over a long open cavity with lateral walls from subcritical to supercritical conditions has been thoroughly investigated. The geometrical ratio is $L:W:D=6:2:1$. The study focuses on both the flow structure evolution and losing of stability existed in this three-dimensional open cavity. The main contributions of the present work are an in-depth analysis of the linear stability on the three-dimensional cavity and a cross-validation of topological changes exerted by the global mode. Additionally, spanwise-periodic counterpart has been investigated as the comparison of different global mode mechanics.

This thesis combines three-dimensional Direct Numerical Simulations (DNS) and TriGlobal instability analysis, using time-stepping methods for solving a large eigenvalue problems associated with three-dimensional flows. Adjoint-based analysis is performed for the first time in the context to provide theoretical basis for flow control.

Along the global stability analysis, it is shown that: the critical condition for the onset of the first instability: $Re_{cr} \sim 1080$, for the flow over a cavity with aspect ratio $L:W:D=6:2:1$.

A more complex flow structure associates with higher $Re$. DNS results show that the steady state flow structure reaches nonlinear saturated state with a three-dimensional wake instability shedding along downstream of cavity as $Re$ increases.

Linear instability analysis is carried on the steady state base flow obtained at $Re=900$, 950, 1000 and 1050. The least stable mode corresponds to a centrifugal stationary mode at $Re = 900$ and $Re = 950$, related to the recirculation region in the cavity. At $Re = 1000$, the least stable mode is still a centrifugal stationary mode, but there a high frequency shear layer mode also appears in the spectrum. As the Reynolds number increases, $Re = 1050$, leading eigenmode becomes a shear layer mode. This shear layer mode $\lambda = -0.010 \pm i \cdot 1.4543$ becomes the first instability for the three-dimensional open cavity flow. Spatial distribution of this three-dimensional shear layer mode resembles the wake shedding structure at supercritical condition $Re=1100$. Hence, the analysis results are in good agreement with DNS results. According to the spatial visualization of the shear
layer mode, the first instability is symmetric.

Because bifurcation is always associated to loss of symmetries, there is no other bifurcation happening before the shear layer mode. This is different to the sphere case, in which the wake loses azimuthal symmetry via a pitchfork bifurcation, i.e. a steady mode. As interpreted in the experiments of Zhang and Naguib [125], it was found that alteration of the geometry may cause the cavity shear layer oscillation to be attenuated or disappear altogether. This means that if we vary the flow parameters, i.e. $L/D$, $W/D$, $Re$, we may find some parameter subspaces in which the centrifugal steady mode is the first instability. In that case we would observe the same pitchfork bifurcation as in the sphere.

The nature and topological bifurcations induced by the first three leading stable eigenmode (ST mode, CS mode and CT mode) on the base flow at $Re = 1050$ have been investigated herein. The methodology follows the previous work of [97, 98]. For construction of the composite flow field, the eigenfunction is normalized to have the maximum streamwise velocity component equal to unity, and is then added linearly to the base flow, with a given small amplitude $\epsilon$. The reconstruction flow streamline trajectories of ST mode over the base flow show a large influence in the downstream of the cavity, the surface streamlines are symmetric to the center plane of the cavity. The flow near the downstream exhibits a large lateral motion, with a vortex shedding past the cavity. This flow feature is in excellent qualitative agreement with the experimental observations [31, 125].

From the reconstruction of the flow streamline of CS and CT modes, it is observed that the largest impact can be found inside of the cavity. Since both of the modes are non-symmetric and break the center-plane symmetry. As the consequence of the growth of mode CS, the front and rear vortices displaces to one side of the cavity. Whereas the effect of CT mode is that the large unsymmetrical flow feature occurs on the front vortex, thus the rear vortex breaks up and merges to the front vortex.

The structure sensitivity are studied to provide applicable information in further development of flow control. The analysis reveals that the spatial region where a modification of the base flow produces the greatest drift of the eigenvalues is located above the cavity, which is related to the shear layer region. The strongest impact region, upstream of the cavity, which is consistent with the experimental observation from Dudley and Ukeiley [36].

Motivation exists to identify the lateral walls effects in three-dimensional open cavity flow structures and instabilities, thus the spanwise-periodic open cavity flow studied at $Re = 1000$. Linear instability analysis has been performed on two-dimensional cavity base flow that are homogeneous in the spanwise direction. The different instability properties are compared with previous real three-dimensional cases by the aid of perturbation flow field. The totally different spatial structures indicate a new global mode emerges with a more complex structure owing to in the influence of side walls. Also, spanwise-periodic open cavity flow loses stability at lower Reynolds number than the three-dimensional cavity flow. This result reveals that the present of lateral walls suppress the instability and delay transition.

