Límite cuántico de la fotodetección

1- Introducción

2- Estadística de Poisson

3- BER

4- Sensibilidad

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1- Introducción

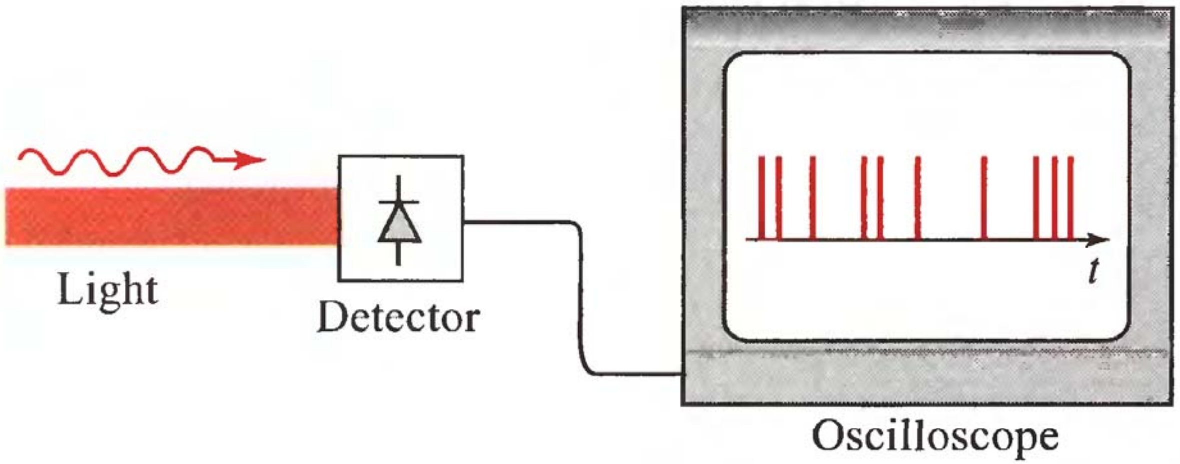
Para potencias muy bajas, aparecen los efectos cuánticos

Energía de un cuanto de enegía electromagnética (fotón)

$$E_{fotón} = \underbrace{h}_{Js}\ \underbrace{\nu}_{1/s}\ (J)$$

Potencia de un grupo de fotones

$$\boxed{\underbrace{P}_{Watios} = \underbrace{N}_{Fotones/seg}\ \underbrace{h\nu}_{Julios}}$$



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$$\text{Flujo de fotones} \rightarrow \frac{P}{h\nu}\left[\frac{\text{Fotones}}{\text{seg}}\right]$$

Número medio de fotones detectados en un tiempo T

$$\boxed{\bar{n} = \frac{P}{h\nu}T\,[\text{Fotones}]}$$

Mean photon-flux density for various sources of light.

Source	Mean Photon-Flux Density (photons/s-cm^2)
Starlight	10^6
Moonlight	10^8
Twilight	10^{10}
Indoor light	10^{12}
Sunlight	10^{14}
Laser light[a]	10^{22}

[a] A 10-mW He–Ne laser beam at $\lambda_o = 633$ nm focused to a 20-μm-diameter spot.

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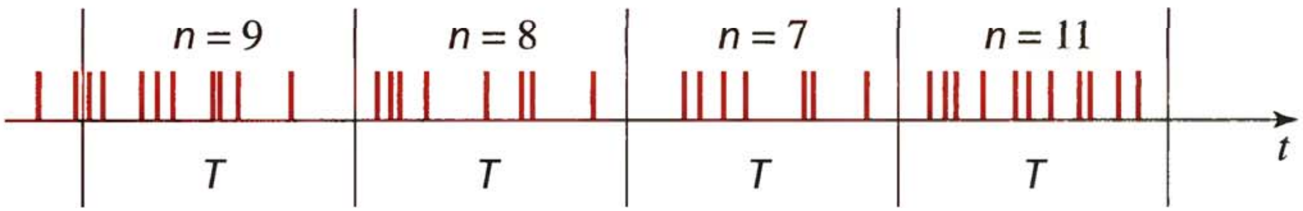

Random arrival of photons in a light beam of constant power P within during intervals of duration T. Although the optical power is constant, the number n of photons arriving within each interval is random.

