UNIVERSIDAD POLITÉCNICA DE MADRID
ESCUELA TÉCNICA SUPERIOR DE INGENIEROS DE TELECOMUNICACIÓN

NEW METHODS FOR THE SYNTHESIS OF
RADIATION PATTERNS OF COUPLED ANTENNA
ARRAYS

NUEVOS MÉTODOS DE SÍNTESIS DE DIAGRAMAS
DE RADIACIÓN DE AGRUPACIONES DE ANTENAS
ACOPLADAS

TESIS DOCTORAL

José Ignacio Echeveste Guzmán

Ingeniero de Telecomunicación

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DEPARTAMENTO DE SEÑALES, SISTEMAS Y
RADIOCOMUNICACIONES
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Autor:

José Ignacio Echeveste Guzmán

Ingeniero de Telecomunicación

Tutor:

Miguel Ángel González de Aza

Doctor Ingeniero de Telecomunicación

Profesor Titular de Universidad

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TESIS DOCTORAL: New methods for the synthesis of radiation patterns of coupled antenna arrays (Nuevos métodos de síntesis de diagramas de radiación de agrupaciones de antenas acopladas)

AUTOR: José Ignacio Echeveste Guzmán
Ingeniero de Telecomunicación

DIRECTOR: Miguel Ángel González de Aza
Doctor Ingeniero de Telecomunicación
Profesor Titular de Universidad

DEPARTAMENTO: Departamento de Señales, Sistemas y Radiocomunicaciones
Universidad Politécnica de Madrid

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Abstract

The main objective of this thesis is the development of optimization methods for the radiation pattern synthesis of array antennas in which a rigorous electromagnetic characterization of the radiators and the mutual coupling between them is performed. The electromagnetic characterization is usually overlooked in most of the available synthesis methods in the literature, this is mainly due to two reasons. On the one hand, it is argued that the radiation pattern of an array is mainly influenced by the array factor and that the mutual coupling plays a minor role. As it is shown in this thesis, the mutual coupling and the rigorous characterization of the array antenna influences significantly in the array performance and its computation leads to differences in the results obtained. On the other hand, it is difficult to introduce an analysis procedure into a synthesis technique. The analysis of array antennas is generally expensive computationally as the structure to analyze is large in terms of wavelengths. A synthesis method requires to carry out a large number of analysis, this makes the synthesis problem very expensive computationally or intractable in some cases.

Two methods have been used in this thesis for the analysis of coupled antenna arrays, both of them have been developed in the research group in which this thesis is involved. They are based on the finite element method (FEM), the domain decomposition and the modal analysis. The first one obtains a finite array characterization with the results obtained from the infinite array approach. It is specially indicated for the analysis of large arrays with equispaced elements. The second one characterizes the array elements and the mutual coupling between them with a spherical wave expansion of the radiated field by each element. The mutual coupling is computed using the properties of translation and rotation of spherical waves. This method is able to analyze arrays with elements placed on an arbitrary distribution. Both techniques provide a matrix formulation that makes them very suitable for being integrated in synthesis techniques, the results obtained from these synthesis methods will be very accurate.

The array synthesis stands for the modification of one or several array parameters looking for some desired specifications of the radiation pattern. The array parameters used as optimization variables are usually the excitation weights applied to the array elements, but some other array characteristics can be used as well, such as the array elements positions or rotations. The desired specifications may be to steer the beam towards any specific direction or to generate shaped beams with arbitrary geometry.

Further characteristics can be handled as well, such as minimize the side lobe level in some other radiating regions, to minimize the ripple of the shaped beam, to take control over the cross-polar component or to impose nulls on the radiation pattern to avoid possible interferences from specific directions.

The analysis method based on the infinite array approach considers an infinite array with a finite number of excited elements. The infinite non-excited elements are physically present and may have three different terminations, short-circuit, open circuit and match terminated. Each of this terminations is a better simulation for the real environment of the array. This method is used in this thesis for the development of two synthesis methods. In the first one, a multi-objective radiation pattern synthesis is presented, in which it is possible to steer the beam or beams in desired directions, minimizing the side lobe level and with the possibility of imposing nulls in the radiation pattern. This method is very efficient and obtains optimal solutions as it is based on convex programming. The same analysis method is used in a shaped beam technique in which an originally non-convex problem is transformed into a convex one applying symmetry restrictions, thus solving a complex problem in an efficient way. This method allows the synthesis of shaped beam radiation patterns controlling the ripple in the mainlobe and the side lobe level.

The analysis method based on the spherical wave expansion is applied for different synthesis techniques of the radiation pattern of coupled arrays. A shaped beam synthesis is presented, in which a convex formulation is proposed based on the phase retrieval method. In this technique, an originally non-convex problem is solved using a relaxation and solving a convex problems iteratively. Two methods are proposed based on the gradient method. A cost function is defined involving the radiation intensity of the coupled array and a weighting function that provides more degrees of freedom to the designer. The gradient of the cost function is computed with respect to the positions in one of them and the rotations of the elements in the second one. The elements are moved or rotated iteratively following the results of the gradient. A highly non-convex problem is solved very efficiently, obtaining very good results that are dependent on the starting point. Finally, an optimization method is presented where discrete digital phases are synthesized providing a radiation pattern as close as possible to the desired one. The problem is solved using linear integer programming procedures obtaining array designs that greatly reduce the fabrication costs.

Results are provided for every method showing the capabilities that the above mentioned methods offer. The results obtained are compared with available methods in the literature. The importance of introducing a rigorous analysis into the synthesis method is emphasized and the results obtained are compared with a commercial software, showing good agreement.

Resumen

El principal objetivo de esta tesis es el desarrollo de métodos de síntesis de diagramas de radiación de agrupaciones de antenas, en donde se realiza una caracterización electromagnética rigurosa de los elementos radiantes y de los acoplos mutuos existentes. Esta caracterización no se realiza habitualmente en la gran mayoría de métodos de síntesis encontrados en la literatura, debido fundamentalmente a dos razones. Por un lado, se considera que el diagrama de radiación de un array de antenas se puede aproximar con el factor de array que únicamente tiene en cuenta la posición de los elementos y las excitaciones aplicadas a los mismos. Sin embargo, como se mostrará en esta tesis, en múltiples ocasiones un riguroso análisis de los elementos radiantes y del acoplo mutuo entre ellos es importante ya que los resultados obtenidos pueden ser notablemente diferentes. Por otro lado, no es sencillo combinar un método de análisis electromagnético con un proceso de síntesis de diagramas de radiación. Los métodos de análisis de agrupaciones de antenas suelen ser costosos computacionalmente, ya que son estructuras grandes en términos de longitudes de onda. Generalmente, un diseño de un problema electromagnético suele comprender varios análisis de la estructura, dependiendo de las variaciones de las características, lo que hace este proceso muy costoso.

Dos métodos se utilizan en esta tesis para el análisis de los arrays acoplados. Ambos están basados en el método de los elementos finitos, la descomposición de dominio y el análisis modal para analizar la estructura radiante y han sido desarrollados en el grupo de investigación donde se engloba esta tesis. El primero de ellos es una técnica de análisis de arrays finitos basado en la aproximación de array infinito. Su uso es indicado para arrays planos de grandes dimensiones con elementos equiespaciados. El segundo caracteriza el array y el acoplo mutuo entre elementos a partir de una expansión en modos esféricos del campo radiado por cada uno de los elementos. Este método calcula los acoplos entre los diferentes elementos del array usando las propiedades de traslación y rotación de los modos esféricos. Es capaz de analizar agrupaciones de elementos distribuidos de forma arbitraria. Ambas técnicas utilizan una formulación matricial que caracteriza de forma rigurosa el campo radiado por el array. Esto las hace muy apropiadas para su posterior uso en una herramienta de diseño, como los métodos de síntesis desarrollados en esta tesis. Los resultados obtenidos por estas técnicas de síntesis, que incluyen métodos rigurosos de análisis, son consecuentemente más precisos.

La síntesis de arrays consiste en modificar uno o varios parámetros de las agrupaciones de antenas buscando unas determinadas especificaciones de las características de radiación. Los parámetros utilizados como variables de optimización pueden ser varios. Los más utilizados son las excitaciones aplicadas a los elementos, pero también es posible modificar otros parámetros de diseño como son las posiciones de los elementos o las rotaciones de estos. Los objetivos de las síntesis pueden ser dirigir el haz o haces en una determinada dirección o conformar el haz con formas arbitrarias. Además, es posible minimizar el nivel de los lóbulos secundarios o del rizado en las regiones deseadas, imponer nulos que evitan posibles interferencias o reducir el nivel de la componente contrapolar.

El método para el análisis de arrays finitos basado en la aproximación de array infinito considera un array finito como un array infinito con un número finito de elementos excitados. Los elementos no excitados están físicamente presentes y pueden presentar tres diferentes terminaciones, corto-circuito, circuito abierto y adaptados. Cada una de estas terminaciones simulará mejor el entorno real en el que el array se encuentre. Este método de análisis se integra en la tesis con dos métodos diferentes de síntesis de diagramas de radiación. En el primero de ellos se presenta un método basado en programación lineal en donde es posible dirigir el haz o haces, en la dirección deseada, además de ejercer un control sobre los lóbulos secundarios o imponer nulos. Este método es muy eficiente y obtiene soluciones óptimas. El mismo método de análisis es también aplicado a un método de conformación de haz, en donde un problema originalmente no convexo (y de difícil solución) es transformado en un problema convexo imponiendo restricciones de simetría, resolviendo de este modo eficientemente un problema complejo. Con este método es posible diseñar diagramas de radiación con haces de forma arbitraria, ejerciendo un control en el rizado del lóbulo principal, así como en el nivel de los lóbulos secundarios.

El método de análisis de arrays basado en la expansión en modos esféricos se integra en la tesis con tres técnicas de síntesis de diagramas de radiación. Se propone inicialmente una síntesis de conformación del haz basado en el método de la recuperación de fase resuelta de forma iterativa mediante métodos convexos, en donde relajando las restricciones del problema original se consiguen unas soluciones cercanas a las óptimas de manera eficiente. Dos métodos de síntesis se han propuesto, donde las variables de optimización son las posiciones y las rotaciones de los elementos respectivamente. Se define una función de coste basada en la intensidad de radiación, la cual es minimizada de forma iterativa con el método del gradiente. Ambos métodos reducen el nivel de los lóbulos secundarios minimizando una función de coste. El gradiente de la función de coste es obtenido en términos de la variable de optimización en cada método. Esta función de coste está formada por la expresión rigurosa de la intensidad de radiación y por una función de peso definida por el usuario para imponer prioridades sobre las diferentes regiones de radiación, si así se desea. Por último, se presenta un método

en el cual, mediante técnicas de programación entera, se buscan las fases discretas que generan un diagrama de radiación lo más cercano posible al deseado. Con este método se obtienen diseños que minimizan el coste de fabricación.

En cada uno de las diferentes técnicas propuestas en la tesis, se presentan resultados con elementos reales que muestran las capacidades y posibilidades que los métodos ofrecen. Se comparan los resultados con otros métodos disponibles en la literatura. Se muestra la importancia de tener en cuenta los diagramas de los elementos reales y los acoplos mutuos en el proceso de síntesis y se comparan los resultados obtenidos con herramientas de software comerciales.

Contents

List of Figures	xi
List of Tables	xvii
Abbreviations	xix
Symbols	xxi
1 Introduction	1
1.1 Motivation and objectives	1
1.2 Text organization	3
2 Theoretical background	5
2.1 Introduction	5
2.2 Finite Element Method	6
2.2.1 Brief description of the hybrid method of finite elements, modal analysis and generalized scattering matrix	7
2.3 Analysis of an array of coupled antennas	10
2.3.1 Infinite array model	10
2.3.1.1 Application of the hybrid method of FEM-MA-GSM to the analysis of infinite arrays.	11
2.3.1.2 Computation of the active element pattern	13
2.3.2 Application of the hybrid method FEM-MA-GSM to the analysis of finite arrays via the spherical wave expansion.	15
2.4 Optimization methods	19
3 Finite array analysis from the infinite array approach	23
3.1 Introduction	23
3.2 Computation of the mutual coupling of the finite array	24
3.3 Radiation pattern of the finite array	27
3.4 Analysis examples	31
4 Multi-objective optimization of coupled arrays using the infinite array approach and convex programming	35
4.1 Introduction	35
4.2 Proposed optimization method	37
4.2.1 Convex optimization problem	38
4.2.2 Real-valued linear problem.	39
4.2.3 Computational aspects and remarks.	42

4.3	Numerical results	43
4.3.1	Broadside pattern with different side lobe regions with a square planar array of square apertures	43
4.3.2	Dual beam pattern with a square array of circular and cavity-backed microstrip antennas	45
4.3.3	Steered pattern with a square planar array of aperture coupled patch antennas	47
5	Shaped-beam synthesis of real arrays using the infinite array approach and convex programming	51
5.1	Introduction	51
5.2	Proposed synthesis method	53
5.3	Numerical results	56
5.3.1	Circular flat-top pattern with a square planar array of square apertures	58
5.3.2	Circular flat-top pattern with a square planar array of rectangular patch antennas	58
5.3.3	Rectangular flat-top pattern with a circular planar array of square and cavity-backed patch antennas	60
5.3.4	Circular-sector flat-top pattern with a circular planar array of square and cavity-backed patch antennas	62
6	Shaped-beam synthesis of coupled antenna arrays using the phase retrieval	65
6.1	Introduction	65
6.2	Proposed optimization method	66
6.3	Numerical results	71
6.3.1	Linear array of isotropic elements: A comparison with optimal synthesis methods	71
6.3.2	Linear array of hemispherical dielectric resonator antennas	72
6.3.3	Planar array of cavity-backed circular microstrip antennas	75
7	Gradient-based array synthesis of real arrays with uniform amplitude excitation including mutual coupling	79
7.1	Introduction	79
7.2	Pattern synthesis of aperiodic and coupled antenna arrays	80
7.2.1	Proposed synthesis method	82
7.2.1.1	Cost function, constraints and optimization procedure	82
7.2.1.2	Weighting functions and initial distribution	84
7.2.1.3	Gradient of the cost function of the coupled array	85
7.2.2	Numerical results	89
7.2.2.1	Linear array of hemispherical dielectric resonator antennas	89
7.2.2.2	Linear array of truncated tetrahedral dielectric resonator antennas	91
7.2.2.3	Planar array of tetrahedral dielectric resonator antennas	93
7.2.2.4	Planar array of microstrip patch antennas	96
7.3	Pattern synthesis of coupled antenna arrays via element rotation	97
7.3.1	Analysis method accounting for elements radiation	99

7.3.2	Proposed synthesis method	100
7.3.3	Results	103
7.3.3.1	Linear array with linear polarization composed of square and cavity-backed patch antennas	103
7.3.3.2	Planar array with linear polarization composed of hemi- spherical dielectric resonator antennas	106
7.3.3.3	Planar array with circular polarization composed of hemi- spherical dielectric resonator antennas	108
8	Synthesis of coupled antenna arrays using digital phase control via integer programming	111
8.1	Introduction	111
8.2	Proposed optimization method	113
8.3	Numerical results	116
9	Conclusions	121
9.1	Original contributions	121
9.2	Future research lines	123
9.3	Framework	124
9.4	Publications	125
9.4.1	Journal articles	125
9.4.2	Conference proceedings	125
9.4.2.1	International	125
9.4.2.2	National	126
A	Results from the synthesis process	127
	Bibliography	131

List of Figures

2.1	Infinite array of rectangular patch antennas placed in a dielectric substrate.	11
2.2	Unit cell of the infinite array.	13
2.3	(a): GSM characterization of the individual elements of the array . (b): GSM characterization of the coupled array where the mutual coupling is rigorously take into account.	16
2.4	(a): Isolated cavity-backed and probe-fed patch antenna analyzed with the present method. (b): Array of antennas with the mutual coupling accounted with translation and rotation of spherical waves.	18
3.1	Example of an ideal finite array of 3×3 radiating elements in an infinite environment of non-excited elements terminated by short circuits, open circuits or matched loads. The dark elements stand for the finite array while the grey ones represent the infinite array environment.	24
3.2	Planar array of $L(M \times N)$ elements. In each element there is an incident (v) and a reflected (w) power.	26
3.3	Inter-element distances in a planar array with equispaced elements.	27
3.4	AEP cuts of one element placed in a corner of a 9×9 -array of open-ended square waveguides, considering three different infinite array environment of non-excited elements: open circuit, short circuit, and match termination. The AEP obtained with the full-wave method presented in Chapter 2 for arrays on an infinite metallic plane is also represented.	31
3.5	E-plane (a) and (b) H-plane radiation pattern of the co-polar component of the 9×9 aperture array.	33
4.1	H -plane cuts of the radiation patterns of a 15×15 -array of open-ended square waveguides in the three different infinite array environment of non-excited elements, and the excitations optimized considering shorted non-excited apertures.	44
4.2	Geometry of a dual-coaxial probe-fed circular microstrip antenna enclosed in a cylindrical metallic cavity recessed in a metallic plane, used as array element. $R_1 = 30$ mm, $R_2 = 24.75$ mm, $c_1 = 1.524$ mm, $c_2 = 3.976$ mm, $s = 6.2$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$. Coaxial feeds (SMA connectors): $r_i = 0.64$ mm, $r_o = 2.05$ mm and $\epsilon_{rx} = 1.951$	44
4.3	Co- and cross-polar components of a two-beam pattern in the $\phi = 0^\circ$ and 90° planes of a 15×15 -element cavity-backed patch antenna array considering two cases: (a) The SLL is minimized setting a quadrature phase difference between the probe feeds in each antenna. (b) The SLL is minimized and the maximum cross-polar component is set to -40 dB optimizing each coaxial excitation independently.	46

4.4	Three dimensional representation of the optimized two-beam pattern for the 15x15-element array of cavity-backed microstrip antennas.	47
4.5	Geometry of a aperture coupled patch antenna used as array element: square periodicity $a = 3.0$ cm. Patch dimensions $W_m = 2.37$ cm and $L_m = 1.68$ cm. Aperture $W_p = 1.18$ cm and $L_p = 0.76$ mm. Microstrip feed $W_s = 1.94$ mm and $L_s = 8.3$ mm. Dielectric substrate $C_1 = 0.8$ mm, $C_2 = 1.6$ mm, $\epsilon_{r1} = 3.2$ and $\epsilon_{r2} = 2.2$	48
4.6	H-plane cuts of the broadside field pattern of a 10×10 -patch array obtained with the optimized excitations considering open-circuit non-excited CDRAs, and with the <i>Cheng – Tseng</i> excitations scheme for isotropic sources. The ideal isotropic Cheng-Tseng pattern is also represented. The dashed grey line represents the desired SLL level.	48
5.1	E -plane cut of a circular flat-top pattern ($ \theta \leq 20^\circ$) for a 15×15 element square array of open-ended square waveguides obtained with the three finite array approaches in this work and from the method in detailed in Chapter 2. Excitations synthesized considering an environment of shorted non-excited apertures.	57
5.2	Geometry of the coaxial probe-fed rectangular patch antenna used as array element in example IV.A. $\epsilon_r = 4.32$, $l = 38$ mm, $a = 18$ mm, $b = 12$ mm, $c_1 = 6.5$ mm, $c_2 = 6$ mm and $d = 0.8$ mm. Coaxial feed (SMA connector): $r_{in} = 0.65$, $r_{out} = 2.05$ and $\epsilon_{rx} = 1.951$	57
5.3	(a) Finite array (dark elements) in an infinite array environment of non-excited and match, open or short-terminated elements (grey elements). (b) Geometry of a finite array of microstrip antennas in Fig. 5.2 surrounded by a substrate with circular contour, simulated with a full-wave electromagnetic solver (CST) in example 5.3.2.	59
5.5	Layout of the 525–element array of cavity backed microstrip antennas with a circular contour and square mesh of $0.5\lambda_0$ -equispaced elements.	60
5.6	Geometry of the coaxial probe-fed and cavity-backed square patch antenna used as array element in example IV.C. $l = 1.35$ cm, $r = 1.815$ cm, $c_1 = 2.42$ mm, $c_2 = 2.9$ mm, $x_0 = 1.7$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$. Coaxial feed (SMA connector): $\epsilon_{rx} = 1.951$, $r_{in} = 0.65$ mm and $r_{out} = 2.05$ mm.	61
5.7	Rectangular flat-top pattern in E – and H –planes for the 525-element array in Fig. 5.5 made-up of cavity-backed microstrip antennas of Fig. 5.6, and synthesized considering an infinite shorted element array environment. Continuous line: co-polar component, dashed line: cross-polar component.	61
5.8	Three dimensional representation of the co-polar component of the synthesized rectangular flat-top pattern for the array in Fig. 5.5 with the cavity-backed microstrip antennas of Fig.5.7.	62
5.9	Color map representation of the co-polar component of the synthesized sector flat-top pattern for the array in Fig. 5.5 with the cavity-backed microstrip antennas of Fig.5.7.	63
6.1	Comparison between different optimization methods. The results from the proposed method and from the method with which it is being compared are represented by the black line and the dashed red line respectively.	71

6.2	Geometry of the hemispherical dielectric resonator antenna used as array element in example IV.B. 50Ω coax. $r_i = 0.5$ mm and $r_o = 1.05$ mm with $\epsilon_r = 1.74$. Parameters: $R = 12.7$ mm, $s = 6.4$ mm and $h = 6.5$ mm with $\epsilon_r = 9.5$	72
6.3	Synthesized flat-top pattern for an E-plane linear array of 15 HDRAs with the optimization performed using real and isotropic array elements. The case with real elements is compared with a commercial electromagnetic software.	73
6.4	Synthesized cosecant squared pattern (b and c) for an E-plane non-uniform random linear array of 15 HDRAs (a) with the optimization performed using real and isotropic array elements.	74
6.5	Geometry of the circular and cavity backed patch antenna used as array element in example IV.C. with the following characteristics: $R_1 = 30$ mm, $R_2 = 24.75$ mm, $c_1 = 1.524$ cm, $c_2 = 3.976$ cm, $t = 2$ mm $x_0 = y_0 = 6.2$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$, $\epsilon_{rx} = 1.9$, $r_1 = 0.65$ mm and $r_0 = 2.05$ mm. . .	75
6.6	E- and H-plane cuts of the synthesized square-shaped footprint pattern for a 6×6 square array of cavity-backed circular microstrip antennas. The results obtained with the present method are compared with those of CST.	76
6.7	Rectangular and triangular masks used in the synthesis with planar arrays of cavity-backed circular microstrip antennas in Section 6.3.	76
6.8	Color map representation of the synthesized triangular-shaped flat-top pattern for a square array of 20×20 cavity-backed circular microstrip antennas.	77
7.1	Different weighting functions used in this work.	85
7.2	Radiated field and the appearance of secondary lobes for using an array of isotropic elements placed with the initial distribution obtained with the density synthesis. The study is performed for a fixed array length ($20 \lambda_0$) and varying the average distance between elements, $0.71 - 1.67\lambda_0$ (indicated in the graphics), and consequently the number of elements, $28 - 12$	86
7.3	Coordinates of antennas j and k separated a distance d and definition of the angle ϕ_{kj}	87
7.4	Synthesized positions of the 18 HDRAs represented with blue dots. The red circles stand for the hemispheres in which the resonators are enclosed.	89
7.5	Radiation pattern of an 18-element HDRA linear array in E-plane with optimized positions obtained from realistic HDRAs (black line) and from isotropic elements (red line). The dashed gray line stands for the maximum SLL obtained in the first case.	90
7.6	Geometry of the truncated tetrahedral dielectric resonator antenna used as array element: $h = 2.4$ cm, $L_u = 6.4$ and $L_l = 2.5$ cm with $\epsilon_r = 12$. The feed properties are $w_x = 0.55$ mm and $w_l = 1.15$ mm. Coaxial probe feed (50Ω): $r_i = 0.5$ mm, $r_o = 1.51$ mm and $\epsilon_{rx} = 1.73$	91
7.7	Value for the cost function versus the number of iterations at each frequency and the total value for the synthesis of the wideband array.	91
7.8	Synthesized field radiation patterns versus frequency, in steps of 50 MHz, of the 40-element linear array of truncated TDRAs along the E-plane. . .	92

7.9	Field radiation patterns at 2.45 GHz of the 40-element linear array of truncated TDRAs along the E-plane with optimized positions obtained with the proposed method. The resulting pattern is compared with the obtained from the commercial software CST.	92
7.10	(a-c): Initial configurations considered for the synthesis of a 40-element planar array of truncated TDRAs: circular, random and sunflower distributions respectively. (d-f): Synthesized distributions obtained with the present method. (g-h) Color map representation of the synthesized field radiation patterns at 2.5 GHz for the three different initial configurations and weighting functions.	94
7.11	Comparison between the radiation patterns at 6.1 GHz of a 40-element planar array of truncated TDRAs with synthesized positions obtained from isotropic elements (represented in red), and from realistic antennas (in blue). Three initial array configurations are considered: circular (a-b), random (c-d) and the sunflower (e-f).	95
7.12	Synthesized field radiation pattern at 6.1 GHz of a 30-element planar array of cavity-backed microstrip antennas scanned at $u_{x0} = 0.3$ and $u_{y0} = 0.75$	97
7.13	Scheme of the elements rotation.	101
7.14	Synthesized rotations of the 14 patch antennas.	103
7.15	Radiation pattern of an 14-element patch linear array in E-plane with optimized rotations obtained from realistic patch antennas. The un-rotated co-polar component and the co-and cross-polar components are represented for the synthesized array with rotated elements. The weighted functions applied to both components are also represented.	104
7.16	Comparison between the array pattern obtained with the proposed method and the radiated field pattern obtained for an array with the synthesized rotations simulated in the commercial software CST.	105
7.17	Cost function per iteration of the synthesis of the 14-element patch linear array.	105
7.18	: 360 phi-cuts of the co-polar, (a), and cross-polar, (b), components of the radiation pattern of the synthesized array made of 10×10 HDRAs.	106
7.19	Synthesized rotations of the 10×10 HDRAs.	107
7.20	(a): 6×6 sequentially rotated elements used as an initial configuration. (b): Synthesized rotations.	108
7.21	(a), (b): Co- and cross-polar component of the radiation pattern of the sequentially rotated array. (c), (d): Co- and cross-polar component of the radiation pattern of the synthesized array.	109
8.1	Synthesized radiation patterns of a 15-element linear array of cavity-backed circular microstrip antenna using discretized phase variables (continuous line) and continuous phase variables (dashed line) in the optimization procedure. The synthesized phases for the first case are also represented.	116
8.2	Synthesized radiation patterns of a 21-element linear array of HDRAs when mutual coupling effects (MC) are considered, and not considered, in the optimization procedure. The synthesized phases for the first case are also represented.	117
8.3	Array feed configurations.	118

8.4	Synthesized radiation patterns of a linear array of 16 HDRAs divided in four-element sub-arrays. The digital phase sequence of the TTDs and phase shifters are also represented.	118
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List of Tables

4.1	Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)	43
5.1	Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)	63
6.1	Number of elements (No. Elem.), analysis time (An. Time), synthesis time, (Sy. time) and a maximum ripple obtained (Ripple).	72
6.2	Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)	77
7.1	Synthesized positions of the HDRAs in example IV.A.	89
7.2	Initial cost function (CF_i), final CF (CF_f), initial SLL (SLL_i), final SLL (SLL_f), number of iterations (It.) and time in the analysis and synthesis process (Ti.)	93
7.3	The number of elements (N.) and their correspondence synthesized rotation (Φ_{rot}) in degrees.	103
7.4	Number of variables of optimization (No. Var.), time required array analysis in seconds: of the isolated element (Elem.) and of the coupled array (Array), number of iterations of the synthesis (No. it.) and time in the optimization process in minutes (Opt.)	108
A.1	Synthesized excitation amplitudes of the 15×15 array elements in Subsection 4.3.1. which generates the radiation pattern shown in Fig. 4.1.	127
A.2	Synthesized excitation phases of the 15×15 array element in Subsection 4.3.1. which generates the radiation pattern shown in Fig. 4.1.	128
A.3	Synthesized excitation amplitudes of the 9×9 array elements in Subsection 5.3.2. which generates the radiation pattern shown in Fig. 5.4.	128
A.4	Synthesized excitation phases of the 9×9 array elements in Subsection 5.3.2. which generates the radiation pattern shown in Fig. 5.4.	128
A.5	Synthesized excitation amplitudes and phases of the 6×6 array elements in Subsection 6.3.3. which generates the radiation pattern shown in Fig 6.6.	129
A.6	Synthesized positions of the 40 elements computed in Subsection 7.2.2.3 and represented in Fig 7.10 (d).	129
A.7	Synthesized rotations of the 36 elements of Subsection 7.3.3.3 and represented in Fig 7.20 (b).	129

A.8 Synthesized discrete phases of the 16 array elements in Section 8.3 which generates the radiation pattern shown in Fig 7.10 (d).	129
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Abbreviations

AEP	A ctive E lement P attern
AF	A rray F actor
APA	A lternating P rojections A lgorithm
ARC	A ctive R eflection C oefficient
CDRA	C ylindrical D ielectric R esonator A ntenna
CP	C onvex P rogramming
Cp	C o-polar
DRA	D ielectric R esonator A ntenna
DRR	D ynamic R ange R atio
FDTD	F inite D ifference T ime D omain
FEM	F inite E lement M ethod
FIR	F inite I mpulse R esponse
GSM	G eneralized S cattering M atrix
HDRA	H emispherical D ielectric R esonator A ntenna
IP	I nteger P rogramming
IPM	I nterior P oint M ethods
LP	L inear P rogramming
MA	M odal A nalysis
MT	M atch T erminated
MBF	M acro B asis F unction
MILP	M ixed I nteger L inear P rogramming
MoM	M ethod o f M oments
OC	O pen C ircuit
PO	P hysical O ptics
QP	Q uadratic P rogramming

SAR	S ynthetic A perture R adar
SC	S hort C ircuit
SDP	S emi D efinite P rogramming
SLL	S ide L obe L evel
SMA	S ub M iniature version A
TDRA	T etrahedral D ielectric R esonator A ntenna
TTD	T rue T ime D elay
UTD	U niform T heory of D iffraction
Xp	X for Cross- p olar

Symbols

$\vec{a}$	Complex coefficients of the incident modes on the spherical (or Floquet) port
$\vec{b}$	Complex coefficients of the scattered modes on the spherical (or Floquet) port
$\vec{v}$	Complex coefficients of the incident modes on the feeding port
$\vec{w}$	Complex coefficients of the reflected modes on the feeding port
$\mathbf{\Gamma}$	Reflection matrix of the isolated elements
$\mathbf{T}$	Transmission matrix of the isolated elements
$\mathbf{S}$	Scattering matrix of the isolated elements
$\mathbf{I}$	Identity matrix
$\mathbf{T}_G$	Transmission matrix of the coupled array
$\mathbf{T}_{G_F}$	Transmission matrix of the coupled array accounting for element rotation
$\mathbf{G}$	General translation matrix
$\mathbf{F}$	Rotation of the array elements
$\vec{e}_m$	Electric field of the m -th spherical mode
$\vec{e}_l^h$	Complex coefficients of the TE component of the l harmonic
$\vec{e}_l^e$	Complex coefficients of the TM component of the l harmonic
$\mathbf{G}_F$	General translation matrix accounting for rotation
$\mathbf{W}$	Weighting function
ν	Voltage
$\vec{E}_a$	Active element pattern
$\vec{E}_A$	Radiated field by the coupled array

Chapter 1

Introduction

1.1 Motivation and objectives

Array antennas are made up of at least two elements radiating with phase coherence. They have been an important research topic since the 40s and they are still being the focus of a lot of works published nowadays. Array antennas have several advantages compared with a single element. One of the most important characteristic is the possibility of reinforcing the radiation pattern in desired directions and suppressing it in undesired directions. This capability allows to create directive and narrow beams or synthesizes radiation patterns with arbitrary shape. Phased array antennas also permit to steer the beam without any mechanical movement. The analysis and synthesis of array antennas are still important and challenging research topics since rigorous and efficient methods are proposed nowadays.

