Universidad Politécnica de Madrid
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COHERENT STRUCTURES IN STATISTICALLY-STATIONARY HOMOGENEOUS SHEAR TURBULENCE

Tesis Doctoral

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Why?

Javier Jiménez

We must know-we will know!

David Hilbert

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Nomenclature

Mathematical operators

$\widehat{(\cdot)}$	Variable in Fourier space
$(\cdot)^+$	Variable in wall-units, normalized using u_τ and ν
$(\cdot)'$	Root mean square of $(\cdot)$
$\langle(\cdot)\rangle$	Average of $(\cdot)$ in time and in the homogeneous spatial directions
$\overline{(\cdot)}$	Average of $(\cdot)$ in the homogeneous spatial directions
$\overline{(\cdot)}_{xz}$	Average of $(\cdot)$ in the horizontal plane
$\langle\cdot\rangle_b$	Time-average over the branch lifetime of $(\cdot)$
∇^2	Laplacian operator

Greek symbols

α	Thresholding parameter for vortex clusters
Δt_s	Time step between contiguous snapshots to track structures in time
Δt	Time step determined by CFL condition
Δ_x	Streamwise length of the coherent structure
Δx	Grid size in the streamwise direction
Δ_y	Vertical length of the coherent structure
Δy	Grid size in the vertical direction
Δ_z	Spanwise length of the coherent structure
Δz	Grid size in the spanwise direction
ε	Kinetic-energy dissipation rate
η	Kolmogorov length scale
κ	von Kármán constant
λ	Taylor microscale
λ_x	Wavelength in the streamwise direction
λ_z	Wavelength in the spanwise direction

ν	Kinematic viscosity
ρ	Density of the fluid
ρ_g	Density of holes per unit volume in a structure
τ	Tangential Reynolds stress
τ_m	Average tangential Reynolds stress of the flow within the coherent structure
Π	The second invariant of the velocity gradient tensor

Roman symbols

A_{xz}	Streamwise aspect ratio of the simulation box
A_{yz}	Vertical aspect ratio of the simulation box
D	Discriminant of the velocity gradient tensor
d	Diagonal length of the circumscribed box to a structure
D_f	Fractal dimension of coherent structures
E_{uv}	Cospectrum density of $u-v$
$E_{\omega\omega}$	Spectrum density of enstrophy
g	Genus of a coherent structure
h	Channel half-height
H	Thresholding parameter for Qs
k	Turbulent kinetic energy
k_x	Streamwise wavenumber
k_z	Spanwise wavenumber
L_c	Corrsin's length scale
L_x	Streamwise length of the simulation box
L_y	Vertical length of the simulation box
L_z	Spanwise length of the simulation box
L_ε	Integral length scale
N_Q	Numbers of Q ⁻ s
N_C	Numbers of vortex clusters
N_x	Number of collocation points in the streamwise direction
N_y	Number of collocation points in the vertical direction
N_z	Number of collocation points in the spanwise direction
p	Pressure
q	Magnitude of the velocity fluctuations
r_1	Inner length of the coherent structures or estimated thickness

r_2	Intermediate length of the coherent structures
r_3	Outer length of the coherent structures
Re	Reynolds number
Re_z	Reynolds number based on S and L_z
Re_τ	Friction Reynolds number, $Re_\tau = h^+$
Re_λ	Taylor-microscale Reynolds number
S	Mean shear, $\partial_y U$
S^*	Shear parameter
t	Time
t_η	Kolmogorov time-scale
T	Lifetime of coherent structures
T_s	Box period, $T_s = L_x / SL_y$
u	Fluctuating velocity in the streamwise direction
u_m	Average streamwise velocity of the flow within the coherent structure
u_η	Kolmogorov velocity-scale
u_τ	Friction velocity
U	Mean streamwise velocity profile
v	Fluctuating velocity in the vertical direction
v_m	Average vertical velocity of the flow within the coherent structure
V	Volume of a coherent structure
V_o	Overlapped volume between structures at contiguous times
V_b	Time-averaged volume of the structures over the branch lifetime
w	Fluctuating velocity in the spanwise direction
x	Streamwise direction
x_1	Streamwise direction
x_2	Vertical or wall-normal direction
x_3	Spanwise direction
$\mathbf{x}$	Position vector containing x , y and z
y	Vertical or wall-normal direction
y_{min}	Minimum distance to the wall of a structure
y_{max}	Maximum distance to the wall of a structure
y_c	Center of gravity of a structure, $(y_{min} + y_{max})/2$
z	Spanwise direction

Abbreviations

HST	Homogeneous shear turbulence
SS-HST	Statistically-stationary homogeneous shear turbulence
CFL	Courant-Friedrichs-Lewy number
DNS	Direct Numerical Simulation
LES	Large Eddy Simulation
p.d.f.	Probability density function
j.p.d.f.	Joint probability density function
Q1	Outward interaction
Q2	Ejections
Q3	Inward interaction
Q4	Sweeps
Qs	Coherent structures based on uv
Q^- s	‘Gradient’ structures Q2 and Q4
Q^+ s	‘Counter-gradient’ structures Q1 and Q3
r.m.s.	Root-mean-square

Resumen

La región cerca de la pared de flujos turbulentos de pared ya está bien conocido debido a su bajo número de Reynolds local y la separación escala estrecha. La región lejos de la pared (capa externa) no es tan interesante tampoco, ya que las estadísticas allí se escalan bien por las unidades exteriores. La región intermedia (capa logarítmica), sin embargo, ha estado recibiendo cada vez más atención debido a su propiedad auto-similares. Además, de acuerdo a Flores *et al.* (2007) y Flores & Jiménez (2010), la capa logarítmica es más o menos independiente de otras capas, lo que implica que podría ser inspeccionado mediante el aislamiento de otras dos capas, lo que reduciría significativamente los costes computacionales para la simulación de flujos turbulentos de pared. Algunos intentos se trataron después por Mizuno & Jiménez (2013), quien simulan la capa logarítmica sin la región cerca de la pared con estadísticas obtenidas de acuerdo razonablemente bien con los de las simulaciones completas. Lo que más, la capa logarítmica podría ser imitado por otra turbulencia sencillo de cizallamiento de motor. Por ejemplo, Pumir (1996) encontró que la turbulencia de cizallamiento homogéneo estadísticamente estacionario (SS-HST) también irrumpe, de una manera muy similar al proceso de auto-sostenible en flujos turbulentos de pared.

Según los consideraciones arriba, esta tesis trata de desvelar en qué medida es la capa logarítmica de canales similares a la turbulencia de cizalla más sencillo, SS-HST, mediante la comparación de ambos cinemática y la dinámica de las estructuras coherentes en los dos flujos. Resultados sobre el canal se muestran mediante Lozano-Durán *et al.* (2012) y Lozano-Durán & Jiménez (2014b). La hoja de ruta de esta tarea se divide en tres etapas.

En primer lugar, SS-HST es investigada por medio de un código nuevo de simulación numérica directa, espectral en las dos direcciones horizontales y compacto-diferencias finitas en la dirección de la cizalla. Sin utiliza remallado para imponer la condición de borde cortante periódica. La influencia de la geometría de la caja computacional se explora. Ya que el HST no tiene ninguna longitud característica externa y tiende a llenar el dominio computacional, las simulaciones a largo plazo del HST son 'mínimos' en el sentido de que contiene sólo unas pocas estructuras media a gran escala. Se ha encontrado que el

límite principal es el ancho de la caja de la envergadura, L_z , que establece las escalas de longitud y velocidad de la turbulencia, y que las otras dos dimensiones de la caja debe ser suficientemente grande ($L_x \gtrsim 2L_z$, $L_y \gtrsim L_z$) para evitar que otras direcciones estando limitado también. También se encontró que las cajas de gran longitud, $L_x \gtrsim 2L_y$, par con el paso del tiempo la condición de borde cortante periódica, y desarrollar fuertes ráfagas linealizadas no físicos. Dentro de estos límites, el flujo muestra similitudes y diferencias interesantes con otros flujos de cizalla, y, en particular, con la capa logarítmica de flujos turbulentos de pared. Ellos son exploradas con cierto detalle. Incluyen un proceso auto-sostenido de rayas a gran escala y con una explosión cuasi-periódica. La escala de tiempo de ruptura es de aproximadamente universales, $\sim 20S^{-1}$ (S es la velocidad de cizallamiento media), y la disponibilidad de dos sistemas de ruptura diferentes permite el crecimiento de las ráfagas a estar relacionado con algo de confianza a la cizalladura de turbulencia inicialmente isotrópico. Se concluye que la SS-HST, llevado a cabo dentro de los parámetros de cálculo apropiados, es un sistema muy prometedor para estudiar la turbulencia de cizallamiento en general.

En segundo lugar, las mismas estructuras coherentes como en los canales estudiados por Lozano-Durán *et al.* (2012), es decir, grupos de vórticidad (fuerte disipación) y Qs (fuerte tensión de Reynolds tangencial, $-uv$) tridimensionales, se estudia mediante simulación numérica directa de SS-HST con relaciones de aspecto de cuadro aceptables y número de Reynolds hasta $Re_\lambda \approx 250$ (basado en Taylor-microescala). Se discute la influencia de la intermitencia de umbral independiente del tiempo. Estas estructuras tienen alargamientos similares en la dirección sentido de la corriente a las familias separadas en los canales hasta que son de tamaño comparable a la caja. Sus dimensiones fractales, longitudes interior y exterior como una función del volumen concuerdan bien con sus homólogos de canales. El estudio sobre sus organizaciones espaciales encontró que Qs del mismo tipo están alineados aproximadamente en la dirección del vector de velocidad en el cuadrante al que pertenecen, mientras Qs de diferentes tipos están restringidos por el hecho de que no debe haber ningún choque de velocidad, lo que hace Q2s (eyecciones, $u < 0, v > 0$) y Q4s (sweeps, $u > 0, v < 0$) emparejado en la dirección de la envergadura. Esto se verifica mediante la inspección de estructuras de velocidad, otros cuadrantes como la uw y vw en SS-HST y las familias separadas en el canal. La alineación sentido de la corriente de Qs ligada a la pared con el mismo tipo en los canales se debe a la modulación de la pared. El campo de flujo medio condicionado a pares Q2-Q4 encontró que los grupos de vórticidad están en el medio de los dos, pero prefieren los dos cizalla capas alojamiento en la parte superior e inferior de Q2s y Q4s respectivamente, lo que hace que la vortici-

dad envergadura dentro de los grupos de vórticidad hace no cancele. La pared amplifica la diferencia entre los tamaños de baja- y alta-velocidad rayas asociados con parejas de Q2-Q4 se adjuntan como los pares alcanzan cerca de la pared, el cual es verificado por la correlación de la velocidad del sentido de la corriente condicionado a Q2s adjuntos y Q4s con diferentes alturas. Grupos de vórticidad en SS-HST asociados con Q2s o Q4s también están flanqueadas por un contador de rotación de los vórtices sentido de la corriente en la dirección de la envergadura como en el canal. La larga 'despertar' cónica se origina a partir de los altos grupos de vórticidad ligada a la pared han encontrado los del Álamo *et al.* (2006) y Flores *et al.* (2007), que desaparece en SS-HST, sólo es cierto para altos grupos de vórticidad ligada a la pared asociados con Q2s pero no para aquellos asociados con Q4s, cuyo campo de flujo promedio es en realidad muy similar a la de SS-HST.

En tercer lugar, las evoluciones temporales de Qs y grupos de vórticidad se estudian mediante el uso de la método inventado por Lozano-Durán & Jiménez (2014b). Las estructuras se clasifican en las ramas, que se organizan más en los gráficos. Ambas resoluciones espaciales y temporales se eligen para ser capaz de capturar el longitud y el tiempo de Kolmogorov puntual más probable en el momento más extrema. Debido al efecto caja mínima, sólo hay un gráfico principal consiste en casi todas las ramas, con su volumen y el número de estructuras instantáneo seguien la energía cinética y enstrofia intermitente. La vida de las ramas, lo que tiene más sentido para las ramas primarias, pierde su significado en el SS-HST debido a las aportaciones de ramas primarias al total de Reynolds estrés o enstrofia son casi insignificantes. Esto también es cierto en la capa exterior de los canales. En cambio, la vida de los gráficos en los canales se compara con el tiempo de ruptura en SS-HST. Grupos de vórticidad están asociados con casi el mismo cuadrante en términos de sus velocidades medias durante su tiempo de vida, especialmente para los relacionados con las eyecciones y sweeps. Al igual que en los canales, las eyecciones de SS-HST se mueven hacia arriba con una velocidad promedio vertical u_τ (velocidad de fricción) mientras que lo contrario es cierto para los barridos. Grupos de vórticidad, por otra parte, son casi inmovil en la dirección vertical. En la dirección de sentido de la corriente, que están advección por la velocidad media local y por lo tanto deforman por la diferencia de velocidad media. Sweeps y eyecciones se mueven más rápido y más lento que la velocidad media, respectivamente, tanto por $1.5u_\tau$. Grupos de vórticidad se mueven con la misma velocidad que la velocidad media. Se verifica que las estructuras incoherentes cerca de la pared se debe a la pared en vez de pequeño tamaño.

Los resultados sugieren fuertemente que las estructuras coherentes en canales no son especialmente asociado con la pared, o incluso con un perfil de cizalladura dado.

Summary

Since the wall-bounded turbulence was first recognized more than one century ago, its near wall region (buffer layer) has been studied extensively and becomes relatively well understood due to the low local Reynolds number and narrow scale separation. The region just above the buffer layer, i.e., the logarithmic layer, is receiving increasingly more attention nowadays due to its self-similar property. Flores *et al.* (2007b) and Flores & Jiménez (2010) show that the statistics of logarithmic layer is kind of independent of other layers, implying that it might be possible to study it separately, which would reduce significantly the computational costs for simulations of the logarithmic layer. Some attempts were tried later by Mizuno & Jimenez (2013), who simulated the logarithmic layer without the buffer layer with obtained statistics agree reasonably well with those of full simulations. Besides, the logarithmic layer might be mimicked by other simpler shear-driven turbulence. For example, Pumir (1996) found that the statistically-stationary homogeneous shear turbulence (SS-HST) also bursts, in a manner strikingly similar to the self-sustaining process in wall-bounded turbulence.

Based on these considerations, this thesis tries to reveal to what extent is the logarithmic layer of channels similar to the simplest shear-driven turbulence, SS-HST, by comparing both kinematics and dynamics of coherent structures in the two flows. Results about the channel are shown by Lozano-Durán *et al.* (2012) and Lozano-Durán & Jiménez (2014b). The roadmap of this task is divided into three stages.

First, SS-HST is investigated by means of a new direct numerical simulation code, spectral in the two horizontal directions and compact-finite-differences in the direction of the shear. No remeshing is used to impose the shear-periodic boundary condition. The influence of the geometry of the computational box is explored. Since HST has no characteristic outer length scale and tends to fill the computational domain, long-term simulations of HST are ‘minimal’ in the sense of containing on average only a few large-scale structures. It is found that the main limit is the spanwise box width, L_z , which sets the length and velocity scales of the turbulence, and that the two other box dimensions should be sufficiently large ($L_x \gtrsim 2L_z$, $L_y \gtrsim L_z$) to prevent other directions

to be constrained as well. It is also found that very long boxes, $L_x \gtrsim 2L_y$, couple with the passing period of the shear-periodic boundary condition, and develop strong unphysical linearized bursts. Within those limits, the flow shows interesting similarities and differences with other shear flows, and in particular with the logarithmic layer of wall-bounded turbulence. They are explored in some detail. They include a self-sustaining process for large-scale streaks and quasi-periodic bursting. The bursting time scale is approximately universal, $\sim 20S^{-1}$ (S is the mean shear rate), and the availability of two different bursting systems allows the growth of the bursts to be related with some confidence to the shearing of initially isotropic turbulence. It is concluded that SS-HST, conducted within the proper computational parameters, is a very promising system to study shear turbulence in general.

Second, the same coherent structures as in channels studied by [Lozano-Durán et al. \(2012\)](#), namely three-dimensional vortex clusters (strong dissipation) and Qs (strong tangential Reynolds stress, $-uv$), are studied by direct numerical simulation of SS-HST with acceptable box aspect ratios and Reynolds number up to $Re_\lambda \approx 250$ (based on Taylor-microscale). The influence of the intermittency to time-independent threshold is discussed. These structures have similar elongations in the streamwise direction to detached families in channels until they are of comparable size to the box. Their fractal dimensions, inner and outer lengths as a function of volume agree well with their counterparts in channels. The study about their spatial organizations found that Qs of the same type are aligned roughly in the direction of the velocity vector in the quadrant they belong to, while Qs of different types are restricted by the fact that there should be no velocity clash, which makes Q2s (ejections, $u < 0$, $v > 0$) and Q4s (sweeps, $u > 0$, $v < 0$) paired in the spanwise direction. This is verified by inspecting velocity structures, other quadrants such as $u-w$ and $v-w$ in SS-HST and also detached families in the channel. The streamwise alignment of attached Qs with the same type in channels is due to the modulation of the wall. The average flow field conditioned to Q2–Q4 pairs found that vortex clusters are in the middle of the pair, but prefer to the two shear layers lodging at the top and bottom of Q2s and Q4s respectively, which makes the spanwise vorticity inside vortex clusters does not cancel. The wall amplifies the difference between the sizes of low- and high-speed streaks associated with attached Q2–Q4 pairs as the pairs reach closer to the wall, which is verified by the correlation of streamwise velocity conditioned to attached Q2s and Q4s with different heights. Vortex clusters in SS-HST associated with Q2s or Q4s are also flanked by a counter rotating streamwise vortices in the spanwise direction as in the channel. The long conical ‘wake’ originates from tall attached vortex clusters

found by [del Álamo *et al.* \(2006\)](#) and [Flores *et al.* \(2007b\)](#), which disappears in SS-HST, is only true for tall attached vortices associated with Q2s but not for those associated with Q4s, whose averaged flow field is actually quite similar to that in SS-HST.

Third, the temporal evolutions of Qs and vortex clusters are studied by using the method invented by [Lozano-Durán & Jiménez \(2014b\)](#). Structures are sorted into branches, which are further organized into graphs. Both spatial and temporal resolutions are chosen to be able to capture the most probable pointwise Kolmogorov length and time at the most extreme moment. Due to the minimal box effect, there is only one main graph consist by almost all the branches, with its instantaneous volume and number of structures follow the intermittent kinetic energy and enstrophy. The lifetime of branches, which makes more sense for primary branches, loses its meaning in SS-HST because the contributions of primary branches to total Reynolds stress or enstrophy are almost negligible. This is also true in the outer layer of channels. Instead, the lifetime of graphs in channels are compared with the bursting time in SS-HST. Vortex clusters are associated with almost the same quadrant in terms of their mean velocities during their life time, especially for those related with ejections and sweeps. As in channels, ejections in SS-HST move upwards with an average vertical velocity u_τ (friction velocity) while the opposite is true for sweeps. Vortex clusters, on the other hand, are almost still in the vertical direction. In the streamwise direction, they are advected by the local mean velocity and thus deformed by the mean velocity difference. Sweeps and ejections move faster and slower than the mean velocity respectively, both by $1.5u_\tau$. Vortex clusters move with the same speed as the mean velocity. It is verified that the incoherent structures near the wall is due to the wall instead of small size.

The results suggest that coherent structures in channels are not particularly associated with the wall, or even with a given shear profile.

Chapter 1

Introduction

Shear-driven turbulence is one of the most ubiquitous flow patterns in our daily life. Typical examples include wall-bounded turbulence like turbulent channels, pipes and boundary layers and free shear turbulence like turbulent jets, wakes and mixing layers. The key feature is the mean shear due to the mean-velocity difference. Figure 1.1(a-c) show three typical shear-driven turbulence with different length scales. Turbulent state is preferred when transporting different petroleum products one after another in a single pipeline for less mixing. The atmosphere turbulent boundary layer is the key consideration for the optimal performance of wind farm design. The accretion disk in the universe can be approximated by homogeneous shear turbulence (HST) with rotation.

Figure 1.1: Typical shear-driven turbulence.

In engineering applications, often other phenomena like combustion, acoustics, and heat convection also take place, leading to a unprecedented complex system. However, even the shear-driven turbulence itself is not totally understood yet since it was first recognized more than one century ago by [Reynolds \(1894\)](#), especially the turbulent boundary layer, which is of crucial importance to aeronautic, auto and shipbuilding industries, etc.. The present thesis focuses on the simplest shear-driven turbulence, HST, to address the effect of the mean shear by comparing it with channel turbulence. Some basic facts about HST are introduced first in this section.

1.1 Homogeneous shear turbulence, idealized model

Throughout this thesis, the velocity $\tilde{\mathbf{u}}$ and vorticity $\tilde{\boldsymbol{\omega}}$ of incompressible turbulence are expressed respectively as fluctuations $\mathbf{u}$ and $\boldsymbol{\omega}$ plus ensemble averages $\mathbf{U} = \langle \tilde{\mathbf{u}} \rangle$ and $\mathbf{\Omega} = \langle \tilde{\boldsymbol{\omega}} \rangle$ implemented here as spatial (homogeneous directions) and temporal averaging, so that $\tilde{\mathbf{u}} = \mathbf{U} + \mathbf{u}$ and $\tilde{\boldsymbol{\omega}} = \nabla \times \tilde{\mathbf{u}} = \mathbf{\Omega} + \boldsymbol{\omega}$. The streamwise, vertical and spanwise directions are, respectively, $(x, y, z) = (x_1, x_2, x_3)$ and the corresponding fluctuating velocity and vorticity components are $(u, v, w) = (u_1, u_2, u_3)$ and $(\omega_x, \omega_y, \omega_z) = (\omega_1, \omega_2, \omega_3)$. Mean velocity and vorticity components are denoted by capital letters. The time-dependent average over the homogeneous directions of a fluctuating quantity ϕ is denoted by $\bar{\phi}$ and root-mean-squared (rms) intensities are denoted by primes, as in $\phi' = \langle \phi^2 \rangle^{1/2}$. For channels, the buffer, logarithmic and outer layers are defined as $y^+ < 100$, $100\nu/u_\tau < y < 0.2h$ and $y > 0.2h$, respectively, where h is the half-height, ν is the kinematic viscosity and u_τ is the friction velocity. u_τ is defined as $u_\tau^2 = |\nu S - \langle uv \rangle|$ for HST, and as $u_\tau^2 = \nu |S_w|$ for channels, where S_w is the mean shear at the wall. Quantities normalized by ν and u_τ are denoted by '+'. We will use the Taylor-microscale Reynolds number $Re_\lambda = q^2 \sqrt{5/3} \nu \varepsilon$ for both HST and channels, for the latter $Re_\tau = u_\tau h / \nu$ is also used, where $q^2 = \langle u^2 + v^2 + w^2 \rangle$ is twice the kinetic energy and $\varepsilon = \nu \langle \omega_1^2 + \omega_2^2 + \omega_3^2 \rangle$ is the dissipation.

HST has a constant mean velocity gradient (mean shear S) in one direction and its statistics are not functions of space (homogeneity). It does not exist in reality because none of real shear-driven turbulence is homogeneous nor with constant mean shear.

Figure 1.2: Sketch of homogeneous shear turbulence.

Figure 1.2 shows the sketch of HST studied in the present thesis. The mean velocity gradient is perpendicular to the streamwise direction so that $\mathbf{U} = (Sy, 0, 0)$. Consequently, only the spanwise vorticity has non-zero mean, i.e., $\mathbf{\Omega} = (0, 0, -S)$. Its governing equations are

$$\left(\frac{\partial}{\partial t} + Sx_2 \frac{\partial}{\partial x_1}\right)u_i = -Su_2\delta_{i1} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} - \frac{\partial(u_i u_j)}{\partial x_j} + \nu \nabla^2 u_i, \quad (1.1)$$

and

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (1.2)$$

where δ_{ij} is the *Kronecker delta*, ρ is density and p is pressure. Unless stated specially, repeated indices imply summation over 1...3. By taking the curl of (1.1), we can get the evolution equation for vorticity given by

$$\frac{\partial \omega_i}{\partial t} = -Sx_2 \frac{\partial \omega_i}{\partial x_1} - u_k \frac{\partial \omega_i}{\partial x_k} + S\omega_2\delta_{i1} - S \frac{\partial u_i}{\partial x_3} + \omega_k \frac{\partial u_i}{\partial x_k} + \nu \frac{\partial^2 \omega_i}{\partial x_k \partial x_k}. \quad (1.3)$$

1.1.1 Short time shearing of isotropic turbulence

HST is intrinsically infinite, since there is no length scale to constrain its development. Most of the studies about HST, both numerical simulations and experiments, intend to retain this property through short time shearing of isotropic turbulence, when structures are still ‘free’ of the computational domain or experimental facilities. This state is discussed below.

Based on (1.1) we can obtain the transportation equation for Reynolds stresses $\overline{u_i u_j}$ by adding equations

$$u_j \left(\frac{\partial u_i}{\partial t} + Sy \frac{\partial u_i}{\partial x} \right) = -Su_2 u_j \delta_{i1} - u_j \frac{1}{\rho} \frac{\partial p}{\partial x_i} - u_j u_k \frac{\partial u_i}{\partial x_k} + \nu u_j \nabla^2 u_i, \quad (1.4)$$

and

$$u_i \left(\frac{\partial u_j}{\partial t} + Sy \frac{\partial u_j}{\partial x} \right) = -Su_2 u_i \delta_{j1} - u_i \frac{1}{\rho} \frac{\partial p}{\partial x_j} - u_i u_k \frac{\partial u_j}{\partial x_k} + \nu u_i \nabla^2 u_j, \quad (1.5)$$

together, which yields

$$\begin{aligned} & \frac{\partial u_i u_j}{\partial t} + Sy \frac{\partial u_i u_j}{\partial x} = \\ & -Su_2(u_i \delta_{j1} + u_j \delta_{i1}) - \frac{1}{\rho} \left(u_i \frac{\partial p}{\partial x_j} + u_j \frac{\partial p}{\partial x_i} \right) - \frac{\partial u_i u_j u_k}{\partial x_k} + \nu (u_i \nabla^2 u_j + u_j \nabla^2 u_i). \end{aligned} \quad (1.6)$$

Consider the homogeneity, i.e., $\overline{\partial \bullet / \partial x_i} = 0$, and implement the spatial average, we can have

$$\overline{\frac{\partial u_i u_j}{\partial t} + Sy \frac{\partial u_i u_j}{\partial x}} = \frac{d \overline{u_i u_j}}{dt}, \quad (1.7)$$

$$\frac{1}{\rho} \overline{u_i \frac{\partial p}{\partial x_j}} + u_j \frac{\partial p}{\partial x_i} = \frac{1}{\rho} \overline{\frac{\partial u_i p}{\partial x_j}} + \frac{\partial u_j p}{\partial x_i} - p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = -\frac{1}{\rho} \overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} = -\overline{\mathcal{R}_{ij}}, \quad (1.8)$$

and

$$\begin{aligned} \overline{\nu u_i \nabla^2 u_j + u_j \nabla^2 u_i} &= \overline{\nu u_i \frac{\partial}{\partial x_k} \left(\frac{\partial u_j}{\partial x_k} \right) + u_j \frac{\partial}{\partial x_k} \left(\frac{\partial u_i}{\partial x_k} \right)} = \\ &= \nu \overline{\frac{\partial}{\partial x_k} \left(u_i \frac{\partial u_j}{\partial x_k} \right) + \frac{\partial}{\partial x_k} \left(u_j \frac{\partial u_i}{\partial x_k} \right)} - 2 \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} = -2\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} = -2\overline{\varepsilon_{ij}}, \end{aligned} \quad (1.9)$$

where $\mathcal{R}_{ij}$ is the pressure redistribution term and ε_{ij} is dissipation. So (1.6) turns to be

$$\frac{d\overline{u_i u_j}}{dt} = -S \overline{u_2 (u_i \delta_{j1} + u_j \delta_{i1})} + \overline{\mathcal{R}_{ij}} - 2\overline{\varepsilon_{ij}}. \quad (1.10)$$

The equation for the evolution of the instantaneous spatial mean kinematic energy $\overline{k}(t) = \overline{u_i u_i}/2$ can be easily obtained from the above as

$$\frac{d\overline{k}}{dt} = \underbrace{\overline{\mathcal{P}}}_{\text{Production}} - \underbrace{\overline{\varepsilon}}_{\text{Dissipation}}, \quad (1.11)$$

where

$$\overline{\mathcal{P}} = -S \overline{u_1 u_2} \quad (1.12)$$

is the kinetic energy production term and $\overline{\varepsilon} = \overline{\varepsilon_{ii}}$. The sum of the term with pressure is cancelled due to continuity for incompressible flows.

(1.11) can also be written as

$$\frac{\tau}{\overline{k}} \frac{d\overline{k}}{dt} = \frac{\overline{\mathcal{P}}}{\overline{\varepsilon}} - 1, \quad (1.13)$$

where $\tau = \overline{k}/\overline{\varepsilon}$ is the characteristic time for kinetic energy to be transferred from large scales to small scales then get dissipated. The initial shearing of isotropic turbulence is self-similar (Pope, 2000; Cambon & Sagaut, 2008), with statistics independent on time when normalized by the mean shear rate S and kinetic energy $\overline{k}$. So that τ , $\overline{\mathcal{P}} \sim \overline{k}$ and $\overline{\varepsilon} \sim S\overline{k}$ are roughly constant, which leads to

$$\overline{k}(t) = \overline{k}(0) \exp\left[\frac{t}{\tau} \left(\frac{\overline{\mathcal{P}}}{\overline{\varepsilon}} - 1\right)\right]. \quad (1.14)$$

If the initial condition is chosen to be $\overline{\mathcal{P}} > \overline{\varepsilon}$, kinetic energy will increase exponentially, so do the instantaneous integral length $L_\varepsilon(t) = (2\overline{k}/3)^{3/2}/\overline{\varepsilon} = (2/3)^{3/2} \overline{k}^{1/2} \tau$. Note that (1.14) is based on statistics after simulations or experiments so that it cannot be used to predict the statistics in advance.

The equation for $\overline{\omega_i \omega_j}$ (Mathieu & Scott, 2000), however, is much more complicated and not so straightforward as (1.10). In order to obtain the equation for the evolution

of instantaneous enstrophy $\overline{\omega_i \omega_i}$, we start directly from the equation of $\tilde{\omega}^2 = \tilde{\omega}_i \tilde{\omega}_i$ (Pope, 2000), which is

$$\frac{\partial \tilde{\omega}^2}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\omega}^2}{\partial x_j} = \nu \nabla^2 \tilde{\omega}^2 + 2\tilde{\omega}_i \tilde{\omega}_j \frac{\partial \tilde{u}_i}{\partial x_j} - 2\nu \frac{\partial \tilde{\omega}_i}{\partial x_j} \frac{\partial \tilde{\omega}_i}{\partial x_j}. \quad (1.15)$$

In the situation of very high Reynolds number, the mean vorticity is negligible compared with its fluctuation, i.e., $\Omega_i/\omega_i \approx 0$. In this way, we can omit Ω_i in the above equation so that we get

$$\frac{\partial \omega^2}{\partial t} + \tilde{u}_j \frac{\partial \omega^2}{\partial x_j} = \nu \nabla^2 \omega^2 + 2\omega_i \omega_j \frac{\partial \tilde{u}_i}{\partial x_j} - 2\nu \frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}. \quad (1.16)$$

Similarly, take the spatial average of the above equation and take homogeneity into account, we obtain

$$\overline{\tilde{u}_j \frac{\partial \omega^2}{\partial x_j}} = \overline{\frac{\partial \tilde{u}_j \omega^2}{\partial x_j}} - \omega^2 \overline{\frac{\partial \tilde{u}_j}{\partial x_j}} = 0, \quad (1.17)$$

and

$$\overline{\nu \nabla^2 \omega^2} = \nu \overline{\frac{\partial}{\partial x_i} \left(\frac{\omega^2}{\partial x_i} \right)} = 0, \quad (1.18)$$

which lead to

$$\frac{d\overline{\omega^2}}{dt} = \underbrace{2\overline{S\omega_1\omega_2}}_{\text{Production}} + \underbrace{2\overline{\omega_i s_{ij}\omega_j}}_{\text{Stretching}} - \underbrace{2\nu \overline{\frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}}}_{\text{Destruction}}, \quad (1.19)$$

where

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (1.20)$$

is the *rate-of-strain* tensor. Note that (1.19) can also be obtained with similar process by multiplying (1.3) with ω_i and assume that $-S/\omega_3 \approx 0$. If $-S \sim \omega_3$ there will be another term $-\overline{S\omega_i \partial u_i / \partial x_3}$ on the right hand side. $\overline{\omega_1 \omega_2}$ works similarly as $-\overline{uv}$ and statistically it is positive (Isaza & Collins, 2009). $\overline{\omega_i s_{ij} \omega_j}$ is the stretching effect and its sign depends on the orientation of vortices. Since $\bar{\varepsilon} \sim S\bar{k}$, $\bar{\varepsilon}$ will also grow exponentially as kinetic energy, so does enstrophy since $\bar{\varepsilon} = \nu \bar{\omega}^2$ in HST. It means that the enstrophy production and vortex stretching effect are dominant over the destruction. This makes the instantaneous Kolmogorov scale $\eta(t) = (\nu^3/\bar{\varepsilon})^{1/4}$ keep decreasing, resulting in wider and wider scale separation $L_\varepsilon(t)/\eta(t)$.

Figures 1.3(a-b) show the time histories of kinetic energy and dissipation for short time shearing of isotropic turbulence from both numerical simulations and experiments. Both of them grow exponentially with time after the transitional period and the kinetic energy grows faster than the dissipation. In fact, the growth rates are not so different with different initial conditions.

Figure 1.3: Time histories of kinetic energy and dissipation for short time shearing of isotropic turbulence from both experiments including Vanderwel & Tavoularis (2011) (★) and Tavoularis & Corrsin (1981) (×) and numerical simulations including Gerz *et al.* (1989) (—), Rogers & Moin (1987) (○ and □ with the mean shear rates $S = 28.284$ and 56.568 respectively), Sarkar (1995) (◇) and Isaza & Collins (2009) (+, ▽ and △ from with initial shear parameter $S_0^* = 3, 15$ and 27 respectively, where $S^* = Sq/\varepsilon$). The dash lines represent $\sim (St)^\epsilon$ with $\epsilon = 1.8$ in (a) and 1.5 in (b).

1.1.2 Statistically-stationary state

In reality, the kinetic energy, enstrophy, integral length and scale separation cannot be infinite. For example, the above quantities of HST with long time simulation in any finite box with length L_x , width L_z and height L_y will be constrained by the box, leading to a statistically-stationary state (SS-HST), which is studied in this thesis.

SS-HST was first studied by Pumir (1996) by numerical simulations. Successive spikes of kinetic energy and enstrophy were found, reminiscent of the bursting process and self-sustaining process (SSP) in wall-bounded turbulence, which inspires the present thesis to compare coherent structures related with the bursting process in SS-HST and channels. As an example, figure 1.4 gives time histories of kinetic energy in SS-HST and in the logarithmic layer of a minimal channel (Flores & Jiménez, 2010). It can be found that both intensities and bursting periods in shear unit roughly agree between the two different flows. This bursting process will be studied later in §2.

Based on (1.11), the statistically-stationary state $\langle dk/dt \rangle = 0$ requires

$$-S \langle uv \rangle = \langle \varepsilon \rangle. \quad (1.21)$$

From dimensional analysis, $\langle \varepsilon \rangle \sim \langle k \rangle^{3/2} / \ell$ and $\langle uv \rangle \sim \langle k \rangle$, (1.21) yields

$$\langle k \rangle^{1/2} \sim S\ell, \quad (1.22)$$

Figure 1.4: Time histories of kinetic energy (a) in SS-HST with $Re_\lambda = 250$ and (b) in a minimal channel with $Re_\tau = 1880$ averaged in the slab $0.1 < y/h < 0.2$ (Flores & Jiménez, 2010), normalized by the ensemble average. Time is normalized by the mean shear. The mean shear in the channel is taken as the value at $y/h = 0.15$.

with ℓ determined by the size of the box in SS-HST. The reason is that any finite box for SS-HST is *minimal*, i.e., structures in SS-HST will always 'fill' the whole box.

This minimal effect is shown clearly by the instantaneous streamwise velocity structures in boxes with different sizes but similar Re_λ as displayed in figure 1.5. The vertical size of the box L_y is set as constant 2 but the streamwise and spanwise periods change from $L_x \times L_z = \pi/2 \times \pi/4$ to $2\pi \times \pi$. For both SS-HST and channels, only one spanwise-paired low- and high-speed streamwise streaks can be accommodated in the small box. As the box size increases, more structures can be contained in channels thus the flow becomes more complicated. Details about the box effect for channels can be found in Lozano-Durán & Jiménez (2014a). However, for SS-HST, streaks just grow proportionally with the box and there is almost no difference with those in the smaller box. Note that for HST, the streamwise velocity is averaged over the vertical direction. It suggests that u spans the whole vertical direction. Later in §2 we will see that the vertical size for HST with given streamwise length is not able to capture u properly. Figure 1.5 suggests that SS-HST behaves in a similar way as in minimal channels and that its statistics should not be free of the effect of box, which will be discussed in §2.

Note that (1.21) is also one of the most fundamental properties of the logarithmic layer in wall-bounded turbulence, which is one of the reasons for emphasizing the comparison between SS-HST and the logarithmic layer for the rest of the thesis. Generally, in fully

Figure 1.5: Low- (blue) and high-speed (red) streamwise velocities in SS-HST (top) and channels (bottom) with streamwise and spanwise periods $L_x \times L_z = \pi/2 \times \pi/4$ (a,c) and $2\pi \times \pi$ (b,d). L_y is constant 2. For SS-HST, the streamwise velocity is averaged over the vertical direction while in channels the streamwise velocity is at the plane with $y/h \approx 0.4$. Re_λ for all the cases are roughly 80-98. Velocity is normalized by u_τ , in the range $[-4.0 \ 4.0]$.

developed wall-bounded turbulence, at each y/h , the kinetic energy equation evolves by

$$-\overline{uv} \frac{\partial U}{\partial y} - \frac{\partial}{\partial y} \left(\frac{\overline{pv}}{\rho} + \overline{kv} - \nu \frac{\partial \bar{k}}{\partial y} \right) = \bar{\varepsilon}. \quad (1.23)$$

However, compared with other two components, in the logarithmic layer, the diffusion term is negligible. So that (1.21) and (1.22) also hold in the logarithmic layer. But in big channels, ℓ will not be the box sizes. A more relevant one in shear-driven turbulence is the Corrsin length $L_c = (\varepsilon/S^3)^{1/2}$ (Corrsin, 1958), below which eddies are isotropically oriented and can not feel the effect of the mean shear. It turns out that $k^{1/2}/SL_c$ in the logarithmic layer is almost constant, but it depends on the Reynolds number.

