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Robust Infrastructure design in Rapid Transit Rail systems

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Abstract

Incidents and rolling stock breakdowns are commonplace in rapid transit rail systems and may disrupt the system performance imposing deviations from planned operations. A network design model is proposed for reducing the effect of disruptions less likely to occur. Failure probabilities are considered functions of the amount of services and the rolling stock's routing on the designed network so that they cannot be calculated a priori but result from the design process itself. A two recourse stochastic programming model is formulated where the failure probabilities are an implicit function of the number of services and routing of the transit lines.

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1. Introduction

Designing a Rapid Transit Network (RTN) or even extending one that is already functioning, is a vital subject due to the fact that they reduce traffic congestion, travel time and pollution. Usually a RTN is in operation with other transportation systems such as private transportation (car) and this makes that the design must take into account this factor. Another factor that needs to be considered is the capability of the newly designed system to keep operating under more or less suitable conditions under a set of predictable disruptions.

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In Bruno G. *et al.* (2002), a RTN design model is presented where the user cost is minimized and the coverage of the demand by a public transport network is made as large as possible. Laporte G. *et al*. (2007) extend the previous model by incorporating the station location problem, the alternative of several lines and the budget constraints as side constraints. The model is defined using the maximum coverage of the public demand as an objective function. Marín (2007), studies the inclusion of a limited number of lines. Also, in Laporte G. *et al* (2011) a design model is developed to build robust networks that provide several routes to passengers, so in case of failure part of the demand can be rerouted.

Liebchen, C. *et al.* (2009) applied the recoverable robustness (RR) concept in railway networks with the focus on finding recoverable solutions in a limited number of steps. In case of disruption, they allow a feasible solution to be modified by a recovery algorithm. They use the maximum deviation of the recovered solutions from the planned solution, where the maximum is taken over a set of disruption scenarios. Another classical approach is two stage stochastic programming for which the disruption scenarios have an associated probability.

Connections between two-stage stochastic programming network design and RR in railway networks planning models have been studied in Cicerone *et al.* (2009), Caprara et al. (2008) and in Cacchiani *et al.* (2011). Also, in Cadarso and Marín (2012) a two-stage stochastic programming model for rapid transit network design is developed in which disruption probabilities are assumed known a priori, illustrating some of its recoverable robustness properties.

This paper presents a conceptual scheme that permits to incorporate a probability model for the disruptions of a RTN. The network modeling framework followed is that of Marín (2007) and Cadarso and Marín (2012). It is assumed that disruptions arise when transportation units present some failure during operation leaving a link blocked. Other sources of disruption with their associated scenarios could be added, but this is not done for ease of exposition. As a consequence of this, the disruption probabilities will depend on the level of traffic on the network links. The probabilities of failure follow the following hypothesis: a) disruptions are due to a single event and scenarios with several simultaneous disruptions are discarded a priori as they are assumed to have a much lower probability, b) a preselected set of scenarios is considered, c) the number of failures that a train unit may experience along a large number of services distributes accordingly to a geometrical law and the individual probability of failure of a service is constant along the planning horizon and depends only on the train unit characteristics (e.g., quality of material and maintenance). The resulting model has a bilevel structure and it is solved by a specific heuristic method.

The paper is organized as follows. In section 2 a two-stage stochastic model is presented for the design of a RTN. Section 3 describes a probability model for the disruptions. In Section 4 the probability model is integrated in the two-stage stochastic model resulting into a bilevel scheme solved heuristically by means of the method of successive averages (MSA). Finally the model recoverability features are analyzed in section 5.

2. Rapid transit network design model

In this RTND model it is assumed that the locations of the potential stations are known. There already exists a current mode of transportation (for example, private cars or an alternative public transportation is already operating in the area) competing with the new RTN to be constructed. The aim of the model is to design a network, i.e. to decide at which nodes to locate the stations and how to connect them covering as many trips by the new network as possible.

- A potential network (N, A) is considered from which the optimum rapid transit network is selected. The node set is composed by centroids (N_c) and stations at RTN (N_r), the node set is then $N = N_c \bigcup N_r$. Links will be denoted either by a single subscript (e.g., a) or by a double subscript (i.e., (i, j)) when considered convenient. Because both riding directions are always considered, the set of potential links is so that $(i, j) \in A \Leftrightarrow (j, i) \in A$. Let $N(i) = \{ j \in N \mid (i, j) \in A\}$ be the set of nodes adjacent to node i.

