Transition prediction in incompressible boundary layer with finite-amplitude streaks

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Research efforts focused on transition delay

- Strategy: modulate boundary layer velocity profile
- Introducing streamwise streaks

- 3D flow structures
- Streamwise long, wall-normal and spanwise thin
- Alternating regions of high (HS) and low (LS) speed
Estabilizing effect of streaks

STREAKS off

STREAKS on

Fransson, Talamelli, Brandt & Cossu, PRL 2006

Estabilizing effect increased with streak amplitude until secondary instability develops for high intensity streaks

Andersson & al., JFM 2001
**Streak generation**

**Roughness elements**

Fransson, Brandt, Talamelli & Cossu, PoF 2005

\[ A_s = \frac{1}{2U_\infty} \max_y \left[ \max_z (U(y, z)) - \min_z (U(y, z)) \right] \]

\(~12\%~

**Miniature Vortex Generators (MVG’s)**

Shahinfar, Sattarzdeh, Fransson & Talamelli, PoF 2012

\(~32\%~

**Motivation to compute high intensity streaks**

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Transition prediction in bl with finite-amplitude streaks
Objective: cheap CPU method

Incompressible, steady, streaky base flow

Reduced Navier Stokes equations (RNS)

- Simplified boundary layer-like formulation (BRE’s) – Fletcher 1990
- High Reynolds number limit \( Re = U_\infty L/\nu \gg 1 \)
- Two short scales, one long

\[
\begin{align*}
    u_x + v_y + w_z &= 0 \\
    uu_x + vu_y + wu_z &= dP_\infty/dx + u_{yy} + u_{zz} \\
    uv_x + vv_y + wv_z &= p_y + v_{yy} + v_{zz} \\
    uw_x + vw_y + ww_z &= p_z + w_{yy} + w_{zz}
\end{align*}
\]

- Parabolic in \( x \)
- Marching scheme, 2nd order accuracy
- Fast and robust
- Re-independent
Formulation

- **Objective**: cheap CPU method

  Incompressible, steady, streaky base flow

- **Stability analysis**

  3D Parabolized Stability Equations (PSE-3D)

  » 3D linear stability equations for base flows with a single slowly-varying spatial direction

  \[
  \mathbf{q} = \hat{\mathbf{q}} + \varepsilon \hat{\mathbf{q}}
  \]

  \[
  \hat{\mathbf{q}}(x, y, z, t) = \sum_{n=-N}^{N} \hat{q}_n(x, y, z) \exp(-i\omega t)
  \]

  \[
  \hat{q}(x, y, z) = \hat{q}(x, y, z) \exp \left[ i \int_{x}^{x'} \alpha(x') dx' \right]
  \]

- **Initial condition**: Spatial Bi-Global analysis
RNS results: nonlinear streaks

- Downstream evolution of optimal streaks

\[ A_s(x) = \frac{1}{2}(\max_{y,z}(u - U_b) - \min_{y,z}(u - U_b)), \]

\[ \beta = 0.45 \text{ Andersson \& al., PoF 1999} \]

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**RNS** Martin & Martel, FDR 2012

**DNS** Cossu & Brandt, PoF 2002

EJFM/B 2004
PSE-3D results: TS energy

- Kinetic energy evolution of a TS wave
  \[ w = 0.0358 \]

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Bagheri & Hanifi, PoF 2007
PSE-3D streak stability analysis

Neutral stability curves

\[ Re = 400 \]

- \( A_s \max(A) = 0\% \)
- \( A_s \max(B) = 14\% \)
- \( A_s \max(C) = 20\% \)
- \( A_s \max(D) = 25\% \)

Critical streak amplitude

\( A_s \max \sim 26\% \)

\[ Andersson \ & al., \ JFM \ 2001 \]

\[ A_s \max(E) = 28\% \]
\[ A_s \max(F) = 32\% \]
\[ A_s \max(G) = 36\% \]
Parametric analysis optimal streaks

- Varying spanwise wave number $\beta$
- Optimization criteria: max energy at $x = 1$
- Gives the velocity profile for initial condition
- Family of streaks with same maximum intensity

$A_s \text{max} = 15\%$

$A_s \text{max} = 10\%$

$A_s \text{max} = 5\%$
Parametric analysis optimal streaks

- Varying spanwise wave number $\beta$
- Optimization criteria: max energy at $x = 1$
- Gives the velocity profile for initial condition
- Family of streaks with same maximum intensity

![Graph showing the variation of $N_k$ with $\beta$ for different $A_{max}$ values]

- Existence of $\beta$ that minimizes energy growth (results inline with NPSE analysis of Bagheri & Hanifi PoF 2007)
- Kept constant with streak amplitude
Based on $N$-factor as $N_k = \ln \left( \frac{K(x_{ii})}{K(x_i)} \right)$

Transition is assumed to occur when $N_k > 9$

$Re = 1000$

$Re_x = Re^2$

Estimation of maximum streak amplitude for delaying transition

$A_{s_{max}} \sim 33\%$
Concluding remarks

- RNS to describe the downstream evolution of streaks
- PSE-3D to analyze the stability of streaks
- Cheap CPU method – desktop Workstation
  - ~ minutes to obtain streaky flow
  - ~ hours to compute a neutral curve (40 frequencies)
  - ~ days to do the complete parametric study
- Neutral stability curves of 3D-modified TS and shear layer streak instabilities
- Optimum spanwise wavenumber for TS damping (constant with streak intensity)
- Displacement of transition location with streak intensity

Thanks for your attention!!

– Questions??