For the understanding of the evolution of the first instability, a DNS of the supercritical case of $Re=1100$ is considered. The flow becomes periodic and saturated with two frequencies ($St_{D,1} \sim 0.230$, $St_{D,2} \sim 0.004$). The higher frequency corresponds to the ST mode. Visualization of the fluctuating velocity of the DNS shows a excellent resemblance with the flow structure associated with the ST mode. We believe the low frequency corresponds to a centrifugal mode because the associated flow structure is confined inside the
cavity.

Numerical simulations of passive control by means of small square cylinders are performed to validate the theoretical model adopted. DNS results show that a small three-dimensional spatial modification in the structural sensitivity region substantially reduces the amplitude of oscillations and inhibits the shear layer instability. In particular, the control system with larger span almost completely eliminates the oscillation.
7. Final Remarks
Appendix A

Adjoint equations

The adjoint framework appears very naturally in optimization theory since it yields a very efficient way to determine the sensitivities of an objective with respect to control variables. The goal of this section is to set out clearly the relationship between the adjoint linearized Navies-Stokes equations and its associated evolution operator $L^*$, also the issues associated with boundary conditions of the adjoint system.

A.0.1 Variables and inner products

For considering the equations expressed in primitive variables, it is convenient to define the quantities: $	ilde{\mathbf{u}} = (\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p})$ , $	ilde{\mathbf{v}} = (\tilde{u}^*, \tilde{v}^*, \tilde{w}^*, \tilde{m})$ where the $	ilde{\mathbf{u}}$ are the perturbation variables and $	ilde{\mathbf{v}}$ are the adjoint perturbation variables, all the field are real.

We define the space–time domain on which the equations are posed, $\Gamma = \Omega \times (0, \tau)$, the $\Omega$ is spatial domain and $\tau$ is time. The inner product defined as

$$<\tilde{\mathbf{v}}, \tilde{\mathbf{u}}> = \int_0^\tau \int_{\Omega} \tilde{\mathbf{v}}^H \cdot \tilde{\mathbf{u}} \, dV dt$$  \hspace{1cm} (A.1)

A.0.2 Operators and PDEs

In general, for any linear operator $A$, its adjoint operator $A^*$ is defined by the adjoint identity, so transpose the linear operator from the inner product,

$$<\tilde{\mathbf{v}}, A \tilde{\mathbf{u}} > = < A^* \tilde{\mathbf{v}}, \tilde{\mathbf{u}} > + B(\tilde{\mathbf{v}}, \tilde{\mathbf{u}})$$  \hspace{1cm} (A.2)

where B is a boundary term. Here we perform the inner products along all the linear Navier-Stokes equations. Considering an example of Two-dimensional linearized Navier-Stokes, convective forms are shown following,

$$M_x : \frac{\partial \tilde{u}}{\partial t} + \tilde{u} \frac{\partial \tilde{u}}{\partial x} + \tilde{v} \frac{\partial \tilde{u}}{\partial y} + \tilde{\nu} \frac{\partial \tilde{u}}{\partial x} + \tilde{\nu} \frac{\partial \tilde{u}}{\partial y} = \frac{1}{Re} (\frac{\partial^2 \tilde{u}}{\partial x^2} + \frac{\partial^2 \tilde{u}}{\partial y^2}) - \frac{\partial \tilde{p}}{\partial x}$$  \hspace{1cm} (A.3a)

$$M_y : \frac{\partial \tilde{v}}{\partial t} + \tilde{u} \frac{\partial \tilde{v}}{\partial x} + \tilde{v} \frac{\partial \tilde{v}}{\partial y} + \tilde{\nu} \frac{\partial \tilde{v}}{\partial x} + \tilde{\nu} \frac{\partial \tilde{v}}{\partial y} = \frac{1}{Re} (\frac{\partial^2 \tilde{v}}{\partial x^2} + \frac{\partial^2 \tilde{v}}{\partial y^2}) - \frac{\partial \tilde{p}}{\partial y}$$  \hspace{1cm} (A.3b)

$$C : \frac{\partial \tilde{u}}{\partial t} + \frac{\partial \tilde{v}}{\partial t} = 0$$  \hspace{1cm} (A.3c)

The $M_x$, $M_y$ and $C$ mean momentum equation in the x direction, y direction and continuity equation.

$$<\tilde{\mathbf{v}}, A \tilde{\mathbf{u}} >= < \tilde{u}^*, M_x > + < \tilde{v}^*, M_y > + < \tilde{m}, C >$$  \hspace{1cm} (A.4a)

$$= < M_x^*, \tilde{u} > + < M_y^*, \tilde{v} > + < C^*, \tilde{p} > + B(\tilde{u}, \tilde{v})$$  \hspace{1cm} (A.4b)
A. Adjoint equations