- Each feasible link (i, j) has a generalized travel cost which may depend on the scenario of disruption. This is further discussed in section 3.

- The nodes and alignments are connected with a finite number of transit lines: $L = \{1, .., |L|\}$.

- The total demand is given by the trip matrix $G = (g_w)$, where g_w is the number of users willing to travel from origin $o(w)$ to destination $d(w)$. Users may choose between two transportation modes: a private (and current) mode or the public transportation mode made up by the set of new lines that are to be build up. The generalized cost satisfying the demand of o-d pair w through the current network is given by the matrix $U^c = (u_c^w)$. The model assumes an all or nothing modal choice for each o-d pair, i.e., trips g_w will use the new public transport network if its generalized cost is smaller than μ_w times the cost of the current transport network. μ_w can be considered a congestion factor for each $w \in W$. By c_a it will be denoted the location cost of arc $a \in A$ and by c_i the cost of locating a station in node $i \in N$. c_x and c_ψ will denote the arc vector costs and the node vector of location costs respectively.

The design model is subdivided in two stages or levels: a) in the first "planning" stage, the decision variables x, y are chosen, i.e., the topology of the network is decided and b) in a second "recoverability" stage, at a given scenario, the passenger flows make use of the network designed in the first stage, taking into account the characteristics of that scenario.

2.1 Description of the 1 st stage. Variables and constraints

For simplicity, in this model it will be assumed that the planners have selected a priori a very large number $|\hat{L}|$ of candidate lines within a set $\hat{L}$ from which only $|L| << |\hat{L}|$ will be finally included in the solution. Thus, a link-line incidence matrix $(\delta_{a,r})$ will be assumed known with elements $\delta_{a,r} = 1$ if candidate line r contains link a and 0 otherwise. Let h_r, $r \in \hat{L}$ be a binary variable indicating whether candidate line r is chosen or not. Let also χ_a be a binary variable so that $= 1$ if arc a is located and $= 0$, otherwise. The following constraints force that link a must be built if some line r using it is chosen:

$$M\chi_a \geq \sum_{r \in \hat{L}} \delta_{a,r} h_r, \; a \in A \tag{1}$$

These constraints must be complemented by other linking flows on links with variables h_r. Because these constraints involve variables of the 2[nd] stage, they are described in next subsection 2.2. A limitation on the number of lines can be imposed by $\sum_{r \in \hat{L}} h_r \leq |L|$. Let now ψ_i be a binary variable so that $= 1$ if station i is located and $= 0$, otherwise. The following constraints should also be included:

$$\begin{aligned} \chi_a &\leq \psi_i, \quad \forall i \in N, \; \forall a = (i,j) \in A \\ \chi_a &\leq \psi_j, \quad \forall j \in N, \; \forall a = (i,j) \in A \end{aligned} \tag{2}$$

2.2 Description of the 2 nd stage. Variables and constraints

The scenario set will be denoted by $S = \{0, 1, 2, ..., |S|-1\}$. The basic scenario $s = 0$ is the scenario without disruptions; the scenarios of disruption will be associated to the failure of a single link $a \in A$, for which the cost d_{ij}^s will be set to a large value or its flow will be banned directly on the model. Let the subset of links where disruptions may occur be designated by $\hat{A}$. For each $a \in \hat{A}$, $s(a)$ will denote the associated scenario and for each scenario $s \in S \setminus \{0\}$, $a(s)$ will denote the disrupted link. Then, $|\hat{A}| = |S| - 1$. Variables and constraints of the model are described.

- $f_a^{w,s} = 1$, if the demand w uses arc a in the RTN under scenario s and $f_a^{w,s} = 0$ otherwise. By $f^{w,s} = (..., f_a^{w,s}, ...; a \in A)$ it will be denoted an arc flow vector per o-d w pair and scendario s.
- $f_c^{w,s} = 1$, if the demand g_w uses the current network in scenario s and 0 otherwise. By $f_c^s = (..., f_c^{w,s}, ...; w \in W)$ it will be denoted an excess flow vector per o-d pair $w = (o,d)$ and scenario *s*. It will also be convenient to consider the vector $f^s = (..., f^{w,s}, ...; w \in W)$.