1.2 Reality of unrealistic homogeneous shear turbulence

HST is an ideal model to study the effect of mean shear without other complications due to its simplicity although it is unrealistic. Like isotropic turbulence, whose large scales are totally artificial, is used widely to study the cascade process. But HST is more natural than isotropic turbulence. It contains the basic energy-production and Reynolds stress transfer mechanisms of shear-induced turbulence in general, and of the logarithmic layer of wall turbulence in particular.

Compared with (1.10), the Reynolds stress transportation equation for inhomogeneous turbulence is given by

$$\frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} + \frac{\partial}{\partial x_k} (\overline{u_i u_j u_k} + (\overline{u_i p} \delta_{jk} + \overline{u_j p} \delta_{ik}) / \rho - \nu \frac{\partial \overline{u_i u_j}}{\partial x_k}) = \overline{\mathcal{P}}_{ij} + \overline{\mathcal{R}}_{ij} - 2\overline{\varepsilon}_{ij}. \quad (1.24)$$

where

$$\overline{\mathcal{P}}_{ij} = -\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} - \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \quad (1.25)$$

is the production tensor. For wall-bounded turbulence, there is Reynolds stress flux in the vertical direction due to the second and third terms on the left hand side, which are absent in (1.10). But (1.10) still retains the key features of wall-bounded turbulence.

The production term $-S\overline{uv}$ for kinetic energy only appears in the equation for $\overline{u_1^2}$. And the production term $\overline{\mathcal{P}}_{12} = -S\overline{u_2^2}$ for Reynolds stress $\overline{uv}$ is only determined by the vertical velocity and the mean shear, which is also true for wall-bounded turbulence. The pressure redistribution terms do not contribute to the kinetic energy except redistribute energy between different velocity components. Even if $\overline{u_1^2}$, $\overline{u_1 u_2}$ and $\overline{u_3^2}$ are zero at the initial moment but not $\overline{u_2^2}$, Reynolds stress will be generated, so does $\overline{u_1^2}$, and $\overline{u_3^2}$ by pressure redistribution term. Such kind of kinetic energy production and redistribution happens all the time.

The generation of streaks is transient, of time S^{-1} . This transient growth is due to the non-normality of the linearized Navier-Stokes equation, which makes laminar shear flows transfer to turbulent state with Reynolds numbers much smaller than critical values computed from modal stability analysis. This transient growth exists in all shear flows.

Omit nonlinear terms, the linearized equations of (1.1) are given by

$$\left(\frac{\partial}{\partial t} + Sy \frac{\partial}{\partial x}\right) u_i = -Su_2 \delta_{i1} - \frac{1}{\rho} \frac{\partial p^{(r)}}{\partial x_i} + \nu \nabla^2 u_i, \quad (1.26)$$

and

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (1.27)$$

where $p^{(r)}$ is the fast pressure sensitive to the mean shear satisfying

$$\frac{1}{\rho} \nabla^2 p^{(r)} = -2S \frac{\partial u_2}{\partial x_1}, \quad (1.28)$$

according to the Poisson equation of p . (1.26) and (1.28) are usually written as the Orr-Sommerfeld equation

$$\frac{\partial}{\partial t} \nabla^2 u_2 = -U \frac{\partial}{\partial x_1} \nabla^2 u_2 + \nu \nabla^2 (\nabla^2 u_2), \quad (1.29)$$

and the Squires equation

$$\frac{\partial \omega_2}{\partial t} = -U \frac{\partial \omega_2}{\partial x_1} - S \frac{\partial v}{\partial x_3} + \nu \nabla^2 \omega_y, \quad (1.30)$$

or in the form of Fourier modes in the streamwise and spanwise directions as

$$\frac{d}{dt} \begin{vmatrix} \hat{u}_2 \\ \hat{\omega}_2 \end{vmatrix} = A \begin{vmatrix} \hat{u}_2 \\ \hat{\omega}_2 \end{vmatrix}, \quad (1.31)$$

where

$$A = \begin{vmatrix} L_{os} & 0 \\ L_c & L_{sq} \end{vmatrix}. \quad (1.32)$$

L_{os} , L_{sq} and L_c are the Orr-Sommerfeld, Squire and the coupling terms given by

$$L_{os} = L_{sq} = -ik_1 U + \nu(-k_1^2 - k_3^2 + \frac{\partial^2}{\partial y^2}), \quad (1.33)$$

$$L_c = -ik_3 S. \quad (1.34)$$

L_c couples u_2 and ω_2 together when $k_3 \neq 0$, introducing the nonnormality to the eigenvectors of A . This nonnormality leads to the transient growth of kinetic energy, which makes the non-modal stability analysis become more practical than the modal stability for long time behaviour (Schmid, 2007). Kim & Lim (2000) further showed that L_c is responsible for the generation of streaks, that is why streaks can be found in HST (Rogers & Moin, 1987) and streaks disappear when passing grid turbulence to a plate moving with the mean velocity (Uzkan & Reynolds, 1967).

The details about transient or optimal growth mentioned above can be better understood by the analytical solution of linearized equations (1.26)-(1.28). Assume that

$$u_i = \hat{u}_i(\mathbf{k}_0, t) \exp[\mathbf{k}(t) \cdot \mathbf{x}], \quad (1.35)$$

and

$$p^{(r)} = \hat{p}_i^{(r)}(\mathbf{k}_0, t) \exp[\mathbf{k}(t) \cdot \mathbf{x}], \quad (1.36)$$

where $\mathbf{k} = (k_1, k_2, k_3)$ is the wave number vector with initial value $\mathbf{k}_0 = (k_{10}, k_{20}, k_{30})$, then (1.26) and (1.28) turn to be

$$\frac{d\hat{u}_i(t)}{dt} + ik_1 U \hat{u}_i(t) + i\hat{u}_i(t) x_j \frac{dk_j}{dt} = -ik_i \hat{p}^{(r)}(t) - S\hat{u}_2(t) \delta_{i1} - \nu k^2 \hat{u}_j(t), \quad (1.37)$$

and

$$\frac{1}{\rho} \hat{p}^{(r)}(t) = 2iSk_1(t) \hat{u}_2(t) / k^2(t). \quad (1.38)$$

Substitute the pressure term in (1.37) by (1.38). It can be found that the evolution of wave number $\mathbf{k}$ is in the imaginary part and the evolution for velocity $\hat{\mathbf{u}}$ is in the real part, which means that the evolutions of wave number and velocity are independent except that they are connected by the continuity relation $\mathbf{k} \cdot \hat{\mathbf{u}} = 0$. The equation for the evolution of k_i and $\hat{u}_i$ are thus

$$\frac{dk_i}{dt} = -k_1 \frac{\partial U}{\partial x_i}, \quad (1.39)$$

and

$$\frac{d\hat{u}_i(\mathbf{k}_0, t)}{dt} = -S\hat{u}_2 \delta_{i1} + \frac{2Sk_1 k_i \hat{u}_2}{k^2} - \nu k^2 \hat{u}_i, \quad (1.40)$$

consider that U can be written as

$$U = \frac{\partial U}{\partial x_i} x_i. \quad (1.41)$$

Apparently, velocities with different initial wave numbers $\mathbf{k}_0$ evolve independently. The streamwise and spanwise wave numbers k_1 and k_3 are constant but the vertical wave number evolves by $dk_2(t)/dt = -Sk_1$.

The solutions for the above equation are given by [Townsend \(1976\)](#), which are

$$\hat{u} = k_0^2 / (k_1^2 + k_3^2) \mathcal{D} \mathcal{Q}_1 v_0 + \mathcal{D} u_0 \quad (1.42)$$

$$\hat{v} = \frac{k_1^2 + k_{20}^2 + k_3^2}{k_1^2 + (k_{20} - Sk_1 t)^2 + k_3^2} \mathcal{D} v_0 \quad (1.43)$$

$$\hat{w} = k_1 k_3 / (k_1^2 + k_3^2) \mathcal{D} \mathcal{Q}_2 v_0 + \mathcal{D} w_0 \quad (1.44)$$

where

$$\mathcal{Q}_1 = -\frac{k_3^2}{k_1 \sqrt{k_1^2 + k_3^2}} \varphi + \psi k_1^2 / k_0^2 \quad (1.45)$$

$$\mathcal{Q}_2 = \frac{k_0^2}{k_1 \sqrt{k_1^2 + k_3^2}} \varphi + \psi \quad (1.46)$$

$$\mathcal{D} = \exp^{-\nu(k_0^2 - Stk_{20}k_{10} + (Stk_{10})^2/3)t} \quad (1.47)$$

and

$$\varphi = \text{atan} \frac{Stk_1 \sqrt{k_1^2 + k_3^2}}{k_0^2 - Stk_1 k_{20}} \quad (1.48)$$

$$\psi = \frac{St(k_0^2 - 2k_{20}^2 + Stk_1k_{20})}{k^2} \quad (1.49)$$

Jimenez (2013b) found that during the transient growth (Orr burst) of v , its intensity v^2 as a function of the averaged tilting angle γ with respect to the direction $x < 0$ given by

$$\pi/2 - \arctan \left(\operatorname{Im} \frac{\int_0^h \hat{u}_2^* \partial_2 \hat{u}_2 dx_2}{(k_1^2 + k_3^2) \int_0^h |\hat{u}_2|^2 dx_2} \right) \quad (1.50)$$

in the channel agrees quite well with that of analytical solutions given above, with

$$\gamma = \pi/2 + \arctan \left(\frac{Sk_1t}{\sqrt{k_1^2 + k_3^2}} \right). \quad (1.51)$$

Equations for Reynolds stress transportation in the linearized situation have the same form as (1.10), the only difference is pressure in the pressure redistribute terms

$$\overline{\mathcal{R}}_{ij} = \overline{p^{(r)} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} / \rho. \quad (1.52)$$

Remind that $p^{(r)}$ is proportional to u_2 from (1.38). Even though there are long time lived streaks created by u_2 when $k_3 \neq 0$, u_2 cannot get energy from u_1 by pressure since $\overline{\mathcal{R}}_{22} < 0$. When u_2 gives energy back to the flow totally, i.e., $u_2 = 0$, $\overline{\mathcal{R}}_{ij}$ will also be zero. So that this Orr burst cannot be activated again in linearized situation. But once nonlinear terms are important, initial conditions for u_2 can be amplified will be generated thus the transient growth will continue. Later we will see that the linear periodic bursts of u_2 for very long boxes ($L_x/L_y > 2$) is exactly due to this reason.

1.3 Aims

Turbulence is often induced by shear, and one of the most fundamental problems in fluid dynamics is to reveal the interaction between the mean flow and the kinetic energy of the turbulent fluctuations.

The thesis seeks to answer two questions about shear-dominated turbulence: a) do all shear-induced turbulent flows with linearly stable mean velocity profiles share common mechanisms for momentum transfer and energy generation? (Linearly unstable profiles are known to behave differently). b) Do the coherent structures in turbulence undergo a cascade as proposed by Richardson (1920), not only as a conceptual model but as an actual physical process? Similar questions have been studied in recent years for wall turbulence, mainly in turbulent channels. That work involved simulating the flow, isolating individual

vortex clusters (del Álamo *et al.*, 2006) and structures of the momentum transfer (ejections and sweeps) (Lozano-Durán *et al.*, 2012), storing them often enough to be considered as temporally resolved, and organizing their evolution and interactions in time and space (Lozano-Durán & Jiménez, 2014b). Especial attention has been paid to the logarithmic layer of wall turbulence (Jiménez, 2012), which is a region of particular theoretical and practical interest. That work has been carried in Reynolds numbers high enough for the cascade to be present.

It turns out that the channel dynamics is controlled by structures that are attached to the wall and it becomes clear that the mean shear is more important than the wall. But still more features have to be investigated to find if they are determined by the mean shear or not. Thus we intend to separate the effect of the mean shear from that of the wall by applying the same techniques as in channels to SS-HST with Reynolds number similar to that available for channels ($Re_\lambda \approx 200$), and to compare the results in both flows. We have already mentioned that SS-HST bursts intermittently (Pumir, 1996; Gualtieri *et al.*, 2002) in a very similar manner to wall turbulence (Flores & Jiménez, 2010; Jiménez, 2012; Jimenez, 2013b). And that any box for long time HST is minimal, so that we can further explore how ejections, sweeps and vortex clusters evolve during the bursting process by relating them with the intermittent kinetic energy and enstrophy.

If successful, the results would simplify considerably our understanding of the dynamics of shear flows in general, but they would also point the way to simpler boundary conditions for large-eddy simulations of wall-bounded flows. The practical application of those simulations is limited at present by the resolution requirements near the wall, but if it could be confirmed that the details of the wall are unimportant, simpler boundary conditions could be designed. In fact, large-eddy simulations of wall-less channels have been recently performed with promising results (Mizuno & Jimenez, 2013). Besides, Toschi *et al.* (2000) and Lvque *et al.* (2007) proposed a modified Smagorinsky model considering the effect of mean shear. That model, although based on the structure function of a much simpler flow, HST, achieved more accurate results in channel turbulence than the classical Smagorinsky model compared with results of direct numerical simulations (DNS).

1.4 Contents and organization of the thesis

Beyond the present chapter, all the results are organized into three chapters.

In chapter 2, the numerical method of a novel code for DNS of HST is introduced.

By using that code, around 100 cases with different Reynolds numbers and a wide range of aspect ratios of simulation boxes are conducted. A region of aspect ratios in which SS-HST is as free as possible from the box effect is found and the characteristics of SS-HST including scaling length, scaling velocity and bursting process are discussed. Also some preliminary comparisons between SS-SHT and the logarithmic layer in channels are presented. Some special attention is paid on unacceptable long boxes with $L_x/SL_y > 2$. The contents of this chapter have been published in *Physics of Fluids* under the title ‘*Direct numerical simulation of statistically stationary and homogeneous shear turbulence and its relation to other shear flows*’, volume 28, number 3, eid 035101, with Atsushi Sekimoto and Javier Jiménez as coauthors.

In chapter 3, based on the above results, three DNS of SS-HST with Re_λ comparable to those of channels are conducted to study the kinematics of three-dimensional sweeps, ejections and vortex clusters by comparing them with those in channels. Both similarities presumably due to the mean shear and differences due to the wall or inhomogeneity are discussed. But the latter does make the spatial organization of structures in SS-HST so different from channels found by [Lozano-Durán et al. \(2012\)](#). The results show that these structures in SS-HST are promising to represent eddies in general shear turbulence. The contents of the chapter have been submitted to the *Journal of Fluid Mechanics* under the title ‘*Kinematics of coherent structures in homogeneous shear turbulence*’, with Adrián Lozano-Durán, Atsushi Sekimoto and Javier Jiménez as coauthors.

In chapter 4, the results in the previous chapter are extended by temporal tracking of sweeps, ejections and vortex clusters during several burst periods as [Lozano-Durán & Jiménez \(2014b\)](#) to study their dynamics. Due to the minimal box effect, the temporal evolution of these structures and the intermittent of kinetic energy can be related together. While the latter is missing in big channels. Thus the results provide a detailed description about how do these structures evolve during the burst process. Besides, some efforts are devoted exclusively to the second question proposed in the §1.3 by relating turbulent inverse and direct cascade with the merge and split of these structures compared with those in channels.

Finally, conclusions and future work are given in chapter 5.

Chapter 2

DNS of SS-HST and its relation to other shear flows^{*}

2.1 Introduction

The simulation of ever higher Reynolds numbers is a common practice in modern turbulence research, mainly to study the multi-scale nature of the energy cascade and similar phenomena. A basic property of these processes is that they are nonlinear and chaotic, which makes their theoretical analysis difficult. Direct numerical simulations offer a promising tool for their study because they provide exceptionally rich data sets. They have also been moving recently into the same range of Reynolds numbers as most experiments, especially when similar levels of observability are compared.

On the other hand, simulations are not without problems, some of which they share with experiments. One of those problems is forcing. Turbulence is dissipative, and unforced turbulence quickly decays. The highest resolution simulations available, with Taylor-microscale Reynolds number $Re_\lambda \sim O(1000)$, are nominally isotropic flows with artificial forcing in a few low wavenumbers (Kanedo *et al.*, 2003), and it is difficult to judge how far into the cascade the effect of the forcing extends. The forcing of wall-bounded shear flows is well characterized and corresponds to physically realizable situations, and they have often been used as alternatives to study high-Reynolds number turbulence. However, a consequence of their more natural large scales is that their cascade range is much shorter than in the isotropic boxes. Current simulations only reach $Re_\lambda \sim 200$ (Lozano-Durán & Jiménez, 2014a; Bernardini *et al.*, 2014; Lee & Moser, 2015). They

^{*}Part of the contents of this chapter have been published in *Physics of Fluids*, volume 28, number 3, eid 035101, with Atsushi Sekimoto and Javier Jiménez as coauthor.

are also inhomogeneous, and it is difficult to distinguish which properties are due to the inhomogeneity, to the presence of the wall, or to inertial turbulence itself.

A compromise between the two limits is HST, which shares the natural energy-generation mechanism of shear flows with the simplicity of homogeneity. This flow is believed not to have an asymptotic statistically stationary state, but it is tempting to use it as a proxy for general shear turbulence in which high Reynolds numbers can be reached without the complications of wall-bounded flows. Up to recently, HST has mostly been used to study the generation of turbulent fluctuations during the initial shearing of isotropic turbulence, where one of the classic challenges is to determine the growth rate with a view to developing turbulence models, avoiding as far as possible artifacts due to numerics and initial conditions. A question closer to our interest in this chapter is whether some aspects of HST can be used as models for other shear flows. For example, [Rogers & Moin \(1987\)](#) showed typical ‘hairpin’ structures under shear rates comparable to those in the logarithmic layer of wall-bounded flows, and [Lee *et al.* \(1990\)](#) found that higher shear rates, comparable to those in the buffer layer, result in structures similar to near-wall velocity streaks ([Jimenez, 2013b](#)). Those structures are known to play crucial roles in transition and in maintaining shear-induced turbulence ([Jiménez & Moin, 1991](#); [Jiménez, 1994](#); [Hamilton *et al.*, 1995](#); [Waleffe, 1997](#)), and [Kida & Tanaka \(1994a\)](#) proposed a generation mechanism for the streamwise vortices in transient HST that recalls those believed to be active in wall turbulence.

Ideal HST in unbounded domains grows indefinitely, both in intensity and length scale ([Rogers & Moin, 1987](#); [Tavoularis & Karnik, 1989](#); [Isaza & Collins, 2009](#)). During the initial stages of shearing an isotropic turbulent flow, linear effects result in algebraic growth of the turbulent kinetic energy, which is later transformed to exponential due to non-linearity ([Cambon & Scott, 1999](#)). These simulations are typically discontinued as the growing length scale of the sheared turbulence approaches the size of the computational box, but [Pumir \(1996\)](#) extended the simulation to longer times and reached a statistically stationary state (SS-HST) in which the largest-scale motion is constrained by the computational box and undergoes a succession of growth and decay of the kinetic energy and of the enstrophy reminiscent of the bursting in wall-bounded flows ([Jiménez *et al.*, 2005](#)), suggesting that bursting is a common feature of shear-induced turbulence, not restricted to wall-bounded situations. In fact, previous investigations of SS-HST have suggested that the growth phase of bursts is qualitatively similar to the initial shearing of isotropic turbulence ([Pumir, 1996](#); [Gualtieri *et al.*, 2002](#)).

It should be emphasized that the main goal of this chapter is not to investigate the

properties of unbounded HST, which is difficult to implement both experimentally and numerically for the reasons explained above. Simulations in a finite box introduce a length scale that, without interfering with homogeneity, is incompatible with an unbounded flow. It is precisely in the effect of this extraneous length scale, which may make simulations more similar to flows in which a length scale is enforced by the wall, that we are more interested. As first pointed out in [Pumir \(1996\)](#), this is what SS-HST provides.

In this chapter we present a numerical code optimized to perform the long simulations required to study SS-HST. We characterize the code and the influence of the numerical parameters on the physics, and draw preliminary conclusions about the physics itself. Since it will be seen that bursting times are $O(20S^{-1})$, where S is the mean velocity gradient, the required simulation times are of the order of several hundreds shear times. The most commonly used code for HST is due to [Rogallo \(1981\)](#), and involves remeshing every few shear times. Some enstrophy is lost in each remeshing ([Lee *et al.*, 1990](#)), and the concern about the cumulative effect of these losses has led to the search for improved simulation schemes. [Schumacher & Eckhardt \(2000\)](#) and [Horiuchi & Ozawa \(2011\)](#) introduced artificial body forces to drive a mean shear gradient between stress-free surfaces, but this creates thin layers near the surfaces, and the impermeability condition prevents large-scale motions from developing. A different strategy for avoiding discrete remeshings in fully spectral codes is that of [Brucker *et al.* \(2007\)](#), who used an especially developed Fourier transform essentially equivalent to remeshing at each time step.

Another approach to avoid periodic remeshing was pioneered by [Baron \(1982\)](#) and later by [Schumann \(1985\)](#) and [Gerz *et al.* \(1989\)](#) who used a ‘shear-periodic’ boundary condition in which periodicity is enforced between shifting points of the upper and bottom boundaries of the computational box by a central-finite-differences scheme. Similar boundary conditions have been used in simulations of astrophysical disks ([Balbus & Hawley, 1998](#); [Johansen *et al.*, 2009](#); [Salhi *et al.*, 2014](#)) under the name of ‘shearing-boxes’. The code in this paper belongs to this family, but uses higher-order approximations and a vorticity representation similar to that in [Kim *et al.* \(1987\)](#).

A substantial part of the present chapter is devoted to the choice of the dimensions of the computational box. The previous discussion about length scales suggests that this choice should influence the properties of the resulting SS-HST, but the matter has seldom been addressed in detail. We also spend some effort comparing the bursts in SS-HST with those in the logarithmic layer, and with the initial shearing of isotropic turbulence.

The organization of this chapter is as follows. The numerical technique and the shear-periodic boundary condition is introduced and analyzed in §2.2. The effect of the compu-

tational domain is studied in §2.3, resulting in the identification of an acceptable range of aspect ratios in which the flow is as free as possible of computational artifacts, as well as in the determination of the relevant length and velocity scales. § 2.4 contains preliminary comparisons of the results of SS-HST with other shear flows, with emphasis on the logarithmic layer of wall-bounded turbulence. Conclusions are offered in §2.5. Several appendices in §2.A, contain details of the numerical implementation and validations of the code.

It has to be stated that the algorithm was developed by Atsushi Sekimoto. I only implemented the analytical integration. It requires 9 steps at each time step. Then Atsushi Sekimoto improved it as shown in §2.A.2, which only needs five steps per time step. The code was originally MPI parallelized, using *plane-plane* communication with `MPI_SENDRECV`. I changed it to MPI+OpenMP and tuned its performance on supercomputing centers. The previous communication was done between streamwise ($z-y$ plane) and spanwise directions ($x-y$ plane), but as we will see that the spanwise length is always the smallest in the region of box sizes that we are interested, so that the number of nodes that can be used is restricted. Therefore the spanwise direction is replaced by the vertical one ($x-y$ plane). And `MPI_SENDRECV` is replaced by `MPI_ALLTOALL(V)`. The details about the performance of the code are given in §2.A.5.

2.2 Numerical method

2.2.1 Governing equations and integration algorithm

The Navier–Stokes equations of motion, including continuity, are reduced to evolution equations for $\phi = \nabla^2 v$ and ω_y (Kim *et al.*, 1987), with the advection by the mean flow explicitly separated,

$$\frac{\partial \omega_y}{\partial t} + Sy \frac{\partial \omega_y}{\partial x} = h_g + \nu \nabla^2 \omega_y, \quad \frac{\partial \phi}{\partial t} + Sy \frac{\partial \phi}{\partial x} = h_v + \nu \nabla^2 \phi. \quad (2.1)$$

Defining $\mathbf{H} = \mathbf{u} \times \boldsymbol{\omega}$,

$$h_g \equiv \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - S \frac{\partial v}{\partial z}, \quad h_v \equiv -\frac{\partial}{\partial y} \left(\frac{\partial H_x}{\partial x} + \frac{\partial H_z}{\partial z} \right) + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_y. \quad (2.2)$$

In wall-bounded flows, the mean velocity difference is generated by the wall and its profile is Reynolds number dependent. Therefore the mean shear can only be obtained after the simulation. But in HST there is no physical mechanism to maintain the constant and has

to be added artificially. Appendix 2.A.1 gives details about how is the mean shear added in the simulation.

In addition, the governing equation for $\bar{\mathbf{u}}_{xz}$ are

$$\frac{\partial \bar{u}_{xz}}{\partial t} = -\frac{\partial \bar{u} \bar{v}_{xz}}{\partial y} + \nu \frac{\partial^2 \bar{u}_{xz}}{\partial y^2}, \quad \frac{\partial \bar{w}_{xz}}{\partial t} = -\frac{\partial \bar{w} \bar{v}_{xz}}{\partial y} + \nu \frac{\partial^2 \bar{w}_{xz}}{\partial y^2}. \quad (2.3)$$

Continuity requires that the mean vertical velocity should be independent of y , and it is explicitly set to $\bar{v}_{xz} = 0$. We consider a periodic computational domain, $0 \leq x \leq L_x$, $0 \leq z \leq L_z$, and use two-dimensional Fourier-expansions with 3/2 dealiasing in those directions. The grid dimensions, N_x and N_z , and their corresponding resolutions, $\Delta x = L_x/N_x$ and $\Delta z = L_z/N_z$, will be expressed in terms of Fourier modes. The domain is shear-periodic in $-L_y/2 \leq y \leq L_y/2$, as explained in the next section. The y direction is not dealiased, and $\Delta y = L_y/N_y$. Its discretization is spectral compact finite differences with a seven-point stencil on a uniform grid. Its coefficients are chosen for sixth-order consistency, and for spectral-like behavior at $k_y \Delta y / \pi = 0.5, 0.7, 0.9$ (Lele, 1992).

Time stepping is third-order explicit Runge–Kutta (Spalart *et al.*, 1991) modified by an integrating factor for the mean-flow advection (see appendix 2.A.3). This uncouples the CFL condition from the mean flow, and is especially helpful in tall computational domains. The resulting time step is

$$\Delta t \leq \text{CFL} \min \left(\frac{\Delta x}{\pi |u|}, \frac{\Delta y}{2.85 |v|}, \frac{\Delta z}{\pi |w|}, \frac{(\Delta x)^2}{\pi^2 \nu}, \frac{(\Delta y)^2}{9 \nu}, \frac{(\Delta z)^2}{\pi^2 \nu} \right), \quad (2.4)$$

where the denominators 2.85 and 9 in (2.4) are the maximum eigenvalues of our finite differences for the first and second derivatives. An implicit viscous part of the time stepper is not required because only the advective CFL matters in turbulent flows with uniform grids of the order of the Kolmogorov viscous scale. Our simulations use $\text{CFL} = 0.5 - 0.8$.

The problem has three dimensionless parameters that are chosen to be two aspect ratios of the computational box, $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$, and the Reynolds number based on the box width, $Re_z = S L_z^2 / \nu$.

2.2.2 The shear-periodic boundary condition

The boundary condition used in this study is that the velocity is periodic between pairs of points in the top and bottom boundaries of the computational box, which are shifted in time by the mean shear (Baron, 1982; Schumann, 1985; Gerz *et al.*, 1989; Balbus & Hawley, 1998). For a generic fluctuation g ,

$$g(t, x, y, z) = g[t, x + m S t L_y + l L_x, y + m L_y, z + n L_z], \quad (2.5)$$

Figure 2.1: Sketch of the shear-periodic boundary condition, as discussed in the text. (a) Physical space. (b) Fourier space.

where l, m and n are integers. In terms of the spectral coefficients of the expansion,

$$g(t, x, y, z) = \sum_{k_x} \sum_{k_z} \widehat{g}(t, k_x, y, k_z) \exp[i(k_x x + k_z z)], \quad (2.6)$$

the boundary condition becomes

$$\widehat{g}(t, k_x, y, k_z) = \widehat{g}(t, k_x, y + mL_y, k_z) \exp[ik_x m S t L_y], \quad (2.7)$$

where $k_i = n_i \Delta k_i$ ($i = x, z$) are wavenumbers, n_i are integers, and $\Delta k_i = 2\pi/L_i$. This condition is implemented by a few complex off-diagonal elements in the compact-finite-difference matrices (see [Schumann \(1985\)](#) and appendix in [2.A.3](#)).

Because of the streamwise periodicity of the domain, the shift $SL_y t$ of the upper boundary with respect to the bottom induces a characteristic time period, $ST_s = L_x/L_y$ (hereafter, box period), in which the flow is forced by the passing of the shifted-periodic boxes immediately above and below ([Jiménez, 2013a](#)). Since this forcing acts at the box scale, small turbulent structures can be expected to be roughly independent of the box geometry, but care should be taken to account for resonances between the largest flow scales and T_s (see § [2.3.2](#)). Note that (2.5) implies that the flow becomes strictly periodic in y whenever t/T_s is integer. From now on, we will use those moments as origins of time, and refer to them as the ‘top’ of the box cycle.

Typically, codes that depend on shearing grids remesh them once per box period ([Rogallo, 1981](#)). The differences between those codes and the present one are sketched in figure 2.1. Consider the two-dimensional finite-differences grid in figure 2.1(a), which is drawn at some negative time, before the top of the box cycle. The solid rectangle is the fundamental simulation box, and the boundary condition (2.5) is that the endpoints of the inclined solid line have the same value. Without losing generality, this can be expressed

by writing the solution as $g = \tilde{g}(x - Sty, y)$, where the dependence of $\tilde{g}$ on its second argument has period L_y . The first argument represents the time-dependent tilting of the solution. Consider the representation of $\tilde{g}$ in terms of Fourier modes,

$$\tilde{g} \sim \exp[ik_x(x - Sty) + i\tilde{k}_y y] = \exp[ik_x x + i(\tilde{k}_y - Stk_x)y], \quad (2.8)$$

where periodicity requires that $\tilde{k}_y = n_y \Delta k_y$. The effective Fourier grid, $(k_x, k_y) = (n_x \Delta k_x, n_y \Delta k_y - Stn_x \Delta k_x)$ is skewed (Rogallo, 1981), as represented by the upwards-sloping dashed lines in figure 2.1(b). Finite-differences formulas always use the Fourier modes within the thick dashed rectangle (ABCD, plus its complex conjugate) in figure 2.1(b), whose limits are defined by the grid resolution (Moin, 2001). As the Fourier grid is sheared downwards for increasing time, some modes leave the lower boundary of the resolution limit and are substituted by others entering through the upper one. The overall resolution is therefore maintained. In contrast, shearing-grid codes (Rogallo, 1981) let the Fourier modes skew outside the resolution box between the periodic remeshings. The triangular region (DcC) in figure 2.1(b) is thus not used, and the enstrophy contained in it is either lost or aliased. In contrast, the triangular region (AbB) is over-resolved, because it contains little useful information if the resolution has been chosen correctly. Note that the Fourier modes just described do not correspond to a unique time. It is easy to show that t and $t + T_s$ result in exactly the same Fourier grid, and are numerically indistinguishable. They typically correspond to forward- and backwards-tilted pairs of boundary conditions, as shown by the dashed inclined line in figure 2.1(a) and by the downwards-sloping single Fourier grid line in figure 2.1(b). In particular, it will become important in §2.3.3 that $t = \pm T_s/2$ form one such pair of numerically-conjugate times.

Validations collected in §2.A.4 confirm that it maintains sixth-order accuracy in space and third-order in time. It is also confirmed that it produces results that are consistent with the well-known Rogallo's (Rogallo, 1981) three-dimensional spectral remeshing method, both in the initial shearing of isotropic turbulence (AGARD, 1998), and in longer simulations of SS-HST (Pumir, 1996; Gualtieri *et al.*, 2002, 2007). In all those cases, our enstrophy and other small-scale statistics are slightly higher than those in remeshing codes using the same Fourier modes, probably because the lack of remeshing prevents the loss of some enstrophy.

2.3 Characterization of SS-HST

We have already mentioned that HST in an infinite domain evolves towards ever larger length scales (Tavoularis & Karnik, 1989; Rogers & Moin, 1987), so that simulations in a

finite box are necessarily controlled to some degree by the box geometry. In this section, we examine the dependence of the turbulence statistics on the geometry and on the Reynolds number. Our aim is to identify a range of parameters in which the flow is as free as possible from box effects and can be used as a model of shear-driven turbulence in general.

A wide range of aspect ratios was sampled at Reynolds numbers between $Re_z = 1000$ and 48000. Each simulation accumulates statistics over more than $St = 100$, after discarding an initial transient of $St \approx 30$. The lowest Reynolds number at which statistically steady turbulence can be achieved is $Re_z \approx 500$, but we do not consider the transitional low-Reynolds number regime, and focus on the fully-developed cases in which turbulence does not laminarize for at least $St > 100$. Most of our simulations have well-resolved numerical grids with effective resolutions $\Delta x_i \lesssim 2\eta$ in the three coordinate directions, where the Kolmogorov length $\eta = (\nu^3/\varepsilon)^{1/4}$. A few very flat boxes with $A_{yz} < 0.5$ have coarser grids with $\Delta x_i \approx 6\eta$, since it will be seen that those geometries are severely constrained by the box, and are not interesting for the purpose of this paper. One of those cases was repeated at $\Delta x_i \approx 2\eta$ to confirm that the one-point statistics vary very little for the coarser grids. Also, because of the computational cost, some long- and tall-box cases, $A_{xz} = 8$, $A_{yz} \gtrsim 1$ and $Re_z > 3000$, were run at $\Delta x_i \approx 3\eta$, which is empirically the marginal value to investigate small scales (Jiménez *et al.*, 1993). In all cases, the energy input $\mathcal{P} = -S \langle uv \rangle$ balances the dissipation rate ε within less than 5%.

Figure 2.2: (a) The aspect ratios, (A_{xz}, A_{yz}) , of the DNSes used in the paper, and the definition of the classification of the computational boxes into different regimes. (b) Integral scale L_ε/L_z as a function of A_{xz} in semi-logarithmic scale. The inset shows L_ε/L_y in logarithmic scale. Symbols as in table 2.1, but $\triangleleft$ from Pumir (1996) and $\triangleright$ from Gualtieri *et al.* (2002, 2007).

Zone	Symbol		
Short	$\times$	$A_{xz} \leq 2$	$A_{yz} \geq 1$
Flat	$\square$	—	$A_{yz} < 1$
Acceptable	$\bigcirc$	$2 < A_{xz} (\lesssim 5)$	$A_{xz} < 2A_{yz}$
Long	$\triangle$	$A_{xz} \geq 2A_{yz}$	$A_{yz} \geq 1$
Tall and long	∇	$A_{xz} (\gtrsim 5)$	$A_{xz} < 2A_{yz}$

Table 2.1: Parameter ranges in the space of aspect ratios for the SS-HST simulations

It turns out that the effect of the Reynolds number is relatively minor, at least for the larger scales, but that the flow properties depend strongly on the aspect ratios. A summary of the geometries tested is figure 2.2(a). The plane is divided into several regions, whose properties will be shown to be different. Those ranges, as well as the associated symbols used in the figures and the short name by which we refer to them below, are summarized in table 2.1. The characteristics of a few representative simulations can also be found in table 2.2 in §2.4.

Figure 2.3: Velocity fluctuation intensities as functions of A_{xz} , scaled with SL_z : (a) u'/SL_z . (b) v'/SL_z . (c) w'/SL_z . (d) Tangential Reynolds stress, given as a friction velocity. Symbols as in table 2.1, but $\triangleleft$ from Pumir (1996) and $\triangleright$ from Gualtieri *et al.* (2002, 2007).

Figure 2.4: Velocity fluctuation intensities as functions of A_{xz} , scaled with the friction velocity. (a) u'^+ . (b) v'^+ . (c) w'^+ . (d) Structure coefficient $C_{uv} = -\langle uv \rangle / u'v'$. Symbols as in table 2.1, but $\triangleleft$ from Pumir (1996).

2.3.1 The characteristic length and velocity

Although it is reasonably well understood that the time scale of sheared turbulence is S^{-1} , there is surprisingly little information on the length scales. Sheared turbulence itself has no length scale, except for the viscous Kolmogorov length, and properties like the integral scale grow roughly linearly in time (Rogers & Moin, 1987; Tavoularis & Karnik, 1989). In statistically stationary simulations, the question is which of the box dimensions limits the growth and imposes the typical size of the turbulent structures. figure 2.2(b) shows the integral scale, $L_\varepsilon = (q^2/3)^{3/2}/\varepsilon$, normalized by L_z as a function of A_{xz} . The inset in that figure tests the scaling of L_ε with L_y , and it is clear that L_z is a much better choice. The ordinates in the inset are logarithmic, and L_ε/L_y spans two orders of magnitudes, while L_ε/L_z is always around 0.5, except for a few short-flat cases. This suggests that the flow is ‘minimal’ in the spanwise direction, and visual inspection confirms that there is typically a single streamwise streak of u that fills the domain (see figure 2.13 in § 2.4). A similar structure is found in minimal channels, both near the wall (Jiménez & Moin, 1991) and in the logarithmic layer (Flores & Jiménez, 2010), and Pumir (1996) reports that most of the kinetic energy in his SS-HST is contained in the first spanwise wavenumber. The scaling with L_x , not shown, is even worse than with L_y .

The scaling of the length suggest that velocities should scale with SL_z . This is tested in figure 2.3(a-c) for the velocity fluctuation intensities. They collapse reasonably well except for very flat boxes, $A_{yz} < 1$. The scaling with SL_y , not shown, is very poor.

