VIRTUAL MIMO RADAR USING OFDM-CDM WAVEFORMS

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ABSTRACT
This paper addresses a new perspective on the exploitation of diversity resembling recent seminal proposals with multiple antennas known as MIMO radar. Our focus pursues similar advantages as spatial MIMO systems but intending to achieve the desired resistance over fading or SNR increase without relying on multiple antennas: We design an OFDM-CDM waveform well inspired in modern communications systems that creates a virtual MIMO system operating on the artificial 2D domain formed by a set of well separated carriers (OFDM) and several OFDM symbols each one modulated by orthogonal codes (CDM). We consider the most general scenario with moving targets and large size targets originating an equivalent time variant and frequency selective channel model. Our proposal proceeds in two steps, a first one to reorthogonalize the transmitted set of OFDM signal by proper time and frequency synchronization (this stage provides range and velocity estimators), and a second one based on Neyman Pearson detection improved by the diversity gain.

1. MOTIVATION
Application of spatial MIMO (Multiple Input – Multiple Output) principles has attracted large research interest in the communication field due to the great improvements predicted by recent theoretical results [Te99, Fo98]. Presently all important applications from mobile to WLANs, xDSL or sensor networks and working towards practical implementations to increase their performances exploiting diversity at reception by optimum combining, space time coding to emulate the diversity gain at transmitter side, beamforming gain and even also spatial multiplexing [Pa03]. Therefore, it was just a question of time that these advantages may be applied to the radar field. Several recent papers are revisiting the classical radar objectives applying these concepts claiming much higher resolution capabilities and improved detection probabilities [Fi04, Fi06, Le07]. Although these contributions are clearly opening a new paradigm in the radar field [Le04], we would like to enforce two key issues: on one hand, deployment of several multi site antennas may not be applicable in many scenarios and any case the cost increase may be unaffordable. On the other, the statistical MIMO (S-MIMO in the sequel) radar is based on the assumption of orthogonality among the received waveforms in order to create many independent radars. This last hypothesis is probably limited to very simple cases and in practice correlation among received waveforms will decrease performance.

From our point of view, we propose a different means to achieve the same appealing features of spatial MIMO systems using just one antenna (single site or multisite) per link inducing diversity and array gain exploiting the OFDM-CDM characteristics. This architecture is robust to work in moving and large size target scenarios using the well known capability of OFDM systems to deal with multipath channels. Doppler robustness is compensated by frequency estimation in an equivalent way as offsets are reduced in standard communications.

2. SIGNAL MODEL
The received signal in the S-MIMO radar is based on a simple model as described by equation (1)

\[ r_{S-MMIMO}(t) = \sqrt{P} \sum_{k=1}^{L} H_{rs}(t-kT) + n(t) \]  

where \( n(t) \) is a AWGN noise signal with \( \sigma^2 \) power, \( P \) is the transmitted power (assumed 1 in the sequel), \( H_{rs} \) is the instantaneous mixture among the set of transmitted waveforms \( s(t) \) from every transmitting antenna to every receiving antenna (subscript \( T \) means that we are talking of MIMO systems in the time domain. In OFDM, it is more convenient to express the process in the frequency domain where we will drop the subscript for clarity reasons). Statistics of matrix \( H_{rs} \) may range from fully correlated (related to close located antennas and beamforming principles) to uncorrelated typically related to sparse configurations. While the former provides SNR gain by the coherence of the array factor, that is range increase, the latter provides diversity gain meaning robustness against target scintillations.

Equation (1) shows two important simplifications that are clearly conditioning the large improvement of S-MIMO radar in front of standard approaches. Let us show a more general model in equation (2) including Doppler effect \( f_{D} \) and multipath L-size (larger size reflectors)

\[ r_{o}(t) = \sqrt{P} \sum_{l=1}^{N} H_{nl} s(t-lT) + n(t) \]  

Performance of S-MIMO is based on simple receivers per antennas as a bank of matched filters to the transmitted waveforms. As they are supposedly orthogonal, you have \( MN \) independent radars. This orthogonality preservation applies if channel model follows equation (1) but clearly fails in the more general case. To the best of our knowledge, there is no evaluation of the S-MIMO performance degradation if equation (2) applies.