The balance constraints for flows at a given scenario *s* must verify:

$$\sum_{k\in N(i)} f_{ki}^{w,s} - \sum_{j\in N(i)} f_{ij}^{w,s} = \begin{cases} f_c^{w,s}-1 & if\ i=o(w) \\ 1-f_c^{w,s} & if\ i=d(w), \forall i\in N, \forall w\in W, \forall s\in S \\ 0 & otherwise \end{cases}$$

Using the proper node-link incidence matrix B and the vector e^w definced as: $(e^w)_i = 1$ if $i = d$, $(e^w)_i = -1$, if $i = o$ and $= 0$ otherwise, the polytope for feasible flows on scenario $s \in S$ can be expressed as:

$$V^s = \{(f^s, f_c^s) \in \{0,1\}^{|A|\cdot|W|} \mid Bf^{w,s} = (1-f_c^{w,s})e^w, w\in W\}, s\in S \tag{3}$$

In order to take into account the competing mode, the following constraints can be included in the model:

$$\sum_{a\in A} d_a^s f_a^{w,s} \le \mu_w u_c^{w,s}(1-f_c^{w,s}), \forall w\in W, \forall s\in S \tag{4}$$

These constraints have the effect of conmuting to the competing mode when the cost for travelling in the RTN is higher than $\mu_w u_c^{w,s}$ which is assumed to be smaller than $\gamma + u_c^{w,s}$, where γ is an ugmenting cost for users of the competing mode.

The following location-allocation constraints prevent from using a link not included in the design:

$$f_a^{w,s} \le \chi_a, \ \ a\in A, \ \ s\in S, \ \ w\in W \tag{5}$$

If the probabilities p_s for each scenario $s \in S$ are known, then the RTND model can be expressed as follows:

$$\begin{aligned} Min_{\chi,\Psi,\phi,h} \quad & c_x^{\mathrm{T}}\chi + c_\psi^{\mathrm{T}}\Psi + \vartheta\sum_{s\in S} p_s \,(\sum_{w\in W} g_w \left\{ d^{s\,\mathrm{T}} f^{w,s} + (\gamma + u_c^{w,s}) f_c^{w,s} \right\}) \\ & \phi^s = (f^s, f_c^s) \in V^s, \ \ s\in S \\ & + \ constraints\ (1),(2),(4),(5) \end{aligned} \tag{6}$$

where ϑ is the value of time for the users of the rapid transit system, It has the effect of increasing the use of RTN network. Building costs Λ are given are assumed linear in the number of new links and stations $\Lambda = c_x^{\mathrm{T}}\chi + c_\psi^{\mathrm{T}}\Psi$.

For each probability vector p, the previous problem (6) has a solution set parametrized by the probability vector p: $\chi^*(p)$, $\Psi^*(p)$ and $\hat{\Phi}^*(p)$, where $\hat{\Phi}^*(p) = \Phi^*(\chi^*(p), \Psi^*(p))$. Notice that if variables χ, Ψ are fixed to $\overline{\chi}, \overline{\Psi}$, then previous problem (6) decomposes into network flow subproblems of the type (7) and thus $\phi = \Phi^*(\overline{\chi}, \overline{\Psi})$

$$\begin{aligned} Min_\phi \quad & \sum_{w\in W} g_w \left\{ d^{s\,\mathrm{T}} f^{w,s} + u_c^{w,s\mathrm{T}} f_c^{w,s} \right\} \\ & \phi^s = (f^s, f_c^s) \in V^s \qquad\qquad s\in S \\ & + \ constraints\ (4),(5)\ \ with\ \ \chi = \overline{\chi}, \ \ \Psi = \overline{\Psi} \end{aligned} \tag{7}$$