Mutual coupling plays a main role in the radiation, reflection and polarization characteristics of the array, and thus it should be seriously considered in the electromagnetic analysis. In classic theory, the radiation pattern of an array antenna is computed multiplying the array factor by the radiation pattern of the isolated element. These simplifications may lead to unwanted results.

There is a huge number of electromagnetic procedures to analyze array antennas. Although infinite arrays do not exist, some procedures are based on their analysis because of their simplicity and because most of the scanning characteristics are present in this type of arrays. The infinite array approach has been demonstrated to be a powerful tool to analyze and design large planar arrays of equal and equispaced elements. In those arrays, the inner elements behave in a similar way since they are equally affected by the mutual coupling. They see an alike array environment and also very close comparing to

an element embedded in an infinite array [1]. In this way, the radiation pattern in this case is computed as the multiplication of the array factor by the active or embedded element pattern, the radiation pattern of the array when just one element is excited, being the rest of the elements match-terminated. The mutual coupling is inherently considered in this formulation.

Finite arrays of equally and equispaced elements can also be analyzed from the results obtained by the infinite array approach with a convolution or windowing technique [2–6] or methods that characterize the mutual coupling of the finite array with impedance, admittance or scattering matrices [7]. Rigorous and efficient analysis methods that can be used for large and sparse arrays have been proposed over the last decade. Some of them are based on Macro Basis Function, [8, 9], where a reduction in the number of unknowns is accomplished with a negligible error. Another approach was presented in [10] where the elements are enclosed in spheres or hemispheres, while the field radiated by each element is expressed as an expansion of spherical waves. In that way, the elements can be individually analyzed obtaining an electromagnetic problem with a much lower computational burden.

Array synthesis is usually carried out looking for the desired excitation weights that accomplish some desired radiation characteristics, but it can also be formulated with other array properties as the variable of the problem, such as the number of elements or the elements positions or rotations. Analytical methods for array synthesis were proposed firstly for arrays with minimized sidelobes [11–13], shaped beam radiation patterns [14–17] or phase-only synthesis [18]. Numerical methods have also been proposed for different kinds of applications, such as [19–21], which offer very good results. Methods based on convex programming [22] have arisen as very powerful optimization procedures when the desired characteristics of the array antenna can be expressed as convex, such as [23, 24].

The radiation pattern of an array antenna may be approximated as the pattern multiplication of the array factor and the element pattern. Generally, the element factor has a broad behaviour while the array factor behaves as a more rapidly varying function. That is why the radiation pattern synthesis of an array antenna is usually approximated as the array factor synthesis, neglecting in this way the element factor in some cases or the mutual coupling between elements in others. Just a few methods take into account the radiation pattern of the elements and the mutual coupling between them in the synthesis process.

In this thesis, different optimizations procedures for the radiation pattern of array antennas are proposed in which a rigorous and efficient electromagnetic analysis method is merged into the synthesis technique, obtaining in this way more reliable results.

1.2 Text organization

This thesis is organized as follows:

- Chapter 1 contains the introduction to the thesis and describes the motivation and objectives of the present work. The organization of the thesis is also detailed.
- Chapter 2 introduces some theoretical background related to the analysis of array antennas which are used in the synthesis methods presented in this thesis. The analysis methods have been developed in the research group in the last decades.
- Chapter 3 addresses the finite array analysis with results obtained from the infinite array approach. This analysis is included in the synthesis techniques presented in Chapters 4 and 5.
- Chapter 4 proposes an efficient optimization methodology of antenna array excitations accounting for the effects of mutual couplings and real radiating patterns of array elements with complex geometry. The method is formulated as a convex programming problem, that can be reduced to a linear programming problem, providing multi-beam patterns with specified beamwidths, constraints on sidelobe and cross-polar levels, and null pointing directions. A matrix formulation is used to integrate the optimization procedure and a finite array analysis based on the infinite array approach proposed in Chapter 3. Numerical examples of arrays of open-ended waveguides, microstrip antennas and dielectric resonator antennas are presented.
- Chapter 5 presents a shaped beam synthesis methodology for planar arrays with control of the ripple amplitude in the shaped region, and constraints on the sidelobe and cross-polar levels. A previously developed synthesis formulation for designing FIR filters by transforming a non-convex synthesis problem into a convex optimization scheme enforcing conjugate symmetric excitation weights is extended to real and coupled radiating elements of complex geometry, taking into account mutual coupling effects. The optimization procedure is integrated with a finite array approach proposed in Chapter 3. Numerical results of different synthesized beam patterns are presented for arrays of open-ended apertures and microstrip antennas.
- Chapter 6 proposes the application of the phase retrieval algorithm to the shaped beam synthesis of real antenna arrays through convex optimization. Near-optimal solutions are obtained with this method that do not yield in local minima and do not need initial points to converge. The method takes into account the real

radiation pattern of the elements that composes the array and the mutual coupling between them with the analysis method based on the spherical wave expansion. Numerical results for microstrip and dielectric resonator antennas are presented.

- Chapter 7 proposes two synthesis methods of arrays of realistic antennas excited with uniform amplitude, where the mutual coupling between elements is rigorously taken into account. The first one is a position-based synthesis method while the second of them is a rotation-based method. Both methods iteratively minimize the sidelobe level with a gradient method. A cost function that involves the expression of the radiated field of the coupled array is obtained. In the first method, the gradient w.r.t. the elements position is evaluated in order to move the elements in the corresponding direction at each iteration. In the second method, the gradient w.r.t. the elements rotation is computed and they are iteratively rotated following some desired characteristics. Both synthesis methods involve the calculation of the gradient of the radiation intensity by the coupled array, which is obtained analytically by using a full-wave method based on the spherical mode expansion and rotation and translation properties of spherical waves.
- Chapter 8 presents a pattern synthesis technique for real and coupled antenna arrays by using digital phase-only excitation weights. The method is formulated as an integer linear programming problem incorporating a full-wave analysis of the array, based on the spherical mode expansion of the radiated field obtained in Chapter 2. The method achieves radiation patterns with specified pointing directions, main lobe widths, minimum side lobe level and prescribed nulls, controlled by digital phase shifters.
- In Chapter 9 the conclusions of the present thesis are enumerated. The original contributions, the future research lines, the framework of the present thesis and the publications achieved with the present work are listed in this chapter.

Chapter 2

Theoretical background

2.1 Introduction

There is a huge variety of methods to solve electromagnetic problems, which solve the Maxwell's equations in different geometries and boundary conditions. These methods can be classified in three big groups:

- Analytical methods: In some occasions it is possible to solve the Maxwell's equations in an analytical way. The solution is exact and extremely efficient. However, a structure with arbitrary shape cannot be analyzed in this way.
- Numerical methods: These methods can be applied to any electromagnetic problem, independently of the structure to be analyzed. They work discretizing the analysis domain and they obtain a system of equations from the Maxwell's equations. They are usually computationally expensive because they work with a high number of equations in order to obtain an accurate solution. The most common full-wave numerical methods are the finite difference method, in the frequency domain (FDFD) and in the time domain (FDTD), the method of moments (MoM) or the finite element method (FEM). There are other methods which are more efficient but less rigorous, such as physical optics (PO) or the uniform theory of diffraction (UTD).
- Semi-analytical methods or hybrids (combination of numerical and analytical methods): The semi-analytical methods use specific characteristics of the structures analyzed, as radial symmetries, periodicities or a constant behaviour in one of their dimensions, being in this way a more efficient analysis. These methods are only applicable to the structures with the characteristics that they were designed for.

2.2 Finite Element Method

The employment of discretization methods, spatial and temporal, and the numerical approximation for finding solutions to engineering problems have existed for long time ago. The finite element method (FEM) was first proposed in the 40s [25], applied to the structure calculus. The method was used for electromagnetic problems for the first time in 1969 [26].

The method has evolved becoming one of the most common procedures to solve electromagnetic problems. The FEM is a numerical procedure which allows the discretization of continuous physic problems. It divides a continuous problem in a number of finite sub-domains, whose behaviour is modeled by a number of unknown parameters, from which the differential equations are transformed into algebraical equations. The solution of the complete system is obtained assembling all the elements. A deeper explanation of the FEM applied to solve electromagnetic problems can be found in [27].

In the Departamento de Señales, Sistemas y Radiocomunicaciones of the Escuela Técnica Superior de Ingenieros de Telecomunicación, where this thesis has been developed, a hybrid analytical-numerical method has been carried out which is able of joining the velocity and accuracy of analytical methods with the versatility of the FEM. The method combines:

- FEM
- Modal analysis
- Generalized scattering matrix (GSM)
- Domain decomposition

The method has been developed and improved in the last decades. Different electromagnetic problems have been approached with the present method, such as waveguide discontinuities [28], combination of the method for 2D and 3D geometries for the analysis of microwave passive circuits [29, 30], analysis of cavity-backed antennas [31], finite array antennas computing the mutual coupling and radiation pattern using an spherical mode expansion, [10], analysis of periodic structures [32] or arrays antennas using periodic conditions and Floquet harmonics [33, 34].

2.2.1 Brief description of the hybrid method of finite elements, modal analysis and generalized scattering matrix

Any arbitrary volume is univocally electromagnetically characterized if the tangential component of the electric or magnetic field is specified in all of its boundaries. Generally, the boundaries of the volume are delimited by a perfect conductor and modal ports, in which the field can be expressed as a summation of modes. In the perfect conductor the tangential component of the electrical field is null and in the modal ports they are expressed as a summatory of infinite modes. In the case of antennas, it is possible to employ spherical ports, Floquet harmonics or perfectly matched layers, as well.

It is also possible to define ports inside the structure in order to allow the segmentation of the problem (domain decomposition). This segmentation usually makes the analysis more efficient, specially when there are segments of the structure that can be analyzed analytically.

In the ports with canonical geometric structure the modes are obtained numerically. These ports can be rectangular, circular, coaxial, spherical or radial. There are also plane ports of arbitrary geometry in which the modes are obtained numerically with the finite element method in two dimensions.

Independently of the geometry, it is also possible to assign different characteristics to the defined ports, such as symmetry-plane or rotational symmetry [35], that simplifies the resolution of the problem. It is also possible to define Floquet ports, in which the field is expressed as a summatory of Floquet harmonics or Mortar ports, used for imposing periodic conditions without constrained meshes [32]. The application of the present method provides a matrix formulation which relates the coefficients of the modal expansion of the fields of the different ports defined in the structure.

The hybrid method of the three dimensional FEM, modal expansion, domain decomposition and the generalized scattering matrix is going to be briefly summarized. A deeper explanation can be found in [10] or [30].

The structure to be analyzed is divided in tetrahedrons and the field is approximated as a summation of vectorial functions defined over the tetrahedron. An arbitrary volume V is considered, homogeneous or inhomogeneous, delimited by a surface S made up of magnetic walls, electric walls and ports. Applying the Galerkin's method and taking $\vec{H}$ as an expansion function and $\vec{W}$ as a weight function, the vectorial wave equation for the magnetic field can be transformed in:

$$\int_V \vec{W} \cdot (\nabla \times [\epsilon_r]^{-1} \cdot \nabla \times \vec{H} - w^2 \mu_0 \epsilon_0 [\mu_r] \cdot \vec{H}) dV = 0 \quad (2.1)$$

where $[\epsilon_r]$ and $[\mu_r]$ are respectively the complex relative permittivity and permeability tensors, and w is the angular frequency. Applying vectorial identities:

$$\int_V (\nabla \times \vec{W} \cdot [\epsilon_r]^{-1} \cdot \nabla \times \vec{H} - k^2 \vec{W} \cdot [\mu_r] \cdot \vec{H}) dV - \int_S (\vec{W} \times [\epsilon_r]^{-1} \cdot \nabla \times \vec{H}) \cdot \mathbf{n}_0 dS = 0 \quad (2.2)$$

where $\mathbf{n}_0$ is the normal vector to the surface S and k is the wave number. Applying boundary conditions in the electric and magnetic walls and performing in every port p , a modal expansion of the electric field depending on the voltage coefficients V_j^p [30]:

$$\vec{E}_t^p = \sum_{j=1}^{\infty} V_j^p(\nu_p) \vec{e}_{tj}^p(\xi_p, \eta_p) \quad (2.3)$$

where (ν_p, ξ_p, η_p) is a local coordinate system that makes reference to each port, (2.2) results in:

$$\begin{aligned} & \int_V (\nabla \times \vec{W} \cdot [\epsilon_r]^{-1} \cdot \nabla \times \vec{H} - k^2 \vec{W} \cdot [\mu_r] \cdot \vec{H}) dV \\ &= jw\epsilon_0 \sum_{p=1}^P \sum_{j=1}^{\infty} V_j^p(\nu_p) \int_{S_p} \vec{W} \cdot (\mathbf{n}_p \times \vec{e}_{tj}^p) dS_p \end{aligned} \quad (2.4)$$

where $\mathbf{n}_p$ is a normal vector to each port.

Performing the discretization process from the finite element method, the following system of equations is obtained:

$$[\mathbf{K} - k^2 \mathbf{M}] \mathbf{h}_c = jw\epsilon_0 \mathbf{B} \mathbf{v} \quad (2.5)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the sparse symmetric matrices, $\mathbf{v}$ is a column vector with the voltage coefficients, $\mathbf{h}_c$ is a column vector with the degrees of freedom and $\mathbf{B}$ is a matrix made up of the following coefficients:

$$b_{jk}^p = \int_{S_p} \mathbf{F}_k \cdot (\mathbf{n}_p \times \vec{e}_{tj}^p) dS_p \quad (2.6)$$

where $\mathbf{F}_k$ are vectorial interpolation functions.

On the other hand, if the modal expansion of the magnetic field is carried out in the ports:

$$\vec{H}_t^p = \sum_{j=1}^{\infty} I_j^p(\nu_p) \vec{h}_{tj}^p(\xi_p, \eta_p) \quad (2.7)$$

and considering the orthogonality between modes, it is determined:

$$\int_{S_p} \mathbf{H} \cdot (n_p \times \vec{e}_{tj}^p) dS_p = I_j^p(\nu_p) \Delta_j^p \quad (2.8)$$

with:

$$\Delta_j^p = \int_{S_p} h_{tj}^p \cdot (n_p \times \vec{e}_{tj}^p) dS_p \quad (2.9)$$

a system of equations defined by:

$$\mathbf{B}^T \mathbf{h}_c = \mathbf{\Delta} \mathbf{I} \quad (2.10)$$

being $\mathbf{\Delta}$ a diagonal matrix whose coefficients are defined by (2.9) and $\mathbf{I}$ is the identity matrix.

Expressions (2.5) and (2.10) allow to obtain the admittance matrix that relates the field expansion in the ports:

$$\mathbf{Y} = jw\varepsilon_0 \mathbf{B}_n^T \mathbf{G}^{-1} \mathbf{B}_n \quad (2.11)$$

being:

$$\mathbf{B}_n = \mathbf{B} \mathbf{\Delta}^{-\frac{1}{2}}. \quad (2.12)$$

The scattering matrix of the structure is directly obtained as:

$$\mathbf{S} = 2(\mathbf{I}_d + \mathbf{Y})^{-1} - \mathbf{I}_d \quad (2.13)$$

where $\mathbf{I}_d$ is the identity matrix with the same number of rows as the number of modes used in each port.

Once the scattering matrix of the structure has been obtained, its behaviour can be modeled in terms of the relation between the incident and reflected modes in the defined ports over the multiport structure. For a two ports structure:

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{d} \\ \mathbf{c} \end{bmatrix} \quad (2.14)$$

where $\mathbf{d}$ and $\mathbf{c}$ are the incident and reflected modes in the incoming port, $\mathbf{b}$ and $\mathbf{a}$ are the incident and reflected modes in the outgoing port.

2.3 Analysis of an array of coupled antennas

The main characteristics of the array antennas are radiation patterns with high gain, a good illumination efficiency and high angular resolution that cannot be obtained with isolated elements. In the 40s, mechanical steering was used in applications where it was necessary to steer the beam. However, in the 60s there was an increased interest in antennas capable of steering the beam electronically [36]. They show several advantages comparing with the mechanical steering. They are more robust and the time of commutation is orders of magnitude lower.

The concept of periodic array arises where identical elements are placed equispaced in a linear distribution (linear arrays), a bi-dimensional distribution (planar arrays) or a distribution over a curved surface (conformal arrays). Planar arrays are able to steer the beam in every direction of the half-space and to shape the beam in arbitrary geometries. At the beginning, the elements that compose the array were mainly dipoles or apertures. In the last decades microstrip patches have become very popular due to their low cost and weight.

The array antennas are analyzed in classic theory without taking into account the mutual coupling between elements. The radiation pattern is obtained in terms of the radiation pattern of the isolated element and the array factor. This approximation will lead to unwanted results specially for elements that are placed with an inter-element spacing close to half wavelength.

There are two main techniques to analyze an array rigorously, carrying out a characterization element by element or applying periodic boundaries in the case that the array is made up of identical elements equispaced. In the element by element approach, the mutual coupling has to be computed between every pair of elements of the array, in terms of mutual impedances or admittances. This method computes the mutual coupling between elements one by one and its complexity grows exponentially with the size of the problem.

2.3.1 Infinite array model

The infinite array approach considers that the array is made up of infinite elements equispaced, as it is shown in Fig. 2.1. It behaves as an array of infinite sources, thus it is possible to apply the Floquet theorem and analyze the unit cell of the array assuming that it is in an infinite array environment. The mutual coupling is inherently considered in this formulation.

FIGURE 2.1: Infinite array of rectangular patch antennas placed in a dielectric substrate.

This method offers very good results when large planar arrays are analyzed in which the elements placed in the inner part of the array behave very similar to each other and very similar to an element of an infinite array as well. The elements placed near the edge of the array will behave differently because they are not equally influenced by the remaining elements. In some applications, where the main beam is in broadside direction and a taper amplitude distribution is used in the excitation weights, the edge effects do not affect significantly the array performance and it is possible to consider this effect with windowing methods. In some other applications they play an important role in the design process.

2.3.1.1 Application of the hybrid method of FEM-MA-GSM to the analysis of infinite arrays.

Large arrays are usually analyzed using the infinite array approach, considering that every element in the array behaves equally as they were surrounded by infinite elements. In this way, just the unit cell that composes the infinite array needs to be analyzed.

In order to characterize the array, only one element is full-wave analyzed, as shown in Fig. 2.2 for a microstrip array. This element is meshed and analyzed by a full-wave method, applying periodic boundaries in the lateral walls and considering the upper wall as a Floquet port.

This method is an approximation because it characterizes finite arrays using infinite array data that do not take into account two factors. On the one hand, the method considers that every element radiates in the same way. Actually this is not the case because elements placed in the center of the array will behave differently than elements placed closed to the edge of the array, which will be less influenced by the mutual coupling. On the other hand, the edge effects are not considered and the real array environment is not taken into account in the analysis. These effects can be taken into

account with windowing techniques or methods based on the impedance mutual coupling, as shown in Chapter 3.

The periodic element of the array is analyzed with a hybrid and modular full-wave method based on the three dimensional FEM, modal analysis and the domain decomposition [32].

Considering an infinite array of periodicities d_x and d_y respectively, and with a Floquet excitation (uniform amplitude and linear phase) in both directions defined by:

$$V_{m,n} = V_{0,0} e^{-jk_0(d_x m \sin \theta_0 \cos \phi_0 + d_y n \sin \theta_0 \sin \phi_0)} \quad (2.15)$$

if the double summation is reduced, m and n by a unique summatory l , ordering the Floquet harmonics by cut-off frequency, the radiated field of the array can be expressed as a summation of TE(h) and TM(e) Floquet harmonics:

$$\vec{E}_T = \sum_{l=1}^{\infty} [b_l^h \vec{e}_l^h(k_{xl}, k_{yl}) + b_l^e \vec{e}_l^e(k_{xl}, k_{yl})] e^{jk_{xl}x} e^{jk_{yl}y} e^{jk_{zl}z} \quad (2.16)$$

where:

$$\begin{aligned} \vec{e}_l^h(k_{xl}, k_{yl}) &= \frac{1}{\sqrt{k_{xl}^2 + k_{yl}^2}} (-k_{yl} \hat{x} + k_{xl} \hat{y}) & TE \\ \vec{e}_l^e(k_{xl}, k_{yl}) &= \frac{1}{\sqrt{k_{xl}^2 + k_{yl}^2}} (k_{xl} \hat{x} + k_{yl} \hat{y} + E_{zl} \hat{z}) & TM \end{aligned} \quad (2.17)$$

where b_l^h and b_l^e are respectively the complex amplitudes of the TE and TM components of the l harmonic. The components of the wave vector are:

$$k_{xl} = k_{xm} = \frac{2m\pi}{d_x} - k_0 \sin \theta_0 \cos \phi_0 \quad (2.18a)$$

$$k_{yl} = k_{yn} = \frac{2n\pi}{d_y} - k_0 \sin \theta_0 \sin \phi_0 \quad (2.18b)$$

$$k_{zl} = k_{zm,n} = \sqrt{k_0^2 - k_{xl}^2 - k_{yl}^2} \quad (2.18c)$$

Applying the hybrid method FEM-MA described in the previous section, the infinite array analysis is carried out considering two ports. A feeding port where a modal expansion is performed and a radiation port in which an expansion in Floquet harmonics is performed. The array is characterized by its generalized scattering matrix, $\mathbf{S}^{fl}$. It would be also possible to characterize it by its impedance or admittance matrices $\mathbf{Z}^{fl}$ or $\mathbf{Y}^{fl}$.

The generalized scattering matrix (GSM) is a multi-mode and multi-port matrix that relates the incident and reflected modes in the transmission feed line $\mathbf{v}$ and $\mathbf{w}$, with

FIGURE 2.2: Unit cell of the infinite array.

the incident and reflected Floquet modes in the free space, $\mathbf{a}$ and $\mathbf{b}$:

$$\begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{Fl} & \mathbf{S}_{12}^{Fl} \\ \mathbf{S}_{21}^{Fl} & \mathbf{S}_{22}^{Fl} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} = [\mathbf{S}^{Fl}] \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} \quad (2.19)$$

In this way, it is possible to obtain the active reflection coefficient, or any array characteristic, such as impedance, coupling, scattering and radiating characteristics. This formulation will be used in Chapter 3 to obtain a finite array analysis based on the infinite array approach.

2.3.1.2 Computation of the active element pattern

It is well known the variation of gain and input impedance of the antenna elements of a phase array when the beam is steered in a desired direction. The interaction between array elements depends on the applied excitation phase shifts. The classic pattern multiplication of the isolated element is not valid on those cases. The active, or embedded, element pattern (AEP) stands for the radiation pattern of an array of radiators when just one element is excited and the remaining elements are non-excited and terminated with a matched load. The mutual coupling between elements is inherently considered. In a finite array, each element has a different AEP as they see a different array environment. In infinite arrays the AEP is the same for every element and the pattern multiplication is valid using the AEP instead of the isolated element. Furthermore, the analysis of the infinite array is also valid for finite arrays as it will be shown in Chapter 3. In this subsection the computation of the AEP from (2.19) is detailed, a deeper explanation can be found in [33].

The radiated field by an array is computed as a superposition of the radiated field of each element of the array. Applying the infinite array approach every element is assumed to have the same radiation pattern, it is considered that every element of the array is surrounded by an infinite array environment and consequently they are equally affected by the mutual coupling. In this way, it is possible to express the radiated field as a product of the AEP and the array factor (AF):

$$\vec{E}_A(r, \theta, \phi) = \vec{E}_a(\theta, \phi) \frac{e^{-jkr}}{r} AF(\theta, \phi) \quad (2.20)$$

where AF is defined as:

$$AF(\theta, \phi) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{jk_{xm}x} e^{jk_{yn}y} \quad (2.21)$$

The AF can be expressed as a double summation of Dirac deltas and the spherical wave can be decomposed as the plane wave spectrum [33], the expression (2.20) is transformed into:

$$\vec{E}_A(r, \theta, \phi) = \vec{E}_a(\theta, \phi) \frac{2\pi}{d_x d_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{e^{jk_{zm,n}z}}{k_{zm,n}} e^{jk_{xm}x} e^{jk_{yn}y} \quad (2.22)$$

where

$$k_{x_l} = k_{x_m} = \frac{2m\pi}{d_x} - k_0 \sin \theta_0 \cos \phi_0 \quad (2.23a)$$

$$k_{y_l} = k_{y_n} = \frac{2n\pi}{d_y} - k_0 \sin \theta_0 \sin \phi_0 \quad (2.23b)$$

$$k_{z_l} = k_{z_{m,n}} = \sqrt{k_0^2 - k_{x_l}^2 - k_{y_l}^2} \quad (2.23c)$$

where if the m and n indices are substituted by l , ordering the Floquet harmonics by cut-off frequency. The AEP is included inside the summations and the following expression is obtained:

$$\vec{E}_A(r, \theta, \phi) = \frac{2\pi}{d_x d_y} \sum_{l=-\infty}^{\infty} \vec{E}_a(\theta_l, \phi_l) \frac{e^{jk_{z_l}z}}{k_{z_l}} e^{jk_{x_l}x} e^{jk_{y_l}y} \quad (2.24)$$

Comparing expressions (2.16) and (2.24), the following relation is stated for every Floquet harmonic:

$$\begin{aligned} [b_l^h \vec{e}_l^h(k_{x_l}, k_{y_l}) + b_l^e \vec{e}_l^e(k_{x_l}, k_{y_l})] e^{jk_{x_l}x} e^{jk_{y_l}y} e^{jk_{z_l}z} \\ = \frac{2\pi}{d_x d_y} \vec{E}_a(\theta_l, \phi_l) \frac{e^{jk_{z_l}z}}{k_{z_l}} e^{jk_{x_l}x} e^{jk_{y_l}y} \end{aligned} \quad (2.25)$$

It is possible to obtain the AEP for a desired direction (θ_0, ϕ_0) and $l = 1$ where, if there are not diffraction lobes, the only propagation mode is the fundamental harmonic ($m = 0, n = 0$) and the remaining are no homogeneous or evanescent modes. The radiated field is then:

$$\vec{E}_a(\theta_0, \phi_0) = \frac{d_x d_y}{2\pi} k_{z1} [b_1^h \vec{e}_1^h(k_{x1}, k_{y1}) + b_1^e \vec{e}_1^e(k_{x1}, k_{y1})] \quad (2.26)$$

b_1^h and b_1^e are respectively the complex amplitudes of the TE and TM components of the main harmonic. The AEP in spherical coordinates is obtained as:

$$\vec{E}_a(\theta, \phi) = \frac{d_x d_y}{2\pi} k_0 [\cos \theta b_1^h(\theta, \phi) \hat{\phi} + b_1^e(\theta, \phi) \hat{\theta}] \quad (2.27)$$

If N ports of excitation are considered in each unit cell, or N modes in each port, with incident voltage coefficients given by the column vector $\mathbf{v} = (v_1, v_2, \dots, v_N)^T$, the AEP may be expressed as a function of $\mathbf{v}$ by substituting in (2.27) the coefficients a_1^h and a_1^e computed from $\mathbf{a} = \mathbf{S}_{21}^{fl} \mathbf{v}$:

$$\begin{aligned} \vec{E}_a(\theta, \phi) &= \sum_{n=1}^N v_n \frac{d_x d_y}{2\pi} k_0 [\cos \theta S_{21}^{fl}(1, n) \hat{\phi} + S_{21}^{fl}(2, n) \hat{\theta}] = \\ &= \sum_{n=1}^N v_n \vec{e}_{an} = \vec{e}_a \mathbf{v} \end{aligned} \quad (2.28)$$

where:

$$\vec{e}_{an} = \frac{d_x d_y}{2\pi} k_0 [\cos \theta S_{21}^{fl}(1, n) \hat{\phi} + S_{21}^{fl}(2, n) \hat{\theta}]$$

and $\vec{e}_a$ is a row vector with elements $\vec{e}_{an}$.

2.3.2 Application of the hybrid method FEM-MA-GSM to the analysis of finite arrays via the spherical wave expansion.

In this section, the validated analysis method of coupled antennas arrays developed in [10], and integrated in the synthesis methodologies proposed in Chapters 6, 7 and 8, is briefly summarized. This method provides a rigorous characterization of antenna arrays whose elements can be described by means of spherical waves, such as planar arrays on an infinite ground plane. The analysis methodology consists of two processes. In the first one, each element of the array, considered as isolated, is characterized by a full-wave and modular procedure based on the three dimensional FEM, a modal analysis and a domain decomposition technique [31]. A modal expansion is used on the feeding ports and a spherical mode expansion on a hemisphere surface (spherical port) is used to characterize the radiating region. The analysis provides for each antenna of the array a

FIGURE 2.3: (a): GSM characterization of the individual elements of the array . (b): GSM characterization of the coupled array where the mutual coupling is rigorously take into account.

generalized scattering matrix (GSM) that relates the coefficients of the modal expansions in these ports:

$$\begin{bmatrix} \mathbf{\Gamma}_i & \mathbf{R}_i \\ \mathbf{T}_i & \mathbf{S}_i - \mathbf{I}_i \end{bmatrix} \begin{bmatrix} \mathbf{v}_i \\ \mathbf{a}_i \end{bmatrix} = \begin{bmatrix} \mathbf{w}_i \\ \mathbf{b}_i \end{bmatrix} \quad (2.29)$$

$\mathbf{v}_i$, $\mathbf{w}_i$, $\mathbf{a}_i$ and $\mathbf{b}_i$ are column vectors containing, respectively, the complex amplitudes of incident and reflected modes on the feeding ports, and the incoming and scattered spherical modes on the spherical ports as defined in [37]. The sub-matrices $\mathbf{\Gamma}_i$, $\mathbf{R}_i$, $\mathbf{T}_i$ and $\mathbf{S}_i$ are respectively the individual reflection, reception, transmission and scattering matrices of the antenna i , and $\mathbf{I}_i$ is the identity matrix.

