UNIVERSIDAD POLITÉCNICA DE MADRID

**ESCUELA TÉCNICA SUPERIOR DE INGENIEROS DE
TELECOMUNICACIÓN**

**STATISTICAL TECHNIQUES AND ALGORITHMS APPLIED
IN SATELLITE COMMUNICATIONS ANTENNA ARRAYS FOR
DIRECTION OF ARRIVAL ESTIMATION AND CALIBRATION**

DOCTORAL THESIS – TESIS DOCTORAL

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ABSTRACT

Orbiting satellites and other spatial vehicles have complex trajectories that can usually be precisely approximated with analytical or numerical trajectory estimation algorithms. However, some scenarios, such as LEOP (Launch and Early Orbit Phase) or critical manoeuvres, present greater angular uncertainty, around $\pm 1^\circ$ in elevation and cross-elevation axes, and beyond. During these, large dish antennas used for TT&C (Telemetry, Tracking & Command) may have too narrow a beamwidth to perform a reliable and fast acquisition.

Traditionally, this problem has been solved using Conical Scan techniques, where a circular motion is superimposed to the main movement of the antenna so that the true DoA (Direction of Arrival) of the desired signal can be interpolated from the variation of its power levels. More recently, in X-band, a new system has been proposed by Astrium, termed XAA (X-band Acquisition Aid). In this case, a smaller dish is attached to the rim of the main reflector and, taking advantage of its broader beam, is tasked with the initial acquisition of the signal.

However, Conical Scan on the one hand takes some time to converge, around 1 minute typically, and thus may be unsuitable in critical scenarios, where signal losses must be kept to a minimum. On the other hand, while XAA converges fast enough, below 10 seconds, it suffers from two drawbacks: the initial acquisition is thermal-noised limited due to the smaller G/T (antenna gain over noise temperature) of the auxiliary antenna, and its placement on the rim of the main reflector has a non-negligible mechanical impact on the whole system. Moreover, there is a trade-off between both drawbacks, since increasing the G/T of the auxiliary antenna can only be done at the expense of larger and heavier dishes, thus exacerbating mechanical concerns. In light of these, and taking into account the current trend to resort to ever smaller TT&C antennas in X-band, down to 4 - 5 metres in diameter so as to decrease costs, it would seem that the only solution is to increase the transmitted power of the satellite, at least during critical phases. While this option is currently contemplated, it carries with it normative and legal restrictions, mainly due to interference caused to other systems. Besides, this extra power is mostly unused during normal stages, and thus it places a wasted burden on requirements on the spatial segment.

It is in this context that a novel acquisition aid system, termed SARAS and based on distributed array elements placed on the rim of the main antenna's reflector, has been implemented and successfully tested with real satellite signals. The use of antenna arrays clearly reduces the mechanical impact on the main antenna, since the weight is distributed over the whole receptor. Moreover, the large separation between array elements and the use of digital super-resolution algorithms allows the system to work in very noisy scenarios, so that the radiating elements themselves can have very low G/T and be small-sized. Finally, since the array search for the true signal DoA with electronic scan techniques, rather than mechanical ones, convergence is much faster than Conical Scan. Thus, the proposed solution takes advantages from both Conical Scan and XAA, combining them in a versatile way that opens up the possible applications of SARAS.

The first prototype has been implemented in S-band, mainly keeping in mind the acquisition of launchers, but a migration to X-band has also been considered for future designs, so that the system can cope with ever smaller TT&C antennas.

This Thesis starts with a brief analysis of array history in various applications and, notably, in satellite communications. It also includes the theoretical basis in array analysis that will be needed to assert SARAS performance, covering the mathematical signal model that is used and various DoA estimation and calibration algorithms, which are the core of the system.

The solution is then presented in detail from a system engineering perspective, including subsystem decomposition, external and internal interfaces between them, and physical allocation to hardware elements, all of this stemming from high-level needs and requirements stated by ESA (European Space Agency) at the start of the project.

Simulations in Matlab in a variety of scenarios, which motivated some of the design criteria, are exposed and compared with the actual results obtained in field campaigns with real satellites from ESA missions, thus presenting the final performance of the system, along with possible future improvements, including the aforementioned X-band migration, and commercial applications.

KEYWORDS

Antenna arrays, satellite communications, acquisition aid, spacecraft tracking, DoA estimation, calibration, TT&C, ground stations, launchers, LEOP, MUSIC, WSF, super-resolution, digital signal processing, FPGA, system engineering.

RESUMEN

Los satélites y demás vehículos espaciales siguen órbitas complejas que habitualmente pueden ser aproximadas con precisión utilizando técnicas analíticas y algoritmos de estimación numérica. Sin embargo, existen situaciones, como es el caso de LEOP (Fase de Lanzamiento y Órbita Temprana) o cuando se producen maniobras críticas, que presentan una mayor incertidumbre en la posición angular del vehículo, pudiendo la misma alcanzar cotas de $\pm 1^\circ$ en los ejes de elevación y elevación cruzada, o incluso mayores. En estos casos, las grandes antenas reflectoras utilizadas para comunicaciones de TT&C (Telemetría, Seguimiento y Control) tienen anchos de haz demasiado estrechos como para garantizar una adquisición rápida y fiable.

Tradicionalmente se ha recurrido a la técnica de Escaneo Cónico, en la cual se superpone un movimiento circular al que efectúa la antena mientras intenta seguir al satélite en cuestión, de tal forma que la verdadera DoA (Dirección de Llegada) de la señal del satélite puede ser interpolada a partir del patrón de variación de sus niveles de potencia. Más recientemente, Astrium ha presentado un nuevo sistema en banda X, llamado XAA (Ayuda a la Adquisición en banda X), que consiste en un reflector auxiliar de menor tamaño adherido al principal. Gracias a un ancho de haz más amplio, puede efectuar la adquisición inicial de la señal deseada.

Sin embargo ambos sistemas presentan desventajas. El Escaneo Cónico tiene una convergencia lenta, alrededor de 1 minuto típicamente, y puede resultar inapropiado en situaciones críticas, en las cuales se debe minimizar la cantidad de datos perdidos. Por otro lado, aunque el XAA converge rápidamente, por debajo de los 10 segundos, la adquisición inicial está limitada por el ruido térmico, con mayor influencia teniendo en cuenta que la G/T (ganancia de antena sobre temperatura de ruido) de la antena auxiliar es menor. Además, su colocación en el borde del reflector principal tiene un impacto mecánico considerable. Ambas desventajas se complementan puesto que, para incrementar la G/T, se necesita recurrir a reflectores de mayor tamaño y más pesados. Teniendo en cuenta que, para reducir costes, actualmente las agencias espaciales tienden a utilizar antenas de TT&C en banda X de menor tamaño, en torno a 4 o 5 metros de diámetro, parece que la única solución sería aumentar la potencia transmitida por el satélite, al menos durante las fases críticas. Si bien en las misiones actuales se recurre a esta opción, no está exenta de restricciones normativas y legales, principalmente relacionadas con las interferencias que se puedan causar a otros sistemas. Además, la potencia añadida no se utiliza en las fases normales de operación del satélite, e implica por lo tanto una restricción desaprovechada en los requisitos del segmento espacial de la misión.

Es en este contexto donde se presenta la solución SARAS, un novedoso sistema de ayuda a la adquisición basado en arrays de elementos radiantes distribuidos sobre el perímetro del reflector principal, que ha sido implementado y probado con éxito con señales de satélites reales. El uso de arrays reduce claramente el impacto mecánico en la antena principal, dado que el peso se reparte por todo el reflector. Además, teniendo en cuenta que las antenas del array tienen una gran separación entre ellas, y que se procesa la señal recibida con algoritmos digitales de superresolución, el sistema puede operar en entornos muy ruidosos, permitiendo que los elementos

radiantes puedan tener bajas G/T y menor tamaño. Por último, puesto que la búsqueda de la DoA de la señal deseada se efectúa mediante escaneo electrónico y no mecánico, la convergencia es mucho más rápida que la del Escaneo Cónico. La solución propuesta obtiene por lo tanto ventajas tanto del Escaneo Cónico como del XAA, combinándoles versátilmente de tal forma que ofrece nuevas aplicaciones para SARAS.

El primer prototipo se ha implementado en banda S, principalmente para su uso en la adquisición de lanzadores. Sin embargo, se ha considerado igualmente una migración futura a banda X, para así permitir que el sistema pueda acoplarse con antenas de TT&C con tamaños cada vez más reducidos.

Esta Tesis comienza con un breve análisis de la historia del uso de arrays en varias aplicaciones y, naturalmente, en comunicaciones por satélite, e incluye las bases teóricas del análisis de arrays que serán necesarias para evaluar el desempeño de SARAS. Dichas bases incluyen el modelo matemático de la señal recibida que será considerado, así como los algoritmos de estimación de DoA y de calibración que son el núcleo del sistema.

Se presenta a continuación la solución desde una perspectiva de ingeniería de sistemas, incluyendo su descomposición en subsistemas, con interfaces internas y externas, y el mapeado de los mismos a elementos hardware, todo ello siguiendo los criterios y necesidades de alto nivel que planteó la ESA (Agencia Espacial Europea) al comienzo del proyecto.

Se exponen igualmente simulaciones en el entorno de programación de Matlab, en una variedad de escenarios, y que sirvieron en su momento para tomar decisiones relativas al diseño del sistema SARAS. También se comparan dichas simulaciones con los resultados obtenidos en campo con satélites reales de distintas misiones de la ESA, confirmando por lo tanto el desempeño final del sistema, y esbozando posibles mejoras al mismo, incluyendo la migración a banda X, así como sus aplicaciones comerciales.

PALABRAS CLAVE

Arrays de antenas, comunicaciones por satélite, ayuda en la adquisición, seguimiento de vehículos espaciales, estimación de DoA, calibración, TT&C, segmento terreno, lanzadores, LEOP, MUSIC, WSF, superresolución, procesamiento digital de señales, FPGA, ingeniería de sistemas.

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ACRONYMS

AC	Anechoic Chamber
AC/DC	Alternating Current/Direct Current
ACU	Antenna Control Unit
ADC	Analogue-to-Digital Converter
AF	Array Factor
AGC	Automatic Gain Control
AIC	Akaike Information Criterion
AIV	Assembly, Integration and Verification
ALMA	Atacama Large Millimetre/sub-millimetre Array
AM	Amplitude Modulation
AML	Asymptotic Maximum Likelihood
ASIC	Application-Specific Integrated Circuit
BB	Base-Band
BPF	Band Pass Filter
BPSK	Binary Phase-Shift Keying
BRAM	Block Random Access Memory
BW	BandWidth
CCSDS	Consultative Committee for Space Data Systems
CIC	Cascaded Integrator-Comb
CML	Conditional Maximum Likelihood
CNF	Cascaded Noise Figure
CNR	Carrier-to-Noise Ratio
ConOps	Concept of Operations
CONSCAN	Conical Scan
COTS	Commercial Off-The-Shelf
cPCI	Compact Peripheral Component Interconnect
CPU	Central Processing Unit

CRLB	Cramér-Rao Lower Bound
DC	Down-Converter
DDC	Digital Down Converter
DDS	Direct Digital Synthesizer
DoA	Direction of Arrival
DoD	Department of Defense
DSAN	Deep Space Array-based Network
DSCS	Defense Satellite Communication System
DSN	Deep Space Network
DSP	Digital Signal Processor/Processing
DUC	Digital Up Converter
ECEF	Earth-Centred Earth-Fixed
ECI	Earth-Centred Inertial
EIRP	Equivalent/Effective Isotropic Radiated Power
EKF	Extended Kalman Filter
EM	Expectation-Maximisation
ESA	European Space Agency
ESAC	European Space Astronomy Centre
ESPRIT	Estimation of Signal Parameter via Rotational Invariance Techniques
ETSIT	Escuela Técnica Superior de Ingenieros de Telecomunicación
F&T	Frequency and Time
FDMA	Frequency Division Multiple Access
FEC	Front-End Controller
FFT	Fast Fourier Transform
FIM	Fisher Information Matrix
FIR	Finite Impulse Response
FM	Frequency Modulation
FoM	Figure of Merit
FoV	Field of View

FPGA	Field Programmable Gate Array
FSS	Fixed Satellite Service
FT	Fourier Transform
GBBF	Ground-Based Beam Forming
Gbps	Gigabits per second
GDPA	Geodesic Dome Phased Array Antenna
GEO	Geostationary Earth Orbit
GHz	Giga-Hertz
GOCE	Gravity field and steady-state Ocean Circulation Explorer
GPIO	General Purpose Input/Output
GPS	Global Positioning System
GPU	Graphics Processing Unit
GUI	Graphical User Interface
HMI	Human Machine Interface
HPBW	Half-Power Beam-Width
IF	Intermediate Frequency
IFT	Inverse Fourier transform
INSA	Ingeniería y Servicios Aeroespaciales
INTEGRAL	INTE rnational G amma- R ay A strophysics L aboratory
ISDEFE	Ingeniería de Sistemas para la Defensa de España
JPL	Jet Propulsion Laboratory
LAN	Local Area Network
LEO	Low Earth Orbit
LEOP	Launch and Early Orbit Phase
LF	Loading Factor
LHCP	Left Hand Circular Polarization
LNA	Low Noise Amplifier
LO	Local Oscillator
LS	Least Squares

Acronyms and Symbols

M&C	Monitoring and Control
MAP	Maximum A Posteriori
MCOP	Maximum Combined Output Power
MCOSINR	Maximum Combined Output SINR
MCOSNR	Maximum Combined Output SNR
MDL	Minimum Description Length
MDP	Multichannel Digital Processor
MEO	Medium Earth Orbit satellite
MER	Main Equipment Room
MHz	Mega-Hertz
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimation
MMCX	Micro-Miniature Coaxial
MMSE	Minimum Mean Square Error
MPDR	Minimum Power Distortionless Response
MSE	Mean Square Error
Msp/s	Mega samples per second
MUSIC	MULTiple Signal Classification
MV	Minimum Variance
MVDR	Minimum Variance Distortionless Response
NASA	National Aeronautics and Space Administration
NCO	Numerically Controlled Oscillator
NF	Noise Figure
NTP	Network Time Protocol
OSAT	On-Site Acceptance Test
PAWS	Phased Array Warning System
PDF	Probability Density Function
PFD	Power Flux Density

PLL	Phase-Locked Loop
PPS	Pulse Per Second
PROS	Projection-ROtation-Scaling
QPSK	Quadrature Phase-Shift Keying
RADAR	RAdio Detection And Ranging
RAM	Random Access Memory
RF	Radio Frequency
RHCP	Right Hand Circular Polarization
RLS	Recursive Least Squares
RMS	Root Mean Square
RMSE	Root Mean Square Error
SAGE	Space-Alternating Generalised Expectation-maximisation
SARAS	Spanish acronym for Fast Satellite Acquisition System in S band
SARAX	Spanish acronym for Fast Satellite Acquisition System in X band
SBX	Sea-Based X-band RADAR
SC	SARAS/SARAX Computer
SDMA	Spatial Division Multiple Access
SINR	Signal-to-Interference and Noise Ratio
SKA	Square Kilometre Array
SLL	Side Lobe Level
SMI	Sample Matrix Inversion
SNR	Signal-to-Noise Ratio
SONAR	SOund Navigation And Ranging
SSA	Space Situational Awareness
STAP	Space-Time Adaptive Processing
STARS	Surveillance and Target Attack Radar System
std	Standard deviation
STDM	Spacecraft Trajectory Data Message
SV	Steering Vector

SVD	Singular Value Decomposition
SVE	Steering Vector Estimation
TCP/IP	Transmission Control Protocol/Internet Protocol
TDRSS	Tracking and Data Relay Satellite System
TLE	Two-Line Element
TM	Telemetry
TRL	Technology Readiness Level
TT&C	Telemetry, Tracking and Command
UCA	Uniform Circular Array
UHF	Ultra High Frequency
UKF	Unscented Kalman Filter
ULA	Uniform Linear Array
UMTS	Universal Mobile Telecommunications System
UPM	Universidad Politécnica de Madrid
US	United States
UTC	Universal Time Coordinated
VHDL	VHSIC Hardware Description Language
VHSIC	Very High Speed Integrated Circuit
VLA	Very Large Array
VSAT	Very Small Aperture Terminal
VSWR	Voltage Standing Wave Ratio
Wi-Fi	Wireless Fidelity
WiMax	Worldwide Interoperability for Microwave Access
WSF	Weighted Subspace Fitting
XAA	X-band Acquisition Aid
XMM	X-ray Multi-Mirror Mission
XPD	Cross-Polar Discrimination

SYMBOLS

C/No	Carrier over noise floor level
G/T	Antenna gain over noise temperature
Rx	Receiver
S/S	Subsystem
Tx	Transmitter

CHAPTER 1 INTRODUCTION

1.1 Array Technology

Phased arrays are a burgeoning topic in multiple fields of knowledge, including of course satellite communications. From their starting point in RADAR (RADio Detection And Ranging) systems, where they were first used, they have evolved within multiple fields of knowledge, birthing new concepts each year. The present section and the following one are a panoramic summary of the array applications, starting from their practical beginnings shortly before World War II, all the way down to the most innovative solutions such as Geodesic Phased Arrays Domes or Ground Based Beamforming.

1.1.1 RADAR and SONAR

Regarding RADAR, the first patents and papers on the topic were published between 1899 and 1926 (Fourikis 1997), while the first use was reported in 1937. However, major advances would have to wait to the 1960s. The initial justification of the implementation of array systems in RADAR applications was the search for higher angular resolutions capabilities, in order to improve the recognition capabilities of the system.

Conformal arrays made up of small radiating elements have also been considered for aircrafts or vehicles, while fulfilling aerodynamics restrictions or reducing their RADAR Cross Section, features of interest in Electronic Warfare. On the other hand, non-military applications have been mainly centred in air traffic control radars.

Classical examples of military RADAR include the PAVE PAWS (Phased Array Warning System) System, which consists of two phased arrays with 1792 antenna elements and a FoV (Field of View) of 120° each. It was designed as an early warning of attacks by submarine-launched ballistic missiles and to aid in tracking satellites at the UHF (Ultra High Frequency) band (Fourikis 1997).

Figure 1.1 PAVE PAWS System¹

Array RADAR systems have also moved up the spectrum band. For example, SBX-1 (Sea-Based X-band RADAR) is the largest X-band antenna array in the world, and is part of the US Ballistic Missile Defense System that tracks and identifies long-range missiles. It has almost full hemispherical coverage, with 45000 GaAs transmit/receive modules (Haupt 2010).