The adjoint linear Navier–Stokes equations are defined as below,

\[(M_x)^* = 0;\]
\[(M_y)^* = 0;\]
\[C^* = 0\]

which is in condition of \(B(\tilde{v}, \tilde{u}) = 0\), so a proper boundary conditions must be proposed.

\[
(M_x)^* = \frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} - \tilde{u} \frac{\partial u^*}{\partial x} - \tilde{v} \frac{\partial u^*}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) + \frac{\partial m^*}{\partial x} = 0 \quad (A.6a)
\]

\[
(M_y)^* = \frac{\partial v^*}{\partial t} + u^* \frac{\partial v^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} - \tilde{u} \frac{\partial v^*}{\partial x} - \tilde{v} \frac{\partial v^*}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right) + \frac{\partial m^*}{\partial y} = 0 \quad (A.6b)
\]

\[C^* = \frac{\partial u^*}{\partial t} + \frac{\partial v^*}{\partial t} = 0 \quad (A.6c)\]

(A.6) are the adjoint linear Navier–Stokes equation. Also we can write with the operator way.

\[\nabla \cdot \tilde{v} = 0 \quad (A.7a)\]
\[\frac{\partial \tilde{v}}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{u} - (\tilde{u} \cdot \nabla) \tilde{v} = -\frac{1}{Re} \nabla^2 \tilde{v} - \nabla \tilde{p}^* \quad (A.7b)\]

Here we mainly interested in the adjoint modes, i.e. non-trivial solutions of the adjoint linearized Navies-Stokes equations of the form

\[\tilde{v}^*(x,y,t) = \tilde{u}^* e^{\lambda t} \quad (A.8a)\]
\[m^*(x,y,t) = \tilde{p}^* e^{\lambda t} \quad (A.8b)\]

if \(\tilde{u}^* = \tilde{u} e^{\lambda t}\) is the a global mode of LNSE corresponding to the eigenvalue \(\lambda\), we define \(\tilde{v} = \tilde{v}^* e^{-\lambda t}\) as its adjoint global mode. Substitute equation (A.8) to (A.6), 2–D incompressible adjoint Orr–Sommerfeld equation come out.

\[\lambda \tilde{u}^* + \tilde{u} \frac{\partial \tilde{u}^*}{\partial x} + \tilde{v} \frac{\partial \tilde{u}^*}{\partial y} - \tilde{\tilde{u}} \frac{\partial \tilde{u}^*}{\partial x} - \tilde{\tilde{v}} \frac{\partial \tilde{u}^*}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 \tilde{u}^*}{\partial x^2} + \frac{\partial^2 \tilde{u}^*}{\partial y^2} \right) + \frac{\partial \tilde{m}^*}{\partial x} = 0 \quad (A.9a)\]

\[\lambda \tilde{v}^* + \tilde{u} \frac{\partial \tilde{v}^*}{\partial x} + \tilde{v} \frac{\partial \tilde{v}^*}{\partial y} - \tilde{\tilde{u}} \frac{\partial \tilde{v}^*}{\partial x} - \tilde{\tilde{v}} \frac{\partial \tilde{v}^*}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 \tilde{v}^*}{\partial x^2} + \frac{\partial^2 \tilde{v}^*}{\partial y^2} \right) + \frac{\partial \tilde{m}^*}{\partial y} = 0 \quad (A.9b)\]

\[\frac{\partial \tilde{\tilde{u}}^*}{\partial x} + \frac{\partial \tilde{\tilde{v}}^*}{\partial y} = 0 \quad (A.9c)\]

A.0.3 Boundary and initial conditions

We can now address the initial and boundary conditions for the direct and adjoint systems. The part \(B((\tilde{v}, \tilde{u})\) can be writted as below,

\[B(\tilde{v}, \tilde{u}) = \int \int |B(\tilde{v}, \tilde{u})| dx dy dt + \int \int |B(\tilde{v}, \tilde{u})| dy dx dt + \int \int |B(\tilde{v}, \tilde{u})| dx dy \quad (A.10)\]

Inorder to keep the \(B(\tilde{v}, \tilde{u})\) goes to zero, every parts of \(B(\tilde{v}, \tilde{u})\) goes to zero respectively. Combine with the direct boundary condition, we can get the adjoint problem boundaries.