In the following step of the analysis method, the overall GSM of the coupled finite array is analytically computed from the GSMs of the single elements by applying properties of rotation and translation of spherical waves. If an antenna array with N elements is considered, and if it is first assumed that the array elements are uncoupled, the whole array would be characterized by means of a GSM as follows:

$$\begin{bmatrix} \mathbf{\Gamma} & \mathbf{R} \\ \mathbf{T} & \mathbf{S} - \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} \quad (2.30)$$

where $\mathbf{\Gamma}$, $\mathbf{R}$, $\mathbf{T}$ and $\mathbf{S} - \mathbf{I}$ are diagonal block-matrix defined by:

$$\begin{aligned}\mathbf{\Gamma} &= \text{diag}(\mathbf{\Gamma}_i), & \mathbf{R} &= \text{diag}(\mathbf{R}_i) \\ \mathbf{T} &= \text{diag}(\mathbf{T}_i), & \mathbf{S} - \mathbf{I} &= \text{diag}(\mathbf{S}_i - \mathbf{I}_i)\end{aligned}\tag{2.31}$$

and:

$$\begin{aligned}\mathbf{v} &= \begin{pmatrix} v_1 \\ \vdots \\ v_N \end{pmatrix} & \mathbf{w} &= \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} \\ \mathbf{a} &= \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} & \mathbf{b} &= \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix}.\end{aligned}\tag{2.32}$$

In order to account for the mutual coupling, the incident field on each antenna of the array is now considered as the superposition of the field coming from outside the array and the field scattered by the remaining elements of the array. In this way, if each one of these fields is expanded into spherical modes, the complex amplitudes of the incoming modes in element i , $\mathbf{a}_i$, are obtained as:

$$\mathbf{a}_i = \mathbf{a}_{di} + \sum_{\substack{j=1 \\ j \neq i}}^N \mathbf{a}_i^j\tag{2.33}$$

where $\mathbf{a}_{di}$ and $\mathbf{a}_i^j$ are, respectively, column vectors containing incoming modes from outside the array in element i , and $\mathbf{a}_i^j$ scattered modes from the element j translated to the position of element i . Each one of the column vectors $\mathbf{a}_i^j$ is then related to the scattered spherical mode coefficients in the antenna j by means of the following relation, as shown in [37]:

$$\mathbf{a}_i^j = \mathbf{G}_{ij} \mathbf{b}_j\tag{2.34}$$

$\mathbf{G}_{ij}$ is the General Translation Matrix between antennas i and j obtained by using rotation and translation properties of spherical waves. This matrix includes in the most general case, rotations, axial translation and inverse rotations of spherical waves [10]. By substituting (2.34) in (2.33) for each radiating element, the incoming modes for all the antennas of the array are obtained:

$$\mathbf{a} = \mathbf{a}_d + \mathbf{G} \mathbf{b}\tag{2.35}$$

where $\mathbf{a}_d$ is given by:

$$\mathbf{a}_d = (\mathbf{a}_{d1}^T \dots \mathbf{a}_{di}^T \dots \mathbf{a}_{dN}^T)^T\tag{2.36}$$

FIGURE 2.4: (a): Isolated cavity-backed and probe-fed patch antenna analyzed with the present method. (b): Array of antennas with the mutual coupling accounted with translation and rotation of spherical waves.

and $\mathbf{G}$ is a square matrix obtained from submatrices $\mathbf{G}_{ij}$ as follows:

$$\mathbf{G} = \begin{bmatrix} 0 & \mathbf{G}_{12} & \cdots & \cdots & \mathbf{G}_{1N} \\ \mathbf{G}_{21} & 0 & \ddots & \mathbf{G}_{ij} & \cdots \\ \cdots & \ddots & \ddots & \ddots & \cdots \\ \cdots & \mathbf{G}_{ji} & \ddots & 0 & \mathbf{G}_{N-1N} \\ \mathbf{G}_{N1} & \cdots & \cdots & \mathbf{G}_{NN-1} & 0 \end{bmatrix}. \quad (2.37)$$

Finally, the overall GSM of the finite array, including mutual coupling between elements, and defined as:

$$\begin{bmatrix} \mathbf{\Gamma}_G & \mathbf{R}_G \\ \mathbf{T}_G & \mathbf{S}_G - \mathbf{I}_G \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{a}_d \end{bmatrix} = \begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} \quad (2.38)$$

is obtained substituting (2.35) in (2.30) after matrix operations. The sub-matrices $\mathbf{\Gamma}_G$, $\mathbf{R}_G$, $\mathbf{T}_G$ and $\mathbf{S}_G$, stand respectively for the reflection, reception, transmission and scattering matrices of the finite array and are given by:

$$\begin{aligned} \mathbf{\Gamma}_G &= \mathbf{\Gamma} + \mathbf{R}\mathbf{G}[\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}\mathbf{T} \\ \mathbf{R}_G &= \mathbf{R} + \mathbf{R}\mathbf{G}[\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}(\mathbf{S} - \mathbf{I}) \\ \mathbf{T}_G &= [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}\mathbf{T} \\ (\mathbf{S}_G - \mathbf{I}_G) &= [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}]^{-1}(\mathbf{S} - \mathbf{I}). \end{aligned} \quad (2.39)$$

Therefore, a closed-form GSM which rigorously describes the array as a circuit is obtained providing the impedance, coupling, radiating and scattering characteristics of the array for any arbitrary excitation. The method is valid for arrays with different

elements, placed in arbitrary positions and with complex geometries, since they are characterized using the FEM.

The radiated far-field of an array for a desired excitation $\mathbf{v}$ will be obtained as an expansion of spherical modes weighted by complex coefficients $\mathbf{b}$. In this way, considering a planar array of N antennas placed in the xy -plane, and M spherical modes for each antenna, and denoting by $\vec{e}_m(\theta, \phi)$ the electric field corresponding to the m -th spherical mode on each antenna, and by b_{im} the coefficient corresponding to this mode on the antenna i , the radiated field is expressed as:

$$\vec{E}(\hat{u}) = \sum_{i=1}^N \vec{e}(\hat{u}) \mathbf{b}_i e^{jk\hat{u} \cdot \vec{u}_i} \quad (2.40)$$

where $\vec{e}(\hat{u})$ is a row vector given by $\vec{e} = (\vec{e}_1, \vec{e}_2, \dots, \vec{e}_m)$, and $\mathbf{b}_i$ is a column vector defined by $\mathbf{b}_i = (b_{i1}, b_{i2}, \dots, b_{im})^T$, k is the wave number in the free space, $\hat{u} = (u_x, u_y, u_z)$ is the unitary vector in spherical coordinates, and $\vec{u}_i = x_i\hat{x} + y_i\hat{y}$ is the position vector of the antenna i , developing the summation in (2.40) yields:

$$\vec{E}(\hat{u}) = (\vec{e} e^{jk\hat{u} \cdot \mathbf{u}}) \mathbf{b} \quad (2.41)$$

where

$$(\vec{e} e^{jk\hat{u} \cdot \mathbf{u}}) = (\vec{e} e^{jk\hat{u} \cdot \vec{u}_1}, \vec{e} e^{jk\hat{u} \cdot \vec{u}_2}, \dots, \vec{e} e^{jk\hat{u} \cdot \vec{u}_N}) \quad (2.42)$$

and $\mathbf{b}$ is the column vector containing the vectors $\mathbf{b}_i$. Assuming that there is no incident field from the exterior, $\mathbf{a}_d = 0$ in (2.38), the coefficients $\mathbf{b}$ are obtained from the transmission matrix applying a desired excitation, $\mathbf{b} = \mathbf{T}_G \mathbf{v}$, where, without loss of generality, a single excitation mode in each feeding port is considered $\mathbf{v} = (v_1, v_2, \dots, v_N)^T$. Substituting $\mathbf{b}$ in (2.41) results in:

$$\vec{E}(\hat{u}) = (\vec{e}(\hat{u}) e^{jk\hat{u} \cdot \vec{u}}) \mathbf{T}_G \mathbf{v}. \quad (2.43)$$

This expression provides the field radiated by the coupled array in a rigorous way and will be used in the synthesis methods proposed in Chapters 6, 7 and 8.

2.4 Optimization methods

Analytical methods are very efficient and in some occasions they obtain optimal solutions. However, they are not very versatile, each method is only valid for the purpose it was designed for, variations of the problem are not generally valid and the inclusion of electromagnetic procedures is difficult. They have been used in the array synthesis

problem since the beginning of the forties for very different purposes, like to steer the beam with low sidelobes [11–13, 38–40], to obtain flat-top radiation patterns [14, 16, 17] or to perform a phase only synthesis [18].

Array synthesis methods have been handled with numerical optimizations for some decades. An optimization problem generally looks for minimizing, or maximizing, an objective function subject to a set of constraints. In the array synthesis problem, the objective function or the restrictions of the problem can be the sidelobe or the cross-polar levels, the ripple on the mainbeam, the dynamic range ratio, nulls, etc.

Optimization methods can be divided in two big groups depending on the nature of their solution: local and global. Local methods are usually very efficient but they may be trapped in local solution of a problem that can be far from the global solution of the problem. Examples of this kind of method are the simplex, newton or gradient methods [41]. Global methods are very expensive computationally, they are supposed to find the global solution of the problem but depending on the problem's size and behavior this cannot be assured. Examples of global methods applied to array synthesis problems are: based on genetic algorithms [42], particle swarm [43] or simulated annealing [44].

A new class of optimization methods have reached the attention of researchers: convex optimization [22]. A classic optimization problem can be described as:

$$\min f_0(\mathbf{x}) \quad (2.44a)$$

$$s.t. \quad f_i(\mathbf{x}) \leq b_i \quad i = 1 \cdots m. \quad (2.44b)$$

If the objective function, f_0 , and the restrictions applied to the objective function, f_i satisfy:

$$f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y) \quad (2.45)$$

Then the problem is linear. However if they satisfy the following equation:

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y) \quad (2.46)$$

Then the problem is convex, as it can be observed linear programming is a special case of convex programming. Before, there was a dividing line between linear and non-linear problems. This line has been moved between convex and non-convex problems because knowing if a problem is convex will tell its complexity and whether if an optimal solution can be found in polynomial time.

There is no point in solving the problem with global methods because they will obtain the same result as local methods, but orders of magnitude slower. The local methods cannot be trapped in local solutions as they do not exist in that problem.

Convex optimization also deals with non-convex problems. There are several ways of facing non-convex problems: transform it into a convex problem, with restrictions or relaxations, or solve it directly with a global or local method although an optimal solution will not be assured and the computation time will grow exponentially with problem's size.

Chapter 3

Finite array analysis from the infinite array approach

3.1 Introduction

The direct application of numerical methods to antenna array analysis based on the finite element method (FEM), method of moments or finite difference time domain method provides precision at the expense of large computational memory space and time requirements, so they are well suited for small-sized arrays. An alternative in case of large periodic arrays is the infinite array approach, where the analysis is reduced to the characterization of a periodic radiating element or unit cell by applying appropriate periodic boundary conditions. Mutual coupling between array elements is inherently considered although the edge effects are not taken into account. It provides however a reasonable approximation in particular cases, as in the boresight and near-side lobe regions for array excitation with large taper.

In addition to these techniques, numerous intermediate approaches in which different approximations are carried out have been proposed for analyzing large arrays. For example the method proposed in [2] and [3], and extended in [4], approximates the behaviour of finite arrays under arbitrary excitation through convolution or windowing techniques applied to infinite array results. The same principle has been applied in other works, [45] or [46], to account for edge effects in finite arrays. An alternative formulation for finite array analysis, equivalent to the convolution technique, based on the mutual coupling coefficients between the array elements and the active or embedded element pattern (AEP) of the associate infinite array, is described in [7]. It is suitable for a matrix characterization of the unit cell and it is straightforward for complex radiating elements.

FIGURE 3.1: Example of an ideal finite array of 3×3 radiating elements in an infinite environment of non-excited elements terminated by short circuits, open circuits or matched loads. The dark elements stand for the finite array while the grey ones represent the infinite array environment.

In this chapter, a finite array approach is obtained based on the infinite array results obtained in the previous chapter. Considering a finite array as an infinite array with a finite number of excited elements, the remaining elements are match-terminated, open-circuited or short-circuited. An example of this kind of array is represented in Fig. 3.1 for a finite array of 3×3 elements. These terminations are useful to approximate the actual array environment of the real array.

3.2 Computation of the mutual coupling of the finite array

It is well known that impedance and admittance mutual coupling between elements in an infinite array can be derived from a modal-based Floquet analysis [1], [7].

As shown in the previous chapter, applying the hybrid method FEM-MA, the element of the periodic infinite array has been characterized by a scattering matrix as:

$$\begin{bmatrix} \mathbf{w} \\ \mathbf{b} \end{bmatrix} = \begin{bmatrix} S_{11}^{Fl} & S_{12}^{Fl} \\ S_{21}^{Fl} & S_{22}^{Fl} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} = [S^{Fl}] \begin{bmatrix} \mathbf{v} \\ \mathbf{a} \end{bmatrix} \quad (3.1)$$

in terms of the coefficients of the incident and reflected modes in the transmission feed line, $\mathbf{v}$ and $\mathbf{w}$ respectively, and the coefficients of the incident and reflected Floquet's harmonics in the half-space, $\mathbf{a}$ and $\mathbf{b}$ respectively. In the following:

$$S^{Fl} = S_{11}^{Fl}(1, 1) \quad (3.2)$$

where $S_{11}^{Fl}(1,1)$ makes reference to the active reflection coefficient (ARC) for the excitation mode. The ARC is obtained when all the antennas are excited with uniform amplitude and a linear phase.

The active impedance and active admittance can be obtained via the active reflection coefficient as:

$$Z^{fl} = Z_0 \frac{1 + S^{Fl}}{1 - S^{Fl}} \quad (3.3)$$

$$Y^{fl} = Y_0 \frac{1 - S^{Fl}}{1 + S^{Fl}} \quad (3.4)$$

where Z_0 and Y_0 are the characteristic impedance and admittance of the transmission feed line.

Without loss of generality, a linear array with one mode of excitation per element is considered at first to obtain the mutual coupling of a finite array in an infinite array environment. The expressions are then generalized for planar arrays with the possibility of analyze elements with multi-mode or multi-port excitations. An infinite linear array is considered placed in the x axis with an inter-element distance of d_x . A Floquet excitation is assumed with uniform amplitude and linear phase being the excitation for the n element in terms of the input voltage:

$$V_n = V_0 e^{-jn\psi} \quad (3.5)$$

with $\psi = k_0 d_x \sin \theta$.

The input current for the element placed at $x_n = 0$ is:

$$I_0(\psi) = \sum_{n=-\infty}^{+\infty} Y_{0n} V_n \quad (3.6)$$

where Y_{0n} is the mutual admittance between elements located at $x = 0$ and $x = nd_x$. Taking into account (3.5):

$$I_0(\psi) = \sum_{n=-\infty}^{+\infty} Y_{0n} V_0 e^{-jn\psi} \quad (3.7)$$

The input admittance of the element in the origin of coordinates:

$$Y_0(\psi) = \frac{I_0}{V_0} = \sum_{n=-\infty}^{+\infty} Y_{0n} e^{-jn\psi} \quad (3.8)$$

For a Floquet excitation, the input admittance is equal to the active admittance, which is also called the Floquet admittance and it is the same for every element of the

FIGURE 3.2: Planar array of $L(M \times N)$ elements. In each element there is an incident (v) and a reflected (w) power.

array:

$$Y^{Fl}(\psi) = Y_0(\psi) = \sum_{n=-\infty}^{+\infty} Y_{0n} e^{-jn\psi} \quad (3.9)$$

In this way, the expression (3.9) is the Fourier series expansion of the Floquet admittance where the Fourier coefficients are equal to the mutual admittances. These coefficients can be obtained as:

$$Y_{0n} = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y^{Fl} e^{jn\psi} d\psi \quad (3.10)$$

The mutual coupling between two arbitrary elements m and n placed in $x_m = md_x$ and $x_n = nd_x$ is:

$$Y_{mn} = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y^{Fl} e^{-j(m-n)\psi} d\psi = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y^{Fl} e^{-jp\psi} d\psi \quad (3.11)$$

where the term $p = (m - n)$, stands for the distance between both elements.

In the case of a planar array as shown in Fig. 3.2, the mutual coupling, of two elements separated by a distance $p d_x$ and $q d_y$, as shown in Fig. 3.3, where p and q are integer numbers, is computed as:

$$Y_{pq} = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} Y_{11}^{fl} e^{j(p\psi_x + q\psi_y)} d\psi_x d\psi_y. \quad (3.12)$$

where $\psi_x = k_0 d_x \sin \theta \cos \phi$ and $\psi_y = k_0 d_y \cos \theta \cos \phi$.

Considering the case of N ports of excitation in each array element, and denoting by $Y_{pq}(i, j)$ the mutual admittance between two ports, i and j , situated in two elements of the array separated by a distance $p d_x$ and $q d_y$, the elements $Y_{11}^{fl}(i, j)$ of the $N \times N$ active admittance matrix $\mathbf{Y}_{11}^{fl}$ of the infinite array can be expanded as a Fourier series

FIGURE 3.3: Inter-element distances in a planar array with equispaced elements.

in ψ_x and ψ_y , with coefficients $Y_{pq}(i, j)$. In this way the mutual admittance is obtained as:

$$Y_{pq}(i, j) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} Y_{11}^{fl}(i, j) e^{j(p\psi_x + q\psi_y)} d\psi_x d\psi_y. \quad (3.13)$$

The double integral in (3.13) is computed numerically from a discrete number of $Y_{11}^{fl}(i, j)$ sampled in the space. Following an analogous procedure, the following expressions could be obtained for the mutual coupling in terms of the impedance and scattering parameters:

$$Z_{pq}(i, j) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} Z_{11}^{fl}(i, j) e^{j(p\psi_x + q\psi_y)} d\psi_x d\psi_y. \quad (3.14)$$

$$S_{pq}(i, j) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} S_{11}^{fl}(i, j) e^{j(p\psi_x + q\psi_y)} d\psi_x d\psi_y. \quad (3.15)$$

3.3 Radiation pattern of the finite array

The infinite array approach applied to finite array analysis is a well established technique. The mutual admittance, $\mathbf{Y}$, and impedance, $\mathbf{Z}$, matrices of a finite planar array may be approximated from coefficients $Y_{pq}(i, j)$ and $Z_{pq}(i, j)$ obtained in the previous section, neglecting edge effects. These matrices would be exact for an ideal finite planar array surrounded by an infinite number of non-excited elements short circuited or open circuited, respectively. In the same way the radiated far-field of these ideal finite planar arrays with arbitrary excitations may be computed from the results of the infinite array approach (full details can be found in [7, Chapter 4]). An example of this antenna array is shown in Fig. 3.1. In this section, the basics for obtaining the radiated field in case of

multiport and/or multimode excitation are outlined, since they will be used in the optimization formulation proposed in Chapters 4 and 5. Firstly, a finite array surrounded by an infinite number of non-excited and match-terminated elements is considered. It is well known that in this case the radiated field with respect to incident voltages, for an arbitrary excitation, is determined exactly from the AEP using superposition. For an array made up of L elements with N feeding ports in each one, it is determined as:

$$\vec{E}_A = \sum_{l=1}^L \vec{E}_{al} e^{j(u_{xl}+u_{yl})} \quad (3.16)$$

where $u_{xl} = pk_0 d_x \sin \theta \cos \phi$ and $u_{yl} = qk_0 d_y \sin \theta \sin \phi$. p and q are the indexes to obtain the position of each element, and l is used to reduce every couple (p, q) to a single index. $\vec{E}_{al}$ is the AEP of element l obtained from (2.28), that substituted in (3.16) yields:

$$\vec{E}_A = \sum_{l=1}^L \vec{e}_a \mathbf{v}_l e^{j(u_{xl}+u_{yl})} \quad (3.17)$$

where $\mathbf{v}_l$ contains incident voltages applied to the ports of element l . Developing the expression in (3.17) results in:

$$\vec{E}_A = (\vec{q}_1, \vec{q}_2, \dots, \vec{q}_L)(\mathbf{v}_1^T, \mathbf{v}_2^T, \dots, \mathbf{v}_L^T)^T \quad (3.18)$$

where $\vec{q}_l$ is a row vector defined as:

$$\vec{q}_l = \vec{e}_a e^{j(u_{xl}+u_{yl})}. \quad (3.19)$$

Expression (3.18) may be written in a more compact form as $\vec{E}_A = \vec{q} \mathbf{v}$ where $\vec{q}$ and $\mathbf{v}$ are vectors of dimension $N \cdot L$ containing $\vec{q}_l$ and $\mathbf{v}_l$ elements respectively.

Next, the expression of the radiated field of a finite array in an infinite array environment of short-circuited elements $\mathbf{E}_A^{sc}$ will be derived. For this purpose the radiated field of an infinite array under Floquet excitation, $\mathbf{E}_A^{fl}$, is computed from the AEP (2.28) as follows:

$$\vec{E}_A^{fl} = \sum_{l=-\infty}^{\infty} \vec{e}_a \mathbf{v}_l^{fl} e^{j(u_{xl}+u_{yl})}. \quad (3.20)$$

Incident voltages in cell l , $\mathbf{v}_l^{fl}$, are related to the input voltages, $\boldsymbol{\nu}_l^{fl}$, in the form:

$$\mathbf{v}_l^{fl} = \frac{1}{2} \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y}_{11}^{fl} \right] \boldsymbol{\nu}_l^{fl} \quad (3.21)$$

where $\mathbf{Z}_0$ is a diagonal matrix whose elements are the internal impedance of the sources,

and $\mathbf{I}$ is the identity matrix. $\vec{E}_A^{fl}$ is obtained in terms of the input voltages by substituting (3.21) in (3.20):

$$\vec{E}_A^{fl} = \sum_{l=-\infty}^{\infty} \frac{1}{2} \vec{e}_a \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y}_{11}^{fl} \right] \boldsymbol{\nu}_l^{fl} e^{j(u_{xl} + u_{yl})}. \quad (3.22)$$

In the same way as $\vec{E}_A^{fl}$ is computed from the AEP in terms of the incident voltages in (3.20), the following term can be identified in (3.22):

$$\vec{E}_a^{sc} = \frac{1}{2} \vec{e}_a \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y}_{11}^{fl} \right] \boldsymbol{\nu}, \quad (3.23)$$

from which $\vec{E}_A^{fl}$ is obtained in terms of the input voltages. $\vec{E}_a^{sc}$ represents the radiation pattern of an element in an infinite array environment of short-circuited elements. The radiated field of a finite array of L elements in the same environment is obtained applying superposition:

$$\vec{E}_A^{sc} = \sum_{l=1}^L \frac{1}{2} \vec{e}_a \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y}_{11}^{fl} \right] \boldsymbol{\nu}_l e^{j(u_{xl} + u_{yl})}. \quad (3.24)$$

Developing this expression results in:

$$\vec{E}_A^{sc} = (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_L) (\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T, \dots, \boldsymbol{\nu}_L^T)^T \quad (3.25)$$

where

$$\vec{r}_l = \frac{1}{2} \vec{e}_a \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y}_{11}^{fl} \right] e^{j(u_{xl} + u_{yl})}. \quad (3.26)$$

$\vec{E}_A^{sc}$ may be expressed in terms of the incident voltages by substituting in (3.25) the known relationship:

$$(\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T, \dots, \boldsymbol{\nu}_L^T)^T = 2 \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y} \right]^{-1} \mathbf{v} \quad (3.27)$$

yielding:

$$\vec{E}_A^{sc} = (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_L) \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y} \right]^{-1} \mathbf{v} \quad (3.28)$$

Following a dual procedure to that detailed above, an expression of the radiated field of a finite array in an infinite array environment of open-circuited elements, can also be derived from the infinite array approach:

$$\vec{E}_A^{fl} = \sum_{l=-\infty}^{\infty} \vec{e}_a \mathbf{v}_l^{fl} e^{j(u_{xl} + u_{yl})}. \quad (3.29)$$

Incident voltages in cell l , v_l^{fl} , are related to input currents, i_l^{fl} , as:

$$v_l^{fl} = \frac{1}{2} [Z_{11}^{fl} + Z_0] i_l^{fl}. \quad (3.30)$$

$\vec{E}_A^{fl}$ is obtained substituting (3.30) in (3.29):

$$\vec{E}_A^{fl} = \sum_{l=-\infty}^{\infty} \frac{1}{2} \vec{e}_a [Z_{11}^{fl} + Z_0] i_l^{fl} e^{j(u_{xl} + u_{yl})}. \quad (3.31)$$

The following term can be identified in (3.31):

$$\vec{E}_a^{oc} = \frac{1}{2} \vec{e}_a [Z_{11}^{fl} + Z_0] i, \quad (3.32)$$

from which $\vec{E}_A^{fl}$ is obtained in terms of the input currents. $\vec{E}_a^{oc}$ represents the radiation pattern of an element in an infinite array environment of open circuited elements. The radiated field of a finite array of L elements is obtained applying superposition:

$$\vec{E}_A^{oc} = \sum_{l=1}^L \frac{1}{2} \vec{e}_a [Z_{11}^{fl} + Z_0] i_l e^{j(u_{xl} + u_{yl})}. \quad (3.33)$$

Developing (3.33) yields:

$$\vec{E}_A^{oc} = (\vec{s}_1, \vec{s}_2, \dots, \vec{s}_L) [\mathbf{Z} + \mathbf{Z}_0]^{-1} \mathbf{v} \quad (3.34)$$

where

$$\vec{s}_l = \frac{1}{2} \vec{e}_a [Z_{11}^{fl} + Z_0] e^{j(u_{xl} + u_{yl})} \quad (3.35)$$

The radiation pattern computed from (3.18), (3.28) and (3.34) may be particularized for one port/mode of excitation, reported in [7], in each radiating element ($N = 1$) as:

$$\vec{E}_A^{mt} = \vec{E}_a \vec{p} \mathbf{v} \quad (3.36)$$

$$\vec{E}_A^{sc} = \vec{E}_a (\mathbf{Y} + \mathbf{Y}^{fl}) \vec{p} (\mathbf{Y} + \mathbf{Y}_0)^{-1} \mathbf{v}. \quad (3.37)$$

$$\vec{E}_A^{oc} = \vec{E}_a (\mathbf{Z} + \mathbf{Z}^{fl}) \vec{p} (\mathbf{Z} + \mathbf{Z}_0)^{-1} \mathbf{v} \quad (3.38)$$

where $\vec{p}$ stands for the exponential functions:

$$\vec{p} = \sum_{l=1}^L e^{j(u_{xl} + u_{yl})} \quad (3.39)$$

FIGURE 3.4: AEP cuts of one element placed in a corner of a 9×9 -array of open-ended square waveguides, considering three different infinite array environment of non-excited elements: open circuit, short circuit, and match termination. The AEP obtained with the full-wave method presented in Chapter 2 for arrays on an infinite metallic plane is also represented.

3.4 Analysis examples

The analysis method is firstly tested with a 9×9 -array of open-ended square waveguides on a ground plane, with a 4 cm side length, filled with a dielectric of $\epsilon_r = 4$, and with an inter-element spacing of $0.5\lambda_0$ in x and y directions, at the resonant frequency of 2.14 GHz. The AEP of one element placed in a corner of the array considering the three different finite array approaches described in this chapter is shown in Fig. 3.4 for E and H planes. They are computed from (3.18), (3.28) and (3.34), particularized for one excitation port ($N = 1$), and only exciting the considered array element. The results show the expected asymmetry due to the edge effects for the case of open-circuited and short-circuited non-excited elements. The AEP obtained from the full wave methodology in [10], and briefly detailed in Chapter 2, based on the 3D-FEM and spherical mode expansion is also represented. This method rigorously simulates arrays with elements placed on an infinite metallic plane. As observed, this pattern agrees very well with that computed from (3.28), modelling the infinite metallic plane with shorted waveguide apertures which are minimum scatterers with respect to the admittance parameters.

In the following example, a 9×9 -array of open-ended square waveguides on a ground plane, with an aperture length of $0.6\lambda_0$ and an inter-element distance of $0.7\lambda_0$. The radiation pattern in E and H-planes for the three different array environments is represented

in Fig. 3.5. The same example is carried out in [7] showing good agreement. As the inter-element distance is larger than $0.5\lambda_0$ a blind spot is produced in the E-plane, the angle where it occurs can be computed as [7]:

$$\theta_c \approx \arcsin\left(\frac{2\pi}{ak_0} - 1\right) = 25.4^\circ \quad (3.40)$$

The proposed array analysis will be applicable to planar arrays with a double periodic grid (rectangular, or with a certain grid angle). It is not suitable for non-regular arrays, although planar arrays with arbitrary contours may be considered. The accuracy of the proposed approximation will depend on the array element and on the real array environment. If the array elements considered are minimum scattering antennas [47, 48], with respect to the admittance or impedance parameters, they do not scatter when their local ports are terminated by short- or open-circuited elements respectively [6], and the corresponding finite array model will be a good approximation of the real array. For example, open-ended waveguide arrays or slot arrays on an infinite ground plane are minimum scatterers with respect to the admittance parameters and the ground plane may be modeled by short-circuited non-excited elements; dipoles are minimum scatterers with respect to the impedance parameters and the non-excited elements open circuited behave as if they were not present. These approaches are also found to be suitable for describing mutual coupling and radiating performance of more complex radiating elements which are not good minimum scatters, such as blade antennas [49] or probe-fed microstrip antennas [47, 50]. On the other hand, expression (3.18) applied to real finite arrays assumes minimum scattering with respect to S parameters and, although it does not fit into real situations, it may be a good approximation for moderate and large arrays [50].

(a)

(b)

FIGURE 3.5: E-plane (a) and (b) H-plane radiation pattern of the co-polar component of the 9×9 aperture array.

Chapter 4

Multi-objective optimization of coupled arrays using the infinite array approach and convex programming

4.1 Introduction

The antenna array synthesis problem has been a significant area of research for several decades. The problem can be described as finding the parameters of the array of radiators, usually the excitation weights, which provide a radiation pattern as close as possible to a desired response. There are a huge amount of synthesis methods that can be classified in three categories: analytical, [12, 51], local [52], or global optimization methods [43, 53]. Analytical methods may be really efficient for a particular objective but they are not very versatile. Global methods are supposed to find the global minimum, but their convergence is generally slow and the complexity grows exponentially with the size of the problem, becoming unfeasible for large arrays. Local methods usually yield local minima that can be far from the global solution of the problem. However, if the optimization problem is convex, local methods will find the global solution as well as global methods but orders of magnitude faster. For this reason, if the problem is convex there is no point in using global methods to yield the global solution. Convex programming [22] has received much attention recently. Formerly, there was a dividing line, representing the difficulty of a problem, between linear and non-linear optimization. Now, the dividing line has been established between convex and non-convex problems because knowing whether a problem is convex or not will reveal the complexity of the problem.

A non-convex problem is np-hard to solve and even if a solution is found there is no proof that it is actually the global solution to the problem. Large problems are usually intractable and problems with a few tens of variables can be extremely challenging to solve.

Many optimization problems can be expressed as convex. Otherwise, they can be dealt with in different ways: additional restrictions can be added to the original problem as certain symmetries, or in some cases the problem can be relaxed in order to make it convex. If additional restrictions are imposed, the new problem will become a different one or a restricted version of the original. This option can lead to a non-optimal solution or to a solution that is only valid for specific cases. With the second choice, a relaxation of the problem can be obtained to yield the global solution whenever the relaxation is really tight [54]. In other cases the solution obtained from the relaxation can be used as an initiation point for a local solver, or to narrow the space of solutions in a branch and bound algorithm. Some specifications related to array synthesis are not convex, such as the shaped beam or phase only synthesis.

One of the most common application is the optimization of the complex excitation weights of periodic arrays looking for steering the beam towards a desired direction and minimize the maximum value of the sidelobe level, with the possibility of imposing nulls in the radiation pattern or fixing a maximum value for the cross-polar component. Other array configurations and radiation patterns are also possible. This synthesis problem, for arrays with fixed positions has been demonstrated to be a convex problem. A lot of convex optimization techniques applied to radiation pattern synthesis problems have been presented in recent years [23, 55, 56]. Some other methods solve this problem using genetic algorithms [57, 58] or using some other heuristic methods [59]. However, when the problem is convex there is no point in using global methods as they will obtain the same result but orders of magnitude slower.

On the other hand, the optimization and synthesis of antenna array excitations usually work with isotropic sources or analytical element patterns, and neither inter-element coupling between array elements nor real radiated fields are considered. The reason is the difficulty associated with including electromagnetic analysis methods into optimization procedures when considering these aspects. Some works consider the actual radiation pattern of the array elements. In [60] the AEP, obtained with the method proposed in [61] is also used in a synthesis method. Some other methods use the measured or calculated element-pattern data as in [62].

In this chapter, an approximate antenna array analysis procedure accounting for real array elements and mutual coupling effects between them, is integrated with a pattern synthesis technique formulated as a convex programming problem that can be

transformed into a linear programming problem. It provides multi-beam patterns with constraints on beamwidths, peak sidelobe and cross-polar levels in prescribed regions, as well as fixed null pointing directions. The analysis method, detailed in Chapters 2 and 3, is based on the infinite array approach through Floquet modal analysis, providing a rigorous characterization of finite arrays with arbitrary excitations in an infinite array environment of non-excited radiating elements with different load conditions [7]. A matrix formulation combining these approaches and a full wave FEM-based generalized scattering matrix (GSM) procedure for infinite arrays proposed in [32] or [63] is developed here. The proposed methodology will provide synthesized array patterns that will be good approximations when non-excited elements with a given load condition are minimum scatterers [6]. It will be also a reasonable approximation for other radiating elements that are not good minimum scatterers, such as microstrip antennas [47, 50], or blade antennas [49]. In other cases the different approaches considered for finite arrays provide an estimation of the optimized radiation pattern for real finite arrays.