The tangential Reynolds stress is given in figure 2.3(d), in the form of a friction velocity. As was the case for the intensities, it scales well with SL_z , but it turns out that most of the residual scatter in figure 2.3 is removed using u_τ as a velocity scale, as shown in figure 2.4(a-c). It is interesting that, when the intensities are scaled in this way, they are reasonably similar to those in the logarithmic layer of wall-bounded flows, which are $(u'^+, v'^+, w'^+) \approx (2, 1.1, 1.3)$ in channels (Hoyas & Jiménez, 2006). The usual argument to support this ‘wall-scaling’ is that the velocity fluctuations have the magnitude required to generate the tangential Reynolds stress, u_τ^2 , which is fixed by the momentum balance. The argument also requires a constant value of the anisotropy of the Reynolds-stress tensor, whose most important contribution is the structure coefficient $C_{uv} = -\langle uv \rangle / u'v'$. It is given in figure 2.4(d), and also collapses well among the different cases, although it varies slowly with A_{xz} . Note the good agreement of the intensities in figure. 2.3 and 2.4 with the results included from previously published simulations (Pumir, 1996; Gualtieri *et al.*, 2002, 2007).

We defer to the next section the discussion of the slow growing trend of v'^+ with the aspect ratio, but the behavior of the three intensities for short boxes is interesting and probably has a different origin. For $A_{xz} \lesssim 1$, u^+ (v^+ , w^+) is stronger (weaker) than the cases with more equilateral boxes. A similar tendency was observed in simulations of turbulent channels in very short boxes by Toh & Itano (2005). It can also be shown that the inclination angle of the two-point correlation function of u tends to zero for these very short boxes, while it is around 10° in longer boxes (not shown), and in wall-bounded flows (Gualtieri *et al.*, 2002). The behavior of the intensities suggests that the streaks of the streamwise velocity become more stable and break less often when the box is short, which is reasonable if we assume that the box limits the range of streamwise wavenumbers available to the instability. Indeed, the linear stability analysis of model streaky flows shows that they are predominantly unstable to long wavelengths, and that shorter instabilities require stronger streaks (Waleffe, 2003).

Figures 2.5 and 2.6 display the two-point correlation functions of the velocities. The streamwise correlations in figure 2.5(a-c) are the longest, especially $R_{uu}(r_x)$, as already noted by Rogers & Moin (1987). This is common in other shear flows (Sillero *et al.*, 2014b), and figure 2.5(a) shows that at least $A_{xz} = 2$ is required for the streamwise velocity to become independent of the box length. This roughly agrees with the observation of

Figure 2.5: Two-point velocity correlation functions: (a,d,g) R_{uu} ; (b,e,h) R_{vv} ; (c,f,i) R_{ww} , in (a-c) streamwise, (d-f) vertical, and (g-i) spanwise directions, for $Re_z = 2000$ and $A_{yz} = 1$. $-\nabla-$, $A_{xz} = 1.5$; $-\circ-$, 3; $-\triangle-$ with Δ , 8. The arrows are in the sense of increasing A_{xz} .

Figure 2.6: Two-point velocity correlation functions: (a) R_{uu} , (b) R_{vv} , and (c) R_{ww} , for $Re_z = 2000$ and $A_{xz} = 3$. $-\nabla-$, $A_{yz} = 0.5$; $-\square-$, 1; $-\circ-$, 2; $-\circ-$, 4; $-\triangle-$, 8.

Reynolds-stress structures in channels, which have aspect ratios of the order of $L_x/L_y \approx 3$ (Lozano-Durán *et al.*, 2012), and with our discussion on the velocity intensities in short boxes. Pumir (1996) likewise reported that the turbulence statistics become independent of the aspect ratio for $A_{xz} \gtrsim 3$, and Rogers & Moin (1987) used $A_{xz} = 2$ for their transient-shearing experiments after a qualitative inspection of the correlation functions. It is interesting that R_{vv} in figure 2.5(b,e,h) becomes longer in the three directions for longer boxes, even if v is typically a short variable in wall-bounded flows (Jimenez, 2013b; Sillero *et al.*, 2014b). This agrees with the common notion that the limit on the size of v in wall-bounded turbulence is due to the blocking effect of the wall, and suggests that the effect observed in the longer boxes is related to structures that span several vertical copies of the shear-periodic flow. Otherwise, the central peak of all the correlations collapses well

when normalized with L_z , even if A_{xz} spans a factor of over 20 in our simulations. Note that none of the curves in figure 2.5(g,h,i) shows signs of decorrelating at the box width, supporting the conclusion that the flow is constrained by the spanwise box dimension.

figure 2.6 shows two-point velocity correlation functions in the y direction, for different vertical aspect ratios. The collapse with L_z of the different correlations is striking; taller boxes lead to more structures of a given size, but not to taller structures. The figure also provides a minimum vertical aspect ratio, $A_{yz} \approx 2$, for structures that are not vertically constrained, ruling out what we have termed above ‘flat’ boxes.

The correlation functions in figures 2.5 and 2.6 depend on the Reynolds number in a natural way. The inner core narrows as the Reynolds number increases and the Taylor microscale decreases, but the outer part of the correlations stays relatively unaffected.

The Taylor-microscale Reynolds number satisfies $Re_\lambda \approx Re_z^{1/2}$ except for very flat or short boxes. This agrees approximately with Pumir (1996) and the latest high-Reynolds-number DNS by Gualtieri *et al.* (2007), although their earlier data (Gualtieri *et al.*, 2002) are 10–20% smaller than in our simulations. The difference is too small to decide whether the reason is numerical or statistical.

2.3.2 Long boxes, the bursting period, and box resonances

The results up to now show that statistically-stationary shear flow is mostly limited by the spanwise dimension of the box, but that very short, $A_{xz} \lesssim 2$, or very flat boxes, $A_{yz} \lesssim 1$, should be avoided because they are also minimal in the streamwise or vertical direction. Figures 2.4 and 2.5 show that there is an additional effect of very long boxes, which have stronger and longer fluctuations of v and, to some extent, of u . This is the subject of this section.

The most obvious property of the SS-HST flows is that they burst intermittently. This is seen in the time histories of the box-averaged velocity fluctuations, given in figure 2.7(a–c) for three boxes of different streamwise elongation. The character of the bursting changes with A_{xz} . For the relatively short box in figure 2.7(a) the bursts are long, and predominantly of the streamwise velocity $\overline{u^2}$. They are followed some time later by a somewhat weaker rise of $\overline{v^2}$ and $\overline{w^2}$. This relation between velocity components is reminiscent of the bursting in minimal channels analyzed in Jiménez (2013a), and the bursting time scale ($ST \approx 15$ – 20) is also of the same order. The longer boxes in figure 2.7(b,c) undergo a different kind of burst, sharper ($ST \approx 2$), and predominantly of $\overline{v^2}$. These sharp bursts always occur at the top of the box cycle, which is marked by the vertical dashed ticks in the upper part of the figures. There is little sign of interaction of the box cycle with the

Figure 2.7: (a-c) The time history of box-averaged velocity intensities, normalised by $(SL_z)^2$: ---, $u^2(t)$; —, $v^2(t)$; -·-·-, $w^2(t)$. (a) $A_{xz} = 1.5$; (b) 4; (c) 8. In all cases, $Re_z = 2000$, $A_{yz} = 1$. The vertical dashed lines mark the top of the box cycle. (d) Premultiplied frequency spectra of the vertical velocity fluctuation histories in (a-c), $fE_{v2}(f)$, as functions of the period $T = 2\pi/f$. —, (a); ---, (b); -·-·-, (c). The inset is a zoom of the region $1.3 \leq ST \leq 1.7$, and defines the amplification in figure 2.9(c).

history in figure 2.7(a).

The frequency spectra of the histories of $\overline{v^2}$ are given in figure 2.7(d). They have two well-differentiated components: a broad peak around $ST = 25$ that corresponds to the width of the bursts in figure 2.7(a), and sharp spectral lines that represent the average distance between the shorter bursts in figure 2.7(b,c). We will see below that these lines are at the box period and its harmonics. Note that these are not frequency spectra of the velocity v , but of the temporal fluctuations of its box-averaged energy, and that their dimensions are proportional to v^4 .

Note also that the broad ‘turbulent’ frequency peak is still present in the resonant cases, although it weakens progressively as the resonant component takes over.

The resonant lines correspond to the narrow bursts of figure 2.7(b,c), and get stronger with increasing A_{xz} . figure 2.8(a) shows that they are characterized by a temporary increase in the two-dimensionality of the flow. The open symbols in that figure show the fraction of $\overline{v^2}$ contained in the first few two-dimensional x -harmonics ($k_z = 0, k_x = 2\pi n_x/L_x, n_x = 1, \dots, 6$), plotted against $ST_s = A_{xz}/A_{yz}$. They account for a relatively constant fraction (approximately 20%) of the total energy. The solid symbols are the same quantity computed at the top of the box cycle, $t = nT_s$. It begins to grow at elongations of the order of $ST_s \approx 2$, and accounts for almost 60% of the total energy for the longest

Figure 2.8: (a) Energy in the first six two-dimensional modes ($k_z = 0$) of v as a function of the box period $ST_s = A_{xz}/A_{yz}$. Several values of A_{yz} are included. Open symbols are averaged over the whole history. Solid ones are conditionally averaged at the top of the box cycle. Symbols are gray-colored by the Reynolds number, from $Re_z = 1000$ (light-gray) to 3200 (dark-gray). (b) Bursting time scale, ST_b , obtained from the width of the temporal autocorrelation function of $\overline{v^2}$. The diagonal dashed line is $T_b = T_s$; the horizontal one is the bursting width of a single linearized Orr burst. Symbols as in table 2.1.

boxes. Note that the data collapse relatively well with ST_s , rather than with A_{xz} .

The growth of the two-dimensionality interferes with the overall behavior of the flow. The average length T_b of individual bursts is given in figure 2.8(b), computed as the width of the temporal autocorrelation function of $\overline{v^2}$, measured at $C_{v^2v^2} = 0.5$ (Jiménez, 2013a). For short boxes ($ST_s \lesssim 2$), the period stays relatively constant, $ST_b \approx 15$. But it decreases when the two-dimensionality begins to grow in figure 2.8(a). The diagonal dashed line in figure 2.8(b) is $T_b = T_s$, and strongly suggests that the effect of the long boxes is associated with the interaction of the box period with the bursting. Note that the bursting width for $ST_s \gg 2$ tends to be that of a linearized two-dimensional Orr burst (Orr, 1907).

That the spikes are associated with the box period is shown in figure 2.9(a), which displays the total spectral energy of the temporal evolution of $\overline{v^2}$ at period $T = T_s$ and its harmonics $T_{sn} = T_s/n_x$. It is negligible for boxes with $ST_s \lesssim 1$, but increases rapidly after that threshold. It then keeps increasing slowly until most of the $\overline{v^2}$ fluctuations are due to the spectral spikes.

The reason for the residual growth after resonance is seen in figure 2.9(b), where the energy in each harmonics is plotted separately versus T_{sn} . Each harmonic resonates with

Figure 2.9: (a) Total spectral energy in the harmonics of the box period in the frequency spectrum of $\bar{v}^2$, normalized with the total energy, and versus the box period. Symbols as in table 2.1. (b) Spectral energy in each of the harmonics of the box period, $T_{sn} = T_s/n_x$ ($n_x = 1, 2$ and 4), as in (a). Open symbols are for $n_x = 1$; grey, $n_x = 2$; black, $n_x = 4$. (c) The amplification G of the individual harmonics of the box period, as defined in the inset in figure 2.7(d). Symbols as in (b). The dashed line is (2.10). (d) Lines without symbols are the segment of the history of integrated intensities in figure 2.7(b), around $St = 35$. For comparison, the lines with symbols are the inviscid linearized solution (see 2.9) of the fundamental mode $k_x = 2\pi/L_x, k_z = 0$, scaled to the same amplitude of the peak of the corresponding mode, $\hat{v}_{10}$, of DNS. ---, $\bar{u}^2$; —, $\bar{v}^2$; -.-.-, $\bar{w}^2$.

its own box period, $ST_{sn} \approx 1$, after which its amplitude decreases slightly (note the wide vertical scale of these two figures). The slow growth of the spike energy beyond the resonance in figure 2.9(a) is due to the accumulation of new resonant harmonics.

In fact, if we accept that the spikes are due to the amplification of pre-existing background fluctuations, figure 2.9(c) shows that the maximum amplification takes place at

$ST_{sn} \approx 1$. This is true for each individual harmonic. The quantity $G \equiv E_{v2}(T_{sn})/\tilde{E}_{v2}(T_{sn})$ in this figure is sketched in the inset of figure 2.7(b), and is the ratio between the energy in the spectral spike, and the energy in the background spectrum at the same frequency when the spike is removed by linear interpolation across the sharp spectral line, $\tilde{E}_{v2}(T_{sn}) = (E_{v2}(T_{sn} + \Delta T) + E_{v2}(T_{sn} - \Delta T))/2$.

It is tempting to hypothesize that a phenomenon with the same periodicity as the box-passing period is due to the interaction between structures from neighboring shear copies. Figure 2.9(d) shows that this is not the case. It displays the details of the velocity amplitudes during one of the spikes, and it agrees almost exactly with the linearized Orr burst included in the figure as a comparison. Orr (1907) The overtaking of shear-periodic copies advected by the mean velocities in neighboring boxes is not very different from the shearing mechanism in Orr's. For example, the characteristics dip in u^2 during the burst of v^2 is the same in both cases (Umurhan & Regev, 2004; Jiménez, 2007, 2013a), and the essence of the interaction is the tilting of the structures due to the shear. However, the details are different. In the case of the shear-periodic boundary condition, the interacting structures pass each other at a constant vertical distance. It can then be shown that both the period and the width of the velocity fluctuations are of order T_s , and that their amplitude decays very fast as L_y/L_x increases, essentially because the shear-periodic copies move farther from each other (Jiménez, 2013a). In the case of the shear, the mechanism is local tilting, and the time scale of the amplification is S^{-1} . The width of each burst is $St = O(1)$, but the distance between bursts is still T_s . It is clear from figure 2.9(d) that this is the case here. Inspection of the sharp bursts in figure 2.9(b,c) shows that their width is a few shear times, independently of T_s .

On the other hand, Orr bursts are not periodic. The Orr mechanism is a linear amplification process that works on preexisting initial conditions of the right shape (backwards tilting wavefronts), even when they are only one component of a more complex flow. In turbulence, these conditions are created 'randomly' by nonlinear processes, and Orr-like bursts, not necessarily two-dimensional, occur whenever that happens. But the observed two-dimensional quasiperiodic bursting with the passing frequency of the box requires a more deterministic explanation. We have seen that the flow during the spikes is two-dimensional in the (x, y) plane. Taking the origin of time at the top of the burst, the inviscid solution of the linearized equations (Jiménez, 2013a) for two-dimensional perturbations with $k_z = 0$ and $k_y(0) = 0$ is a single burst (Moffatt, 1967; Townsend, 1976),

$$\hat{v}(t)/\hat{v}_0 = (k_x^2 + k_{0y}^2 + k_z^2)/(k_x^2 + k_y^2 + k_z^2) = (1 + S^2 t^2)^{-1}, \quad (2.9)$$

where $\hat{v}_0$ is the initial vertical velocity at the top of the burst, k_{0y} and $k_y = k_{0y} - Sk_x t$

Figure 2.10: Two-point spatial correlation function of $\hat{v}_{10}$. Frame (c) is the bottom of the box cycle, $t = T_s/2$. Time is from top to bottom, separated by $St = 0.5$ between frames. Isolines are $R_{vv} = (0.1:0.2:0.9)$. $Re_z = 2000$, $A_{xz} = 8$, $A_{yz} = 1$.

are the initial and time-evolving wave numbers. Figure 2.10 shows the evolution near the bottom of the box cycle ($t \approx T_s/2$) of the two-point correlation function of the $(n_x = 1, n_z = 0)$ harmonic $\hat{v}_{10}$ within the fundamental numerical box. A band of v is tilted forward, as in an infinite shear, and can only extend across the box boundaries by connecting with one of the several periodic copies of itself in the boxes immediately above and below the fundamental box. It does so linking with the closest copy. Consider the top boundary. In the first part of the box cycle, $t < T_s/2$, the upper-box structure closest to the fundamental one is the one ahead of it, and the band keeps evolving as an infinitely long forward-tilted sheet. At $t = T_s/2$ the distance from the fundamental to the two copies ahead and behind it is the same, and after that moment the trailing copy is the closest one. The figure shows that the correlation condenses into an oval structure and reconnects itself to the trailing copy. The result is a backwards-tilting layer that restarts the shearing amplification cycle.

A rough estimate of the amplifications in figure 2.9(c) can be obtained from this mechanism. The amplitude of v in an Orr burst, $v \propto \cos^2 \psi$, depends almost exclusively of the front inclination angle $\psi = \arctan(k_y/k_x)$ (Jiménez, 2013a). The tilting of the velocity fronts at the moment of reconnection is given by $\tan \psi_r = -L_x/2L_y = -ST_s/2$. The vertical velocity v_r is amplified as the layer moves towards the vertical ($\psi = 0$), and

the maximum amplification is $v/v_r = \cos^{-2} \psi_r = 1 + (ST_s/2)^2$. Remembering that the amplification in figure 2.9(c) is proportional to v^4 , we obtain

$$G \approx [1 + (ST_s/2)^2]^4. \quad (2.10)$$

In essence, longer boxes have stronger two-dimensional bursts because the longer box period gives the primary wavelength more time to amplify. (2.10) is plotted as a dashed line in figure 2.9(c), and captures reasonably well the rise of the amplification. Note that a similar estimate holds for the higher two-dimensional Fourier harmonics if L_x is substituted by the wavelength L_x/n_x , and T_s by T_{sn} . It is harder to explain why the spikes cease to be amplified beyond $ST_s \approx 2$. The most likely reason is nonlinearity. Figure 2.9(b) shows that the amplitude of the spikes at $ST_s \approx 2$ is already about 10% of the total, and it is probably inconsistent to describe them with a linear theory. Also, the spectra in figure 2.7(d) show that the spectrum of the turbulent bursts begins to grow beyond $ST \approx 3$ –4, so that the spikes and the normal turbulence begin to interact directly. In fact, the reason why the amplitude of the spikes does not decrease beyond $ST_s \approx 2$ in figure 2.9(b), even if the amplifications in figure 2.9(c) decrease, is that the Orr mechanism has stronger baseline perturbations on which to work. It is conceivable that the background turbulence acts as a eddy viscosity that prevents further growth of the Orr bursts.

2.3.3 The numerical regeneration of linear Orr bursting

Although it is tempting to interpret the reconnection process in physical terms, it is clear that it is an artifact of simulating the flow in a shear-periodic finite box. Homogeneous shear flow has no implied temporal periodicity, even for flows that are periodic in x , and the interactions described above do not exist. It is important to stress that only the averaged correlations in figure 2.10 are smooth. The instantaneous flow has smaller scales and is less ordered. It should also be noted that this two-dimensional linear bursting is unrelated with the streak instability in the self-sustaining process of shear turbulence (Waleffe, 1997), which is intrinsically three-dimensional.

To understand the numerical regeneration process we should go back to the sketch of the implied Fourier grid in figure 2.1(b). Note that the temporal evolution of the Fourier modes in (2.8), $k_y = \tilde{k}_y - Stk_x$, is the same as for those in the linearized RDT solution in (2.9). Therefore, the amplitude of v in each grid mode approximately satisfies the linearized RDT, which can be recast as $v \propto k_x^2/(k_x^2 + k_y^2)$. The decrease in k_y^2 as

the boundary condition approaches the top of the box cycle is the Orr amplification mechanism.

Consider the first streamwise Fourier mode, $n_x = 1$. Near the bottom of the box cycle, the two modes responsible for the largest-scale structures are those marked as Λ and Λ' in figure 2.1(b). The top mode, Λ , whose wave-vector points upwards ($k_y > 0$), is a backwards-tilted front such as the one in figure 2.10(e). The bottom mode Λ' is a forward-tilted front. Statistically, both modes are similarly forced by the nonlinearity, and their respective amplitudes are due to the linear processes associated with their changing wavevector. For $t < T_s/2$, mode Λ' is closest to $k_y = 0$ and is therefore the strongest of the two. As time increases, Λ' moves away from $k_y = 0$ and weakens, while mode Λ moves closer and strengthens. At $t = T_s/2$ both modes are equally intense on average. After that moment, the upper, backwards-tilted mode predominates, and the structures flip backwards.

The linear Orr bursting is not restricted to long boxes. Whenever a particular Fourier mode crosses the $k_y = 0$ axis, the linear amplification process occurs to some extent, even at moderate box aspect ratios. There are two time scales involved that correspond to the two spectral components in figure 2.7(d). Two-dimensional linear bursts have widths of the order $St \approx 2$, and are triggered by the boundary conditions at longer intervals determined by the aspect ratio λ_x/L_y of the particular wavelength involved. The strongest resonance is for wavelengths of the order of L_x , because they are the largest ones, and interact most strongly with their periodic copies. They are triggered at intervals determined by the box aspect ratio L_x/L_y , and are the ones discussed in this section. Shorter harmonics also burst linearly, but they are seldom two-dimensional, and their behavior tends to be dominated by nonlinearity. The direct detection of the linear bursting in minimal channels is investigated in Jiménez (2015). The result is that, once a burst is initiated by other causes, its evolution can be predicted linearly over times of the order of 10–20% of an eddy turnover, corresponding to a tilting interval $\psi = -\pi/4$ to $\psi = \pi/4$. Two-dimensional linear bursts become dominant in long boxes because there are few nonlinear structures at the scale of the box length, and those that exist are only the ones triggered by the two-dimensional boundary conditions.

2.4 Comparison with other shear flows

The previous section delimits the set of box aspect ratios in which the flow is as free as possible from the artifacts of a finite simulation domain. The ‘acceptable’ region is

Run	Re_z	N_x, N_y, N_z	ST_{stat}	Re_λ	S^*	ω'/S	L_ϵ/L_z	ξ	ζ
L32	2000	190, 192, 94	1706	47	6.82	5.36	0.41	0.092	0.104
L34	2000	190, 384, 94	1581	48	6.97	5.30	0.42	0.095	0.105
L38	2000	190, 768, 62	3177	48	7.02	5.34	0.43	0.097	0.106
L44	2000	190, 384, 94	1570	48	7.10	5.28	0.43	0.093	0.100
M32	12500	766, 512, 254	1029	105	7.53	10.9	0.38	0.101	0.108
M34	12500	766, 1536, 382	1953	111	7.70	11.2	0.41	0.105	0.111
H32	48000	2046, 2048, 1022	334	243	7.57	24.8	0.45	0.096	0.102

Table 2.2: Parameters and statistics for the SS-HST simulations within the ‘balanced’ range. Runs are labelled by their aspect ratios. Thus, L32 has $A_{xz} = 3$ and $A_{yz} = 2$. $L_\epsilon = (q^2/3)^{3/2}/\epsilon$ is the integral scale, and ξ and ζ are the modified Lumley invariants, as explained in the text.

summarized in figure 2.2(a) as the trapezoidal central region in which cases are marked as circles (see also table 2.1). Note that very tall and long boxes are defined by a further uncertain limit that has not been discussed up to now. Very long and tall boxes satisfy all the above criteria, but the single example tested in that region ($A_{xz} = A_{yz} = 8$ at $Re_z = 3000$ marked as a down-pointing triangle in figure 2.2(a)) behaved strangely, with very deep three-dimensional bursts that could not be described as Orr bursts. Those simulations are expensive to run. They are still minimal in the spanwise direction, but otherwise very far from our desired range of minimal flows, and this range was not pursued further.

A full discussion of what can be learned about the physics of shear turbulence from such simulations is left to other chapters, but we briefly review in this section how similar the statistically stationary HST is to other shear-dominated flows, particularly the logarithmic layer of wall-bounded turbulence and the initial shearing of isotropic turbulence.

Some of those comparisons have already been made in previous sections or publications. The numerical parameters and some basic statistics of the acceptable cases among our runs are summarized in table 2.2. When discussing figure 2.4, we mentioned that the velocity fluctuation intensities are similar in SS-SHT and in the logarithmic layer. The correspondence improves if we center our attention on the acceptable cases marked as circles, with the exception of v'^+ , which is stronger in the SS-HST than in channels ($v'^+ \approx 1.2$ – 1.3 against 1.1). The relation between the different intensities is best expressed by the Lumley invariants of the Reynolds-stress anisotropy tensor $b_{ij} = \langle u_i u_j \rangle / \langle u_i u_i \rangle - \delta_{ij}/3$, which are shown in figure 2.11(a) in the modified form, $6\zeta^2 = b_{ij}b_{ji}$, $6\zeta^3 = b_{ij}b_{jk}b_{ki}$ (Pope, 2000).

Figure 2.11: (a) Modified Lumley invariants of the Reynolds-stress-anisotropy tensor. Pope (2000) The dark solid curve at the top of the figure, and the right-hand vertical axis are the realizability limit. (b) S/ω' as a function of Re_λ . - - - - , $SRe_\lambda/\omega' = 11$. Lines are large-box simulations of channels at $Re_\tau = 934$ (del Álamo & Jiménez, 2004), 2003 (Hoyas & Jiménez, 2006), and 4200 (Lozano-Durán & Jiménez, 2014a). Those in (a) go from the wall to the channel center, and those in (b) go from $y^+ = 50$ to $y/h = 0.4$, with y increasing from left to right. Higher Reynolds numbers correspond to longer lines. The solid segments represent the logarithmic layer between $y^+ = 100$ and $y/h = 0.2$. Circles are SS-HST in table 2.2: (white) L32, L34, L38, L44 from left to right; (grey) M32, M34; (black) H32.

The figure includes results for large channels. They vary from roughly axisymmetric turbulence, dominated by u' , near the wall, which is mapped in the upper-left part of the figure, to approximate isotropy in the center of the channel, mapped in the lower-right corner. The logarithmic layer is characterized by an approximately constant second-order invariant, which is also where the SS-HST tends to concentrate. The tendency of the latter to be more isotropic than the channel reflects its stronger v' . The velocities tend to be more isotropic as the Reynolds number increases, although there is a residual effect of the aspect ratios for the SS-HST. Numerical values are given in table 2.2.

The strongest connection between the SS-HST and the logarithmic layer is that both are approximately equilibrium flows in which the energy production equals dissipation. The local energy balance can then be expressed as $u_\tau^2 S = \nu \omega'^2$ (Jimenez, 2013b), and links quantities that are usually associated with large and small scales. One example is the Corrsin parameter $S^* = Sq^2/\langle \varepsilon \rangle$, which was shown in Jimenez (2013b) to be $O(10)$ in various shear flows, including some of the present SS-HST simulations (see table 2.2). Another example is the ratio S/ω' , also listed in table 2.2, which is shown in figure 2.11(b) as a function of Re_λ . The SS-HST and the channels agree well. For both quantities, the

Figure 2.12: (a) Normalised premultiplied two-dimensional spectrum $k_x k_z E_{vv}/v'^2$. Contours are $[0.045:0.02:0.085]$. (b) Two-point correlation function of the vertical velocity, $R_{vv}(\delta_x, \delta_y)$. Contours are $[0.1:0.2:0.9]$. Shaded contours are a minimal channel (Flores & Jiménez, 2010): $Re_\tau = 1840$, $L_x = \pi h/2$, $L_z = \pi h/4$; solid, full channel (Hoyas & Jiménez, 2006): $Re_\tau = 2003$, $L_x = 8\pi h$, $L_z = 3\pi h$; dashed, SS-HST (M32). The spectra for both channels are at $y/h \approx 0.15$, where $Re_\lambda \approx 100$ as in the HST case. This is also the reference height for the correlations. Lengths are normalised with the Corrsin scale $L_c = L_\epsilon(3/S^*)^{3/2}$.

energy equation can be manipulated to give exact expressions: $S^* = q^{2+}$ and $SRe_\lambda/\omega' = \sqrt{5/3}q^{2+}$. The agreement in figure 2.11(b) and the approximate universality of S^* are then equivalent to the agreement of the intensities in figure 2.4. Nonequilibrium flows, such as the initial shearing of isotropic turbulence (Rogers & Moin, 1987; Lee *et al.*, 1990; Sarkar, 1995; Isaza & Collins, 2009), do not agree either with the channels or with the SS-HST. Most of them do not even fall within the ranges represented in figure 2.11(a,b).

A more diagnostic property is the geometry of the structures. Figure 2.12(a) shows the premultiplied two-dimensional spectrum of the vertical velocity of the SS-HST (M32 in table 2.2), compared with those of a large channel (Hoyas & Jiménez, 2006) and of a minimal one (Flores & Jiménez, 2010). The wall distance of the channels ($y/h \approx 0.15$) is chosen to match the three Reynolds numbers to $Re_\lambda \approx 100$, and falls within the logarithmic layer. The boxes of the SS-HST simulation and of the minimal channel are too small to capture the largest scales of v , but the agreement of the three cases is surprisingly good. The small scales of the spectrum of u and w (not shown) also agree well, although both quantities, which are typically larger than v , are severely truncated in the two minimal boxes.

It is hard to define spectra in the non-periodic vertical direction, but figure 2.12(b) displays the two-point correlation function $R_{vv}(\delta_x, \delta_y)$ for the same three cases, centered at

the same location as in figure 2.12(a). The upper part of the correlations agrees relatively well for the SS-HST and the large channel, even if the top of the outermost solid contour is at $y/h \approx 0.45$, where the shear in the channel is three times weaker than at the reference point. The upper part of the minimal channel agrees worse, probably because its computational box is too narrow to allow good statistics above $y \approx 0.3h$ (Flores & Jiménez, 2010).

The lower part of the correlations agree well for the two channels, but the SS-HST extends much farther than any of the other two. The lower part of the outermost contour of the two channels is very near the wall, but the SS-HST has no wall, and is symmetric with respect to its center. It is interesting that, in spite of the large differences in their lower part, the geometry of the upper part of the turbulent structures is so similar between the three flows. The extra freedom of the vertical velocity in the absence of the wall is probably responsible for the slightly higher intensities of this velocity component in the SS-HST case. As in the spectra, the small scales of u and w agree well, but the differences of the simulation boxes are too large to allow a useful comparison between these larger structures.

The length scale to be used in normalizing figure 2.12 is not immediately obvious. Viscous wall units are not relevant for the SS-HST, and it is indeed found that they collapse the correlations poorly. The integral length L_ε works better but, after some experimentation, the best normalization is found to be the Corrsin scale. This is probably physically relevant. That length was introduced in Corrsin (1958) as the limit for small-scale isotropy in shear flows, and it is roughly the scale of the smallest tangential Reynolds stresses. It can be interpreted as the energy-injection scale for shear flows, and it is related to the integral scale by $L_c = (\varepsilon/S^3)^{1/2} = L_\varepsilon(3/S^*)^{3/2}$. We have mentioned that S^* is fairly similar in SS-HST and channels, but the differences are enough to substantially improve the collapse of the correlations and of the spectra.

We finally compare the bursting behavior of SS-HST and wall-turbulence. Figure 2.13 shows the evolution of the streamwise-velocity streak in SS-HST during a burst. Comparison with the logarithmic layer of a minimal channel in figure 3 of Flores & Jiménez (2010) strongly suggests that the two phenomena are related.

Previous investigations of SS-HST have suggested that the growth phase of bursts is qualitatively similar to the shearing of initially isotropic turbulence (Pumir, 1996; Gualtieri *et al.*, 2002). Figure 2.14(a) provides some quantitative evidence and qualifications. It contains probability density functions (p.d.f.) for the logarithmic growth rate of the kinetic energy, $\Lambda = d(\log \overline{q^2})/d(St)$, in SS-HST and minimal channels. This quantity

Figure 2.13: (a) Temporal evolution of $-\overline{uv}_{xz}$, conditionally-averaged for $v > 0$ over wall-parallel planes. Strong bursting events are represented by grey-contour. (b) Isosurface of the total streamwise velocity $u + Sy = -2SL_z$ at three moments of a bursting event marked by red circles in (a). Only one fourth of the vertical domain is shown, centered at $y/L_z = -2$. The isosurfaces are coloured by the mean streamwise velocity Sy . Case L38.

has been widely discussed for the shearing of initially isotropic turbulence, and is believed to settle asymptotically to $\Lambda \approx 0.1 - 0.15$ in weakly sheared cases ($S^* \lesssim 10$) (Rogers & Moin, 1987; Sarkar, 1995), and to somewhat higher values, $\Lambda \approx 0.2$, in strongly sheared ones (Lee *et al.*, 1990; Isaza & Collins, 2009). In statistically stationary flows, the mean value of Λ vanishes, but the distribution of its instantaneous values can be used as a measure of how fast bursts grow during their generation phase, or otherwise decay.

The p.d.f.s for the SS-HST flows in figure 2.14(a) are for three cases with the same aspect ratios, $A_{xz} = 3$, $A_{yz} = 2$, but different Reynolds numbers. Those for the minimal channels are compiled over the range of height $y/L_z = 0.15 - 0.3$, which is known to be roughly minimal for the largest structures of the logarithmic layer (Flores & Jiménez, 2010). By changing Re_τ and the width L_z of the simulation box, the Taylor-microscale Reynolds number of the three channels are also made different. The agreement of the

Figure 2.14: (a) P.d.f. of the instantaneous growth rates of the kinetic energy. Lines without symbols are minimal channels (Flores & Jiménez, 2010) in the band $y/L_z \approx 0.13$ – 0.25 : —, $(Re_\tau, Re_\lambda) = (1830, 125)$; - - - - , $(1700, 80)$; - - - , $(950, 90)$. Lines with symbols are SS-HST from table 2.2: $\circ$, L32; $\triangle$, M32; ∇ , H32. The horizontal bar is the range of growth rates for weak initial shearing of isotropic turbulence. (b) Temporal autocorrelation function of $\overline{v^2}$. The channels are as in (a), but the SS-HST are now: $\circ$, L32; $\triangle$, M32; ∇ , M34.

different p.d.f.s again supports that the dynamics of bursting in minimal channels and in SS-HST is similar. The range of initial growth rates found in weakly sheared turbulence is marked in figure 2.14(a) by a horizontal bar. It spans the near-tail of the p.d.f.s, consistent with the interpretation mentioned above that the growth of individual bursts is similar to the weak shearing of initially isotropic turbulence. The values of S^* in table 2.2 confirm that SS-HST is in the weakly sheared regime.

The p.d.f. of Λ depends on the size of the computational box. Taller SS-HST boxes or channel heights closer to the wall produce narrower distributions. The common property of the boxes in figure 2.14(a) is that they are minimal, in the sense that they are expected to contain a single large structure on average. Taller boxes can be expected to contain several structures, evolving roughly independently. The same is true of near-wall planes in the channel, where several small near-wall structures fit within the simulation box. In those cases, the energy of the individual structure add to the total energy and the standard deviation of the total growth rate should decrease roughly as the square root of the expected number of structures. This is confirmed by the SS-HST simulations. The standard deviation of Λ in cases L32, L34 and L38, expected to contain one, two and four structures, are in the ratio 1.96, 1.39 and 1, which are almost exactly powers of $\sqrt{2}$.

The situation is the opposite for the temporal correlation functions used to compute the bursting period in figure 2.8(b), a few of which are displayed in figure 2.14(b). It

was shown in [Jimenez \(2013b\)](#) that the width of the temporal correlation of v is of the same order of magnitude in both flows, but figure 2.8(b) shows that the correlations of the SS-HST are somewhat wider than in the channels. They also widen with increasing Reynolds number. Two of the SS-HST simulations in figure 2.8(b) (L32 and M32) have the same aspect ratios and different Re_λ . The higher Reynolds number is substantially wider than the lower one. Two other simulations (M32 and M34) have similar Reynolds numbers and different aspect ratios, and their correlations agree. It can be shown that the wider correlations are due to the different behavior of the higher Fourier modes. If these are filtered out, the temporal correlations of the longest Fourier mode ($n_x = 1$) collapse for all Reynolds numbers.

The width of individual bursts, $St \approx 20$, is thus determined by the linear transient amplification of random nonlinear perturbations, as already shown in [Jiménez \(2013a\)](#); [Jimenez \(2013b\)](#). On the other hand, the time between consecutive bursts is determined by how often these particular initial conditions are generated, and is beyond the scope of the thesis. The long-term behavior of sheared turbulence is a controversial subject ([Cambon & Scott, 1999](#)), but the wider consensus seems to be that it grows indefinitely, both in intensity and in length scale ([Rogers & Moin, 1987](#); [Tavoularis & Karnik, 1989](#); [Isaza & Collins, 2009](#)). In simulations, the growth of the length scale will eventually interfere with any finite computational box, and simulations of the initial shearing have traditionally been discontinued at that moment. It is generally believed that the periodic bursting of statistically stationary HST is due to the periodic filling of the computational box by the growing length scales. We have seen in §2.3.3 a particular numerical artifact leading to the regeneration of linear bursts, and there is circumstantial evidence from our simulations that some of the length scales reach the box limit before the collapse of the intensities. On the other hand, figure 2.8(b) shows that the temporal width of the bursts is relatively independent of the box dimensions within certain limits, and that it scales well with the shear. A similar argument about length scales could be made about channels, although inhomogeneity and the wall may play a role similar to geometry in that case. However, we have just shown that channels share many properties with SS-HST, and it was shown in [Jimenez \(2013b\)](#) that the bursting period of minimal channels, which are geometrically constrained, is similar to the lifetime of individual structures in larger simulations ([Lozano-Durán & Jiménez, 2014b](#)), which are not. Both are of the same order as the bursting width of SS-HST (figure 2.14b). In summary, it appears that bursting is a property of shear flows in general, not linked to the presence of a wall, but its properties require further work.

2.5 Conclusions

We have performed direct numerical simulations of HST to explore the parameter range of SS-HST. Our code uses a shear-periodic boundary condition in the vertical direction that requires no periodic remeshings, and that is implemented directly on the compact finite difference. The other directions are spectral. Validations collected in § confirm that the code maintains its designed accuracy, and reproduces well previous simulations. In all those cases, our small-scale statistics are slightly higher than those in remeshing codes using the same Fourier modes (Rogallo, 1981), probably because the lack of remeshing prevents the loss of some enstrophy. We have given a simple theoretical analysis for why that should be so.