Figure 1.2 SBX-1 RADAR²

As of civilian applications, ESA (European Space Agency) Space Situational Awareness (SSA) Programme constitutes a notable example. The Programme seeks to obtain independent and accurate information of hazards to infrastructures in space and on the ground, such as asteroids or man-made space debris. Part of the concept is based on a phased array RADAR which is to catalogue and track objects orbiting the Earth. A testbed of the system was successfully tested in Spain in November 2013, detecting objects of around 1m in size, including the Cygnus cargo service craft from the ISS (International Space Station)³. This RADAR demonstrator, operating in L-

¹ From <http://mostlymissiledefense.com/2012/04/12/pave-paws-and-bmews-radars-april-12-2012/>

² From <http://www.naval-technology.com/projects/sea-based-x-band-radar-1-sbx-1/>

³ From http://www.esa.int/Our_Activities/Operations/Space_Situational_Awareness

band, was developed by Indra and Fraunhofer FHR. The receiving array was made of 128 elements, with a resulting maximum directivity of 28.8 dB (Wilden et al. 2012).

Advanced systems with recent developments such as Space-Time Adaptive Processing (STAP) have also been implemented. This is the case of the Joint Surveillance and Target Attack Radar System (Joint STARS), which uses a phased array antenna mounted on the forward underfuselage of an Air Force E-8A (Monzingo, Haupt, and Miller 2011). STAP is well suited to detect targets in the presence of clutter and jamming.

Arrays are also widely present in SONAR (SOund Navigation And Ranging) systems, which process received echoes from acoustic transmissions. The main difference with RADAR systems is that propagation of the acoustic wave is more complex to model (Van Trees 2002). The most important application is the detection and tracking of submarines.

1.1.2 Radioastronomy

As in RADAR systems, the stringent resolution requirements were the main reason behind the use of arrays in radioastronomy. As stated in (Fourikis 1997), resolutions of the order of 1 arc second were the long-sought objective of radio astronomers for several decades. Such a requirement, without the use of arrays, would be translated into monolithic antennas with 6.188 km in diameter, for a frequency of operation of 10 GHz.

The first array development was made during the early 1950s for interferometric applications (Fourikis 1997), and led to the implementation of advanced array systems such as the VLA (Very Large Array). Commissioned in the early 1980s, the VLA is situated in the Plains of San Augustin (west-central New Mexico). It is made up of 27 antennas, each one of these with a diameter of 25m. The antennas can be moved to several predetermined positions, so that variable spatial resolutions can be obtained, and thus the u-v observation space can be more densely completed.

Figure 1.3 VLA⁴

⁴ From <http://www.vla.nrao.edu/>

Arrays are ever-present in newer generations of radiotelescopes. The Atacama Large Millimetre/sub-millimetre Array (ALMA), located in the Chajnantor plain in the Chilean Andes, uses 66 dish antennas, 54 of them 12 metres across, and 12 of them 7 metres, and seeks to catch never before seen details about the first galaxies and stars, probe the centre of the Milky Way, and directly image the formation of planets ⁵.

Figure 1.4 ALMA⁶

One of the most ambitious radioastronomy projects of all times, the SKA (Square Kilometre Array) will become operative in the near future. SKA is an international effort from 10 member countries (as of 2015) to construct an array with hundreds of thousands of radio-telescopes, which will give sky surveying capabilities with unprecedented level of detail and scan speeds. Two systems will be implemented, in South Africa and in Australia. Pre-construction started in 2012, with two project phases in mind. At the end of phase 1, Australia will host the low-frequency telescopes with more than 900 stations, each one with almost 300 dipole antennas, as well as 96 dishes, while South Africa will host an array of 254 dishes. Phase 2 will complete both implementations, becoming fully operational in the mid-2020s ⁷.

Figure 1.5 SKA System Architecture (Dewdney 2013)

⁵ From <https://public.nrao.edu/telescopes/alma/alma-basics>

⁶ From <http://www.eso.org/public/chile/teles-instr/alma/>

⁷ From <https://www.skatelescope.org/>

1.1.3 Wireless Communications

The development of communications array systems ran more or less in parallel with those in RADAR in the 1930s (Van Trees 2002). Wireless communications are driven by an increasing demand of commercial system capacities, made by an increasing number of end users. These two phenomena drive the evolving technological improvements with surprising speed, exacerbated by the globalisation of such services, even in rural and developing areas and countries, and by the multimedia revolution experienced in mobile systems.

Since the electromagnetic spectrum is a scarce resource, higher capacities must be translated into a more efficient uses of it. Several ways have been proposed, including new source and channel coding schemes, cognitive radio solutions and multiple access techniques. Among those, the latest form of SDMA (Spatial Division Multiple Access) usually employs adaptive antenna arrays (Imbriale 2003).

It was in mobile communications where the term “smart antennas” was first coined to refer to arrays with adaptive signal processing. According to (Molisch 2010), smart antennas can be used for various purposes:

- Increase of coverage.
- Increase of capacity.
- Improvement of link quality.
- Decrease of delay dispersion.
- Improvement of user position estimation.

A MIMO (Multiple Input Multiple Output) system emerges if antenna arrays are used both at the transmit and the receiver side of a given link, as originally suggested in (Winters 1987). MIMO systems have become a very promising technology and, since the beginning of the 21st century, studies in their stead have gone from theoretical to practical implementations. Indeed, MIMO concepts have been integrated into Third Generation cellular systems and beyond, as well as into WiMax (Worldwide Interoperability for Microwave Access) and Wi-Fi (Wireless Fidelity) standards (Oestges 2007).

1.2 Arrays and Satellite Communications

Many satellite systems use phased arrays. Examples include the Defense Satellite Communication System (DSCS III), with a 61-beam receive array, and a 19-beam transmit array (Van Trees 2002), and the Iridium satellites, where the main mission antenna consists of three phased array panels in L-band. Each phased array produces 16 fixed simultaneous beams from a total of 48 beams per satellite (Allen and Ghavami 2005).

Next subsections will give some more examples of actual and future array satellite systems, divided into three categories, TT&C (Telemetry, Tracking and Command), DSN (Deep Space Network) and GBBF (Ground-Based Beam Forming).

1.2.1 TT&C

Implementations of array systems for TT&C applications have a dilated history. The Pluton Complex, built in the 1960s in Yevpatoria, in Crimea, supported all the Soviet space programmes until 1978, and was the world's highest capacity deep space communications system prior to Goldstone in 1966. The dishes were placed on two hulls of diesel submarines, welded together and laid down on railway bridge trusses⁸.

Figure 1.6 Pluton Complex in Crimea

Newer solutions, though, have replaced the assembly of reflector concept with pure phased arrays. Indeed, an initiative financed by the US DoD is currently on-going to develop such a phased array antenna for tracking of more than 90 defence satellites, with a mean capacity of 9 simultaneous links. Hemispheric coverage is achieved in this case using a geodesic dome geometry. The antenna, named GDPAA (Geodesic Dome Phased Array Antenna), was proposed by Ball Aerospace in 2001, and some of its achievements can be consulted in (Henderson, Davis, and Huisjen 2010). The antenna will consist of 1675 panels with two Tx/Rx beams in S-band. Digital beamforming will be at the core of the signal processing, with a maximum bandwidth of 5 MHz. The global budget exceeds 30 M\$ and a prototype with 56 panels has already been successfully implemented, as shown below:

⁸ From http://mentallandscape.com/V_Telemetry.htm

Figure 1.7 GDPAA Antenna in Colorado

A similar system, named GEODA, has been proposed by ISDEFE and the UPM to ESA. GEODA is an architecture based on antenna arraying of ground stations which offers the opportunity to receive and transmit signals from several satellites in S band. These antenna structures are an alternative to traditional parabolic dishes since they permit to process several satellites simultaneously, they do not have mechanical steering components, and they can adapt their radiation pattern to mitigate the presence of interferers and multipath. The final system will have between 40 and 60 triangular and square panels, to obtain hemispherical coverage for transmission and reception, with the capability to process more than eight missions simultaneously and with adaptive robustness against jammers.

Figure 1.8 GEODA final concept

A prototype made of 6 triangular panels is currently in development:

Figure 1.9 GEODA prototype

1.2.2 DSN

Next generation Deep Space Networks already seek to eliminate single point of failures inherent to single reflector solutions, while also increasing bandwidth requirements for future missions, such as would be needed for manned flights to Mars.

As indicated by NASA (Imbriale 2003), studies have been conducted where arrays have been considered as an alternative to 70 meters antennas. Proposed solutions included an array of four 34-m aperture antennas, an array of small reflector antennas, and an array of flat-plane antennas.

A similar study was carried out by former INSA (now ISDEFE) company, for the definition of ESA new deep space network architectures. Proposed alternatives included arrays of four 34/35-m antennas, as in the NASA case, and arrays of forty 12-m antennas.

While downlink arrays face important technological challenges, uplink array, due to their inherent lack of a simple feedback loop, are even more disruptive. Initial experiments have been carried out by JPL (Davarian 2011), using reflections from the Moon (Vilnrotter et al. 2010), and an operative system was firstly implemented by Harris Corporation, as explained in (Martin and Minear 2014).

The next figure shows future bandwidth requirements for manned missions to Mars. As can be deduced, the one order of magnitude increase with respect to present requirements implies that new solutions must be implemented, and arrays represent a viable alternative.

	User	Channel Content	Latency	No. of Channels	Bit rate/channel	Total Bit Rate
Operational	Base	Speech	NRT	2	10 kbps	20 kbps
		Engineering	NRT	1	100 kbps	100 kbps
	Astronauts	Speech	NRT	4	10 kbps	40 kbps
		Helmet Camera	NRT	4	100 kbps	400 kbps
		Engineering	NRT	4	20 kbps	80 kbps
	Human Transports	Video	NRT	2	1.5 Mbps	3 Mbps
		Engineering	NRT	2	20 kbps	40 kbps
	Robotic Rovers	Video	NRT	8	1.5 Mbps	12 Mbps
		Engineering	NRT	8	20 kbps	160 kbps
	Science Orbiters	Quick Look	NRT	4	1 Mbps	4 Mbps
		Engineering	NRT	4	20 kbps	80 kbps
	Human Transport	HDTV	NRT	2	20 Mbps	40 Mbps
		Hyperspectral imaging	1 day	1	150 Mbps	150 Mbps
High Rate	Base	HDTV	1 day	1	20 Mbps	20 Mbps
	Robotic Rovers	Surface Radar	1 day	2	100 Mbps	200 Mbps
		Hyperspectral imaging	1 day	1	150 Mbps	150 Mbps
	Science Orbiter	Orbiting Radar	1 day	2	100 Mbps	200 Mbps
		Hyperspectral imaging	1 day	2	150 Mbps	300 Mbps
Total	Design Reference Mission (DRM) and HDTV – operational					80 Mbps
	Add robotic operations and DRM Hyperspectral imaging					480 Mbps
	Add science orbiters					980 Mbps

Figure 1.10 Bandwidth requirements for future manned missions to Mars (Vyas et al. 2006)

1.2.3 GBBF

In satellite communications, SDMA is implemented using spot beams with a frequency re-use factor. Although limited in scope, future systems have already been proposed with digital beamforming and GBBF (Ground Based Beam Forming), which will enable the implementation of HTS (High Throughput Satellites) for 5G networks and satellite Internet.

As traffic demand for the next generation of satellite services continue to increase, the advent of new paradigms which contribute to the improvement of the spectral efficiency and the bandwidth occupancy of satellite signals will become critical.

A classical solution in this sense is the use of multi-spot array antennas in the satellite, both for the Forward and the Return Links. This spatial multiplexing increases the theoretical maximum throughput of the system for the same occupied bandwidth. Moreover, smaller and slightly overlapping beams reduce the signal loss experienced at the coverage's edge. However, if adjacent beams share the same frequency bands, inter-spot interference will take place and degrade the availability and final performance of the communications links. This inter-spot interference, as well as the intra-spot one, severely limits the practical increase of traffic throughput.

The ability to increase the frequency bands re-use among spots is thus mandatory to fully recollect the advantages that arrays bring to satellite communications. For this, Multi-User Detection and Interference Mitigation techniques

in the Return Link, and Linear Pre-coding with low cross-correlation codes have been proposed and successfully implemented in some real satellite payloads. The main drawback of these solutions, which will be exacerbated in the coming years, is that the intense digital signal processing that ensues implies more complex designs of satellites, which translate into higher power and mass requirements and a higher cost of the satellite segment.

In this context, GBBF (Ground Based Beam Forming) is being considered as an alternative to alleviate the complexity imposed on the satellite payloads, and migrate it to the gateways in the ground segment, which would perform the beamforming and pre-coding of signals received and transmitted by the satellite array antenna. In order to maintain the spatial differentiation of each feed signal, a frequency multiplexing scheme is devised in the link between gateway and satellite.

The advantages of GBBF have already been assessed in the Mobile Satellite Service scenario and successfully implemented in TDRSS (Tracking and Data Relay Satellite System). However, in order to increase the attractiveness of such an innovation, new markets and synergies are to be explored, taking into account for example both the Fixed and Broadcasting Satellite Services.

More information on these solutions can be found, among other references, in (Cecca et al. 2014).

1.3 Framework for SARAS Application

The encompassing presence of arrays and arraying algorithms in satellite communications has been demonstrated in the preceding section. In the present one, another innovative application will be discussed, and the proposed solution will be the centre of this Thesis.

1.3.1 Motivation

Ground stations aimed to receive or transmit data from/to orbiting satellites and other spatial vehicles need to know their angular location with great precision and accuracy. Indeed, large antennas in the ground segment are characterised by their narrow beams, which allows having a high gain. Such narrow beams become a problem in the initial phases of satellite acquisition, especially when dealing with LEOP (Launch and Early Orbit Phase). As the angular uncertainty window during this phase is wide it can take a long time before the ground station acquires the satellite, causing data losses and obviously inability to operate the spacecraft.

In many situations, the problem can be solved through complex mathematical models of the vehicle trajectory. In particular, for satellites, orbital propagators are commonly used for calculating expected positions. However, those techniques are generally designed for stable or final orbits, with decreasing precision in some critical scenarios where too much inherent uncertainty makes the estimation system less reliable. Thus the initial estimate of the vehicle DoA (Direction of Arrival) must be improved with additional means. During LEOP, critical manoeuvres, malfunctioning or

anomalies in the nominal orbit and, of course, with any complex trajectory which cannot be fitted into the mentioned mathematical models, TT&C (Telemetry, Tracking and Command) becomes critical.

An obvious solution to this problem is the reduction of the size of the ground station, in order to increase its beamwidth. When this is not possible, due to link budget considerations, an auxiliary antenna can be included, thus retaining the high gain of the main antenna, and allowing an increased angular search window with the smaller one. This auxiliary system can be completely independent, with a single interface with the main antenna to send the estimated DoA. On the other hand, the auxiliary antenna can be attached to the rim of the main one in order to share the steering and coordinate frame systems.

In general terms, this solution suffers from one main drawback: a trade-off between thermal noise robustness, acquisition range and mechanical impact. In order to broaden the beam to cope with the DoA dispersion window, the G/T (Gain over Noise Temperature) must be significantly reduced, and this fact poses a lower bound to the incoming power flux density of the signal and its data rate. To partially overcome this limit, state of the art technologies can be used, such as very low noise amplifiers, waveguide feeding systems and compact outdoor converters, all of them focused on decreasing the system noise temperature. However, the improvement of equipment is also limited by the mechanical interface between the main and auxiliary antennas, and by weight considerations. In addition, this solution has no interferer discrimination capabilities.

Figure 1.11 Conventional Solution: XAA (X-band Acquisition Aid System) (Migl et al. 2010)

1.3.2 Objectives

A fast and robust system for the localisation and tracking of space-borne vehicles in scenarios with great angular uncertainty and highly dynamical characteristics is thus needed. In the frame of SARAS (Spanish acronym for Fast

Satellite Acquisition System) Project, co-funded by ESA (European Space Agency) under a GSTP A (General Support Technical Programme), a new concept of acquisition aid system based on an array solution has been developed and tested. The system is composed of 8 sensors distributed in the rim of a 15 meter diameter antenna. Indeed, SARAS has been validated using Vil-1 antenna in ESAC station, as shown below:

Figure 1.12 SARAS system in VIL1 antenna, ESA/ESAC station

Its main characteristics are:

- **High white noise robustness:** the system can operate at very negative SNR (Signal-to-Noise Ratio), an essential feature taking into account the reduced G/T of the individual radiating elements. This robustness is achieved through dual space-frequency estimators.
- **Fast acquisition** through an electronically steered array.
- **Tracking and prediction** features using STDN (Spacecraft Trajectory Data Message) files and EKF (Extended Kalman Filters).
- **Interferer discrimination** capabilities, both in space and frequency.

1.4 Thesis Structure

The remainder of this Thesis has been structured in the following chapters:

- **Chapter 2: Antenna Array Processing.** This chapter will be dedicated to the development of the mathematical tools and algorithms that will be required for the analysis of SARAS system. From general arraying theory and signal processing, specific algorithms will be presented for the problem at hand, namely, the estimation of the DoA of a desired signal.
- **Chapter 3: Fast Acquisition of Satellite Systems.** This will be the core of the Thesis, where SARAS system will be formally presented and analysed in detail, from a systemic point of view. Actual implementations of the algorithms presented in chapter 2 will be discussed, along with their performance using Monte Carlo simulations.

- **Chapter 4: Calibration.** This chapter will deal with the complement of DoA estimation in an advanced array system, i.e., the calibration of the array response, or array manifold, so that phase offsets among antenna elements can be accurately translated into azimuth and elevation angles. As was the case with the previous chapter, Monte Carlo simulations will demonstrate the effective performance and limitations of the proposed algorithms.
- **Chapter 5: Test Campaigns and Validation.** Beyond the simulations presented in chapters 3 and 4, actual test results with real satellite signals will be shown and compared with theoretical and simulated FoM (Figures of Merit).
- **Chapter 6: Conclusions.** The last chapter will summarise the contributions of this Thesis, and will propose the next steps to be taken for the upgrade of SARAS system to a fully operational product.