4.2 Proposed optimization method

The approximate analysis of real and coupled antenna arrays described in Chapter 3 is integrated in this section with a pattern synthesis procedure. The general objective is to synthesize the excitation weights of the antenna array providing a multi-beam pattern with specified direction and beamwidth mainlobes, optimized sidelobe and cross-polar levels in different radiating regions, as well as prescribed null pointing directions.

It is observed that the expressions of the radiated field of a finite array in an infinite array environment for the three different load conditions of non-excited elements considered in Chapter 2:

$$\vec{E}_A = (\vec{q}_1, \vec{q}_2, \dots, \vec{q}_L)(\vec{v}_1^T, \vec{v}_2^T, \dots, \vec{v}_L^T)^T \quad (4.1)$$

$$\vec{E}_A^{sc} = (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_L) \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y} \right]^{-1} \mathbf{v} \quad (4.2)$$

$$\vec{E}_A^{oc} = (\vec{s}_1, \vec{s}_2, \dots, \vec{s}_L) \left[\mathbf{Z} + \mathbf{Z}_0 \right]^{-1} \mathbf{v} \quad (4.3)$$

may be expressed in a similar way as: $\vec{E}_A = \vec{t} \mathbf{v}$ where $\vec{t}$ represents the resulting row vector multiplying the excitation coefficient vector $\mathbf{v}$ in each case:

$$\vec{t} = \begin{cases} (\vec{q}_1, \vec{q}_2, \dots, \vec{q}_L) \\ (\vec{r}_1, \vec{r}_2, \dots, \vec{r}_L) \left[\mathbf{I} + \mathbf{Z}_0 \mathbf{Y} \right]^{-1} \\ (\vec{s}_1, \vec{s}_2, \dots, \vec{s}_L) \left[\mathbf{Z} + \mathbf{Z}_0 \right]^{-1} \end{cases}, \quad (4.4)$$

corresponding to match, short and open terminations respectively. In this way, the optimization formulation will be the same for the three cases. The co- and cross-polar components of the radiated field are obtained directly by taking the corresponding components in $\vec{t}$, $\vec{t}^{cp}$ and $\vec{t}^{xp}$, obtaining respectively:

$$\vec{E}_A^{cp} = \vec{t}^{cp} \mathbf{v} \quad (4.5a)$$

$$\vec{E}_A^{xp} = \vec{t}^{xp} \mathbf{v} \quad (4.5b)$$

$\vec{t}^{cp}$ and $\vec{t}^{xp}$ are in turn obtained by taking the co- and cross-polar components of $\vec{e}_a$ in (3.19), (3.26) or (3.35).

4.2.1 Convex optimization problem

One way of establishing the desired optimization problem is to minimize simultaneously the maximum absolute value of the co-polar component in the sidelobe region, R_{SL} , and of the cross-polar component in another specified space region, R_{XP} , while maintaining a constant value of the co-polar component in the direction of the main beam (or main beams):

$$\min \max \{ |\vec{t}^{cp} \mathbf{v}|, |\vec{t}^{xp} \mathbf{v}| \} \quad (4.6a)$$

$$s.t \quad |\vec{t}^{cp} \mathbf{v}| = c_k \quad (\theta_k, \phi_k), k = 1 \cdots K \quad (4.6b)$$

This minimax problem may be redefined as a constrained optimization problem, stated in (4.7), by minimizing with $\mathbf{v}$ the sum of two real positive quantities: α which fixes the maximum co-polar field level in R_{SL} ; and β , establishing the maximum cross-polar field level in R_{XP} ; setting complex constant values, c_k , of the co-polar field level in specified main beam directions. Additional constraints may be added to impose field intensity to zero in a region R_{NU} or at discrete pointing directions.

$$\min \alpha + \beta \quad (4.7a)$$

$$s.t \quad |\vec{t}^{cp}(\theta, \phi) \mathbf{v}| \leq \alpha \quad (\theta, \phi) \in R_{SL} \quad (4.7b)$$

$$|\vec{t}^{xp}(\theta, \phi) \mathbf{v}| \leq \beta, \quad (\theta, \phi) \in R_{XP} \quad (4.7c)$$

$$\vec{t}^{cp}(\theta_k, \phi_k) \mathbf{v} = c_k, k = 1 \cdots K \quad (4.7d)$$

$$|\vec{t}^{cp}(\theta, \phi) \mathbf{v}| = 0, \quad (\theta, \phi) \in R_{NU} \quad (4.7e)$$

Other optimization problems may be formulated, depending on the desired specifications, minimizing only one of the objective functions, crosspolar component or SLL,

and introducing the other as a restriction by imposing a prescribed maximum value; or imposing different SLL regions with different maximum values.

The constraints in (4.7) are then discretized for the numerical implementation of the antenna array synthesis problem. For this purpose, the regions defined in (4.7) are sampled in (θ, ϕ) yielding:

$$\text{Min} \quad \alpha + \beta \quad (4.8a)$$

$$\text{s.t.} \quad |\vec{t}^{cp}(\theta_i, \phi_i)\mathbf{v}| \leq \alpha, \quad i = 1 \cdots I \quad (4.8b)$$

$$|\vec{t}^{xp}(\theta_j, \phi_j)\mathbf{v}| \leq \beta, \quad j = 1 \cdots J \quad (4.8c)$$

$$\vec{t}^{cp}(\theta_k, \phi_k)\mathbf{v} = c_k, \quad k = 1 \cdots K \quad (4.8d)$$

$$|\vec{t}^{cp}(\theta_q, \phi_q)\mathbf{v}| = 0, \quad q = 1 \cdots Q \quad (4.8e)$$

where I , J and Q are the number of pointing directions in which each corresponding space domain is discretized. The grids in (4.8) have to be dense enough to cover all possible sidelobe and cross-polar local maxima. Coefficients $\vec{t}(\theta_m, \phi_m)$ are computed from (4.4) after a Floquet modal analysis of the considered array, using the full-wave analysis methods in [32] or [63]. The excitation weights in (4.8), $\mathbf{v}$, are the optimization variables, α and β are the dependent variables, and complex values c_k fixing the peaks of the main beams are predefined constants. $\mathbf{v}$, $\vec{t}^{cp}$ and $\vec{t}^{xp}$ are $L \cdot N$ -dimensional complex vectors, where L is the number of elements and N is the number of excitation ports in each one. The number of constraints is established by the sum of the number of pointing directions I , J , K and Q in which each region has been discretized.

The resulting optimization problem (4.8) is non-linear due to the norm in the constraints, and is stated with a complex-valued formulation. It can be proven that the upper bound constraints on sidelobes and cross-polar components are convex [23]. Therefore, it may be optimally and efficiently solved using well-established convex programming techniques [22]. Interior-point method-based open solvers which deal with complex-valued variables, constraints and objective functions, e.g. SDPT3 [64] integrated into the CVX software package [65], can be used to solve it.

4.2.2 Real-valued linear problem.

The optimization problem in (4.8) can be converted into an equivalent standard linear programming problem stated with a real-valued formulation. By taking the real and imaginary parts of $\mathbf{v}$, $\vec{t}^{cp}$ and $\vec{t}^{xp}$ in (4.5a) and (4.5b), the co- and cross-polar components

of the radiated field may be expressed as:

$$E_A^\chi(\theta_i, \phi_i) = \zeta_r^\chi(\theta_i, \phi_i)\mathbf{x} + j\zeta_i^\chi(\theta_i, \phi_i)\mathbf{x} \quad (4.9)$$

where χ stands for *cp* or *xp*, and:

$$\zeta_r^\chi(\theta_i, \phi_i) = \left(\Re\{\vec{\mathbf{t}}^\chi(\theta_i, \phi_i)\}, -\Im\{\vec{\mathbf{t}}^\chi(\theta_i, \phi_i)\} \right) \quad (4.10)$$

$$\zeta_i^\chi(\theta_i, \phi_i) = \left(\Im\{\vec{\mathbf{t}}^\chi(\theta_i, \phi_i)\}, \Re\{\vec{\mathbf{t}}^\chi(\theta_i, \phi_i)\} \right) \quad (4.11)$$

$$\mathbf{x} = \begin{bmatrix} \Re\{\mathbf{v}\} \\ \Im\{\mathbf{v}\} \end{bmatrix} \quad (4.12)$$

The real vector $\mathbf{x}$ now contains the optimization variables. By exploiting the triangle inequality principle in (4.9), $|E_A^\chi| \leq |\zeta_r^\chi\mathbf{x}| + |\zeta_i^\chi\mathbf{x}|$, it is possible to minimize separately $|\zeta_r^\chi\mathbf{x}|$ and $|\zeta_i^\chi\mathbf{x}|$, the norm of the real and imaginary parts of E_A^χ , assuring that $|E_A^\chi|$ will be less than the addition of $|\zeta_r^\chi\mathbf{x}|$ and $|\zeta_i^\chi\mathbf{x}|$. In this way, the optimization problem in (4.8) may be reformulated, as stated in (4.13), by minimizing with $\mathbf{x}$ the sum of four real positive quantities, α_1 , α_2 , β_1 and β_2 , which fix the maximum of the real and imaginary parts of the co- and cross-polar components of the radiated field, while setting constant values, $\Re(c_k)$ and $\Im(c_k)$, of the real and imaginary parts of the co-polar field component, respectively, in the main beam directions; and as well as setting to zero both real and imaginary parts of $\vec{E}_A^{cp}$ in null directions:

$$\text{Min} \quad \alpha_1 + \alpha_2 + \beta_1 + \beta_2 \quad (4.13a)$$

$$\text{s.t} \quad |\zeta_r^{cp}(\theta_i, \phi_i)\mathbf{x}| \leq \alpha_1, \quad i = 1 \cdots I \quad (4.13b)$$

$$|\zeta_i^{cp}(\theta_i, \phi_i)\mathbf{x}| \leq \alpha_2, \quad i = 1 \cdots I \quad (4.13c)$$

$$|\zeta_r^{xp}(\theta_j, \phi_j)\mathbf{x}| \leq \beta_1, \quad j = 1 \cdots J \quad (4.13d)$$

$$|\zeta_i^{xp}(\theta_j, \phi_j)\mathbf{x}| \leq \beta_2, \quad j = 1 \cdots J \quad (4.13e)$$

$$\zeta_r^{cp}(\theta_k, \phi_k)\mathbf{x} = \Re(c_k), \quad k = 1 \cdots K \quad (4.13f)$$

$$\zeta_i^{cp}(\theta_k, \phi_k)\mathbf{x} = \Im(c_k), \quad k = 1 \cdots K \quad (4.13g)$$

$$\zeta_r^{cp}(\theta_q, \phi_q)\mathbf{x} = 0, \quad q = 1 \cdots Q \quad (4.13h)$$

$$\zeta_i^{cp}(\theta_q, \phi_q)\mathbf{x} = 0, \quad q = 1 \cdots Q \quad (4.13i)$$

The resulting optimization problem now deals with real-valued coefficients and variables, and the norm in (4.13 b-e) can then be cleared by splitting each inequality in two. The number of variables will be twice the number of complex unknowns of the

problem in (4.8). In the same way, the number of restrictions is also duplicated. The problem in (4.13) can now be solved with any standard linear programming procedure, such as LCONF included in the commercially available numerical software IMSL, or an algorithm in the family of interior point methods, such as SDPT3.

$$\text{Min} \quad \mathbf{f}^T \mathbf{y} \quad (4.14a)$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{y} \leq \mathbf{b} \quad (4.14b)$$

$$\mathbf{C}\mathbf{y} = \mathbf{d} \quad (4.14c)$$

being:

$$\mathbf{y} = \begin{bmatrix} \mathbf{x} \\ \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \beta_2 \end{bmatrix} \quad (4.15a)$$

$$\mathbf{f} = \begin{bmatrix} \mathbf{0} \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (4.15b)$$

$$A = \begin{bmatrix} \zeta_{Ir} & -1 & 0 & 0 & 0 \\ -\zeta_{Ir} & -1 & 0 & 0 & 0 \\ \zeta_{Ii} & 0 & -1 & 0 & 0 \\ -\zeta_{Ii} & 0 & -1 & 0 & 0 \\ \chi_{Qr} & 0 & 0 & -1 & 0 \\ -\chi_{Qr} & 0 & 0 & -1 & 0 \\ \chi_{Qi} & 0 & 0 & 0 & -1 \\ -\chi_{Qi} & 0 & 0 & 0 & -1 \end{bmatrix} \quad (4.16a)$$

$$b = \begin{bmatrix} 0 \end{bmatrix} \quad (4.16b)$$

$$C = \begin{bmatrix} \zeta_{Jr} & 0 & 0 & 0 & 0 \\ \zeta_{Ji} & 0 & 0 & 0 & 0 \\ \zeta_{Kr} & 0 & 0 & 0 & 0 \\ \zeta_{Ki} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.16c)$$

$$d = \begin{bmatrix} \Re(\mathbf{c}_j) \\ \Im(\mathbf{c}_j) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.16d)$$

where ζ or χ are matrices formed by the discretized vectors $\zeta(\theta, \phi)$ and $\chi(\theta, \phi)$ respectively, for the secondary lobes (ζ_{Ir} and ζ_{Ii}), the mainbeams (ζ_{Jr} and ζ_{Ji}), the nulls (ζ_{Kr} and ζ_{Ki}) and the crosspolar component (χ_{Qr} and χ_{Qi}).

In [56], it is proposed a technique for transforming a convex problem related to the array synthesis into an equivalent linear programming problem. In this section, the classical triangular inequality is used to obtain a linear representation of the problem. Although the problem is over-restricted in this step, the simplicity of the method makes it useful in case that a linear programming representation of the problem is needed.

4.2.3 Computational aspects and remarks.

The proposed synthesis methodology requires a Floquet analysis of the infinite array model, from the FEM-based procedure in [32] or [63], for building the optimization problem, before the resolution process. It must be performed for each sampled angle in which the space is discretized for obtaining vectors $\vec{\mathbf{t}}(\theta_m, \phi_m)$ or $\zeta(\theta_m, \phi_m)$ in (4.8) or (4.13) respectively. The computational burden for this step depends on the size of the array since, besides determining the number of optimization variables, a greater

number of array elements generally corresponds to a higher angular variation in the array patterns, and a greater sampling density is required. Each FEM simulation may take from less than a second, for simple array elements such as apertures, or when the domain decomposition technique may be applied in the analysis, to around fifteen seconds for more complex elements such as DRAs. Applying the same reasoning, as the array size grows, the number of constraints, determined by the sampling density, increases, and thus so does the computational cost for the resolution of the optimization problem.

The proposed array pattern synthesis method will be applicable to planar arrays with a double periodic grid (rectangular, or with a certain grid angle), since it is based on the Floquet modal analysis of the infinite array model. Although the proposed array pattern synthesis provides a global optimum, it is performed from approximated finite array models. The degree of approximation will depend on the array characteristics commented in Chapter 3.

4.3 Numerical results

In order to illustrate the capability of the proposed optimization procedure, three different radiating elements are considered in several array pattern synthesis problems. They are optimized from the linear programming problem in (4.13) using the open solver SDPT3.

4.3.1 Broadside pattern with different side lobe regions with a square planar array of square apertures

TABLE 4.1: Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)

Example	No. Var.	No. Res.	An.(min)	No. It.	Opt.(s)
Apertures	450	6526	10	23	121
Circ. Patch 1	450	6844	15	27	283
Circ. Patch 2	900	14164	15	26	1948
DRA	200	2074	50	10	12

The analysis method presented in Chapter 3 is employed to optimize a 15×15 -element waveguide array of open-ended square waveguides on a ground plane, with a 4 cm side length, filled with a dielectric of $\epsilon_r = 4$, and with an inter-element spacing of $0.5\lambda_0$ in x and y directions, at the resonant frequency of 2.14 GHz. The excitation

FIGURE 4.1: H -plane cuts of the radiation patterns of a 15×15 -array of open-ended square waveguides in the three different infinite array environment of non-excited elements, and the excitations optimized considering shorted non-excited apertures.

FIGURE 4.2: Geometry of a dual-coaxial probe-fed circular microstrip antenna enclosed in a cylindrical metallic cavity recessed in a metallic plane, used as array element. $R_1 = 30$ mm, $R_2 = 24.75$ mm, $c_1 = 1.524$ mm, $c_2 = 3.976$ mm, $s = 6.2$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$. Coaxial feeds (SMA connectors): $r_i = 0.64$ mm, $r_o = 2.05$ mm and $\epsilon_{rx} = 1.951$

coefficients are optimized to achieve a broadside pattern with a first-null beamwidth of 20° , minimizing the maximum SLL in the region $10^\circ \leq |\theta| \leq 25^\circ$, while keeping a maximum SLL of -20 dB in the region $|\theta| \geq 25^\circ$. The resulting field pattern fulfills the required restrictions, with a maximum SLL in the region besides the mainlobe below -35 dB. The cross-polar component is negligible. Fig. 4.1 compares the H plane radiation pattern computed with previously optimized excitations applied to the three different finite array approaches and to the analysis procedure in [10]. It is observed that although all of them are very similar, the one obtained from the Y-matrix formulation (3.28) and the method in the reference are almost coincident.

In order to evaluate the computational performance and dimensionality of the problem for the examples in this section, the number of variables of optimization, number

of restrictions and iterations, and the time required for the analysis and the optimization process are shown in Table 4.1. Every computation time shown in this thesis is accomplished on a personal laptop (i7 16 GB RAM).

4.3.2 Dual beam pattern with a square array of circular and cavity-backed microstrip antennas

In the following example, the array element is a cavity-backed circular microstrip antenna with super-strate and dual coaxial probe feeding for circular polarization. The cavity enclosure is used in this type of antenna to prevent surface-wave excitation. The geometry and dimensions of the antenna, reported in [66] as a benchmark, is detailed in Fig. 4.2. All the dimensions and dielectric constants are considered in the finite element analysis. The array is made up of 15×15 -microstrip antennas with an inter-element distance of $0.5\lambda_0$ at the resonant frequency of 1.96 GHz. A circularly polarized radiation pattern with two main lobes at $\theta = \pm 20^\circ$ in the $\phi = 0$ plane, with a first-null beamwidth of 20° , is optimized considering a short-circuited element environment (expression (3.28) with $N = 2$). As stated in the Section 4.2, depending on the real array environment, one of the proposed finite array approaches will be more suitable than others. If the real array of microstrip antennas is supposed to be surrounded by a ground plane, which is a practical situation, shorted elements will provide a better approximation as shown in [50].

Two different optimizations are considered with this array. Firstly, the maximum SLL of the co-polar component is minimized in the side lobe region, setting a quadrature phase difference between the two orthogonal coaxial probe feeds in each microstrip antenna to obtain circular polarization. The cross-polar component is not included in the optimization process. Fig. 4.3(a) shows the resulting co- and cross-polar components of the radiation pattern in $\phi = 0^\circ$ and 90° . A maximum SLL and cross-polar component below -26.5 dB and -25 dB, respectively, are obtained. This example is identified by "Circ. Patch 1" in Table 4.1.

Then, the optimization is performed by minimizing the maximum SLL of the co-polar component while the cross-polar component is introduced as a restriction in the formulation, setting a maximum value of -40 dB in the whole half-space. The excitation in each coaxial is optimized independently using the double of degrees of freedom than in the previous case. The resulting field pattern represented in Fig. 4.3(b) fulfills the required specifications achieving a maximum SLL below -26.2 dB. In Fig. 4.4, the field pattern is represented in a three dimensional plot. In these examples, the synthesized

FIGURE 4.3: Co- and cross-polar components of a two-beam pattern in the $\phi = 0^\circ$ and 90° planes of a 15×15 -element cavity-backed patch antenna array considering two cases: (a) The SLL is minimized setting a quadrature phase difference between the probe feeds in each antenna. (b) The SLL is minimized and the maximum cross-polar component is set to -40 dB optimizing each coaxial excitation independently.

FIGURE 4.4: Three dimensional representation of the optimized two-beam pattern for the 15x15-element array of cavity-backed microstrip antennas.

patterns with two main lobes and minimum SLL are obtained at the expense of a non-uniform amplitude excitation, which is associated to a decrease in the aperture efficiency. These effects are increased in the second case, where in addition, the excitations are optimized independently in each coaxial feed to fix a low cross polarization level, giving rise to a highly non-uniform amplitude distribution and a low aperture efficiency.

4.3.3 Steered pattern with a square planar array of aperture coupled patch antennas

Finally, an array of aperture coupled patch antennas placed on an infinite ground plane with a resonant frequency of 5 GHz is considered. The geometry and dimensions of the aperture coupled patch antenna are given in Fig. 4.5. As before, all dimensions and dielectric permittivities are considered in the optimization. A 10×10 -element square array with a $0.5\lambda_0$ spacing has been chosen to illustrate the effect of real element patterns and coupling in the optimization process. The excitations to achieve a steered pattern in the E -plane towards $\theta = 45^\circ$ with a first-null beam width of 20 degrees are optimized considering the array surrounded by non-excited open-ended elements (3.34) obtaining a maximum SLL of -15.5 dB. For comparison purposes, an optimization with the same mainlobe width and SLL constraint is also performed with the classic-analytical Cheng-Tseng synthesis procedure for isotropic sources [67] and then steered applying the

FIGURE 4.5: Geometry of a aperture coupled patch antenna used as array element: square periodicity $a = 3.0$ cm. Patch dimensions $W_m = 2.37$ cm and $L_m = 1.68$ cm. Aperture $W_p = 1.18$ cm and $L_p = 0.76$ mm. Microstrip feed $W_s = 1.94$ mm and $L_s = 8.3$ mm. Dielectric substrate $C_1 = 0.8$ mm, $C_2 = 1.6$ mm, $\epsilon_{r1} = 3.2$ and $\epsilon_{r2} = 2.2$.

FIGURE 4.6: H-plane cuts of the broadside field pattern of a 10×10 -patch array obtained with the optimized excitations considering open-circuit non-excited CDRAs, and with the *Cheng–Tseng* excitations scheme for isotropic sources. The ideal isotropic Cheng-Tseng pattern is also represented. The dashed grey line represents the desired SLL level.

correspondence phase shift. Fig. 4.6 shows the optimized aperture coupled patch array pattern and the resulting isotropic Cheng-Tseng pattern in the E-plane. As observed, both are practically coincident. However, the Cheng-Tseng excitation scheme applied to the aperture coupled patch array results in a field pattern, also shown in the figure, that clearly does not fulfill the SLL constraint because of its degradation due to mutual coupling and real element patterns.

Chapter 5

Shaped-beam synthesis of real arrays using the infinite array approach and convex programming

5.1 Introduction

In satellite communications where the antenna arrays are supposed to illuminate a wide range of angles, or in applications where the direction of arrival of a signal is unknown or estimated with some errors, it is important to design radiation patterns with an arbitrary shape, fixed response ripple and negligible power radiated to the remaining directions. A shaped beam synthesis problem involves finding the amplitude and phase distribution of the array excitation, and sometimes the positions of the array elements, satisfying as closely as possible this set of specifications on the beam pattern. The desired radiation pattern can be specified for the field, including amplitude and phase, or for the power (only amplitude). For most applications, the phase of the pattern is not specified, giving more degrees of freedom to the designer, but making the problem more difficult to deal with. This kind of problem has been widely addressed in the last decades, from analytical approaches applied to idealized designs problems at first [11, 15, 68] to the large number of standard or innovative numerical techniques available in the literature [19, 20, 69–71].

In most of the general numerical synthesis techniques reaching an optimum solution is not guaranteed, like methods based on alternative projections [19], which have drawn

considerable attention from antenna designers, but may converge to a local minimum if a good starting point is not used. Global optimization methods, such as simulated annealing, genetic algorithm, particle swarm optimization or branch-and-bound techniques [43, 53, 72], may achieve optimal solutions. However, they are orders of magnitude less efficient and more time consuming.

Although without the generality of these techniques, convex optimization methods [22] guarantee that the global solution to an optimization problem is reached both efficiently and very reliably. Convex optimization has been widely applied for a long time to many engineering fields, as well as being proposed to solve array pattern synthesis problems [23, 56]. It has traditionally been considered computationally expensive. However, the increasing computer processing power and the advances in algorithms, like recently developed interior-point methods, make real-time convex optimization possible [73]. Antenna array beam pattern shaping is not in general a convex optimization problem. There are non-convex specifications, as the lower bound constraint applied to the radiated power, impossible or hard to solve exactly in a reasonable time. Different techniques to yield a convex formulation, using additional constraints or reformulating the initial problem, and originally developed for designing finite impulse response (FIR) digital filters, have been adapted to antenna array beam forming. Thus, the same efficient methodology proposed in [74], using a change of variables and spectral factorization, is implemented for array weight design in several works [74, 75]. However, it is only applicable for uniform linear arrays with isotropic elements. Other methods face np-hard shaped beam synthesis problems by relaxing [76] or reducing [24] the original problem, and solving iteratively convex optimizations. A methodology, proposed for designing FIR linear phase digital filters [77, 78] consisting of imposing symmetric weights, has also been adapted to shaped beam synthesis of arrays of elements with isotropic or analytical patterns, [75, 79, 80]. This technique has also been applied to other areas of engineering such as photonics [81] or to design blocking filters [82]. The new constraint gives rise to a real-valued array pattern, transforming the lower bound constraint on the beam pattern in a convex problem, as well as reducing the dimension of the optimization formulation. The synthesized pattern will not in general be the global solution to the original problem, because the new constraints added consume degrees of freedom. However, it is a trade-off because a satisfactory solution to an np-hard problem is obtained in polynomial time.

On the other hand, antenna array synthesis methods usually work with isotropic or analytical element patterns. This simplification can lead to unwanted synthesized array patterns, given that mutual coupling between array elements or real element patterns are not included in the optimizations. In the analysis of large arrays of complex antennas, the element-by-element approach requires large computational time and memory space.

In case of uniformly spaced arrays, the infinite array approach is found to be more convenient since the analysis is reduced to a periodic radiating element using the Floquet modal analysis [1, 7, 83]. Mutual coupling is inherently considered with this formulation, although the edge effects caused by the finiteness of the array is not accounted for. It is possible however to approximate the behaviour of the infinite array under arbitrary excitations from infinite array data, applying windowing or convolution techniques [2–4], or by means of other approaches based on the truncation of the Floquet modes [45, 84].

This chapter proposes a shaped beam synthesis procedure for real and coupled antenna arrays of complex radiating elements, with control of the ripple amplitude in the shaped region, optimized side-lobe and cross-polar levels in prescribed radiation regions, as well as fixed null pointing directions. The optimization is expressed as a convex problem based on the formulation introduced in [77] for FIR digital filter design and used in [75, 79, 80] for isotropic antennas. The synthesis approach incorporates the analysis method based on the infinite array approach presented in Chapter 2 and the finite array analysis presented in Chapter 3.

5.2 Proposed synthesis method

A formulation to integrate the finite array analysis obtained in Chapter 3 and a shaped beam synthesis procedure via a convex optimization, achieved by enforcing an additional constraint on the optimization problem, is developed in this section. For this purpose, the radiated field corresponding to different load conditions of non-excited elements in the infinite array environment, considering one mode and one port of excitation:

$$\vec{E}_A^{mt} = \vec{E}_a \vec{p} \mathbf{v} \quad (5.1)$$

$$\vec{E}_A^{sc} = \vec{E}_a (\mathbf{Y} + \mathbf{Y}^{fl}) \vec{p} (\mathbf{Y} + \mathbf{Y}_0)^{-1} \mathbf{v}. \quad (5.2)$$

$$\vec{E}_A^{oc} = \vec{E}_a (\mathbf{Z} + \mathbf{Z}^{fl}) \vec{p} (\mathbf{Z} + \mathbf{Z}_0)^{-1} \mathbf{v} \quad (5.3)$$

for the matched, the short-circuited and open-circuited array environments, respectively. They may be expressed in a similar way as follows:

$$\vec{E}_A = \vec{E}_0 \vec{p} \boldsymbol{\gamma} = \vec{E}_0 \sum_{m=1}^M \sum_{n=1}^N e^{j(mu_x + nu_y)} \gamma_{m,n} \quad (5.4)$$

where the following terms have been grouped together in each case as:

$$\vec{E}_0 = \begin{cases} \vec{E}_e & \text{(match-terminated)} \\ \vec{E}_e (\mathbf{Z}_0 + \mathbf{Z}^{fl}) & \text{(open-circuited)} \\ \vec{E}_e (\mathbf{Y}_0 + \mathbf{Y}^{fl}) & \text{(short-circuited)} \end{cases} \quad (5.5)$$

and a different column vector, γ , obtained from the incident voltages has been defined in each case:

$$\gamma = \begin{cases} \mathbf{v} & \text{(match-terminated)} \\ (\mathbf{Z} + \mathbf{Z}_0)^{-1} \mathbf{v} & \text{(open-circuited)} \\ (\mathbf{Y} + \mathbf{Y})^{-1} \mathbf{v} & \text{(short-circuited)} \end{cases} \quad (5.6)$$

The coefficients $\gamma_{m,n}$ become the variables to be optimized. Assuming M and N are odd numbers (the process for even numbers is similar) and rewriting (5.4) with a common factor, outside the summatory, the exponential corresponding to the central element of the array, $m = (M + 1)/2$ and $n = (N + 1)/2$, the following expression is obtained:

$$\begin{aligned} \vec{E}_A(\theta, \phi) &= \vec{E}_0(\theta, \phi) e^{j(\frac{M+1}{2}u_x + \frac{N+1}{2}u_y)} \times \\ &\left(\gamma_{1,1} e^{-j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)} + \dots + \gamma_{(\frac{M+1}{2}),(\frac{N+1}{2})} + \dots + \gamma_{M,N} e^{j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)} \right) \end{aligned} \quad (5.7)$$

Next, looking for achieving a convex formulation, conjugate symmetric optimization variables are imposed in (5.7), $\gamma_{i,j} = \gamma_{M-i+1,N-j+1}^*$. This assumption has been used earlier in array pattern synthesis, [75, 79, 80], and other areas, [77, 78], achieving a significant simplification in the synthesis process. Applying this condition, the term between brackets is transformed into a real expression, given that imaginary parts cancel out:

$$\begin{aligned} \vec{E}_A(\theta, \phi) &= \vec{E}_0(\theta, \phi) e^{j(\frac{M+1}{2}u_x + \frac{N+1}{2}u_y)} \times \\ &\left(\gamma_{1,1} e^{-j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)} + \dots + \gamma_{(\frac{M+1}{2}),(\frac{N+1}{2})} + \dots + \gamma_{1,1}^* e^{j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)} \right) \end{aligned} \quad (5.8)$$

In this way, the magnitude of the radiated field can be expressed as:

$$\begin{aligned} |\vec{E}_A(\theta, \phi)| &= |\vec{E}_0(\theta, \phi)| 2\Re \left(\gamma_{1,1} e^{-j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)} + \dots + \frac{1}{2} \gamma_{(\frac{M+1}{2}),(\frac{N+1}{2})} \right) \\ &= |\vec{E}_0(\theta, \phi)| 2\Re(\vec{\mathbf{p}}_s \gamma_s) \end{aligned} \quad (5.9)$$

where the row and column vectors, $\vec{\mathbf{p}}_s$ and γ_s , stand respectively for the exponential terms and for the coefficients $\gamma_{m,n}$ in (5.9), corresponding to a symmetric half of the radiating elements, as well as the central one, whose weight is forced to be real:

$$\begin{aligned} \vec{\mathbf{p}}_s(\theta, \phi) &= (e^{-j(\frac{M-1}{2}u_x + \frac{N-1}{2}u_y)}, e^{-j(\frac{M-1}{2}u_x + \frac{N-2}{2}u_y)}, \dots, \frac{1}{2}) \\ \gamma_s &= (\gamma_{(1,1)}, \gamma_{(1,2)}, \dots, \gamma_{(\frac{M+1}{2}),(\frac{N+1}{2})})^T. \end{aligned}$$

The co- and cross-polar components of the radiated field are obtained by taking the corresponding components in $\vec{E}_0$, $\vec{E}_0^{cp}$ and $\vec{E}_0^{xp}$ respectively, in (5.9), yielding:

$$|\vec{E}_A^{cp}| = |\vec{E}_0^{cp}| 2\Re(\vec{p}_s \gamma_s) \quad (5.10)$$

$$|\vec{E}_A^{xp}| = |\vec{E}_0^{xp}| 2\Re(\vec{p}_s \gamma_s). \quad (5.11)$$

$|\vec{E}_0^{cp}|$ and $|\vec{E}_0^{xp}|$ are in turn obtained by taking the co- and cross-polar components of the AEP, $\vec{E}_e$, in (5.5).