3. A probability failure model

The probability p_s of each scenario cannot be considered constant but dependent on the use that is made on the designed network. Thus, it is absurd that if failure scenario *s* is associated with a disruption of a link *a*, a positive probability p_s could be assigned to the scenario without knowing a) whether or not the link will be built and b) the number of services that will load link *a*. With these considerations in mind, a model that states the number of services that must operate on each link is required. Let v^0 be the passenger vector flow on each of the network links in the normal situation, i.e., the scenario without distuptions. v^0 can be expressed as a function of the decision variables of model (6) as $v_a^0 = \sum_{w \in W} f_a^{w,0} g_w,\ a \in A$. Consider also the binary variables x_a^ℓ which state whether link *a* is used by line ℓ. Let γ^ℓ the individual cost of a service on line $\ell \in L$. Then, the following simple covering model will be used to determine the number of services for each line:

$$\begin{aligned} Min_z \quad & \sum_{\ell \in L} \gamma^\ell z^\ell \\ & m_{pax} \sum_{\ell \in L} z^\ell x_a^\ell \geq v_a^0, \quad a \in A \\ & Z \ni z^\ell \geq 0 \end{aligned} \tag{8}$$

where m_{pax} is maximum number of passengers that a service can allocate. If z^* is the vector for the optimal number of services for the lines, then the total number of services θ_a on link a will be given by:

$$\theta_a(f^0) = \sum_{\ell \in L} z^{*,\ell} x_a^\ell \tag{9}$$

By means of a failure model it will be possible to find an expression for the probability that a link presents a disruption during the operational horizon of the transit network. It will be assumed that the probability of failure of a service is mainly determined by the type of units operating in the service and the characteristics of the link. Let T be the set of type units operating on the network. Let $\pi_{a,\tau}$ be the joint individual probability that a service carried out by a unit of type $\tau \in T$ presents a disruption on link $a \in A$. By examining annual disruption reports from transit operators, the fraction of disrupted services with a disruption time of 20 minutes or more over the total number of services on a line is between $1.5 \cdot 10^{-4}$ to $5.0 \cdot 10^{-4}$, i.e. 1 disruption each 2000 or 6600 services. Assume that the probabilities $\pi_{a,\tau}$ have been determined by analyzing statistically the previous mentioned annual disruption reports. Let now $\theta_{a,\tau}$ be the total number of services of type τ carried out on link a during the operational horizon used for our planning model (for instance, peak morning period or one day). Let $T(a)$ be the set of unit types that operate on link $a \in A$. Let also $\tilde{\theta}_{a,\tau}$ be the total number of services with a relevant disruption out of the $\theta_{a,\tau}$ and $\tilde{\theta}_a = \sum_{\tau \in T(a)} \tilde{\theta}_{a,\tau}$ the aggregated number of disrupted services on link *a*. Our modeling hypothesis assumes that $\tilde{\theta}_{a,\tau}$ follows a binomial distribution with probability $\pi_{a,\tau}$, i.e.: $\tilde{\theta}_{a,\tau} \sim Bino(\theta_{a,\tau}, \pi_{a,\tau})$. Thus, the probability $\hat{P}_a$ that link $a \in A$ has at least one disrupted service from any unit type $\tau \in T(a)$, as a function of the number of services $\theta_{a,\tau}$ of type τ operating on that link is:

$$\begin{aligned} \hat{P}_a &\stackrel{\Delta}{=} P(\tilde{\theta}_a \geq 1) = 1 - \prod_{\tau \in T(a)} P(\tilde{\theta}_{a,\tau} = 0) = \\ &= 1 - \prod_{\tau \in T(a)} (1 - \pi_{a,\tau})^{\theta_{a,\tau}} = 1 - \exp\Big(- \sum_{\tau \in T(a)} \alpha_{a,\tau} \theta_{a,\tau}\Big) \end{aligned} \tag{10}$$

where $\alpha_{a,\tau} = -\log(1 - \pi_{a,\tau})$. For small probabilities $\pi_{a,\tau}$, then $\alpha_{a,\tau} \approx \pi_{a,\tau}$. Also, the probability of having no disruption on link *a* of any of the type units $\tau \in T(a)$ is $Q_a = 1 - \hat{P}_a$.

Because the probability of more than one link with simultaneous disruptions is small, the set S of scenarios with disruption that will be considered is made up of scenarios s associated with the failure of a single link *a* within the set of selected links A. All the links with positive flow may be considered, each one of them defining a disruption scenario, or a subset of the links may be selected because they are critical or because their high traffic volume. The scenario with no disruptions at all (scenario 0) is always included in the set S. Let $s(a)$ denote scenario associated to failure of link $a \in \hat{A}$ and let $a(s)$ denote the link associated with scenario $s \in S \setminus \{0\}$. For ease of notation let $\hat{A}_a = \hat{A} \setminus \{a\}$. The probability of each scenario s corresponding to a disruption in link $a(s)$ will be evaluated now by a given function $F : \Re^{|\hat{A}|} \to \Re^{|S|}$ of the number of services θ_a, $a \in \hat{A}$ on the links candidates for a disruption.