CHAPTER 2 ANTENNA ARRAY PROCESSING

2.1 Mathematical Framework

In the generic case, an array of N radiating elements is modelled using three position coordinates per element. The coordinate system is chosen such as to simplify the equations of the problem at hand, and thus may depend on the specific form of the array. The most typical case is a Cartesian system, where the position of the n -th element is modelled as:

$$\mathbf{p}_n = \begin{pmatrix} x_n \\ y_n \\ z_n \end{pmatrix}, \quad n = 0, \dots, N - 1 \quad (2.1)$$

The other element in the mathematical formulation of a receiving array problem is the incoming wave, sent by the signal source. In the case of satellite emissions, taking into account to the distances involved, the wavefront can be assumed to be locally flat without loss of generality. This wavefront has an associated direction which can be completely characterised by two spherical angles, θ and ϕ . The relationship between spherical and Cartesian systems is established through the following equations:

$$\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \end{aligned} \quad (2.2)$$

From these, the unitary vector $\mathbf{a}$ associated with the wavefront is defined as:

$$\mathbf{a} = \begin{pmatrix} -\sin\theta \cos\phi \\ -\sin\theta \sin\phi \\ -\cos\theta \end{pmatrix} \quad (2.3)$$

The minus sign appears with impinging waves with respect to the array, as shown in the next figure:

Figure 2.1 Array with a planar wavefront

Each array element will receive the same signal $f(t)$, but with differing delays in each case. Taking as a reference the coordinate origin, all the received signals can be grouped in a single column vector:

$$\mathbf{f}(t, \mathbf{p}) = \begin{pmatrix} f(t - \tau_0) \\ f(t - \tau_1) \\ \vdots \\ f(t - \tau_{N-1}) \end{pmatrix} \quad (2.4)$$

τ_n is the delay in the n -th element of the array, with respect to the origin of the coordinate system, defined as:

$$\tau_n = \frac{\mathbf{a}^T \mathbf{p}_n}{c} \quad (2.5)$$

Where c is the speed of propagation of the wave in the considered medium, which can be approximated by that of the light in the vacuum, and $(\)^T$ is the matrix transposition operator. On the other hand, for pass-band signals, the generic representation of $f(t)$ is as follows:

$$f(t) = \sqrt{2} \Re[\tilde{f}(t)e^{j\omega_c t}] \quad (2.6)$$

ω_c is the carrier frequency and $\tilde{f}(t)$ is the complex envelope. The $\sqrt{2}$ factor is added so that real signal and complex envelope have the same power. The delayed version of the signal is thus defined as:

$$f(t - \tau_n) = \sqrt{2} \Re[\tilde{f}(t - \tau_n)e^{j\omega_c(t - \tau_n)}] \quad (2.7)$$

In the presence of narrow-band signals, time delays can be translated into phase offsets, due to the following approximation:

$$\tilde{f}(t - \tau_n) \approx \tilde{f}(t) \quad (2.8)$$

$$f(t - \tau_n) \approx \sqrt{2} \Re[\tilde{f}(t)e^{j\omega_c t} e^{-j\omega_c \tau_n}] = f(t)e^{-j\omega_c \tau_n} = f(t)e^{-j\alpha_n} \quad (2.9)$$

For this to be valid the following condition must be met:

$$B_s \Delta T_{max} \ll 1 \quad (2.10)$$

Where B_s is the bandwidth of the pass-band signal and ΔT_{max} is the maximum delay between any two elements of the array. Since this is a heuristic condition, a more detailed analysis of this challenging problem can be found in (Zatman 1998).

In the narrow-band model, phase offsets α_n are defined as:

$$\alpha_n = -k [\sin(\theta) \cos(\varphi) x_n + \sin(\theta) \sin(\varphi) y_n + \cos(\theta) z_n] \quad (2.11)$$

Where k is the wavenumber, which can be put as a function of the carrier frequency, or its wavelength λ_c :

$$k = \frac{\omega_c}{c} = \frac{2\pi f_c}{c} = \frac{2\pi}{\lambda_c} \quad (2.12)$$

Thus, the vector $\mathbf{f}(t, \mathbf{p})$ can be redefined as:

$$\mathbf{f}(t, \mathbf{p}) = f(t) \times \begin{pmatrix} e^{-j\alpha_0} \\ e^{-j\alpha_1} \\ \vdots \\ e^{-j\alpha_{N-1}} \end{pmatrix} \quad (2.13)$$

This equation shows two components: the generic pass-band function $f(t)$, and a vector entity which summarises the geometrical characteristics of the array. This vector is known as the Steering Vector (SV) or the Array Manifold Vector:

$$\mathbf{V} = \begin{pmatrix} e^{-j\alpha_0} \\ e^{-j\alpha_1} \\ \vdots \\ e^{-j\alpha_{N-1}} \end{pmatrix} \quad (2.14)$$

When defining the SV in the narrow-band model, the only signal-dependent information that is required is its central frequency and its angular orientation with respect to the array.

2.2 Array Factor and Gain

Typically, the signals obtained in each array branch are multiplied by a complex weight w , and then combined in a single adder. In this, a complex envelope representation of the signal is implied, and the narrow-band condition is assumed to be true.

Figure 2.2 Array processing: weighting and sum

This processing can be expressed in vector form:

$$y = \mathbf{w}^H \mathbf{f}(t, \mathbf{p}) = \mathbf{w}^H \times (\mathbf{f}(t) \times \mathbf{V}) = \mathbf{AF} \times \mathbf{f}(t) \quad (2.15)$$

The $()^H$ operator represents the hermitic transposition. The weights are grouped into a single column vector:

$$\mathbf{w} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_{N-1} \end{pmatrix} \quad (2.16)$$

AF is the Array Factor, which is the equivalent for arrays of a single antenna radiation pattern, and thus contains information of its spatial discrimination characteristics.

$$\mathbf{AF}(\theta, \varphi) = \mathbf{w}^H \times \mathbf{V} \quad (2.17)$$

The shape of the AF depends on the implemented weights, whereas the array response to a given signal depends on its angular direction with respect to the array. By modifying the weights appropriately, the maximum of the AF can be shifted towards the direction of the desired signal, which justifies the fact that the weighting and sum processes are commonly known as array *beamforming*.

For uniform arrays, i.e., arrays where the separation amongst antenna elements is the same, it can be shown that the AF is periodic when considering the appropriate coordinates. This periodic behaviour implies that there are replicas of the AF maxima, known as grating lobes. Since the array can be seen as a spatial sampling, these grating lobes are the equivalent to aliasing in time sampling analysis. Thus, an array with insufficient sampling is not able to discern signals coming from DoA that correspond to the same AF response.

In order to completely eliminate grating lobes in a uniform array, the spatial-equivalent of Nyquist criterion must be met, which sets that the separation amongst array elements must be equal to or lower than $\lambda_c/2$. In SARAS application, though, as will be explored in Chapter 3, there is an a priori knowledge of the signal DoA, which limits the angular acquisition window. In turn, inter-element distances much greater than $\lambda_c/2$ can be considered, as long as grating lobes do not appear within this window.

The next figure illustrates the concept of grating lobes for a 10-element Uniform Linear Array (ULA) with different inter-element distances:

Figure 2.3 AF for different inter-element distances in a 10-element ULA

It can be shown that shifts in the maximum of the AF are obtained with changes in the phases of the weights, which is why these systems are also known as *phased arrays*. The simplest way to do this is known as *conventional beamforming*, which is the starting point of more complex algorithms:

Figure 2.4 Conventional beamformer

Naming the SV of the desired signal as $\mathbf{So}$, the matrix representation of the conventional beamformer is:

$$y = \frac{1}{N} \mathbf{So}^H \times \mathbf{So} \times f(t) = f(t) \quad (2.18)$$

$$\mathbf{w}_{conv} = \frac{\mathbf{So}}{N} \quad (2.19)$$

$$y = \mathbf{w}^H \mathbf{f}(t, \mathbf{p}) = \mathbf{w}^H \mathbf{x} \quad (2.20)$$

In this ideal case, the desired signal goes through the system without distortion. Of course, this non-distortion condition is only a partial objective of the beamforming system, usually complemented by the reduction of white noise, mainly of thermal origin. This constitutes an effective gain in SNR.

In order to quantify this reduction, it is necessary to know the power of the signal components. If they can be modelled as stationary processes with zero mean, the mean power at the array output is:

$$P(\mathbf{w}) = E[|y(t)|^2] = E[y(t) \times y^*(t)] = \mathbf{w}^H E[\mathbf{x}(t) \mathbf{x}^H(t)] \mathbf{w} = \mathbf{w}^H \mathbf{R} \mathbf{w} \quad (2.21)$$

$E[\]$ is the ensemble average operator, $y^*(t)$ is the conjugated output signal, $|\ |$ is the absolute value operator, and $\mathbf{R}$ represents the array correlation matrix, with size $N \times N$, so called because the element R_{ij} denotes the correlation between the i -th and j -th array components. In a simplified model of the array signal, which shall be extended in section 2.3.1, there will be a desired signal and a noise component:

$$\mathbf{x}(t) = \mathbf{x}_s(t) + \mathbf{n}(t) \quad (2.22)$$

The column vector $\mathbf{n}(t)$ is a white noise process with zero mean, uncorrelated among array branches, and $\mathbf{x}_s(t)$ is the desired signal. With this model, two output power estimates can be derived, one for each component:

$$P_s = \mathbf{w}^H E[\mathbf{x}_s(t) \mathbf{x}_s^H(t)] \mathbf{w} = \mathbf{w}^H \mathbf{R}_s \mathbf{w} \quad (2.23)$$

$$P_n = \mathbf{w}^H E[\mathbf{n}(t) \mathbf{n}^H(t)] \mathbf{w} = \mathbf{w}^H \mathbf{R}_n \mathbf{w} \quad (2.24)$$

Accordingly, the array correlation matrix can also be divided into two components:

$$\mathbf{R}_s = \sigma_s^2 \mathbf{S} \mathbf{O} \mathbf{S}^H \quad (2.25)$$

$$\mathbf{R}_n = \sigma_n^2 \mathbf{I} \quad (2.26)$$

σ_s^2 is the input power of the desired signal, assumed to be equal in all array branches, σ_n^2 is the input power of the noise, and $\mathbf{I}$ is the $N \times N$ identity matrix. The array gain in the conventional beamforming architecture is thus:

$$\begin{aligned} G(dB) &= (SNR_{out})_{dB} - (SNR_{in})_{dB} = 10 \log \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}} - 10 \log \frac{\sigma_s^2}{\sigma_n^2} \\ &= 10 \log \frac{\sigma_s^2 (\mathbf{S} \mathbf{O}^H / N) \mathbf{S} \mathbf{O} (\mathbf{S} \mathbf{O} / N)}{\sigma_n^2 (\mathbf{S} \mathbf{O}^H / N) (\mathbf{S} \mathbf{O} / N)} - 10 \log \frac{\sigma_s^2}{\sigma_n^2} = 10 \log N \end{aligned} \quad (2.27)$$

The preceding discussion considered all array antennas to be isotropic radiators, but this is not the case in real systems. However, if all elements are approximately the same, the radiation pattern of the array can be modelled as the multiplication of the radiation pattern of a single antenna element, placed at the origin of the coordinate system, times the AF:

$$|E_T(r, \theta, \varphi)| = |E_{element}(r, \theta, \varphi)| \times |AF(\theta, \varphi)| \quad (2.28)$$

More information on the preceding discussions can be found in (Balanis 2005), (Van Trees 2002) and (Godara 2004).

2.3 Smart Antennas

Adaptive array algorithms will be presently explored, going beyond the conventional beamforming architecture described in 2.2. Firstly, the array signal model will be updated. Then, several ideal optimisation criteria shall be presented, according to which different beamforming algorithms can be implemented.

2.3.1 Signal Model

The array signal model presented in equation (2.22) can be extended, as shown in the following equation:

$$\mathbf{x}[k] = \mathbf{x}_s[k] + \sum_{l=1}^L \mathbf{i}_l[k] + \mathbf{n}[k] = \mathbf{x}_s[k] \times \mathbf{S}\mathbf{o} + \sum_{l=1}^L \mathbf{i}_l[k] \times \mathbf{S}\mathbf{i}_l + \mathbf{n}[k] \quad (2.29)$$

This new signal model incorporates the presence of L interfering signals, $\mathbf{i}_l$, which are represented as column vectors. These interferers have also an associated SV, $\mathbf{S}\mathbf{i}_l$, which will usually be different from $\mathbf{S}\mathbf{o}$, due to the fact that their DoA will differ. This is precisely what motivates the spatial discrimination capabilities of the array.

The preceding equation also explicitly shows the temporal dependence of the involved processes. A digital framework is implied, and thus k represents the k -th sample taken by the digitiser. In the equation the SV are constant, but usually this will not be the case, mainly due to changes with time in the DoA of the signal, as the spacecraft follow a given trajectory. The number of interferers, too, can change with time, as some disappear and others emerge.

In general, a noise process in one of the array branches is uncorrelated with the ones present in the other branches, and with the desired signal. Mathematically, this can be expressed as:

$$E[x_s n_i^*] = 0, \quad i = 1, \dots, N \quad (2.30)$$

$$E[n_i n_j^*] = \begin{cases} 0 & i \neq j \\ \sigma_n^2 & i = j \end{cases} \quad (2.31)$$

The same hypothesis can be applied to interferers:

$$E[x_s i_l^*] = 0, \quad l = 1, \dots, L \quad (2.32)$$

$$E[i_l i_m^*] = \begin{cases} 0 & l \neq m \\ \sigma_{i_l}^2 & l = m \end{cases} \quad (2.33)$$

$\sigma_{i_l}^2$ is the power of the l -th interferer.. With this signal model, it can be shown that the correlation matrix has the following expression:

$$\mathbf{R}_{xx} = \mathbf{V}\mathbf{S}\mathbf{V}^H + \sigma_n^2 \mathbf{I} \quad (2.34)$$

$\mathbf{S}$ is a diagonal matrix with the power of each signal component in its main diagonal:

$$\mathbf{S} = \begin{pmatrix} \sigma_s^2 & 0 & & \mathbf{0} \\ 0 & \sigma_{i_1}^2 & & \\ & & \ddots & 0 \\ \mathbf{0} & & 0 & \sigma_{i_l}^2 \end{pmatrix} \quad (2.35)$$

In this general case, $\mathbf{V}$ is a $N \times (L+1)$ matrix where each column is one of the SV of the signals present. This structure justifies the Eigen processing explained in sections 2.3.2.3 and 2.3.3.3. On the other hand, some of the assumptions of the presented signal model will be challenged and extended in later sections.

2.3.2 Optimisation Criteria

2.3.2.1 MMSE

The MMSE (Minimum Mean Square Error) criterion is based on the assumption that the desired signal is known, which is the case for instance when dealing with training sequences. Thus, an error signal can be estimated, using the following equation:

$$e[k] = x_s[k] - y[k] = d[k] - \mathbf{w}^H \mathbf{x}[k] \quad (2.36)$$

From this function the mean square error can be calculated:

$$E(|e[k]|^2) = E(|x_s[k]|^2) - \mathbf{w}^H E(x_s^*[k] \mathbf{x}[k]) - E(x_s[k] \mathbf{x}^H[k]) \mathbf{w} + \mathbf{w}^H E(\mathbf{x}[k] \mathbf{x}^H[k]) \mathbf{w} \quad (2.37)$$

The optimisation goal, as may be expected, is to minimise this function. Applying the complex gradient with respect to $\mathbf{w}^H$, using the rules found in (Brandwood 1983) and (Kreutz-Delgado 2009), and making it equal to zero, the optimum weights, known as the Wiener-Hopf solution (Sklar 2001), can be found:

$$\nabla_{\mathbf{w}^H} \{E(|e(k)|^2)\} = -\mathbf{r}_{xd} + \mathbf{R}_{xx} \mathbf{w} = \mathbf{0} \quad (2.38)$$

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{r}_{xd} \quad (2.39)$$

$\mathbf{R}_{xx}$ is the usual array correlation matrix, with the following mathematical expression:

$$\mathbf{R}_{xx} = E(\mathbf{x}(k) \mathbf{x}^H(k)) \quad (2.40)$$

On the other hand, $\mathbf{r}_{xd}$ is a cross-correlation vector between the received data and the known desired signal, with size $N \times 1$:

$$\mathbf{r}_{xd} = E(x_s^*(k) \mathbf{x}(k)) \quad (2.41)$$

The algorithms that derive from this optimal criterion are known as temporal reference algorithms, due to the fact that the signal must be known a priori for a number of samples. These algorithms are very useful because they can cancel interfering signals, as long as they are uncorrelated with the desired one. However, the use of training sequences makes it mandatory to partially demodulate the signal prior to the beamforming stage so that sampling times can be correctly estimated. Since at that point the SNR can be quite low (it must not be forgotten that one of the objectives of the array is precisely to improve it), the demodulation can be very challenging.

2.3.2.2 MV

The MV (Minimum Variance) criterion seeks to minimise the variance, i.e. the power, of the output noise, while maintaining a given AF response in the direction of the desired signal. The problem can be formulated in the following way:

$$\text{minimise } (\mathbf{w}^H \mathbf{R}_n \mathbf{w}) \quad (2.42)$$

$$\text{with the restriction: } \mathbf{w}^H \mathbf{S} \mathbf{o} = \alpha$$

This problem can be solved using Lagrange multipliers (Litva 1996):

$$\mathbf{w}^H \mathbf{R}_n \mathbf{w} + \lambda (\mathbf{w}^H \mathbf{S} \mathbf{o} - \alpha) + \lambda^* (\mathbf{S} \mathbf{o}^H \mathbf{w} - \alpha^*) \quad (2.43)$$

λ is a Lagrange multiplier. The presence of its conjugate is due to the fact that the function to be optimised, even if containing complex variables, must be real, since it models the power of a process. Taking, as usual, the complex gradient:

$$\mathbf{w}_{mv} = -\lambda \mathbf{R}_n^{-1} \mathbf{S} \mathbf{o} \quad (2.44)$$

Using the restriction, the Lagrange multiplier is equal to:

$$\lambda = -\alpha \times [\mathbf{S} \mathbf{o}^H \mathbf{R}_n^{-1} \mathbf{S} \mathbf{o}]^{-1} \quad (2.45)$$

Finally, the weights are expressed as:

$$\mathbf{w}_{mv} = \frac{\alpha \mathbf{R}_n^{-1} \mathbf{S} \mathbf{o}}{\mathbf{S} \mathbf{o}^H \mathbf{R}_n^{-1} \mathbf{S} \mathbf{o}} \quad (2.46)$$

Taking α equal to 1 yields the MVDR (Minimum Variance Distortionless Response) or Capon's solution. In this case the desired signal is not distorted by the beamforming stage, as with the conventional architecture.