Our proposal follows the scenario described by equation (2) for the SISO case. Multipath distortion is eliminated by the
cyclic prefix of the OFDM symbol while velocity is estimated designing a specific OFDM feature for this purpose. After correction we retrieve an equivalent MIMO system with orthogonality preservation.

As mentioned above, we will use a waveform based on \( K \) consecutive OFDM symbols each one with \( 2N \) carriers separated \( \Delta \). We will assume that this separation is enough to guarantee uncorrelated fading characteristic. Every arbitrary OFDM symbol has a specific structure where only alternate carriers are active. Active carriers in every OFDM symbol are modulated by one different code belonging to any set of orthogonal functions as Walsh-Hadamard. The reasoning behind this structure is double: modulation of alternate carriers creates time duplicity of the symbol in two identical halves that may be used for robust time (range) and frequency offset (Doppler) estimation following [Sch97]. On the other hand, a certain number of \( K\leq N \) consecutive symbols are transmitted in order to increase the autocorrelation properties of the transmitted signal and also minimize possible interference coming from close targets that may overlap the echoes. This number \( K \) large will provide improved detection and estimation features but also will reduce the target discrimination capabilities. In practice, there should be a trade-off depending on the specific scenario. In the sequel, we will consider these figures as parameters. This system may operate as a pulsed radar or continuous wave depending on the scenario’s target. We will emphasize the pulsed case because it is simpler to explain but it may be extended to the other option with minimum effort. In the frequency domain, the set of transmitted symbols may be represented in a time-frequency matrix \( S \):

\[
S = \sqrt{2} [\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_K]
\]

Where, as mentioned, \( \tilde{c}_k \) represents a \( 2N \) vector with alternate zeroes and values according to the \( k \) Walsh Hadamard code. In the discrete time domain transmitted symbols are represented as the concatenation of the \( 2N \) IDFT (\( F' \) as a matrix operator) of the codes and \( M \) is the linear transformation to add the cyclic prefix (\( (2N+L)x2N \)):

\[
s = \sqrt{2} \begin{bmatrix}
MF^{-1}\tilde{c}_1 \\
MF^{-1}\tilde{c}_2 \\
\vdots \\
MF^{-1}\tilde{c}_K
\end{bmatrix}
\]

Denoting \( h_r \) as the impulse response of the incoming echoes, \( H_r=Toe(h_r) \) is the Toeplitz matrix representation of the linear convolution with \( h_r \), and \( W \) is a diagonal matrix whose elements are the increasing phase shift due to the Doppler frequency normalized at the sampling rate, received signal becomes:

\[
x = WH_r s + n
\]

Figure 1 shows the block diagram of the receiver where clearly we want to remark two stages: a first one devoted to synchronize the received sequence in time and frequency that plays the role of range and velocity estimators. These parameters allow the receiver signal to be reorthogonalized in order to make possible the diversity achievement and/or SNR increase. Second step performs standard detection using Neyman-Pearson criteria that may work under two hypotheses: on one hand, if channel estimator is possible, coherent processing applies with the consequent improvement. If not, detection based on the vector norm may also be applied. From the perspective of communications, channel estimation in OFDM is very simple because after proper synchronization and suitable choice of the cyclic prefix length longer than the maximum expected delay, flat fading applies per carrier. Therefore we would like to emphasize that with minimum extra complexity, this architecture provides coherent detection.