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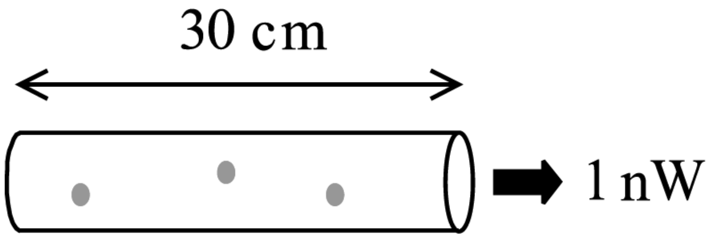
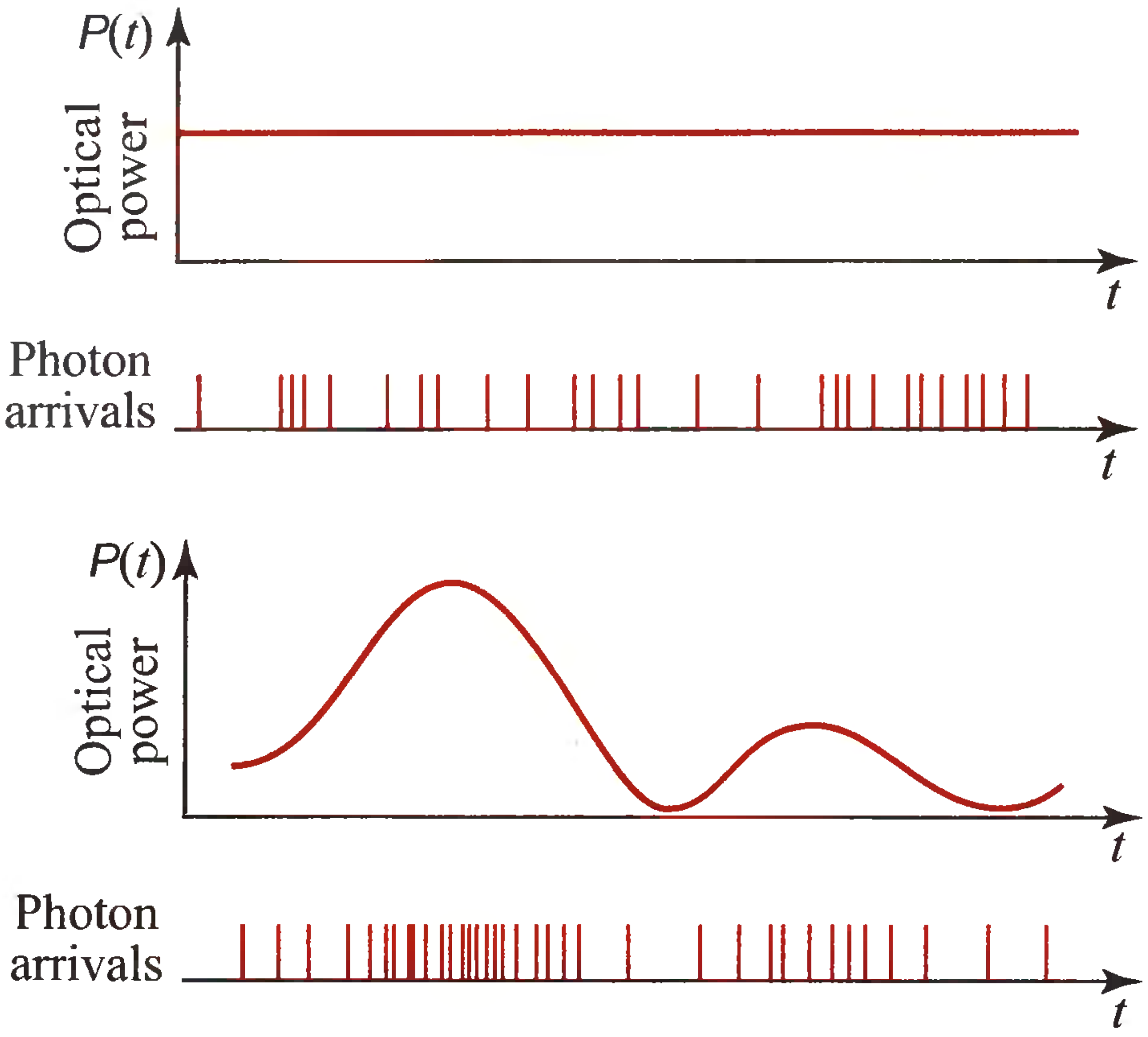
Ejemplo:



A 30 cm section of a beam light at 633 nm with a power of 1 nW contains three photons on average. (3)

$$\left.\begin{aligned} \bar{n} &= \frac{PT}{h\nu} \\ T &= \frac{L}{c} \end{aligned}\right\} \bar{n} = \frac{PL\lambda}{hc^2} = \frac{(10^{-9})(30{\cdot}10^{-2})(0,633{\cdot}10^{-6})}{(6,62{\cdot}10^{-34})(3{\cdot}10^{8})^{2}} = 3,18$$


P(t)
Optical power
t
Photon arrivals
t
P(t)
Optical power
t
Photon arrivals
t

(2)

2- Estadística de Poisson

Para potencias muy bajas $\rightarrow$ El ruido shot no se puede caracterizar con estadística gausiana

$\rightarrow$ Estadística de Poisson

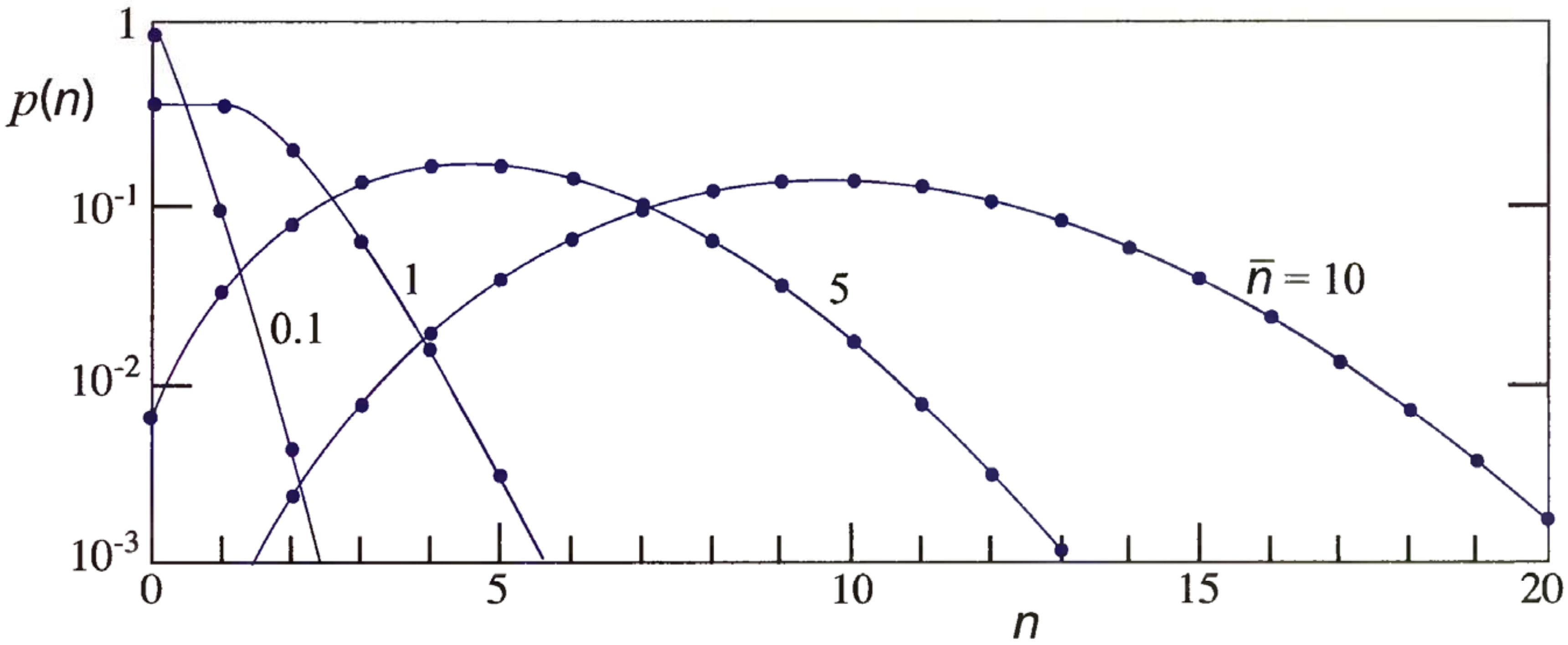
$\rightarrow P(n) \rightarrow$ Probabilidad de recibir n fotones