The statistics of SS-HST are strongly dependent on the geometry of the computational box, represented by its two aspect ratios, $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$. We have shown that the relevant length scale is the spanwise width of the box, L_z , and that the velocity scale is SL_z . The relevant Reynolds number is therefore $Re_z \equiv SL_z^2/\nu$, but the characteristics of the largest-scale motions in SS-HST are found to be fairly independent of Re_z . It is interesting that L_z is also the dimension that determines the range of valid wall distances in minimal turbulent channels (Flores & Jiménez, 2010). Since it is believed that the large scales of theoretical homogeneous shear flow have no characteristic length, flows simulated over long enough times tend to ‘fill’ any computational box, and long-term simulations of SS-HST are always ‘minimal’ in the sense that there are only a few largest-scale structures in the computational box.

The empirical evidence is that the effect of the geometry can be reduced by ensuring that box dimensions other than L_z do not constrain further these minimal structures. The limits identified are $A_{xz} > 2$ and $A_{yz} > 1$. A similar argument can be made for minimal channels (Flores & Jiménez, 2010), even though ‘natural’ structures are always influenced by the wall in that case. The effect of minimal boxes is to override that constraint when the box becomes too narrow far enough from the wall. It follows that the limit in which the channel is ‘just minimal’ ($y \approx L_z/3$) (Flores & Jiménez, 2010) should have similar properties to ‘just minimal’ SS-HST.

That turns out to be approximately true. The one-point statistics of SS-HST in the suitable range of the aspect ratios agree surprisingly well with those of the logarithmic layer in turbulent channel flows, particularly when scaled with the friction velocity derived from the measured Reynolds stresses. The same is true for the wall-parallel spectra of the wall-normal velocity, although the length scales of the other two velocity components are typically too large to be fruitfully compared either with SS-HST or with minimal channels.

The wall-normal spatial correlation of v also agrees well with channels and boundary layers, but only in the direction away from the wall. The correlations of channels in the direction of the wall are limited by impermeability, but those of the SS-HST are not, and extend symmetrically downwards. It is interesting that even this strong difference does not influence the top part of the correlation.

An interesting limit of SS-HST that does not appear to exist in minimal channels is that of very long boxes ($A_{xz} \geq 2A_{yz}$). Their bursting is dominated by a two-dimensional numerical regeneration process associated with the interaction between shear-periodic copies of the numerical box. It can be treated analytically to a large extent, but appears to be unrelated to physics.

A common characteristic of SS-HST and wall-bounded turbulence is quasi-periodic bursting, and we have shown that it shares many common characteristics in both flows. Besides strong similarities of the flow fields, the lifetime of individual bursts, defined from the temporal autocorrelation function, scales with the shear in both cases as $ST_b \approx 20$. In contrast with the numerical two-dimensional bursts described in the previous paragraph, these ones involve the quasi-simultaneous growth of the three velocity components, and presumably originate from the linear amplification of three-dimensional ‘dangerous’ initial conditions (Schmid, 2007; Jiménez, 2013a) randomly found as parts of the nonlinear turbulent field. The probability distribution of the growth rates of the intensities suggests that the amplification phase of the bursts is similar to the weak shearing of initially isotropic turbulence, where the generation of initial conditions is presumably the same. In general, it is concluded that the similarities between SS-HST and other shear flows, particularly with the logarithmic layer of wall turbulence, make it a promising system in which to study shear turbulence in general.

2.A Details about the numerical method

2.A.1 Add the mean shear

Start from the equation for flow fields without the mean value, i.e., $\mathbf{U}=(0,0,0)$ and $\mathbf{\Omega}=(0,0,0)$

$$\frac{\partial \nabla^2 v}{\partial t} = h_v - \nu \nabla^2 \nabla^2 v \quad (2.11)$$

$$\frac{\partial \omega_y}{\partial t} = h_g - \nu \nabla^2 \omega_y \quad (2.12)$$

where

$$h_v = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_2 - \frac{\partial}{\partial y} \left(\frac{\partial H_1}{\partial x} + \frac{\partial H_3}{\partial z} \right) \quad (2.13)$$

and

$$h_g = \frac{\partial H_1}{\partial z} - \frac{\partial H_3}{\partial x}. \quad (2.14)$$

When the mean velocity and vorticity are considered,

$$h'_v = h_v - Sy \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \omega_z - \frac{\partial^2 \omega_y}{\partial y \partial z} \right) + S \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u + \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial \omega_y}{\partial z} \right), \quad (2.15)$$

and

$$h'_g = h_g - S \frac{\partial v}{\partial z} - Sy \frac{\partial \omega_y}{\partial x}. \quad (2.16)$$

Based on the continuity condition, it is quite easy to find

$$\begin{aligned} & -Sy \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \omega_z - \frac{\partial^2 \omega_y}{\partial y \partial z} \right) \\ &= -Sy \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) - \frac{\partial^2}{\partial y \partial z} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \right) = -Sy \frac{\partial}{\partial x} \nabla^2 v, \end{aligned} \quad (2.17)$$

and

$$S \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u + \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial \omega_y}{\partial z} \right) = S \left(\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) u + \frac{\partial^2 v}{\partial x \partial y} - \left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x \partial z} \right) \right) = 0. \quad (2.18)$$

Thus we got the equations (2.1) and (2.2).

2.A.2 Three-step fully-explicit Runge-Kutta with analytically integrations for the shear convective terms

Applying the Fourier transform to the governing equations (2.1), we have in general,

$$\frac{\partial f}{\partial t} + ik_x Sy f = R(t, f), \quad (2.19)$$

where f represent any of $\widehat{\omega}_y$, $\widehat{\phi}$, $\overline{u}_{xz}$, or $\overline{w}_{xz}$. We analytically absorb the linear shear convective term $ik_x Sy f$ in (2.19) by multiplying it by the integrating factor $\exp(ik_x Sy t)$,

$$\frac{\partial(e^{ik_x Sy t} f)}{\partial t} = e^{ik_x Sy t} R(t, f). \quad (2.20)$$

The semi-discrete form of the three-step fully-explicit Runge–Kutta scheme (Spalart *et al.*, 1991) to advance from $f(t)$ to $f(t + \Delta t)$ leads to,

$$\begin{aligned} f^* &= f + \gamma_1 \Delta t R(t, f), \\ f_1 &= e^{-ik_x Sy c_1 \Delta t} f^*, \end{aligned} \quad (2.21)$$

$$R_1 = e^{-ik_x Sy c_1 \Delta t} R(t, f), \quad (2.22)$$

$$\begin{aligned} f^{**} &= f_1 + \gamma_2 \Delta t R(t + c_1 \Delta t, f_1) + \zeta_1 \Delta t R_1, \\ f_2 &= e^{-ik_x Sy (c_2 - c_1) \Delta t} f^{**}, \end{aligned} \quad (2.23)$$

$$R_2 = e^{-ik_x Sy (c_2 - c_1) \Delta t} R(t + c_1 \Delta t, f_1), \quad (2.24)$$

$$\begin{aligned} f^{***} &= f_2 + \gamma_3 \Delta t R(t + c_2 \Delta t, f_2) + \zeta_2 \Delta t R_2, \\ f_3 &= e^{-ik_x Sy (c_3 - c_2) \Delta t} f^{***}, \end{aligned} \quad (2.25)$$

where f^* , f^{**} , f^{***} , f_1 and f_2 represent the intermediate variables at each Runge–Kutta sub-step, $i=\{1,2,3\}$, and $f_3 = f(t + \Delta t)$ corresponds to the next time step. The coefficients are:

$$\gamma_i = \left\{ \frac{8}{15}, \frac{5}{12}, \frac{3}{4} \right\}, \quad \zeta_i = \left\{ -\frac{17}{60}, -\frac{5}{12} \right\}, \quad c_i = \left\{ \frac{8}{15}, \frac{2}{3}, 1 \right\}. \quad (2.26)$$

This scheme is third-order consistent. The additional operations over a traditional integrator are the five ‘unmapping’ multiplications in (2.21)–(2.25) by $\exp[-ik_x Sy (c_{i+1} - c_i) \Delta t]$ ($c_0 = 0$). In our simulations, the cost of mapping is roughly 10% of the total, but it reduces the advective CFL by the ratio $2u'/SL_y$, which can be considerable, especially for tall computational boxes, $A_{yz} > 1$.

A semi-implicit scheme for the viscous term could also be used (Spalart *et al.*, 1991), but it is useful only at very low Reynolds numbers (roughly $Re_z < 1000$ in the present case) for which the viscous CFL leads to a smaller time step than the advective one.

2.A.3 Compact finite difference with the shear-periodic boundary condition

In order to compute derivatives in the vertical direction (y), we use a compact-finite-differences scheme Lele (1992) based on a seven-point stencil with 6th- and 8th-order

resolution accuracy for the first and second derivative, respectively. Exact spectral behavior is enforced at the wavenumbers $k\Delta y/\pi = 0.5, 0.7, 0.9$ for the first derivative, and $k\Delta y/\pi = 0.5, 0.9$ for the second one. The modified wavenumber $k'\Delta y$ estimated by Fourier analysis for the compact finite differences described in this section stays close to the exact differentiation over a range of wavenumbers $k'\Delta y \leq 2.5$, which is used for the estimation of the resolution requirements of the DNS. The consistency errors, $\varepsilon_1 \equiv |k' - k|/k$ for the first derivative, and $\varepsilon_2 \equiv |k'^2 - k^2|/k^2$ for the second one, are $\varepsilon_1 \approx 0.006$ and $\varepsilon_2 \approx 0.005$, respectively, at the adopted resolving efficiency $k\Delta y = 2.5$.

The discretized form of the n -th derivative of $f(y_j) \approx F_j$ in the y -direction, where $y_j \equiv (j - 1)L_y/N - L_y/2$, $j = 1, \dots, N$, is written as

$$BF^{(n)} = AF, \quad (2.27)$$

where $F^{(n)}$ represents the n -th derivative of F . Assuming an even derivative, the structure of the matrix B is

$$B = \begin{pmatrix} \delta & \alpha & \beta & \gamma & 0 & \cdots & 0 & \gamma'^* & \beta'^* & \alpha'^* \\ \alpha & \delta & \alpha & \beta & \gamma & 0 & \cdots & 0 & \gamma'^* & \beta'^* \\ \beta & \alpha & \delta & \alpha & \beta & \gamma & 0 & \cdots & 0 & \gamma'^* \\ \gamma & \beta & \alpha & \delta & \alpha & \beta & \gamma & 0 & \cdots & 0 \\ 0 & \gamma & \beta & \alpha & \delta & \alpha & \beta & \gamma & 0 & \cdots \\ \vdots & & \ddots & & & \ddots & & & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma & \beta & \alpha & \delta & \alpha & \beta & \gamma \\ \gamma' & 0 & \cdots & 0 & \gamma & \beta & \alpha & \delta & \alpha & \beta \\ \beta' & \gamma' & 0 & \cdots & 0 & \gamma & \beta & \alpha & \delta & \alpha \\ \alpha' & \beta' & \gamma' & 0 & \cdots & 0 & \gamma & \beta & \alpha & \delta \end{pmatrix}, \quad (2.28)$$

and A has the same structure of non-zero entries, with different coefficients. Note that α , β , γ and δ are constant real values. The application of the shear-periodic boundary condition

$$F_j(t, k_x, k_z) = F_{j+N}(t, k_x, k_z) \exp[ik_x SL_y t], \quad (2.29)$$

to the compact finite difference matrices appears in its off-band-diagonal elements, which are complex α' , β' , γ' , and their complex conjugates α'^* , β'^* , and γ'^* . Specifically, $\alpha' = \alpha \exp[-ik_x SL_y t]$, etc., which is used to substitute off-grid elements by their shifted copies near the opposite boundary. I.e., $F_0 = F_N \exp[ik_x \Delta U t]$, $F_{N+1} = F_1 \exp[-ik_x \Delta U t]$, etc., where $\Delta U = SL_y$ is the mean velocity difference between the two boundaries. Therefore, A and B are time-dependent Hermitian and need to be updated at each Runge-Kutta sub-step. Odd derivatives are handled similarly with a skew-Hermitian A .

The linear system (2.27) is directly solved by applying the modified Cholesky decomposition $B = LDL^*$,

$$L = \begin{pmatrix} 1 & 0 & & & & & & & & 0 \\ a_2 & 1 & 0 & & & & & & & \\ b_3 & a_3 & 1 & 0 & & & & & & \\ c_4 & b_4 & a_4 & 1 & 0 & & & & & \\ 0 & c_5 & b_5 & a_5 & 1 & 0 & & & & \\ \vdots & & \ddots & & & \ddots & & & \ddots & \vdots \\ 0 & \cdots & 0 & c_{N-3} & b_{N-3} & a_{N-3} & 1 & 0 & & \\ e_1 & e_2 & \cdots & & e_{N-5} & e_{N-4} & e_{N-3} & 1 & 0 & \\ f_1 & f_2 & \cdots & & \cdots & f_{N-4} & f_{N-3} & f_{N-2} & 1 & 0 \\ g_1 & g_2 & \cdots & & & \cdots & g_{N-3} & g_{N-2} & g_{N-1} & 1 \end{pmatrix}, \quad (2.30)$$

and

$$D = \begin{pmatrix} d_1 & & & & 0 \\ & \ddots & & & \\ & & d_i & & \\ & & & \ddots & \\ 0 & & & & d_N \end{pmatrix}. \quad (2.31)$$

The modification in the time-marching in DNS is done only for the three complex lines e_i , f_i and g_i for the matrix L , and their complex-conjugates for L^* . Note that the band-diagonal elements a_i , b_i , c_i and the diagonal elements d_i are real constant.

The one-dimensional Helmholtz equation, expressed generally as $F^{(2)} + \lambda F = R_f$ (where λ is real) leads to a linear system $(A + \lambda B)F = BR_f$, which can be solved for F by applying the modified Cholesky decomposition of the Hermitian operator $(A + \lambda B)$.

2.A.4 Validations

Rapid distortion theory

Figure 2.15 shows the time-integrated relative error, $u_e^2 = t_l^{-1} \int_0^{t_l} \overline{|u - u_{\text{RDT}}|^2} / |\mathbf{u}_0|^2 dt$, between the streamwise velocity in the present DNS and the corresponding RDT solution. Both in DNS and RDT, $S = 1$ and $Re_z = 4 \times 10^4$. The initial condition is a sine wave, $\mathbf{u}_0 = (0, v_0, 0) \sin(k_{0x}x + k_{0z}z)$, with initial wavenumbers $\mathbf{k}_0 L_z = (0.5, 0, 1)$. In the DNS, the initial amplitude is $|v_0|/(SL_z) = 10^{-3}$, and the box aspect ratios are $(A_{xz}, A_{yz}) =$

Figure 2.15: Relative error u_e for the streamwise velocity, compared with the corresponding linear RDT solution. (a) For different grids in y , as a function of the CFL. —○—, $N_y = 16$; —△—, 24; —□—, 32; --○--, 48; --△--, 64. In all cases, $N_x = N_z = 18$. The chain-dotted line has slope 3. For other parameters, see text. (b) As in (a), as a function of N_y . —○—, CFL = 0.02; —△—, 0.05; —□—, 0.1; —▽—, 0.2; --○--, 0.4; --△--, 0.6; --□--, 0.8. The chain-dotted line has slope -6.

$(2, 1/\pi)$, so that the box always contains a single wavelength in the horizontal plane. The simulations run from $t = 0$ to $St_l = 15$, by which time the magnitude of the vertical wavenumber reaches $|k_y \Delta y| \approx 1$ for $N_y = 16$. This is a typical minimum resolution in later turbulence simulations. Note that the cases in figure 2.15 imply run times of roughly 2 box periods, in spite of which the figure shows that the numerical scheme retains its third- and sixth-order consistency in time and space, respectively.

Initial shearing of isotropic flow

To validate the nonlinear terms of the code, the short-term shearing of an initially isotropic turbulent flow is compared with the classical results of Rogers & Moin (1987), as given in the data set HOM23 of the AGARD database (AGARD, 1998), whose naming notation we use. The initial conditions are random isotropic fields with a top-hat one-dimensional energy spectrum, as in Rogallo (1981), adjusted to the same parameters as in Rogers & Moin (1987). They all agree well with the reference data, but the energy dissipation of our DNSes is slightly higher than in the reference cases after $St \approx 5$ (see later figure 2.16(b)), probably because of the periodic loss of the enstrophy in the remeshing process of Rogallo’s Fourier code (Lee *et al.*, 1990; Rogallo, 1981). Note that the reference simulations in Rogers & Moin (1987) were remeshed every $St = 2$. Other quantities,

Figure 2.16: Effect of the grid resolution. Case HOM23U (Rogers & Moin, 1987). (a) The time evolution of the effective resolutions, $\mathbf{k}_{\max} \eta(t)$. Lines with symbols are $k_x \eta$; without symbols are $k_y \eta$. (b) Evolution of the energy dissipation rate. $\circ$, HOM23U (Rogers & Moin, 1987). In both figures, ---, fine grid, (510, 384, 254); —, coarse grid (126, 192, 126).

Figure 2.17: Auto-correlation functions for streamwise (a), vertical (b), and spanwise (c) velocities along x - (black), y - (red), z - (blue). —, present DNS ($192 \times 192 \times 192$ with $CFL=0.6$); --- $\circ$ ---, HOM23U in AGARD (1998), from Rogers & Moin (1987). $St = 10.0$.

such as the two-point velocity correlation functions, were checked in detail against case HOM23U (figure 2.17). They also agree well, confirming that the large scales of the present DNS are consistent with those of the three-dimensional Fourier spectral simulations.

The previous results do not test the effect of dealiasing in y , which is applied in Rogers & Moin (1987) but not in our case. This is tested in figure 2.16 by comparing the results of simulating case HOM23U in its original grid with a much finer grid with $(N_x, N_y, N_z) = (510, 384, 254)$. Figure 2.16(a) shows the temporal evolution of the effective resolutions $\mathbf{k}_{\max} \eta(t)$, where $\eta(t) = (\nu^3 / \bar{\varepsilon})^{1/4}$ is the instantaneous Kolmogorov scale, and the maximum effective wavenumbers are $k_{x\max} = \pi / \Delta x$ and $k_{y\max} = 2.5 / \Delta y$. Figure 2.16(b) shows the evolution of the dissipation rate $\bar{\varepsilon}$ of the two simulations. The finer grid case has a slightly

Run	Re_z	N_x, N_y, N_z	$k_x\eta$	$k_y\eta$	ST	Re_λ	S^*	B_2	$-b_{xy}$	$\mathcal{P}/\langle\varepsilon\rangle$	ω'/S
L11	2600	62, 96, 62	1.5	1.9	802	52.9	6.4	0.452	0.154	0.990	6.38
run2	2632	64^3	1.5	1.5	210	51.6	6.6	0.446	0.152	1.004	6.05
M11	8224	108, 162, 108	1.1	1.40	831	91.0	6.9	0.454	0.142	0.989	10.1
run10	8225	108^3	1.2	1.2	106	83.4	7.1	0.430	0.141	1.000	9.10

Table 2.3: Parameters of present DNS (L11 and M11), CFL=0.6, compared with run2 and run10 in [Pumir \(1996\)](#). In all cases, $A_{xz} = A_{yz} = 1$. The effective resolution is $k\eta$. The total time to accumulate the statistics is ST . $B_2 = (3b_{ij}b_{ij}/2)^{1/2}$ is the second invariant of the Reynolds-stress anisotropy tensor [Lumley \(1979\)](#), $b_{ij} = \langle u_i u_j \rangle \langle u_k u_k \rangle - \delta_{ij}/3$. The ratio of energy input and energy dissipation is $\mathcal{P}/\langle\varepsilon\rangle$.

larger dissipation rate at around $5 \lesssim St \lesssim 10$, but the agreement is excellent considering that the larger grid is at least twice as fine as the coarser one. As an added test, the coarser grid was run at CFLs from 0.05 to 0.6, but the results agree within the thickness of the lines in figure 2.16(b).

Statistically stationary homogeneous shear turbulence

Closer to the subject of this thesis than any of the previous tests is the long-term behaviour of small computational boxes. It was shown in [Pumir \(1996\)](#) that, under those circumstances, turbulence grows in size, fills the box, and collapses intermittently, while reaching a statistically steady state that resembles the bursting cycle of wall-bounded turbulence ([Jiménez et al., 2005](#); [Jimenez, 2013b](#)). We have seen that a typical bursting period is $ST_b \approx 25$, so that reasonable statistics require, depending on the box geometry, running times of the order of hundreds of box periods. In this section we test the ability of our code to run for long times by repeating two of the simulations in [Pumir \(1996\)](#).

Both simulations run in a cubical box ($A_{xz} = A_{yz} = 1$), and start from initially turbulent conditions. In each case, we accumulate statistics for $St \approx 800$, after discarding the initial $St \approx 30$. The energy input by action of the shear on the stress, $\mathcal{P} \equiv -S\langle uv \rangle$, balances the dissipation rate within 1%.

Table 2.3 compares our two simulations (L11 and M11) with those in [Pumir \(1996\)](#). It includes both small- and large-scale quantities, which agree well. It is probably significant that the quantities that depend on the small scales, such as Re_λ or ω'/S , tend to be somewhat higher in our simulations than in [Pumir \(1996\)](#). This is consistent with the loss of enstrophy in the spectral code due to remeshing, although the difference is too small to decide whether the reason in this particular case is numerical or statistical.

2.A.5 Performance of the code on BG/Q in Julich Supercomputing Center, Germany

In order to study high Reynolds number SS-HST. The code has to be ported in a supercomputing center with satisfied performance in order to conduct a large scale simulation. The code has been tuned through two PRACE preparatory accesses with roughly 0.5M CPU hours in total on the machine JUQUEEN (BG/Q system) in Julich Supercomputing Center, Germany. For convenience, the code will be named by "SHEAR" hereinafter.

We have found that `MPI_ALLTOALLV` (`MPI_ALLTOALL`) is much faster than `MPI_SENDRECV`. However, the odd number of spanwise Fourier modes only allow `MPI_ALLTOALLV`, which usually performs worse than `MPI_ALLTOALL`. For that reason, a ghost $x - y$ plane was added. Nevertheless, communication between $x - z$ planes and $x - y$ planes restricts the number of MPI tasks can be used when $A_{yz} > 1$. So that the decomposition in x and y is also developed using `MPI_ALLTOALL`. In this way, depending on the aspect ratios of the boxes, two decomposition methods are available. In order to further exploit the computer resources, a 'fat' *pencil-pencil* decomposition is also implemented by splitting one plane into two equal pieces. The mechanism is the same when splitting them into even more and the limit will be *pencil-pencil*. Data in `MPI_ALLTOALL` are converted to `real*4` to reduce the message size and thus the communication time.

Hybrid MPI-OpenMP parallelization was implemented. It is trivial by just seeding OpenMP directives where there are loops. The only thing that needs care is the matrix solver, which requires `THREADPRIVATE` for constructing the matrix. Table 2.4 shows the thread scalability for computation time. It is clear that there is almost no benefit when using hyper-threading. Even 16 threads (one thread per core) lose 20% efficiency. Tracing of the code has shown that our in-house matrix solvers extract a $\times 3.5$ speedup out of $\times 4$ hyper-threading, but that the other big computational block, the FFT, only obtains a $\times 1.3$ speedup from either $\times 2$ or $\times 4$ hyper-threading using FFTW3. Consultation with similar groups using Mira in Texas and Cineca in Italy confirms that those numbers are already close to optimal. Although hyper-threading can only obtain a $\times 1.5$ speed up, we still enabled it.

Some efforts were made to make FFT scale better when using hyper-threading, but without promising results. There are two ways to thread FFT. One is the default threading by FFTW library. It includes 1D FFT with advanced interface, where users can specify the number of transformation and the 2D FFT. For the latter, the matrix transpose is incorporated. The other one is user customized threading by 1D FFT with basic interface, which is used for the current SHEAR code. The matrix transpose and padding/packing

threads	1	2	4	8	16	32	64
Comp. T	85.2s	43.1s	22.04s	11.71s	6.7s	4.8s	4.56s
E	-	0.988	0.966	0.913	0.795	0.555	0.292

Table 2.4: Thread scalability of computation time. Grid size is $3K \times 2K \times 1.5K$ in physical space. Using 512 nodes.

are done using ZGETM0 supported by the IBM ESSL library. The threading by default does not scale well neither and they are slightly slower than customized one.

The strong scalability of the SHEAR code is given in table 2.5. The efficiency drops

MPI tasks	256	512	1024	2048
Total T	10.48s	5.24s	3.04s	2.04s
E	-	1.0	0.91	0.64
Comp. T	8.15s	4.10s	1.99s	1.10s
Comm. T	1.46s	0.75s	0.81s	0.83s
Data arrange	0.87s	0.39s	0.24s	0.11s

Table 2.5: Strong scalability. Grid size is fixed at $3K \times 2K \times 1.5K$ in physical space. MPI tasks change from 256 to 2K. Each task employs 64 threads.

dramatically, by $\sim 30\%$, from 1K to 2K nodes, which is due to communication since the computation time still drops by roughly a factor of two. Once the code uses more than one mid-plane (512 nodes), the communication time even increases slightly. The technical supports in JUQUEEN attribute it to the network congestion. However, the TBL code in the same machine scales almost perfect up to the whole machine (458,752 nodes, see http://www.fz-juelich.de/ias/jsc/EN/Expertise/High-Q-Club/OpenTBL/_node.html). It is not clear whether the poor performance of the communication is due to the *plane-plane* or not, which needs to be tested in the future. The weak scalability is shown in Table 2.6. It is clear that the weak scaling is almost perfect.

Table 2.7 shows the profile of the SHEAR code. Evidently, communication and FFT are the two most time consuming parts, up to $\sim 67\%$ of the total together, with similar fractions. Matrix solver (§2.A.3) and the mapping process (§2.A.2) are the next two most time consuming parts, also with similar contribution, in total $\sim 24\%$. These four parts

MPI tasks	128	256	512	1024
Total T	3.00s	2.97s	3.03s	3.04s
E	-	1.0	1.0	1.0

Table 2.6: Weak scalability. *DoF* per node is fixed at 9M in physical space. MPI tasks change from 128 to 1K. Each task employs 64 threads.

account roughly 90% of the total. As we mentioned that the algorithm is quite similar to that for channel turbulence. The table also includes results of the channel code developed by Lee *et al.* (2013) (hereinafter CHANNEL) tuned also in IBM BG/Q system on the machine Mira in Argonne Leadership Computing Facility. The speed of the two different codes on the same system are almost the same, slightly faster than that of TBL code in BG/P system, $19\mu\text{s}$, (Borrell *et al.*, 2013). The CHANNEL code spends roughly the same time on communication as SHEAR, similar value is also true for TBL code (Borrell *et al.*, 2013). But its data arrange is more expensive, compensated roughly by FFT, probably due to the *pencil-pencil* decomposition. It is not clear why FFT in Mira is double faster than in JUQUEEN.

Code	Data arrange	Comm.	Matrix solver	FFT	Mapping	Time/ <i>DoF</i>
SHEAR	7.84%	26.57%	13.61%	32.95%	10.50%	$12.4\mu\text{s}$
CHANNEL*	20.70%	34.13%	-	17.22%	-	$12.0\mu\text{s}$

* Private communication with Myoungkyu Lee and N. Malaya at U. of Texas at Austin.

Table 2.7: Profile of the SHEAR code compared with that of CHANNEL (Lee *et al.*, 2013) in BG/Q. Both codes are tested with grid size $3\text{K}\times 2\text{K}\times 1.5\text{K}$ in physical space using 1K MPI tasks and each task with 64 threads (16 cores with $\times 4$ hyper-threading). I/O are not enabled. Data arrange is the preparing work before calling `MPI_ALLTOALL`, to which we refer as communication. The time for transposing matrix and padding/packing for $3/2$ dealiasing is considered into the total time of FFT. Time/*DoF* is defined by the total CPU time per step divided by degree of freedom in terms of Fourier modes. The wall time per step for the two codes is almost the same, $\sim 3.0\text{s}$.

As we will see later in §4 that the SHEAR code is supposed to be I/O intensive for the purpose of time-resolved turbulence. Therefore the original serial I/O is changed to

parallel I/O using HDF5. However, the speed is quite slow, only around 0.5-1.0 GB/s, no matter using MPI IO or MPI POSIX driver. For instance, with 1K MPI tasks, a 9GB file costs around 6s and a 32GB file costs 60s. For the MPI IO driver, the sieve buffer size was also tuned but without better performance. Mira writes faster than JUQUEEN, with 14s to write 37GB data with 1K nodes (private communication with N. Malaya). That I/O interface is specially optimized for pencil-pencil decomposition by the PECOS center (The Center for Predictive Engineering and Computational Science). Nevertheless, this speed is still much slower than the OpenTBL code in BG/P, 16G/s, which is only slightly slower than the read speed ([Borrell *et al.*, 2013](#)). That speed was exactly achieved by tuning the sieve buffer size of MPI IO driver. It is not clear why it did not in JUQUEEN.

At last the SHEAR code consumed roughly 27M CPU hours using 1K nodes in JUQUEEN. The whole history is shown in the figure [1.4\(a\)](#).

Chapter 3

Kinematics of coherent structures in SS-HST

3.1 Introduction

The concept of coherent structures in wall-bounded flows probably originated with [Theodorsen \(1952\)](#). It was soon connected with the phenomenon of bursting, which was first observed in experiments of turbulent boundary layers by [Kline *et al.* \(1967\)](#) as an oscillation and breakdown of what were later recognised to be streaks of low streamwise velocity. Both have received continuing attention since then because they are locally strong events that promise, up to a point, an independent evolution from the rest of the flow. They make chaotic turbulence appear ‘simpler’. It was not initially clear whether the intermittency observed in experiments was due to measurements artifacts or to true temporally recurrent motions. The temporal analysis of direct numerical simulations (DNS) of boundary layers by [Robinson \(1991\)](#), and later of minimal simulations of the buffer ([Jiménez & Moin, 1991](#)) and logarithmic layers ([Flores & Jiménez, 2010](#)) in channels, showed that experimentally observed bursts are passing intense structures, but that these structures grow and wane over longer time scales of the order of their turnover ([Jiménez *et al.*, 2005](#)). More recently, visual inspection has begun to be substituted by automatic machine processing allowing the spatial and temporal characterisation of bursting structures in full-scale channels at reasonably high Reynolds numbers ([Lozano-Durán & Jiménez, 2014b](#)). The structural point of view has become an indispensable and complementary method to statistics, and both are often blended together ([Jiménez, 2013a](#)).

Streaks are created by the interaction of the wall-normal velocity with the shear ([Kim & Lim, 2000](#); [Jiménez, 2013a](#)), and they are known to form even in the absence of walls

(Uzkan & Reynolds, 1967; Lee *et al.*, 1990). Bursting appears to require more restrictive conditions. The simplest shear flow is unbounded homogeneous shear turbulence (HST) which, unfortunately, does not have an asymptotic statistically stationary state. Ideal HST in unbounded domains grows indefinitely, both in intensity and in length scale (Champagne *et al.*, 1970; Harris *et al.*, 1977; Lee *et al.*, 1990; Kida & Tanaka, 1994b) and simulations are typically discontinued as the growing length scale approaches the size of the computational box (Rogers & Moin, 1987). However, Pumir (1996) extended the simulation to longer times and reached a statistically stationary state in which the largest-scale motion is constrained by the computational box and undergoes a succession of growth and decay phases of the kinetic energy and of the enstrophy, reminiscent of the bursts in wall-bounded flows. This suggests that bursting is also a common feature of shear-induced turbulence, but that it requires the restriction of the growing structures before growth can be reinitiated. In SS-HST, this restriction is due to the numerical box. In wall-bounded turbulence it is due to the wall or, when simulated in a small box, to the constrained spanwise direction (Flores & Jiménez, 2010). Note that this implies that any simulation of SS-HST is minimal, especially in the spanwise direction and that its largest scales are not necessarily representative of those in unbounded HST.

Nevertheless, previous investigations have found that the growth rate of kinetic energy during bursts in SS-HST and in the logarithmic layer of minimal channels are very similar, and that the growth phase of their bursts is qualitatively the same as the initial shearing of isotropic turbulence (Pumir, 1996; Gualtieri *et al.*, 2002). Indeed, §2 showed that the evolution of the flow during all the phases of the SS-HST bursts is similar to that in wall turbulence (Flores & Jiménez, 2010; Jimenez, 2013b), undergoing lift-up, instability, break-down and regeneration. Statistics, such as the two-point correlation of the vertical velocity also agree well between both flows.

The purpose of this chapter is to study the formation of coherent flow structures in SS-HST, compare them with those in turbulent channels, and study their behaviour in a simpler setting than in wall turbulence. In particular, by comparing the structures in the two flows, we seek to distinguish which features are dominated by the wall and which ones are general properties of shear-driven turbulence.

One of the best-studied models for bursting is based on the quadrant analysis introduced by Wallace *et al.* (1972). If u and v are the streamwise and wall-normal velocities, Reynolds-stress (uv) events can be classified into Q1s ($u > 0, v > 0$), Q2s (ejections, $u < 0, v > 0$), Q3s ($u < 0, v < 0$) and Q4s (sweeps, $u > 0, v < 0$). Coherent Qs are defined as specially intense (uv) events, classified in terms of their quadrant. Around

70% of the total Reynolds stress in a turbulent boundary layer is contained in strong Q2s and Q4s (Kim *et al.*, 1971; Lu & Willmarth, 1973), and they were early identified as one of the best techniques for burst identification in terms of one-point measurements (Bogard & Tiederman, 1986). More recently, Qs were extended to three-dimensional objects by Lozano-Durán *et al.* (2012, LFJ12 hereinafter), who later studied their temporal evolution (Lozano-Durán & Jiménez, 2014b). They found that they can be divided into wall-attached and detached families, with the former being dominant from the point of view of momentum transfer. Bursting Qs have recently been associated with the linearised transient Orr (1907) amplification mechanism by Jiménez (2013a, 2015).

Also associated with bursting are hairpin vortices (Theodorsen, 1952; Perry & Chong, 1982), although mostly observed at relatively low Reynolds numbers, near the wall, or in transition (Acarlar & Smith, 1987; Zhou *et al.*, 1999; Adrian, 2007). The interest in hairpins is largely due to their simple shape, which makes them easy to visualise as triggers of bursts. The two legs of a hairpin are quasi-streamwise vortices that move low-velocity fluid away from the wall to create streaks that eventually become unstable and break down. In this sense, hairpins are models for the Q2s described in the previous paragraph. A different kind of vortical structure was studied by del Álamo *et al.* (2006, AJZM06 from now on) who examined three-dimensional connected vortex clusters defined by the discriminant of the velocity gradient tensor in the logarithmic layer of channels at relatively high Reynolds number. They found a self-similar hierarchy of structures fitting the attached eddy model of Townsend (1961). As in the case of Qs, vortex clusters separate into wall-attached and detached families but, although the averaged flow field of an attached vortex cluster contains a long conical low-speed streamwise velocity ‘wake’ reminiscent of streaks, headed by a shorter pair of ejections, the instantaneous shapes are irregular and very different from hairpins. The question of whether hairpin vortices persist in high-Reynolds number turbulence, and of whether they should be understood as instantaneous structures or as conditional statistical constructs remains controversial, and is the subject of much current debate (Wu & Moin, 2009; Schlatter *et al.*, 2014).

Streaks, Qs and vortical structures are not unrelated to each other. On average, Q2s and Q4s form spanwise pairs located between high- and low-velocity streaks. The flow field conditioned to these pairs includes a central quasi-streamwise large-scale roller (LFJ12; Jimenez 2013b), reminiscent of the average arrangement of streaks and quasi-streamwise vortices in the buffer layer (Robinson, 1991; Jiménez & Pinelli, 1999; Schoppa & Hussain, 2002). Vortex clusters are also located on average between the two Qs, closer to the Q2. Whether the quasi-streamwise roller is an averaged reflection of these vortex clusters is

unclear, and will be one of the questions considered in this paper. Similarly, it is also unknown whether hairpins are connected with the structure just described.

Another question to be addressed is the relation between wall-attached and detached structures. Most of the properties mentioned above apply mostly to attached Qs and clusters. Hairpins have also been considered mostly in relation to the wall. In wall-bounded flows, detached Qs tend to be small and isotropic, and do not carry net Reynolds stress, and [LFJ12](#) showed that structures with similar properties are found in free-shear flows, and even in flows with no mean shear. Even if it may appear natural to compare eddies in HST with detached ones in wall turbulence, the difference between attached and detached structures has to be clarified and will be examined in this chapter.

In fact, the distinction between the two families is not absolute. There are large detached structures in wall-bounded turbulence, although very large ones tend to hit the wall and become attached. Structures that are detached at a given threshold may be attached at lower ones and, when [Lozano-Durán & Jiménez \(2014b\)](#) followed Qs and vortex clusters in time, they found that a given structure may start life as detached only to attach to the wall later, and vice versa. [Rogers & Moin \(1987\)](#) found that hairpin vortices, or at least hairpin vortex lines, exist in HST, and [Vanderwel & Tavoularis \(2011\)](#) found both ‘upright’ and ‘inverted’ hairpin eddies in experimental HST strong enough to be made visible as concentrations of hydrogen bubbles. The similarities and differences between the statistics of HST and wall turbulence have been examined by [Pumir \(1996\)](#) and [Gualtieri *et al.* \(2002\)](#) but, as far we are aware the relationship between their three-dimensional structure has not been studied in any detail.

The rest of this chapter starts by introducing the data base and the identification method used to extract coherent structures in §4.2, after which detailed comparisons about their characteristics with those of channel turbulence are presented. Specifically, §3.4.2 displays the intensities of these structures in terms of their mean Reynolds stress, enstrophy, anisotropy, etc. §3.4.3 discussed about their size, orientation and shape. Then the spatial organization of them in terms of their relative positions is studied in §??, based on which the conditional averaged flow field to Q2–Q4 pairs and vortex clusters are shown in §3.4.6. Finally the chapter is closed with conclusion and discussion.