![Fig. 1: Block diagram of our system.](image_url)

3. OUR PROPOSAL. ESTIMATION STAGE

3.1 Range and speed estimation

Range and speed estimation is essentially the same problem faced by an OFDM modem when it intends to synchronize in time and frequency. There are several methods to cope with this task but one of the most robust schemes are based on the special time duplicated structure of the training symbol proposed in [Sch97]. As mentioned before, transmission over alternate carriers creates the time duplication structure. The procedure is quite simple working in two steps: first step is just two consecutive sliding windows size \( N \), correlating both identical halves to perform a time metric defined as (for just one symbol):

\[
M(d) = \frac{|P(d)|^2}{(R(d))^2}
\]

\[
P(d) = \sum_{m=0}^{N-1} x_{2m}x_{2m+N} R(d) = \sum_{m=0}^{N-1} |x_{2m+N}|^2
\]

Equation (6) may be easily generalized to match \( K \) consecutive OFDM symbols achieving the corresponding improvements due to the noise decorrelation. Clearly,

\[
\hat{d}_w = \max_d M(d)
\]

Once range has been estimated, the carrier frequency offset (and so target’s speed) is obtained from the phase estimation

\[
\hat{\phi} = \arg\left(P(d_w)\right)
\]

However, it is important to remark the ambiguity of this procedure where maximum Doppler frequency is limited to the intercarrier separation. So, reduced ambiguity means increasing bandwidth or reducing number of carriers that also penalizes in SNR and diversity gain.

3.2 Signal model after time and frequency acquisition

After proper correction in time and frequency, OFDM symbols are orthogonal providing a simple representation in the frequency domain for the \( k \)-OFDM symbol:

\[
z_k = H_k c_k + n_k
\]
We get

\[ \text{Correlated: } \bar{H} = \mathbb{R} \begin{bmatrix} e^H \bar{H}^H \mathbf{e} \end{bmatrix} \]

As mentioned before, we may assume that we are able to estimate \( \bar{H} \) in a very straightforward way.

4. DETECTION STAGE

For the detection stage we will assume to extreme models for the time correlation among the frequency responses: on one hand, fully correlated modeling a Swerling 1 target and on the other, fully uncorrelated to represent the Swerling 2 case. In the sequel we will assume that carriers are separated enough in order to claim for them as uncorrelated. \( \mathbf{c} \) are the scaled \( \sqrt{2} \) Walsh-Hadamard codes when zeroes are eliminated. Stacking the \( K \) symbols in one vector we have

\[
\mathbf{z} = \begin{bmatrix} \mathbf{H}_1 & 0 & 0 & 0 & \cdots & \mathbf{H}_K \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1 & 0 & 0 & \cdots & 0 \end{bmatrix} = \tilde{\mathbf{H}} \mathbf{c} + \mathbf{n}
\]

Defining

\[
\{ H_0 \} \quad T_c = \mathbb{R} \begin{bmatrix} \mathbf{c}^H \bar{H}^H \mathbf{c} \end{bmatrix} \quad \mathcal{N}(0, \sigma^2)
\]

\[ \{ H_1 \} \quad T_c = m + \mathbb{R} \begin{bmatrix} \mathbf{c}^H \bar{H}^H \mathbf{c} \end{bmatrix} \quad \mathcal{N}(m, \sigma^2)
\]

This simple test follows the criteria [BarOS]

\[
T_c > \sigma^2 \ln \eta + m = \gamma
\]

The corresponding probability of detection \( P_D \) and probability of false alarm \( P_{fa} \) are well known

\[
P_{fa} = \frac{Q(\gamma)}{\sigma}
\]

\[
P_D = \frac{Q\left(\frac{\gamma - m}{\sigma}\right)}{\sigma}
\]

Applying the Neyman-Person strategy, for a certain \( \gamma \) we get a \( \gamma \) given by

\[
P_D = Q(Q^{-1}(P_D) - \frac{m}{\sigma})
\]

In practice, assuming normalized \( \tilde{\mathbf{H}} \), \( m = 2\sqrt{NK} \) relying on the ergodicity concept where time averaging approximates statistical mean.