(siendo $\bar{n}$ el número de fotones promedio esperados)

Distribución de Poisson $\rightarrow \boxed{P(n) = \dfrac{e^{-\bar{n}}\bar{n}^n}{n!}}$ $\quad [n = 0, 1, 2, \ldots]$

Limite Clásico $\rightarrow$ Distribución Gaussiana

$$P(n) = \frac{e^{-\bar{n}}\bar{n}^n}{n!} \qquad [n \gg] \qquad \rightarrow \qquad P(n) = \frac{1}{\sqrt{2\pi\bar{n}}} e^{-\frac{(n-\bar{n})^2}{2\bar{n}}}$$



Poisson distribution $p(n)$ of the photon number n.

(2)

3- BER

$$P(n) = \frac{e^{-\bar{n}}\bar{n}^n}{n!} \qquad [n = 0, 1, 2, \ldots]$$

Probabilidad de recibir $\underbrace{1}_{n}$ fotón, cuando $\bar{n} = 0 \rightarrow \underbrace{P(1/0)}_{\text{Probabilidad de decidir 1, cuando es 0}} = 0$

Probabilidad de recibir $\underbrace{0}_{n}$ fotones, cuando $\bar{n} \neq 0 \rightarrow \underbrace{P(0/1)}_{\text{Probabilidad de decidir 0, cuando es 1}} = e^{-\bar{n}}$

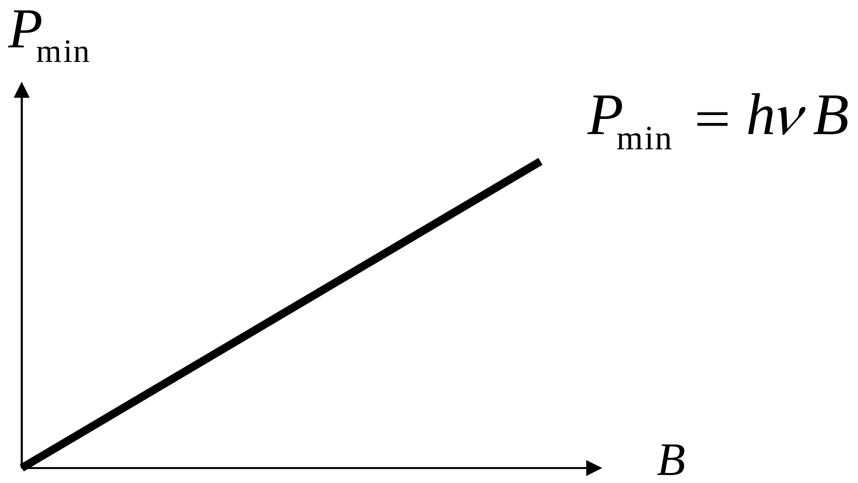
$$BER = \frac{P(0/1) + P(1/0)}{2} \rightarrow \boxed{BER = \frac{1}{2} e^{-\bar{n}}}$$