Different shaped beam synthesis problems can be established depending on the selected objective functions and set of constraints. For instance, it may be formulated as a side-lobe level minimization, setting a real and positive value, α , which represents the maximum allowed co-polar field level in the side-lobe region, R_{SL} ; while imposing a mask constraint in the shaped region, R_{ML} , using upper and lower limits of real values, L and U , respectively; fixing a maximum cross-polar field level, β , in another specified region R_{Xp} ; and imposing a null on the radiation pattern in discrete directions or in a specified region R_{Nu} :

$$\text{Min } \alpha \quad (5.12a)$$

$$\text{s.t. } L \leq |\vec{E}_0^{cp}| 2\Re(\vec{p}_s \gamma_s) \leq U \quad (\theta, \phi) \in R_{ML} \quad (5.12b)$$

$$|\vec{E}_0^{cp}| 2\Re(\vec{p}_s \gamma_s) \leq \alpha, \quad (\theta, \phi) \in R_{SL} \quad (5.12c)$$

$$|\vec{E}_0^{cp}| 2\Re(\vec{p}_s \gamma_s) = 0, \quad (\theta, \phi) \in R_{Nu} \quad (5.12d)$$

$$|\vec{E}_0^{xp}| 2\Re(\vec{p}_s \gamma_s) \leq \beta, \quad (\theta, \phi) \in R_{Xp} \quad (5.12e)$$

The same shaped beam synthesis problem could be formulated minimizing the maximum cross-polar level, or the ripple of the co-polar component in the shaped region, or a linear combination of some of them, establishing the remaining specifications as constraints. Next, each spatial region defined in (5.12) is discretized into a grid of directions giving rise to a linear programming problem in γ_s (or semi-definite program formulation) stated as:

$$\text{Min } \alpha \quad (5.13a)$$

$$\text{s.t. } L \leq |\vec{E}_{0_h}^{cp}| 2\Re(\vec{p}_h \gamma_s) \leq U \quad h = 1 \dots H \quad (5.13b)$$

$$|\vec{E}_{0_i}^{cp}| 2\Re(\vec{p}_i \gamma_s) \leq \alpha, \quad i = 1 \dots I \quad (5.13c)$$

$$|\vec{E}_{0_j}^{cp}| 2\Re(\vec{p}_j \gamma_s) = 0, \quad j = 1 \dots J \quad (5.13d)$$

$$|\vec{E}_{0_k}^{xp}| 2\Re(\vec{p}_k \gamma_s) \leq \beta, \quad k = 1 \dots K \quad (5.13e)$$

where the subscripts (h, i, j, k) make reference to the set of pointing directions in each region. For example expression (5.13b) is equivalent to: $L \leq |\vec{E}_0^{cp}(\theta_h, \phi_h)| \ 2\Re(\vec{p}_s(\theta_h, \phi_h)\gamma_s) \leq U$, $h = 1, \dots, H$ with $(\theta_h, \phi_h) \in R_{ML}$. The grid in each region should be dense enough to cover all possible side-lobe and cross-polar local maxima, as well as the beam pattern behaves according to the imposed variations in the shaped region. As the minimization problem is formulated with coefficients γ_s as optimization variables, and therefore with variables γ defined in (5.6), the element excitation weights providing the desired shaped beam pattern are computed directly by solving expressions (5.6) for γ_s , once the optimization is carried out. Thus, for the last two cases, corresponding to short- and open-circuited load conditions, the excitations weights $\mathbf{v}$ will not be symmetric, although they are computed from symmetric coefficients γ .

The resulting optimization problem (5.13) is a semi-definite program (SDP) that can be solved very efficiently by means of interior-point methods such as SeDuMi [85] that can be handled from the software package [86]. The optimization is very efficient and always converges to the best possible solution. However, in general, it will be a suboptimal solution to the original problem, because symmetric conjugate constraints are imposed, reducing the space of solutions. The method constitutes a trade-off between efficiency and complexity because a satisfactory solution to an np-hard problem is obtained in polynomial time. With the same formulation other shaped beam design problems may be established, such as imposing different side-lobe regions with different maximum levels, or synthesizing beam patterns with arbitrary shape, imposing mask constraints with the desired form.

As the method is based on the Floquet modal analysis, it will be applicable to arrays with a rectangular periodicity or with a certain grid angle. Moreover, the array elements must be distributed symmetrically about the center of the array. However, planar arrays with any contour fulfilling the previous conditions may be considered.

5.3 Numerical results

In order to illustrate the validity and effectiveness of the proposed synthesis procedure, four examples considering different radiating elements, array contours and shaped beam patterns are described below.

FIGURE 5.1: E -plane cut of a circular flat-top pattern ($|\theta| \leq 20^\circ$) for a 15×15 element square array of open-ended square waveguides obtained with the three finite array approaches in this work and from the method in detailed in Chapter 2. Excitations synthesized considering an environment of shorted non-excited apertures.

FIGURE 5.2: Geometry of the coaxial probe-fed rectangular patch antenna used as array element in example IV.A. $\epsilon_r = 4.32$, $l = 38$ mm, $a = 18$ mm, $b = 12$ mm, $c_1 = 6.5$ mm, $c_2 = 6$ mm and $d = 0.8$ mm. Coaxial feed (SMA connector): $r_{in} = 0.65$, $r_{out} = 2.05$ and $\epsilon_{rx} = 1.951$.

5.3.1 Circular flat-top pattern with a square planar array of square apertures

In the first example a footprint pattern with a circular contour, covering the region $|\theta| \leq 20^\circ$, with a maximum allowed ripple of 0.5 dB in the shaped region, has been synthesized by minimizing the maximum SLL in the region $|\theta| \geq 30^\circ$. A 15×15 element array made up of open-ended square waveguides filled with dielectric material of $\epsilon_r = 4$, and embedded in a ground plane, is considered. The aperture side and the inter-element spacing in x - and y -directions are, respectively, $\lambda_0/3$ and $\lambda_0/2$ at the resonance frequency of 2.14 GHz. The optimization is performed using the finite array model considering non-excited elements terminated by short-circuits, expression (3.37). The minimization converges correctly satisfying the prescribed specifications (marked by a dashed line in Fig. 5.1) and achieving a maximum SLL of -20 dB and a ripple of 0.5 dB. The optimized radiation pattern in H -plane is plotted in the figure, together with the resulting patterns computed with the optimized excitation weights applied to the other two finite array approaches, expressions (3.36) and (3.38). The cross-polar component is negligible.

The radiation pattern computed with the analysis procedure in [10] is also represented. This method is based on the 3D-FEM and spherical mode expansion, and performs a full-wave characterization of planar arrays with elements on an infinite metallic plane. As observed, the radiation pattern computed from the three finite array models are very similar. However, the pattern computed from (3.37) is closer to that obtained with the method in [10] since, as pointed out in Chapter 3, this approach simulates an infinite ground plane surrounding the finite array.

5.3.2 Circular flat-top pattern with a square planar array of rectangular patch antennas

In this example the array element is a rectangular microstrip antenna with coaxial probe-feeding. All the dimensions and dielectric constants, detailed in Fig. 5.2 and reported in [4], are considered in the FEM analysis and in the optimization. A 9×9 element square array with inter-element spacing of $0.5067\lambda_0$ at the resonant frequency of 4 GHz, is used. The excitations to synthesize a circular-shaped main lobe, defined by $|\theta| < 30^\circ$, with a maximum ripple of 0.5 dB, and minimizing the maximum SLL in the region $|\theta| > 40^\circ$, are optimized considering an infinite open-circuited element environment, expression (3.38). The optimized radiation pattern fulfills the proposed requirements with a SLL below -17 dB. Fig. 5.4 shows the co-polar component in E -plane. The same microstrip array has also been analyzed using a full-wave electromagnetic solver (CST Microwave Studio

FIGURE 5.3: (a) Finite array (dark elements) in an infinite array environment of non-excited and match, open or short-terminated elements (grey elements). (b) Geometry of a finite array of microstrip antennas in Fig. 5.2 surrounded by a substrate with circular contour, simulated with a full-wave electromagnetic solver (CST) in example 5.3.2.

FIGURE 5.4: Co-polar field pattern in E plane of a circular flat-top pattern ($|\theta| \leq 30^\circ$) for a 9×9 element square array of rectangular microstrip antennas in Fig. 5.2, obtained with the models with non-excited open and short-terminated elements, and with the array surrounded by a finite substrate with circular contour (Fig. 5.3(b)) using CST. The excitations are synthesized considering an infinite array environment of open non-excited elements.

FIGURE 5.5: Layout of the 525–element array of cavity backed microstrip antennas with a circular contour and square mesh of $0.5\lambda_0$ -equispaced elements.

[87]). A circular substrate of radius 0.5 m, more than three times the side of the array, has been considered, as shown in Fig. 5.3(b). The radiation pattern simulated with this solver, and that obtained considering an infinite short-circuited element environment, (3.37), are also depicted in Fig. 5.4, both computed with the optimized excitation weights. As observed, the results are practically coincident in the main lobe region for the three cases. The patterns obtained with (3.37) and (3.38) show a null of radiation at $\theta = 76^\circ$ which is not present in the CST simulation. This is due to the fact that finite array approaches are based on the Floquet analysis model and AEP, and scan blindness effect appears at this direction owing to the propagation of the first TM surface wave. The real array is not large enough to cause this effect. The result considering an open-circuited element environment shows a better agreement with that obtained with the full-wave solver in the side-lobe region near main lobe. For this case, this approach provides a more accurate result when a dielectric plane is surrounding the real array. In both cases a better approximation with a full-wave analysis is expected for array elements and element spacing that do not give rise to the blindness effect.

5.3.3 Rectangular flat-top pattern with a circular planar array of square and cavity-backed patch antennas

In this example a 525-element square-meshed array with $0.5\lambda_0$ spacing between the elements and a circular contour as shown in Fig. 5.5 is used. The array element, described in Fig. 5.6, is a square and cavity-backed probe-fed microstrip antenna with

FIGURE 5.6: Geometry of the coaxial probe-fed and cavity-backed square patch antenna used as array element in example IV.C. $l = 1.35$ cm, $r = 1.815$ cm, $c_1 = 2.42$ mm, $c_2 = 2.9$ mm, $x_0 = 1.7$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$. Coaxial feed (SMA connector): $\epsilon_{rx} = 1.951$, $r_{in} = 0.65$ mm and $r_{out} = 2.05$ mm.

FIGURE 5.7: Rectangular flat-top pattern in E - and H -planes for the 525-element array in Fig. 5.5 made-up of cavity-backed microstrip antennas of Fig. 5.6, and synthesized considering an infinite shorted element array environment. Continuous line: co-polar component, dashed line: cross-polar component.

FIGURE 5.8: Three dimensional representation of the co-polar component of the synthesized rectangular flat-top pattern for the array in Fig. 5.5 with the cavity-backed microstrip antennas of Fig.5.7.

superstrate and resonant frequency of 6 GHz. The metallic cavity is recessed in an infinite metallic plane. Details of this antenna can be found in [10]. All the antenna characteristics are also considered in the synthesis process in this example. A rectangular footprint covering the region defined by $|\psi_x| \leq 0.2$ and $|\psi_y| \leq 0.3$ and a maximal response ripple of 0.5 dB is synthesized by considering an infinite shorted element environment for the array. In the region defined by $|\psi_x| \geq 0.3$ and $|\psi_y| \geq 0.45$ the maximum SLL is minimized. Fig. 5.7 shows the co-polar and cross-polar components of the synthesized radiation pattern in E and H planes. A three dimensional representation of the co-polar component is shown in Fig. 5.8. It fulfills the specifications achieving a $SLL = 25.7$ dB with a maximum cross-polar level below -45 dB.

5.3.4 Circular-sector flat-top pattern with a circular planar array of square and cavity-backed patch antennas

In the last example, and to further demonstrate the applicability of the proposed method to shape more complex patterns, a circular-sector flat-top pattern defined by $\psi_x \geq 0$, $\psi_y \geq 0$ and $\psi_x^2 + \psi_y^2 \leq 0.4^2$ is synthesized. The same radiating element, array layout and infinite array environment of the previous example is considered. The sector beam is synthesized with a maximum ripple of 0.5 dB and minimizing the maximum SLL in the region defined by $\psi_x \leq -0.1$ or $\psi_y \leq -0.1$ and $\psi_x^2 + \psi_y^2 \geq 0.5^2$. The proposed

FIGURE 5.9: Color map representation of the co-polar component of the synthesized sector flat-top pattern for the array in Fig. 5.5 with the cavity-backed microstrip antennas of Fig.5.7.

requirements are fulfilled with a maximum SLL of -24 dB as shown in the color map representation of the co-polar component of the synthesized pattern in Fig. 5.9.

In order to evaluate the computational performance and dimensionality of the optimization problem for the examples in this chapter, the number of variables, and the time required for the analysis and the optimization process are shown in Table 5.1. The number of restrictions in (tal) determined by the number of sampled points in the discretization, which has been carried out with a constant step in θ and ϕ in the radiating region, is also shown.

TABLE 5.1: Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)

Example	No. Var.	No. Res.	An.(min)	No. It.	Opt.(min)
5.3.1	113	6844	10	15	13
5.3.2	41	2074	13	13	7
5.3.3	263	6844	15	20	17
5.3.4	263	6844	15	21	19

Chapter 6

Shaped-beam synthesis of coupled antenna arrays using the phase retrieval

6.1 Introduction

The shaped beam problem, solved in Chapter 5 with additional restrictions can also be formulated as a phase retrieval, which has been studied over the last decades with a greedy algorithm as the alternative projections algorithm at first. Recently this formulation has been relaxed and solved using convex programming. A phase retrieval problem makes reference to recovering the complex phase of a general signal when only its magnitude is known. It arises in many engineering and physical applications such as X-ray crystallography or astronomical imaging, as well as in antenna array synthesis. The problem of recovering a signal from the magnitude of its Fourier transform is a common example. Phase retrieval is difficult to solve numerically because it is non-convex and constitutes an np-hard problem. There is a well-known process for solving this kind of problem: the alternating projections algorithm (APA) proposed by Gerchberg and Saxton [88] and later improved by Fienup [89]. Good results can be obtained using this method but it has two main drawbacks. The APA will always converge to a local minimum but as the set is not convex it is not guaranteed to be the global solution to the problem. Actually, different initial points will yield different solutions because the APA is highly dependent on the initial point. The second drawback is that the convergence rate is known to be slow. This method, or variations of the same, have also been applied to array synthesis problems as in [19, 90].

Some papers have recently been published in which a convex formulation of the phase retrieval is stated [91–93]. Based on the relaxation of the np-hard restrictions, they obtain really good results. These methods have also been applied to array antennas as in [94], where a similar formulation is obtained for mono-pulse pattern notching via phase-only excitations, or in [24] where the semi-definite relaxation is used to obtain shaped-beam and phase-only synthesis of linear arrays of isotropic elements. In [95], the phase retrieval via the semi-definite relaxation is also used to seek the excitations of a linear array from phase-less far-field data.

On the other hand, shaped beam synthesis procedures usually consider array elements idealized to be isotropic or with analytical element patterns. There is not an analysis of the radiators, and the coupling between them is not taken into account. These simplifications can lead to undesired results. However, more and more analysis procedures are integrated with array optimization methods to consider these effects. Global optimization techniques, such as genetic algorithm [96] or particle swarm optimization [43], do not guarantee an optimal solution and are usually extremely time consuming. The same applies to methods based on measured or calculated element-pattern data as in [62]. Other methods, such as those based on the infinite array approach [97], require less memory space and computational time, but they do not take into account the real array environment or it is only considered approximately.

In this chapter, the integration of a fast full wave electromagnetic analysis method and a shaped beam synthesis procedure for antenna arrays based on the convex relaxation of the phase retrieval algorithm is proposed. The array is rigorously characterized in a matrix form using a hybrid method that combines the finite elements method (FEM), modal analysis and the expansion of the radiated field in spherical modes. The shaped beam synthesis procedure is based on phase retrieval, semi-definite relaxation and convex programming.

6.2 Proposed optimization method

In this section a matrix formulation which merges a convex approach of the algorithm and the analysis method described in Chapter 2, for the analysis of finite arrays with an spherical wave expansion, is developed to obtain shaped-beam designs of real antenna arrays.

A shaped beam synthesis problem may be established from the expression of the radiation intensity of real and coupled antenna arrays is obtained, from the expression

of the radiated field expressed in (2.43), as a Hermitian form:

$$|\vec{E}_a|^2 = \mathbf{v}^H \mathbf{T}_G^H (\vec{e} e^{jk\hat{\mathbf{r}} \cdot \vec{r}_i})^H \cdot (\vec{e} e^{jk\hat{\mathbf{r}} \cdot \vec{r}_i}) \mathbf{T}_G \mathbf{v} = \mathbf{v}^H \mathbf{P}(\theta, \phi) \mathbf{v} \quad (6.1)$$

The general objective is to find the array excitations $\mathbf{v}$ providing a radiation intensity that fulfills a given objective function or mask, $Ma(\theta, \phi)$, defined in the radiation space. The problem is stated as follows:

$$\begin{aligned} & \underset{\mathbf{v}}{\text{find}} && \mathbf{v} \\ & \text{subject to} && |\mathbf{v}^H \mathbf{P}(\theta, \phi) \mathbf{v} - Ma(\theta, \phi)| \leq \varepsilon \end{aligned} \quad (6.2)$$

ε is a real constant that stands for the maximum ripple in the shaped region, and for the maximum SLL in sidelobe region with $Ma = 0$. This optimization problem may be considered as a phase retrieval problem. This kind of problem deals with the recovery of a signal, the complex array elements excitation in this case, from the magnitude of a mathematical projection, represented here by the radiation intensity.

The quadratically constrained optimization problem in (6.2) is known to be non-convex and np-hard to solve exactly, owing to the lower bound constraint applied to the radiated intensity. It may be reformulated by turning the non-convex quadratic constraint into a convex constraint via the semi-definite relaxation, as proposed in [91]. For this purpose, the problem of recovering a complex vector from quadratic constraints is transformed into recovering a rank-one matrix from affine constraints as shown below.

The radiation intensity in (6.2) can be expressed as follows given that $\mathbf{P}$ is an Hermitian matrix:

$$|\vec{E}_a|^2 = \mathbf{v}^H \mathbf{P} \mathbf{v} = \text{Tr}(\mathbf{v}^H \mathbf{P} \mathbf{v}) = \text{Tr}(\mathbf{P} \mathbf{v} \mathbf{v}^H) = \text{Tr}(\mathbf{P} \mathbf{V}) \quad (6.3)$$

where Tr stands for the trace of the matrix, which is computed as the sum of the elements of the main diagonal of the matrix. The property that $\text{Tr}(AB) = \text{Tr}(BA)$ has been applied. $\mathbf{V} = \mathbf{v} \mathbf{v}^H$ is a symmetric positive semi-definite matrix of rank one. In this way, the synthesis problem in (6.2) can be stated as:

$$\begin{aligned} & \underset{\mathbf{V}}{\text{find}} && \mathbf{V} \\ & \text{subject to} && |\text{Tr}(\mathbf{P}(\theta, \phi) \mathbf{V}) - Ma(\theta, \phi)| \leq \varepsilon, \\ & && \mathbf{V} \succeq 0. \\ & && \text{rank } \mathbf{V} = 1 \end{aligned} \quad (6.4)$$

The symbol $\succeq$ represents the restriction that $\mathbf{V}$ has to be positive semi-definite. Both problems, (6.2) and (6.4), are equivalent, but the quadratic function of $\mathbf{v}$ has been transformed into a linear function of $\mathbf{V}$, although the optimization problem remains non-convex owing to the rank constraint. Given that, by definition, there exists a rank-one solution to the optimization problem in (6.4), it can be formulated equivalently as:

$$\begin{aligned} & \underset{\mathbf{V}}{\text{minimize}} && \text{rank } \mathbf{V} \\ & \text{subject to} && | \text{Tr}(\mathbf{P}(\theta, \phi)\mathbf{V}) - Ma(\theta, \phi) | \leq \varepsilon, \\ & && \mathbf{V} \succeq 0. \end{aligned} \tag{6.5}$$

which belongs to the field of low-rank matrix completion or matrix recovery, a well-known class of optimization problems.

The resulting optimization problem is np-hard to solve exactly owing to the rank minimization and it is not guaranteed to find an optimal solution in polynomial time. Global stochastic optimization methods, such as simulated annealing or genetic algorithms may achieve optimal solutions. However, this kind of method suffers from large memory cost, low speed of convergence and high computational burden, becoming very inefficient as the number of variables increases. As shown in [98], there are several techniques for transforming (6.5) and solve it approximately but efficiently from heuristic solution methods. One of them consists in substituting the rank functional for the trace norm, giving rise to a convex relaxation in such a way that a near-optimal solution is obtained [91]:

$$\begin{aligned} & \underset{\mathbf{V}}{\text{minimize}} && \text{Tr } \mathbf{V} \\ & \text{subject to} && | \text{Tr}(\mathbf{P}(\theta, \phi)\mathbf{V}) - Ma(\theta, \phi) | \leq \varepsilon, \\ & && \mathbf{V} \succeq 0. \end{aligned} \tag{6.6}$$

The trace of a function, also called the nuclear norm, can be interpreted here as a convex envelope of the rank function. It means that the trace is the best convex approximation of the rank function [98]. The trace minimization problem over a convex set can be solved iteratively to encourage low rank solutions as explained in the reference.

The first constraint in (6.6) is then discretized for the numerical implementation of the optimization problem. In this way, the radiating region is sampled in (θ, ϕ) yielding:

$$\begin{aligned} & \min_{\mathbf{V}} && \text{Tr } \mathbf{V} \\ & \text{s. t.} && | \text{Tr}(\mathbf{P}(\theta_q, \phi_q)\mathbf{V}) - Ma(\theta_q, \phi_q) | \leq \varepsilon, \quad q = 1 \dots Q, \\ & && \mathbf{V} \succeq 0. \end{aligned} \tag{6.7}$$

where Q is the number of pointing directions in which the radiated region has been discretized. The grid of the discretization should be dense enough to cover all possible side-lobe local maxima, as well as the beam pattern behaving according to the variations imposed in the shaped region. Therefore, the number of constraints in (6.7) is determined mainly by the number of sampling points, Q , in which the radiating region is discretized. This problem is solved iteratively as shown in [98]:

$$\begin{aligned} \min_{\mathbf{V}} \quad & \text{Tr}((\mathbf{V}_0 + \delta \mathbf{I})^{-1} \mathbf{V}) \\ \text{s. t.} \quad & |\text{Tr}(\mathbf{P}(\theta_q, \phi_q) \mathbf{V}) - Ma(\theta_q, \phi_q)| \leq \varepsilon, \quad q = 1 \dots Q, \\ & \mathbf{V} \succeq 0. \end{aligned} \tag{6.8}$$

where $\mathbf{V}_0$ is the result of the previous iteration, in the first iteration it can be fixed to an arbitrary value. δ is a constant to select the speed convergence, a high value will provide a more rapidly convergence but will have more convergence problems.

Once the minimization problem in (6.8) is solved, the excitation weights must be obtained from $\mathbf{V}$. As shown in [92], the np-hard problem (6.4) and the convex problem (6.6) are formally equivalent under certain conditions; in this case, $\mathbf{V}$ is of rank one and the optimal solution of the original shaped beam problem (6.2) is directly obtained by factorizing $\mathbf{V}$ as $\mathbf{v}\mathbf{v}^H$. However, this is not the general case, and it cannot be assured that the solution to (6.8), $\mathbf{V}'$, is going to be of unitary rank because, although (6.8) derives from a convex formulation which is solved optimally, this one is a relaxation of the original problem. In order to attain a feasible solution to (6.2), an approximation of $\mathbf{V}'$ of rank one, $\mathbf{V}''$, may be achieved via a singular value decomposition and the excitation coefficients directly obtained from this one.

Therefore, the original problem (6.2) is solved approximately because a near optimal solution is attained after a semi-definite relaxation and a rank one approximation. However, it is performed very efficiently and reliably because the resulting semi-definite programming problem is solved optimally in polynomial time.

Semi-definite problems are usually addressed efficiently using interior point methods (IPM) [99], such as SeDuMi [85]. The IPM are fast and robust for small and medium problems and can be handled with very powerful modeling systems, such as CVX [65] or Yalmip [86]. For large problems (a few thousands of variables and a few tens of thousands of constraints) their complexity grows and the previously cited methods cannot handle it. Larger problems are better solved using first order methods where the cost per iteration is smaller than in IPM. The algorithms performed in this work are implemented using TFOCS [100] and modifications to the TFOCS template files. Its use is especially

recommended for planar arrays where the number of variables and equations becomes higher.

The proposed shaped beam synthesis procedure allows different array sizes, array configurations, grids or layouts to be considered very efficiently as new design variables thanks to the analysis procedure of Chapter 2. Thus, a prescribed beam pattern with a desired maximum ripple, maximum SLL and beam pattern slope may be achieved very efficiently with the appropriate number of array elements or array arrangement in an iterative procedure. This is possible because the implementation of each new optimization problem (6.6) for a new array configuration only requires the analytical computation of the transmission matrix of the array, $\mathbf{T}_G$, from the GSM of the isolated element (2.29), and the calculation from expression (6.1) of the radiation intensity in the sample angles to construct the optimization problem (6.7). These processes just involve analytical computations using the analysis procedure used in this work and it takes seconds for small or medium sized arrays.

On the other hand, the expression of the radiated field obtained in Chapter II (2.43):

$$\vec{E}(\hat{u}) = (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{T}_G \mathbf{v} \quad (6.9)$$

may be formulated in the form:

$$\vec{E}(\hat{u}) = \vec{\mathbf{A}}(\theta, \phi) \mathbf{v} \quad (6.10)$$

where each element of vector $\vec{\mathbf{A}}(\theta, \phi)$ represents the active element pattern of a specific element of the array. This vector, sampled in the required angular directions ($\vec{\mathbf{A}}(\theta_q, \phi_q), q = \dots Q$), may be obtained from other full wave analysis procedures after the characterization of the whole array, and from these samples, the optimization problem in (6.7) may be constructed. However, it should be noted that, in general, the characterization of a new array size or array arrangement using other methods, such as those used in commercial software, means that the whole analysis has to be repeated at once, requiring much more computation time and making an iterative design process very time-consuming or even computationally intractable.

A large variety of shaped beam synthesis problems can be established using the suitable mask $Ma(\theta, \phi)$ in the optimization formulation. It will be possible to specify different sidelobe and shaped-zone ripple levels as well as footprint patterns with arbitrary contours, or synthesizing beam patterns with an arbitrary shape, imposing mask constraints with the desired form.

FIGURE 6.1: Comparison between different optimization methods. The results from the proposed method and from the method with which it is being compared are represented by the black line and the dashed red line respectively.

6.3 Numerical results

Various examples of shaped beam synthesis considering different radiating elements are presented now in order to illustrate the capabilities of the method developed in previous sections.

6.3.1 Linear array of isotropic elements: A comparison with optimal synthesis methods

As it is hard to check the conditions that the optimization problem has to accomplish in order to obtain an optimal solution (see [101], and references therein), a comparison between the proposed optimization method and known optimal solutions for isotropic elements is first accomplished in order to show the strength of the method. Thus, the matrix formulation in Chapter 2 is particularized for isotropic elements.

A linear array of 15 isotropic radiators with an inter-element spacing of $\lambda/2$ is considered. In the first example the excitation coefficients for achieving a broadside beam pattern with a first-null beamwidth of 10 degrees, minimizing the maximum sidelobe level are optimized. The resulting radiation pattern is compared in Fig. 6.1(a) with the Dolph optimization scheme [12] which provides optimal solutions in terms of the ratio between SLL and beamwidth. A very good agreement is observed except for a deviation in the far sidelobes. In the second example a flat-top pattern covering the region $|\theta| \leq 20$ degrees, a maximum allowed ripple of 0.25 dB and SLL below -18 dB in the region $|\theta| \geq 25$ degrees is designed. A comparison is carried out with the method proposed in [74], which uses the autocorrelation of the excitations as optimization variables, obtaining the optimum solution. This methodology is very efficient but it is only valid

FIGURE 6.2: Geometry of the hemispherical dielectric resonator antenna used as array element in example IV.B. 50Ω coax. $r_i = 0.5$ mm and $r_o = 1.05$ mm with $\epsilon_r = 1.74$. Parameters: $R = 12.7$ mm, $s = 6.4$ mm and $h = 6.5$ mm with $\epsilon_r = 9.5$.

TABLE 6.1: Number of elements (No. Elem.), analysis time (An. Time), synthesis time, (Sy. time) and a maximum ripple obtained (Ripple).

No. Elem.	11	12	13	14	15	16
An. time (s)	0.6	0.7	0.9	1.0	1.0	1.1
Sy. time (s)	27	35	39	43	51	59
Ripple (dB)	1.05	0.75	0.37	0.23	0.1	0.08

for linear arrays of equispaced isotropic elements. As shown in Fig. 6.1(b), the synthesized patterns with both methods are very close in the shaped region and first sidelobe, and diverge as they move away from the shaped region. However, a similar result with regard to the compliance with the design specifications (maximum ripple, beam pattern slope, maximum SLL) is observed. These results demonstrate that the proposed method obtains nearly optimal solutions and will be applicable to the formulation considering antenna arrays with real and coupled elements.

6.3.2 Linear array of hemispherical dielectric resonator antennas

In the following examples real elements are considered in the synthesis process. In this example the array element is a hemispherical dielectric resonator antenna (HDRA). All the dimensions and dielectric constants, detailed in Fig. 6.2 and obtained from [102], are considered in the FEM analysis and in the synthesis process. A linear equispaced array of 15 HDRAs along the E -plane with an inter-element spacing of 0.4λ at the resonant frequency of 3.64 GHz is considered. A synthesis to achieve, with the minimum number of antennas, a flat-top pattern defined by $|\theta| \leq 20$ degrees, with a maximum ripple of 0.1 dB and a maximum SLL of -18 dB in the region $|\theta| \geq 30$ degrees has been performed. The optimization problem in (6.7) is solved in an iterative procedure for different numbers of array elements. Table 6.1 shows the maximum ripple attained under the required SLL constraint, according to different array sizes; as well as the computation time for the array analysis and for the optimization procedure in each

(a) E-plane

(b) Ripple in the main lobe

FIGURE 6.3: Synthesized flat-top pattern for an E-plane linear array of 15 HDRA with the optimization performed using real and isotropic array elements. The case with real elements is compared with a commercial electromagnetic software.

case. The analysis time stands for the analytical computation of the overall GSM of the array (2.38) from the GSM of the isolated element (2.29). The FEM simulation of the HDRA, which has to be performed only once, takes 20 seconds. As observed, the proposed requirements are fulfilled with 15 antennas. Fig. 6.3 shows the radiation pattern obtained with the optimized excitations in this case. It is also represented the resulting radiation pattern when these excitations are applied to the array analyzed with the commercial software CST Microwave Studio [87] considering a finite metallic ground plane. An excellent agreement with that obtained from the present method is observed.

In order to illustrate the influence of mutual coupling and real element patterns, the same shaped beam synthesis has been performed with isotropic sources and the

(a) Linear array of HDRAs with random distribution

(b) E-plane

(c) E-plane Csc^2

FIGURE 6.4: Synthesized cosecant squared pattern (b and c) for an E-plane non-uniform random linear array of 15 HDRAs (a) with the optimization performed using real and isotropic array elements.