3.2 Numerical experiments

We are already familiar that the largest scales in the SS-HST simulations are always constrained by their numerical box, and §2 concluded that they are only useful as an approximate model for other shear flows when $2 < A_{xz} < 5$ and $A_{yz} > A_{xz}/2$. These results were used to design the three simulations analysed in this chapter, for which $A_{xz} = 3$ and $A_{yz} \geq 2$. Their properties are summarized in table 3.1. Three Reynolds numbers are investigated, $Re_\lambda \approx 50, 100$ and 250 , labelled hereinafter as L38, M32 and H32 for ‘low’, ‘medium’ and ‘high’ Re_λ . We compare them with two channels in ‘large’ domains ($L_x = 8\pi h$, $L_z = 3\pi h$), at $Re_\tau = u_\tau h/\nu = 934$ (del Álamo *et al.*, 2004) and $Re_\tau = 2003$ (Hoyas & Jiménez, 2006), respectively labelled C950 and C2000. Although not used in detail in this chapter, other simulations of SS-HST are occasionally mentioned here to clarify the effect of the computational box on the statistics. They are labelled in the obvious way by their Reynolds number and aspect ratios, as in L32, M34, etc. Some of their basic flow statistics are compiled in table 3.1.

In the logarithmic layer of channels, $Re_\lambda \approx 7y^{+1/2}$ (Jimenez, 2013b), so that the Reynolds number of L38 is similar to that in the buffer layer, $y^+ \approx 50$, while those of M32 and H32 are equivalent to $y^+ \approx 200$ and $y^+ \approx 1300$, respectively. This comparison allows us to estimate the effective range of scales in our simulations, and how close they are to fully developed turbulence. Although the integral scale $L_\epsilon = q^3/3\sqrt{3}\epsilon$ is usually taken to represent the largest flow scales, it was argued by Corrsin (1958) that a more relevant measure in shear flows is $L_c = (\epsilon/S^3)^{1/2}$, below which eddies are isotropically oriented and decouple from the effect of the mean shear. In this sense, L_c is a length scale for the small-scale limit of the energy-injection range, in the same way that the Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$ characterises the beginning of the dissipative one. It follows from their definition that $L_c = L_\epsilon(3/S^*)^{3/2}$. Since $S^* \approx 7-9$ both in HST (§2), and in the logarithmic layer of wall-bounded flows (Jimenez, 2013b), it follows that $L_c \approx 0.25L_\epsilon \approx 0.1L_z$ and $L_c/\eta \approx 0.033Re_\lambda^{3/2}$ in our HST, and $L_c \approx 0.3y$ and $L_c/\eta \approx 0.4y^{3/4}$ in the logarithmic layer of channels.

Both L_c and η are characteristics scales, rather than absolute limits. Figure 3.1(a) shows enstrophy spectra in Kolmogorov scaling for the three HST and for the two channels, and figure 3.1(b) shows co-spectra of the tangential Reynolds stress in terms of the Corrsin scale. Both collapse well, although the constricting effect of the numerical box on the Reynolds stress of the HSTs is obvious. It follows from these spectra that the true lower limit of the inertial range is approximately $\ell_\epsilon \approx 40\eta$, while the largest flow scales are $L_0 \approx 15L_c$, so that the effective range of scales is approximately $L_0/\ell_\epsilon = 0.4L_c/\eta$. Using

the values in table 3.2, $L_0/\ell_\varepsilon \approx 5, 14$ and 50 for L38, M32 and H32, respectively, suggesting that the Reynolds number of L38 is probably too low to be considered as fully turbulent, and to be compared to the other two simulations.

Due to the strong intermittency of HST (Pumir, 1996), the instantaneous $\bar{\eta}(t)$ undergoes strong fluctuations. This is shown in table 3.1 by the standard deviation of the numerical resolution, which has to be set fine enough to capture well the smallest scales over the whole history. Our grids are adjusted to uniformly satisfy $\Delta/\bar{\eta}(t) \lesssim 1.5$, which is the usual requirement for the study of vortex filaments in isotropic turbulence (Jiménez *et al.*, 1993), but it is probably true that the resolution is marginal in that respect at isolated points during the most intense bursts. This is likely to be a local problem in most high-Reynolds number simulations of turbulence, but it is made more obvious in minimal boxes because intermittency affects the whole flow at the same time.

Figure 3.1: (a) Premultiplied one dimensional spectra of the vorticity magnitude, normalised with the total enstrophy, as a function of the streamwise wavelength $\lambda_x = 2\pi/k_x$, in Kolmogorov scaling. The vertical line is $\lambda_x/\eta = 40$. (b) One dimensional co-spectra of u and v normalized with u_τ and with the Corrsin length. The vertical line is $\lambda_x/L_c = 15$. Symbols are as in table 3.1.

3.3 Structure identification

The three-dimensional structure identification method used in this paper was first introduced for strong vorticity and strain-rate in isotropic turbulence by Moisy & Jiménez (2004), and extended by AJZM06 and LFJ12 to vortex clusters and to Qs in channels. Defining the point-wise Reynolds stress as $\tau = -uv$, the Qs are defined as connected

Case	Sym.	Re_z	Re_λ	A_{xz}	A_{yz}	$\Delta x/\eta$	$\Delta y/\eta$	$\Delta z/\eta$	L_c/η	σ_u/u'	σ_v/v'	σ_Π/Π'
L38	$\diamond$	2000	48	3	8	1.6 ± 0.06	1.1 ± 0.04	1.6 ± 0.06	12	0.06	0.07	0.17
M32	$\square$	12500	104	3	2	1.4 ± 0.11	1.4 ± 0.11	1.4 ± 0.11	36	0.15	0.15	0.34
H32	$\circ$	48000	248	3	2	1.6 ± 0.18	1.0 ± 0.12	1.0 ± 0.12	124	0.20	0.23	0.53
C950	∇		80	2.7	0.2	2.4	1.6	1.0	24			
C2000	$\triangle$		127	2.7	0.2	1.8	1.5	0.9	46			

Table 3.1: Parameters for three DNSs of SS-HST. Re_λ is the Reynolds number based on the Taylor microscale; A_{xz} and A_{yz} are the box aspect ratios; $\Delta x/\eta$, $\Delta y/\eta$ and $\Delta z/\eta$ are the average resolutions in terms of the Kolmogorov scale η , with their standard deviations due to intermittency. Δx and Δz are computed from Fourier modes; L_c is the Corrsin scale; σ_u/u' is the ratio of the rms of $\sqrt{u^2}(t)$ to the rms of the point-wise fluctuations of u , with similar definitions for σ_v/v' and σ_Π/Π' , where Π is the second invariant of the velocity gradient tensor. The two channels used for comparison are included for reference, with properties given at $y/h \approx 0.4$, where Re_λ is maximum.

Case	$(u', v', w', u_\tau)/SL_z$				$(\eta, \lambda, L_\varepsilon, L_c)/L_z$				ω'/S	S^*
L38	0.214	0.160	0.164	0.118	0.0097	0.133	0.420	0.118	5.34	7.02
M32	0.186	0.134	0.138	0.097	0.0027	0.055	0.388	0.098	10.9	7.53
H32	0.214	0.161	0.161	0.113	0.0009	0.028	0.455	0.110	24.8	7.57

Table 3.2: Flow parameters for the three DNSs of HST.

regions in which all the points satisfy the intensity criterion,

$$|\tau(\mathbf{x})| > Hu'v'. \quad (3.1)$$

Vortex clusters are similarly defined as connected regions where the second-order invariant Π of the velocity-gradient tensor (Chong *et al.*, 1990) exceeds a fraction α of its rms,

$$\Pi(\mathbf{x}) > \alpha\Pi'. \quad (3.2)$$

Note that Π' , u' and v' are functions of y in channels, but not in HST.

Note also that Π is often denoted by Q in many publications, but we avoid that notation here to prevent confusion with the Q-structures. The reason for choosing Π to define vortex clusters instead of the discriminant D (AJZM06; LFJ12) is the different resolution requirements of the two quantities. The discriminant is a sixth power of the velocity derivatives, while Π is only quadratic, and it was shown by Lozano-Durán *et al.* (2015) that high-order quantities can quickly become numerically meaningless when the resolution is marginal. Chakraborty *et al.* (2005) showed that there are few statistically significant differences between the various vortex identification methods.

The thresholds H and α are determined by a percolation analysis similar to that in Moisy & Jiménez (2004), AJZM06 and LFJ12. Briefly, the ratio V_{lar}/V_{tot} between the volume of the largest connected structure and the total volume contained in all the structures is computed as a function of the threshold. For a sufficiently low threshold, the volume fraction of the largest structure increases sharply, and would reach unity for structures of a single kind if the threshold were chosen low enough. The nominal threshold for identifying structures is chosen near the middle of the percolation transition.

The resulting percolation diagrams are shown in figures 3.2(a) and 3.2(b), where V_{lar}/V_{tot} is given as a function of the threshold, normalized with its maximum value at very low thresholds. The definition of individual structures becomes more computationally expensive for the lower thresholds because structures are larger, and it sometimes

Figure 3.2: Percolation diagram of the volume fraction of the largest structure V_{lar}/V_{tot} as a fraction of the total volume of all identified structures. All curves are normalized with their maximum. (a) Qs. (b) Vortex clusters. The vertical dashed lines indicate the nominal thresholds, $H=1.75$ for Qs, and $\alpha = \Pi_{thr}/\Pi'=1.5$ for vortex clusters. Symbols are as in table 3.1. (c) Time history of the kinetic energy $\overline{q^2}(t)/\langle q^2 \rangle$ of M32, divided into: —, energetic part ($\overline{q^2}(t) > \langle q^2 \rangle$); ---, quiescent part ($\overline{q^2}(t) < \langle q^2 \rangle$). (d) Percolation diagram for vortex clusters in: $\triangleleft$, energetic (h); $\triangleright$, quiescent (l) parts in (c). In each case the threshold is defined with respect to the corresponding rms, Π'_h or Π'_l . The line without symbols is the full average of M32, from (b).

becomes impractical to perform the percolation analysis over the whole computational box. For example, the analysis of the vortex clusters of H32 for $\alpha < 0.25$ could only be done in our servers for a quarter of the computational box (AJZM06), but we tested that this did not make any difference for thresholds within the percolation transition.

There is a subtle difference between the processing of Qs here and in previous papers. In LFJ12, all the points satisfying (3.1) were first isolated and separated into individual structures, and each structure Ω was then classified into a quadrant according to the mean values of its two velocity components, defined as in

$$u_m = \frac{\int_{\Omega} u dV}{\int_{\Omega} dV}, \quad (3.3)$$

with similar definitions for other variables.

For perfectly resolved simulations, finding points of different quadrants within a single contiguous structure would imply a discontinuity in the flow, and it was explicitly tested in [LFJ12](#) that the number of structures with ‘mixed’ points was negligible. However, when the resolution is limited, it is possible for different quadrants to coexist within the a structure defined in this way. Even in [LFJ12](#), about 5% of the total volume of the largest Qs belongs to quadrants that are different from the average quadrant of their structure.

To avoid ambiguities, this thesis classifies points individually into quadrants before clustering them into contiguous structures. This could not be done in channels, where the points in a Q change quadrant when they cross the central plane, and has the added advantage of requiring less computational resources, since the structures of each quadrant involve less points than the total. For example, the classification of Qs in H32 could not be performed in our home servers when treating all the Qs together during strong bursts.

The differences with the older method were tested in HST by comparing the results of both schemes. The difference in the number of Qs identified by the two methods was about 4% in L38 and less than 1% in M32, but the volume of the largest structure found when identifying all the quadrants together could be twice larger than when treating them separately. The effect in channels was tested in C950. The difference in the number of structures was relatively small, consistent with the percentage of misclassified points mentioned above, but the volume of the largest Qs found using the old method was found to be up to 50% larger than by the new one in some flow fields. In both HST and channels, statistical properties such as the fraction of Reynolds stress carried by the Qs and its distribution among Q-classes, differ little between the two methods.

An unwelcome consequence of computing the four Q-classes independently is that their percolation behaviour is not exactly the same. To avoid using multiple thresholds and to facilitate the comparison with channels, figure [3.2](#) is computed using accumulated volumes for the four Q-classes, even if each of them is computed independently. Moreover, it is clear from figure [3.2\(a\)](#) that the threshold found in [LFJ12](#) for channels ($H = 1.75$) is within the percolation transition in HST, at least for the two highest Reynolds numbers. This is the value used here as nominal threshold. The analyses presented below were repeated for the range $H = 1.25 - 2.25$ with similar results, and we will only refer from now on to structures identified using the nominal threshold, unless stated otherwise.

Most of these problems do not apply to vortex clusters, which have no classes. Figure

3.2(b) suggests a percolation transition in the range $\alpha = 0.2$ – 3 . We chose $\alpha = 1.5$ as our nominal threshold, and tested the range $\alpha = 0.75$ – 2.5 . Although the quantity used here to define clusters is different from that used in [AJZM06](#) and [LFJ12](#) for channels, the results are compared below without recomputing the channels with Π . It was tested in the two previous papers that clusters based on the discriminant and on Π are visually and statistically indistinguishable.

A more serious problem has to do with the intermittency associated with the small computational boxes of HST. The reference u' , v' and Π' in (3.1)–(3.2) are averaged over the whole flow history. In the larger boxes of the computational channels the intermittency is spatial, and temporal and spatial averages are roughly equivalent. On the other hand, the intermittency in the minimal computational boxes of HST is temporal, and it is unclear whether a single threshold is adequate to isolate structures at different stages of the flow.

Table 3.1 gives the ratio of the standard deviation of $\sqrt{u^2}(t)$, $\sqrt{v^2}(t)$ and $\sqrt{\Pi^2}(t)$ with respect to their long-time mean. They are substantial and appear to increase with increasing Re_λ , although part of that increase is probably statistical. The case with the lowest Reynolds number (L38) also has a larger box that is not minimal in the vertical direction, chosen precisely to improve its statistics. The standard deviations for a smaller box at the same Reynolds number ($A_{yz} = 2$, case L32 in §2) are very similar to those of M32, and those in the large channels, which contain many structures, are less than one percent of the means. Figure 3.2(c) displays an example of the temporal history of the kinetic energy in M32, classified into ‘energetic’ and ‘quiescent’ parts depending on whether the instantaneous $\overline{q^2}(t)$ is respectively higher or lower than the overall average $\langle q^2 \rangle$. In order to verify whether the percolation results in figure 3.2(b) depend on the state of the flow, we plot in figure 3.2(d) the percolation diagrams for clusters in the quiescent and energetic flow periods. When a uniform threshold is used at all times there are fewer structures in quiescent periods than in energetic ones, but figures 3.2(d) shows that, when the percolation is plotted with respect to a rescaled threshold, $\Pi_{thr} = \alpha \Pi'_j$, where Π'_j is $j = h$ or $j = l$ according to whether the statistics are compiled over the energetic or quiescent periods, the percolation diagrams collapse well even if $\Pi'_h/\Pi'_l = 1.7$. This strongly suggests that the physics of the structures differs little between the two periods, and that most of the differences in their identification could be accounted for by a temporally variable threshold. However, doing so would prevent us from studying the bursting dynamics and complicate the comparison with the spatially intermittent channels, and we use a constant threshold for all times.

Case	ST	T_{eto}	N_F	N_C	N_Q	N_1	N_2	N_3	N_4	V_1	V_2	V_3	V_4
L38	8.7	3.73	1815	1.1	0.43	24.0	26.0	23.9	26.1	4.8	45.3	4.8	45.1
M32	1.0	0.41	574	2.8	0.60	24.5	25.3	24.6	25.5	6.0	44.1	5.9	43.9
H32	0.1	0.04	95	7.5	0.93	22.5	27.7	22.4	27.4	6.7	43.0	6.9	43.4

Table 3.3: Parameters of the structures used in the paper. $ST \times 1000$ is the total shear time of the simulation; $T_{eto} \times 1000$ is the total eddy turn-over time defined by $L_\varepsilon/\sqrt{q^2/3}$; N_F is the number of flow fields used to extract the structures; N_C and N_Q are the number of vortex clusters and Qs identified with $\alpha=1.5$ and $H=1.75$, respectively. For Qs with the nominal threshold, N_i and V_i are the percentages of Q_i in terms of their number and volume.

Table 3.3 contains information about the coherent structures identified, after structures with volume smaller than $(5\eta)^3$ are discarded to avoid resolution issues. These small fragments typically represents a third of the total number of structures but they only account for a negligible fraction of the total volume of the Qs, and for 1%–2% of the volume of the vortex clusters.

The Qs and clusters account, respectively, for roughly 10% and 3.5% of the volume of the computational box at the reference threshold. Although the number of Qs is split roughly equally between the four quadrants at $H=1.75$, their volume is mainly contributed by the Q^- s, distributed equally in Q2s and Q4s as expected from symmetry.

We mentioned in the introduction that structures in channels can be classified as attached or detached according to whether their root reaches or not the neighbourhood of the wall, and that most of the volume and the Reynolds stresses of Qs are contained in the attached family (AJZM06, LFJ12). There are no walls in HST, but an equivalent classification can be established between large and small structures. We have already seen that structures smaller than the Corrsin (1958) scale decouple from the shear. For example, vorticity becomes more isotropic as it moves away from the wall to scales smaller than L_c (Jimenez, 2013b). A rough classification of the structures in HST can thus be based on whether some characteristic size is larger or smaller than L_c . Large and small structures can be expected a-priori to be roughly equivalent to the attached and detached ones in channels.

The probability density function (pdf) of the volume of the Qs is roughly proportional to $V^{-5/3}$, which approximately agrees with the exponents (1.5 – 1.6) found by LFJ12 for several shear flows, including channels. A consequence of this distribution is that most Qs

are small, but that most of their volume is contained in large structures. For example, Qs with $V > L_c^3$ contribute roughly 20% of the number of structures in L38, and an almost negligible fraction in H32, but they account for 96.5% and 87.6% of the total volume, respectively. In channels, about 70% of the total Q volume is in tall attached Qs at all Reynolds numbers (LFJ12).

The pdf of the volume of the clusters is steeper, V^{-4} , so that small eddies dominate both in number and in volume. Large ($V > L_c^3$) vortex clusters account for 8.6% of the total number of clusters in L38, and for a negligible fraction in the two higher-Reynolds-number cases. Their contribution to the total volume also becomes less important as the Reynolds number increases, (61.7%, 3.0% and 0.5% for the three HST). Attached vortex clusters only occupy 10–15% of the total cluster volume in channels, although their association with attached Q2s (AJZM06, LFJ12) has made them the subject of intensive study as markers of the momentum and energy cascades across different scales.

It will be found convenient in the rest of the paper to characterise the size of eddies by the ‘box diagonal’, d , of the circumscribing parallelepiped aligned to the three coordinate directions (see figure 3.3). The relation between d and the volume of the structure will be discussed in §3.4.3. It depends on the type of structure and weakly on the Reynolds number, but $V = L_c^3$ corresponds to $d/L_c \approx 4$ for Qs, and $d/L_c \approx 7$ for vortex clusters in M32.

Examples of an instantaneous Q2 and of a vortex cluster from M32 are shown in figure 3.3. They can be compared with equivalent figures of structures in channels in AJZM06 and LFJ12, and with the larger online collection of channel structures in <http://turbulentbeautycontest.appspot.com/>. Although visual impressions should not substitute statistical analysis, the comparison reinforces the idea that the structures in HST are very similar to those in channels, especially to those detached from the wall.

3.4 Properties of the structures in HST and in channels

One of the motivations of this thesis is to explore how coherent structures in HST are related to those in channels. Both flows draw their energy from the mean shear, but the channel is inhomogeneous in y and has walls, while HST is homogeneous and has no walls. It is then reasonable to infer that properties shared by the structures in both flows are primarily due to the local shear, while those that are different are related either to inhomogeneity or to the presence of the wall. In this section, we study the geometrical

Figure 3.3: Instantaneous structures extracted from case M32. (a) Q2. (b) vortex cluster. Coloured by the mean streamwise velocity.

and flow properties of the structures defined above, and enquire about their similarities and differences with the rest of the flow, and with the channel.

Before doing so, it is important to understand some differences between the two flows, even when considering channels far from walls. The first one concerns symmetry. If we denote a transformation by its effect on the flow variables, both channels and HST are symmetric with respect to reflections across vertical planes, written as

$$S1 : (x, y, z, u, v, w, \omega_x, \omega_y, \omega_z) \rightarrow (x, y, -z, u, v, -w, -\omega_x, -\omega_y, \omega_z).$$

This symmetry was used in [LFJ12](#) to simplify some statistics, and will be used here for the same purpose. A symmetry of HST that is not shared by the channel is reflection across horizontal planes, which changes the sign of u and v ,

$$S2 : (x, y, z, u, v, w, \omega_x, \omega_y, \omega_z) \rightarrow (-x, -y, z, -u, -v, w, -\omega_x, -\omega_y, \omega_z).$$

The product of these two symmetries is an inversion with respect to the origin, also exclusive to HST,

$$S3 : (x, y, z, u, v, w, \omega_x, \omega_y, \omega_z) \rightarrow (-x, -y, -z, -u, -v, -w, \omega_x, \omega_y, \omega_z).$$

These symmetries can be used to constraint which relations among structures are statistically possible. For example, it was found in [AJZM06](#) and [LFJ12](#) that clusters in channels tend to be associated with Q2s rather than with Q4s, but S3 leaves the definition of clusters invariant and transforms Q2s into Q4s. This implies that any preference of clusters

for one particular flavour of Q^- is statistically impossible in HST, and has to be a consequence of the differential advection of vorticity towards and away from the wall. It was also found in LFJ12 that the volume occupied by Q_2 s in channels is larger than that of the Q_4 s, especially near the wall, but the previous argument shows that the two should be similar in HST (as well as those of Q_1 s and Q_3 s). This is confirmed by the simulations, and statistics for the HST will mostly be given in terms of Q^+ s and Q^- s from now on.

Another important difference between channels and our HST simulations is that the largest scales in the latter are limited in the spanwise direction by the computational box (§2), while the flow in the large channels is only constrained vertically by the wall. The consequence is that only relatively small scales (empirically smaller than about $L_z/2 \approx L_\epsilon$) should be expected to behave similarly in the two cases.

3.4.1 Reynolds stress

The fraction of the total Reynolds stress carried by Q s in HST is listed in the first column of table 3.4. The next two columns give the distribution into Q -classes, which roughly agrees with Pumir (1996). At least within our range of Reynolds numbers, there is a weak trend towards a smaller overall fraction of Reynolds stress in intense Q s as the Reynolds number increases, due both to stronger counter-gradient Q^+ s and to weaker co-gradient Q^- s. A consequence is that the correlation coefficient $c_{uv} = -\langle uv \rangle / u'v'$ between u and v decreases from roughly 0.41 in L38 to 0.37 in H32. A similar trend of c_{uv} is found in wall-bounded flows as the Reynolds number increases. It is usually attributed to the increased importance of the ‘inactive’ motions due to the blocking effect of the wall (Townsend, 1961), but the present results suggest that there is at least some contribution from an increased isotropy of the stress-carrying eddies.

An important property of channels is that only large attached Q^- s carry net Reynolds stress, and that there are basically no large attached Q^+ s (LFJ12). For smaller detached structures, the counter-gradient contribution of the Q^+ s cancels the co-gradient contribution of the Q^- s. The simplest interpretation is that a net Reynolds stress can only be produced by large anisotropic eddies that couple with the shear (Corrsin, 1958), and that those eddies are mostly co-gradient. Observations in channels cannot easily distinguish whether the asymmetry between Q^+ and Q^- is due to the size of the eddies or to the presence of the wall.

This is tested for HST in the right-hand part of table 3.4, which lists the fraction of the total stress carried by ‘active’ Q_C with $d > L_c$. Comparing them with the left-most columns of the table, it is clear that large structures are also responsible for most of the

Case	Q	Q ⁺	Q ⁻	Q _C ⁺	Q _C ⁻	C
L38	61.2	-5.8	67.1	-5.8	67.0	26.0
M32	57.7	-7.3	65.1	-7.1	64.8	27.2
H32	53.7	-8.0	61.7	-7.4	60.9	33.8

Table 3.4: Fraction of the Reynolds stress and enstrophy contained in structures with the nominal threshold. The column Q is the percent of the total Reynolds stress in Qs. The columns Q[±] refer the stress in each Q-class, while the subscript ‘C’ refer to Qs whose diagonal is $d > L_c$ where L_c is the Corrsin scale. The last column is the percent of total enstrophy within vortex clusters.

momentum transfer in the absence of walls. In fact, it can be shown that there are very few Q⁺s larger than $d \approx 5L_c$ (see also figure 3.6b below). Although not strictly equivalent, Pumir (1996) mentions that 60–70% of the Reynolds stress in a cubic HST box is carried by the first spanwise mode, whose wavelength is approximately $10L_c$.

This is seen in the right-hand part of table 3.4, which lists the fraction of the total stress carried by ‘active’ Q_C, defined as those with $d > L_c$. Comparing them with the left-most columns of the table, it is obvious that these large structures are also responsible for most of the momentum transfer in the absence of walls. In fact, the counter-gradient contribution of the Q_C⁺ is even lower than for the general Q population. Although not strictly equivalent, Pumir (1996) mentions that 60–70% of the Reynolds stress in a cubic HST box is carried by the first spanwise mode, whose wavelength is approximately $10L_c$.

The last column of table 3.4 shows that the fraction of enstrophy carried by vortex clusters increases slowly with the Reynolds number.

3.4.2 Flow anisotropy

A measure of the spacial extent of the correlation of u and v responsible for the Qs is the conditional two-point cross-correlation function, defined for any two quantities $\psi(\mathbf{x})$ and $\phi(\mathbf{x}')$ as

$$C_{\psi\phi}(\mathbf{x}, \mathbf{x}')|_{\Gamma(\mathbf{x})} = \frac{\langle \psi(\mathbf{x})\phi(\mathbf{x}') \rangle_{\Gamma(\mathbf{x})}}{[\langle \psi^2(\mathbf{x}) \rangle_{\Gamma(\mathbf{x})} \langle \phi^2(\mathbf{x}') \rangle]^{1/2}}, \quad (3.4)$$

where $\langle \rangle_{\Gamma}$ is an average conditioned to the event Γ . Along homogeneous directions, this correlation is only a function of the displacement $\boldsymbol{\delta} = \mathbf{x}' - \mathbf{x}$, but along the vertical direction of channels, both end points have to be included. We write $C_{\psi\phi}(\mathbf{x}, \mathbf{x}')|_{\Gamma(\mathbf{x})}$ as

$$C_{\psi\phi}(\delta_x, \delta_y, y, \delta_z)|_{\Gamma(\mathbf{x})}.$$

Consider the correlation conditioned to Q2s, i.e. $\Gamma(\mathbf{x})$: $u(\mathbf{x})v(\mathbf{x}) < -Hu'v'$ with $u(\mathbf{x}) < 0, v(\mathbf{x}) > 0$ and $H = 1.75$. The cross section of $C_{uv}|_{Q2}$ in the plane of the shear ($\delta_z = 0$) is shown in figure 3.4. We plot data from C2000 at $y \approx 0.15h$ and from M32, both of which have $Re_\lambda \approx 100$. The maximum of $-C_{uv}|_{Q2}$ is about 0.87 in both cases, and is located at $\delta = 0$. When the correlations are scaled with L_c at the reference point, they agree reasonably well in the horizontal direction, and even for $\delta_y > 0$, but the correlation of the channel towards the wall ($\delta_y < 0$) is much steeper than for the HST, presumably because of the constricting effect of the wall. Similar results are found for $C_{uv}|_{Q4}$, and for the autocorrelation C_{vv} (§2). Note that the correlations in figure 3.4 are different from the unconditioned C_{uv} (not shown), which has two peaks for the HST, located at $(\delta_x/\eta, \delta_y/\eta) \approx (\pm 10, \mp 10)$. Channels only have the peak at $(-10, 10)$, and the unconstrained maximum is only $C_{uv} \approx -0.4$ in both cases.

Figure 3.4: Vertical section of the (u, v) cross correlation conditioned to ejections, $-C_{uv}|_{Q2}$. Lengths are normalized by L_c . —, M32; ---, C2000 at $y \approx 0.15h$ to match Re_λ . Contour levels are $[0.2:0.2:0.8]$.

Another question is whether the flow within Qs is different from the flow as a whole. Figure 3.5(a) displays the Lumley invariants, which are defined from the anisotropy tensor of the Reynolds stresses, $b_{ij} = \langle u_i u_j \rangle / \langle u_i u_i \rangle - \delta_{ij}/3$, as $\zeta^2 = b_{ij}b_{ji}/6$ and $\xi^3 = b_{ij}b_{jk}b_{ki}/6$ (Pope, 2000). The dashed curve at the top of the figure is the two-component realisability limit, and the right-hand vertical axis is the axisymmetric one. The point where the two lines meet in the upper-right part of the plot correspond to one-component flows. Isotropic flow is at $\xi = \zeta = 0$, below the lower-right corner of the plot. The two solid symbols in the lower part of the plot correspond to unconditioned statistics of HST, with the Reynolds

Figure 3.5: (a) Shifted Lumley invariants of the Reynolds-stress anisotropy tensor. $\square$, M32; $\circ$, H32; $\triangle$, channel C2000 above the buffer layer. The symbol marks the lower limit, $y^+ = 100$. Lines with closed symbols are unconditional statistics. Those with open symbols are conditioned to Q^- s. For the two HST, the identification threshold increases in the direction of the arrow from $H = 0.25$ to 4. The channel is only drawn for $H = 1.75$. The inset is a zoom of the region around the nominal threshold $H = 1.75$, marked by symbols. The thin dashed line at the top of the figure is the realisability limit for the invariants. The thicker dashed line that follows the thresholded HSTs is the result for a thresholded velocity field with joint-gaussian statistics and a correlation coefficient $c_{uv} = -0.4$. (b) The thin solid elliptical contours are the joint pdf, $p_G(u/u', v/v')$, of two gaussian variables with $c_{uv} = 0.4$. They contain 10%, 50% and 90% of the points. The thicker solid ones are $-p_G uv/u'v'$, with contours $[0.2, 0.5, 0.8]$ of its maximum. The dashed ones are the same quantity for M32, and the chaindotted are for C2000 at roughly the same Reynolds number ($y/h = 0.15$). The two hyperbolas are $H = -1.75$.

number increasing towards the right. The line marked with the solid triangle is C2000 for $y^+ > 100$, moving towards isotropy as y increases towards the right. The kink in the line is $y/h \approx 0.2$, and the HSTs approximate the channel near the top of the logarithmic layer, $y/h \approx 0.3 - 0.4$.

The shorter lines in the upper-right part of the plot are conditioned to points within Q^- s. The two roughly diagonal solid curves are the HST cases, with thresholds that increase from $H = 0.25$ to $H = 4$ in the direction of the arrow. The kinked line marked with a triangle is the channel ($H = 1.75$), with the kink at $y/h \approx 0.2$ and y increasing away from the triangle. The inset is an enlargement of this part of the plot. The symbols of the HSTs in the inset are for the nominal threshold $H = 1.75$, and show that the conditional statistics in HST also approximate those in the logarithmic layer of channels. It is clear from the figure that Q^- s are more dominated by a single velocity component than the flow in general, and that this dominance is higher for the stronger events. Both

Figure 3.6: (a) Anisotropy of the Reynolds-stress tensor within Qs, as a function of their box $H = 1.75$. Open symbols are for Q_{uv} , and closed ones are for Q_{uw} . (b) Pdf of the box diagonal of Qs defined from different variable pairs. Case M32 and $H = 1.75$. —, Q_{uv} ; ---, Q_{uw} ; -·-·-, Q_{vw} ; —△—, Q_{uv} of C2000 centred in $y/h \in (0.1, 0.2)$ ($Re_\lambda \approx 100$). In the case of Q_{uv} , open symbols are Q^+ and closed ones are Q^- s.

within and outside the structures, the dominant velocity component is u . A similar figure can be drawn for C950.

However, we should be careful to distinguish between actual properties of the flow and artifacts created by the definition of the Qs. It was shown by [Antonia & Atkinson \(1973\)](#) and [Lu & Willmarth \(1973\)](#) that the joint pdf of u and v in a boundary layer is essentially that of two gaussian variables with the correct cross-correlation coefficient, $c_{uv} \approx 0.4$. The thick dashed line in figure 3.5(a), largely hidden by the thresholded results, is the locus of the two Lumley invariants of such a joint-gaussian distribution, thresholded in the same way as the Q^- s. It agrees very well with the simulation results.

Figure 3.5(b) plots the joint pdf, $p(u, v)$, of two gaussian variables, together with the $H = -1.75$ threshold for Q^- s. It also includes the product $p(u, v)uv$, whose integral is the Reynolds stress, plotted both for the joint-gaussian distribution and for M32. They agree very well, suggesting that most properties computed for these two velocity components using a joint-gaussian distribution should agree well with the simulation results. The chaindotted lines in the figure are for C2000. They also agree reasonably well with the HST and with the gaussian model, although the asymmetry between Q2s and Q4s suggests that the latter are weaker because they come from the outer part of the channel, where the shear is weaker.

Large and small structures are different from the point of view of the flow anisotropy. The open symbols in figure 3.6(a) show the mean second Lumley invariant of the flow

within Qs, as a function of their size. This quantity is proportional to the euclidean norm of b_{ij} and measures the anisotropy of the Reynolds-stress tensor. It ranges from $\zeta = 0$ for full isotropy to $\zeta = 1/3$ for one-component flows. It is clear that the flow is roughly isotropic within structures with $d \lesssim L_c$, which are decoupled from the shear, and only becomes anisotropic within large Qs.

At this point, it may be useful to reflect on the meaning of defining Qs that are much smaller than L_c . There is no problem in understanding small vortices, because enstrophy is a scalar quantity independent of the orientation of the frame of reference. But Qs are defined in terms of the product of two velocity components, u and v , that can only be defined with reference to a given frame orientation. In a shear flow, the frame of reference is linked to the shear. The streamwise velocity u is the component along the direction of the mean flow, and v is the component in the direction of the shear. Small structures that do not couple with the shear cannot have any statistically preferred direction, and there is no reason for either u or v to be especial. From the point of view of these structures, the flow is isotropic and all the velocity components are equivalent. It was indeed found in [LFJ12](#) that the statistics of small Qs are indistinguishable among many different turbulent flows, including some without local shear, implying that many apparently meaningful statistics of small Qs are artifacts of their definition, and cannot be specific to a particular kind of eddy. For example, the closed symbols in figure [3.6\(a\)](#) correspond to Q_{uw} structures, which are defined from points in the flow for which $|uw| > Hu'v'$ (note that the threshold has been kept as in Q_{uv} , for consistency). For small sizes, the anisotropy within these new structures is essentially the same as within the Q_{uv} . Both types of Qs only differ when they are large, in which case the Q_{uv} couple with the shear, but the Q_{uw} (or Q_{vw} , not shown) do not. Note that the thresholding operation induces some anisotropy even on perfectly isotropic Reynolds-stress tensors, because it tends to select velocity pairs along the diagonal of the quadrant plot (see figure [3.5b](#)). It can be shown by direct calculation that a threshold $H = 1.75$ results in $\zeta \approx 0.1$ for an isotropic joint-gaussian velocity distribution with $c_{uv} = 0$, and in $\zeta \approx 0.25$ for $c_{uv} = 0.4$. This is approximately the range of ζ in figure [3.6\(a\)](#). Figures [3.5\(a\)](#) and [3.6\(a\)](#) are consistent with a model of the flow with mostly joint-gaussian velocity components, in which most of the velocity cross-correlation is contained in large Q^- s.

Figure [3.6\(b\)](#) further supports the distinction between small and large Qs. It shows the probability distribution of the size of the Qs defined for the three possible combinations of velocity components. They agree almost exactly below $d \approx 5L_c$, and only diverge for structures above that limit. The figure also plots the size distribution for detached Qs in

C2000 centred at the band of wall distances that matches the Reynolds number of M32, with similar results. The pdfs of Q_{uv} in HST and channels have been separated into Q^+ and Q^- . Only the latter reach large sizes, presumably because they can grow by drawing energy from the shear. We have already mentioned that there are essentially no large Q^+ .

We will mostly restrict ourselves for the rest of the thesis to Qs that are not much smaller than L_c . Smaller ones may perhaps be useful tools for the study of the ‘universal’ inertial range, but they should probably be defined in terms of rotationally invariant properties of the Reynolds-stress tensor. It makes little sense to classify them into quadrants.

Vortex clusters are almost isotropic in terms of the vorticity anisotropy tensor, and essentially identical in this respect to the flow in general. As in channels, vortex clusters in HST tend to be associated with Q^- s. Contrary to channels, they are equally distributed between Q2s and Q4s in HST, as required by symmetry. The number fraction of co-gradient vortex clusters ($\tau_m > 0$) is approximately 60%, almost independent of the Reynolds number and of the threshold. However, the fraction of the total cluster volume contained in co-gradient clusters decreases with increasing threshold, from nearly 100% for $\alpha \approx 0.1$, to approximately 70% at the nominal $\alpha = 1.5$. The fastest decay happens around the percolation crisis, and is more marked for the higher Reynolds numbers.

3.4.3 Geometry and shapes

The shape and orientation of the structures can be characterised by the aspect ratios $a_{ij} = \Delta_i/\Delta_j$, where the three Δ_i are the dimensions of the circumscribing parallelepiped. It was found in LFJ12 that the aspect ratios of large wall-attached Q^- s are $a_{xy} \approx 3$ and $a_{zy} \approx 1$, while the smaller detached Qs are roughly isotropic. Figure 3.7(a) shows the aspect ratios of the Q^- s in HST as a function of their box-diagonal, scaled with the Corrsin (1958) length. The longitudinal aspect ratio increases weakly below $d \approx 5L_c \approx L_z/2$, which is the scale at which structures begin to feel the effect of the computational box. The spanwise aspect ratio decreases slowly with the size of the structures. The figure also includes data for detached structures in the channel C2000, scaled in the same way. They agree well with HST. In both cases, small structures are roughly isotropic while larger ones tend to become longer (increasing a_{xy}) and narrower (decreasing a_{zy}), but there is a range of linear dimensions of about an order of magnitude, $1 \lesssim d/L_c \lesssim 10$, in which the aspect ratios remain approximately constant and the Qs can be considered geometrically self-similar.

Figure 3.7(b) shows the same data for vortex clusters. They behave similarly, although the clusters in channels tend to be slightly more elongated than the Qs.