Ejemplo: $BER = 10^{-9} \rightarrow \bar{n} = 20,7 \rightarrow 21$

4- Sensibilidad

Limite Ideal

$$\left.\begin{aligned} &P = \frac{E}{t} \rightarrow P_{\min} = \frac{E_{\min}}{T_B} = \frac{h\nu}{T_B} \\ &\frac{1}{T_B} = B \end{aligned}\right\} \boxed{P_{\min} = h\nu B}$$



Límite cuántico

Número promedio de fotones contenidos en un bit "1" $\rightarrow \bar{n}\left[\dfrac{\text{fotones}}{\text{bit "1"}}\right]$

Número promedio de fotones contenidos en un bit ("1" o "0") $\rightarrow \bar{N}_{ph}\left[\dfrac{\text{fotones}}{\text{bit}}\right]$

$$\bar{N}_{ph} \rightarrow \frac{"1"+"0"}{2} \rightarrow \frac{\bar{n}+0}{2} \rightarrow \frac{\bar{n}}{2}$$

$$\left.\begin{array}{l} \bar{P}_{rec} = \dfrac{P_1+P_0}{2} = \dfrac{P_1}{2} \\ P_1 = \bar{n}h\nu \underbrace{B}_{\frac{1}{T}} \end{array}\right\} \rightarrow \boxed{\bar{P}_{rec} = \frac{1}{2}\bar{n}h\nu B = \bar{N}_{ph}h\nu B}$$

Ejemplo: $BER = 10^{-9} \rightarrow \bar{n} = 20,7 \rightarrow \bar{N}_{ph} = \dfrac{20,7}{2} \approx 10$

$$\lambda = 1,55\mu m,\ B = 10Gb/s \rightarrow \bar{P}_{rec} \approx 10h\nu B = 13nW \rightarrow (-48,9dBm)$$

En la práctica este límite se excede $\sim 20dB \rightarrow \bar{N}_p = 10 \cdot 100 = 1000$