If there is a single type of units operating in the network then, the function $F(\cdot)$ for the probabilities p_s and p_0 that will be adopted is:

$$p_s \overset{\Delta}{=} F_s(\theta) = \frac{\exp(\alpha_{a(s)}\theta_{a(s)}) - 1}{1 + \sum_{b \in \hat{A}} (\exp(\alpha_b \theta_b) - 1)}, \quad s \in S \setminus \{0\} \tag{11}$$

$$p_0 \overset{\Delta}{=} F_0(\theta) = (1 + \sum_{b \in \hat{A}} (\exp(\alpha_b \theta_b) - 1))^{-1} \tag{12}$$

It must be noticed that if probabilities $\pi_{a,\tau}$ are very small, the probability p_0 of no disruption is much higher than the probabilities p_s associated with the disruption on the corresponding links $a(s)$.

4. A consistent stochastic 2-stage model and a heuristic solution

Taking into account that the number of services given by (9) depends on the solutions of model (6) which in turn, is parametrized by probabilities p_s, $s \in S$, and that the probability model is given by (11), (12), the following fix point relationships should be verified:

$$p_s = \frac{\exp\{\alpha_{a(s)}\theta_{a(s)}(f^0(\chi(p), \Psi(p)))\} - 1}{1 + \sum_{b \in \hat{A}} (\exp\{\alpha_b \theta_b (f^0(\chi(p), \Psi(p)))\} - 1)}, \quad s \in S \setminus \{0\} \tag{13}$$

$$p_0 = (1 + \sum_{b \in \hat{A}} (\exp(\alpha_b \theta_b (f^0(\chi(p), \Psi(p)))) - 1))^{-1} \tag{14}$$

The previous fix point problem can be stated as the following bilevel programming problem:

$$\begin{aligned}
\operatorname{Min}_{\substack{p,\chi,h \\ \Psi,\phi,z}} \quad & c_x^{\mathrm{T}}\chi + c_\psi^{\mathrm{T}}\Psi + \vartheta \sum_{s \in S} (p_s \sum_{w \in W} g_w) \left\{ d^{s\,\mathrm{T}} f^{w,s} + u_c^{w,s} f_c^{w,s} \right\} \\
& \phi^s = (f^s, f_c^s) \in V^s, \quad s \in S \\
& + \ \mathit{constraints}\ (1), (2), (4), (5) \\
& p_s = F_s(\ldots, \sum_{\ell \in L} x_a^\ell z^\ell, \ldots; a \in \hat{A}), \quad s \in S \\
& z^\ell, \ \ell \in L, \mathit{solves\ the\ lower\ level\ problem}\,[LL(x, f^0)]
\end{aligned} \tag{15}$$

$$
\begin{array}{ll}
Min_z & \sum_{\ell \in L} \gamma^{\ell} z^{\ell} \\
 & m_{pax} \sum_{\ell \in L} z^{\ell} x_a^{\ell} \geq \sum_{w \in W} f_a^{w,0} g_w, \; a \in A \\
[LL(x, f^0)] & Z \ni z^{\ell} \geq 0
\end{array}
\tag{16}
$$

The previous BLP problem will be solved using the following heuristic algorithm, which uses the construction costs as stopping criterion:

0. Set initial probabilities $p^{(0}$; set $\Lambda^{(-1} = 0$; $k = 0$.
1. Solve problem (6) using probability vector $p^{(k}$. Let $\chi_a^{(k}$, $\psi_i^{(k}$, $x_a^{\ell,k}$. Also let $\Lambda^{(k} = c_x^{\mathrm{T}} \chi^{(k} + c_\psi^{\mathrm{T}} \Psi^{(k}$ be the building costs.
2. If $|\Lambda^{(k} - \Lambda^{(k-1}| \leq \varepsilon \Lambda^{(k-1}$ then STOP
3. With the solutions $x_a^{\ell,k}, \ell \in L,\ a \in A$, for the lines on the network links and the passenger flows at iteration k for the undisrupted scenario 0 given by $v_a^{0,(k} = \sum_{w \in W} (f_a^{w,0})^{(k} g_w$, calculate the required number of services $\theta_a^{(k}$ at each network link $a \in A$. This implies to solve the lower level problem (16) $[LL(x^{(k}, f^{0,(k})]$. From the solutions $z^{\ell,(k}$ of (16), the number of services will be given using (9) by: $\theta_a^{(k} = \sum_{\ell \in L} z^{\ell,(k} x_a^{\ell,(k}$.
4. Taking into account the number of services $\theta_a^{(k}$, reevaluate the failure probabilities $\hat{p}_s^{(k} = F_s(\ldots, \theta_a^{(k}, \ldots; a \in \hat{A}),\ \ s \in S$ and
5. Perform an MSA step specified in (17) (using, for instance, $\eta^{(k} = 1/(k+1)$). Then, increase the iteration counter $k = k + 1$.