The algorithms based on this criterion are useful because they can cancel interfering signals, as was the case with the MMSE criterion. They are known as spatial reference algorithm, because knowledge of the SV of the desired signal, and thus its

DoA, is mandatory. It can be shown that this criterion coincides with the ML (Maximum Likelihood) (Monzingo, Haupt, and Miller 2011).

One important disadvantage, beyond the required knowledge of the SV, is the fact that the noise correlation matrix may not be easily obtained from the received samples, which also contain a desired signal component. It is usual, therefore, to replace the MVDR with an MPDR (Minimum Power Distortionless response) solution (Van Trees 2002). In this case, the complete correlation matrix is used:

$$\mathbf{w}_{mpdr} = \frac{\mathbf{R}_{xx}^{-1} \mathbf{S}_o}{\mathbf{S}_o^H \mathbf{R}_{xx}^{-1} \mathbf{S}_o} \quad (2.47)$$

The main disadvantage of MPDR versus MVDR is that it is very sensitive to calibration and DoA errors, and thus requires a very precise modelling of the SV of the desired signal. More on calibration will be considered in section 2.5.

2.3.2.3 *MCOSNR, MCOSNIR and MCOP*

The objective of MCOSNR (Maximum Combined Output SNR) and MCOSINR (Maximum Combined Output Signal-to-Interference and Noise Ratio) criteria consists in maximising the output SNR and SINR, respectively. These can be mathematically defined as:

$$SNR = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_n \mathbf{w}} \quad (2.48)$$

$$SINR = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{n+i} \mathbf{w}} \quad (2.49)$$

$\mathbf{R}_{n+i}$ includes noise and interferers and, with the uncorrelation assumption, is equal to:

$$\mathbf{R}_{n+i} = \mathbf{R}_n + \mathbf{R}_i \quad (2.50)$$

The interferer correlation matrix is defined as:

$$\mathbf{R}_i = \sum_{l=1}^L \sigma_{i_l}^2 \mathbf{S}_{i_l} \mathbf{S}_{i_l}^H \quad (2.51)$$

Considering the MCOSNR criterion, the complex gradient with respect to $\mathbf{w}^H$ is calculated and made equal to zero, yielding (Lee et al. 2002):

$$\frac{\mathbf{R}_s \mathbf{w} \mathbf{w}^H \mathbf{R}_n \mathbf{w} - \mathbf{R}_n \mathbf{w} \mathbf{w}^H \mathbf{R}_s \mathbf{w}}{(\mathbf{w}^H \mathbf{R}_n \mathbf{w})^2} = \mathbf{0} \quad (2.52)$$

$$\Leftrightarrow \mathbf{R}_s \mathbf{w} = SNR \times \mathbf{R}_n \mathbf{w} = \lambda \mathbf{R}_n \mathbf{w} \quad (2.53)$$

$$\Leftrightarrow [\mathbf{R}_n^{-1} \mathbf{R}_s] \mathbf{w} = \lambda \mathbf{w} \quad (2.54)$$

Thus, the weight vector which maximises the output SNR is the eigenvector of the dominant eigenvalue from matrix $\mathbf{R}_n^{-1}\mathbf{R}_s$ (Cheung 1996). It is evident that for the MCOSINR criterion, the same reasoning applies, using the dominant eigenvalue from matrix $\mathbf{R}_{n+i}^{-1}\mathbf{R}_s$.

As with the MVDR/MPDR dilemma, in most cases it is not easy, or even possible, to separate desired signal and noise components of the correlation matrix, even if they are uncorrelated. In such situations, an alternative is the MCOP (Maximum Combined Output Power) criterion, where the goal is to maximise the output power. This can be written as:

$$P_{out} = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \quad (2.55)$$

An eigen-decomposition can be carried out for matrix $\mathbf{R}_{xx}$:

$$\mathbf{R}_{xx} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H \quad (2.56)$$

$\mathbf{U}$ is a set of orthonormal eigenvectors of size $N \times 1$ each:

$$\mathbf{U} = [\mathbf{u}_1 \quad \mathbf{u}_2 \quad \dots \quad \mathbf{u}_N] \quad (2.57)$$

$\mathbf{\Lambda}$ is a diagonal matrix with the eigenvalues in its main diagonal:

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \lambda_N \end{pmatrix} \quad (2.58)$$

In the presence of $L+1$ uncorrelated signals, L interferers and one desired signal, there will be $L+1$ eigenvalues associated to them, and $N-L-1$ eigenvalues associated to the white noise. Ideally, these will all be equal to the noise variance σ_n^2 .

Taking into account the eigen-decomposition of $\mathbf{R}_{xx}$, the weight vector can be written as a linear combination of the eigenvectors of said matrix:

$$\mathbf{w} = \sum_{i=1}^N \alpha_i \mathbf{u}_i = \mathbf{U} \boldsymbol{\alpha} \quad (2.59)$$

$\mathbf{u}_i$ is the i -th eigenvector and $\boldsymbol{\alpha} = [\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_N]^T$. Besides, in order to avoid unstabilities, the weight vector is considered to be unitary, and thus:

$$\sum_{i=1}^N |\alpha_i|^2 = 1 \quad (2.60)$$

Replacing $\mathbf{w}$ in equation (2.55), the expression becomes:

$$P_{out} = (\mathbf{U} \boldsymbol{\alpha})^H \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H (\mathbf{U} \boldsymbol{\alpha}) = \boldsymbol{\alpha}^H \mathbf{\Lambda} \boldsymbol{\alpha} = \sum_{i=1}^N \lambda_i |\alpha_i|^2 \leq \sum_{i=1}^N \lambda_{max} |\alpha_i|^2 \leq \lambda_{max} \quad (2.61)$$

Therefore, the weight vector which satisfies the MCOP criterion is the eigenvector associated with the dominant eigenvalue of matrix $\mathbf{R}_{xx}$, which can also include interferers. No separation between matrix components is required in this case.

The algorithms that stem from these criteria are known as blind reference algorithms, because they do not require any a priori information about the desired signal or the array calibration parameters. This makes them more easily implementable. However, in the MCOP case, interferers cannot be eliminated or discriminated from the desired signal.

2.3.3 Adaptive Beamforming Algorithms

The main issue with optimum algorithms is that they require perfect knowledge of a correlation matrix, be it the noise-only or the complete version. Of course, in practical implementations, this matrix must be approximated with the received signal samples, hence the adaptive version of the optimum algorithms. Important criteria when devising a specific algorithm are convergence speed, estimation RMSE (Root Mean Square Error) and computational complexity.

2.3.3.1 Iterative and Block Estimation

As already specified, the ideal definition of the spatial correlation matrix is:

$$\mathbf{R} = E[\mathbf{x}(t)\mathbf{x}^H(t)] \quad (2.62)$$

If the stochastic processes that conform $\mathbf{x}(t)$ are ergodic, ensemble averages can be approximated by temporal ones, which converge to the true values as the integration time goes to infinity. Taking this into account, a temporal iterative approximation can be devised:

$$\hat{\mathbf{R}}[0] = \mathbf{x}[0]\mathbf{x}^H[0] \quad (2.63)$$

$$\hat{\mathbf{R}}[k+1] = \frac{k\hat{\mathbf{R}}[k] + \mathbf{x}[k+1]\mathbf{x}^H[k+1]}{k+1} \quad (2.64)$$

The matrix is estimated in all temporal iterations or snapshots, with the subsequent computational load. Alternatively, K-block estimations can be implemented:

$$\hat{\mathbf{R}}[k, K] = (1/K) \sum_{i=k \times K}^{k \times K + K - 1} \mathbf{x}[i]\mathbf{x}^H[i] \quad (2.65)$$

By adjusting K, this method can react more rapidly to changes in the electromagnetic environment than the purely iterative one. It requires more memory, though, and does not take advantage of a priori information to generate a new block estimate.

The RLS (Recursive Least Squares) algorithm seeks to combine the advantages of the preceding two solutions. In a fast-varying signal scenario, prior

snapshots can be multiplied by a positive parameter lower than 1 called the forgetting factor, and denoted as γ . With this factor, the most immediate snapshots have a greater weight in the current estimation of the correlation matrix. Typical values are between 0.9 and 1, and the quantity $1/(1-\gamma)$ is known as the memory of the algorithm. The estimation itself can be iterative or block-wise:

$$\hat{\mathbf{R}}[k+1] = \gamma \hat{\mathbf{R}}[k] + \mathbf{x}[k+1] \mathbf{x}^H[k+1] \quad (2.66)$$

$$\hat{\mathbf{R}}[k, K] = \sum_{i=k \times K}^{k \times K + K - 1} \gamma^{k \times K + K - 1 - i} \mathbf{x}[i] \mathbf{x}^H[i] \quad (2.67)$$

More details on RLS can be found in (Van Trees 2002).

2.3.3.2 Inversion of Correlation Matrices

As in the MV criterion, the inversion of the correlation matrix is sometimes required. The most straightforward way of doing this is by estimating $\mathbf{R}$ using some of the methods described in the previous section, and directly calculating its inverse. This technique is known as SMI (Sample Matrix Inversion). The main drawback of such approach is that direct inversion may imply high computational load, especially for arrays with a large number of elements. In such cases iterative estimations can be designed:

$$\hat{\mathbf{R}}^{-1}(k+1) = \hat{\mathbf{R}}^{-1}(k) - \frac{\hat{\mathbf{R}}^{-1}(k) \mathbf{x}(k+1) \mathbf{x}^H(k+1) \hat{\mathbf{R}}^{-1}(k)}{1 + \mathbf{x}^H(k+1) \hat{\mathbf{R}}^{-1}(k) \mathbf{x}(k+1)} \quad (2.68)$$

With a forgetting factor, an RLS version is obtained:

$$\hat{\mathbf{R}}^{-1}(k+1) = \gamma^{-1} \hat{\mathbf{R}}^{-1}(k) - \frac{\gamma^{-2} \hat{\mathbf{R}}^{-1}(k) \mathbf{x}(k+1) \mathbf{x}^H(k+1) \hat{\mathbf{R}}^{-1}(k)}{1 + \gamma^{-1} \mathbf{x}^H(k+1) \hat{\mathbf{R}}^{-1}(k) \mathbf{x}(k+1)} \quad (2.69)$$

Initialisation values are usually defined as follows, where $\mathbf{I}$ is the identity matrix, and δ a design parameter known as regulation factor:

$$\mathbf{R}^{-1}(0) = \delta^{-1} \mathbf{I} \quad (2.70)$$

2.3.3.3 Eigen Algorithms

MCOSNR, MCOSINR and MCOP criteria must solve an eigen-decomposition problem, which, as in the matrix inversion case, may have a high computational load. However, since usually only the dominant eigenvector, and the associated eigenvalue, must be obtained, the overall estimation process can be simplified, using the Power Method solution.

The weight vector, which is assimilated to the dominant eigenvector, is iteratively estimated as follows:

$$\mathbf{w}[k, i] = \frac{\mathbf{R}[k] \mathbf{w}[k, i - 1]}{\|\mathbf{R}[k] \mathbf{w}[k, i - 1]\|} \quad (2.71)$$

$\|\cdot\|$ represents the Euclidean vector norm. Several initial estimates for the weight vector can be considered. The preceding equation considers the MCOP criterion, but is applicable to the other criteria by replacing the correlation matrix accordingly.

Once more, spatial correlation matrices must be approximated, using methods described in 2.3.3.1. Some versions of the algorithm seek to further decrease the computational load, using matrix-free implementations (Lee, Cheung, and Vilenrotter 2003) (Ahn, Choi, and Sarkar 2001):

$$\mathbf{w}[k] = \frac{\mathbf{u}[k]}{\|\mathbf{u}[k]\|} \quad (2.72)$$

$$\mathbf{u}[k] = \mathbf{R}[k] \mathbf{w}[k - 1] = E(\mathbf{x}[k] \mathbf{x}^H[k]) \mathbf{w}[k - 1] \quad (2.73)$$

Vector $\mathbf{u}$ can then be adaptively approximated as:

$$\mathbf{u}[k + 1] = \frac{(k + 1) \mathbf{u}[k] + \mathbf{x}[k + 1] \mathbf{x}^H[k + 1] \mathbf{w}[k]}{k + 2} \quad (2.74)$$

$$\mathbf{u}[0] = \mathbf{x}[0] \quad (2.75)$$

In these simplifications, the parameter i in $\mathbf{w}[k, i]$ has been limited to 1.

2.4 DoA Estimation

As expressed in equation (2.11), the phases of the SV have the following mathematical expression under the narrowband assumption:

$$\alpha_n(\theta, \varphi) = -k [\sin(\theta) \cos(\varphi) x_n + \sin(\theta) \sin(\varphi) y_n + \cos(\theta) z_n] \quad (2.76)$$

The equation explicitly states that there is a non-linear dependence between the phases of the SV and the spherical angles θ, φ that define the DoA of the desired signal. Thus, the DoA estimation process is the resolution of the inverse problem that obtains these spherical angles, or any other combination in a suitable coordinate system, from the estimated SV phases. More details on the discussions below can be found in (Van Trees 2002), chapters 8 and 9.

2.4.1 Statistical Bounds

Before proceeding to actual DoA estimation algorithms, this section will briefly revise the CRLB (Cramér-Rao Lower Bound) of the DoA estimation problem in a plurality of scenarios. The CRLB is a lower bound of the variance of all unbiased estimators in a given statistical problem. The formulas presented hereunder rely on a set of assumptions:

- Desired signals and interferers are point-like, with no angular spread.
- Noise samples are supposed to have a complex Gaussian distribution and to be statistically white. Also, they are supposed to be uncorrelated with the desired signal and among array branches.

2.4.1.1 One Received Signal

This first scenario considers a single received signal. The CRLB matrix in this case becomes:

$$\mathbf{C}_{CR}(\varphi, \theta) = \frac{1}{2K} \left(\frac{1}{snr} + \frac{1}{Nsnr^2} \right) \left[\mathbf{D}^H \left(\mathbf{I} - \frac{\mathbf{V}\mathbf{V}^H}{N} \right) \mathbf{D} \right]^{-1} \quad (2.77)$$

K is the number of received uncorrelated snapshots, i.e., digitised signal samples including a desired and a noisy component; snr is the input Signal-to-Noise Ratio, in natural units; and $\mathbf{D}$ is the derivative of the array manifold with respect to the angular variables:

$$\mathbf{D} = \begin{bmatrix} \frac{\partial \mathbf{V}}{\partial \varphi} & \frac{\partial \mathbf{V}}{\partial \theta} \end{bmatrix} \quad (2.78)$$

Matrix $\mathbf{C}_{CR}(\varphi, \theta)$ is a 2 x 2 covariance matrix, divided as:

$$\mathbf{C}_{CR}(\varphi, \theta) = \begin{bmatrix} C_{CR\varphi\varphi} & C_{CR\varphi\theta} \\ C_{CR\theta\varphi} & C_{CR\theta\theta} \end{bmatrix} \quad (2.79)$$

$C_{CR\theta\theta}$, for example, is the variance associated with the estimation of the θ angle.

2.4.1.2 Two Uncorrelated Received Signals

In this case, the CRLB must be expressed as a function of its inverse, the FIM (Fisher Information Matrix) $\mathbf{J}$:

$$\mathbf{J} = \begin{bmatrix} J_{\varphi\varphi} & J_{\varphi\theta} & J_{\varphi s} & J_{\varphi n} \\ J_{\theta\varphi} & J_{\theta\theta} & J_{\theta s} & J_{\theta n} \\ J_{s\varphi} & J_{s\theta} & J_{ss} & J_{sn} \\ J_{n\varphi} & J_{n\theta} & J_{ns} & J_{nn} \end{bmatrix} = \begin{bmatrix} J_{ww} & J_{wu} \\ J_{uw} & J_{uu} \end{bmatrix} \quad (2.80)$$

The four terms in the upper left corner of matrix $\mathbf{J}$, grouped in $\mathbf{J}_{ww}$, are in turn sub-matrices with the wanted parameters in a DoA estimation problem. Diagonal sub-matrices can be expressed as:

$$J_{\varphi\varphi} = 2KRe \left[(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\varphi) \odot (\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\varphi)^H + (\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V} \mathbf{S}) \odot (\mathbf{D}_\varphi^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\varphi)^H \right] \quad (2.81)$$

$$J_{\theta\theta} = 2KRe \left[(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\theta) \odot (\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\theta)^H + (\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V} \mathbf{S}) \odot (\mathbf{D}_\theta^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\theta)^H \right] \quad (2.82)$$

$Re[\]$ is the real part operator, $\odot$ denotes the matrix Hadamard, or element-wise, product, and:

$$\mathbf{D}_\varphi = \begin{bmatrix} \frac{\partial \mathbf{V}_1}{\partial \varphi} & \frac{\partial \mathbf{V}_2}{\partial \varphi} \end{bmatrix} \quad (2.83)$$

$$\mathbf{D}_\theta = \begin{bmatrix} \frac{\partial \mathbf{V}_1}{\partial \theta} & \frac{\partial \mathbf{V}_2}{\partial \theta} \end{bmatrix} \quad (2.84)$$

The mn -th element of the cross-diagonal sub-matrices is expressed as:

$$J_{\varphi\theta mn} = K * tr[\mathbf{R}_{xx}^{-1}(\mathbf{D}_\varphi \mathbf{e}_m \mathbf{e}_m^T \mathbf{S} \mathbf{V}^H + \mathbf{V} \mathbf{S} \mathbf{e}_m \mathbf{e}_m^T \mathbf{D}_\varphi^H) \mathbf{R}_{xx}^{-1}(\mathbf{D}_\theta \mathbf{e}_n \mathbf{e}_n^T \mathbf{S} \mathbf{V}^H + \mathbf{V} \mathbf{S} \mathbf{e}_n \mathbf{e}_n^T \mathbf{D}_\theta^H)] \quad (2.85)$$

$$J_{\theta\varphi mn} = K * tr[\mathbf{R}_{xx}^{-1}(\mathbf{D}_\theta \mathbf{e}_m \mathbf{e}_m^T \mathbf{S} \mathbf{V}^H + \mathbf{V} \mathbf{S} \mathbf{e}_m \mathbf{e}_m^T \mathbf{D}_\theta^H) \mathbf{R}_{xx}^{-1}(\mathbf{D}_\varphi \mathbf{e}_n \mathbf{e}_n^T \mathbf{S} \mathbf{V}^H + \mathbf{V} \mathbf{S} \mathbf{e}_n \mathbf{e}_n^T \mathbf{D}_\varphi^H)] \quad (2.86)$$