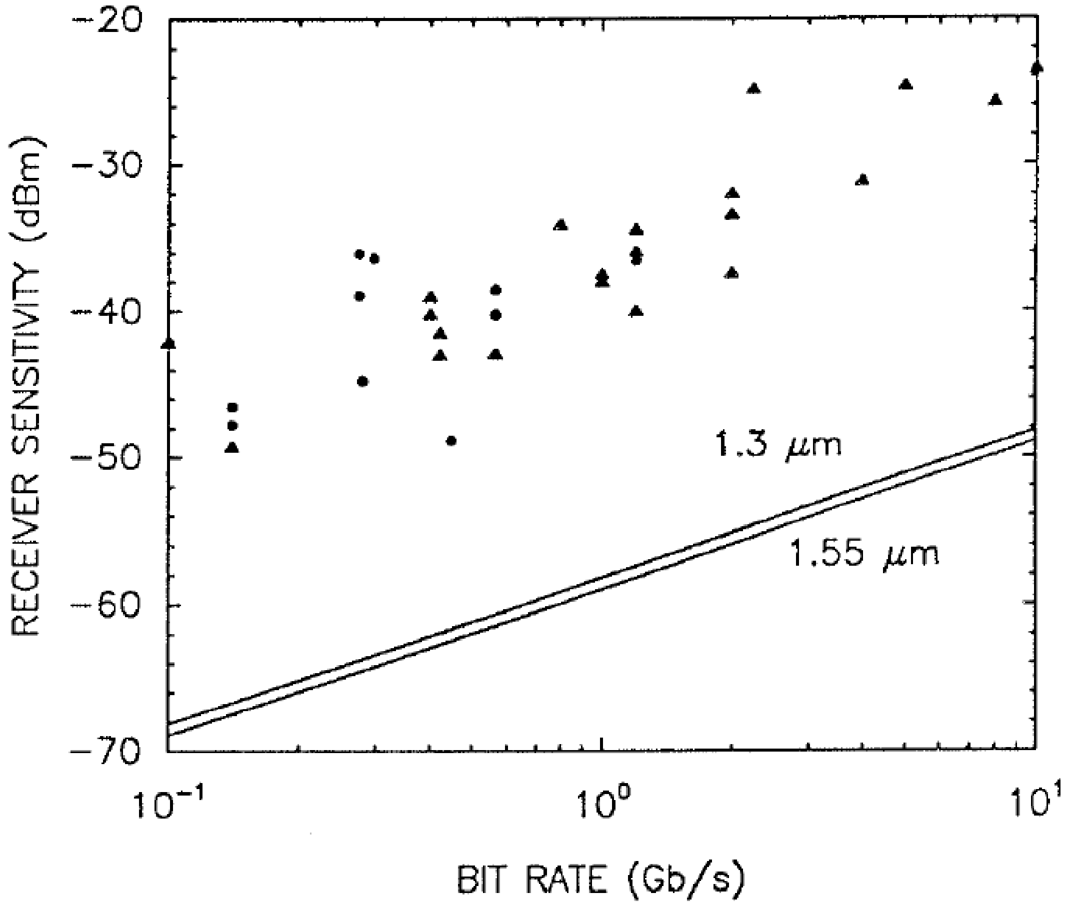


Figure 4.23: Measured receiver sensitivities versus the bit rate for *p–i–n* (circles) and APD (triangles) receivers in transmission experiments near 1.3- and 1.55-μm wavelengths. The quantum limit of receiver sensitivity is also shown for comparison (solid lines).

(1)

Typical Sensitivities (Mean Number of Photons per Bit) of Some Optical Receivers Operating at Bit Rates in the Range 1 Mb / s to 2.5 Gb / s

Receiver	Receiver Sensitivity (photons/bit)
Photon-limited ideal detector	10
Si APD	125
Er-doped silica-fiber preamplifier/ InGaAs *p-i-n* photodiode	215
InGaAs APD	500
p-i-n photodiode	6000

(2)

$$P_1 = \overline{n} h\nu B$$

$$\left.\begin{array}{l} \text{NRZ} \rightarrow \Delta f \approx \dfrac{B}{2} \rightarrow P_1 = 2\overline{n} h\nu \Delta f \\ \text{Recordando que } SNR = \dfrac{\eta_R P_1}{2h\nu\Delta f} \end{array}\right\} \rightarrow \underline{SNR = \eta_R \overline{n}} \xrightarrow{SNR=Q^2} Q = \sqrt{\eta_R \overline{n}}$$

$$\left.\begin{array}{l} \text{RZ} \rightarrow \Delta f \approx B \rightarrow P_{in} = \overline{n} h\nu \Delta f \\ \text{Recordando que } SNR = \dfrac{\eta_R P_1}{2h\nu\Delta f} \end{array}\right\} \rightarrow \underline{SNR = \frac{1}{2} \eta_R \overline{n}} \xrightarrow{SNR=Q^2} Q = \sqrt{\frac{1}{2} \eta_R \overline{n}}$$

$$\text{Ejemplo} \rightarrow \left.\begin{array}{l} SNR \approx 20dB \rightarrow \overline{n} \approx 100 \\ \lambda = 1{,}55\mu m \\ B = 10Gb/s \end{array}\right\} P_1 \approx 130\ nW$$

(1) Agrawal, "Fiber-Optic Communication Systems", 3rd Ed., Wiley, 2002

(2) Saleh and Teich, "Fundamentals of Photonics", 1st Ed., Wiley, 2007

(3) Fox, "Quantum Optics", Oxford University Press, 2006