\[B1(\tilde{v}, \tilde{u}) = [\tilde{u} \tilde{u}^* + \tilde{u} \frac{\partial \tilde{u}^*}{\partial x} + \tilde{v} \tilde{v}^* + \tilde{v} \frac{\partial \tilde{v}^*}{\partial x} + \tilde{m} + \tilde{p}^*](A.11a)\]
\[ B2(\tilde{v}, \tilde{u}) = [\tilde{v}^* \tilde{v} + \tilde{v}^* \frac{\partial \tilde{v}}{\partial y} - \tilde{v^*} \frac{\partial \tilde{v}^*}{\partial y} + \tilde{u}^* \tilde{v} + \tilde{u}^* \frac{\partial \tilde{u}}{\partial y} - \tilde{u}^* \frac{\partial \tilde{u}^*}{\partial y} + m \tilde{v} + \tilde{v}^* \tilde{p}] \] (A.11b)

\[ B3(\tilde{v}, \tilde{u}) = [\tilde{u}^* \tilde{u} + \tilde{v}^* \tilde{v}] \] (A.11c)

Obviously, the boundary conditions for the adjoint system must be compatible to the direct system, according to (A.2). For the Dirichlet boundary, direct and adjoint system are the same, while a mix boundary conditions are derived by the Neumann outflow condition of direct system. Some times, Dirichlet boundary is imposed for convenience.
A. Adjoint equations
Staggered grid technique has been used for the spatial discretization, based on two sets of Chebyshev points.

The Chebyshev polynomial of degree $p$, $T_p(x)$, is given explicitly in terms of trigonometric functions by

$$T_p(x) = \cos(p\theta),$$
$$x = \cos(\theta).$$

The Chebyshev Gauss-Lobatto (CGL) points distribution is

$$x_j = \frac{j\pi}{n}, (j = 0, ..., N),$$

(B.2)

While the Chebyshev Gauss (CG) points discretization is

$$x_j = \frac{\pi(2j + 1)}{2n}, (j = 0, ..., N - 1),$$

(B.3)

We can express a function via a polynomial:

$$f(x_j) = \sum_{p=0}^{N} a_p T_p(x_j),$$

(B.4)

where $a_k$ are the series coefficient and $p$ is the order of Chebyshev polynomial.

The first derivative of on the CGL points are calculated using the collocation derivative matrix, see the derivation in [17].

$$\frac{df(x_j)}{dx} = \sum_{p=0}^{N} D_{i,p} a_p$$

(B.5)

$$D_{i,j} = \begin{cases} 
\frac{2N^2+1}{6} & i = j = 0 \\
\frac{-2i}{N^2} & i = j, j \neq 0, N \\
\frac{\bar{c}_j (-1)^{i+j}}{6} & i \neq j \\
\frac{-\bar{c}_j x_i - x_j}{6} & i = j = N 
\end{cases}$$

where

$$\bar{c}_j = \begin{cases} 
2 & j = 0, N \\
1 & 1 \leq j \geq N - 1 
\end{cases}$$

with $x_j$ defined by (B.2).

The second and higher derivatives can be defined via Kronecker product,

$$D'' = DD,$$
B. Chebyshev Polynomial and Staggered grid

\[ D^{(m)} = D^m, \]  
\[ (B.6b) \]

The collocation first derivative matrix in CG points are calculates using

\[ D_{i,j} = \begin{cases} 
0.5x_j, & i = j \\
\frac{(-1)^{i+j}}{1-x_i^2} \sqrt{1-x_i^2}, & i \neq j 
\end{cases} \]

with \( x_j \) defined by (B.3).

The second and higher derivatives follow the algorithm B.6. Data may be transferred between the grids using the interpolation arrays, the algorithm can be viewed in [113].

\[ I_{GL \to G} \] represents interpolation arrays transferring data from the CGL to CG points, while \( I_{G \to GL} \) is interpolation arrays transferring data from CG to CGL points.

\[ I_{GL \to G} = C_{G}^{-1} C_{GL}, \quad (B.7a) \]
\[ I_{G \to GL} = C_{GL}^{-1} C_{G}, \quad (B.7b) \]

where for \( i = 0, ..., N, \)

\[ (C_{GL})_{i,j} = \frac{2}{N_c c_j} \cos \frac{ij\pi}{N}, j = 0, ..., N, \]
\[ (C_{GL})_{i,j}^{-1} = \cos \frac{ij\pi}{N}, j = 0, ..., N, \]
\[ (C_{GL})_{i,j} = \frac{2}{N_c c_i} \cos \frac{ij + \frac{1}{2} \pi}{N}, j = 0, ..., N - 1, \]
\[ (C_{GL})_{i,j} = \cos \frac{ij + \frac{1}{2} \pi}{N}, j = 0, ..., N - 1, \]

in which \( c_0 = c_N = 2 \) and \( c_{i,j} = 1 (i, j = 1, ..., N - 1) \)
Bibliography


Bibliography