4.2 Non coherent processing

This case shows a different behavior for the correlated and uncorrelated scenarios. If \( \mathbf{H} \) is assumed Gaussian, received signal is averaged over time in the full correlated case to reduce noise variance but also reducing the vector dimension while in the uncorrelated case there is no preprocessing before the test application. Let us define \( \mathbf{c} \) as this Gaussian vector with normalized variance,

\[
\text{Uncorrelated: } \mathbf{w}(\mathbf{z}) = \mathbf{z} = \sqrt{2} \mathbf{a} + \mathbf{n}
\]

\[
\text{Correlated: } \mathbf{w}(\mathbf{z}) = \sqrt{2} \mathbf{K} \mathbf{a} + \mathbf{n}
\]

Clearly, we have [Fis06]

\[
\text{Uncorrelated: } \|\mathbf{w}(\mathbf{z})\|^2 \rightarrow \begin{cases} 
H_0 & \mathbf{c}^H \mathbf{c} \geq \frac{\sigma^2}{2} \chi^2_{2NK} \\
H_1 & \left(1 + \frac{\sigma^2}{2}\right) \chi^2_{2NK}
\end{cases}
\]

\[
\text{Correlated: } \|\mathbf{w}(\mathbf{z})\|^2 \rightarrow \begin{cases} 
H_0 & \mathbf{c}^H \mathbf{c} \geq \frac{\sigma^2}{2} \chi^2_{2N} \\
H_1 & \left(K + \frac{\sigma^2}{2}\right) \chi^2_{2N}
\end{cases}
\]

In this case, \( P_{fa} \) and \( P_D \) are

\[
\text{Uncorr.: } \begin{cases} 
P_{fa} = P[T > \delta | H_0] = P\left[\chi^2_{2NK} > \frac{2\delta}{\sigma^2}\right] \\
P_D = P[T > \delta | H_1] = P\left[\chi^2_{2NK} > \frac{2\delta}{\sigma^2 + 2}\right]
\end{cases}
\]

\[
\text{Corr.: } \begin{cases} 
P_{fa} = P[T > \delta | H_0] = P\left[\chi^2_{2N} > \frac{2\delta}{\sigma^2}\right] \\
P_D = P[T > \delta | H_1] = P\left[\chi^2_{2N} > \frac{2\delta}{\sigma^2 + 2K}\right]
\end{cases}
\]

Threshold \( \delta \) is obtained in a similar way,
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using oversampling it is possible to reduce this ambiguity $N$ times without requiring more bandwidth [Zazo01]. The use of different codes per OFDM symbol intends to increase the robustness of the system in case that close targets cause waveforms overlapping. In principle, the target discrimination limit will be the OFDM symbol length ($1/\Delta f$) because this is the minimum signal waveform from the first target that should be received with reduced interference from the second one in order to proceed with proper acquisition. Once this first one is detected, we propose to use interference cancellation schemes like those used in multiuser communications as BLAST [Fos03]. These ideas will be described in a forthcoming paper.

![Fig. 4](image-url)

**Fig. 4.** Probability of detection as a function of the SNR for a given probability of false alarm equal to $10^5$.

![Fig. 5](image-url)

**Fig. 5.** Probability of detection as a function of the false alarm probability for a target SNR = -5 dB.

The interplay between diversity and SNR increase is well known in communications: as observed in previous figures, diversity increases the slope of the probability curves while correlation shifts them. Depending on the scenario SNR, improvement due to one or other factor may be more valuable. In this case, at low SNR, correlated target provides more benefit.

### 6. CONCLUSIONS

This paper presents a new approach to the radar MIMO topic where a virtual multiple input multiple output system created with just one antenna using OFDM-CDM waveforms. This way, we are able to cope with more realistic scenarios including large size moving targets. Our proposal is based on a two steps procedure where range and velocity are estimated with improved resolution in order to restore the original orthogonality of OFDM waveforms and a second step where this diversity improves coherent or non coherent standard Neyman-Pearson detection. One of the key issues of this proposal is the single antenna motivation in order to reduce costs but clearly these ideas are perfectly compatible with having more antennas to increase diversity and more indeed become able to retrieve angular information. Eventually, it is important to remark that this approach may be interpreted as a detection procedure in two stages: a first one is the range detector that plays the role of "activity detector" in order to decide if it is worth to extract the time-frequency characteristic of the signal due to the multipath and moving target. Second stage outperforms this first one extracting all the available diversity provided by the bandwidth (several well separated frequencies) and time duration.

### REFERENCES