$$
p_s^{(k+1} = p_s^{(k} + \eta^{(k} (\hat{p}_s^{(k} - p_s^{(k}) \tag{17}
$$

5. Computational tests

The computational proofs have been carried out on the test network shown in figure 1, with 9 nodes, 15 edges, 72 origin-destination pairs and a total demand of 1044 trip units. The network parameters (construction costs for nodes and links, i.e. c_i and c_a, respectively) are shown on links of the network in figure 1. The o-d demand matrix and the o-d costs for the alternative mode of transportation, ($u_c^w + \gamma$), are shown in figure 2. The parameter γ has been set to $\gamma = 1$ for all o-d pairs. Table 2 shows the list of 16 scenarios and the links associated to it. In all computational tests a maximum of $|L| = 5$ lines has been allowed in the solution and no limitation in the budget has been included. A value of ϑ =0.09 for the value of time has been used in all the tests shown. Additionally, constraints (4) have not been taken into account and then commutation to the competing mode is made only when costs in the RTN network are higher than $\gamma + u_c^{w,s}$. Other tests performed including constraints (4) show identical conclusions.

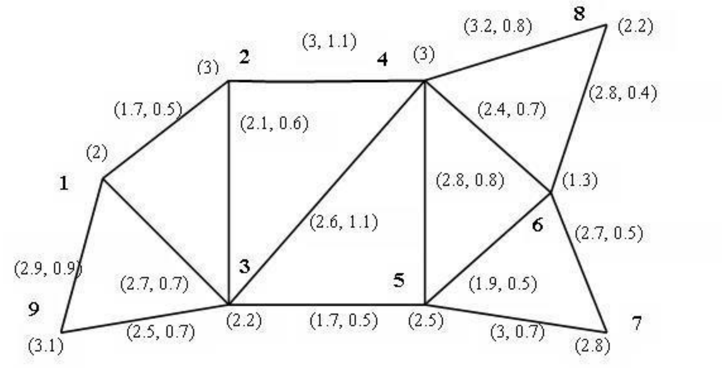

Figure 1. The test network, the total trip demand o-d matrix and trip travel times for the current mode.

$$U_{curr} = \begin{pmatrix} - & 1.6 & 0.8 & 2 & 2.6 & 2.5 & 3 & 2.5 & 0.8 \\ 2 & - & 0.9 & 1.2 & 1.5 & 2.5 & 2.7 & 2.4 & 1.8 \\ 1.5 & 1.4 & - & 1.3 & 0.9 & 2 & 1.6 & 2.3 & 0.9 \\ 1.9 & 2 & 1.9 & - & 1.8 & 2 & 1.9 & 1.2 & 2 \\ 3 & 1.5 & 2 & 2 & - & 1.5 & 1.1 & 1.8 & 1.7 \\ 2.1 & 2.7 & 2.2 & 1 & 1.5 & - & 0.9 & 0.9 & 2.9 \\ 2.8 & 2.3 & 1.5 & 1.8 & 0.9 & 0.8 & - & 1.3 & 2.1 \\ 2.8 & 2.2 & 2 & 1.1 & 1.5 & 0.8 & 1.9 & - & 0.3 \\ 1 & 1.5 & 1.1 & 2.7 & 1.9 & 1.8 & 2.4 & 3 & - \end{pmatrix} \qquad G = \begin{pmatrix} - & 9 & 26 & 19 & 13 & 12 & 13 & 8 & 11 \\ 11 & - & 14 & 26 & 7 & 18 & 3 & 6 & 12 \\ 30 & 19 & - & 30 & 24 & 8 & 15 & 12 & 5 \\ 21 & 9 & 11 & - & 22 & 16 & 25 & 21 & 23 \\ 14 & 14 & 8 & 9 & - & 20 & 16 & 22 & 21 \\ 26 & 1 & 22 & 24 & 13 & - & 16 & 14 & 12 \\ 8 & 6 & 9 & 23 & 6 & 13 & - & 11 & 11 \\ 9 & 2 & 14 & 20 & 18 & 16 & 11 & - & 4 \\ 8 & 7 & 11 & 22 & 27 & 17 & 8 & 12 & - \end{pmatrix}$$