$tr[\]$ is the trace operator and $\mathbf{e}_m$ is a 2×1 vector with 1 in the m -th position and 0 in the other. On the other hand, sub-matrices involving desired and undesired parameters are defined as:

$$J_{\varphi s} = 2KRe[(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V}) \odot (\mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\varphi)^H] \quad (2.87)$$

$$J_{\theta s} = 2KRe[(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V}) \odot (\mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{D}_\theta)^H] \quad (2.88)$$

$$J_{\varphi n} = 2Kdiag[Re(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{-1} \mathbf{V} \mathbf{D}_\varphi)] \quad (2.89)$$

$$J_{\theta n} = 2Kdiag[Re(\mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{-1} \mathbf{V} \mathbf{D}_\theta)] \quad (2.90)$$

$diag(\)$ is the matrix diagonal operator. Finally, sub-matrices with unwanted parameters are defined as:

$$J_{ss} = K[(\mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V}) \odot (\mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V})^H] \quad (2.91)$$

$$J_{sn} = Kdiag(\mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{-1} \mathbf{V}) \quad (2.92)$$

$$J_{nn} = Ktr(\mathbf{R}_{xx}^{-1} \mathbf{R}_{xx}^{-1}) \quad (2.93)$$

It is important to remember that $J_{mn} = J_{nm}^H$. With this in mind, the Cramér-Rao matrix can be derived:

$$\mathbf{C}_{CR}(\varphi, \theta) = [\mathbf{J}_{ww} - \mathbf{J}_{wu}(\mathbf{J}_{uu})^{-1} \mathbf{J}_{wu}^H]^{-1} \quad (2.94)$$

2.4.1.3 Calibration errors

Calibration errors refer to errors in the knowledge of the positions of the array elements, or residual amplitude and phase offsets among branches. Starting with position errors, let the perturbation vector $\boldsymbol{\rho}$ be defined as:

$$\boldsymbol{\rho} = [x_2 \ y_2 \ z_2 \ \cdots \ x_N \ y_N \ z_N]^T \quad (2.95)$$

It is implied as a convention that there is no perturbation in the first element, which is chosen as a reference. The nominal value of $\boldsymbol{\rho}$ is $\boldsymbol{\rho}_0$, which implies an error-

free situation. The errors are supposed to be Gaussian, with zero mean vector and covariance matrices defined as:

$$\mathbf{\Lambda}_x = \begin{bmatrix} \sigma_{x_2}^2 & 0 & \dots & 0 \\ 0 & & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \dots & 0 & \sigma_{x_N}^2 \end{bmatrix} \quad (2.96)$$

$$\mathbf{\Lambda}_y = \begin{bmatrix} \sigma_{y_2}^2 & 0 & \dots & 0 \\ 0 & & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \dots & 0 & \sigma_{y_N}^2 \end{bmatrix} \quad (2.97)$$

$$\mathbf{\Lambda}_z = \begin{bmatrix} \sigma_{z_2}^2 & 0 & \dots & 0 \\ 0 & & \ddots & \vdots \\ \vdots & & & 0 \\ 0 & \dots & 0 & \sigma_{z_N}^2 \end{bmatrix} \quad (2.98)$$

$$\mathbf{\Lambda}_p = \begin{bmatrix} \mathbf{\Lambda}_x & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Lambda}_z \end{bmatrix} \quad (2.99)$$

The following auxiliary matrices are defined, with $\otimes$ representing the matrix Kronecker product:

$$\mathbf{P}_V^\perp = \mathbf{I} - \mathbf{V}(\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H = \mathbf{I} - \mathbf{P}_V \quad (2.100)$$

$$\mathbf{\Sigma} = \mathbf{S} \mathbf{V}^H \mathbf{R}_{xx}^{-1} \mathbf{V} \mathbf{S} \quad (2.101)$$

$$\mathbf{D}_x = \sum_{n=1}^N \frac{\partial \mathbf{V}}{\partial x_n} \quad (2.102)$$

$$\mathbf{D}_y = \sum_{n=1}^N \frac{\partial \mathbf{V}}{\partial y_n} \quad (2.103)$$

$$\mathbf{D}_z = \sum_{n=1}^N \frac{\partial \mathbf{V}}{\partial z_n} \quad (2.104)$$

$$\mathbf{F}_{x,y,z} = 2 \frac{K}{\sigma_n^2} \text{Re} \left\{ (\mathbf{D}^H \mathbf{P}_V^\perp) \odot \left[(\mathbf{D}_{x,y,z} \mathbf{\Sigma})^T \otimes \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] \right\} \quad (2.105)$$

$$\mathbf{F} = [\mathbf{F}_x \quad \mathbf{F}_y \quad \mathbf{F}_z] \quad (2.106)$$

$$\mathbf{G}_{ij} = 2 \frac{K}{\sigma_n^2} \text{Re} \left\{ (\mathbf{D}_i \mathbf{\Sigma} \mathbf{D}_j^H) \odot (\mathbf{P}_V^\perp)^T \right\} \quad (2.107)$$

By deleting the first column of matrices $\mathbf{F}_{x,y,z}$, and the first row and column of matrices $\mathbf{G}_{ij}$, bearing in mind the fact that the first element in the array is taken as reference, the CRLB is obtained:

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{xx} & \mathbf{G}_{xy} & \mathbf{G}_{xz} \\ \mathbf{G}_{xy}^H & \mathbf{G}_{yy} & \mathbf{G}_{yz} \\ \mathbf{G}_{xz}^H & \mathbf{G}_{yz}^H & \mathbf{G}_{zz} \end{bmatrix} + \Lambda_p^{-1} \quad (2.108)$$

$$\mathcal{C}_{CR}(\varphi, \theta) = \left\{ 2 \frac{K}{\sigma_n^2} \text{Re} \left\{ (\mathbf{D}^H \mathbf{P}_V^\perp \mathbf{D}) \odot \left[\boldsymbol{\Sigma}^T \otimes \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right] \right\} - \mathbf{F} \mathbf{G}^{-1} \mathbf{F}^H \right\}^{-1} \quad (2.109)$$

The derivation is very similar for array phase and amplitude offsets, replacing the definition of matrices $\mathbf{F}$ and $\mathbf{G}$ accordingly.

2.4.2 Maximum Likelihood and MAP Estimation

MLE (Maximum Likelihood Estimation) can be seen as the optimal procedure, i.e., the one which achieves the CRLB, for unbiased non-random parameters above a given SNR threshold, while MAP (Maximum A Posteriori) estimators are used for random variables.

MLE is derived from the maximisation of the log-likelihood function, for a given number K of received samples. In complex Gaussian scenarios with zero-mean processes, it takes the following form:

$$L_{MLE}(\boldsymbol{\Psi}) = -\ln \det(\mathbf{K}_{xx}(\boldsymbol{\Psi}) - \text{tr}[\mathbf{K}_{xx}^{-1}(\boldsymbol{\Psi}) \mathbf{R}_{xx}]) \quad (2.110)$$

$\mathbf{R}_{xx}$, already defined, is the K -block sample correlation matrix estimate, $\mathbf{K}_{xx}$ is the actual covariance matrix of the Gaussian random vector $\mathbf{x}$, which contains the samples for all array branches, $\boldsymbol{\Psi}$ is a general vector containing all desired unknown parameters, in this case, φ and θ , $\ln(\cdot)$ is the natural logarithm function, and $\det(\cdot)$ is the matrix determinant operator. For MAP, the function to be maximised is the a posteriori density, defined as:

$$L_{MAP}(\boldsymbol{\Psi}) = -\ln \det(\mathbf{K}_{xx}(\boldsymbol{\Psi}) - \text{tr}[\mathbf{K}_{xx}^{-1}(\boldsymbol{\Psi}) \mathbf{R}_{xx}]) + \ln p_{\boldsymbol{\Psi}}(\boldsymbol{\Psi}) = L_{MLE}(\boldsymbol{\Psi}) + \ln p_{\boldsymbol{\Psi}}(\boldsymbol{\Psi}) \quad (2.111)$$

$p_{\boldsymbol{\Psi}}(\boldsymbol{\Psi})$ is the multivariate probability density function of random parameter vector $\boldsymbol{\Psi}$. Several estimators can be derived from the preceding equations. Considering the MLE case, the AML (Asymptotic Maximum Likelihood) estimator is:

$$\hat{\boldsymbol{\Psi}}_{AML} = \arg \min_{\boldsymbol{\Psi}} \left\{ \det \left[\mathbf{P}_V \mathbf{R}_{xx} \mathbf{P}_V + \frac{\text{tr}(\mathbf{P}_V^\perp \mathbf{R}_{xx}) \mathbf{P}_V^\perp}{N - D} \right] \right\} \quad (2.112)$$

D is the known number of input signals, and $\arg \min$ is the minimisation operator. An alternative estimator, known as CML (Conditional Maximum Likelihood), follows the simplified expression:

$$\hat{\boldsymbol{\Psi}}_{CML} = \arg \min_{\boldsymbol{\Psi}} \{ \text{tr}(\mathbf{P}_V^\perp \mathbf{R}_{xx}) \} \quad (2.113)$$

As further discussed in chapter 3, the DoA estimation algorithm used in SARAS, WSF (Weighted Subspace Fitting), is a slight modification of the CML estimator.

2.4.3 Sub-Optimum and High-Resolution Algorithms

The preceding algorithms have very good performance but can be computationally demanding, especially if the number of input signals increases. Computationally simpler algorithms, termed as quadratic, have been designed to compensate this.

Bartlett beamformer, for instance, computes the output power of the array for different scan angles, using conventional weights, and averaging for K samples:

$$\hat{P}_B(\boldsymbol{\psi}) = \mathbf{V}^H(\boldsymbol{\psi}) \left(\frac{1}{K} \sum_{k=1}^K \mathbf{x}[k] \mathbf{x}^H[k] \right) \mathbf{V}(\boldsymbol{\psi}) \quad (2.114)$$

$\mathbf{V}(\boldsymbol{\psi})$ is the general SV for the DoA defined by $\boldsymbol{\psi}$, with size $N \times 1$. Once the power is known for all angles of interest, peaks in the obtained spatial power spectrum will correspond to the presence of desired signals and interferers.

An alternative to Bartlett is known as Capon beamformer. The principle is the same, but instead of using conventional weights, the ones from MPDR (cf. section 2.3.2.2) are considered:

$$\hat{P}_C(\boldsymbol{\psi}) = \frac{1}{\mathbf{V}^H(\boldsymbol{\psi}) \mathbf{R}_{xx}^{-1} \mathbf{V}(\boldsymbol{\psi})} \quad (2.115)$$

Although the Capon beamformer has better spatial resolution than Bartlett's, its performance is well below the MLE. In order to cope with this, while maintaining a reasonable computational load, sub-space DoA estimation algorithms with high-resolution capabilities were originally derived. They are based upon the orthogonality between signal and noise subspaces, $\mathbf{U}_S$ and $\mathbf{U}_N$ respectively, of the array correlation matrix. In the presence of D signals, the signal subspace is defined with the eigenvectors corresponding to the D largest eigenvalues, as:

$$\mathbf{U}_S = [\mathbf{u}_1 \ \mathbf{u}_2 \ \dots \ \mathbf{u}_D] \quad (2.116)$$

While the noise subspace is defined as:

$$\mathbf{U}_N = [\mathbf{u}_{D+1} \ \mathbf{u}_{D+2} \ \dots \ \mathbf{u}_N] \quad (2.117)$$

Taking into account the signal model assumptions, the following condition is met:

$$\|\mathbf{V}_i^H \mathbf{U}_N\|^2 = 0, \quad i = 1, 2, \dots, D \quad (2.118)$$

$\mathbf{V}_i$ is the SV of the i -th signal. Based on this, the well-known MUSIC (Multiple Signal Classification) algorithm seeks to maximise the following function (Schmidt 1986):

$$\hat{Q}_{MU}(\boldsymbol{\psi}) = \frac{1}{\mathbf{V}^H(\boldsymbol{\psi}) \hat{\mathbf{U}}_N \hat{\mathbf{U}}_N^H \mathbf{V}(\boldsymbol{\psi})} = \frac{1}{\mathbf{V}^H(\boldsymbol{\psi}) [\mathbf{I} - \hat{\mathbf{U}}_S \hat{\mathbf{U}}_S^H] \mathbf{V}(\boldsymbol{\psi})} \quad (2.119)$$

MUSIC can be slightly modified to yield the Min-Norm algorithm. The modification is based upon the fact that, instead of taking all the eigenvectors from the noise subspace, a linear combination is considered. The optimisation minimises the norm of the chosen vector, while the zeros of the noise subspace, associated with the desired DoA, remain in the unitary circle. This criterion implies the definition of a weight matrix $\mathbf{W}$ in the spatial spectrum:

$$\hat{Q}_{Min-Norm}(\boldsymbol{\psi}) = \frac{1}{\mathbf{V}^H(\boldsymbol{\psi}) \hat{\mathbf{U}}_N \mathbf{W} \hat{\mathbf{U}}_N^H \mathbf{V}(\boldsymbol{\psi})} \quad (2.120)$$

A proper definition of $\mathbf{W}$ reduces the probability of false alarm. By defining it as:

$$\mathbf{W} = \hat{\Lambda}^2 \hat{\Lambda}_S^{-1} \quad (2.121)$$

$$\hat{\Lambda} = \hat{\Lambda}_S - \hat{\sigma}_n^2 \mathbf{I} \quad (2.122)$$

$$\hat{\sigma}_n^2 = \frac{1}{N-D} \sum_{i=D+1}^N \hat{\lambda}_i \quad (2.123)$$

Where $\hat{\lambda}_i$ is an estimate of the eigenvalues of the array correlation matrix, while $\hat{\Lambda}_S$ is a diagonal matrix with the estimated eigenvalues corresponding to the D largest eigenvectors. In the preceding equations, D is the number of incoming signals in the array, and is supposed to be known a priori.

By modifying the mathematical expression of the spatial spectrum, one obtains the WSF algorithm, which is a modification of the CML one (Van Trees 2002), as commented in the preceding section:

$$\hat{Q}_{WSF}(\boldsymbol{\psi}) = \frac{1}{\text{tr}[\mathbf{P}_V^\perp \hat{\mathbf{U}}_S \mathbf{W} \hat{\mathbf{U}}_S^H]} \quad (2.124)$$

Another popular super-resolution algorithm is ESPRIT (Estimation of Signal Parameter via Rotational Invariance Techniques). In uniform linear arrays, it is based upon the concept of forming subarrays that verify invariability properties with respect to translations. The DoA is estimated using Least Squares techniques (Roy and Kailath 1989).

2.5 Array Calibration

Spatial beamforming algorithms, along with DoA estimation techniques, make use in their equations of the array manifold vector $\mathbf{V}$. In the first case this vector must be known only for a set of DoA, corresponding to desired and, sometimes, interfering signals, while in the case of WSF or MUSIC, for example, $\mathbf{V}$ is taken as a parametric vector, to be evaluated in a given optimisation window, which may be 1- or 2-dimensional.

It is precisely the estimation of this vector what constitutes the array calibration problem and, from the discussion in the previous paragraph, two main procedures can be defined:

- A non-parametric approach, which seeks to directly obtain the numerical values of the vector for a set of steering directions of the array. This set can be made as exhaustive as required.
- A parametric approach, where the array manifold is expressed as an equation which depends on a given set of DoA-independent unknowns. If estimated, numerical values of $\mathbf{V}$ can be obtained for any desired DoA.

Both approaches will be reviewed in the next sub-sections.

2.5.1 Non-Parametric Algorithms

As commented, non-parametric algorithms, in their simplest form, are straightforward from a software point of view. By using a given source with a known angular position ψ_c , i.e., a beacon, and processing the received samples with the Eigen algorithm, as in section 2.3.3.3, one may obtain $\mathbf{V}(\psi_c)$ as the dominant eigenvector of the spatial correlation matrix. This step can be repeated for different steering directions, either using several fixed sources or a moving one. If possible, a single fixed source can also be considered by moving the array with respect to it.

This technique presents a number of challenges which may render it inappropriate as it is for real systems, though. One of the main concerns is the fact that there may not be enough degrees of freedom in the election of the calibration angles ψ_c . For example, if a set of fixed beacon sources is used, the available $\mathbf{V}(\psi_c)$ is obviously limited. A moving source has a predetermined trajectory as well, which may not be compatible with the calibration requirements. Thus, in order to obtain $\mathbf{V}(\psi_c)$ for arbitrary, or at least, wider angular ranges, an interpolation technique may be mandatory. Such interpolations are also useful to limit the memory required to store the values of $\mathbf{V}(\psi_c)$, especially for large arrays.

The interpolation of $\mathbf{V}(\psi)$, be it linear, spline, etc., may not be easy, though, since it is not guaranteed that the array manifold is a smooth function. In order to cope with this, it is suggested in (Lanne, Viberg, and Lundgren 2007) to interpolate instead the following function:

$$\mathbf{q}(\psi) = \mathbf{V}(\psi) ./ \mathbf{V}_{nom}(\psi) \quad (2.125)$$

Where an element-wise vector division is used. $\mathbf{V}_{nom}(\psi)$ is an a priori known array manifold model, at least locally, and based upon equation (2.11).

The interpolation of the manifold function is not the only issue with non-parametric algorithms. A non-exhaustive list of impairments is provided below:

- Strong beacon sources are required, so that the SNR is large. This is also true for parametric algorithms.
- The calibration process is very sensitive to the presence of interfering signals.
- Even with interpolation, a dense set of calibration points is usually required, with the ensuing memory problems, and the fact that the calibration process takes a long time to be carried out.

- Dynamic calibration errors are not compensated. Moreover, if large enough, the whole calibration process is endangered since the array manifold will change for different ψ_c values.
- Depending on the beacon source, the calibrated array manifold may significantly differ from the true one. For example, if a near-field source is considered, a suitable near- to far-field transformation must be included in the calibration process.

2.5.2 Parametric Algorithms

Once more referring to equation (2.11), parametric algorithms try to estimate the parameters thereof, by also making use of beacon signals. Generally, five parameters are required per array branch:

- Three Cartesian coordinates, in the case of 3-dimensional arrays, which indicate the position of the phase centres, or a common geometrical point, of each of the array antennas.
- An amplitude and a phase offset, mainly due to the different responses of the analogue components in the array branches.