FIGURE 6.5: Geometry of the circular and cavity backed patch antenna used as array element in example IV.C. with the following characteristics: $R_1 = 30$ mm, $R_2 = 24.75$ mm, $c_1 = 1.524$ cm, $c_2 = 3.976$ cm, $t = 2$ mm $x_0 = y_0 = 6.2$ mm, $\epsilon_{r1} = 2.62$, $\epsilon_{r2} = 1.0$, $\epsilon_{rx} = 1.9$, $r_1 = 0.65$ mm and $r_0 = 2.05$ mm.

optimized excitations are applied to the 15-element HDRA array. The resulting radiation pattern, also shown in the figure, clearly does not fulfill the specified maximum ripple, and the first sidelobe level is bigger than that obtained using the present method. These discrepancies are the result of the electromagnetic performance of practical antenna elements.

The proposed synthesis methodology will be directly applicable to aperiodic arrays. For example, the non-uniform random linear array of 15 HDRA in Fig. 6.4(a) is considered to achieve a cosecant squared pattern with a maximum ripple of 0.5 dB in the shaped beam region defined by $|\theta| \leq 20$ degrees, minimizing the SLL in the region $|\theta| \geq 30$ degrees. The synthesized pattern, given in Fig. 6.4(b) and Fig. 6.4(c), fulfills these specifications obtaining an SLL of less than -17 dB. It is also represented the resulting radiation pattern when the same optimization problem is solved using isotropic elements and the optimized excitations are applied to the array of resonators. As observed, the exclusion of the real radiation patterns and the inter-element coupling in the synthesis procedure results again in an undesirable result with a ripple of 0.75 dB and a maximum SLL of -15.5 dB.

The previous optimization problem may be stated in different ways, depending on the desired specifications; for example, by fixing the maximum SLL and minimizing the maximum ripple in the shaped region, or by fixing the maximum SLL and ripple and maximizing the slope in the transition between shaped and sidelobe regions.

6.3.3 Planar array of cavity-backed circular microstrip antennas

The cavity-backed circular patch antenna with the characteristics detailed in Fig. 6.5 is used as array element to perform the following examples. All the antenna characteristics are also considered in the synthesis process. In the first case, a planar array of 6×6 elements with an inter-element spacing of 0.5λ at the resonant frequency of 1.97 GHz is considered. The excitations to synthesize a square-shaped footprint pattern, represented

FIGURE 6.6: E- and H-plane cuts of the synthesized square-shaped footprint pattern for a 6×6 square array of cavity-backed circular microstrip antennas. The results obtained with the present method are compared with those of CST.

FIGURE 6.7: Rectangular and triangular masks used in the synthesis with planar arrays of cavity-backed circular microstrip antennas in Section 6.3.

FIGURE 6.8: Color map representation of the synthesized triangular-shaped flat-top pattern for a square array of 20×20 cavity-backed circular microstrip antennas.

TABLE 6.2: Number of variables of optimization (No. Var.), number of restrictions (No. Res.), time required for the array analysis (An.), number of iterations (No. it.) and time in the optimization process (Opt.)

Example	No. Var.	No. Res.	An.(s)	No. It.	Opt.(s)
A.1 (Fig. 1(a))	15	179	-	4	27
A.2 (Fig. 1(b))	15	179	-	5	43
B.1 (Fig. 3)	15	179	20.3	7	51
B.2 (Fig. 4)	15	179	20.3	8	55
C.1 (Fig. 6)	36	3650	35	23	1348
C.2 (Fig. 8)	400	8212	167	34	4342

in Fig. 6.7(a), covering the region R_{SB} , defined by $|u| \leq 0.35$ and $|v| \leq 0.35$ ($u = \sin \theta \cos \phi$ and $v = \sin \theta \sin \phi$) with a maximal response ripple of 0.75 dB have been optimized by minimizing the maximum SLL in the region R_{SL} , defined by $|u| \geq 0.57$ or $|v| \geq 0.57$. Fig. 6.6 shows the optimized radiation patterns in E - and H -planes together with the resulting patterns obtained with the optimized excitation weights applied to a simulation carried out with CST. As observed, they satisfy the imposed

constraints achieving a maximum SLL of -14 dB, and comparing very well with the analysis obtained using the commercial software.

The shaped beam patterns of antenna arrays in satellite communications usually have to illuminate a small range of angles with a small ripple, while radiating a negligible power to the remainder of the directions. As an example, a larger planar array of 20×20 microstrip patch antennas with an inter-element spacing of 0.5λ is considered. A triangular-shaped flat-top pattern represented in Fig. 6.7(b) covering the region R_{SB} , defined by $|u| \leq 0.12$, $|v| \leq 0.12$ and $v \geq u + 0.6$, with a maximum ripple of 0.5 dB is synthesized by minimizing the maximum SLL in the region R_{SL} , defined by $|u| \geq 0.17$, $|v| \geq 0.17$ and $v \leq u + 0.8$. These specifications are fulfilled achieving a maximum SLL of -13 dB as shown in the color map representation of the synthesized pattern in Fig. 6.8.

In order to evaluate the computational performance and dimensionality of the optimization problem (6.7) for the examples in this section, the number of variables, the number of iterations, and the time required for the analysis and the optimization process are shown in Table 6.2. The number of restrictions in (6.7) determined by the number of sampled points in the discretization, which has been carried out with a constant step in θ and ϕ in the radiating region, is also shown.

Chapter 7

Gradient-based array synthesis of real arrays with uniform amplitude excitation including mutual coupling

7.1 Introduction

Well behaved problems, or convex problems, are optimally and efficiently solved with convex optimizations as shown in Chapter 4. Some other problems can be transformed into convex with some additional restrictions, as in Chapter 5, or relaxing the non-convex restrictions, as in Chapter 6. Most of the array synthesis problems, such as shaped beam designs, phase-only optimizations, position-based synthesis, array thinning or rotation-based designs are non-convex problems and consequently they are difficult to solve. Most of them are solved using evolutionary programming, such as genetic algorithms [57, 58, 103–105]. These kinds of methods remains a viable approach for small arrays. However, their populations grow with problem’s size and requires very high computation time or even the problem become infeasible for certain applications. Some other works based on density taper techniques, such as [106] have been presented as well. A deeper understanding of the problem may lead to acceptable starting points that are iteratively improved with local methods, which have a computational cost much lower than global methods. Some gradient-based methods have also been presented for array synthesis problems in [107, 108].

In this chapter, two local methods for the radiation pattern synthesis of coupled arrays are proposed. In the first one, described in Section 7.2, the variables of the problem are the positions of the elements and in the second one, described in Section 7.3, the variables of the problem are the rotations of the elements. Different starting points are iteratively improved minimizing a cost function that involves the radiation intensity of the coupled array. They are based on the gradient-based method presented in [109] for a radiation pattern synthesis of isotropic elements.

7.2 Pattern synthesis of aperiodic and coupled antenna arrays

Although sparse or aperiodic arrays were first studied more than four decades ago [110], they recently demonstrated to be a promising, and a challenging, technology for different kinds of applications, such as low frequency radio telescopes, [111, 112], satellite communications [113] or SAR observations [114]. Sparse array antennas are adequate for large aperture radio telescopes because they are required to work in an ultra-wide frequency band. In order to avoid an over-sampled array at the lower frequencies, the array has to be sparse at the upper ones [115]. Two main drawbacks appear when a phased array is considered: really high cost due to the high number of elements and the poor efficiency induced by the amplitude tapering. Using sparse arrays the number of elements, and consequently the number of control points, can be drastically reduced decreasing the cost of conventional phased arrays. With a uniform amplitude excitation, allowing a phase variation to steer the beam, the second drawback is also avoided because every amplifier works at its optimal level.

Sparse arrays have also some drawbacks, compared with regular array distribution. The number of elements can be reduced without modifying the beam width but the aperture efficiency is going to be poorer than the one of a regular array fully populated and uniformly excited, independently of the design process. There are some approaches in order to mitigate this effect such as designing elements of different sizes [116] or interleaved sub-arrays [117]. Another difficulty arises from the designer point of view: the complexity grows. Some approaches used in the analysis of equispaced arrays, as imposing periodic boundaries, cannot be used here, and a large full wave analysis may be needed in order to have an accurate characterization of the radiating structure. Rigorous and efficient analysis methods that can be employed for large and sparse arrays have been proposed over the last decade. Some of them are based on Macro Basis Function, [8, 9], where a reduction in the number of unknowns is accomplished with a negligible error. Another approach was presented in [10] where the elements are enclosed in spheres

or hemispheres, while the field radiated by each element is expressed as an expansion of spherical waves. In that way, the elements can be individually analyzed obtaining a much smaller electromagnetic problem.

The optimization of aperiodic arrays induces also a more complex problem from the synthesis point of view, due to the sparse distribution of the problem. Indeed, most of the classical synthesis methods usually applied to periodic arrays, e.g. [12, 14], cannot be applied here. But the complexity of the problem is not only due to its novelty. In an equispaced or periodic array, the degrees of freedom are the excitation weights which have a linear behavior and for which optimal solutions can be found efficiently [23].

However, sparse arrays, where the location of the elements are the variables of the problem, have a highly non-convex behavior which causes an np-hard problem that is difficult to deal with [22]. Non uniformly excited sparse arrays can accomplish very stringent specifications but their reduction in the cost of the phased array is not as big as when the elements are uniformly excited, and they still suffer from poor efficiency due to the amplitude variation. Thinned arrays can reduce the number of elements but they may suffer from periodic distribution associated drawbacks if the thinning is not very high. Here we will focus on uniformly excited sparse arrays. Several methods are available in the literature for synthesizing aperiodic, sparse or thinned arrays. Some of these methods are based on global, and computationally intensive approaches such as in [118], and some others are based on density tapering techniques proposed by [106], as [119]. On the other hand, in most of these methods the mutual coupling and the full wave analysis of realistic elements are not considered because it is not easy to merge an electromagnetic analysis into an array synthesis process. In some other works, it is argued that as the elements are placed in a sparse grid, the mutual coupling is not significant and can be avoided [119].

The analysis methods mentioned above deal with fixed positions and the synthesis techniques often consider the elements as isotropic sources. Other array synthesis methods account for mutual coupling but with fixed positions. A synthesis method is presented in [120] where the authors proposed a sparse array synthesis that can incorporate the mutual coupling represented with an impedance matrix, based on the work presented in [121]. But as commented in the latter reference, this kind of simplification is valid only for certain kinds of antennas that can be cast as minimum-scattering antennas. In [122] the mutual coupling effect is computed after the synthesis in order to evaluate its importance, but it is not part of the optimization process. In [123] a method for sparse arrays is presented where a convex problem is solved iteratively in order to get some desired specification with the minimum number of elements. In [124] the positions of the elements of an aperiodic array, uniformly excited in amplitude, are

synthesized solving iteratively convex problems as well. In both papers, the elements are considered to be isolated and there is not a mutual coupling study. In [125] a similar iterative procedure is exploited, where at each iteration the mutual coupling of a linear array is computed but it is not taken into account at the decision step regarding the direction in which the elements should be moved.

In this section a local optimization procedure for the synthesis of aperiodic arrays is proposed, it is based on the gradient algorithm presented in [109] for isotropic sources, and on the analysis method presented in [10] and described in Section 2. The analysis method is based on a description of the radiated field as an expansion of spherical waves. The array elements are characterized from a full wave analysis technique and the mutual coupling between them is rigorously taken into account. In [109], the gradient method is part of a three step algorithm. At each iteration a cost function, involving the array factor, is obtained and its gradient, w.r.t. the coordinates of the elements, is computed. According to the result of the gradient, the elements are moved (a desired step) in the appropriate direction, checking at each movement that the elements do not overlap and that the area covered by the array satisfies the imposed limits. As the method is local and the problem is not convex the solutions obtained will not be claimed to be the global solutions of the problem since this solutions will depend on the starting point. Taking this into account, different starting positions will be considered in order to provide the best possible solution.

7.2.1 Proposed synthesis method

In the following section the synthesis method is detailed. A cost function that involves the radiation intensity of the coupled array is obtained, a weighting function depending on the direction is computed and analytical expressions are obtained for the gradient of the cost function w.r.t. elements positions.

7.2.1.1 Cost function, constraints and optimization procedure

The optimization method looks for the synthesis of the radiation pattern of antenna arrays, while accounting for realistic radiating elements as well as the mutual coupling between them. The elements distribution in the non-regular array that minimizes the sidelobe level (SLL), while fixing a mainbeam with a fixed width, is optimized. The secondary lobes are minimized via a cost function that involves an average of the SLL weighted by a desired function. The local method proposed here iteratively moves a given number of elements in a defined area (circular for the planar array), while fixing a minimum distance between elements in order to avoid overlapping. The elements are

uniformly excited in amplitude, and a linear phase distribution can be imposed in order to steer the beam.

As proposed in [109], instead of looking for minimizing exclusively the maximum SLL, the minimization of a certain type of average measure of the sidelobe level is considered. The cost function is obtained by averaging the radiation intensity and using a weighting function, $W(\hat{u})$, to focus on selected regions. In this way the cost function to be minimized is defined as:

$$CF = \left(\int_U [W(\hat{u}) |\vec{E}(\hat{u})|^2]^p d\hat{u} \right)^{1/p} \quad (7.1)$$

where U is the desired integrating region where the SLL has to be minimized and p stands for the L_p -norm. A low number of p is used when an average of the SLL is desired while a higher value will make more emphasis on peaks of the radiated intensity in the sidelobe region.

The cost function for the coupled antenna array is obtained by substituting in (7.1) the radiation intensity of the coupled antenna array which is obtained directly from the expression of the radiated field (2.43) as:

$$\begin{aligned} |\vec{E}(\hat{u})|^2 &= |(\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{T}_G \mathbf{v}|^2 = \\ &= \mathbf{v}^H \mathbf{T}_G^H (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^H \cdot (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{T}_G \mathbf{v} = \mathbf{v}^H \mathbf{P} \mathbf{v} \end{aligned} \quad (7.2)$$

where $\cdot$ stands for the dot-product and the superscript H stands for the Hermitian transpose and

$$\mathbf{P} = \mathbf{T}_G^H (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^H \cdot (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^T \mathbf{T}_G \quad (7.3)$$

has been defined.

Finally the cost function reads:

$$CF = \left(\int_U [W(\hat{u}) (\mathbf{v}^H \mathbf{P} \mathbf{v})]^p d\hat{u} \right)^{1/p}. \quad (7.4)$$

This expression will be differentiable with respect to the array-elements position in such a way that the gradient-based local optimization method proposed in [109] for isotropic elements will be applicable for realistic and coupled array elements. As shown in [109], the first step of the optimization process consists of the computation of the global gradient of the cost function with respect to the coordinates of every antenna of the array. The elements are then moved iteratively in the direction opposite to its corresponding partial gradient, along a distance obtained by the multiplication of the gradient by a constant step δ . A larger δ will require fewer iterations to obtain the

solution but if it is too large, the convergence can be modified. A minimum distance between elements is selected in order to make the array physically realizable and a maximum allowable distance with respect to the array center is selected. In every movement, these constraints are verified by checking that the elements do not overlap and do not get out of the dimension limits. If any of those happens, the elements are moved to the limit not allowing any violation of the constraints.

7.2.1.2 Weighting functions and initial distribution

The local method proposed here depends on the initial distribution of the array elements and the selected weighting function. A good starting point and a proper weighting function will lead to better results or require fewer iterations. Different initial distributions can be considered, as for example the sunflower distribution proposed in [126], those obtained with the classical method proposed by [106], with the gradient-based method for isotropic sources proposed in [109] or just a random or a regular distribution. The beam width of an array is generally insensitive to the distribution and to the number of elements. It directly depends on the aperture length of the array, defined by the two most distant elements of the array. A small beam width can be obtained with a small number of elements if they are placed non uniformly, but the secondary lobes will increase if the distance between elements grows. For an equispaced array the highest secondary lobes are usually placed close to the main beam, this is not the case for non uniformly distributed arrays, especially when they are sparse.

The utilization of the weighting function $W(\hat{u})$ provides more degrees of freedom to the designer because, depending on the desired radiation pattern and on the initial distribution, more importance can be imposed to specific ranges of directions. $W(\hat{u})$ has to be selected wisely in order to minimize the radiated field in desired regions. Some prior knowledge can be applied to the weighting function if the number of elements and the maximum allowed size of the array is known in advance. If the starting point of the synthesis is a uniform distribution and the objective is to minimize the highest secondary lobe level, a weighting function that emphasizes lobes near the mainlobe would be the most adequate. In Fig. 7.1 different weighting functions used in this work are represented. If a sparse distribution is employed as an initial distribution, depending on the distance between elements, the highest secondary lobes will appear in different directions. For example, if the density method presented in [106] is selected to distribute the elements, the following study may be useful to know in advance where the highest secondary lobes are situated. As a proof of concept, the example of a linear array of isotropic sources with a total length of $20 \lambda_0$ is considered. Different array configurations can be designed by varying the number of elements, and consequently

FIGURE 7.1: Different weighting functions used in this work.

the average distance between elements varies. In Fig. 7.2 the results from the density synthesis of linear arrays with an average distance between $0.71 - 1.67$ wavelengths, corresponding to $28 - 12$ elements, are represented. In examples with a low number of elements, it can be observed that the secondary lobes are higher in the *non-coherent region* [109]. Depending on the average distance between elements, these secondary lobes appear for smaller or bigger angles while the beam width of the main lobe is not modified. Taking this into account, different weighting functions will be chosen in order to focus on different regions depending on the starting configuration. Similar procedures can be performed for the different starting points.

7.2.1.3 Gradient of the cost function of the coupled array

The cost function was defined in (7.1) in terms of the position of the elements of the array. Next, the gradient of the cost function w.r.t. the coordinates of every antenna (x for a linear array, x and y for a planar array) is computed as follows:

$$\frac{\partial CF}{\partial x_i} = (CF)^{1-p} \int_u W(\hat{u})^p [\mathbf{v}^H \mathbf{P} \mathbf{v}]^{p-1} \frac{\partial (\mathbf{v}^H \mathbf{P} \mathbf{v})}{\partial x_i} d\hat{u}. \quad (7.5)$$

FIGURE 7.2: Radiated field and the appearance of secondary lobes for using an array of isotropic elements placed with the initial distribution obtained with the density synthesis. The study is performed for a fixed array length ($20 \lambda_0$) and varying the average distance between elements, $0.71 - 1.67\lambda_0$ (indicated in the graphics), and consequently the number of elements, $28 - 12$.

The derivative of the radiation intensity in (7.5), is computed by substituting (7.3) and reads:

$$\frac{\partial |\vec{E}(\hat{u})|^2}{\partial x_i} = \mathbf{v}^H \frac{\partial \mathbf{P}}{\partial x_i} \mathbf{v} = \mathbf{v}^H \frac{\partial \left[\mathbf{T}_G^H (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^H \cdot (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{T}_G \right]}{\partial x_i} \mathbf{v}. \quad (7.6)$$

This expression is computed applying properties of the derivatives of products and taking into account the factors in $\mathbf{P}$ that depend on the positions. Two derivatives are computed separately. On the one hand, the term that relates the exponential function and the spherical modes is derived as:

$$\frac{\partial (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})}{\partial x_i} = jk (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \frac{\partial (\hat{u} \cdot \vec{u})}{\partial x_i} \quad (7.7)$$

On the other hand, the transmission matrix of the finite array, $\mathbf{T}_G$, which has been previously defined in (2.39), can be rewritten for simplicity as $\mathbf{T}_G = \mathbf{M}^{-1}\mathbf{T}$, where $\mathbf{M}$ is:

$$\mathbf{M} = \left[\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G} \right] \quad (7.8)$$

FIGURE 7.3: Coordinates of antennas j and k separated a distance d and definition of the angle ϕ_{kj} .

and the gradient of the transmission matrix is computed as:

$$\frac{\partial \mathbf{T}_G}{\partial x_i} = \frac{\partial \mathbf{M}^{-1}}{\partial x_i} \mathbf{T} = -\mathbf{M}^{-1} \frac{\partial \mathbf{M}}{\partial x_i} \mathbf{M}^{-1} \mathbf{T} = -\mathbf{M}^{-1} \left[-(\mathbf{S} - \mathbf{I}) \frac{\partial \mathbf{G}}{\partial x_i} \right] \mathbf{M}^{-1} \mathbf{T}. \quad (7.9)$$

The expression of the general translation matrix, $\mathbf{G}$, defined in (2.37) and which needs to be derived in (7.33) is provided in [10]. The particularization of the translation matrix between antennas i and j for planar arrays located as indicated in Fig. 7.3 on the xy plane, and with elements without rotation, is expressed as:

$$\mathbf{G}_{jk} = \left[\mathbf{R}_k(\phi_{kj}) \mathbf{D}_k\left(\frac{\pi}{2}\right) \mathbf{C}\left(\frac{d}{\lambda}\right) \mathbf{D}_j\left(-\frac{\pi}{2}\right) \mathbf{R}_j(-\phi_{kj}) \right]^T \quad (7.10)$$

where d and ϕ_{kj} are the distance between the antennas and the angle formed between a line joining them and a reference line, respectively. The matrices $\mathbf{R}_k$, $\mathbf{D}_k$ and $\mathbf{C}$ contain respectively the exponential function that relates the φ -dependence of spherical modes, the rotation coefficient and the axial translation coefficient, as detailed in [37].

The gradient of $\mathbf{G}_{jk}$ is then computed from (7.10) as:

$$\begin{aligned} \frac{\partial \mathbf{G}_{jk}}{\partial x_k} = & \left[\frac{\partial \mathbf{R}_k(\phi_{kj})}{\partial \phi_{kj}} \frac{\partial \phi_{kj}}{\partial x_k} \mathbf{D}_k\left(\frac{\pi}{2}\right) \mathbf{C}\left(\frac{d}{\lambda}\right) \mathbf{D}_j\left(-\frac{\pi}{2}\right) \mathbf{R}_j(-\phi_{kj}) \right]^T + \\ & \left[\mathbf{R}_k(\phi_{kj}) \mathbf{D}_k\left(\frac{\pi}{2}\right) \frac{\partial \mathbf{C}\left(\frac{d}{\lambda}\right)}{\partial d} \frac{\partial d}{\partial x_k} \mathbf{D}_j\left(-\frac{\pi}{2}\right) \mathbf{R}_j(-\phi_{kj}) \right]^T + \\ & \left[\mathbf{R}_k(\phi_{kj}) \mathbf{D}_k\left(\frac{\pi}{2}\right) \mathbf{C}\left(\frac{d}{\lambda}\right) \mathbf{D}_j\left(-\frac{\pi}{2}\right) \frac{\partial \mathbf{R}_j(-\phi_{kj})}{\partial \phi_{kj}} \frac{\partial \phi_{kj}}{\partial x_k} \right]^T. \end{aligned} \quad (7.11)$$

The derivative with respect to the y -coordinate is computed following the same procedure. In case of linear arrays, the expression (7.11) can be simplified because ϕ_{kj} is 0°

or 180° degrees depending on the relative position between the antennas. It then reads:

$$\frac{\partial \mathbf{G}_{jk}}{\partial x_k} = \left[\mathbf{R}_k(\phi_{kj}) \mathbf{D}_k \left(\frac{\pi}{2} \right) \frac{\partial \mathbf{C}(\frac{d}{\lambda})}{\partial d} \frac{\partial d}{\partial x_k} \mathbf{D}_j \left(-\frac{\pi}{2} \right) \mathbf{R}_j(-\phi_{kj}) \right]^T. \quad (7.12)$$

Although just translations are considered, the rotation matrices $\mathbf{R}_k$ and $\mathbf{R}_j$ also needs to be considered. This is because the translation is performed as a combination of rotation-axial translation-rotation, which has been proven to be computationally more efficient in [127]. The elements of matrix $\mathbf{R}_k$ are computed straightforwardly as they are composed of exponential functions [37].

The coefficients of the axial translation matrix as defined in [37] are:

$$C_{\sigma\mu\gamma}^{sn(c)}(kA) = \sqrt{\frac{(2n+1)(2\gamma+1)}{n(n+1)\gamma(\gamma+1)}} \sqrt{\frac{(\gamma+\mu)!(n-\mu)!}{(\gamma-\mu)!(n+\mu)!}} (-1)^\mu \frac{1}{2} i^{n-\gamma} \quad (7.13)$$

$$\sum_{p=|n-\gamma|}^{n+\gamma} \left[i^{-p} (\delta_{s\sigma} \{n(n+1) + \gamma(\gamma+1) - p(p+1)\} + \right.$$

$$\left. \delta_{3-s,\sigma} \{2i\mu kA\}) a(\mu, n, -\mu, \gamma, p) h_p^{(c)}(kA) \right]$$

for a translation of a distance A in the z axis. Where $a(\mu, n, -\mu, \gamma, p)$ is a linearization coefficient defined by the expansion of two unnormalized associated Legendre functions. s distinguishes between the two spherical wave functions depending on if it has a radial component or it is purely transverse. n indicates the degree of the wave function and m stands for the order of the spherical wave function. c indicates the particular function in the radial dependencies, in this work $c = 1$ for spherical Hankel functions, $h_p^{(c)}$, of the first kind and $c = 2$ for spherical Hankel functions of the second kind depending on the direction of the propagated wave. They are respectively:

$$h_n^{(1)}(kA) = j_n(kA) + in_n(kA) \quad (7.14)$$

$$h_n^{(2)}(kA) = j_n(kA) - in_n(kA) \quad (7.15)$$

In (7.13) just two factors are distance-dependent: $h_p^{(c)}(kA)$ and $(kA)h_p^{(c)}(kA)$. The derivative of the spherical Hankel functions of first and second kind are computed as:

$$\frac{\partial h_n^{(1)}(kA)}{\partial(kA)} = \frac{n}{kA} j_n(kA) - j_{n+1}(kA) + i \frac{n}{kA} n_n(kA) - in_{n+1}(kA) \quad (7.16)$$

$$\frac{\partial h_n^{(2)}(kA)}{\partial(kA)} = \frac{n}{kA} j_n(kA) - j_{n+1}(kA) - i \frac{n}{kA} n_n(kA) + in_{n+1}(kA) \quad (7.17)$$

TABLE 7.1: Synthesized positions of the HDRAs in example IV.A.

No. elem.	1	2	3	4	5	6	7	8	9
Pos. (cm)	-57.7	-52.2	-43.6	-36.3	-29.6	-22.6	-16.0	-9.2	-3.3
No. elem.	10	11	12	13	14	15	16	17	18
Pos. (cm)	2.5	8.7	15.5	22.2	29.2	36.3	43.6	52.2	57.7

FIGURE 7.4: Synthesized positions of the 18 HDRAs represented with blue dots. The red circles stand for the hemispheres in which the resonators are enclosed.

The derivative of the product $(kA)h_p^{(c)}(kA)$ is computed as [37]:

$$\frac{\partial}{\partial(kA)} \left((kA)h_n^{(c)}(kA) \right) = (n+1)h_n^{(c)}(kA) - (kA)h_{n+1}^{(c)}(kA) \quad (7.18)$$

Following the steps presented in this sub-section, the gradient of the proposed cost function is obtained. The cost function takes the mutual coupling between elements into account in a rigorous way, in which the radiated field is expressed as spherical wave expansions. In this way, the gradient is obtained and solved very efficiently. At each iteration, the general transmission matrix (2.37) and its derivatives (7.11) versus each element's positions is obtained. These are the most time consuming computations of the method but, as they are obtained analytically, each computation is performed very efficiently. The synthesis of a linear array of 10 to 20 elements can be performed between less than 1 and 3 minutes on a personal laptop. Larger linear arrays of 50 elements can be synthesized in 15 – 20 minutes, while planar arrays between 20 and 60 elements will be optimized within 1 – 6 hours.

7.2.2 Numerical results

In order to validate and demonstrate the capabilities of the present method, some linear and planar arrays will be synthesized using different initial distributions, array specifications, weighting functions and array elements.

7.2.2.1 Linear array of hemispherical dielectric resonator antennas

In this example, an E-plane linear array of 18 hemispherical dielectric resonator antennas is synthesized, over a maximum length of $14 \lambda_0$ at the resonance frequency of 3.64 GHz.

FIGURE 7.5: Radiation pattern of an 18-element HDRA linear array in E-plane with optimized positions obtained from realistic HDRAs (black line) and from isotropic elements (red line). The dashed gray line stands for the maximum SLL obtained in the first case.

The geometry of the array radiator, obtained from [102], is detailed in Fig. 6.2. The array is designed with a main beam width of $R_i = 0.06$ and the weighting function used in this example is [109]:

$$W(\hat{u}) = \frac{1}{2} \left[1 - \sin \left(\frac{(\beta - \frac{1+R_i}{2})\pi}{1 - R_i} \right) \right] \frac{1}{\beta^q} \quad (7.19)$$

β is the norm of the vector (u_x, u_y) defined as $\beta = ||u_x, u_y||$, and q allows a softer or sharper variation of W . For this example, $q = 0.5$ and $p = 2$ have been selected. This function gives more importance to the secondary lobes that are close to the mainbeam and it has demonstrated to be the best weighting function for this design.

The classical density taper technique [106] is employed as starting point. In order to emphasize the importance of the electromagnetic analysis in the synthesis process, this synthesis is carried out in two situations. In the first case, the proposed method is used to synthesize the coupled array, and in the second case the synthesis method is applied considering isotropic elements. The optimized positions in the first case are detailed in Table 7.1 and represented in Fig. 7.4. The result of the synthesis in both cases are used to distribute real coupled arrays of HDRAs. The resulting radiation patterns are compared in Fig. 7.5. As observed, the simplification in case of the synthesis with isotropic sources

FIGURE 7.6: Geometry of the truncated tetrahedral dielectric resonator antenna used as array element: $h = 2.4$ cm, $L_u = 6.4$ and $L_l = 2.5$ cm with $\epsilon_r = 12$. The feed properties are $w_x = 0.55$ mm and $w_l = 1.15$ mm. Coaxial probe feed (50Ω): $r_i = 0.5$ mm, $r_o = 1.51$ mm and $\epsilon_{r_x} = 1.73$.

FIGURE 7.7: Value for the cost function versus the number of iterations at each frequency and the total value for the synthesis of the wideband array.

leads to unwanted results with a higher SLL. The cross-polar component of the field is negligible in all considered cases.

7.2.2.2 Linear array of truncated tetrahedral dielectric resonator antennas

It is well known that the bandwidth of a periodic array is inversely proportional to the array size [128, Ch. 8. Sect. 3]. Indeed, it is difficult to preserve some array characteristics, as the SLL, over a large bandwidth. The arbitrary distribution of the elements makes this method suitable for synthesizing wideband arrays. To synthesize the array over a wide frequency band, the cost functions analyzed at different frequencies are combined with an emphasis on the upper and lower frequencies of the desired band. This leads to a multiplication of the number of points in the cost function by the number of frequencies considered.

FIGURE 7.8: Synthesized field radiation patterns versus frequency, in steps of 50 MHz, of the 40-element linear array of truncated TDRAs along the E-plane.

FIGURE 7.9: Field radiation patterns at 2.45 GHz of the 40-element linear array of truncated TDRAs along the E-plane with optimized positions obtained with the proposed method. The resulting pattern is compared with the obtained from the commercial software CST.

TABLE 7.2: Initial cost function (CF_i), final CF (CF_f), initial SLL (SLL_i), final SLL (SLL_f), number of iterations (It.) and time in the analysis and synthesis process (Ti.)