Figure 3.7: Average aspect ratios of the circumscribing boxes for (a) Q⁻s and (b) Vortex clusters, as functions of the box diagonal of the structure, scaled with Corrsin length. The channel only includes structures whose minimum distance to the wall is $y^+ > 100$, and their diagonal is scaled with L_c at the centre of gravity of the structures. —, a_{xy} ; ---, a_{zy} . (c,d) Average inner and outer dimensions of the structures, as functions of their volume. Kolmogorov scaling. —, r_3 ; ---, r_1 . (c) Q⁻s. The two dashed straight lines have slope 1.15 and 0.57, respectively. (d) Vortex clusters. The dashed straight line has slope 1.35.

A more intrinsic indicator of the shape of the structures is provided by the inner and outer dimensions, r_1 and r_3 , which are, respectively, the diameter of their largest inscribed and of their smallest circumscribed sphere (here implemented as grid-aligned cubes). Roughly speaking, r_3 is the diameter of the structure, which typically coincides in our case with Δ_x , and r_1 is the thickness of the constituent substructures. These dimensions were used by [Catrakis & Dimotakis \(1996\)](#), [Moisy & Jiménez \(2004\)](#) and [LFJ12](#) to characterise structures in various turbulent flows. A third dimension can be defined from the volume of the structure as $r_2 = V/r_1 r_3$, and can be used to classify simple shapes ([Moisy & Jiménez, 2004](#)), but its meaning in very irregular ones is unclear. The

two intrinsic dimensions of Q^- s and of vortex clusters in HST are given in figures 3.7(c,d) as functions of the structure volume. They follow fairly good power laws, especially for the higher Reynolds numbers. Note that $r_3 \sim V^{\gamma/3}$ can be rewritten as $V \sim r_3^{3/\gamma}$, which can be interpreted as a statistical estimate, $D_f = 3/\gamma$, of the fractal dimension of the structures in a family (LFJ12). Figures 3.7(c,d) imply $D_f = 2.6$ for Q^- and $D_f = 2.2$ for clusters. These are slightly higher than the estimates in LFJ12 for channels (2.3 and 2.0, respectively), but the difference is consistent with the fairly broad distributions that are obtained when estimating the fractal dimensions of individual structures by various methods.

A consequence of figures 3.7(c,d) is that the bounding box of all the structures is fairly empty, and becomes emptier for the larger volumes. Thus, if we define a ‘fill fraction’ as V/r_3^3 , it varies from approximately 0.1 for the smallest Q s and clusters, to 3×10^{-3} and 3×10^{-4} , respectively, for the largest ones.

The inner dimension of clusters in figure 3.7(d) is $r_3 \approx 6\eta$, independently of the cluster volume, which approximately agrees with the diameter of individual filamentary vortices in turbulence (Jiménez & Wray, 1998; Tanahashi *et al.*, 2001; del Álamo *et al.*, 2006; Pirozzoli *et al.*, 2008; Stanislas *et al.*, 2008). It is also consistent with the description of clusters as ‘sponges of strings’ (LFJ12).

The inner dimension of the Q s in figure 3.7(c) is more interesting because it depends on the volume, and is therefore a property of the structures rather than of the flow. These structures are described in LFJ12 as ‘sponges of flakes’, mainly from visual observation and from their relatively high fractal dimension. It follows from the definition of r_2 and from the power laws in figure 3.7(c) that $r_2 \sim r_3$ in Q s, which also suggests flakes (see figure 3.3a). A rough model that at least reproduces the functional dependence of r_1 can be constructed if we interpret these flakes as viscosity-dominated layers of thickness r_1 . We know from the definition of Q s that the velocity difference across them is of the order of u' , so that we can approximate the resulting structure-dependent mean strain as $s \approx u'/r_3$ in a coarse-grained sense. Some algebra shows that the corresponding viscous equilibrium thickness is $(\nu/s)^{1/2} \approx (\nu/u'\eta)^{1/2}(\eta r_3)^{1/2}$. The prefactor in this estimate is proportional to $Re_\lambda^{-1/4}$, and varies between 0.28–0.4 in our DNSs. The lower dashed line in 3.7(c) is drawn with a prefactor 0.35 and, given the qualitative nature of the above argument, agrees reasonable well with the measured r_1 .

It is clear that this argument can only be approximate, particularly regarding the estimation of the ambient strain. It follows from well-known integral constraints that the predominant strain in turbulence is of the order of $\omega' \sim u'/\lambda$, and that the associated

viscous length is the Kolmogorov scale η (Pope, 2000). But Hunt *et al.* (2006) argued that strong shear layers exclude the small-scale strain, leaving only the strain associated with the depth of the sheltering layer. In our case, the thresholding operation used to define the structures guarantees that they represent relatively strong velocity gradients, but we will see in §3.4.5 that Qs only have velocity differences of order u' across their largest dimensions, if at all, justifying our strain estimate in the previous paragraph. Smaller details, such as those of size $O(r_3)$, correspond to weaker velocity fluctuations that happen to cross the sharp threshold used in the identification. In the limit of very large structures with $r_3 \sim L_\epsilon$, the previous estimate becomes $r_1 \sim \lambda$, and the argument reduces to the one in Hunt *et al.* (2006), but the present results require extending the analysis to the full range of scales.

Attached ones can be considered as just a particular case of detached ones when the latter get attached to the wall. This is confirmed by figure 3.8(a), which shows the probability distribution of the vertical dimensions of Q4s and Q2s in channels and in HST. Those in the channel have been separated in bands by the position of their farthest point from the wall. The probability of their vertical dimension decreases for larger Q⁻s, roughly as $p(\Delta_y) \sim \Delta_y^{-2.75}$, until it reaches $\Delta_y = y_{max}$. Q⁻s then hit the wall, and the p.d.f. accumulates into the distribution of attached Q⁻s. Choosing a higher band simply extends the probability tail to taller Q⁻s before they reach the wall. The figure also contains data from HST, where Q⁻s follow the same probability distribution, except little disparity from case L38 due to the low Reynolds number effect, but never accumulate at the (non-existent) wall. In other words, there is very large ‘detached’ Q⁻s in HST. Figure 3.8(b) show the same plot for vortex clusters. It is more difficult to find attached vortex clusters with such big y_{max} and they drop much faster, with $p(\Delta_y) \sim \Delta_y^{-5}$. However, the p.d.f.s of the height for vortex clusters defined by different criterion agree with each other reasonably well, despite the slower drops for HST with increased Reynolds number at large scales.

Three-dimensional objects are composed by degraded surfaces and lines, which, although simple, are useful in situations where three-dimensional structures cannot be obtained, like in the early experiments. In such cases, usually the streamwise length l_x is acquired, through Taylor’s advection hypothesis by converting consecutive time sequences satisfying $\tau > Hu'v'$ obtained from hot-wire traces. This is equivalent to intersect structures with random streamwise lines and l_x is consequently the length of segments with connected points belonging to those structures.

These one-dimensional intersections can supply more insights especially when we al-

Figure 3.8: Probability density function of the vertical size of Q^- s (a) and vortex clusters (b) in HST and channels for both attached and detached families with a given maximum height, $y_{max}/h = 0.3 \pm 0.02$ and 0.7 ± 0.03 , increasing in the direction of the arrow. Symbols mark $\Delta_y = y_{max}$. The arrows are $p(\Delta_y) \sim \Delta_y^\beta$ with $\beta = -2.75$ in (a) and -5 in (b). Δ_y is in Kolmogorov unit. For channels, η is chosen at $y/h = 0.3$ and 0.7 respectively.

ready had a general impression about these structures. [LFJ12](#) compared the p.d.f. of l_x in Kolmogorov unit between C2000 at $y \approx 0.4h$ where Re_λ is the maximum and experiments in jets and atmosphere surface layer, finding that l_x is not scaled by η . Here we compare l_x of Q^- s in the buffer layer of C950 where $Re_\lambda \approx 50$ and in the logarithmic layer of C2000 where $Re_\lambda \approx 100$ between H38 and M32, respectively. Results are shown in figure 3.9(a). It can be found that channels and HST with similar Re_λ do not agree with each other well. Specifically, in the buffer layer, channels have lower probability for short segments but the opposite for longer segments, while in the logarithmic layer the probability of all length are lower. Remind that Q2s are longer than Q4s in channels, but even only consider Q2s in C2000, they are still shorter than those in HST. In HST the p.d.f. is almost determined by structures with $V > L_c^3$ while in channels by attached families. The results might imply that attached Q^- s in the logarithmic or outer layer have more holes than Q^- s in HST. But in their near wall region, buffer layer for instance, holes are inhibited by the wall.

To confirm this hypothesis, we computed the number of holes or genus g of these structures with the algorithm developed by [Lozano-Durán & Borrell \(2015\)](#). Genus are computed from two-dimensional surfaces embedded in the three-dimensional space. The number of holes g for Q^- s in HST as a function of their volumes V/η^3 is shown in figure 3.9(b). The figure also includes results from channels. Generally, in both flows, the larger structures are, the more holes they have. However, the density of genus per

volume $\rho_g = g/V$ is not constant in HST in that g increases exponentially with V by $g = 2 \times 10^{-6}(V/\eta^3)^{1.18}$ instead of linearly like in channels. For the same volume, Q^- s in channels have more holes than those in HST. However, it can only explain the difference between the logarithmic layer and HST in figure 3.9(a), but not the buffer layer. And the difference decreases gradually with volume. It is foreseeable that the opposite will be true when $V \gtrsim 10^8 \eta^3$. For vortex clusters in HST, $g = 10^{-6}(V/\eta^3)^{1.45}$ (not shown), suggesting that number of holes grows much faster with volumes than Q^- s. The constant genus density ρ_g of Q^- s in channels might due to their attached property, which can find its evidence from the the same results for vortex clusters, which, as we mentioned in §4.2, are dominated by detached family. Obviously, g of vortex clusters is not a perfect linear function of V as Q^- s, but more like an exponential one (Lozano-Durán & Borrell, 2015). It is not clear why in channels there are more holes. One possibility can be that SS-HST is quite intermittent and H is not time dependent.

Figure 3.9: (a) Probability density function of the one-dimensional intersection of Q^- s in the streamwise direction for HST and channel. ∇ , C950 in the buffer layer where $Re_\lambda \approx 50$; $\triangle$, C2000 in the logarithmic layer where $Re_\lambda \approx 100$. Other solid lines are for HST with the arrow points to higher Re_λ . P.d.f.s are normalized by the maximum value and using the same band. $\times$ represents L_ε/η . (b) Average number of holes of Q^- s as a function of volume in Kolmogorov scale. Solid lines with open symbols correspond to HST and dashed lines with closed symbols to channels. The dash-dot lines are $g = 2 \times 10^{-6}(V/\eta^3)^{1.18}$ for the former and $g = 4 \times 10^{-5}(V/\eta^3)$ for the latter. Symbols are ∇ , C950; $\triangle$, C2000; $\star$, L4000; $\diamond$, L38; $\square$, M32 and $\circ$, H32.

3.4.4 Attached versus detached eddies in channels

Up to now, we have compared the structures in HST with the detached eddies in channels. Most of the larger eddies in channels are attached, because the wall limits how high can they grow, but not their wall-parallel dimensions. Attached and detached eddies behave differently, particularly in their geometry. It was shown in LFJ12 that attached Q⁻s in channels have $a_{xy} \approx 3$, instead of $a_{xy} \approx 1.2$, as in HST. The higher elongations of the larger channel eddies in figures 3.7(a) and 3.7(b) correspond to attached structures.

The origin of this higher elongation is explored in figure 3.10(a), which contains the pdf of a_{xy} in C2000 as a function of the minimum distance of the structures to the wall. This is the dimension originally used in AJZM06 to differentiate between attached and detached objects. The figure shows that only strictly attached structures ($y_{min}^+ \lesssim 20$) are elongated. Those farther from the wall have very similar elongations to those in HST.

The relatively good scaling of figures 3.7(a) and 3.7(b) with L_c suggests that the elongation of the structures is a consequence of their interaction with the mean shear. It was shown by Corrsin (1958) that the relevant parameter is $S^* = Sq^2/\varepsilon$, which measures the ratio between the eddy turnover and the shear deformation time. When $S^* \gg 1$ the structures are controlled by the shear, but for lower S^* , the nonlinear effects of the turnover isotropizes them. Jimenez (2013b) showed that S^* is everywhere moderate in channels except for $y^+ \lesssim 25$, where it grows to $S^* \approx 40$. The reason for the large elongation of attached Qs in channels could thus be the high S^* in the viscous layer. In fact, it was shown by Lozano-Durán & Jiménez (2014b) that large attached Qs have two components with different properties: a coherent one above $y^+ \approx 100$ that approximately advects with the local mean velocity, and a more elongated ‘root’ near the wall that moves dispersively, constantly being shredded by the shear and recreated by viscosity (see sketch in figure 3.10b).

Figure 3.10(c) shows that the flow within attached eddies is slightly more isotropic than in detached ones, but that this is mostly due to the near-wall root. When the root is removed from the average, the two anisotropies become more similar. In any case, the effect is slight in agreement with the conclusion from figure 3.5 that the flow anisotropy is mostly due to the thresholding operation. This apparently extends to the buffer layer.

Whether the near-wall root is the reason for the higher elongation of the attached Qs is tested in figure 3.10(d), which compares a_{xy} for the attached eddies and for their fragments above $y^+ = 100$, as functions of the overall eddy height. It shows that relatively small attached eddies owe their higher elongation to their near-wall tail, while their upper coherent component has an aspect ratio similar to those of detached eddies. But this

effect decreases for larger eddies and is negligible for the larger ones. The dashed line in figure 3.10(d) is a model in which the extra elongation is approximated by a fixed tail of length $\Delta_x^+ = 200$, and fits the data reasonable well. Figure 3.10(d) is drawn for C2000, but similar dimensions are obtained in C950. This is the approximate length of a minimal buffer-layer flow unit (Jiménez & Moin, 1991), suggesting that the near-wall root is formed by multiple such units, some of which accidentally stick in front or behind the coherent outer component. Visual inspection of individual structures confirm that this is a reasonable description (see, for example, figures 8 and 12 in LFJ12). Since these minimal structures have a fixed size in viscous units, their effect on the large eddies of the logarithmic and outer layers is negligible. The implied model is that large Qs in the channel grow until they hit the wall. Once they do, the outer part of the eddy keeps growing, but its bottom is continually truncated by the high shear at the wall and by viscosity. Thus, the eddy grows in length but not in height, and its elongation increases. It is interesting in this respect that a_{zy} is similar for attached and detached Qs (not shown), and that Lozano-Durán & Jiménez (2014b) showed that large attached Qs grow predominantly by merging with other eddies in front and behind, but not sideways.

In the rest of this section, we will mostly restrict ourselves to comparisons of HST with detached eddies in channels, particularly with detached ones with $y_{min}^+ \geq 100$.

3.4.5 Spatial organization

The relative position of tall attached structures was studied for channels by LFJ12, who concluded that they group into spanwise pairs of a Q2 and a Q4, and that the pairs are arranged in streamwise trains, roughly associated with the large-scale streamwise-velocity streaks. Vortex clusters tend to be located in the gap between the two components of the pair, predominately associated with the Q2. In the following we repeat their analysis for HST, in part to compare the two flows, but also because it is possible in HST to extend the study to the third dimension without interference from the wall. In particular, we have seen that the symmetry of HST precludes the preferential association of the clusters with only one flavour of Q^- , and we expect to find more symmetric composite structures in HST than in channels. We define three-dimensional joint p.d.f.s $p^{ij}(\delta_x, \delta_y, \delta_z)$ of the relative distances between structures of type j with respect to i , where $i = 2$ refers to Q2s, $i = 4$ refers to Q4s, and $i = C$ to vortex clusters. The vector of relative distances is defined as

$$\boldsymbol{\delta}^{(ij)} = 2 \frac{\mathbf{r}^{(ij)}}{d^{(j)} + d^{(i)}} = 2 \frac{\mathbf{x}_c^{(j)} - \mathbf{x}_c^{(i)}}{d^{(j)} + d^{(i)}}, \quad (3.5)$$

Figure 3.10: (a) Pdf of a_{xy} of Q^- in C2000 as function of their minimum distance to the wall. Each vertical section is the individual pdf at one wall distance. Contours contain 50% and 95% of the data. ---, Average aspect ratio. The vertical dashed line is $y_{min}^+ = 20$. Symbols are mean and standard deviations of a_{xy} in HST, as in table 3.1. They are plotted at arbitrary y -locations. (b) Sketch of the root and body of a large attached Q_2 . Flow is from left to right. (c) Mean anisotropy of the Reynolds stress tensor within Q^- s in C2000, as function of their diagonal size. The solid line is for attached Q^- s; closed symbols are for detached ones, and open symbols are averaged over the the part of the attached Q^- s above $y^+ = 100$, as sketched in (b). (d) Average a_{xy} of attached Q^- s in C2000 as a function of their vertical dimension. See sketch in (b) for definitions. —, Δ_x/Δ_y ; $\triangle$, $\Delta_{x,100}/\Delta_y$; ---, $(\Delta_x - \Delta_{x,100})^+ = 200$. The short horizontal dashed line is $a_{xy} = 1.2$, as in the detached and HST structures in figure 3.7(a).

where $\mathbf{x}_c^{(i)}$ and $d^{(i)}$ are respectively the centre of gravity and the diagonal length of the circumscribing box. When $i \neq j$, we generally take advantage of the spanwise symmetry S1 to define $\delta_z > 0$ as always pointing towards the closest structure of type j relative to i , as in LFJ12. We only consider two structures as related if their size is similar enough

to satisfy

$$1/2 \leq d^{(j)}/d^{(i)} \leq 2. \quad (3.6)$$

In channels, LFJ12 use a condition on the height of the centre of gravity of attached structures instead of (3.6), and only study relative positions in the (x, z) plane. It can be shown that their criterion implies $|\delta_y| \leq 0.2$. We have recomputed their results using (3.6) and three-dimensional positions but, for the sake of comparison, all the two-dimensional sections of our three-dimensional pdfs are presented averaged over slabs of thickness $\delta = \pm 0.2$ normal to the plane of the plot.

For the purpose of this section, we disregard structures that are ‘infinite’ in the sense that their size in any direction equals to that of the simulation box, because their centre of gravity is not well defined. They are much more common in HST simulations than in channels because of the minimal simulation box. For the same reason, the distances between structures are limited to $[-L_x/2 : L_x/2] \times [-L_y/2 : L_y/2] \times [-L_z/2 : L_z/2]$, which are implemented by copying, translating and shifting structures whenever they cross the boundaries of the fundamental simulation box. To test the soundness of this procedure, we repeated the analysis of case L38, whose vertical dimension is not minimal, considering only structures with $y_c \in [L_y/3 : 2L_y/3]$ relative to those with $y_c \in [0 : L_y]$, without using the shifted periodicity in y . The results did not change.

Finally, to simplify the presentation, we will not generally use case L38 from now on. We have seen in several occasions that its Reynolds number is comparable to the buffer layer in channels, rather than to the logarithmic or outer layers, complicating comparisons between the two flows. However, to confirm that its differences with the other HSTs are a Reynolds number effect, rather than due to its different vertical aspect ratio, most of the analysis was repeated for some simulations not otherwise used in this paper. Two low-Reynolds number cases ($Re_\lambda = 50$) in flat boxes, L32 ($A_{yz} = 2$) and L34 ($A_{yz} = 4$), behaved as L38. A higher-Reynolds number case in a taller box ($A_{yz} = 4$, $Re_\lambda = 100$) behaved as M32 or H32.

Relative positions of structures of the same type

Figure 3.11(a) displays the mean value of the absolute distance, $r^{(ij)} = |\mathbf{r}^{(ij)}|$, from a Q to its closest Q of the same kind, as a function of the box diagonal of the reference structure. Below $d \approx 50\eta$, the mean distance is constant, $r^{(44)} \approx 50\eta$, which is roughly the wavelength of the maximum of the dissipation spectrum in most turbulent flows (Jimenez, 2013b), and the size of the largest structures that behave as coherent predictable units in the dissipative end of the turbulence spectrum (Yoshida *et al.*, 2005). These are essentially

Figure 3.11: (a) Average distance between closest pairs of Qs of the same kind, $r^{(44)}$, defined as in (3.5), as a function of the diagonal size of the reference structure. The dashed line is $r^{(44)} \approx d$. (b-d) Cross sections of the three-dimensional pdf of the relative distance between eddies of the same kind, integrated over $\delta = \pm 0.2$ normal to the plane of the plot. (b) $p^{44}(\delta_x, \delta_y)$ for $d < 4L_c$. (c) $p^{11}(\delta_x, \delta_y)$ for $d < 4L_c$. (d) $p^{44}(\delta_x, \delta_y)$ for $d \geq 4L_c$. The contours in (b-d) contain the highest 15% of the data. In all cases, open symbols are for Q_{uv} : $\square$, M32; $\circ$, H32; $\triangle$, Qs in C2000 with $y_{min}^+ \geq 100$. The closed squares are Q_{uv} in M32, in which case the vertical axis is δ_z .

Kolmogorov viscous eddies, and the figure shows that they are densely packed on average, separated by other eddies of the same size. Beyond $r^{(44)} \approx 50\eta$, the distance to the closest comparable structure is $r^{(44)} \approx d$ up to the largest available scale, $d \approx L_\varepsilon$. These are the self-similar inertial eddies, which are also densely packed. It follows from their definition that the velocity fluctuations in Qs are $O(u')$, but their close packing suggests that they are pieces of larger structures that appear when the Reynolds stress gets locally slightly above or below the chosen threshold, but which are not necessarily separated from each other by fluctuations of $O(u')$.

Figure 3.11(b) displays two-dimensional pdfs of the relative position of neighbouring

small Q4s ($d < 4L_c$). In channels, these are arranged streamwise in the (x, z) plane (LFJ12). This is still true here but, in addition, the pdf is inclined downwards in the streamwise direction. Note that the pdfs included for detached Qs in the channel agree well with the HST, and are also inclined downwards. Because of the symmetry of HST, the same inclination is found for Q2s in HST, but the downward tilt of the Q2s is slightly less marked in the channels (not shown). Since the vertical velocity of the Q2s is coming from below, even these relatively small detached Qs are apparently influenced by the wall.

It is tempting to associate the tilting of the pdf with a systematic vertical meandering of the streaks, but the explanation is probably simpler. We have seen that small eddies are basically decoupled from the shear and from the mean velocity. On the other hand, Q4s are defined by $u > 0$ and $v < 0$, so that the flow velocity within them is tilted downwards, and we have seen in figure 3.11(a) that their separation suggests that they are pieces of larger structures whose connectivity is broken by weaker velocity fluctuations. It is then natural to expect that the closest Q of the same kind is located in the direction of the predominant velocity fluctuations which, in the case of Q4s, is the downwards diagonal. This is confirmed by figure 3.11(c), which contains the relative position of Q1s, which are predominantly arranged along the upwards diagonal. It is even more clearly shown by the lines with closed squares in figures 3.11(b,c), which correspond to small Q_{uv} eddies in M32. Their separations agree well with those of the Q_{uv} , and their positions are aligned along their corresponding diagonals, although in the (x, z) , rather than the (x, y) plane.

Finally, figure 3.11(d) presents the pdf of the relative position of neighbouring large Q4s ($d \geq 4L_c$). It is still true that the eddies are aligned streamwise, but the predominant relative separation is now almost horizontal, presumably reflecting the anisotropy of the large-scale velocities. As we saw in the discussion of figures 3.5(a) and 3.6(a), u is the predominant velocity component, and the flow anisotropy is mainly associated with the largest structures.

Relative positions of structures of different types

Figure 3.12(a) displays the mean distance between Qs of different kind (from a reference Q2 to its closest Q4). It contains the same viscous range as in the previous case, $d \lesssim 50\eta$, but the distance between these Kolmogorov structures scales with the Taylor microscale $r^{(24)} \approx 3\lambda$. In addition, the inertial range is not self-similar any more, with $r^{(24)} \sim d^{(0.5-0.7)}$. Both issues are presumably related, and admit dynamical explanations of the type given in §3.4.3 for the intrinsic dimensions of the Qs, but the reason is again probably simpler, and geometrical. It follows from the distribution of the volume of the

Figure 3.12: Relative position of the closest $Q2_{uw}$ to a given $Q4$. $\square$, M32; $\circ$, H32; $\triangle$, C2000 with $y_{min}^+ \geq 100$. The solid squares are Q_{uw} in M32. (a) Average closest distance $\langle r^{(42)} \rangle(d)$, as a function of the diagonal size of the reference $Q4$. The dashed line corresponds to $\langle r^{(42)} \rangle \sim d^{2/3}$. (b) Pdf of the $r^{(42)}$ for different bands of d , centred at $d/\eta = 50, 100, 200, 400$, with the widest tails corresponding to the smaller d . Case M32. Each pdf is normalised with its mean value. The dashed line is the nearest-neighbour distribution for a Poisson point set in three-dimensional space. (c) Probability of ‘isolated’ $Q4$ s, defined as those with no neighbour closer than $r^{(24)} = 2d$, as a function of d . (d) Anisotropy of the position of the closest $Q2$ with respect to $Q4$ s, as a function of d .

Qs given in §3.3, $p(V) \sim V^{-5/3}$, that the probability distribution of $d \sim V^{1/3}$ is $p(d) \sim d^{-\beta}$ with $\beta = 3$. In practice, the exponent ranges from $\beta = 2.8$ for L38 to $\beta = 3.3$ for H32 and $\beta = 4$ for C2000. When looking for nearest neighbours satisfying condition (3.6), the number of available candidates per unit volume is therefore proportional to

$$n \sim \int_{d^{(i)}/2}^{2d^{(i)}} \alpha^{-\beta} d\alpha \sim d^{(i)1-\beta}.$$

Assuming a uniform distribution of candidate structures, the average distance from a randomly chosen point to the closest structure would be $\langle r^{(ij)} \rangle \sim n^{-1/3} \sim d^{(i)(\beta-1)/3}$. The dashed line in figure 3.12(a) is drawn with slope $2/3$ ($\beta = 3$). It represents the data well, although the slope increase from M32 to C2000 is clear. In particular, note that $\beta = 4$ results in $\langle r \rangle \sim d$, which agrees with the steeper slope of C2000 in figure 3.12(a), and with the results in [Lozano-Durán et al. \(2012\)](#). This dependence also explains the scaling of the minimum distance with λ . Invoking the large-scale independence of turbulence with respect to the Reynolds number, i.e., structures of $d = O(L_c)$ should be separated by distances $r = O(L_c)$, the previous relation should be written as

$$\langle r^{(ij)} \rangle \sim L_c^{(4-\beta)/3} d^{(i)(\beta-1)/3} \quad (3.7)$$

Substituting $d \sim O(\eta)$ in this equation, we obtain $r \sim O(\lambda)Re_\lambda$, which is very close to λ when $\beta \approx 3$.

The rest of figure 3.12 tests this model. Figure 3.12(b) displays the pdf of the distance among Q4s and Q2s for several size bands in the ‘inertial’ range, $50\eta < d < 4L_c$. They are compared with the distribution of closest distances, $p(r) = (dN/dr) \exp(-N)$ for a Poisson set in which the number of points within a sphere of radius r is $N(r)$ ([Feller, 1971](#)). They agree reasonably well among themselves and with the Poisson model when normalised by their respective average separation.

A consequence of the two previous figures is that the separation between small Qs of different kind is much larger than their size, so that these Qs cannot be considered to be paired. It was found by [LFJ12](#) that tall attached Q2s in channels tend to be associated with similar Q4s within a distance of $O(d)$, and that such pairs behave like units. For example, they organise the flow between the Qs into a coherent quasi-streamwise ‘roller’. Structures which are much further apart cannot be considered as pairs, and they are unlikely to influence each other. Figure 3.12(c) shows the probability that a Q4 is isolated in the sense that its closest Q2 is at $r^{(24)} > 2d^{(4)}$. It shows that Qs smaller than $d/\lambda \approx 5$ are most likely isolated, but that larger ones are not. The same holds for detached Qs in channels. Several scalings of d were tested for figure 3.12(c) and the one with λ works best. This is not surprising, because we have seen that the smallest distance between different kinds of Qs is also $O(\lambda)$.

Finally, figure 3.12(d) tests the preferential orientation of the location of Q2s with respect to Q4s. The Poisson model suggests that mutually isolated structures should be isotropically distributed with respect to each other, while those that form pairs may be organised in particular ways ([LFJ12](#)). If $\mathbf{r}$ is the separation vector between two structures,

a separation tensor can be defined as $\rho_{ij} = \langle r_i r_j \rangle$. As in the case of the Reynolds stress, an anisotropy tensor can be formed as $A_{ij} = \rho_{ij}/\rho_{ii} - \delta_{ij}/3$, and its anisotropy measured by the second Lumley invariant of that tensor, ζ_{rr} . Figure 3.12(d) shows that small Qs are essentially isotropically distributed with respect to each other, as required by the Poisson mode, but that the isotropy breaks down for sizes of the same order as those in which the structures begin to pair. The behaviour of channels is different in this respect, presumably because the influence of the wall, and the preferential orientation begin for much smaller eddies that do not necessarily form pairs.

Note that these arguments do not apply to pairs of Qs of the same kind, where we have seen that neighbouring structures are separated by relatively weak fluctuations that happen to cross the detection threshold, and candidate neighbours are not uniformly distributed in space. On the other hand, Qs of different kinds are necessarily separated by velocity differences $O(u')$ and, if we restrict the velocity gradients to at most $O(\omega')$, such velocity jumps are unlikely to happen across distances smaller than λ .

Figure 3.13 displays sections of the three-dimensional pdfs of the relative positions of Q4s with respect to a Q2. As in figure 8, statistics can only be compiled by scaling distances properly and, following (3.7), the similarity scaling (3.5) is substituted by

$$\boldsymbol{\mu}^{(ij)} = \frac{\mathbf{x}^{(j)} - \mathbf{x}^{(i)}}{d^{(i)2/3} L_c^{1/3}} \quad (3.8)$$

Note that the same scale is used for all the flows, even if figure 3.11(a) shows that their actual average distance is not exactly the same, especially for the channel. Figure 3.12(a), which displays the section of the pdfs in the (z, x) plane, shows that the three flows collapse well. The result is similar to that found by LFJ12 for attached Qs in channels. There is essentially no probability of finding a Q4 in front or behind the Q2, and the closest Q4s are found in the spanwise direction. Note that the symmetry of this pdf is statistical. It was shown in LFJ12 that, when the direction of the closest partner is defined as positive, the groups are found to be pairs, not trios. The second-closest partner is either much farther away, missing, or in the same direction as the closest one. The same was found to be true here.

However, the Qs in figure 3.13(a) are relatively small ($d < 4L_c$), and arguments similar to the ones used in the previous section for $\mathbf{r}^{(44)}$ suggest that some of the properties of the pdf might be kinematic. Since the perturbation velocity in a Q2 is directed along the downwards streamwise diagonal, $\boldsymbol{\mu} \sim [-1, 1, 0]$, it would tend to exclude from that direction structures with an opposing velocity ($\boldsymbol{\mu} \sim [1, -1, 0]$ for a Q4). In fact, the pdf of $\mathbf{r}^{(24)}$ should be complementary to that for $\mathbf{r}^{(44)}$; empty along the downwards sloping

Figure 3.13: Cross sections of the three-dimensional joint pdf of the relative position of the closest Q4 to a given Q2. $\square$, M32; $\circ$, H32; $\triangle$, C2000 with $y_{min}^+ \geq 100$. All figures are in the similarity variable $\boldsymbol{\mu} = (\mathbf{x}^{(4)} - \mathbf{x}^{(2)})d^{(2)-2/3}L_c^{-1/3}$, and pdfs are integrated over $\delta = \pm 0.2$ normal to the plane of the plot. (a) In the (z, x) plane, for small Q2s ($50\eta < d^{(2)} < 4L_c$). (b) In the diagonal plane orthogonal to $\mathbf{r} = [1, -1, 0]$, for small Q2s. (c) As in (b) for closest Q4 to reference small Q2 but for Q_{uw} (open squares) and Q_{vw} (closed squares) in the diagonal plane orthogonal to $\mathbf{r} = [1, 0, -1]$ and $\mathbf{r} = [0, 1, -1]$ respectively. Data from M32. In all the panels, $\mu_{ij} = (\mu_i + \mu_j)/\sqrt{2}$ and contours contain the highest 15% of the data in the $(-2, 2)^2$ region in the plane of the pdf.

diagonal, and concentrated on the equatorial plane normal to that direction. This turns out to be true. Figure 3.13(b) shows a section along this inclined equatorial plane and reveals that the two high-probability regions in figure 3.13(a) are sections of an inclined arch or semicircle, whose dimensions also collapse well among the different flows. It can be shown that the fact that this equatorial section is not a full torus is not due to the misalignment of its plane. It should be emphasized again that the symmetry of the distributions in figure 3.13 is statistical, due to the symmetry S1 in §3.4. In fact, it is explicitly enforced on the plot to improve statistics. It should also be made clear

that these distributions do not represent the shape of any structure, even on the average. They describe probabilities of the relative position of the centres of gravity of different structures.

The arched shape is different from that between attached Qs in channels, where the probability distribution is formed by a Q2 and a Q4 located spanwise from one another (a three-dimensional representation can be found in figure 12a of LFJ12). The shape of the pdf depends on the structures involved. It follows from symmetry that the arches in figure 10 become Us for the position of Q2s relative to a Q4 in HST. The same is true for the detached Qs in channels. The interpretation of these shapes is interesting. The Q4s in figure 3.13 are structures of high streamwise velocity. The figure shows that they tend to be located above or to the side of the low-streamwise-velocity Q2s, but not below. The complementary shape of the pdf of $\mathbf{r}^{(24)}$ carries the same message: high velocity is above low velocity in the presence of a Q^- . This is the direction of the ambient shear and, since the Qs are defined from perturbation velocities, the implication is that they tend to be located in regions in which the shear is higher than the mean. This is reasonable, since they have to draw their energy from the local velocity difference, and has been mentioned often in the context of vortical structures (see, for example, Adrian (2007)). These high-shear regions can be interpreted as the boundaries of streamwise-velocity streaks, which are known to be layers of high activity both in the buffer region (Kim *et al.*, 1987; Jiménez & Pinelli, 1999; Schoppa & Hussain, 2002) and in the logarithmic layer (Meinhart & Adrian, 1995; Flores & Jiménez, 2006). In the buffer layer, the high shear close to the wall substitutes the horizontal segment of the arch in figure 3.13, and the attached Qs are only concentrated in the lateral boundaries between low- and high-velocity streaks (although see the enveloping high-velocity streak in figure 12d of LFJ12). What figure 3.13 implies is that, away from the wall or in HST, the Qs concentrate either in the side or top boundaries of the low-velocity streaks. A little thought shows that it also implies that only the top part of the low-velocity streaks and the bottom part of high-velocity ones are sharp. Both of them are co-shear layers. The bottom part of the low-velocity streaks and the top of high-velocity ones, which would require layers of counter-shear vorticity are either not sharp, or contain no Qs. A similar arrangement of zones of relatively uniform velocity bounded by sharp interfaces has been discussed often in the literature (Meinhart & Adrian, 1995). Conditional flow fields in the vicinity of pairs of Qs are presented in the next section.

The high probability region of finding closest Q4 to small reference Q2 is in the plane that is orthogonal to the velocity vector formed by the velocity components by which the

quadrant analysis is implemented tends to be universal. Figure 3.13(d) shows the joint pdf of the relative distance of closest Q4 to reference Q2 for Q_{uw} and also Q_{vw} . The high probability region is in the plane perpendicular to $\boldsymbol{\mu} \sim [1,0,-1]$ and $\boldsymbol{\mu} \sim [0,1,-1]$ for Q_{uw} and Q_{vw} respectively. Both of the high probability regions are also of arch shape as in figure 3.13(b), but they are not symmetric respect to the residual direction (vertical direction for Q_{uw} and streamwise direction for Q_{vw}).

At last, we are going to discuss the relative position between vortex clusters and Qs. Figure 3.14(a) is the mean three-dimensional distance between Q2s (Q4s) and the closest vortex cluster. It behaves similarly to the inter-Q4 distance in 3.11(a), but details are different. The distance in channels agrees poorly with HST, almost doubled compared with HST. The reason appears to be that the relative geometry is different in the two flows. Vortex clusters in channels tend to be located close to the wall (LFJ12) and tend to stay vertically away from detached Qs of either kind. While vortex clusters in HST have no preferred vertical location, and their average vertical position is level with the closest Q^- . There might be another reason responsible. Remind that both vortex clusters and Qs here are detached. Since attached Qs are dominant to detached Qs, most of the detached vortex clusters are supposed to be associated with attached Qs, thus the distance between detached vortex clusters and detached Qs is roughly the same as looking for a pair inside attached Qs to detached Qs. We have verified that for channels with different Reynolds number, the distance is scaled with η , so the disagreement between HST and the channel is just because they are different flows.

The viscous range is still roughly below $d < 50\eta$, but the mean distance in that range is $r^{CQ} \approx 20\eta$ for HST, roughly half of that between inter-Q4 pairs. The inertial range is not self-similar, but behaves opposite to Q2-Q4 pairs, i.e., $r^{(CQ)} \sim d^{1.25}$. We have seen that the number of vortex clusters drops much faster than Qs, so that for the same number of Qs, the paired vortex clusters are fewer, thus one vortex cluster would be paired with several Qs, which will lead to larger mean distance. Because of the symmetry of HST, the mean distance from Q2s and Q4s to clusters is similar. Somewhat surprisingly, figure 3.14(a) shows that the same is true in channels, except in the viscous range, even if it is known that clusters are predominantly associated with Q2s in them. The fact that vortex clusters are more associated more with Q2s is based on the quadrant of u_m and v_m , as shown by figure 4(a) in LFJ12. But it does not mean that they are inside strong Qs.

Figure 3.14(b) shows the relative horizontal location of Q^- s and clusters. As in channels, clusters are located spanwise from the Qs, and their position in this plane is roughly the same. Note that the spanwise axis in this figure is always oriented towards the closest

cluster.