Usually, one of the antennas in the array is taken as reference, and the parameters in the remaining branches are considered with respect to this reference. This arbitrary positioning of the origin of the coordinate system is viable due to the fact that DoA and beamforming algorithms are insensitive to multiplications of the SV by a complex constant.

The main advantage over non-parametric algorithms is the fact that the SV may be obtained for arbitrary DoA, once the calibration process has been carried out for a limited set of them. The other key advantage is that a parametric model can be obtained, even if the beacon signals have unknown or imperfectly known DoA. A particular case would be if calibration and DoA estimation are to be performed from the same desired signal, in one phase, a problem known as on-line calibration or complete array auto-calibration. More details on this will be given in section 2.5.2.1.

On the other hand, these processes are sensitive to modelling errors. Indeed, the model in (2.11) may be incomplete in real systems, since it does not take into account the following aspects:

- Cartesian coordinates may not be constant parameters, but may change with the DoA due to the variation of the phase centre of each individual antenna. Cf. for example (Best 2004).
- Phase and amplitude offsets may change with time, frequency and DoA. This last case can be seen as an alternative representation of the variation of the positions of the phase centres.
- Coupling between antennas can change the SV modelling. These are usually DoA-independent, though. In any case, coupling is not an issue in SARAS system, due to the high separation between radiating elements.

With these considerations in mind, an improved model of the array manifold response would take the form below, where α_n is the phase response, β_n the amplitude one, and $\phi(t, f, \theta, \varphi)$ is a residual phase offset:

$$\alpha_n = -k [\sin(\theta) \cos(\varphi) x_n(\theta, \varphi) + \sin(\theta) \sin(\varphi) y_n(\theta, \varphi) + \cos(\theta) z_n(\theta, \varphi)] + \phi(t, f, \theta, \varphi) \quad (2.126)$$

$$\beta_n = \beta_n(t, f, \theta, \varphi) \quad (2.127)$$

The associated CRLB bounds will not be stated, because the derivation follows the same principles as in section 2.4.1, with slightly more cumbersome matrix algebra. It is important to notice, though, that in the general calibration model, there will be wanted and unwanted parameters, so that a generalised version of equation (2.94) will apply, and that the beacon signals may not be present simultaneously. In such situations the CRLB matrix is obtained as:

$$\mathbf{C}_{CR} = [\mathbf{J}_1 + \mathbf{J}_2 + \dots + \mathbf{J}_D]^{-1} \quad (2.128)$$

$\mathbf{J}_d$ is the FIM associated to the calibration process with the d-th beacon signal. Its (m,n) component follows the general equation:

$$(\mathbf{J}_d)_{mn} = K_d \operatorname{tr} \left(\mathbf{R}_{xx}^{-1} \frac{\partial \mathbf{R}_{xx}}{\partial \theta_m} \mathbf{R}_{xx}^{-1} \frac{\partial \mathbf{R}_{xx}}{\partial \theta_n} \right) \quad (2.129)$$

K_d is the number of statistically independent samples considered in the d-th calibration process, and θ_m and θ_n are general estimation parameters. More information on this can be found in (Antón et al. 2010).

The remaining sub-sections will deal with the techniques that can be used for the estimation of calibration parameters, with the assumption of the simplified array manifold model.

2.5.2.1 Maximum Likelihood Calibration

Equation (2.110) can be generalised, as follows:

$$L_{MLE}(\mathbf{p}) = -\ln \det(\mathbf{K}_{xx}(\mathbf{p}) - \operatorname{tr}[\mathbf{K}_{xx}^{-1}(\mathbf{p})\mathbf{R}_{xx}]) \quad (2.130)$$

In this case the vector of unknown DoA angles, $\boldsymbol{\psi}$, has been replaced with a vector of unknown calibration parameters $\mathbf{p}$. Of course, the specific functional dependences change, but the optimisation rationale remains the same. See for example (Weiss and Friedlander 1989) for more details on the specific formulation.

One important aspect, though, is the increased computational complexity, since the dimensionality of $\mathbf{p}$ is usually much higher than that of $\boldsymbol{\psi}$. In order to cope with this, efficient implementations of MLE/MAP non-linear optimisation have to be devised. One of such is the EM (Expectation-Maximisation) algorithm (Dempster, Laird, and Rubin 1977). The idea is to decouple the original multi-parametric optimisation into simpler sub-optimisations with reduced dimensionality. Decoupling is performed from the available, or “incomplete”, data snapshots $\mathbf{x}[k]$, which are transformed into “complete” data $\mathbf{y}[k, i]$, where i is the sub-optimisation index. This transformation is not unique,

and therefore must be designed for each particular problem. The technique reduces to maximizing the conditioned likelihood function of $y[k, i]$ given observed data $x[k]$ and the current parameter estimation $\hat{\mathbf{p}}^{(n)}$:

$$\hat{\mathbf{p}}^{(n+1)} = \underset{\mathbf{p}}{\operatorname{argmax}} E[\ln p_{y[k, i]}(y[k, i]: \mathbf{p} | x[k]: \hat{\mathbf{p}}^{(n)})] \quad (2.131)$$

This simplification can be further improved using the SAGE (Space-Alternating Generalised Expectation-maximisation) algorithm. While EM decouples parameters but estimates them simultaneously, SAGE defines several processing cycles where only one subgroup of parameters is optimised. The advantage behind this is that each subgroup can be associated with a different signal space, called “hidden or augmented space” (Fessler and Hero 1994), a clearly desirable feature when different kinds of parameters must be obtained. It can be proved that SAGE preserves the stability and computational complexity of EM, while it can significantly increase its convergence speed in certain scenarios (Chung and Böhme 2001).

In (Chung and Wan 2008) SAGE is used for array position calibration, although a generalisation as a global calibration algorithm can be deduced defining appropriate cycles. This is especially interesting for the already mentioned auto-calibration problem. Supposing calibration is performed using D beacon signals from unknown DoA, two estimation cycles are defined: the DoA of the beacons and the positions of the antennas. For this problem to be solvable, some restrictions must be met, though, as detailed in (Rockah and Schultheiss 1987). The overall process follows the equations and explanations below.

Starting from an initial estimation $\hat{\mathbf{p}}^{(n)}$ of the calibration parameters, and $\hat{\boldsymbol{\psi}}^{(n)}$ of for the DoA, the algorithm evaluates D array manifolds, one per beacon signal, and estimates the following modified samples:

$$s_i[k] = \frac{\mathbf{V}_i^H \mathbf{Z}_i^{(n)}[k]}{(\mathbf{V}_i^H \mathbf{V}_i)}, i = 1 \dots D \quad (2.132)$$

s_i are estimated temporal signal samples from the augmented space $\mathbf{Z}_i^{(n)}$. This augmented space is not known a priori, and, thus, for the first cycle of the algorithm, it is made equal to the received samples $x[k]$.

Since the algorithm starts with the DoA estimation sub-cycle, beacon signals are separated from one another in this augmented space, which can be updated in sub-subsequent cycles as follows:

$$\mathbf{Z}_i^{(n+1)}[k] = s_i[k] \mathbf{V}_i + x[k] - \mathbf{V} \mathbf{s}[k] \quad (2.133)$$

In the equation $\mathbf{V}$ is a $N \times D$ matrix, where each column is one of the signal SVs, and $\mathbf{s}[k]$ is a $D \times 1$ column vector, with all the estimated s_i . The n -dependence of the array manifolds has been obviated for notation clarity.

Using K samples, an augmented correlation matrix, $\mathbf{R}_{zz}^{(i)}$, for each D signal, can be obtained. The optimisation function for the i -th DoA estimation becomes thus:

$$\hat{\psi}_i^{(n+1)} = \underset{\psi}{\operatorname{argmax}} \left\{ V^H(\hat{\rho}^{(n)}, \psi) R_{zz}^{(i)} V(\hat{\rho}^{(n)}, \psi) / [V^H(\hat{\rho}^{(n)}, \psi) V(\hat{\rho}^{(n)}, \psi)] \right\} \quad (2.134)$$

$\hat{\psi}_i^{(n+1)}$ is the set of angular estimates associated to the i -th beacon, in the n -th optimisation cycle. $V(\hat{\rho}^{(n)}, \psi)$, not to be confused with the matrix in equation (2.133), is the updated parametric model of the array response, for a general steering direction ψ . It is important to notice that an updated version of $R_{zz}^{(i+1)}$ can be obtained, by re-evaluating equations (2.132) and (2.133) with the updated $\hat{\psi}_i^{(n+1)}$, before estimating $\hat{\psi}_{i+1}^{(n+1)}$.

Once the DoA estimation sub-cycle has been carried out, the calibration parameters are estimated. In this case the augmented space is directly associated with the received signal samples $x[k]$. The optimisation thus becomes:

$$\rho^{(n+1)} = \underset{\rho}{\operatorname{argmax}} \left\{ \operatorname{tr} \left[P_{V(\rho, \psi^{(n+1)})} R_{xx} \right] \right\} \quad (2.135)$$

$P_{V(\rho, \psi^{(n+1)})}$ is the projection matrix into the columns of matrix $V(\rho, \psi^{(n+1)})$, where each column is the SV of one of the D signals, using the updated DoA information, for a given value of the search vector ρ .

While SAGE may greatly simplify the MLE in the DoA sub-cycle, by decoupling each estimation, in the calibration sub-cycle the multi-dimensional optimisation implied in equation (2.135) is still very complex. To mitigate this, the multidimensional optimisation can be further simplified by dividing the search into different stages, one for antenna positions, another one for phase offsets, and a final one for amplitude offsets. And of course, direct search can be substituted in each case by iterative procedures such as Newton-Raphson. The next sub-section, though, will explore a simpler calibration routine that tries to circumvent with a different approach the complexity of the last equation.

2.5.2.2 Least Squares

This technique relies on linear LS (Least Squares), thus eliminating the non-linearity of MLE. The assumption is that the beacon DoA is known, so that no auto-calibration is required. As in the non-parametric case, several measurements are performed, at least as many as unknowns, with a different DoA for the beacon signal (or different beacons) in each one of them. Phase parameters are estimated following the matrix equation below:

$$- \begin{pmatrix} \frac{2\pi f}{c} \times \sin \theta_1 \cos \varphi_1 & \frac{2\pi f}{c} \times \sin \theta_1 \sin \varphi_1 & \frac{2\pi f}{c} \times \cos \theta_1 & -1 \\ \frac{2\pi f}{c} \times \sin \theta_2 \cos \varphi_2 & \frac{2\pi f}{c} \times \sin \theta_2 \sin \varphi_2 & \frac{2\pi f}{c} \times \cos \theta_2 & -1 \\ \frac{2\pi f}{c} \times \sin \theta_3 \cos \varphi_3 & \frac{2\pi f}{c} \times \sin \theta_3 \sin \varphi_3 & \frac{2\pi f}{c} \times \cos \theta_3 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{2\pi f}{c} \times \sin \theta_M \cos \varphi_M & \frac{2\pi f}{c} \times \sin \theta_M \sin \varphi_M & \frac{2\pi f}{c} \times \cos \theta_M & -1 \end{pmatrix} \begin{pmatrix} x^{(1)} & x^{(2)} & \dots & x^{(N)} \\ y^{(1)} & y^{(2)} & \dots & y^{(N)} \\ z^{(1)} & z^{(2)} & \dots & z^{(N)} \\ \phi^{(1)} & \phi^{(2)} & \dots & \phi^{(N)} \end{pmatrix} \quad (2.136)$$

$$\begin{aligned}
&= \begin{pmatrix} \alpha_1^1 & \alpha_1^{(2)} & \dots & \alpha_1^{(N)} \\ \alpha_2^1 & \alpha_2^{(2)} & \dots & \alpha_2^{(N)} \\ \vdots & \vdots & \dots & \vdots \\ \alpha_M^1 & \alpha_M^{(2)} & \dots & \alpha_M^{(N)} \end{pmatrix} \\
&\Leftrightarrow \mathbf{A}\mathbf{X} = \mathbf{B}
\end{aligned} \tag{2.137}$$

The matrix equation is derived from equation (2.126), obviating DoA, time and frequency dependencies of the unknowns. $\mathbf{A}$ is the matrix of known parameters, $\mathbf{X}$ the matrix of unknowns, and $\mathbf{B}$ the matrix of phase coefficients obtained from the received samples using the Eigen technique, as already discussed. M different measurements are carried out and, since there are four unknowns per array branch, the condition $M \geq 4$ must be verified.

Amplitude offsets can be obtained directly from the amplitude information of the dominant eigenvectors, performing a simple average. This decoupling between amplitude and phase parameters is the reason why linear optimisation can be considered in this case. The optimal solution of the problem in equation (2.137), in the LS sense, is obtained with the Moore-Penrose pseudo-inverse:

$$\mathbf{X} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{B} \tag{2.138}$$

Since matrix $\mathbf{A}$ can have errors due to imperfect knowledge of the DoA of the beacons, a more correct approach may be Total Least Squares or, even, Partial Total Least Squares, as the last column of $\mathbf{A}$ has no errors. More details can be found in (Moon and Stirling 2000).

Another important consideration is that, in order to truly have an over-determined system of equations, the DoA of the beacons cannot be chosen arbitrarily. In the ideal case, to avoid multicollinearity, it is enough that they differ in an arbitrarily small quantity. In real computations, fixed point and quantisation errors, along with the finite resolution of the array, impose greater restrictions which cannot be ignored.

Finally, if in a given measurement more than one beacon signal is present, the calibration can still be carried out, but some additional processing is required. This will be further explored in section 2.6.2, in a different, albeit related, scenario: the DoA estimation problem in the presence of interferers.

2.6 Additional Topics

2.6.1 Estimation of the Number of Incoming Signals

In the preceding considerations it was always implied that the number of incoming signals was known. Since this is not usually the case, especially in DoA estimation, this parameter must be obtained. A number of techniques based on the eigenvalues of the spatial correlation matrix have been devised, taking advantage of the fact that the eigenvalues corresponding to noise-only eigenvectors are ideally equal among themselves and to the noise variance.

Two popular likelihood functions, AIC (Akaike Information Criterion) and MDL (Minimum Description Length), are given below (Van Trees 2002):

$$AIC(d) = K(N - d) \ln \left\{ \frac{\frac{1}{N-d} \sum_{i=d+1}^N \hat{\lambda}_i}{\left(\prod_{i=d+1}^N \hat{\lambda}_i \right)^{1/(N-d)}} \right\} + \frac{1}{2} d(2N - d + 1) \quad (2.139)$$

$$MDL(d) = K(N - d) \ln \left\{ \frac{\frac{1}{N-d} \sum_{i=d+1}^N \hat{\lambda}_i}{\left(\prod_{i=d+1}^N \hat{\lambda}_i \right)^{1/(N-d)}} \right\} + \frac{1}{2} \{d(2N - d) + 1\} \ln K \quad (2.140)$$

d is the hypothesis on the number of incoming signals, K the number of received snapshots and $\{\hat{\lambda}_i\}$ the estimated eigenvalues of the correlation matrix. The number of incoming signals is estimated as:

$$\hat{D}_{AIC} = \arg \min_d [AIC(d)] \quad (2.141)$$

$$\hat{D}_{MDL} = \arg \min_d [MDL(d)] \quad (2.142)$$

AIC, though, tends to overestimate the number of signals, even at high SNR, whereas MDL tends to underestimate them, especially at low SNR (Wong et al. 1990). To cope with this, a modified AIC algorithm has been considered, as specified in (Nadler 2010):

$$MODAIC(d) = K(N - d) \ln \left\{ \frac{\frac{1}{N-d} \sum_{i=d+1}^N \hat{\lambda}_i}{\left(\prod_{i=d+1}^N \hat{\lambda}_i \right)^{1/(N-d)}} \right\} + 2C_n d \left(N + 1 - \frac{d + 1}{2} \right) \quad (2.143)$$

C_n is a penalty coefficient which can be arbitrarily chosen, although in (Nadler 2010) some indications are given as to its optimal value, which is set to:

$$C_n = 2 + 0.001 \ln K \quad (2.144)$$

2.6.2 Interferers

If the algorithms from 2.6.1 determine that more than one signal is present, the algorithms need to sort out interferers from desired components. In this section only the DoA estimation problem is considered, although the principles can be extended to a calibration scenario.

One obvious solution to the multi-signal DoA estimation problem would be to take as many eigenvectors as detected signals and used them as a basis for the signal subspace $\mathbf{U}_s$. This will work as long as $D < N$. However, taking more eigenvectors adds noise to the estimation process, and this is unnecessary if the end user is only interested in the DoA of a single signal. In such cases, it is desirable to treat each signal separately, computing as many spatial spectrums as signals presents, and obtaining thus completely uncoupled DoA estimates.

This cannot be directly performed from the eigenvectors of the spatial correlation matrix, because this subspace and the signal subspace are equal except for a unitary rotation, which must be estimated. This pre-compensation is known as the Procrustes problem, and several solutions have been proposed in the scientific literature.

The mathematical formulation is as follows. Given two matrices, $\mathbf{A}$ and $\mathbf{B}$, of equal size, it is sought to solve the following equation:

$$\min_Q \|\mathbf{A} - \mathbf{BQ}\|_F \quad \text{subject to} \quad \mathbf{Q}^H \mathbf{Q} = \mathbf{I} \quad (2.145)$$

The Frobenius norm has been used. It can be shown that if the SVD (Singular Value Decomposition) is used to decompose the matrix $\mathbf{B}^H \mathbf{A}$:

$$\mathbf{B}^H \mathbf{A} = \mathbf{U} \mathbf{S} \mathbf{V}^H \quad (2.146)$$

Then it follows that:

$$\mathbf{Q} = \mathbf{U} \mathbf{V}^T \quad (2.147)$$

One algorithm based on these considerations is the PROS (Projection-ROtation-Scaling), which uses a series of additional suppositions regarding the structure of the SV subspace (Tseng, Feldman, and Griffiths 1995). A similar method, named SVE (Steering Vector Estimation), but with better stability performance, is presented in (Weiss and Friedlander 1995). This last algorithm carries out a series of successive optimisations with respect to several modelling unknowns, with the following generic equation:

$$\min_Q \|\mathbf{GCS}^{0.5} - \mathbf{BQ}\|_F \quad (2.148)$$

$\mathbf{G}$ includes the amplitude offsets information, $\mathbf{C}$ the phase offsets, $\mathbf{S}$ is the power correlation matrix of the incoming signals, and $\mathbf{B}\mathbf{B}^H$ is the signal subspace, without the noise component.