Example	CF_i	CF_f	SLL_i (dB)	SLL_f (dB)	It.	Ti.(min)
Circular	4.70	2.01	-7.3	-16.1	30	125.6
Random	8.83	4.32	-7.2	-15.5	22	82.8
Sunflower	8.4	3.9	-10.24	-15.0	26	100.1

The element that composes the array is the wideband truncated tetrahedral dielectric resonator antenna (TDRA), obtained from [129], and represented in Fig. 7.6. It has been designed to operate between 2 and 3 GHz. The array is designed to work with a 26% bandwidth (from 2 to 2.6 GHz). The specifications of the array comprise a linear configuration of 40 elements along the E-plane distributed over a maximum length of $60 \lambda_0$ at its central frequency. For this synthesis a uniform weighting function has been selected, for angles bigger than $R_i = 0.03$. The synthesis process is carried out computing the cost function for three different frequencies: 2, 2.3 and 2.6 GHz. The initial positions are obtained with the classical density taper technique [106] with a maximum SLL of -9 dB. The cost function is iteratively minimized as it is represented in Fig. 7.7. The result of the synthesis is represented in Fig. 7.8 where one can observe the radiated field over the complete frequency band, with steps of 50 MHz, obtaining a SLL lower than -14.5 dB for every frequency and angle.

For validation purposes, a comparison with the CST Microwave Studio [87] commercial software has been carried out with the elements placed with the distribution obtained in the synthesis process. The complete array has been analyzed at once with CST at the frequency of 2.45 GHz. In Fig. 7.9 the radiation pattern obtained with the presented method and with CST are compared. A very good agreement is observed between both simulation results.

7.2.2.3 Planar array of tetrahedral dielectric resonator antennas

In the following example, the tetrahedral resonator antenna represented in Fig. 7.6, used in the previous example, is also employed. In this case the planar array comprises 40 elements placed in a disk of radius $5\lambda_0$ at a frequency of 2.5 GHz. The desired radiation pattern is a mainlobe at broadside direction, with a beam width defined by $R_i = 0.125$. For this example different initial configurations and weighting functions have been selected looking for the best possible configuration. The initial distributions considered in this example are: the circular distribution (Fig. 7.10(a)), a random distribution (Fig. 7.10(b)) and the sunflower distribution [126] (Fig. 7.10(c)). In those representations, the black circle stands for the radius delimiting the array surface, the

FIGURE 7.10: (a-c): Initial configurations considered for the synthesis of a 40-element planar array of truncated TDRAs: circular, random and sunflower distributions respectively. (d-f): Synthesized distributions obtained with the present method. (g-h) Color map representation of the synthesized field radiation patterns at 2.5 GHz for the three different initial configurations and weighting functions.

FIGURE 7.11: Comparison between the radiation patterns at 6.1 GHz of a 40-element planar array of truncated TDRAs with synthesized positions obtained from isotropic elements (represented in red), and from realistic antennas (in blue). Three initial array configurations are considered: circular (a-b), random (c-d) and the sunflower (e-f).

blue dots stands for the exact position of each antenna while the red circles stand for the minimum separation between elements that is fixed in the design process. Each case has been synthesized with three different weighting functions: the function expressed in (7.19) and represented in Fig. 7.1(a), a slight modification of this one represented in Fig. 7.1(b):

$$W(\hat{u}) = \frac{1}{2} \left| 1 + \cos^q \left(\left(\beta + \beta_0 - \frac{1}{2} \right) \pi \right) \right| \quad (7.20)$$

where a focus on the region of secondary lobes centered at β_0 is performed, and a uniform distribution, represented in Fig. 7.1(c). In (7.20), $\beta_0 = 0.6$ is used following a previous study for selecting the proper weighting function, as commented in subsection 7.2.1.2.

A high p has been selected in order to focus on the highest secondary lobes ($p = 8$).

The SLL obtained with the circular, the random and the sunflower initial distributions are -7.3 , -7.2 and -10.24 dB respectively. Each initial distribution has been optimized with the three weighting functions; for every case the initial SLL has been improved but, as the method is non convex, different solutions have been obtained for the different configurations. For the circular distribution, the best solution has been obtained with the weighting function (7.19), obtaining a SLL of -16.1 dB. In the case where the random distribution has been selected as a starting point, the best possible solution has been obtained with a uniform weighting function, obtaining a SLL of -15.5 dB. However, for the sunflower distribution, the lowest SLL has been obtained with the weighting function (7.20), obtaining in this case a SLL of -15.0 dB. The convergence parameters are represented in Table 7.2, for each of them the analysis process takes between 6 and 7 seconds. For the synthesis process $\delta = \lambda_0/50$ has been selected and the convergence was terminated when the improvement become lower than 1%. The final distributions synthesized by the present method are represented in Figs. 7.10(d), (e) and (f), respectively. The field radiation patterns of the synthesized arrays are represented in Figs. 7.10(g), (h) and (i). Although the solutions obtained for each initial distribution are different, the maximum difference in SLLs is of 1.1 dB for this example.

The same array synthesis has been carried out considering the elements as isotropic and not taking into account the mutual coupling between them. In Figs. 7.11(a), (c) and (e) are represented the TDRA array radiation patterns, for phi-cuts in step of 1 degree, obtained with the optimized positions from isotropic sources, for the circular, random and sunflower initial distributions, respectively. For comparison, in Figs. 7.11(b), (d) and (f) are represented the radiation patterns synthesized from coupled array elements. As observed, the results obtained from the synthesis with isotropic elements have higher secondary lobes than the present method. It is important to notice that the mutual coupling significantly affects the array performance even for cases where the elements are not very close to each other, as in this example where the average distance between elements is higher than one wavelength. The SLL obtained from the circular, random and sunflower initial distributions with the isotropic synthesis are -13.2 , -12.7 and -11.3 dB respectively.

7.2.2.4 Planar array of microstrip patch antennas

For the latter example, the elements are excited with a uniform amplitude distribution and a linear phase taper in order to steer the beam towards a desired direction. The array element is a probe-fed and cavity-backed square microstrip patch antenna, obtained from

FIGURE 7.12: Synthesized field radiation pattern at 6.1 GHz of a 30-element planar array of cavity-backed microstrip antennas scanned at $u_{x0} = 0.3$ and $u_{y0} = 0.75$.

[10], and detailed in Fig. 5.6. The array is made up of 30 elements placed in a disk of radius $3\lambda_0$ at the resonant frequency of 6.1 GHz. A phase shift is obtained with the classic theory of array factor for scanning the array towards $u_{x0} = 0.3$ and $u_{y0} = 0.75$. At each iteration, an excitation phase is assigned to every element depending on its position $\mathbf{u}_i$ according to $\phi = k\hat{u}_0\mathbf{u}_i$. The elements positions are optimized to achieved a beam width defined by $R_i = 0.2$ while minimizing the average SLL. The starting point used in this example is the sunflower distribution and the weighting function that provides the best result is the one expressed in (7.20) and represented in Fig. 7.1(d). In this case $\beta = ||u_x - u_{x0}, u_y - u_{y0}||$. Fig. 7.12 represents the field radiation pattern synthesized with the present method, obtaining a maximum SLL of -14.5 dB.

7.3 Pattern synthesis of coupled antenna arrays via element rotation

Array synthesis stands for changing any array parameter looking for obtaining some desired characteristic in the array performance. The most common variables changed in the array synthesis are the excitation weights applied to the array elements. Very

good results can be obtained but it has some drawbacks, such as incremented costs or a decrease of array efficiencies. However, the excitation weights are not the only possible variables of the synthesis. In recent years, position-based synthesis, as the method presented in the previous section, have been presented [109, 113]. In the excitation synthesis, as well as in the position synthesis, the relationship between the co- and cross-polar components is mostly defined by the array element, both variations of the array elements equally affect this ratio, making it very difficult to control one of them without affecting the other one. There is also another degree of freedom to carry out the array synthesis: the elements rotation.

Elements rotation is useful for different applications, in this thesis it has been applied for two problems. On the one hand, the sidelobe level can be minimized while steering the mainbeam in a desired direction with a fixed width controlling the cross-polar component. On the other hand, an array with circular polarization can be obtained from properly rotated, and phased, linearly polarized elements.

Modifications on the radiation pattern can be accomplished without increasing the array costs or without decreasing the aperture efficiency with elements rotation. It is also interesting because the rotation significantly affects the array polarization, making it possible to decrease the cross-polar component in desired directions. This type of synthesis can also be carried out in combination with an excitation or position synthesis. A low sidelobe synthesis via a dipole rotation is proposed in [130] via genetic algorithms. A synthesis of planar arrays via element rotation is also presented in [131] through differential evolution and considering the elements as isolated.

Arrays made up of elements with circular polarization usually have more complex feed chains and losses in the polarizer circuits, if they are fed with more than one excitation port, or they suffer from narrower bandwidth if the circular polarization is accomplished with some modification of the radiators. A well-known alternative is to use sequentially-rotated linearly-polarized radiating elements [132, 133]. This method has the disadvantages of a poor diagonal plane behaviour, a limited bandwidth and lower gain comparing with an array made of circularly polarized elements [134]. The first two disadvantages are reduced with a random rotated array in [135] and the third can be enhanced as proposed in [136]. Elements rotation is also a well established technique in reflectarray antennas [137].

In this section, a synthesis method for array antennas via element rotation is presented. The array is rigorously and efficiently analyzed through a full-wave formulation which analytically includes the rotation of the elements and the mutual coupling effects [10], explained in Chapter 2. Some modifications to the original formulation must be performed in order to account to the elements rotation. The synthesis is carried out

with a modification of the gradient method proposed in [109] for a positions-based synthesis of isotropic elements, and in the previous section for coupled arrays. The rigorous expression of the radiation intensity of the coupled array is part of the cost function and its gradient w.r.t. the elements rotation is efficiently obtained, as it is computed analytically.

7.3.1 Analysis method accounting for elements radiation

The transmission matrix (2.39) and the expression of the field radiated by a coupled array (2.43) obtained in Chapter 2 does not account for element rotation. In this subsection, some modifications are performed to account to this characteristics rigorously. The transmission matrix of the array when there exists certain rotation between elements is:

$$\mathbf{T}_{G_F} = [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}_F]^{-1}\mathbf{T} \quad (7.21)$$

The general transmission matrix, $\mathbf{G}_F$, containing the rotation of the elements is computed as a matrix multiplication as:

$$\mathbf{G}_F = \mathbf{F}^H \mathbf{G} \mathbf{F} \quad (7.22)$$

where $\mathbf{F}$ is a diagonal matrix with exponential functions accounting for the rotation of the elements, having in the main diagonal $F_i = e^{-jm_s\Phi_i}$, where m_s is the order of the spherical wave function and Φ_i is the rotation angles for the antenna i .

The radiated far-field of the array is obtained in a rigorous way, accounting for coupling effects, as an expansion of spherical modes weighted by their complex coefficients, which in turn are computed from the transmission matrix of the array for an arbitrary excitation. In this way, considering a planar array of N antennas placed on the xy -plane, a common number M of spherical modes on the spherical ports for each antenna, and denoting by $\mathbf{e}_m(\theta, \phi)$ the expression of the electric field corresponding to the m -th spherical mode, and by b_{im} the complex coefficient for mode m in the antenna i , the radiated field is obtained applying superposition as:

$$\vec{E}(\theta, \varphi) = \sum_{i=1}^N \vec{e}(\theta, \varphi) \mathbf{b}_i e^{jk\hat{\mathbf{u}} \cdot \vec{\mathbf{u}}_i} \quad (7.23)$$

where $\vec{e}$ is a row vector given by $\vec{e} = (\vec{e}_1, \vec{e}_2, \dots, \vec{e}_M)$, $\mathbf{b}_i$ is a column vector containing the coefficients b_{iq} . $\mathbf{b}$ is now defined as $\mathbf{b} = \mathbf{F}\mathbf{T}_{G_F}\mathbf{v}$, as it accounts for the rotation of elements. $\hat{\mathbf{u}} = (u_x, u_y, u_z)$ is the unitary vector in spherical coordinates, and $\vec{\mathbf{u}}_i = x_i\hat{x} + y_i\hat{y}$ is the position vector of the antenna i . k is the wave number in free space.

The expression in (7.23) may be expressed in matrix form as:

$$\vec{E} = (\vec{e}(\theta, \varphi) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{b} \quad (7.24)$$

where

$$(\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) = (\vec{e} e^{jk\hat{u}\cdot\vec{u}_1}, \vec{e} e^{jk\hat{u}\cdot\vec{u}_2}, \dots, \vec{e} e^{jk\hat{u}\cdot\vec{u}_N}) \quad (7.25)$$

has been defined. The resulting expression of the radiated field is:

$$\vec{E}(\hat{u}) = (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{F} \mathbf{T}_{G_F} \mathbf{v} \quad (7.26)$$

This expression is valid for arrays with arbitrary spatial distributions and Φ -rotations.

7.3.2 Proposed synthesis method

In this section, a synthesis method via rotation of elements for coupled arrays, characterized by expression (7.26), is detailed. Starting from an initial configuration, a local and gradient-based optimization is established in which some desired characteristics, represented by a cost function, are iteratively pursued.

The proposed radiation pattern synthesis method aims at fixing a mainbeam with a desired width, while minimizing the sidelobe level (SLL) and cross-polar level in the defined areas. This is accomplished with successive rotations of the elements. A cost function is built up from the expression of the radiation intensity (7.26). Two cost functions are defined to separately deal with the co- and the cross-polar components. Depending on the desired polarization these components will be stated based on the Ludwig's third definition or by the left and right handed circular polarizations. Instead of looking for an average of the radiation intensity in every direction, a weighting function, $W(\hat{u})$, is introduced in order to impose distinct specifications for different radiation sectors. Consequently, different weighting functions are defined for the co- and cross-polar components looking for minimize them in desired directions, based on L_p -norm values as in [109], obtaining:

$$CF_{cp} = \left(\int_U [W_{cp}(\hat{u}) |\vec{E}_{cp}(\hat{u})|^2]^p d\hat{u} \right)^{1/p} \quad (7.27)$$

$$CF_{xp} = \left(\int_U [W_{xp}(\hat{u}) |\vec{E}_{xp}(\hat{u})|^2]^p d\hat{u} \right)^{1/p} \quad (7.28)$$

The global cost function is obtained then as the addition of (7.27) and (7.28).

The expression of the cost function is differentiable w.r.t. the elements' rotation angle Φ . A local and gradient-based optimization method is performed, similar to the one

FIGURE 7.13: Scheme of the elements rotation.

proposed in [109] for aperiodic arrays of isotropic elements, for coupled arrays where the array elements and the mutual coupling between them is rigorously taken into account. Firstly, the cost function is computed for an initial configuration, the gradient w.r.t. the elements rotation of the cost function is computed and the elements are rotated iteratively along the partial gradient multiplied by a constant step $\Delta\Phi$.

The cost function for the coupled antenna array is obtained by substituting in (7.27) and (7.28) the radiation intensity of the coupled antenna array, which is obtained directly from the expression of the radiated field (7.26), yielding:

$$\begin{aligned}
 |\vec{E}(\hat{u})|^2 &= |(\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{F} \mathbf{T}_{G_F} \mathbf{v}|^2 = \\
 &= \mathbf{v}^H \mathbf{T}_{G_F}^H \mathbf{F}^H (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^H \cdot (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{F} \mathbf{T}_{G_F} \mathbf{v} \\
 &= \mathbf{v}^H \mathbf{P} \mathbf{v}
 \end{aligned} \tag{7.29}$$

where $\cdot$ stands for the dot-product and the superscript H for the Hermitian transpose.

The gradient of the cost function in terms of the rotation of each element of the array is obtained as follows:

$$\frac{\partial C_{F_\chi}}{\partial \Phi_i} = (C_{F_\chi})^{1-p} \int_u W_\chi(\hat{u})^p [\mathbf{v}^H \mathbf{P}_\chi \mathbf{v}]^{p-1} \frac{\partial (\mathbf{v}^H \mathbf{P}_\chi \mathbf{v})}{\partial \Phi_i} d\hat{u} \tag{7.30}$$

where χ stands here for cp or xp .

The derivative of the radiation intensity in (7.29) is computed as:

$$\begin{aligned}
 \frac{\partial |\vec{E}_\chi(\hat{u})|^2}{\partial \Phi_i} &= \mathbf{v}^H \frac{\partial \mathbf{P}_\chi}{\partial \Phi_i} \mathbf{v} = \\
 &= \mathbf{v}^H \frac{\partial \left[\mathbf{F}^H \mathbf{T}_{G_F}^H (\vec{e}_\chi(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \cdot (\vec{e}_\chi(\hat{u}) e^{jk\hat{u}\cdot\vec{u}})^T \mathbf{T}_{G_F} \mathbf{F} \right]}{\partial \Phi_i} \mathbf{v}
 \end{aligned} \tag{7.31}$$

The transmission matrix of the finite array, $\mathbf{T}_{G_F}$, which has been previously defined in (7.21), can be rewritten for simplicity as $\mathbf{T}_{G_F} = \mathbf{M}^{-1}\mathbf{T}$, where $\mathbf{M}$ is the only term which is Φ -dependent, resulting in:

$$\mathbf{M} = [\mathbf{I} - (\mathbf{S} - \mathbf{I})\mathbf{G}_F] \quad (7.32)$$

and the gradient of the transmission matrix can be computed as:

$$\frac{\partial \mathbf{T}_{G_F}}{\partial \Phi_i} = \mathbf{M}^{-1}(\mathbf{S} - \mathbf{I}) \frac{\partial \mathbf{G}_F}{\partial \Phi_i} \mathbf{M}^{-1}\mathbf{T} \quad (7.33)$$

The gradient of the general transmission matrix is then computed as:

$$\frac{\partial \mathbf{G}_F}{\partial \Phi_i} = \frac{\partial \mathbf{F}^H}{\partial \Phi_i} \mathbf{G}_F + \mathbf{F}^H \mathbf{G} \frac{\partial \mathbf{F}}{\partial \Phi_i} \quad (7.34)$$

The matrix $\mathbf{G}$ is Φ independent, which makes the method very efficient.

The initial configurations presented in this work are arrays with uniformly aligned elements for linear polarization, or sequentially rotated arrays, for circular polarization, but some other array configurations, regarding elements position or element rotation, may be used as well. A good starting point will provide better results or will converge with fewer iterations.

A phase variation is needed to obtain the circular polarization from linearly polarized elements. In [132], the angle of rotation of the elements is also employed as the excitation phase. Two different methodologies have been developed in order to consider the phase of the elements, both methods consider the sequentially rotated technique as a starting point. In the first one, the excitation can be fixed following the sequential rotation technique, the successive variation of the rotation of the elements does not affect to the excitation phase which remains fixed. However, better results have been obtained if the excitation phase is fixed to the rotation angle and thus, it has to be considered in the gradient computation. The excitation phase is an exponential function and it is directly derived in the cost function using the derivative's chain rule.

The excitation amplitude weights are not part of the optimization process. The method is especially indicated for arrays uniformly fed in amplitude obtaining in this case high values of the gain of the array. Arbitrary excitation weights can be applied to the array elements, a convex optimization process could be established with fixed positions and rotation angles. Steered beams can also be designed, in the case of circular polarization, the phase applied to the array element is computed as the result of the contribution from the phase accounting for the rotation and the phase needed to steer the beam, yielding [135]: $\Phi = \Phi_{rot} + \Phi_{ste}$.

TABLE 7.3: The number of elements (N.) and their correspondence synthesized rotation (Φ_{rot}) in degrees.

N.	1	2	3	4	5	6	7
Φ_{rot}	2.3	13.6	7.3	12.2	-0.4	-7.8	-0.5
N.	8	9	10	11	12	13	14
Φ_{rot}	5.1	-3.9	3.6	-20.4	48.1	-51.3	14.8

FIGURE 7.14: Synthesized rotations of the 14 patch antennas.

7.3.3 Results

In this section, linear and planar arrays with linear and circular polarizations are designed in order to demonstrate the capabilities of the proposed method.

7.3.3.1 Linear array with linear polarization composed of square and cavity-backed patch antennas

In the first example, an E-plane linear array made of 14 coaxial probe-fed and cavity-backed square patch antennas is synthesized. The radiating elements are uniformly excited and are regularly placed at a distance of $0.6\lambda_0$ at their resonance frequency of 6.1 GHz. The geometry of the radiating element, obtained from [10], is shown in Fig. 6.5. The objective is to minimize the sidelobe level defined for $|u| \geq 0.13$, keeping a constant mainbeam width and controlling the cross-polar level at acceptable levels.

In Fig. 7.17 is represented the cost function of the 14-element patch linear array, it is particularized for the co- and cross-polar components and for the total value.

The weighting functions applied to the co- and cross-polar components, represented in the Fig. 7.15, are:

$$W_{cp}(\hat{u}) = \frac{1}{2} \left[1 - \sin \left(\frac{(\beta - \frac{1+R_i}{2})\pi}{1 - R_i} \right) \right] \frac{1}{\beta^q} \quad (7.35)$$

$$W_{xp}(\hat{u}) = \frac{1}{2} (1 - \beta^2) \quad (7.36)$$

β is the norm of the vector (u_x, u_y) defined as $\beta = ||u_x, u_y||$, and q allows a softer or sharper variation of W , as explained in [109]. For this example, $q = 0.5$ and $p = 6$ have been selected. The weighting function of the co-polar component, W_{cp} , gives more

FIGURE 7.15: Radiation pattern of an 14-element patch linear array in E-plane with optimized rotations obtained from realistic patch antennas. The un-rotated co-polar component and the co-and cross-polar components are represented for the synthesized array with rotated elements. The weighted functions applied to both components are also represented.

importance to the secondary lobes that are close to the mainbeam while the weighting function of the cross-polar component, W_{xp} , focuses on the direction of the mainbeam.

The cost function of the co- and cross-polar components and the total cost function are represented in Fig. 7.17. The synthesized angles of rotation are detailed in Table 7.3 and represented in Fig. 7.14. The circle represents the spherical port in which the antennas are enclosed while the arrow represent the rotation of the corresponding element. In Fig. 7.15, the co-polar component of the non-rotated array (the cross-polar component has a much lower level) and the co- and cross-polar components of the synthesized array are also represented. The un-rotated array has the common sidelobe level for uniformly excited and distributed arrays of -13.3 dB. The maximum co-polar level is minimized to -16.1 dB with the synthesized rotations while the maximum cross-polar component is lower than -18.9 dB in every direction of space. In this example, a considerable reduction in the sidelobe level is accomplished without increasing the fabrication costs at expenses of a slightly higher cross-polar level. The latter caused however has been increased, it has been kept at acceptable levels.

The results are compared with an array simulated with the commercial software CST [87] from the synthesized results. As shown in Fig. 7.16, the results compare very

FIGURE 7.16: Comparison between the array pattern obtained with the proposed method and the radiated field pattern obtained for an array with the synthesized rotations simulated in the commercial software CST.

FIGURE 7.17: Cost function per iteration of the synthesis of the 14-element patch linear array.

well for the co- and cross-polar components. The bigger differences are in angles between 0.8 and 1, this is due to the method presented in [10] considers the array surrounded by an infinite metallic plane while the array simulated in CST is surrounded by a finite metallic plane.

FIGURE 7.18: : 360 phi-cuts of the co-polar, (a), and cross-polar, (b), components of the radiation pattern of the synthesized array made of 10×10 HDRAs.

7.3.3.2 Planar array with linear polarization composed of hemispherical dielectric resonator antennas

In this second example, a linearly polarized planar array of 10×10 hemispherical dielectric resonator antennas (HDRAs) is synthesized. The geometry of the array element, obtained from [102], is detailed in Fig. 6.2. The elements are also uniformly excited

FIGURE 7.19: Synthesized rotations of the 10×10 HDRAs.

and are regularly placed with an inter-element distance of $0.5\lambda_0$ at the resonance frequency of 3.64 GHz. The desired characteristics of the radiation pattern are a broadside main beam while the sidelobe level, defined for $|\mathbf{u}| \geq 0.2$ is minimized. In this example $q = 0.5$ and $p = 8$, a uniform weighting function is used for the co-polar and cross-polar components, looking for a uniform maximum value in both of them.

The starting point is the non-rotated array. The synthesis is carried out obtaining a maximum sidelobe level of -20.1 dB represented in Fig. 7.18(a) and the maximum of the cross-polar component, represented in Fig. 7.18(b), is -19.9 dB. Comparing with the non-rotated array, the sidelobe level has been decreased by almost 7 dB while the cross-polar component has been incremented but kept under acceptable levels.

In these examples, the elements have a low cross-polar component and one may afford a slight increment of that level to optimize some other characteristics. The opposite procedure could also be followed; an array composed of elements with a high cross-polar component could also be synthesized in order to minimize the cross-polar component in desired directions.

FIGURE 7.20: (a): 6×6 sequentially rotated elements used as an initial configuration. (b): Synthesized rotations.

TABLE 7.4: Number of variables of optimization (No. Var.), time required array analysis in seconds: of the isolated element (Elem.) and of the coupled array (Array), number of iterations of the synthesis (No. it.) and time in the optimization process in minutes (Opt.)

Example	No. Var.	Analysis (s)		Synthesis	
		Elem.	Array	No. It.	Opt. (min)
Lin. array	14	15	1	37	0.75
Pla. LP array	100	20	4	25	57
Pla. CP array	36	20	3	21	31

7.3.3.3 Planar array with circular polarization composed of hemispherical dielectric resonator antennas

In the last example, a circularly polarized planar array is obtained from linearly polarized elements. The array is composed of 6×6 antennas, the radiating element is the same as in the previous example. As explained at the end of Sub-section 7.3.1, the phase of the elements is composed as the addition of the necessary phase to steer the beam, and the synthesized phase fixed with the element's rotation.

The mainbeam is steered towards $u_{x0} = 0.24$ and $u_{y0} = 0.24$. The phase needed to steer the beam is obtained, as in classic theory, as $\Phi_{ste} = k\hat{u}\mathbf{u}_i$. The weighting functions used in this example are the same as in the first example, with $\beta = \|u_x - u_{x0}, u_y - u_{y0}\|$. The initial sequentially rotated array is shown in Fig. 7.20(a), while the synthesized array is represented in Fig. 7.20(b).

The co-polar component of the sequentially rotated array, represented in Fig. 7.21(a), has a maximum SLL of -6.5 dB, while the synthesized array, represented in Fig. 7.21(c), obtains a SLL of -13 dB. The cross-polar component level of the sequentially rotated

FIGURE 7.21: (a), (b): Co- and cross-polar component of the radiation pattern of the sequentially rotated array. (c), (d): Co- and cross-polar component of the radiation pattern of the synthesized array.

array, shown in 7.21(b), is -9.5 dB in the direction of the mainbeam and has a maximum level of -2.7 dB. The cross-polar component level of the synthesized array, represented in 7.21(d), has a level of -17 dB in the mainbeam direction and a maximum level of -7.5 dB.

Table 7.4 shows the number of variables, the number of restrictions and the time required for the analysis and synthesis for the three examples shown in this work.

Chapter 8

Synthesis of coupled antenna arrays using digital phase control via integer programming

8.1 Introduction

Array pattern synthesis involves finding the amplitude and phase distribution of the array excitations, and sometimes the positions or the number of array elements, satisfying as close as possible a desired response. For certain design specifications, complex feeding schemes (or even impossible to implement) are achieved if additional restrictions to the excitations weights are not imposed. Some improvements have been accomplished with different methodologies in order to get easier to fabricate arrays. Two main drawbacks appear when these restrictions are applied, the first of them is the limitation in the results obtained due to the restrictions and the second of them is that the addition of the restrictions usually makes the problem non convex and much more difficult to deal with. There are plenty of methods where the dynamic range ratio (DRR) of the excitations is minimized. Minimizing DRR facilitates the fabrication and avoids huge difference between elements weights, however, an array with a small DRR may need the same amount of phase shifters and power dividers that an array with an arbitrary DRR. Phase-only antenna array synthesis with amplitude weights fixed in advance has received large attention due to the simplicity in the resulting feeding network, or the reduction of the excitation errors.

Several procedures of synthesizing phase-only pattern antenna arrays have been described in the literature, based on analytical methods [18], or numerical techniques [19]. Digital phase shifters are widely used in phased arrays rather than continuous ones due

to the technical complexity and cost level. However, the first ones only offer a discrete set of phase states or quantized phase shifts. In this case, if the phase-only optimization problem is formulated with continuous phase variables, the optimized phase shifts must be approximated to the nearest available phase state. Although good solutions are usually obtained, this methodology has two main drawbacks, the discretization process may cause sub-optimal solutions and the appearance of phase discretization errors [138]. For this reason, the formulation of the optimization problem with discrete phase variables is more suitable in this case. The optimal solution is selected within the available discrete number of phases of the digital phase shifters. It is known that phased arrays work with bandwidths inversely proportional to the array size [138]. The use of true time delays (TTD) instead of phase shifts eliminates the restriction due to beam squint. TTD can be used at sub-array level and phase shifters at array element but it is difficult to make the distinction in the synthesis process between the desired phase at sub-array level and desired phase at the element.

Global optimization methods for designing digital phase control in antenna arrays, such as genetic algorithms or particle swarm optimization [139], can be found in the literature. They allow to apply any restriction and may achieve optimal solutions. However, their convergence is generally slow, are time-consuming, and can yield in a local minimum far away from the global solution of the problem. In [21] a randomization method that mitigates the degradations caused by the quantization of amplitude and phase is presented.

A linear optimization problem is the simplest representation of a convex optimization problem which are known to be solved optimally and very efficiently. When some values are restricted to be integer, or binary, the problem is called mixed integer linear program (MILP). This new restriction increments the number of problems that can be modeled but makes the problem more difficult to deal with. MILP problems are usually solved with branch and bound algorithms, a strategy of divide and conquer. The problem is firstly relaxed to a linear problem, where the integer restriction has been dropped. If the solution of the relaxed problem is integer then the optimal solution has been found. As this is not usually accomplished, this solution is considered as the upper bound of the linear program and two sub-problems arises. A non integer value is usually obtained (for example $x_1 = 1.7$) in the first of the sub-problems the value is imposed to be bigger than the immediately upper integer value ($x_1 \geq 2$) and in the second problem, the value is fixed to be lower than the immediately lower value ($x_1 \leq 1$). The enumeration of integer solutions has a tree structure that is solved iteratively with the methodology described. The idea of the branch and bound algorithm is to avoid growing the whole tree as much as possible, because if every possible combination of a binary (0 – 1) problem of just 30 variables would need to be solved $2^{30} \approx 10^9$ times. Some recent advances in the field of

MILP, as pre-solve, cutting planes, heuristics and parallel computing have improved its efficiency considerably.

Most of the array synthesis techniques traditionally assume ideal radiating elements, i.e. there is not mutual coupling between them or they are considered as isotropic sources. This simplification can lead to unwanted results in real array environments. The development of an array synthesis procedure incorporating electromagnetic-based analysis of the radiating elements in the formulation is in general a difficult task.

In this chapter, a hybrid analytical-numerical analysis method of antenna arrays is integrated with an optimization procedure for pattern synthesis using digital phase control. Real radiating patterns and mutual coupling effects are inherently taken into account in the formulation. The method is formulated as a digital integer linear programming problem optimizing quantified phases of the array excitation representing digital phase shifters.

8.2 Proposed optimization method

A method for the synthesis of coupled antenna arrays by optimizing quantized phase excitation coefficients of the array elements is integrated with the analysis procedure based on the spherical wave expansion outlined in Chapter 2. The objective is to synthesize multi-beam patterns with specified directions and beamwidth mainlobes, optimized side-lobe levels and prescribed nulls. The radiated field of the coupled antenna array can be reformulated from the expression (2.43) as:

$$\vec{E}(\hat{u}) = (\vec{e}(\hat{u}) e^{jk\hat{u}\cdot\vec{u}}) \mathbf{T}_G \mathbf{v} = \vec{g}(\theta, \phi) \mathbf{v} \quad (8.1)$$

The optimization problem is initially formulated with continuous excitation variables $\mathbf{v}$ as:

$$\underset{\mathbf{v}}{Min} \quad \alpha \quad (8.2a)$$

$$s.t \quad |\vec{g}(\theta, \phi) \mathbf{v}| \leq \alpha, \quad (\theta, \phi) \in R_{SL} \quad (8.2b)$$

$$|\vec{g}(\theta, \phi) \mathbf{v}| = 0, \quad (\theta, \phi) \in R_{NU} \quad (8.2c)$$

$$|\vec{g}(\theta_0, \phi_0) \mathbf{v}| = c \quad (8.2d)$$

where the maximum allowable field level, α , in the sidelobe region, R_{SL} , is minimized, while fixing a real constant level for the main beam c in the desired direction (θ_0, ϕ_0) and with the possibility of including nulls of the radiation pattern in the selected region R_{NU} .