Figure 3.14: (a) Average distance between closest pairs of vortex clusters and Q⁻s, $r^{(CQ)}$, as a function of the diagonal size of the reference structure. The dashed line is $r^{(CQ)} \approx d^{1.25}$. Both distance and diagonal are in Kolmogorov unit. (b) Cross sections of the three-dimensional pdf of the distance of vortex clusters relative to Q2s with $d^{(2)} > L_c$, integrated over $\delta = \pm 0.2$ normal to the plane of the plot. In both panels, vortex clusters and Qs in C2000 are detached families with $y_{min}^+ > 100$.

3.4.6 Q2-Q4 pairs

As shown in the above section that Q2s and Q4s are dominantly paired in the spanwise direction. This section is devoted to the averaged flow field of these pairs. Considering that only those pairs with $r \approx d$ are supposed to be influenced by each other. Pairs are defined by the closest neighboring Q4 satisfying

$$|\delta_x| < 1.0, |\delta_y| < 0.5 \text{ and } |\delta_z| < 1.0 \quad (3.9)$$

relative to a Q2 in the inertial range $50\eta < d^{(2)} < L_\epsilon$, with diagonals of the two structures satisfying (3.6). With the restriction of (3.9), the mean distance between the closest Q2s and Q4s is self similar for $d \lesssim L_\epsilon$ and the distance is scaled well by η instead of λ (not shown). The same conclusion to what we are showing below can be obtained if pairs are defined by closest Q2s relative to Q4s. The conditional average to Q2-Q4 pairs is computed in the reference

$$\mathbf{r} = 2(\mathbf{x} - \mathbf{x}_c^{(24)})/\varrho d^{(24)}, \quad (3.10)$$

where $d^{(24)} = (d^{(2)} + d^{(4)})/2$ is the mean diagonal length, $\mathbf{x}_c^{(24)}$ is the midpoint of the line connecting the centers of the two bounding boxes and ϱ is the factor that determines the

size of the region Ξ enclosing the pair, from where flow information are accumulated. ϱ has to be chosen properly so that the averaged flow field is converged in each direction. $r_z > 0$ is pointing to Q4s. Although we have seen that the inertial range is only moderate for H32, the following results are from M32.

The average process is implemented by adding any interested flow information in Ξ to a common grid with fixed number of points, thus the grid size is proportional to $d^{(24)}$. All the points in Ξ are mapped into the common grid. Therefore, there are two different ways to do the average. One is add the information of each pair at the points in Ξ to the corresponding grid first and then average these information when this process is finished for all the pairs. The number of samples in each grid is the sum of number of grid points corresponding to that grid. Since large pairs have larger Ξ and more points, results are weighted by them and difficult to converge since large pairs are rare. The other one is first do the average of the flow information of all the points in Ξ corresponding to the same grid and then add the mean value to that grid. Then the number of samples for each grid is the number of pairs. Results in this way are weighted by small scales and converge much faster. The results shown below are obtained by the second method, which is different from LFJ12, AJZM06 and Flores *et al.* (2007a).

Figure 3.15(a) shows the averaged organization of Q2-Q4 pairs. Averaged vortex cluster lies equally between Q2 and Q4, instead of prefer to Q2 as shown by the same results in LFJ12. Averaged Q2 and Q4 are slightly elongated in the streamwise direction, as expected from figure 3.7(a). The averaged streamwise velocity associated with the pair is displayed in figure 3.15(b), where high and low streamwise velocities elongated in the streamwise direction are found. The averaged vertical velocities are organized in a similar way as the averaged streamwise velocities, due to the definition of intense Qs, but they are weaker and shorter (not shown), which is consistent with table 3.2. The averaged spanwise velocity, however, is different, as shown by figures 3.15(c-d). Obviously, spanwise velocities with opposite signs around Q2-Q4 pairs are inclined and aligned in the $x - y$ plane due to the no-clashing requirement as we have discussed in the above section. The inclination angle is roughly 40° , which is very close to the value 32° found in the turbulent channels and boundary layer by Sillero *et al.* (2014a) in terms of the inclination angle of two-point correlation function of w in the vertical plane. In fact, this inclination can be found easily in instantaneous w fields (not shown).

The averaged flow field at the cross section $r_x = 0$ is shown in figure 3.15(e). Note that high and low streamwise velocities are not paired symmetrically in the spanwise direction, but with the high streamwise velocity slightly above the low streamwise velocity, which

is consistent with the joint pdf of the relative position of Q4s with respect to Q2s (figure 3.13) and also with the relative position of three-dimensional velocity structure $u^\oplus$ with respect to $u^\ominus$ (not shown). Besides, the most probable Q4 and Q2 are not symmetrically enclosed by streamwise velocities. There is a quite wide ‘wake’ of high streamwise velocity above Q4 and the opposite is true for Q2. Figure 3.15(e) also displays the averaged streamwise streak with value $U = 0$. And figure 3.15(g) shows its cross section in the plane $r_y = 0$. Q4 and Q2 are roughly at the boundary of the streak, as predicted in §3.4.5. The temporal evolution of the streak and Reynolds stress shown in figure 2.13 reveals that the streak breaks down when the Reynolds stress is very high. Although the average can only give a static wavy and meandering streak, figure 3.15(e) suggests that as Q2 and Q4 move upward and downward, the streak will be more steeper in the vertical direction thus will not be able to sustain such big velocity difference and will finally get breakdown. Dynamically, a given Q2 cannot be always paired with the same Q4. The results in figure 3.15(e) may also suggest that, the forming of a streak is when a Q2 is paired with Q4, but the streak will breakdown when they are no longer paired.

It was conjectured that Q2s and Q4s might be two parts of a common structure in which they are embedded, since they seem to happen at the same time and same distance to the wall during the bursting process (Flores & Jiménez, 2010), which, however, has to be verified further from the time resolved analysis. And they move with the same absolute vertical velocity $|dy_c/dt| = u_\tau$, by temporal tracking the centers of gravity of Q2s and Q4s (Lozano-Durán & Jiménez, 2014b). The averaged velocity (LFJ12) and vorticity (Jimenez, 2013b) fields of attached pairs in channels suggest that this common structure is a large scale streamwise roller, which is also true for HST based on figure 3.15(e).

Figure 3.15(f) shows the averaged ω_x in the same plane as in figure 3.15(e). It can be found that except the positive ω_x in the center of the pair, there are also negative ω_x with intensity roughly half of positive ω_x at the two sides in the spanwise direction of the pair. This is different with the attached Q2-Q4 pairs in channels, as shown by Jimenez (2013b), where the strongest negative ω_x happens at the bottom of the pair. That negative ω_x near the wall is actually the induced secondary vorticity (Jiménez & Pinelli, 1999) due to the wall.

Figure 3.15(g) shows the cross section of averaged streamwise vorticity in the vertical plane $r_z = 0$. It is inclined to the streamwise direction, with angle $\approx 22.5^\circ$, which is roughly half of the typical angle of hairpins (Head & Bandyopadhyay, 1981). Q2 and Q4 are also inclined by the same angle. This streamwise roller probably is the same to the

one proposed by [Sillero *et al.* \(2014b\)](#) based on the two-point self correlation of v and w , in the sense that the positive and negative correlated regions of v are paired in the spanwise direction but in the vertical direction for w , which is also true from figure 3.15(c). Although Qs are defined by only u and v , the streamwise roller incorporates w together, thus connects full velocity field. Note that the streamwise vorticity of attached Q2-Q4 pairs in ([Jimenez, 2013b](#)) is the curl of the averaged velocity field, instead of directly averaged streamwise vorticity. We found that the average of the vertical derivatives of any quantity in the channel is quite noisy due to averaging quantities in a non-uniform grid to a uniform one, which makes the derivative loses its meaning in that direction.

The streamwise rollers composed of Q2-Q4 pairs offer a much simpler model to study shear-induced turbulence, instead of studying Q2s and Q4s separately. For example, [Perry & Chong \(1982\)](#) obtained the logarithmic velocity profile by a hierarchy of hairpins in channels, suggesting that this streamwise roller may also be able to reconstruct statistics in shear-induced turbulence in a similar way. But the identification of such rollers still lacks a proper definition. Apparently only the streamwise vorticity itself is not enough because it is usually for small scales.

Although it is appealing, the streamwise roller is only part of the story and cannot represent full characteristics of the pair. It can be found that there is also ω_y in the center of the pairs with almost the same intensity as ω_x . On the other hand, ω_z is different, both in intensity and location. The mean ω_x and ω_y are both positive in the center of the pair because we have reoriented the z axis based on the symmetry of the flow. But the mean ω_z is not affected by this process and is mainly contributed by $\partial u/\partial y$, which is a shear layer. Figure 3.16(a) shows the averaged ω_z (red) and averaged $\partial u/\partial y$ (magenta) with the same absolute value. They almost coincide with each other, with slight shift in the streamwise direction due to the contribution of $\partial v/\partial x$. They are at the top and bottom of Q2s and Q4s respectively, connected together by vortex clusters, or streamwise and vertical vortices. Figure 3.16(b) shows the flow field at the cross section $r_x = 0$ of figure 3.16(a). If the threshold for the isosurface of vortex clusters decreases, the averaged vortex clusters will be elongated in the diagonal direction where the two shear layers lie. Figure 3.16(c) shows the cross section at $r_z = -0.35$, where $\partial u/\partial y$ is close to its maximum. Based on figures 3.16(b-c) we can roughly estimate the size of the shear layer by $\partial u/\partial y \approx 0.05\omega'$, which is roughly $2d^{(24)}$ in the streamwise and spanwise directions and $1.5d^{(24)}$ in the vertical direction. And the averaged $d^{(24)}$ is roughly $0.2L_z$, which is double of the Corrsin length. Finally figure 3.16(d) shows the organization among total streamwise velocity, shear layers and vortex clusters. It is necessary to mention that the

Figure 3.15: Flow fields conditional averaged to Q2-Q4 pairs in the inertial range in SS-HST from M32. (a) Averaged Q2 (green), Q4 (blue) and vortex clusters (grey). With isosurfaces represent 0.7 times the maximum value of the probabilities; (b) Averaged streamwise velocity with blue for $u^+ = 1.0$ and red for $u^+ = -1.0$; (c) and (d) are the averaged spanwise velocity with views into the vertical and horizontal planes respectively. Blue for $w^+ = 0.5$ and red for $w^+ = -0.5$; (e) Flow field at the cross-section $r_x = 0$. The shaded map for streamwise velocity with $u^+ = [-1.5 \ 1.5]$ and arrows are $(v - w)$. The solid curve is the averaged total streamwise velocity with $U = 0$; (f) The same as (e) but the shaded map is ω_x in the range $[-0.2 \ 0.55]\omega'$; (g) Flow field at the cross-section the cross section $r_y = 0$. The shaded map is the averaged total streamwise velocity U^+ in the range $[-1.5 \ 1.5]$; (h) Averaged ω_x at the cross-section $r_z = 0$, in the range $[0 \ 0.55]\omega'$. In (e-f), enclosed contours represent the same meaning as those isosurfaces in (a). In (g), solid contour is for Q4 taken at $r_z = 0.35$ and projected to $r_z = 0$.

averaged velocity scales with u_τ and the averaged vortices scale with ω' by comparing between M32 and L38.

Figure 3.16: Flow fields conditional averaged to Q2-Q4 pairs in the inertial range in SS-HST from M32. (a) Averaged $\partial u / \partial y$ (magenta), ω_z (red) and vortex clusters (grey). With isosurfaces represent $0.18\omega'$ for $\partial u / \partial y$, $-0.18\omega'$ for ω_z and 0.7 times the maximum value of the p.d.f.s belongs to vortex clusters; (b) At the cross section $r_x = 0$ with shaded map for $\partial u / \partial y$ in the range $[-0.275, 0.275]\omega'$. Other lines are the same as in figure 3.15(c); (c) Averaged $\partial u / \partial y$ at the cross-section $\delta_z = -0.3$, in the same range as in (c). The solid contour is $\omega_z = -0.18\omega'$; (d) Three-dimensional view of the total streamwise velocity $U = 0$, shear layers and vortex clusters. The arrow points to the flow direction. The total streamwise velocity is colored by r_y .

The averaged flow fields conditioned to Q2-Q4 pairs can be understood better by inspecting the averaged flow fields conditioned to Q2s and Q4s separately. In SS-HST Q2s and Q4s are symmetric but not in the channel. Therefore only results for Q2s are shown in the former but for both Q2s and Q4s in the latter. Figures 3.17(a-b) shows the cross sections of the averaged u conditioned to Q2s at $r_z = 0$ and $r_x = 0$, respectively. The intensity of u is roughly doubled compared with that conditioned to Q2-Q4 pairs. The difference probably is because that Q2s used in figures 3.15 and 3.16 are not the same as here due to the restriction (3.9) imposed on the pairs. In average, vortex clusters are lodged at the top and in the downstream of Q2s in the vertical plane. The averaged ω_x

and ω_y (not shown) are at the two sides of Q2s in the spanwise direction with opposite signs and of roughly the same intensity, suggesting that the strong ω_x and ω_y in the center of Q2-Q4 pairs is due to the reorientation of z . Figures 3.17(c-f) show the averaged ω_z and $-\partial u/\partial y$. The location of the averaged vortex cluster almost coincides with the place where $-\omega_z$ is the maximum. The two shear layers are roughly at the top and bottom of Q2. But ω_z are slightly displaced in the streamwise direction due to the contribution of $\partial v/\partial x$, which is in the up and downstream of Q2s. The absolute values for both ω_z and $\partial u/\partial y$ are larger at the boundary of the streak.

Figure 3.18 shows similar results as figure 3.17, but for attached Q2s and Q4s in the logarithmic layer of C950. Specifically, figures 3.18(a-b) show the averaged u . It is observed that the high-speed streak associated with attached Q4s is longer and taller than the low-speed one associated with Q2s. The reason will be stressed later. Besides, the vertical velocity associated with Q2s turns to be a more compact blob than the one associated with Q4s. What is more, the low-speed streak in the upstream of Q2s is of well-know ramp shape, which was found in the boundary layer (Adrian *et al.*, 2000) and channels (Christensen & Adrian, 2001) related with hairpins. And it is also similar to the averaged u conditioned to tall attached vortex clusters ($y_{max}^+ \gtrsim 100$) in the channel with (Flores *et al.*, 2007a) and without roughness (AJZM06). The angle of that ramp is roughly 12° , which is close the value obtained by (Flores *et al.*, 2007a). Figures 3.18(c-d) show the averaged $-\partial u/\partial y$. One difference with HST is that the positive shear layer is no longer totally at the top of Q2s, only true for the strongest part, but also lies in the upstream of Q2s and reaches the wall due to the ramp. Another difference is the negative shear layer at the bottom of Q2s is stronger than the positive shear layer, so does the negative ω_z compared with negative ω_z . This is due to the permeable boundary condition of the wall. We can see that in the channel, $\omega_z < 0$ above the wall is almost exclusively associated with ejections.

We have seen from the averaged flow fields of Q2-Q4 pairs that the most probable position of vortex cluster is in the center of the pair and connects the positive and negative shear layers and spanwise vortices. But figures 3.17(c-f) suggest that they cannot change position from the top (bottom) of Q2s (Q4s) to the center of the pair no matter the z axis is reoriented or not. The reason is that figures 3.17 and 3.18 are only two-dimensional sections. Three-dimensional views are shown in figure 3.19 both for HST and channels. It is found that the averaged vortex clusters associated with Q2s in HST is hairpin. But this hairpin should be the artifact of the average process. Because hairpin is rare if inspecting the averaged vortex cluster at each snapshot. It is argued by Adrian & Moin (1988) and

Figure 3.17: Flow fields conditional averaged to Q2s with $L_c < d < L_\varepsilon$ in M32 at cross-sections $r_z = 0$ (a, c and e) and $r_x = 0$ (b, d and f). In (a) and (b) the shaded map is the averaged u , in the range $[-2.5 \ 0]u_\tau$, arrows in (a) are $(u - v)$ and $(v - w)$ in (b). In (c-d) the shaded map is the averaged ω_z in the range $[-0.75 \ 0.75]\omega'$. In (e-f) the shaded map is the averaged $-\partial u / \partial y$ in the same range as in (c-d). In all the panels, contours are 0.7 times the maximum value of the probability for Q2 (solid) and 0.8 for vortex clusters (dashed).

Figure 3.18: Averaged flow fields conditional averaged to attached Q2s (a, c, e and g) and attached Q4s (b, d, f and h) in the logarithmic layer of C950 at the cross-section $r_z = 0$. The shaded maps are the averaged u in (a-b), $-\partial u / \partial y$ in (c-d), $\partial v / \partial x$ in (e-f) and ω_z in (g-h). Arrows in (a-b) are $(u - v)$. The values for the shaded map are $[-2.5 \ 1.5]u_\tau$ in (a), $[-1.5 \ 2.5]u_\tau$ in (b), $[-2 \ 4]u_\tau / y_{cg}$ in (c), (e) and (g), $[-4 \ 2]u_\tau / y_{cg}$ in (d), (f) and (h). In all the panels, isosurfaces represent 0.7 and 0.9 times the maximum value of the probability for Q2 or Q4 (solid) and vortex clusters (dashed). Derivatives are obtained from averaged velocity fields. The inclined angle of the dash straight line in (a) is 12° .

Vanderwel & Tavoularis (2011) that there are ‘upright’ and ‘inverted’ hairpins in HST, related respectively with ejections and sweeps. Vanderwel & Tavoularis (2011) also points

that in wall-bounded turbulence the latter is absent due to the suppression of the wall. But we know that, although much less than Q2-associated ones, there are Q4-associated vortex clusters in terms of the quadrant of u_m^+ and v_m^+ , but with no clear shapes. However, figure 3.19(b-c) show that the averaged vortex cluster associated with Q4s is hairpin but not for Q2s associated.

Figure 3.19: Three-dimensional view of conditional averaged Q2 or Q4 (green), vortex clusters (grey), positive ω_z (magenta) and negative ω_z (red). For HST, the isosurfaces are 70% and 80% of the maximum probability of points belong to Q2 (Q4) and vortex clusters respectively and $\pm 0.5\omega'$ for ω_z . For C950, isosurfaces are 70% and 90% of the maximum probability of points belong to Q2 (Q4) and vortex clusters and $\omega_z = -1.5u_\tau/y_{cg}$ and $3u_\tau/y_{cg}$ for ω_z in (c) and $\omega_z = -4u_\tau/y_{cg}$ and $1u_\tau/y_{cg}$ for ω_z in (d).

Finally, the enstrophy of the averaged vorticity field in HST is given in figure 3.20. Obviously, the enstrophy of associated with the pair is like a combination of the ‘upright’ and ‘inverted’ hairpins in 3.19(a-b). But it is only clear when the threshold is low, otherwise the enstrophy will lose the two ‘hands’ at the side of the pair contributed by ω_x and ω_y .

Figure 3.20: The enstrophy of averaged vorticity field with $\omega^2 = (0.16\omega')^2$ colored by r_y . With views from the rear (a) and the top (b).

High and low streamwise velocities associated with Q4s and Q2s

One phenomenon that has not been explained in LFJ12 is why the high-speed streak associated with attached Q4s in the logarithmic layer is much longer and taller than the low-speed one in which paired Q2s are embedded. This was already mentioned in figures 3.18(a-b). However, it turns out to be not always true. Figures 3.21(a-c) shows the streaks associated with Q2-Q4 pairs in the buffer, logarithmic and outer layer. Apparently, for Q2-Q4 pairs in the outer layer, there is almost no difference for the associated low- and high-speed streaks. While the closer to the wall of Q2-Q4 pairs, the more disparity for the sizes between streaks, which strongly suggests that the wall or the inhomogeneity is the reason. Note that the flow field is respect to the height of the pair, so figures 3.21(a-c) do not imply that the absolute size of the high-speed streak is getting smaller for taller Q2-Q4 pairs.

However, Jiménez *et al.* (2004) found that the low-speed streak is longer than the high-speed streak by comparing sizes of bounding rectangles for two dimensional u -structures defined by connected regions with $|u|$ beyond the local $u'(y)$ at $y^+ = 15$. Later, it was found that the difference in lengths between the two kinds of streaks diminishes gradually as y increases and almost disappears when $y^+ \approx 60$ (Jimenez & Kawahara, 2013). Similarly, Sillero *et al.* (2014b) found that the self correlation of u in the wall parallel plane conditioned to $u < 0$ is longer than that conditioned to $u > 0$, independent of the wall distance. But results in Lee & Sung (2011) do not show difference of the sizes between low and high streamwise velocities. Apparently, these findings are opposite to what we have observed for averaged streaks for Q2-Q4 pairs. But the above findings

are restricted in the two-dimensional wall parallel plane. Further, [Sillero \(2014\)](#) studied the three-dimensional u -structures in zero pressure gradient boundary layers, finding that high-speed streamwise velocity streaks are only slightly longer than low-speed ones, but it is not clear if there is any dependence on their heights. Remember that streaks in [figure 3.15\(a-b\)](#), [3.17\(a-b\)](#) and [3.18\(a-b\)](#) are related with Q2-Q4 pairs, thus they are highly correlated with the vertical velocity from the previous discussion in [§3.4.2](#). But the vertical velocity moving downward and upward are different in wall-bounded turbulence. It is well accepted that closer to the wall structures are smaller while the opposite when farther away from the wall ([Jiménez, 2012](#)). So the downward moving v is supposed to bring large scales to the near wall region while the opposite for the upward v , i.e. moves small scales to the outer region. This difference is extremely obvious when close to the wall. In this way, the streamwise streaks should behave differently depending on which vertical velocity they are correlated with. To justify our assumption, we investigate the self correlation of u inside Q2s and Q4s, which is equivalent to restrict u with upward and downward v . According to [\(3.4\)](#), the event $\Gamma(\mathbf{x})$ for the self correlation of u conditioned to Q2s $C_{uu}^{Q2}(\mathbf{x}, \mathbf{x}')$ is $v(\mathbf{x}) > -Hu'v'/u(\mathbf{x})$, and $v(\mathbf{x}) < -Hu'v'/u(\mathbf{x})$ for $C_{uu}^{Q4}(\mathbf{x}, \mathbf{x}')$.

The point is to verify if C_{uu}^{Q2} and C_{uu}^{Q4} behave similarly as high- and low-speed streaks conditioned to Q2-Q4 pairs, i.e. in the near wall region the correlation length of C_{uu}^{Q4} is longer than that of C_{uu}^{Q2} but in the outer layer they are of similar size. Thus we computed C_{uu}^{Q2} and C_{uu}^{Q4} at two reference heights crossing attached Q2s and Q4s respectively in the logarithmic and outer layers in channel C950, one at $y/h \approx 0.15$ and another at $y/h \approx 0.4$. So Q2s and Q4s in the two layers have to be taller than the reference height. Results are shown in [figure 3.21\(d-g\)](#) in wall parallel and vertical planes. Solid lines represent results for C_{uu}^{Q2} and dashed for C_{uu}^{Q4} . It is evident that when conditioned to Q⁻s in the logarithmic layer, negative v related streamwise velocity is longer, wider and taller than positive v related one and the difference of the size agrees well with the results in [figure 3.21\(b\)](#). But this difference almost disappears when conditioned to Q⁻s in the outer layer, as indicated by [figure 3.21\(c\)](#). It is quite clear that the low streamwise velocity near the wall contributing to Reynolds stress is not elongated in the streamwise direction. The same analysis is also implemented in C2000 with almost the same results.

The flow field conditioned to close detached Q2-Q4 pairs computed in the same way show similar behaviours to that of attached pairs. The problem is that the volume fraction of detached Q⁻s is only roughly 20% of that of total Q⁻s, thus their averaged flow field is not representative enough. Besides, there is contribution from attached pairs unavoidable in the neighborhood of detached pairs. Even if removing that contribution, the conclusion

does not change. This means that the behaviour of the streaks are not related with whether Q2-Q4 pairs are attached or not, but with where are they with respect to the wall.

Figure 3.21: Isosurfaces of low- and high-speed streaks associated with attached Q2-Q4 pairs in the buffer (a), logarithmic (b) and outer layer (c), with $u = -0.5u_\tau$ ($r_z < 0$) and $u = 0.5u_\tau$ ($r_z > 0$). Two-point self correlation C_{uu} in the vertical (d-e) and horizontal (f-g) plane conditioned to attached Q2s (—) and Q4s (---) with reference heights $y/h \approx 0.15$ for (d) and (f), $y/h \approx 0.40$ for (e) and (g) in C950. In (d) and (f), Q2s and Q4s are in the logarithmic layer crossing the reference height ($y_{max}/h > 0.15$ and $y_{max}/h < 0.2$), while in (e) and (g) they are in the outer layer ($y_{max}/h > 0.4h$). Contour levels are [0.1:0.2:0.9].

3.4.7 Vortex clusters associated with Q⁻s

In this section, the averaged flow fields conditioned to vortex clusters are displayed. Instead of treating all the vortex clusters together as in AJZM06 and Flores *et al.* (2007a), we are more interested with those that associated with Q⁻s and further separate them into Q2 and Q4 associated ones, based on the quadrant of their averaged u_m and v_m . Here ‘Q2’ and ‘Q4’ does not imply three dimensional Qs, but the second and the fourth quadrant, for convenience.

Due to the symmetry in SS-HST, only flow fields conditioned to Q2-associated vortex clusters is given, which is shown in figure 3.22(a) at the cross section $r_z = 0$. The

averaged vortex cluster is ‘fatter’ at its top, which is opposite to that of Q4-associated ones. The averaged flow field is quite very similar to the one conditioned to Q2s. And it is also similar to the one conditioned to detached vortex clusters in the channel (AJZM06) because detached vortex clusters are more associated with Q2s. Although the averaged vortex cluster conditioned to Q2s or Q4s are hairpins, it is not true when conditioned to themselves. It looks like the head of the hairpin at most.

The averaged flow field to tall attached vortex clusters ($y_{max}^+ > 100$) in AJZM06 has a conical shaped long low-speed streamwise streak or ‘wake’ that originates from the upstream of the averaged vortex cluster. However, the results are different for vortex clusters separated by the aforementioned method, even though vortex clusters in channels are more associated with Q2. Figure 3.22(b) and (c) show the results for Q2 and Q4 associated tall attached vortex clusters respectively. By comparing figure 3.22(b) and figure 12 in AJZM06 (which is reproduced but not shown), it is clear the high-speed velocity streak in the upstream of the conical ‘wake’ disappears while the low-speed streak is almost the same. The results for Q4 associated tall attached vortex clusters in figure 3.22(c) is totally different from AJZM06 and Flores *et al.* (2007a) but very similar to the flow field conditioned to attached Q4s (figure 3.18b). Therefore, the results shown in AJZM06 is the combination of the contributions from Q2- and Q4-associated vortex clusters. However, Flores *et al.* (2007a) implemented the same analysis for tall attached vortex clusters in channels with slightly lower Reynolds number and non-smooth wall, and they found that there is no high-speed streak in the upstream direction of the conical wake. It is not clear whether in channels with roughness vortex clusters are exclusively related with Q2s or not.

The result in figure 3.22(c) actually does not depend on the height of vortex clusters while the result for Q2 related attached vortex clusters do, the averaged u in the buffer and the logarithmic layers do not show a conical shape, but true for u conditioned to all attached vortex clusters in the logarithmic layer (not shown).

Based on the similarities between the averaged flow fields conditioned to Q2s (Q4s) and vortex clusters associated with them, we think that the conical wake seen by tall attached vortex clusters is just the low-speed streak they are embedded in, which is already suggested by Flores *et al.* (2007a)

Figure 3.22: Averaged flow fields conditioned to vortex clusters at cross section $r_z = 0$. Shaded maps are streamwise velocity and arrows are (u, v) (a) Vortex clusters associated with Q2 in M32 with $d > L_c$. Contours are 60% and 70% of the maximum probability of points belong to vortex clusters; (b) Tall attached vortex clusters ($y_{min}^+ < 20$, $y_{max}^+ > 100$) associated with Q2 in C950; (c) The same as (b) but conditioned to tall attached vortex clusters associated with Q4. In (a-b) $u^+ = [-1.25 \ 0.15]$ and (c) $u^+ = [-0.15 \ 2]$. In (b-c), contours are 50% and 75% of the maximum probability of the points belong to vortex clusters.

3.5 Conclusions

We have studied the kinematic properties of three-dimensional Qs and vortex clusters responsible for the kinetic production and dissipation respectively in HST with Re_λ comparable to channel turbulence, with special emphasis on comparisons with those structures in channels. The results are supposed to reveal what are the effects of the wall and the mean shear and also work as preparations before the study about their dynamic properties are conducted.

Nearly all the Reynolds stress is carried by Q⁻s that are able to interact with the mean shear ($d \gtrsim L_c$). However, vortex clusters are gradually decoupled from the kinetic energy generation process with Reynolds number, in the sense that the volume fraction of those being able to interact with the mean shear drops dramatically as Re_λ increases. Generally, Qs are large-scale structures while vortex clusters are small-scale ones although they both form a hierarchy of scales from Kolmogorov to integral scale.

In both HST and channels, vortex clusters are more associated with Q⁻s in terms of

the quadrant of u_m and v_m , with equal preference to Q2s and Q4s in the former but more preference Q2s in the later, no matter they are in the buffer, logarithmic or outer layer. u and v in HST are almost Gaussian, no matter thresholded or not, which is also roughly true in the channel. The flow inside intense Qs is different from the flow in general, in terms of the anisotropy of Reynolds stress tensor. Flow in Qs are more anisotropic and the streamwise velocity is dominant over the other two velocity components, especially over w . Further inspecting the size dependence of the anisotropy found that the anisotropy in Qs is largely determined by very large Qs ($d \gtrsim L_\epsilon$). Qs in HST are more like detached Qs in channels, based on the factor that which show almost the same size dependence of the anisotropy. Once detached Qs are large enough to carry net Reynolds stress, they become attached and the anisotropy is almost independent of the size. The spatial correlation between u and v conditioned to Q2s or Q4s in HST and channels agree reasonably well in the direction away from the wall, similar to the self correlation of v , suggesting that the kinetic energy production process in both flows is similar.

Qs in HST are slightly anisotropically oriented with mean aspect ratios $a_{xy} = \Delta_x/\Delta_y \approx 1.25$ for Q⁻s and $a_{xy} \approx 1.4$ for vortex clusters, which agree well with detached families in channels until they are of comparable size to the box, when a_{xy} and a_{zy} are affected artificially by the box aspect ratios. The large elongation in the streamwise direction for attached Qs is not due to their ‘root’ in the near wall region, whose contribution to a_{xy} is negligible for very tall attached Qs. The intrinsic length reveals that large structures are sheet like while small ones are tube like. And as the Reynolds number increases they become more wrinkling and more empty, especially for vortex clusters.

The distance between closest Q-pairs of the same type is scaled well by η . In the viscous range ($d < 50\eta$), the distance is roughly 50η . In the inertial range, the distance is proportional to their sizes, i.e., they are self similarly organized. However, the distance between closest Q2-Q4 pairs scales well with λ . In the viscous range $r \approx 3\lambda$, but the inertial range in HST is not self similar, with $r/\lambda \approx (d/\eta)^{2/3}$. This is well described by a Poisson model assuming that for a given Q2 (Q4), its potentially paired Q4s (Q2s) are uniformly distributed, considering that the probability of the size $p(d) \sim d^{-3}$. In the channel, the distance between both attached and detached closest Q2-Q4 pairs are self-similar. The distance between C-Q⁻ closest pairs also scales well with η but the inertial range is neither self similar ($r \sim d^{1.25}$), which might due to the different probability distribution of sizes of vortex clusters and Qs.

The closest Q-pairs of the same type is aligned in the streamwise direction as in the channel. But the details are different for small ($d < 4L_c$) and large pairs ($d \geq 4L_c$). For

the former, they are more tilted in the vertical plane to the direction of velocity vector $u - v$, while for the latter, they are more separated horizontally as attached Q-pairs. Q2-Q4 pairs are also paired in the spanwise direction, like those in the channel, both attached and detached. But they are not symmetrically paired in the spanwise direction. It is more probable for Q2 to pair with Q4s in the spanwise direction but slightly above it instead of below it, especially for large pairs. The closest vortex clusters to Q⁻s are also paired in the spanwise direction, but their center of gravities can coincide together.

Averaged flow fields conditioned to closest Q2-Q4 pairs in HST are qualitatively similar to those in channels. There are paired low- and high-streamwise velocities. Vortex cluster is in the middle of the pairs in average. But there are more features. It is found that above and below Q2 and Q4 respectively there are strong shear layers with $\partial u / \partial y < 0$. The two shear layers are connected by the vortex cluster. The averaged flow fields conditioned to Q2s show that vortex clusters are lodged at the top and downstream direction of Q2s where ω_z is strong, but the opposite is true when conditioned to Q4s. Streamwise velocities correlated with v moving upward and downward are different in channels, especially close to the wall.

Averaged flow fields conditioned to vortex clusters associated respectively with Q2s or Q4s are very similar to those conditioned to Q2s and Q4s, both in HST and channels. The very long conical low-streamwise-velocity wake starts from the upstream of the averaged vortex cluster only exists for Q2-associated tall attached vortex clusters but not for Q4-associated ones.

Chapter 4

Dynamics of coherent structures in SS-HST

4.1 Introduction

The previous chapter presents details about the time-independent properties of coherent structures in SS-HST. However, it has always been desirable to study the dynamics of turbulence since it was found that turbulence is not totally chaotic but also coherent to some extent ([Theodorsen, 1952](#)). But it only becomes more feasible recently in terms of time-resolved turbulence due to the advancements of both high performance computing and experimental techniques (time-resolved particle image velocimetry for example), especially to the former, which permits inspecting temporal evolutions of as many quantities as possible.

The bursting process is probably the most extensively studied part of turbulence in terms of dynamics, due to its violent intermittency as a function of time. Its dynamics are not only important for understanding turbulence itself, but also for solving engineering problems. For example, it is expectable that the turbulence control schemes for drag reduction, turbulent combustion instability inhibition and mixing enhancement cannot be improved if the dynamics of bursting process is delusive.

Although the fine-temporal-resolved turbulence is only possible nowadays, the study about dynamics of bursting process has never stopped actually. Several dynamical models have been proposed, which improved our understanding to burst significantly. The most widely accepted model is based on the streamwise vortices (or the leg of hairpin vortices) and streaks. Streamwise vortices pump low velocity up and form streaks which later grows, oscillates and finally breaks which in turn generates streamwise vortices ([Jiménez](#)

& Pinelli, 1999; Jiménez & Mark, 2001; Schoppa & Hussain, 2002).

The study about the behaviours of streaks and streamwise vortices during the bursting process can be simplified by the minimal simulation unit. It was invented by Jiménez & Moin (1991) for the buffer layer, which makes it possible to focus only on the evolution of single vortex and streak during the bursting process. Later Flores & Jiménez (2010) extended it to the logarithmic layer and found that the bursting period is proportional to the distance to the wall. Such minimal unit is also employed in Couette flow (Smith *et al.*, 2005)

Since the bursting process is quasi periodic, some exact periodic-like solutions of Navier-Stokes equations are obtained as the dynamical models. Those work includes, for instance, Itano & Toh (2001); Kawahara & Kida (2001); Toh & Itano (2003); Jiménez *et al.* (2005); Viswanath (2007). But these solutions are usually restricted to very low Reynolds numbers and it is still controversy about if they can represent high Reynolds number turbulence or not. Also, it is not clear yet whether a particular periodic solution is able to describe the whole turbulence since there are infinite number of such solutions.

Another dynamical model for the bursting process, instead of based on the streamwise vortex or streaks but on the vertical velocity, was proposed by Jimenez (2013b). In that model, the tilting angle and energy of v during the burst in channels can be approximated quite well by the linear Orr burst of HST. Based on this model, a method to detect and predict the bursting process was also proposed (Jiménez, 2015). Although these studies are only for minimal channels, it is promising to expect its applications in fully turbulent big channels (Miguel & Jiménez, 2015)

The most straightforward way to inspect the bursting process is through time-resolved turbulence, i.e., dump the information of flow field as frequent as possible. Such kind of study is rare, especially for high Reynolds number. Bernard *et al.* (1993) tracked streamwise vortices in the buffer layer of channels and found that they are responsible for the generation of Reynolds stress. Similar conclusion was also obtained by Kim (1985), who studied the evolution of horseshoe vortex in channels. Lozano-Durán & Jiménez (2014b) for the first implemented time-resolved channels with Re_τ up to 4200, by tracking three-dimensional ejections, sweeps and vortex clusters temporally, focusing especially on the logarithmic layer. It was first shown that these structures are not only self-similar in the space but also in time. Also, it reveals that these structures undergo a process with their volumes increase and decay, which is actually the burst.

However, it is difficult to relate the evolution of these structures with the intermittency of kinetic energy and enstrophy in big channels. But this is possible in minimal box, which

is the case of SS-HST. So this chapter is going to implement the same analysis as [Lozano-Durán & Jiménez \(2014b\)](#) but with more emphasize on the relation between the evolution of kinetic energy or enstrophy and coherent structures.

4.2 Numerical experiments and coherent structure tracking method

4.2.1 Numerical experiments

As in the previous chapter, numerical experiments of SS-HST are conducted with the code introduced in the chapter §2.

Table 3.1 shows parameters of simulations. Some other information have already been shown in §2 and §3. Three simulations with the same Reynolds number, i.e., Re_λ around 50, 100 and 250 as in §3 are studied in this chapter. But the vertical aspect ratio for the case with $Re_\lambda = 100$ is doubled, i.e., $A_{yz} = 4$, compared with the one with the same Reynolds number in §3. This case is labelled as M34 hereinafter. The difference of statistics for the two cases are presented in table 2.2. The reason is that, according to §2, M34 is supposed to be less affected by the boundary condition than case M32 considering that the time is important in the time-resolved situation. Other two cases are with the same aspect ratios as in §3 so that they are labelled with the same names. Considering that $\sim 10^4$ snapshots of structure information have to be stored, the vertical aspect ratio is reduced accordingly as Reynolds number becomes higher in order to make the tracking process feasible.