CHAPTER 3 FAST ACQUISITION OF SATELLITE SIGNALS

3.1 Introduction

3.1.1 State of the Art

The main motivations behind the development of acquisition aid systems in satellite TT&C communications in S- and X-band, especially for LEOP, were already analysed in section 1.3.1. Before describing the proposed novel solution to these problems, though, current systems will be analysed in more detail, and their advantages and drawbacks put into context.

When the trajectory data of the satellite to be tracked is not reliable enough, Program Track modes solely based on the ephemerides cannot be used. Ground station antennas include an additional feature, known as Autotrack, which is typically based on monopulse techniques (Sherman and Barton 2011) (Gudim et al. 1999). Monopulse has good tracking performance and does not need pointing predictions, but the initial acquisition is limited by the -3dB beamwidth of the antenna.

An alternative, originally proposed in (Damonte and Stoddard 1956), is to add a circular motion to the tracking antenna, which cause sinusoidal variations in the power of the received signal. These variations in turn can be used to interpolate the true spacecraft position. This technique is known as CONSCAN (Conical Scan).

Figure 3.1 CONSCAN technique with a given spacecraft trajectory (Gawronski and Craparo 2001)

In (Gawronski and Craparo 2001), two variants of CONSCAN are proposed, where the circular motion is replaced by Lissajous and Rosette curves. However, as stated in the reference, the performance achieved by the three of them in estimating the position of a spacecraft is similar, so that CONSCAN is favoured due to its simplicity. The main drawback of CONSCAN, though, is that, since it relies on a mechanical movement, it takes a long time to converge. In fact, it has been widely used

for deep space applications, where spacecraft angular speeds are low, but for launchers or LEO (Low Earth Orbit) satellites, faster estimation techniques should be considered.

CONSCAN belongs to a class of techniques known as Sequential Amplitude Sensing. In this class another popular method is the Step-Track one. Step-Track, also known as Hill-Climbing, is a very simple technique, which seeks the maximum of the received signal power in successive small movements, alternating between two orthogonal axes (Dybdal 1998). The main drawbacks of the technique are its lower noise robustness when compared to monopulse (it is better to track using a sharp null than a beam maximum), slow convergence as was the case of CONSCAN, and a permanent lag between spacecraft and antenna position (Hawkins, Edwards, and McGeehan 1988).

Another technique presented in (Hawkins, Edwards, and McGeehan 1988) is the Electronic Beam Squinting, which is closer to array solutions and relies on electronic steering of the beam. The performance is similar to, but does not surpass that of monopulse.

In 2009, Astrium proposed an innovative solution to this conundrum in X-band. The new system, known as XAA (Migl et al. 2010), was briefly presented in 1.3.1. The XAA is an auxiliary antenna, attached to the reflector of the main one. While the larger reflector antenna performs tracking and full transmission and reception functionalities, the XAA is charged with the initial acquisition and the correction of the error offsets of the main antenna at the start of the pass. Both antennas share the same coordinate system, reducing the effect of parallax errors. A block diagram of the system is shown below:

Figure 3.2 XAA block diagram («X-Band Acquisition Aid - An advanced tool for future LEOP support» OPS-G Forum)

The first implementation of this system used a smaller antenna with a diameter of 1.3 metres, and a G/T greater than 14.5 dB/K. Its broader beam permitted to increase the acquisition range of the overall system beyond 2 degrees in elevation and cross-elevation offsets. A monopulse tracking system was used for the estimation of the DoA of the satellite to be tracked.

Figure 3.3 XAA installation in Perth (Migl et al. 2010)

While XAA fully covers the uncertainty range of LEOP, the use of a smaller antenna increases its sensitivity to thermal noise. On the other hand, this additional dish can have a non-negligible impact on the mechanical performance of the main antenna, especially taking into account the current trend to reduce the overall diameter for cost savings. Both limitations, thermal noise robustness and mechanical impact, are complementary, and a trade-off between the two of them must be established, with no evident solutions in some scenarios.

It is thus in this context that an innovative array-based solution, SARAS, is proposed. The use of a distributed array of antennas, attached to the rim of the main reflector, clearly reduces the mechanical impact when compared to the single dish XAA solution. On the other hand, thermal noise robustness is guaranteed thanks to digital array processing, while the resolution of the system is greatly improved thanks to the great separation among array elements. Finally, hardware implementations in FPGA (Field Programmable Gate Array) or GPU (Graphics Processing Units) guarantee a fast convergence of the estimation algorithms, which do not rely on a mechanical scanning but on an electronic one.

Arrays for increased acquisition ranges have also been proposed in (Mukai et al. 2002), for NASA Deep Space Network. In this case, though, the system relies on multiple feed arrays and neural networks. The idea of extending this to distributed arrays, thus increasing the resolution, is a novel aspect of SARAS, as well as its application in launcher acquisition. The concept of the system, with a uniform distribution of array elements, is shown below:

Figure 3.4 SARAS concept

3.1.2 Contributions

The contributions of this chapter are

- The proposal of a novel acquisition aid system of satellites and launchers, based on array technology and digital signal processing, using a system engineering methodological approach. The approach started with system level requirements from ESA, which have driven the decomposition of SARAS into functional subsystems with well-defined interfaces. Lastly, these logical blocks have been allocated into physical components, and integrated to produce the final prototype.
- The definition of FoM to evaluate the system performance of SARAS.
- The simulation of some of these algorithms in different realistic scenarios to obtain the FoM of the system, based on Monte Carlo techniques.
- The elaboration of a detailed DoA estimation error budget, based on some hypotheses and the results obtained from the previous simulations.

3.2 System Level Requirements

Taking into account the state-of-the-art as explored in section 3.1.1, ESA formulated high level requirements for an innovative solution to the problem of acquiring spacecraft in LEOP and similar critical phases, which can be summarised as follows:

Req. ID	Description	Current status
REQ-1	The new solution shall be an array-based acquisition aid system and make use of digital processing techniques and algorithms.	Compliant

REQ-2	<p>The new solution shall be tested with S-band signals from at least one of the following target mission satellites:</p> <ul style="list-style-type: none"> • Cryosat-2: LEO, non Sun-synchronous orbit, with a maximum angular velocity of the spacecraft in azimuth and elevation of 0.6 °/s. • GOCE (Gravity field and steady-state Ocean Circulation Explorer): low-Earth, Sun-synchronous, near-circular. • XMM-Newton (X-ray Multi-Mirror Mission): 48-hour elliptical orbit around the Earth, with a maximum angular velocity of the spacecraft in azimuth and elevation of 0.39 °/s. • INTEGRAL (INTErnational Gamma-Ray Astrophysics Laboratory): highly eccentric 72-hour orbit around the Earth. <p>More details on these will be given in chapter 5. An S-band implementation has been considered as a first step in the overall viability analysis, with an X-band migration planned in the near future, cf. chapter 6. The S-band bandwidth is limited to 2.2 – 2.3 GHz.</p>	Compliant
REQ-3	<p>It shall minimise or completely eliminate the mechanical impact on the main antenna charged with the actual tracking of the spacecraft. In particular, such mechanical impact shall be lower than the one caused by the XAA system, using lighter components. Total weight of the system shall be below 50 kg, to be evenly distributed between array elements.</p>	Compliant
REQ-4	<p>The solution shall be able to acquire signals with $\pm 1^\circ$ of uncertainty regarding their location in elevation and cross-elevation offsets with respect to the broadside direction of the main antenna. Such uncertainty can be expected in LEOP and other critical phases of the satellite mission.</p>	Partially compliant ($\pm 0.9^\circ$ achieved with the required reliability)

REQ-5	The accuracy and precision of the improved DoA estimates shall be such that the main antenna is able to continue with the tracking, i.e., residual errors shall be within the -3dB beamwidth of this antenna. In the frame of the project the Vil-1 antenna from ESAC (European Space Astronomy Centre) station in Madrid was considered. This antenna has a diameter of 15 metres, and a corresponding S-band -3dB beamwidth of approximately 0.56° (+/- 0.28°).	Compliant
REQ-6	The system shall be able to acquire a spacecraft with elevations equal to or higher than 5° , and with arbitrary azimuth.	Compliant
REQ-7	The time to compute one DoA estimate, from the moment the order is given to the system, shall be below 10 seconds. The requirement imposes an improvement when considering conical scan solutions, which take around 1 minute to converge.	Partially compliant (12s achieved)

Table 3.1 SARAS system level requirements

The logical decomposition of these high-level requirements gave rise to the functional blocks that will be described in the next section.

3.3 System Analysis

The following figure summarises the architecture of SARAS system, along with relevant internal and external interfaces. It is the result of a set of functional requirements that were determined during the first phases of the project.

Figure 3.5 SARAS system architecture

In the figure three antennas are considered for clarity reasons, although, in the actual implementation, eight have been used. The reasons behind this choice will be made clear in section 3.5.2.5. Also, auxiliary subsystems such as power supply or mechanical parts are not explicitly shown. Finally, as discussed afterwards, the Calibrator subsystem may interface directly with the antennas or the RF/IF (Radio Frequency / Intermediate Frequency) chains by generating adequate reference or beacon signals.

3.3.1 Subsystems

3.3.1.1 Antennas and RF/IF Chains

The antennas are the first element in the RF/IF chain. While no specific type is favoured, their design must bear in mind the mechanical limitations of the main antenna, i.e., their size and weight must not exceed predefined limits. Those are set by the main antenna, especially by its rigidity and diameter. Although in this Thesis simple radiating elements will be considered, it is perfectly possible for each one of them to be in turn a compact subarray. The use of subarrays with analogue beamforming and control permits to adapt the G/T of each radiating element to specific circumstances, by turning individual elements on and off, and adds a second layer of control in terms of electronic steering, and thus FoV, and interferer rejection. The main disadvantage, though, is that the number of cables and the overall complexity of the system increase. The concept of variable G/T is illustrated in the figures below:

Figure 3.6 G/T and station visibility trade-off

Figure 3.7 Variable G/T with analogue subarrays

After the antennas come the RF/IF chains, one per array element, with a structure as depicted below:

Figure 3.8 RF/IF architecture

A filter is included before the LNA (Low Noise Amplifier) to eliminate out-of-band signals and thus reduce the possibilities of saturation of the first amplifiers. This is especially useful if SARAS operates when the main antenna is in transmission mode. On the other hand, the aim of the LNA is, as usual, to control the NF (Noise Figure) of the overall system as soon as possible.

The Down-Converter transforms the central frequency of the received signal to IF, which is better suited for narrow filtering and digital processing. In order to do so, it uses an LO (Local Oscillator) signal generated in the F&T (Frequency and Time) subsystem. This stage also includes filtering before the conversion, to improve the image band rejection and eliminate as much noise generated by the prior amplifiers as possible; filtering after the conversion, where more selective filters can be implemented; and various amplification stages. Filtering bandwidth will usually be higher prior to the down-conversion, in order to be able to acquire a greater variety of signals, than afterwards, where noise must be minimised and where the system can benefit from a selectable LO signal.

In the figure all blocks seem to be co-located, but this is not mandatory, though. Indeed, it may be that some components are close to the auxiliary antenna, in an outdoor scenario, while others are in an indoor operating centre, along with remaining subsystems. It is advisable that at least the LNA, and prior components, are close to the individual radiating elements, in order to reduce the NF of the system.

The architecture presented above is thus only one of many possibilities, especially taking into account that not all components are required. For instance, the coupler, which is part of the calibration path, is only required if a test signal injection is contemplated. On the other hand, more than one signal could be received from each radiating element, and be combined in a hybrid structure to be able to obtain either circular polarisation. Finally, interfaces with the M&C (Monitoring and Control) and GUI (Graphical User Interface) subsystem, as shown in Figure 3.5, have not been implemented, but could be considered for monitoring and controlling the Down-Converters or any variable gain amplifier or attenuators.

3.3.1.2 *Digital Processor*

The Digital Processor is the core of the processing algorithms, and performs the specific task of estimating the DoA of desired signals based on digitised samples. The figure below shows its representation for a three element array:

Figure 3.9 Digital Processor architecture

The first block is constituted by the ADCs, which sample the output from the RF/IF chains. There are as many ADC as antennas. Once more, three channels are considered for simplicity's sake. All the ADCs must be synchronised by the F&T subsystem, using a trigger signal to start the acquisition at the same time, and a periodic clock to obtain successive samples in the same manner. The digitised signal can have a varying number of bits. More bits will increment the dynamic range of the ADC, at the expense of higher computing and memory requirements, so that the main advantage of reducing the number of bits is that higher sampling rates can be achieved. A more detailed study of these parameters will be performed in chapter 5.

After these comes the Digital Conditioner stage. This Digital Conditioner can be implemented in a number of ways, or even eliminated. Some tasks assigned to it are the following: digital filtering, baseband down-conversion and decimation. One of the

main advantages of this block is that, if implemented in generic digital processing devices, such as FPGAs, the flexibility in the whole system can be maintained while considering a very specific conditioning of the received signals. Indeed, the RF/IF chains can be made more general, in terms of bandwidth for example, while a more specific, user-selected filtering is performed in the digital realm. Thus the configuration of this stage can be specified in the M&C and GUI subsystem.

Once the signal has been properly conditioned, the statistical processing is carried out. The output of this processing will be either calibration information, or the DoA of the desired signal. It is a statistical processor because it estimates this information based on the statistics of the received digital signal samples. Thus, the Statistical Processor can operate in two modes. In normal acquisition mode, the digital signal samples are processed in order to obtain an improved DoA estimation. In calibration mode, the pertinent statistics of the signal samples are sent to the Calibrator to obtain a calibration model of the array, or update the existing one. This information is used in acquisition mode for proper estimation.

After the Statistical Processor comes the Time Stamping. The estimated DoA parameters are associated with an absolute time reference given by the F&T subsystem. This absolute time reference is also applied to the digital signal samples. The timestamp will be essential for the predicting functionalities of the system.

The last stage is the Signal Tracker and Discriminator. This block operates if iterative estimations are carried out over time in the Statistical Processor, i.e., if instead of a set of DoA for desired signal and interferers, there is a set of DoA vectors, where estimations are carried out in different times, properly referenced in the Time Stamping block. This block seeks to filter the resulting trajectories from the spacecraft, discriminate desired signal from interferers, and predict future values of the DoA of the desired signal.

The specific digital conditioning implemented in SARAS, and the structure of the Statistical Processor and the Signal Tracker and Discriminator, will be covered in section 3.4.

3.3.1.3 *F&T*

The F&T subsystem provides an LO frequency reference for the RF/IF chains, and timing references for the Digital Processor. These timing references include one signal for the ADC (Analogue-to-Digital Converters), so that their sampling is synchronised, and an absolute time reference to act as a timestamp of the received samples. The architecture of the subsystem is shown below:

Figure 3.10 F&T architecture

The start of the acquisition or calibration is signalled through the interface from the M&C and GUI to the Trigger and Clock Generator, and will correspond to the pressing of a button by the End User in the GUI, or to a previously programmed event. This sets the Trigger and Clock signals in the Generator, which are sent to the ADC. The trigger signal is also sent to the Absolute Time Reference Processor to trigger the time reference acquisition. It is important that the cables are of very similar lengths and constitution, to guarantee almost identical propagation times of the triggering signals. The Absolute Time Reference Processor is a GPS (Global Positioning System) receiver which adapts the corresponding signal to the format used by the Digital Processor.

3.3.1.4 *Calibrator*

Next figure summarises the generic architecture of the Calibrator subsystem:

Figure 3.11 Calibrator architecture

The Calibrator is divided into calibrating algorithms and the resulting calibration information. This last data is stored in the Calibration Data Collector, which can adopt several forms, such as files in the processing computer, a database, etc. From this block the pertinent and updated information will be extracted to be used by the Digital Processor. Prior to this, though, the Calibration Data Processor combines and adapts the calibration information obtained from various sources.

The M&C and GUI subsystem will give the indications to the Calibration Data Collector so that it can obtain the required calibration data. For example, for calibration files, the paths of the required files will be specified. Parameters which can change the calibration data to be used include the polarisation, frequency, and a priori known DoA of the desired signal.

The Geometrical Calibrator seeks to estimate the positions of the auxiliary antennas. These will be geometrical references, while in truth the interest resides in the phase centres of each radiating element. The phase centre of an antenna is a reference point which is the origin of the ideal equivalent spherical wave front of the radiated signal. Due to the duality properties of antennas, this also applies for reception. Since in practical implementations geometrical references and phase centres need not be the same, it is interesting to complement the Geometrical Calibrator with an Antenna Calibrator. This block computes amplitude and phase radiation patterns of antennas, typically in anechoic chambers. From these, the position of phase centres for differing DoA can be computed, and thus some of the errors made in the Geometrical Calibrator corrected. The amplitude and phase radiation pattern

response for various DoA, frequencies and polarisations are sent to the Calibration Data Collector.

These calibration methods may not be enough to guarantee the estimation of an accurate parametric model. Dynamical perturbations and the offsets generated by the RF/IF chains, among other errors, require proper compensation. This is done by the Statistical Calibrator, which performs a calibration using the statistics of the incoming signal samples estimated in the Statistical Processor of the Digital Processor subsystem. Combining these statistics with a priori calculated calibration models, the Statistical Calibrator computes and updates an improved model. The parameters of the algorithms in this block are given by the M&C and GUI subsystem.

The signals samples used by the Statistical Calibrator can be of diverse origin. The simplest one is a Test Signal injected to couplers in the RF/IF chains, as already commented in section 3.3.1.1. In this case, the Test Signal is typically a tone with the same frequency as that of the desired one, specified by the End User and sent from the M&C and GUI subsystem. The signal is generated in the Test Signal Generator. This calibration will compute the phase and amplitude offsets of the analogue components beyond the couplers. However, in this computation, two spurious offsets will be included: those of the cables which route the signal to the couplers, and those of the couplers themselves.

In order to compensate these, they must be previously calibrated in the Component Calibrator, using devices such as Vector Analysers, and included in the Calibration Data Collector. The obtained S-parameters describe the electrical behaviour of linear electrical components and networks, and can be used to determine the phase and amplitude changes which apply to an electrical steady-state stimulus as it traverses the corresponding component or network. The cables must also be phase-stable with respect to differences of temperature and bending angles among them, if there are significantly different routing paths.

The Test Signal can also be sent to one or several test antennas, to be received by SARAS system as a beacon signal, as explained in 2.5. In any case, the calibration processing algorithms will be examined in more detail in chapter 4.