The problem in (8.2) is then reformulated in order to accomplish a phase-only synthesis taking the phases as discrete variables. The planar array of M coupled antennas analyzed in the previous section is considered taking, without loss of generality, one single mode of excitation in each antenna, $\mathbf{v} = (v_1, v_2, \dots, v_M)^T$. These ones are restricted to have an equal module, or a fixed amplitude given by the vector $\mathbf{a} = (a_1, a_2, \dots, a_N)^T$. The phase of each excitation is forced to take one of the available quantized values $(\Phi_1, \Phi_2, \dots, \Phi_N)$ of the digital phase shifters used for controlling the element phases. The number N of possible phase states is determined by the number of bits, n , of digital phase shifters, fulfilling $N = 2^n$. In this way, each one of the excitations weights may be expressed in the form:

$$v_i = a_{i1} [d_{i1} e^{j\Phi_1} + \dots + d_{iN} e^{j\Phi_N}] \quad (i = 1, \dots, M) \quad (8.3)$$

where the coefficients d_{ij} are binary values, and for each excitation weight, v_i , only one of this coefficients are equal to one, i.e.: $d_{i1} + d_{i2} + \dots + d_{iN} = 1$ for $i = 1, \dots, M$. From previous definitions the vector of excitation weights may be expressed in matrix form as: $\mathbf{v} = \mathbf{A}\mathbf{D}\mathbf{\Phi}$ where $\mathbf{A}$ is a diagonal matrix whose coefficients are given by the vector of amplitudes $\mathbf{a}$, $\mathbf{D}$ is a binary matrix of dimension $M \times N$ with elements d_{ij} defined as:

$$\mathbf{D} = (\mathbf{d}_{i1}^T \mathbf{d}_{i2}^T \dots \mathbf{d}_{iM}^T) = \begin{bmatrix} d_{11} & \dots & d_{1M} \\ \vdots & \ddots & \vdots \\ d_{N1} & \dots & d_{NM} \end{bmatrix} \quad (8.4)$$

and $\mathbf{\Phi}$ is a column vector given by $\mathbf{\Phi} = (e^{j\Phi_1}, e^{j\Phi_2}, \dots, e^{j\Phi_N})^T$. Substituting the vector of excitation weights in (8.1), the radiated field is expressed as:

$$\vec{E}_a(\theta, \varphi) = \vec{g}(\theta, \varphi) \mathbf{v} = \vec{g}(\theta, \varphi) \mathbf{A}\mathbf{D}\mathbf{\Phi} \quad (8.5)$$

In this way, the optimization problem in (8.2) may be transformed to a digital phase-only array pattern optimization by using the expression of the radiated field in

(8.5) and taking as optimization variables the coefficients d_{ij} of matrix $\mathbf{D}$:

$$\underset{\mathbf{D}}{\text{Min}} \quad \alpha \quad (8.6a)$$

$$\text{s.t.} \quad |\vec{g}(\theta, \phi) \mathbf{A} \mathbf{D} \Phi| \leq \alpha, \quad (\theta, \phi) \in R_{SL} \quad (8.6b)$$

$$|\vec{g}(\theta, \phi) \mathbf{A} \mathbf{D} \Phi| = 0, \quad (\theta, \phi) \in R_{NU} \quad (8.6c)$$

$$|\vec{g}(\theta_0, \phi_0) \mathbf{A} \mathbf{D} \Phi| = c \quad (8.6d)$$

$$d_{ij} = 0 \text{ or } 1 \quad (8.6e)$$

$$\sum_{j=1}^N d_{ij} = 1 \quad i = 1 \cdots M \quad (8.6f)$$

Next, the spatial regions R_{SL} and R_{NU} are discretized in (θ, ϕ) for the numerical implementation of the optimization problem yielding:

$$\underset{\mathbf{D}}{\text{Min}} \quad \alpha \quad (8.7a)$$

$$\text{s.t.} \quad |\vec{g}_k(\theta_k, \phi_k) \mathbf{A} \mathbf{D} \Phi| \leq \alpha, \quad k = 1 \cdots K \quad (8.7b)$$

$$|\vec{g}_l(\theta_l, \phi_l) \mathbf{A} \mathbf{D} \Phi| = 0, \quad l = 1 \cdots L \quad (8.7c)$$

$$|\vec{g}_c(\theta_0, \phi_0) \mathbf{A} \mathbf{D} \Phi| = 1 \quad (8.7d)$$

$$d_{ij} = 0 \text{ or } 1 \quad (8.7e)$$

$$\sum_{j=1}^N d_{ij} = 1 \quad i = 1 \cdots M \quad (8.7f)$$

where K and L are the number of pointing directions in which each domain is sampled and $\{(\theta_k, \phi_k) \in R_{SL}\}$ and $\{(\theta_l, \phi_l) \in R_{NU}\}$. The discretization in (8.7) has to be dense enough to cover all possible sidelobe local maxima.

The resulting optimization problem may be modeled using digital integer programming since the variables can only take values 0 or 1. It can be solved by appropriate and well established integer programming algorithms. In this paper, the efficient mixed integer linear programming solver *Gurobi* [140], based on branch and cut strategy and handled from a toolbox for modeling and optimizing as *Yalmip* [86], is applied.

The proposed synthesis method allows to consider very efficiently different array geometries, what can also be used for optimizing the positions or the number of array elements in an iterative procedure, if a fast analysis method is used. For the analysis procedure considered in this work, summarized in Chapter 2, each new array geometry for building the optimization problem (8.7) only requires the analytical computation of the global transmission matrix of the array, from the already computed individual transmission matrices of the isolated elements. This process is performed in less than

FIGURE 8.1: Synthesized radiation patterns of a 15-element linear array of cavity-backed circular microstrip antenna using discretized phase variables (continuous line) and continuous phase variables (dashed line) in the optimization procedure. The synthesized phases for the first case are also represented.

a second or in few seconds for small or medium-sized arrays, respectively. It should be noted that it would require much more computation time for other analysis methods, such as those used in commercial software, making an iterative design process very time-consuming or even unaffordable.

8.3 Numerical results

Different array synthesis problems based on digital phase control are next considered in order to validate the proposed method. In the first example, a linear equispaced array of 15 cavity-backed circular microstrip antennas recessed in a metallic plane and with coaxial probe feeding is considered. The geometry and dimensions of the antenna, obtained from [10], are detailed in Fig. 5.6. The array elements are placed along the E-plane with an inter-element spacing of $0.5\lambda_0$ at the resonant frequency of 1.96 GHz. In the considered examples in this work, the finite element analysis of the isolated antennas and the analytical computation of the transmission matrix of the array take respectively an average of 20 seconds and less than one second. The synthesis process is established imposing a uniform amplitude distribution and quantized phase variables obtained with

FIGURE 8.2: Synthesized radiation patterns of a 21-element linear array of HDRAs when mutual coupling effects (MC) are considered, and not considered, in the optimization procedure. The synthesized phases for the first case are also represented.

digital phase shifters of 3 bits with an increment step of 45° . The phase of the excitation coefficients is optimized to scan the array at 29° with a first-null beamwidth of 17° , minimizing the maximum SLL. The optimization takes just 1 second. The optimized sequence of phases of the phase shifters and the resulting field radiation pattern obtained from these ones are given in Fig. 8.1. A maximum SLL of -14 dB is achieved. The same pattern optimization has also been performed using analog phase variables in order to illustrate the degradation owing to the discretization of continuous solutions. The synthesized phases are quantized with 45° increment step in order to reach the closest quantized values of the 3-bit digital phase shifters used in the previous case. The resulting radiation pattern obtained from these rounded phases in Fig. 8.1 shows a higher SLL (-12.5 dB) compared with the obtained using quantized phase variables.

In the next two examples the array element is a hemispherical dielectric resonator antenna (HDRA) obtained from [102] and described in Fig. 6.2, placed on a ground plane, with coaxial probe feeding and resonant frequency of 3.64 GHz. A linear array of HDRAs placed along the E-plane with a $0.4\lambda_0$ spacing is firstly considered. A synthesis to achieve, with the minimum number of antennas, a radiation pattern scanned at -12° with a null beamwidth of 16° and a maximum SLL of -16 dB has been performed. The optimization of each new array configuration which comprises the building and resolution of a new optimization problem takes an average of 20 s. A 3-bit phase shifter

FIGURE 8.3: Array feed configurations.

FIGURE 8.4: Synthesized radiation patterns of a linear array of 16 HDRAs divided in four-element sub-arrays. The digital phase sequence of the TTDs and phase shifters are also represented.

with a quantization step of 45° , and an array taper amplitude obtained from classical Dolph-Chebyshev excitation scheme, are used in this example. The required restrictions are achieved with 21 antennas. Fig. 8.2 shows the resulting radiation pattern and the optimized sequence of phases.

In order to illustrate the influence of mutual coupling in the optimization process, the same pattern synthesis has been performed with the same 21-element array of HDRAs without considering the coupling effects. In this case the formulation is identical to the one developed in previous section but the transmission matrix $\mathbf{T}_G$ in (2.28) is replaced by a diagonal block-matrix composed by the transmission matrix of the isolated antennas as shown in [10]. The real radiation patterns of the HDRAs are considered with this formulation. Fig. 8.2 shows the radiation pattern obtained when the optimized excitations in this case are applied to the coupled antenna array. As observed, the non-inclusion of the mutual coupling between array elements in the optimization process results in a distortion of the real pattern both in the main beam and side lobe regions.

It is known that true time delayers (TTDs) may be used instead of phase shifters for suppressing beam-squinting in phased array antennas [138]. A common architecture consists of combining TTDs at sub-array level, and phase shifters at element level. The synthesis formulation proposed in this work allows to cope with this configuration. It is performed by adding as constraints in (8.7) the elements of a same sub-array, which share a TTD, to have the same more significant bits. As an example, a linear array of 16 HDRAs partitioned into 4 sub-arrays with 4 elements each one is considered. A one-bit TTD is applied at sub-array level and 2-bits phase shifters is used at each element. The array is scanned at $\theta = 10^\circ$ with a beamwidth of 15° while the SLL is minimized, achieving a maximum SLL of -14 dB. The optimization takes 2 minutes in this case. The resulting pattern and the synthesized digitalized phase sequence of the TTDs and phase shifters are shown in Fig. 8.4.

Chapter 9

Conclusions

9.1 Original contributions

- A finite array analysis, based on the infinite array approach, which comprises the hybrid method based on the finite elements method, modal analysis and domain decomposition has been developed. The method considers a finite array as an infinite array with a finite number of excited elements. The non-excited elements are physically present and with three different terminations, short-circuited, open circuited or match terminated. An original formulation of the finite array analysis for multi-port and multi-mode elements has been presented. It has been demonstrated to be a powerful tool for large planar arrays composed of minimum scattering antennas with respect to one of the considered load conditions, also providing reasonable approximation, or estimating the uncertainty in the optimization of radiation patterns, of real finite arrays with other types of antennas.
- The finite array analysis based on the infinite array approach has been integrated into an array pattern synthesis methodology based on convex programming. The optimization process is formulated steering the main beam, or beams, towards a desired direction, fixing or minimizing an upper bound for the side lobe region and with the possibility of imposing nulls or a maximum level for the cross-polar component. The problem is solved via convex optimization with a computational cost slightly superior to the case of isotropic elements, obtaining global solutions in polynomial time.
- The same analysis method is used for synthesizing shaped beam patterns, or footprints with arbitrary contour, allowing to control the ripple amplitude on the main beam, the side lobe level or the cross-polar levels. The method gives rise to a sub-optimal design because conjugate symmetric characteristics are imposed.

However, a significant simplification in the synthesis process is carried out since the non-convex initial problem is solved very efficiently as a semi-definite problem, also reducing the dimension of the optimization formulation. For this reason, and also as the integrated finite array analysis is based on Floquet modal expansion, the proposed synthesis technique will be suitable for synthesizing large arrays. The comparison with results from full-wave in-house and commercial software has shown reasonable agreement.

- Another antenna array analysis has been employed in this thesis that includes a fast full wave characterization of the radiators. This method characterizes the array antenna as an expansion of spherical modes, obtaining a rigorous and efficient analysis of array antennas whose elements can be enclosed in spherical or semi-spherical ports, such as cavity-backed antennas, dielectric resonators, horns, apertures, PIFAs or dipoles. This analysis is not limited to arrays of equispaced elements. They can be placed on an arbitrary geometry and with different orientation. This analysis method has been used in the remaining optimization methods, it is firstly integrated in a shaped beam method which efficiently obtains near-optimal solutions of originally np-hard phase retrieval problems, solving convex optimizations iteratively. This method solves the same problem than the previously one, relaxing the restrictions instead of adding new constraints. It obtains better results at the expense of a higher computational time. Using first-order optimization methods, the procedure is able to solve large arrays. Some array synthesis results have been presented showing the strength and the capabilities of the method. The results from the synthesis have been tested with full-wave methods showing good agreement. The importance of merging a full-wave analysis into the optimization process has been emphasized.
- In this thesis, a new synthesis method for sparse arrays has been presented. The array characterization and the variation of the mutual coupling due to the variation of the position of the elements are computed analytically, through the calculation of the gradient, versus positions, of the appropriate cost function. This makes this method very efficient for small arrays and suitable for accurate optimization of middle size array designs. The efficiency of the local method proposed here provides really good results which depends on the starting configuration. It can be employed also to improve the results of global methods available in the literature or to mitigate the unwanted effects that mutual coupling may induce on these methods. Several examples have been presented, where the strength of the method and the importance of a rigorous analysis of the array, in terms of mutual coupling, have been emphasized.

- An array synthesis method for coupled arrays via element rotation has been presented. A rigorous expression of the radiation intensity is obtained and integrated in a cost function which is minimized with a gradient-based method. The gradient of the cost function, and thus the radiation intensity, is efficiently obtained w.r.t. the elements rotation and the cost function is minimized with iterative rotations of the elements. The rotation synthesis has been proved to be an important tool for side lobe minimization and for obtaining circular polarization from linearly polarized elements, without adding significant costs in the fabrication process. A highly non convex problem is solved very efficiently, obtaining very good results. Some examples are presented showing the capabilities of the present method.
- A novel methodology for array pattern synthesis using digital phase control, accounting for real radiating patterns and mutual coupling effects, has been presented. It is based on linear integer programming and provides a powerful tool for array designs, obtaining good results with phases obtained from digital phase shifters with a low number of bits. The synthesis procedure has been tested with linear arrays of complex radiating elements achieving good results, and evidencing the need of the integration of an electromagnetic analysis in the synthesis procedure.

9.2 Future research lines

- Conformal arrays would be handled with the analysis methods, presented in Chapters 2 and 3, and with the synthesis methods presented in Chapters 4-8, with some modifications.
- Combination of the analysis methods for the rigorous analysis of large arrays in where the inner elements are analyzed using the infinite array approach while the elements near to the edge of the array are analyzed with the spherical wave expansion.
- Array synthesis problems that are currently being solved with global, and very time consuming, methods may be handle with convex programming. This would require in some cases to relax, as in Chapter 6, or to constraint, as in Chapter 5, the original problem in order to obtain a convex formulation of the problem.
- The phase retrieval technique used in this work can also be applied in the field of antenna measurements. The excitations can be retrieved from the magnitude of the radiated field discovering possible array errors or faulty elements efficiently.

- An extension of the gradient-based method, developed in Chapter 7, in which the mutual coupling w.r.t. each element is only computed for the elements that are closer than a desired threshold. In that way the computation would be more efficient and larger arrays could be handled.
- Extension of the gradient-based array synthesis of real arrays in Chapter 7 to non-planar arrays. The analysis method described in Chapter 2 based on the spherical modal expansion must be extended to this kind of arrays.
- Extension of the proposed synthesis procedures of coupled antenna arrays to the optimization of other arrays characteristics as aperture efficiency or dynamic range ratio.
- Study of optimization methods that provide affordable arrays making possible to build a demonstrator.

9.3 Framework

The thesis has been developed in the Departamento de Señales, Sistemas y Radiocomunicaciones, at the Escuela Técnica Superior de Telecomunicación of the Universidad Politécnica de Madrid.

Part of this work, the synthesis methods developed in Chapter 7, have been partially developed within a research period in collaboration with Prof. Christophe Craeye at the ICTEAM department of the Université Catholique de Louvain, Louvain-la-Neuve, Belgium. March 15- May 15.

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- *Efficient discretizations and optimizations for analysis and design of filters, antennas and antenna arrays.*, supported by Plan Estatal de Investigación Científica, Desarrollo e Innovación Tecnológica 2013-2016. Ministerio de Economía y Competitividad (Spanish Government). Entity: E.T.S.I. Telecomunicación-U.P.M. Duration: January 2014 - December 2016. Principal Investigator: José María Gil Gil

9.4 Publications

The work developed in this thesis has given rise to the following publications:

9.4.1 Journal articles

- J. Ignacio Echeveste Guzmán, Miguel Á. González and Juan Zapata, "Array Pattern Synthesis of Real Antennas Using the Infinite Array Approach and Linear Programming", in *IEEE Transactions on Antennas and Propagation* vol. 63, no. 12. pp. 5417-5424 December 2015.
- J. Ignacio Echeveste Guzmán, Miguel Á. González and Juan Zapata, "Shaped-Beam Synthesis for Finite Array Antennas via Finite-Element Method, Active Element Pattern and Convex Programming", accepted in *IEEE Transactions on Antennas and Propagation*, to be published in April 2016.
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Jesús Rubio and Juan Zapata, "Near Optimal Shaped Beam Synthesis of Arrays of Real Antennas via Phase Retrieval and Convex Programming.", accepted to be published in *IEEE Transactions on Antennas and Propagation*
- J. Ignacio Echeveste Guzmán, Jesús Rubio, Miguel Á. González and Christophe Craeye, "Gradient-based Aperiodic Array Synthesis of Real Arrays with Uniform Amplitude Excitation Including Mutual Coupling", submitted to *IEEE Transactions on Antennas and Propagation*
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Jesús Rubio and Juan Zapata, "Synthesis of Coupled Antenna Arrays using Digital Phase Control via Integer Programming", submitted to *IEEE Antennas and Wireless propagation Letters*
- J. Ignacio Echeveste Guzmán, Jesús Rubio, Miguel Á. González and Christophe Craeye, "Array Synthesis of Real Arrays Including Mutual Coupling via element rotation", submitted to *IEEE Transactions on Antennas and Propagation*

9.4.2 Conference proceedings

9.4.2.1 International

- J. Ignacio Echeveste Guzmán, Miguel Á. González and Juan Zapata, "Shaped-Beam Synthesis for Microstrip Antenna Arrays via Finite-Element Method, Active Element Pattern and Convex Programming", in *The 7th European Conference on Antennas and Propagation*. Gothemburg, Sweden. 8-11 April 2013.

- J. Ignacio Echeveste Guzmán, Miguel Á. González and Juan Zapata, "AEP and Mutual Coupling Formulation for Antenna Array Synthesis via Convex Optimization", in *IEEE International Symposium on Antennas and Propagation and USNC-URSI*. Memphis, USA. 6-11 July 2014.
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Rafael Gómez and Jesús Rubio, "Shaped Beam Synthesis of Arrays of Real Antennas via Phase Retrieval and Convex Programming", in *The 9th European Conference on Antennas and Propagation*. Lisbon, Portugal. 12-17 April 2015.
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Jesús Rubio and Juan Zapata, "Antenna Array Synthesis using Digital Phase Control via Integer Programming and 3D FEM", in *IEEE International Symposium on Antennas and Propagation and USNC-URSI*. Vancouver, Canada. 18-25 July 2015.
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Jesús Rubio and Christophe Craeye, "Synthesis of Aperiodic Arrays with Uniform Amplitude Excitation Including Coupling Effects", accepted in *The 10th European Conference on Antennas and Propagation*. Davos, Switzerland. 10-15 April 2015.

9.4.2.2 National

- J. Ignacio Echeveste Guzmán, Miguel Á. González and Juan Zapata, "Síntesis de diagramas de radiación de arrays de antenas reales a partir del modelo de array infinito y programación lineal.", in *XXVIII Simposium Nacional de la Unión Científica Internacional de Radio. URSI 2013*. Santiago, Spain. 11-13 September 2013.
- J. Ignacio Echeveste Guzmán, Miguel Á. González, Jesús Rubio and Juan Zapata, "Síntesis de diagramas de radiación de arrays de antenas reales a partir del método de recuperación de fase y optimización convexa.", in *XXIX Simposium Nacional de la Unión Científica Internacional de Radio. URSI 2014*. Valencia, Spain. 3-5 September 2014.

Appendix A

Results from the synthesis process

In this appendix, the results obtained from the synthesis processes from one example of each chapter are indexed. For clarity reasons, the value of each synthesis is shown with two digits. It has been checked that the results obtained with this accuracy are the same than with a higher number of digits.

The tables represents the x dimension horizontally and the y dimension vertically. In the example in which the position of the element is the optimization value, they are represented in the table with the same order for the x and y coordinates.

The amplitudes are given normalized with respect to unity. The phases and rotation angles are given in radians and the distance is given in meters.

TABLE A.1: Synthesized excitation amplitudes of the 15×15 array elements in Sub-section 4.3.1. which generates the radiation pattern shown in Fig. 4.1.

A	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0.01	0.01	0.02	0.00	0.04	0.02	0.04	0.03	0.04	0.02	0.04	0.01	0.02	0.01	0.01
2	0.00	0.02	0.02	0.05	0.03	0.03	0.01	0.02	0.00	0.04	0.03	0.05	0.03	0.02	0.01
3	0.02	0.03	0.07	0.05	0.07	0.04	0.03	0.08	0.02	0.04	0.07	0.04	0.07	0.03	0.03
4	0.00	0.05	0.05	0.09	0.09	0.10	0.22	0.15	0.22	0.09	0.07	0.08	0.04	0.05	0.01
5	0.05	0.04	0.07	0.07	0.13	0.32	0.36	0.49	0.37	0.32	0.13	0.06	0.06	0.03	0.04
6	0.02	0.05	0.01	0.07	0.31	0.46	0.70	0.67	0.70	0.46	0.31	0.06	0.03	0.04	0.02
7	0.05	0.04	0.02	0.19	0.35	0.69	0.81	0.99	0.81	0.69	0.34	0.19	0.03	0.03	0.05
8	0.03	0.05	0.07	0.13	0.47	0.67	0.99	0.96	0.99	0.66	0.46	0.13	0.08	0.05	0.03
9	0.06	0.06	0.08	0.20	0.35	0.70	0.82	1.00	0.82	0.68	0.33	0.19	0.08	0.06	0.05
10	0.03	0.09	0.10	0.10	0.31	0.46	0.71	0.69	0.71	0.45	0.30	0.09	0.08	0.07	0.03
11	0.06	0.09	0.14	0.12	0.13	0.33	0.38	0.51	0.39	0.32	0.11	0.08	0.11	0.06	0.05
12	0.03	0.09	0.12	0.14	0.10	0.11	0.24	0.20	0.24	0.10	0.07	0.10	0.08	0.07	0.01
13	0.03	0.05	0.11	0.09	0.09	0.06	0.08	0.11	0.08	0.05	0.07	0.06	0.09	0.04	0.03
14	0.00	0.03	0.05	0.07	0.04	0.04	0.03	0.05	0.03	0.04	0.03	0.06	0.04	0.03	0.00
15	0.00	0.00	0.03	0.01	0.04	0.02	0.05	0.02	0.05	0.02	0.04	0.01	0.03	0.00	0.00

TABLE A.2: Synthesized excitation phases of the 15×15 array element in Subsection 4.3.1. which generates the radiation pattern shown in Fig. 4.1.

φ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1.86	-1.68	-0.01	-0.36	-0.36	-0.29	-0.02	-0.59	-0.20	0.06	-0.32	-1.88	0.34	-1.42	2.02
2	-0.32	-0.23	2.91	-0.36	2.40	-0.49	-0.54	0.27	-0.29	-0.59	2.60	-0.00	-2.49	0.49	-1.14
3	-0.15	3.06	-0.18	2.53	-0.73	0.78	-2.07	0.04	-2.05	0.91	-0.60	3.04	0.20	-2.47	0.38
4	2.98	-0.11	2.76	-0.63	0.93	-2.28	0.14	-2.97	0.13	-2.36	0.79	-0.38	-2.83	0.33	-1.56
5	-0.05	-3.04	-0.28	0.96	-2.34	0.28	-2.99	0.04	-3.03	0.21	-2.50	0.60	0.05	-2.55	0.11
6	-0.62	0.43	-0.18	-2.27	0.30	-2.92	0.06	-3.13	0.03	-2.99	0.21	-2.63	-0.58	0.58	-0.38
7	0.27	-2.00	0.96	0.14	-2.91	0.11	-3.11	-0.02	3.13	0.04	-3.03	0.02	1.43	-1.74	0.22
8	-0.71	1.02	-0.75	3.12	0.12	-3.07	-0.01	3.08	-0.05	3.13	0.02	2.98	-0.74	1.13	-0.66
9	0.49	-1.78	1.44	-0.16	-3.10	0.01	3.09	-0.09	3.03	-0.06	3.07	-0.26	1.44	-1.74	0.46
10	-1.01	1.07	-1.43	2.13	-0.13	3.03	-0.12	2.95	-0.16	2.97	-0.16	2.14	-1.41	1.02	-0.87
11	0.65	-1.98	1.07	-1.19	2.43	-0.25	2.82	-0.28	2.78	-0.27	2.53	-1.04	0.93	-2.03	0.54
12	-1.67	0.94	-1.97	0.97	-1.08	2.13	-0.47	2.41	-0.47	2.22	-0.78	0.72	-2.14	0.78	-1.64
13	0.54	-2.15	0.79	-2.13	0.80	-1.29	1.61	-0.86	1.65	-1.03	0.52	-2.42	0.61	-2.30	0.43
14	-0.74	0.86	-2.09	0.68	-2.49	0.56	-1.79	1.11	-1.61	0.49	-2.81	0.47	-2.23	0.73	-1.71
15	-2.10	-1.44	0.69	-1.77	0.24	0.40	0.31	-0.19	0.39	0.30	0.17	-2.05	0.58	-2.02	-1.83

TABLE A.3: Synthesized excitation amplitudes of the 9×9 array elements in Subsection 5.3.2. which generates the radiation pattern shown in Fig. 5.4.

A	1	2	3	4	5	6	7	8	9
1	0.02	0.00	0.03	0.04	0.03	0.04	0.03	0.00	0.02
2	0.01	0.06	0.01	0.04	0.06	0.04	0.01	0.06	0.01
3	0.03	0.02	0.10	0.06	0.04	0.07	0.10	0.02	0.03
4	0.03	0.03	0.03	0.17	0.33	0.18	0.03	0.03	0.03
5	0.03	0.09	0.03	0.41	1.00	0.41	0.03	0.09	0.03
6	0.03	0.03	0.03	0.18	0.33	0.17	0.03	0.03	0.03
7	0.03	0.02	0.10	0.07	0.04	0.06	0.10	0.02	0.03
8	0.01	0.06	0.01	0.04	0.06	0.04	0.01	0.06	0.01
9	0.02	0.00	0.03	0.04	0.03	0.04	0.03	0.00	0.02

TABLE A.4: Synthesized excitation phases of the 9×9 array elements in Subsection 5.3.2. which generates the radiation pattern shown in Fig. 5.4.

φ	1	2	3	4	5	6	7	8	9
1	-2.18	-3.10	0.61	0.52	0.43	0.54	0.61	-2.96	-2.18
2	1.98	0.46	0.24	-2.73	-2.76	-2.73	0.24	0.47	2.02
3	0.13	-2.47	-2.73	-2.64	-2.32	-2.64	-2.73	-2.44	0.14
4	0.45	3.13	-2.46	0.32	0.38	0.32	-2.48	-3.13	0.46
5	0.03	-2.75	-2.17	0.53	0.41	0.53	-2.14	-2.76	0.02
6	0.46	3.12	-2.45	0.32	0.38	0.32	-2.44	3.12	0.46
7	0.15	-2.41	-2.75	-2.62	-2.35	-2.61	-2.76	-2.38	0.14
8	2.01	0.47	0.27	-2.72	-2.77	-2.70	0.20	0.48	2.00
9	-2.19	-2.90	0.62	0.52	0.44	0.53	0.63	-2.79	-2.19

TABLE A.5: Synthesized excitation amplitudes and phases of the 6×6 array elements in Subsection 6.3.3. which generates the radiation pattern shown in Fig 6.6.

A	1	2	3	4	5	6	φ	1	2	3	4	5	6
1	0.32	0.22	0.15	0.20	0.09	0.15	1	3.14	3.13	3.13	-3.10	-3.08	0.00
2	0.10	0.11	0.13	0.10	0.22	0.08	2	-3.14	0.07	0.06	-3.07	-3.09	-3.07
3	0.30	0.68	0.65	0.25	0.13	0.14	3	0.06	0.06	0.06	0.06	-3.10	-3.11
4	0.53	1.00	0.95	0.49	0.03	0.14	4	0.05	0.06	0.06	0.07	2.96	3.13
5	0.38	0.81	0.79	0.37	0.07	0.12	5	0.06	0.06	0.06	0.07	3.09	3.13
6	0.04	0.25	0.25	0.03	0.28	0.15	6	3.07	0.07	0.07	3.10	-3.11	-3.13

TABLE A.6: Synthesized positions of the 40 elements computed in Subsection 7.2.2.3 and represented in Fig 7.10 (d).

p_x	1	2	3	4	5	p_y	1	2	3	4	5
1	0.14	0.11	0.01	-0.11	-0.16	1	0.00	0.12	0.17	0.09	-0.02
2	-0.09	0.01	0.11	0.31	0.23	2	-0.10	-0.16	-0.12	0.01	0.12
3	0.20	0.03	-0.08	-0.20	-0.28	3	0.22	0.31	0.21	0.17	0.07
4	-0.26	-0.21	-0.09	0.02	0.18	4	-0.07	-0.18	-0.22	-0.31	-0.24
5	0.24	0.45	0.47	0.35	0.25	5	-0.12	0.02	0.18	0.26	0.37
6	0.09	-0.04	-0.16	-0.31	-0.44	6	0.43	0.48	0.42	0.29	0.26
7	-0.45	-0.44	-0.39	-0.32	-0.21	7	0.11	-0.14	-0.24	-0.34	-0.43
8	-0.05	0.07	0.24	0.32	0.44	8	-0.46	-0.43	-0.36	-0.22	-0.16

TABLE A.7: Synthesized rotations of the 36 elements of Subsection 7.3.3.3 and represented in Fig 7.20 (b).

p_x	1	2	3	4	5	6
1	1.36	2.35	2.94	0.35	2.14	1.99
2	2.79	0.88	1.75	3.07	0.39	0.73
3	0.90	1.53	2.15	0.13	0.40	0.15
4	2.04	0.41	1.08	0.61	1.88	1.21
5	0.96	2.05	2.37	2.49	0.82	2.60
6	2.36	0.42	1.97	0.58	2.35	0.66

TABLE A.8: Synthesized discrete phases of the 16 array elements in Section 8.3 which generates the radiation pattern shown in Fig 7.10 (d).

N. Element	1	2	3	4	5	6	7	8
Phase	1.80	1.80	0.90	0.90	0.00	0.00	-0.90	-0.90
N. Element	9	10	11	12	13	14	15	16
Phase	-1.80	-1.80	-2.69	-2.69	2.69	0.90	2.69	0.90

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