Special attention has to be paid to the resolution requirements, both spatial and temporal ones. When tracking coherent structures, the two resolutions are bonded together. Fine spatial resolution loses its meaning when the temporal resolution is coarse. Coarse spatial resolution can be remedied to some extent by fine temporal resolution. In this subsection we only focus on the spatial resolution. The temporal one will be discussed more properly in §4.2.2, where the tracking method is introduced. In average, the streamwise resolution is 1.5η and one third finer in the other two directions for all the cases. Kinetic energy and enstrophy undergo strong fluctuation, so does the instantaneous resolution. In the worst moments of all cases, the streamwise resolution is around $1.8 - 2.1\eta$, which is marginal. These special moments are discussed below.

As been shown by [Yeung *et al.* \(2015\)](#) that there are rare extreme events with enstrophy and dissipation magnitudes tens of thousands the mean value in very high-Reynolds-

Case	Sym.	Re_λ	A_{xz}	A_{yz}	$\Delta x/\eta$	$(\Delta y, \Delta z)/\eta$	$\Delta t_s/t_\eta$	N_s	ST	T_{eto}
L38	◇	48	3	8	1.61±0.06	1.07±0.04	0.22±0.02	20	1.65	0.70
M34	□	108	3	4	1.45±0.07	0.97±0.05	0.33±0.04	60	0.63	0.25
H32	○	248	3	2	1.57±0.18	1.05±0.12	0.43±0.08	120	0.22	0.088

Table 4.1: Parameters for simulations. Re_λ is the Reynolds number based on the Taylor microscale; $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$ are the two aspect ratios, L_x and L_y are the streamwise and vertical lengths; $\Delta x/\eta$, $\Delta y/\eta$ and $\Delta z/\eta$ are the average spatial resolutions in terms of Kolmogorov scale η with their standard deviations due to intermittency. Δx and Δz are the grid size before dealiasing; $\Delta t_s/t_\eta$ is the mean sampling frequency or temporal resolution with respect to the Kolmogorov time scale $t_\eta = (\nu/\varepsilon)^{1/2}$ with standard deviation; N_s is the constant number of time steps to store structure information; $ST \times 1000$ is the total simulation time in shear unit; $T_{eto} \times 1000$ is the total eddy turn-over-time, which is defined by $L_\varepsilon/\sqrt{q^2/3}$ and $\approx 2.5S^{-1}$ in all the three cases.

number homogeneous isotropic turbulence. Those events are small in space and short-lived in time, which is inevitable for some small structures. Probably it is not realistic and unnecessary to capture all of them. Although our highest Reynolds number here is only roughly one fifth of that in [Yeung *et al.* \(2015\)](#), it is still necessary to find out the difference between the adopted resolution and the extreme small scale.

Figure 4.1(a) shows the p.d.f. of instantaneous pointwise Kolmogorov length η_e normalized by η when the instantaneous resolution is the worst for all the three cases. It is clear that both smaller and larger scales with respect to η appear when the Reynolds number increases. The most probable Kolmogorov length η_e is roughly η , and the probability of length $\eta_e < \eta$ increases as a function of the Reynolds number. The two vertical lines represent spatial resolutions in the streamwise and transverse directions in physical space, in which coherent structures are tracked. It can be found that the spatial resolutions are still capable to capture the most probable small structures even in the worst situation. However, coherent structures usually are not only one point as we can see in §4.2.2.

4.2.2 Coherent structure tracking method

Coherent structures studied in this chapter are three-dimensional Qs and vortex clusters defined in the same way as in §3. Constant thresholds $H = 1.75$ and $\alpha = 1.5$ are

Figure 4.1: Probability density function of instantaneous pointwise Kolmogorov length $\eta_e = (\nu^3/\bar{\epsilon})^{1/4}$ (a) and time $t_{\eta_e} = (\nu/\bar{\epsilon})^{1/2}$ (b) at the extreme moment when $\bar{\epsilon}$ is the largest, both normalized by the ensemble averaged value. Vertical lines in (a) represent spatial resolutions in the streamwise (right) and other two transverse directions (left) for all the cases in physical space (after dealiasing), while in (b), they represent temporal resolutions with increasing Reynolds number from left to right. In both panels arrows point to higher Reynolds number.

employed for Qs and vortex clusters respectively, based on the percolation analysis in §3 for the purpose of inspecting the bursting process. Since there are strong fluctuations of $\sqrt{u^2}(t)$, $\sqrt{v^2}(t)$ and $\sqrt{\Pi^2}(t)$ (figure 1.4 and figure 2.3(a) for example), constant H and α mean time varying thresholds respect to $\sqrt{u^2}(t)$, $\sqrt{v^2}(t)$ and $\sqrt{\Pi^2}(t)$. Later we will compare results between using (u', v', Π') and $(\sqrt{u^2}(t), \sqrt{v^2}(t), \sqrt{\Pi^2}(t))$ when tracking these structures temporally.

Figure 4.2 shows instantaneous vortex clusters and Q⁻s in the three cases in the same box size in each direction. The moments are chosen with the same reference, i.e., $\sqrt{\Pi^2}(t) = \Pi'$ for vortex clusters and $\sqrt{u^2}(t)\sqrt{v^2}(t) = u'v'$ for Q⁻s. Obviously, vortex clusters, which are of Kolmogorov scale and of filament shape, become smaller and smaller with respect to the box size, reflecting the change of L_ϵ/η as a function of Reynolds number. Q⁻s, on the other hand, keep their size, but becomes more wrinkly. It can be easily observed that Q2s and Q4s are of similar size and are paired in the spanwise direction (Lozano-Durán *et al.*, 2012; Lozano-Durán & Jiménez, 2014b) as we discussed in §2. For Q⁻s, the moments happen to be when large Q2s and Q4s with streamwise sizes comparable to L_x because of the minimal box effect.

Figure 4.3(a) shows the temporal correlation between and total number and volume of Q⁻s between the kinetic energy. It is clear that, the number and volume of Q⁻s are highly correlated with kinetic energy, except L38. But there is a lag between them. The total

Figure 4.2: Instantaneous vortex clusters (a-c) and Q^- s (Q2s in blue and Q4s in red) (d-f) at moments $\sqrt{\Pi^2}(t) = \Pi'$ and $\sqrt{u^2}(t)\sqrt{v^2}(t) = u'v'$ respectively. Columns from left to right are from L38, M34 and H32, with vertical region $y/L_z = [-1, 1]$. Arrows indicate the streamwise direction. (a-c) are the in $x - y$ plane. Structures with volume smaller than $(5\eta)^3$ are discarded for better visualization.

volume first arrives ($\approx 0.8S^{-1}$ earlier) to its maximum before the kinetic energy while the opposite is true for the total number ($\approx 2.0S^{-1}$ later, roughly one eddy-turn-over time). It suggests that the increase of the kinetic energy is due to the reformation and growth of structures while its decay is due to the break-down, which leads to more smaller pieces of structures. The same result is also shown for vortex clusters, but correlated with enstrophy. Both total number and volume happen almost the same time with the enstrophy, with total number slightly later. The difference between Q^- s and vortex clusters might be that the former are the energy harvesting structures and their energy needs time to be transferred to smaller scales via direct cascade while vortex clusters are small scales where energy are dissipated. Note that for H32 there are two bumps, which might be the effect of the boundary condition (see §2).

The structure tracking method employed in this chapter was invented by LJ2014 where detailed explanation can be found. For self consistency, key steps are briefly recapitulated.

The prerequisite for tracking individual structure is the information of structures (only lists of points belong to structures with several thresholds for storage reduction) stored at a given frequency, which is the temporal resolution as we mentioned in §4.2.1. Higher

Figure 4.3: Temporal correlation between the total number (—) and volume (---) of Q^- s with kinetic energy (a) and the total number (—) and volume (---) of vortex clusters with enstrophy (b).

frequency is always desirable, but it challenges the storage, I/O, data transfer and the subsequent post-processing, especially for very high Reynolds number with decent eddy-turn-over time. Thus the temporal resolution Δt_s has to be chosen with care to compromise the volume of structures cannot be tracked and other practical considerations mentioned above.

A universal criterion to determine proper Δt_s is always desirable. This is more simple in SS-HST due to homogeneity, nevertheless, we can only give an empirical one shown below. Since structures are advected by the mean velocity in the streamwise direction (see §4.4), they are more probable to get lost in that direction. Therefore, we require that, for instance, all the structures with streamwise length $\Delta_x \geq 5\eta$ have to be tracked, and assume that they move with the characteristic streamwise velocity $\bar{u}(t)$. Then the sampling frequency has to satisfy

$$\bar{u}(t)\Delta t_s < 5\eta. \quad (4.1)$$

In all the cases, the convection CFL determines the time step Δt , which is decoupled from the mean velocity. And

$$\pi|w_{max}(t)|\Delta t/dz \quad (4.2)$$

is nearly always dominant (larger than the other two during $\sim 98\%$ of the time history). In average $|w_{max}(t)|/\bar{u}(t) \approx 4.0$. Considering the spatial resolutions given in table 3.1 before dealiasing and that structures are tracked in physical space, we can obtain

$$\Delta t_s/\Delta t \lesssim 90/\text{CFL}, \quad (4.3)$$

i.e., the temporal resolution is inverse proportional to the CFL number. Note that Δt here should be the smallest one during the history. For $\text{CFL} = 0.5$, which is used for all the cases, Δt_s has to be smaller than roughly 180 time steps. Unfortunately, such a constant sampling frequency is not very practical. Because the ratio between the time step and Kolmogorov time scale $\Delta t/t_\eta \propto (\nu\varepsilon)^{1/4}$ decreases with Reynolds number. So that more structures cannot be tracked when Reynolds number is low while it will be increasingly expensive for higher Reynolds numbers. Therefore, we use frequencies promotional with Re_λ as shown in table 3.1, but we are still able to track structures shorter than 5η and also lose longer ones at the same time. That might because that Δ_x is the length of circumscribing box and the structures may deform or diffused by viscosity.

The p.d.f.s of pointwise Kolmogorov time t_{η_e} at the extreme moment that we mentioned before normalized by t_η are shown in figure 4.1(b). t_{η_e} shows similar trends to η_e as a function of Reynolds number, because both of them are determined by $\bar{\varepsilon}(t)$. Obviously, the adopted temporal resolutions are fine enough to capture the most probable time scales even at extreme moments.

Readers may have noticed that the estimation in (4.1) ignores the mean velocity, that is because we already canceled it, otherwise structures in the region with high mean velocity cannot be tracked, especially for L38. Briefly, the tracking procedure is divided into three steps. First, given any two consecutive snapshots t and $t + \Delta t_s$, structures at $t + \Delta t_s$ are shifted back by $-U(y)\Delta t_s$ in the streamwise direction at each horizontal plane but still in the computational box due to periodicity. Then overlaps of every structure at one moment with all possible others at the neighbouring moment are obtained. One may have overlaps with several or none at all, the latter is called isolated structures. Such process goes forward for all the snapshots. Second, for any two structures at any two consecutive moments with volume V_1 and V_2 and overlap V_o , if the relative overlap $V_{ro} = |V_o/(V_1 - V_2)|$ is maximum for both of them, they are put in the same branch, such branch sorting continues both forward and backward until V_{ro} is not maximum for one of the two structures at two consecutive moments. For each structure in any branch, it may have overlaps with others in different branches but V_{ro} is not maximum, thus different branches can be connected together. So the last step is organize all the branches with connections into a graph.

In order to speed up data transfer and also to reduce the storage assumption, only the lists of points belong to structures defined by (3.1) and (3.2) are stored, with several thresholds.

Branches are further classified into primary, incoming, outgoing, and connector based

Figure 4.4: (a) The temporal evolution of vortex clusters belong to the largest graph during a typical bursting process of case L32 shown in (b) where — for kinetic energy and --- for enstrophy. In (a), time from left to right, as indicated by the arrow and denoted by dots in (b). (c) The largest graph for ejections during the burst in (b). The horizontal solid lines are branches, colored by the volume of individual Q2 in a given branch at each snapshot. The vertical dashed ones are mergers (green) and splits (blue). In (c) for simplicity, $H = 2.25$ and those branches with life less than 10 snapshots are ignored.

on the status of their starts and ends. If a branch starts and ends both from the background of turbulence, it is called primary. Similarly, incomings start from the scratch but end with a merge into other branches. Outgoings are the opposite, i.e., they splits from other branches and end into the background of turbulence. Connectors start and end with other branches which are not necessary different. The sketches for these branches are given in [Lozano-Durán & Jiménez \(2014b\)](#).

As an example, figure 4.4(a) gives a piece of the temporal evolution of the largest graph of vortex clusters during a typical burst shown by figure 4.4(b). Similar to kinetic energy and enstrophy, the graph also undergoes grow and decay, in terms of its total volume and

number of structures. The evolution reminiscent that in channels as shown in figure 5(a) of [Lozano-Durán & Jiménez \(2014b\)](#), where it is difficult to obtain histories of local kinetic energy and enstrophy unless instantaneous flow fields are stored. Evolutions of graphs of ejections or sweeps show similar behaviour. But it is impossible to find how structures are organized in figure 4.4(a). For simplicity, figure 4.4(c) shows connections between branches among the largest graph of ejections with a higher threshold $H = 2.25$ and life time at least 10 snapshots. The complexity of connections is quite inhomogeneous with respect to time, more connections are spawned around the peak of kinetic energy, which is different with the one given by [Lozano-Durán & Jiménez \(2014b\)](#), where splits and merges are almost symmetric as a function of time. The relation between the behaviour of the evolution and burst will be discussed in detail.

4.3 Temporal evolution

The evolution of the volume V of primary branches of Q⁻s is shown in figure 4.5(a) as a function of time started from branch beginnings. It is clear that V is roughly symmetrically distributed during its evolution, i.e., it increases gradually before $t/T = 0.5$, after which it decays with similar speed. The evolution of length at each direction behaves similarly as volume. Intuitively, this phenomenon might suggest the exchange of dominance between split and merge events. And it is proved to be true by results in figure 4.5(b), which displays the p.d.f. of the time when split and merge happen during the evolution of primary branches. In general, merge happens more frequent than split when V is increasing, while the opposite is true when V decreases. The most probable time for merge and split is roughly $t/T = 0.2$ and 0.8 , which is also true in channels (LJ14). Similar results as figure (a-b) for vortex clusters can also be found.

The same analysis for other kinds of branches show that incomings keep growing, outgoings keep decaying while connectors nearly does not change size. The times when splits and merges happen in connectors are roughly the same and independent of time. For incomings, both times increase but the opposite for outgoings. For both incomings and outgoings, merges are dominant before $t/T = 0.5$ and later splits happen more. But the difference between the two times is much smaller than that in primaries. Note that results in figure 4.5(a-b) are in average thus for a single particular branch it might not be true.

Note that the symmetry of the volume evolution in figure 4.5(a) is with respect to time. The one-dimensional p.d.f. of $V/\langle V \rangle_b$ itself, however, is not necessary symmetric,

with the most probable value at its mean value, which is the case for channel (LJ14, figure 7). Imagine that the volume evolves like a sine wave, i.e., $V/\langle V \rangle_b = \sin(t)$ with $t \in [0, \pi]$, the p.d.f. of $V/\langle V \rangle_b$ is not symmetric and quite similar to that of primaries, incomings and outgoings. Actually in SS-HST, we only found connectors have symmetric p.d.f. of $V/\langle V \rangle_b$.

Figure 4.5: (a) Probability density function of volume V of primary branches as a function of time t started from their beginnings. The volume is normalized by the mean value of each branch $\langle V \rangle_b$. Time is normalized by the branch lifetime T . The vertical section at each moment is a p.d.f. with 0.8 of its maximum. (b) Probability density functions of the time when mergers and splits happens for primary branches, normalized by the branch lifetime T . —, splits, ---, mergers.

Previous studies LBJ12 and also §2 show that in both channels and SS-HST, vortex clusters are more associated with ejections and sweeps in terms of the quadrant in which their u_m and v_m belong to. In channels, they are more associated with the former than the latter while in SS-HST the preferences are the same required by the symmetry of the flow. However, this cannot offer any information about the evolution of the associated quadrant in their whole life.

Figure 4.6(a) and 4.6(b) show the percentages of moments in different quadrants for vortex clusters in all branches whose initial moments are associated with Q1 ($u_m^+ > 0$, $v_m^+ > 0$) and Q2 ($u_m^+ < 0$, $v_m^+ > 0$), respectively. Note that the initial moments are not necessary birth. And also that here Q1 and Q2 are the first and second quadrant, rather than intense Qs ($|uv| > Hu'v'$) we have extracted. Apparently, the quadrants of vortex clusters belong to remain the same as the initial moment of the branches. Especially when the initial moment is in Q2, in which case almost 60-80% of the time in the branches are still in Q2, especially for vortex clusters in channels. If the branch starts from Q1, they

will be related with Q1 most of the time, but roughly 20% lower than starts from Q2, and it is more probable for them change to the adjacent quadrants Q2 and Q4, but not for Q3. Similar conclusion holds for vortex cluster branches with initial moments in the third and fourth quadrant.

Figure 4.6(c) displays the joint p.d.f. between u_m^+ and v_m^+ of vortex clusters in branches with initial moments in the first and second quadrants, both in SS-HST and channels. Obviously, due to cancellation of velocities inside vortex clusters, 80% of them are weaker than intense Qs we studied. The intensities of u_m^+ and v_m^+ for vortex clusters in branches start with Q1 in both flows agree quite well. On the other hand, vortex clusters in channels in branches start with Q2 are in stronger low-speed streaks, which might due to the fact that vortex clusters in channels are almost in the buffer layer where the peak of $u'(y)$ happens.

4.4 Velocity

How fast are these structures moving and other related properties are discussed in this section.

4.4.1 Vertical velocity

Figure 4.7(a) and 4.7(b) show the p.d.f. of vertical velocities for individual Q2s, Q4s and vortex clusters. The vertical velocity is defined by the vertical displacement of their centers of gravity y_c divided by the elapsed time between the two consecutive moments, during which they do not undergo split or merge. By definition, Q2s move upwards and Q4s move downwards to cross the mean shear, while vortex clusters can move in both directions. Quantitatively, the most probable vertical velocities for Q2s and Q4s are u_τ and $-u_\tau$ respectively, with the p.d.f. of dy_c/dt quite narrow. The results agree very well with Flores & Jiménez (2010) and LJ2014. The former concluded so by the inclination angles of Reynolds stress averaged in the horizontal plane conditioned separately to ejection and sweep events as a function of time and vertical distance in a minimal channel. The latter used the same definition for time resolved structures in channels. The most probable vertical velocity for vortex clusters, on the other hand, is zero, which means that vortex clusters are almost still in the vertical direction, like vortex clusters in channels (LJ2014). Although vortex clusters in SS-HST are equally distributed with $v_m > 0$ and $v_m < 0$, figure 4.7(b) does not show other two peaks with non zero v_m . It is interesting that small scale vortex clusters are not carried by large scale Qs in the vertical direction.

Figure 4.6: Percentage r of moments of vortex clusters in each quadrant in terms of u_m^+ and v_m^+ when the initial moments of the branches they belong to are associated with Q1 ($u_m^+ > 0, v_m^+ > 0$) (a) and Q2 ($u_m^+ < 0, v_m^+ > 0$) (b). Solid bars from L38 and empty bars for C950. (c) Joint probability density functions between u_m^+ and v_m^+ for vortex clusters in branches whose initial moments are associated with Q1 ($u_m^+ > 0, v_m^+ > 0$) (—) and Q2 ($u_m^+ < 0, v_m^+ > 0$) (---). The shaded contour is the joint p.d.f. of u_m^+ and v_m^+ for all the vortex clusters in L38 and the curves are the thresholds $u_m v_m > H u' v'$ for the same case.

The vertical velocity measured by the center of gravity is roughly equivalent with the mean vertical velocity inside structures. For example, v_m^+ of Q2s with threshold changing from 0.25 to 4 increases from u_τ to $3u_\tau$, which is almost constant considering the range of threshold. v_m^+ for vortex clusters are independent of threshold, with the most probable value around zero as figure 4.7(b).

The vertical velocity of individual Q2s and Q4s can also be approximated by the inclination angle of the two-point correlation function $R(r_y, r_t)$ of the Reynolds stress history profile $\overline{uv}_{xz}(y, t)$ as we mentioned above, which is a more accurate estimation than Flores & Jiménez (2010). One piece of $\overline{uv}_{xz}(y, t)$ conditioned to ejection events

($u < 0, v > 0$) in SS-HST is shown in §3. The correlation results are shown in figure 4.7(c), with the top for ejections and bottom for sweeps. The inclination angle γ is roughly $\arctan \gamma = \pm u_\tau$, which is expected from Flores & Jiménez (2010) and the same as tracking individual Q2s and Q4s. Similar result is shown for vortex clusters in figure 4.7(d), with the correlation for the second invariants $\overline{\Pi}_{xz}(y, t)$ instead of Reynolds stress. It is clear that $\overline{\Pi}_{xz}(y, t)$ is not inclined.

Another information that can be obtained from the above correlations is the lifetime defined by for example, $R = 0.1$. The lifetime defined in this way for Q⁻s is about $30S^{-1}$, which is close to the value by the width of temporal correlation of box averaged vertical velocity. The lifetime of vortex clusters defined in the same way is smaller, around $10S^{-1}$. The above correlations are also computed for strong $\overline{uv}_{xz}(y, t)$ and $\overline{\Pi}_{xz}(y, t)$, by only averaging those points in the horizontal plane satisfying (3.1) and (3.2). Results are represented by the line contours. It can be found that both the lifetime and inclination angle do not change, implying that the lifetime of Q⁻s and vortex clusters are determined by their strong parts. The lifetime of vortex clusters in this way is shorter than the temporal correlation of time history of enstrophy, which is almost of the same width as kinetic energy.

4.4.2 Advection velocity

It is expected that structures in shear-driven turbulence are advected roughly by the mean velocity. Thus the advection velocity can be used as a sanity check for the tracking process.

Figure 4.8 shows the advection velocity in the streamwise direction measured by the phase correlation method, which is employed by PIV in experiments. Generally, Q2s move slightly slower than the mean velocity while the opposite is true for Q4s, as shown by figures 4.8(a-b). This is determined by the fact that Q2s and Q4s are in low- and high-speed streamwise velocities respectively, as we saw in §3. In order to see how different are the advection velocities of Q2s and Q4s from the mean velocity. Figures 4.8(c-d) further show the p.d.f. of $U_a(y)^+ - U(y)^+$. It is found that the most probable advection velocities are roughly $U(y) - 1.5u_\tau$ and $U(y) + 1.5u_\tau$ for Q2s and Q4s respectively, which agree quite well with those of channels. While for vortex clusters, the most probable advection velocity is almost $U(y)$ (not shown), which is different from channels where vortex clusters are almost in Q2s.

The group velocity is also computed. It is defined in a similar way as the aforementioned vertical velocity, but for two-dimensional section, i.e., by the displacement of the

Figure 4.7: (a) P.d.f. of the vertical velocity of the center of gravity v_c^+ for Q2s (—) and Q4s (---). The dashed vertical lines are $dy_c/dt = \pm u_\tau$. (b) The same as (a) but for vortex clusters with the dashed vertical line as $dy_c/dt = 0$. (c) Two-point correlation functions for two-dimensional history of Reynolds stress $\overline{uv}_{xz}(y, t)$ conditioned to ejection (top) and sweep events (bottom). The dash lines represent $r_y/r_t = \pm u_\tau$. Contour levels are [0.1:0.1:1.0]. The shaded contours are the correlation of Reynolds stress with zero threshold, i.e., $|uv| > 0$, whereas contour lines are obtained only for strong events: $|uv| > 1.75u'v'$. (d) The same as (c) but for the second invariant $\overline{\Pi}_{xz}(y, t)$. The shaded contours are the correlation of Π with zero threshold, i.e., $\Pi > 0$, whereas contour lines are obtained only for strong events: $\Pi > 1.5\Pi'$. In both (c) and (d), data come from case L38.

center of the bounding box at each horizontal section. The phase velocity and group velocity are almost the same, which means that they are coherent (not shown). Besides, both of the two velocities are independent of their sizes.

The results of advection velocity suggest that these structures will be distorted by the mean shear due to the difference of the advection velocity in the vertical direction. Figure 4.9 shows the p.d.f. of the relative distance between small split or merge pieces and the one where they split from or merge into at two consecutive moments. The results for Q⁻s

Figure 4.8: Probability density function of the streamwise advection velocity at each horizontal section of individual structures, as a function of section's vertical position. (a) for Q2s and (b) for Q4s. Each horizontal section of the results is a p.d.f., with value 40% of the maximum at that section. The thicker dashed line is the mean streamwise velocity profile. One dimensional p.d.f. of the streamwise advection velocity at each horizontal section subtracting the local mean streamwise velocity $U_a(y)^+ - U(y)^+$. (c) for Q2s and (d) for Q4s. The dashed lines are $-1.5u_\tau$ and $1.5u_\tau$ for Q2s and Q4s respectively. Advection velocity is measured by the phase correlation method.

show that both the split and merge mainly come from the top and bottom of the large one, but split prefers to be along $\delta_x = \delta_y$ while merge follow roughly $\delta_x = -\delta_y$, which are the main stretching and constraining direction of the velocity gradient tensor. However, results for vortex clusters do not show clear difference for split and merge, i.e., always with the most probable position around its center, inclined to $\delta_x = \delta_y$. The reason is that vortex clusters are much more empty so that the split and merge can happen anywhere. Three-dimensional results are also computed and the conclusion does not change, i.e. the high probability region is in the region with $\delta_z \approx 0$.

Figure 4.9: Joint probability density functions of the relative vertical and streamwise distances of the centres of mergers and splits (structures s) with respect to the centre of the big eddy (structure b) normalized with the characteristic size of the latter, Δ_b . (a) Split for Q^- s. (b) Merge for Q^- s. (c) Split for vortex clusters. (d) Merge for vortex clusters. Contours contain 20% and 90% of the total data.

4.5 Conclusions

Temporal evolution of three-dimensional ejections, sweeps and vortex clusters in SS-HST with Reynolds number up to $Re_\lambda = 250$ are investigated with the method invented by [Lozano-Durán & Jiménez \(2014b\)](#).

There is a very large graph with lifetime almost equal to the total simulation time. It connects nearly almost all the branches together, while rest of graphs are consisted by only one branch in average. Connections between branches inside this graph is very asymmetric with respect to time, i.e., there are more connections after the peak of kinetic energy or enstrophy. The volume of primary branches is almost negligible. Thus their life time, which only makes sense for them, loses its meaning here.

It is found that vortex clusters are coherent during their evolutions in terms of the

quadrant they belong to defined by u_m^+ and v_m^+ , i.e., they stay in the same quadrant as the initial moment of the branch they are in. Especially for those vortex clusters in branches with initial moments associated with the second and fourth quadrant.

The vertical velocities of ejection and sweeps are roughly u_τ and $-u_\tau$, respectively. While vortex clusters does not move in the vertical direction, which means that they are generated everywhere. In the streamwise direction, they are advected by the mean velocity, with sweeps $1.5u_\tau$ faster than the mean velocity and the opposite for ejections, and vortex clusters moves roughly the same as the mean velocity, which does not depends on their sizes. Due to the velocity difference, these structures are deformed by the mean shear.

Chapter 5

Conclusions and future work

Simplified models are always desirable in this complicated world. Although simple, most of them are sufficient for engineering applications. Good models are proposed based on extensive and careful studies and they are quite universal. In the channel, by DNS with one of the highest available Reynolds numbers in large box, a new model was proposed by [LFJ12](#) in terms of Q2-Q4 pairs with vortex clusters lying in the middle. Hopefully, this model will promote the development of LES for wall-bounded turbulence. The present thesis works as a complimentary study to [LFJ12](#), by implementing similar analysis in the simplest shear turbulence, i.e., HST, in order to find how versatile is that model and how does it behave in shear turbulence with and without the wall.

Instead of periodic remesh, this thesis uses a newly developed code which imposes the shifting-periodic boundary condition in the vertical direction by adding ‘shifting factors’ in the matrix solver. Validations for both the linear and nonlinear parts of Navier-Stokes equations and comparisons with previous studies recommend this code for simulations of HST in the future. In order to study high Reynolds number HST, the code was tuned in a BG/Q system by hybrid MPI/OpenMP parallelization. Almost perfect scaling was obtained up to 16K cores.

Any box for long-time simulations of HST is ‘minimal’, therefore the statistics for large scales in SS-HST are highly affected by the box, represented by its two aspect ratios, $A_{xz} = L_x/L_z$ and $A_{yz} = L_y/L_z$. Based on the fact that the relevant length scale is the spanwise length of the box, L_z , through around 100 DNSes with boxes in a wider range of aspect ratios. Therefore the relevant velocity scale is SL_z and the relevant Reynolds number is $Re_z = SL_z^2/\nu$. Similarly, in the minimal channel, the height y below which statistics agree well with full channels is also determined by L_z , with $y \approx L_z/3$ ([Flores & Jiménez, 2010](#)).

It is found that the acceptable aspect ratios for SS-HST are $2 < A_{xz} \lesssim 5$ and $A_{xz} < 2A_{yz}$. Boxes shorter either in length or height are not able to contain the large scale velocity structures. While in very long boxes with $A_{xz} \geq 2A_{yz}$, the bursting process is dominated by periodic two-dimensional regeneration process due to the artificial of simulating unbounded flow in a finite box with periodic boundary condition. These bursts agree well with analytical solutions to a large extent and appear to be unrelated to physics. Such bursts do not appear for very long channels with minimal size in the spanwise direction.

With acceptable aspect ratios, SS-HST and the logarithmic layer are similar both in one-point and two-point statistics. For example, u' , v' and w' of SS-HST are of similar intensities to those in the logarithmic layer of channels, especially when scaled with the friction velocity. The streamwise and spanwise velocities in full channels are attached quantities (Sillero *et al.*, 2014a) and they are typically too large to be fairly compared with those either in SS-HST or in minimal channels, where they are artificially affected by the simulation box. But the vertical velocity, which is a detached quantity in channels and relatively smaller than other velocity components in both flows, agrees reasonably well. For example, the wall-parallel spectras of v in both full and minimal channels at the height with comparable Re_λ to SS-HST agree well with that of SS-HST. Besides, although there is blocking effect of the wall in channels, the two-point correlation function in the direction away from the wall agree quite well with that in SS-HST. The dynamics about v was well described by Jiménez (2013a), who found that the bursting process of v in HST and channels are very similar, in terms of the relation between the tilting angle and energy of v . Therefore, v in HST and channels are similar both kinematically and dynamically.

Although the quasi-periodic burst in SS-HST is artificial, it is quite similar with that in the wall-bounded turbulence. For example, it also undergoes the generation, meander and break down of streaks. The lifetime of single burst in shear unit, defined by the width of the temporal autocorrelation function of $\overline{v}(t)$, is $ST_b \approx 20$ in both SS-HST and channels. The probability distribution of the growth rates of the kinetic energy in minimal channels and SS-HST agree with each other quite well, suggesting that the generation of initial conditions for the next burst is probably the same in turbulent flows with stable mean shear profile as shown by Jiménez (2013a) and Jiménez (2015).

Based on the above preliminary comparisons. Time independent characteristics of three-dimensional Qs (or Q_{uv} based on quadrant analysis of u and v) and vortex clusters in SS-HST with Re_λ up to 250 are compared with those in the channel, especially with detached families above the buffer layer. The Reynolds number of HST studied in the

thesis is up to now the highest within the knowledge of the author. Nevertheless, it is still not high enough for a sufficient wide inertial range.

The anisotropy of the Reynolds stress tensor inside Qs in HST is close to that in the channel slightly above the logarithmic layer ($y/h \approx 0.3 - 0.4$), which is also true for the anisotropy of the flow in general. The anisotropy is mainly contributed by large Qs with $d \gtrsim 4L_c$ while small Qs ($d \lesssim 4L_c$) have almost the same anisotropy as Q_{uw} and Q_{vw} defined with the same thresholds. The spatial correlation between u and v conditioned to ejection or sweep events in HST and channels agree reasonably well in the direction away from the wall, although u in channels is much larger than that in HST, implying that the kinetic energy production process in both flows is similar.

The elongation of Qs in HST in the streamwise direction, measured by the aspect ratio of circumscribing boxes, is close to that of detached Qs in the channel, with $\Delta_x/\Delta_y \approx 1.25$, unless they are of comparable size to the simulation box. But Δ_z/Δ_y of Qs in HST is strongly affected by the minimal box effect, decreasing from roughly 1.0 to L_z/L_y once $d > L_c$, which means that Qs in HST are not self-similar. The much larger elongation ($a_{xy} \approx 3$) in the streamwise direction for attached Qs is not due to their ‘root’ in the near wall region, whose contribution to the length of Qs is only ≈ 200 wall units, which is negligible for tall attached Qs. The fractal dimensions measured from intrinsic lengths for structures in HST are slightly higher than those in channels.

The distance between closest Q-pairs of the same type is scaled well by η . There is a viscous range ($d < 50\eta$), where the distance is roughly constant, 50η . In the inertial range, they are packed self similarly. However, the distance between closest Q2-Q4 pairs scales well with λ , which is the distance required by the velocity jump $\sim u'$ between Q2s and Q4s, assuming that the maximum velocity gradient is $\sim \omega'$. In the viscous range $r \approx 3\lambda$, and the inertial range in HST is not self similar, with $r/\lambda \approx (d/\eta)^{2/3}$. The distance between closest C-Q⁻ pairs also scales well with η but the inertial range is neither self similar, $r/\eta \sim (d/\eta)^{1.25}$, which might be because that the population of vortex clusters drops much faster than Qs.

Closest Q-pairs of the same type in HST are aligned in the streamwise direction, forming a ‘tube’. But small Q⁻-pairs ($d < 4L_c$) are more tilted downward in the vertical plane. This alignment is determined by the velocity vector (u, v) inside Q⁻s, which is verified by the alignments of Q⁺s, Q_{uw} and Q_{vw} of the same type. Large pairs ($d \geq 4L_c$) are almost separated horizontally as attached Q-pairs. The relative position between Q2s and Q4s is determined by the non-velocity-clash condition. Therefore, given a reference Q2, the high probability region for finding its closest paired Q4s is perpendicular to the

velocity vector (u, v) in Q2. Both small and large Q2-Q4 pairs are dominantly paired in the spanwise direction, like those in the channel. But the high probability region for Q4s relative to reference small Q2s form an arch. It is more probable for Q2 to pair with Q4s in the spanwise direction and slightly above it, instead of below it, especially for large pairs. The arch disappear for large Q2-Q4 pairs, which are only paired in the spanwise direction.

The already well accepted Q2-Q4 pairs with vortex clusters in the middle also exist in HST. But the streamwise roller formed by these pairs (Flores & Jiménez, 2010, LFJ12, Jimenez, 2013b) is incomplete to describe all the characteristics of the pair. Averaged Q2 and Q4 lie at the boundary of the streak, where there is strong shear layers ($\partial u / \partial y < 0$) connected by the vortex cluster. As the paired Q2 and Q4 passing each other, the streak will be more wavy the finally breakdown. A new streak will be regenerated when another Q2-Q4 pair is formed. The averaged flow fields conditioned to Q2s show that vortex clusters are lodged at the top and downstream direction of Q2s but the opposite is true when conditioned to Q4s. In channels, v moving upward is supposed to bring small scales to the center part of the channel while the opposite for v moving downward. So streamwise velocities correlated with v moving up and down are different, leading to much longer and taller high streamwise velocity associated with Q4 than the low streamwise velocity associated with Q2, but such difference disappear gradually as Q2-Q4 pairs become taller and taller.

The averaged flow field conditioned to vortex clusters associated with Q2s in HST is different from that conditioned to tall attached vortex clusters in the channel, where a very long conical low-streamwise-velocity ‘wake’ originates from the upstream of the averaged vortex cluster exist. Previous models for this ‘wake’ is related with vortical structures (Adrian *et al.*, 2000, AJZM06). Consider that there is strong ω_z at the head of Q2s, it can also be interpreted as a results of a group of self similar Q2s, or further Q2-Q4 pairs.

Even though there are differences between Q2-Q4 pairs in HST and channels, the results should not be interpreted that these pairs in channels are intrinsically different. They rather form a continuum in which structures become more symmetric, and more similar to the structures of HST, as they move farther from the wall. The wall itself does not appear to be required for the generation of Q2s or Q4s, but introduces asymmetry and inhomogeneity. It also modulates those wall attached structures, without which, most of interesting features like the ramps of low streamwise velocity and the induced secondary vorticity in channels will disappear.

At last, the temporal evolution of three-dimensional ejections, sweeps and vortex clusters in SS-HST reveals that velocities of these structures are the same as those in the channel. Specifically, the vertical velocity for Q2s and Q4s are u_τ and $-u_\tau$ respectively. And the streamwise velocities of each horizontal section are $1.5u_\tau$ slower and faster than the local mean streamwise velocity for Q2s and Q4s respectively. Vortex clusters, on the other hand, almost do not move vertically during their lifetimes and they are only advected by the local mean streamwise velocity. Due to the mean velocity difference, these structures are distorted mainly in the vertical plane, so that the streamwise elongation Δ_x/Δ_y is modulated.

These similarities between SS-HST and the logarithmic layer of wall-bounded turbulence make SS-HST a promising system to study shear turbulence in general.

The different mean velocity profiles do not seem to affect the properties of coherent structures. The wall introduces inhomogeneity and asymmetry while providing a support for structures be attached.

We have seen that Qs in SS-HST can reproduce most of the properties of those in channels. Actually, these similarities are not unexpected, because we are inspecting the mean shear itself, which introduces anisotropy into the flow by tangential Reynolds stress uv . But it is not clear how does the box affect the large scale motions or in other words, to what extent are those results artificial. In order to fully understand the effect of the mean shear, it would be worth to extend similar work into those turbulent flows with unstable mean velocity profiles, such as turbulent boundary layers. In such flows, it would be more interesting to inspect the dynamics, since the flow is developing downstream. Another extension would be to find other more proper eddies to study the cascade process. The problem is that Qs are large scale structures responsible for kinetic energy production while vortex clusters are small scale structures responsible for energy dissipation. Therefore, they are at the beginning and the end of the inertial range. Besides, the Reynolds stress does not appear in the Navier-Stokes equations, but their derivatives, or the fluxes of Reynolds stress across different scales do. At last, this thesis is not only for SS-HST itself, but for simplifying our understandings to the much more important wall-bounded turbulence. How can these results be incorporated to more practical models for large-eddy simulation of wall-bounded turbulence still needs more work.

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