Figure 2. The total trip demand o-d matrix and trip travel times for the current mode

The heuristic method has been tested using several starting points and in all cases the final probabilities $p_s,\ s \in S$ that have been obtained are the same. Table 1 shows in column *#it* the number of iterations necessary to converge and column *difpr* displays the error $\left| p^{(k} - p^{(k-1} \right|_1$ in the last iteration.

The tests carried out analyze the recoverability characteristics of the model and the influence of the service probability failure π in the reliability of the designed RTN, as shown in tables 1 and 2. For large values of π (i.e., 0.1) the algorithm seems to oscillate, converging very slowly. Clearly, the more reliable the system (or equivalently, the smaller is π), the smaller the total costs, being these represented in the *objfun* column. As it must be expected, the probability p_0 of no disruption increases as π is smaller. Also, the attractiveness of the public transportation system increases as the system becomes more reliable. This is illustrated in columns *PTUserTime* and *CUserTime*, showing that the total expected time spent by all public transport users increases whereas the expected time spent in the competing mode decreases.

Table 1. Model outputs for different values of the probability π of service failure

π	$objfun$	Λ	$PTUserTime$	$CUserTime$	p_0	$difpr$	$\#it.$
$1.0 \cdot 10^{-1}$	$2.2146 \cdot 10^{2}$	77.51	$9.5470 \cdot 10^{2}$	$6.4481 \cdot 10^{2}$	$6.18 \cdot 10^{-3}$	$3.09 \cdot 10^{-2}$	(*)41
$5.0 \cdot 10^{-2}$	$2.2263 \cdot 10^{2}$	62.71	$9.8669 \cdot 10^{2}$	$7.9035 \cdot 10^{2}$	$3.13 \cdot 10^{-2}$	$1.39 \cdot 10^{-2}$	21
$1.0 \cdot 10^{-3}$	$2.1095 \cdot 10^{2}$	77.51	$1.2152 \cdot 10^{3}$	$2.6752 \cdot 10^{2}$	$7.72 \cdot 10^{-1}$	0.00	7
$1.0 \cdot 10^{-4}$	$2.0677 \cdot 10^{2}$	77.51	$1.2599 \cdot 10^{3}$	$1.7645 \cdot 10^{2}$	$9.72 \cdot 10^{-1}$	$4.33 \cdot 10^{-18}$	6
$1.0 \cdot 10^{-5}$	$2.0624 \cdot 10^{2}$	77.51	$1.2656 \cdot 10^{3}$	$1.6492 \cdot 10^{2}$	$9.97 \cdot 10^{-1}$	$6.93 \cdot 10^{-18}$	6
$1.0 \cdot 10^{-6}$	$2.0619 \cdot 10^{2}$	77.51	$1.2577 \cdot 10^{3}$	$1.7213 \cdot 10^{2}$	$9.99 \cdot 10^{-1}$	$6.11 \cdot 10^{-18}$	6

(*) Stopped at iteration 41

Next table 2 illustrates some characteristics of the scenarios for different values of π . Columns *s* and *A* show the scenario number and its associated link. Columns p_s show the probability distributions for the corresponding probability of service failure $\pi = 0.05$, 0.001, 0.0001, whereas the corresponding columns f_{PT} display the fraction of trips using public transport. Using a high probability of failure of a service, $\pi = 0.05$, i.e., one disruption each twenty services, the most likely scenario is not the scenario 0 but a scenario under disruption (scenario 8 as shown in table 2) and once the probability π is below a given threshold, the no disruption scenario becomes the most likely situation. In our test example this seems to happen for $\pi \approx 10^{-3}$. The tests also show that for $\pi < 0.001$, the topology of the designed network does not change, i.e. it is *as if* the failure scenarios would not need to be taken into account in the design of the transportation system. Notice that this is in practice achieved when $\pi = 10^{-6}$, where disruption scenarios have almost no relevance in the model. Finally, the fractions of public transportation usage showed in columns f_{PT} for the different disruption scenarios show good recoverability characteristics of the model since the level of usage of the RTN remain relatively high in the disruption scenarios.

Table 2. Probabilities for the disruption scenarios and fractions of public transport usage for π=0.05, 0.001, 0.0001.

s	A	p_s ($\pi = 0.05$)	f_{PT}	p_s ($\pi = 10^{-3}$)	f_{PT}	p_s ($\pi = 10^{-4}$)	f_{PT}
0	$--$	$3.13 \cdot 10^{-2}$	$7.92 \cdot 10^{-1}$	$7.72 \cdot 10^{-1}$	$9.25 \cdot 10^{-1}$	$9.71 \cdot 10^{-1}$	$9.25 \cdot 10^{-1}$
1	(9,1)	$2.80 \cdot 10^{-3}$	$7.92 \cdot 10^{-1}$	$\mathbf{0.00}$(†)	$--$	0.00	$--$
2	(1,2)	$8.67 \cdot 10^{-2}$	$7.22 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$8.20 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$8.20 \cdot 10^{-1}$
3	(9,3)	$8.67 \cdot 10^{-2}$	$5.98 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.81 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.81 \cdot 10^{-1}$
4	(1,3)	$9.61 \cdot 10^{-2}$	$6.49 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.45 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.45 \cdot 10^{-1}$
5	(2,3)	$2.80 \cdot 10^{-3}$	$7.92 \cdot 10^{-1}$	0.00	$--$	0.00	$--$
6	(2,4)	$8.67 \cdot 10^{-2}$	$7.22 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.75 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.75 \cdot 10^{-1}$
7	(3,4)	$7.13 \cdot 10^{-2}$	$7.13 \cdot 10^{-1}$	0.00	$--$	0.00	$--$
8	(3,5)	$1.61 \cdot 10^{-1}$	$6.27 \cdot 10^{-1}$	$4.61 \cdot 10^{-2}$	$6.50 \cdot 10^{-1}$	$5.65 \cdot 10^{-3}$	$6.50 \cdot 10^{-1}$
9	(4,5)	$1.62 \cdot 10^{-2}$	$7.92 \cdot 10^{-1}$	0.00	$--$	0.00	$--$
10	(5,6)	$1.53 \cdot 10^{-1}$	$7.27 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.08 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.08 \cdot 10^{-1}$
11	(4,6)	$8.67 \cdot 10^{-2}$	$7.04 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.74 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.74 \cdot 10^{-1}$
12	(4,8)	$8.07 \cdot 10^{-2}$	$6.42 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$8.56 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$8.56 \cdot 10^{-1}$
13	(6,8)	$1.62 \cdot 10^{-2}$	$7.92 \cdot 10^{-1}$	0.00	$--$	0.00	$--$
14	(5,7)	$1.81 \cdot 10^{-2}$	$7.92 \cdot 10^{-1}$	$2.27 \cdot 10^{-2}$	$7.66 \cdot 10^{-1}$	$2.82 \cdot 10^{-3}$	$7.66 \cdot 10^{-1}$
15	(6,7)	$2.80 \cdot 10^{-3}$	$7.92 \cdot 10^{-1}$	0.00	$--$	0.00	$--$

((†) $p_s = 0.00$ correspond to network links not included in the solution of the RTN model)

6. Conclusions

A simplified two-stage stochastic model has been developed for the design of rapid transit systems which is consistent with a probability distribution model for the disruption scenarios that arise as a consequence of failures in the transportation unit services. The model is formulated as a bilevel programming model solved heuristically. The heuristic method has shown to be effective for realistic values of the probability π of service failure observed in practice in railway networks. The probability of service failure π seems to be the most influencing parameter in the model, being possible to detect the threshold of values for π for which the system has good reliability and recoverability characteristics. The probability model permits to evaluate realistic weights of the disruption scenarios being considered in the design and, consequently, not incurr in excessive costs derived by extremely conservative solutions. Without loss of generality the model can be extended to include other sources of disruptions.

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