3.3.1.5 *M&C and GUI*

The last subsystem is the M&C of SARAS, and serves as the main interface for the end user and the main antenna. Its architecture is as follows:

Figure 3.12 M&C and GUI architecture

Using the GUI the user can specify and verify the value of several parameters, and operate the system. The GUI also shows the parameters obtained from other external sources and from the main antenna, given by the External Data Processor and Distributor. Any number of information parameters can be considered here, such as the local time and date for example. Conversely, the External Data Processor and Distributor also processes the external data obtained from the End User through the GUI.

Data from the GUI and the External Data Processor and Distributor are given to the M&C core, which is in charge of the monitoring and control of all pertinent devices. Updated values in this process are sent back. The M&C Core is also in charge of the

DoA Distributor, which takes the calculated DoA and the time stamp from the Digital Processor and sends it to the main antenna. Finally, the M&C Core compiles logging information, obtained from events in other subsystems and those provoked by the End User, and which can be shown in the GUI.

The information which arrives to the M&C Core can be stored in the Data Storage component. This component can be a local or remote database, computer files, etc. The stored information can be used at a later time. A last functionality of the M&C subsystem is the plotting of results in the Data Plotter, obtained from information compiled by the M&C Core.

The structure of the External Data Processor and Distributor will be analysed in more detail in the next subsection.

3.3.2 External Interfaces

As can be derived from Figure 3.5, the main external interfaces of SARAS are:

- The wireless channel between the radiating elements and the spacecraft, which can produce desired signals or interferers.
- The wireless channel between the F&T subsystem and the GPS satellite constellation. This interface may be modified if the absolute time reference is obtained from alternative equipment. For example, it could be provided by a subsystem of the main antenna.
- Some calibration processes make use of external equipment, for the generation of beacon signals. These include a collimation tower and a probe placed in the subreflector of the main antenna. Beacon signals can also be received from orbiting satellites. In all cases, a wireless channel acts as an interface between the calibration sources and the individual radiating elements. More details on calibration are given in chapter 4.
- The GUI serves as a visual interface for the end user.
- The M&C subsystem interacts with the ACU (Antenna Control Unit), or similar unit, of the main antenna, and with external sources which give preliminary spacecraft ephemerides.

As commented in section 3.3.1.5, this last interface is managed by the DoA Distributor and the External Data Processor and Distributor. The structure of the latter is shown below:

Figure 3.13 External Data Processor and Distributor architecture

In this interface manager two important types of data are obtained. On one hand there is the trajectory information of the desired signal, which can be obtained from Trajectory Data Files. These files may be located in the same computer, in the Internet Network, etc. Depending on their format, a mathematical propagation may be required, using a trajectory propagator, such as is the case with TLE (Two-Line Element) files. The complete trajectory is distributed to the GUI and the M&C Core, thus allowing its use in the Digital Processor for interferer discrimination in the Signal Tracker and Discriminator. The distribution of data is performed in the Data Distributor. The complete trajectory information can also be uploaded, in order to have updated values. In any case, these ephemerides do not have initially the required precision, and thus must be considered as preliminary, as it is precisely the role of SARAS to improve the DoA estimates of the desired signal.

On the other hand the steering information from the main antenna is obtained from the Main Antenna Interface, typically the ACU. The steering information can be given in the form of elevation and azimuth angles, for example, coupled with appropriate timestamps. In any case, it is adapted to the format required by SARAS in the Steering and Time Stamp Transformer. The adapted information is once more given to the Data Distributor.

This read interface with the ACU of the main antenna is complemented with a write one, using the DoA Distributor from the M&C subsystem. This block sends to the main antenna the improved DoA estimates, using the appropriate format, and once more coupled with timestamp information.

3.3.3 Physical Allocation

The next figure shows an exemplary representation of the hardware components needed for the Digital Processor, the F&T, the Calibrator, and the M&C and GUI subsystem.

Figure 3.14 Hardware architecture

This computer system includes a CPU (Central Processing Unit), a GPU, a memory, which includes Main and Static Memories, an HMI (Human Machine Interface), FPGA boards, where the analogue-to-digital conversion and some of the digital processing will be carried out, and a Network Interface, which can connect the system to external networks such as the Internet. All these are connected through several buses. Not all the components are mandatory, and additional ones could be included, such as drive units, etc.

Generalisations to the system in Figure 3.14 can be considered. For example, there can be more than one CPU, one for directly controlling the FPGA boards, and another one for performing dedicated statistical algorithms. Also, some or all of these components may be remotely located. The presence of a GPU can be interesting to carry out some resources consuming algorithms. In this instance, the bus between the FPGA boards, the CPU and the GPU should have a high bandwidth. In any case, a GPU has not been considered in the implementation of the first SARAS prototype. The specific hardware elements, and more details of the actual mapping between functional subsystems and physical ones, will be covered in chapter 5.

3.3.4 ConOps

SARAS system may operate in several modes, divided in two main categories:

- Spacecraft acquisition.
- Calibration.

For clarity's sake, calibration operations will be detailed in the next chapter. Spacecraft acquisition is modelled with the following ConOps (Concept of Operations):

Figure 3.15 Acquisition mode ConOps

The preceding ConOps does not take into account the initialisation of the external systems, i.e., ephemerides repository, main antenna and its sub-reflector probe for on-line calibration (cf. chapter 4), which are considered as independent. Configuration parameters mentioned in the figure include the selection of the processing bandwidth, implementation of FFT (Fast Fourier Transform) processing, Kalman filtering, etc. These options are explored in 3.4 and chapter 5.

3.4 Digital Processing Algorithms

This section gives a more detailed decomposition of the Digital Processor, centred on three of its constituent blocks: the Digital Conditioner, the Statistical Processor and the Signal Tracker and Discriminator. The algorithms described hereby are partly based on the theory developed in chapter 2.

3.4.1 Digital Conditioner

While the actual processing of this block can take many forms, the one considered for SARAS prototype will be analysed. Its architecture is shown below:

Figure 3.16 Digital Conditioner architecture

The first stage of the Digital Conditioner includes an amplifier and a filter, the parameters of which may be chosen by the end user by changing the appropriate FPGA register through the M&C link. Since bandpass sampling is performed, a baseband conversion using an NCO (Numerically Controlled Oscillator) is included. The frequency of the NCO is set such that one of the bandpass replicas is shifted towards baseband. The process includes a frequency inversion to compensate the one previously caused by the analogue mixer.

The final stage is the Decimator, which lowers the initial sampling frequency. It has been implemented using a CIC (Cascaded Integrator-Comb) filter (Hogenauer 1981), coupled with a compensation FIR (Finite Impulse Response) one which flattens the frequency response of the system, as explained in the next figure:

Figure 3.17 Decimator frequency responses

Once more, the configuration of the Decimator is controlled by the M&C subsystem. Its specific structure will be presented in chapter 5. The final output of the Decimators, i.e. the decimated signal samples, can be stored in the M&C subsystem, for off-line processing or plotting purposes.

3.4.2 Statistical Processor

The aim of this subsystem is to perform advanced statistical processing of the received signal samples in order to obtain the DoA of the satellites that are to be acquired. The main blocks are shown below:

Figure 3.18 Statistical Processor architecture

The first block is the Frequency Shift Compensator. The idea is that, if frequency estimation is to be performed, as will be described below, it is advisable to minimise the drift in the central frequency of the desired signal during the acquisition time of the signal samples. Main causes of this drift are variations in the PLL (Phase-Locked Loop) of the spacecraft which must be acquired, and the Doppler shift of moving targets. The first one cannot be known in advance, but the second one can be estimated, even if roughly, provided some information of the spacecraft's trajectory is available. The estimated Frequency Rate $\dot{f}_{est}$ is obtained thus from the M&C subsystem, taking into account the interface defined in Figure 3.13, using the following equations:

$$f_{est}(t_k) = f_{TX} \frac{c}{c \pm v_{TX}(t_k)} - f_{TX} \quad (3.1)$$

t_k is a given absolute time, as provided by the time stamp, and $v_{TX}(t_k)$ is the radial speed of the spacecraft at said time. $v_{TX}(t_k)$ is computed as:

$$v_{TX}(t_k) = \left. \frac{dR(t)}{dt} \right|_{t=t_k} \quad (3.2)$$

Derivative calculus can be approximated using two values as close to the time of interest as possible. With this in mind the preceding equation becomes:

$$\left. \frac{dR(t)}{dt} \right|_{t=t_k} \approx \frac{R(t_k + h) - R(t_k)}{h} \quad (3.3)$$

Where h should be as small as possible. In this last equation $R(t)$ is the distance between SARAS and the spacecraft, defined as:

$$R(t) = (R_T + h(t)) \sqrt{1 + \left(\frac{R_T}{R_T + h(t)} \right)^2 - 2 \frac{R_T}{R_T + h(t)} \cos \gamma(t)} \quad (3.4)$$

R_T is the Earth radius, which will depend on the geographical location of SARAS, taking into consideration that the Earth is an oblate spheroid. On the other hand $h(t)$ is the satellite height, defined as:

$$h(t) = \sqrt{x_i^2(t) + y_i^2(t) + z_i^2(t)} - R_T \quad (3.5)$$

$x_i(t)$, $y_i(t)$ and $z_i(t)$ are the inertial coordinates of the spacecraft in the ECI (Earth-Centred Inertial) coordinate system, obtained from the TLE or similar file and, eventually, an orbital propagator algorithm. Other coordinate systems can be used. If no values are available for the desired time t_k , an interpolation or extrapolation shall ensue. In the equation where $R(t)$ is defined, the angle $\gamma(t)$ is equal to:

$$\gamma(t) = \cos^{-1} [\cos(lat_S(t)) \times \cos(lat_{ET}) \times \cos(lon_{ET} - lon_S(t)) + \sin(lat_S(t)) \times \sin(lat_{ET})] \quad (3.6)$$

lon_{ET} and lat_{ET} are respectively the East Longitude and the North Geodetic Latitude of the location of SARAS. $lon_S(t)$ and $lat_S(t)$ are the same concepts for the sub-satellite point. These are obtained from the inertial coordinates, using a rotation matrix so that the Earth coordinate system is fixed, and applying spherical trigonometric formulae. Finally, the Doppler rate, $\dot{f}_{est}(t_k)$, is estimated as follows:

$$\dot{f}_{est}(t_k) \approx \frac{f_{est}(t_k + h) - f_{est}(t_k)}{h} \quad (3.7)$$

The signal samples must be transformed in the following way (Jeng, Tsung, and Lu 2007):

$$x_n[kT] = x_n[kT] * \exp\{-j\pi \dot{f}_{est} \times (kT)^2\} \quad (3.8)$$

$x_n[kT]$ is the signal sample of the n th-array branch, taken at sampling time kT . From this equation, and considering t_0 as the absolute time of the first sample, it is clear that:

$$t_k = t_0 + kT \quad (3.9)$$

This processing also improves the interferer discrimination capabilities, as interfering signals with different frequency shift rates will be sprawled in a wider bandwidth. In any case, this processing is not mandatory or could be implemented in the Digital Conditioner, prior to the Decimator. By doing this, the effective bandwidth of the received signal would be reduced, and thus the final filtering stage could be narrower.

After this comes the Frequency Estimator. This stage is optional, but useful when the desired signal has a non-uniform power distribution in the considered bandwidth. Indeed, for the DoA estimation algorithms, it is optimum to process only those frequency bins where the desired signal's power is the highest. A typical application is the processing of the carrier of a signal, discarding the modulated part. In this context, a frequency bin is a frequency window given by the resolution of frequency processing algorithms.

The simplest way to carry out the frequency estimation is to perform an FFT of the input signal samples, take its absolute value, and locate the peaks where maximum power is present through a direct search algorithm. In order to improve the SNR, the maximum search must be performed with the average of all the incoming digital channels.

In general, the required information from the M&C subsystem will be the number of processed samples K , the frequency range where the estimation will be performed, and the resolution. Range will be determined by the uncertainty in the a priori frequency knowledge of the signal, and limited by the narrowest filter, typically in the digital realm. Resolution, on the other hand, will be determined by the amount of noise and limited by the number of processed signal samples and the residual frequency shift of the signal.

More than one frequency can be estimated, thus taking into account the presence of interfering signals with non-uniform power distribution. Each estimation will be treated in parallel, until the desired signal can be discriminated from interferers in the Signal Tracker and Discriminator block. The maximum number of frequencies that can be processed is the number of frequency bins minus 1. A much lower number is recommended, though, due to computing requirements. Besides, the presence of a very high number of interferers is not foreseeable.

Frequency estimations are used to select the appropriate frequency bins, which are then given to the Statistics Estimator, which computes the autocorrelation matrix as defined in section 2.3.3.1. If the Frequency Estimator is turned off, a block average with the time samples and the block size K is carried out. If frequency bins around a maximum have been selected, the correlation matrix is computed in the frequency domain, as in the following equation:

$$\mathbf{R}_{xx} = \frac{1}{N_{bins}} \mathbf{X}_f \mathbf{X}_f^H \quad (3.10)$$

$\mathbf{X}_f$ groups the samples of all digital channels in the frequency domain, but only including those frequency bins close to a frequency maximum. N_{bins} is the number of selected frequency bins.

In a calibration operating mode, the obtained information is given to the Calibrator for array calibration (cf. chapter 4). In acquisition mode, it is given to the DoA Estimator, which is based on the WSF algorithm as explained in section 2.4.3. The non-linear optimisation is divided into two stages: a direct search coarse optimisation, followed by a fine one based on Newton method and including penalty terms which limit the angular optimisation window to the a priori uncertainty one.

When more than one signal is detected using the algorithms of section 2.6.1, special precautions must be taken. Here the case of two signals will be treated, based on the equations of section 2.6.2, although this can be generalised up to N-1 signals, where N, as commented in the preceding chapter, is the number of array elements. Three different WSF spectrums are computed:

$$\hat{Q}_1(\psi) = \frac{1}{tr[\mathbf{P}_V^\perp \mathbf{A}_1 \mathbf{W} \mathbf{A}_1^H]} \quad (3.11)$$

$$\hat{Q}_2(\psi) = \frac{1}{tr[\mathbf{P}_V^\perp \mathbf{A}_2 \mathbf{W} \mathbf{A}_2^H]} \quad (3.12)$$

$$\hat{Q}_3(\psi) = \frac{1}{tr[\mathbf{P}_V^\perp \mathbf{A} \mathbf{W} \mathbf{A}^H]} \quad (3.13)$$

The first column of $\mathbf{A}$, $\mathbf{A}_1$, is the Steering Vector of the first signal and the second column, $\mathbf{A}_2$, that of the second signal. Each $\mathbf{W}_{opt}$ is computed taking into consideration the modifications in the $\mathbf{U}_S$ matrix for the corresponding spectrum. One maximum is selected from each $\hat{Q}_1$ and $\hat{Q}_2$ spectrums, and two from $\hat{Q}_3$. The four maxima will be given to the Signal Discriminator so that the desired signal can be separated from the undesired ones. The same discrimination is applied if more than one frequency peak has been detected.

For proper operation, the DoA Estimator needs calibration data from the Calibrator. As input data from the M&C, there will be the uncertainty angular window, where the cost function will be considered, centred on the a priori DoA estimation, and the operating frequency of the signal before any down-conversion. As mentioned, the DoA Estimator must be carried for as many frequency estimates as obtained in the Frequency Estimator.

3.4.3 Signal Tracker and Discriminator

The architecture of the last component in the Digital Processor is detailed below:

Figure 3.19 Signal Tracker and Discriminator architecture

The block starts and concludes with coordinate transforms. The first one is motivated by the fact that filtering and prediction operations may require a different coordinate system with respect to the one used by WSF, namely, elevation and cross-elevation offsets. On the other hand, the second transform is included to adapt predicted DoA values, if estimated, to the coordinate system used by the main antenna. If the Signal Tracker and Discriminator is not used, such transformation will not be required, as WSF already produces angular offsets which can be usually read by the ACU of the main antenna.

Regarding the DoA Filtering and Prediction features, in the current implementation of SARAS system these are carried out with an EKF, to take into account non-linearities of the equations involved.

State equations use ECI coordinates, with the following state vector X :

$$X = [p_x \ p_y \ p_z \ v_x \ v_y \ v_z \ a_x \ a_y \ a_z]^T \quad (3.14)$$

The vector includes three position, three speeds, and three acceleration coordinates. The 9x9 state transition matrix is:

$$F(n+1, n) = \begin{bmatrix} 1 & 0 & 0 & \Delta T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta T & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta T & 0 & 0 & 0 \\ & & & 1 & 0 & 0 & \Delta T & 0 & 0 \\ \mathbf{0} & & & 0 & 1 & 0 & 0 & \Delta T & 0 \\ & & & 0 & 0 & 1 & 0 & 0 & \Delta T \\ & & & & & & 1 & 0 & 0 \\ \mathbf{0} & & & & \mathbf{0} & & 0 & 1 & 0 \\ & & & & & & 0 & 0 & 1 \end{bmatrix} \quad (3.15)$$

ΔT is the time offset between measurements, which should be chosen based on the type of trajectory to be tracked, while taking into account computational restrictions of the system. While this state update is a linear equation, measurement variables, elevation and azimuth, are obtained from the ECI coordinates with non-linear ones. Indeed, the transformation from state vector to measurement one undergoes the following stages (Meeus 1998):

- Transformation from ECI to ECEF (Earth-Centred Earth-Fixed) coordinate frame. This is performed with the following rotation matrix:

$$\mathbf{R}_z(\lambda) = \begin{bmatrix} \cos \lambda & \sin \lambda & 0 \\ -\sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.16)$$

λ is the addition of the Greenwich Apparent Sidereal Time and the ground station longitude λ_e :

$$\lambda = \theta_{g0} + \Omega_e t + \lambda_e \quad (3.17)$$

ECI coordinates are taken with the ground station as origin.

- Transformation from ECEF to topocentric geodetic coordinate frame, with the matrix:

$$\mathbf{R}_y\left(\frac{\pi}{2} - \phi\right) = \begin{bmatrix} \sin \phi & 0 & -\cos \phi \\ 0 & 1 & 0 \\ \cos \phi & 0 & \sin \phi \end{bmatrix} \quad (3.18)$$

ϕ is the ground station latitude. This produces a topocentric state vector $\mathbf{Y}$, where the first three parameters $\mathbf{r}_{\text{topo}}$ are position coordinates, and is related to the ECI position vector as:

$$\mathbf{r}_{\text{topo}} = \mathbf{R}_y\left(\frac{\pi}{2} - \phi\right) \mathbf{R}_z(\lambda) \mathbf{r}_{\text{eci}} \quad (3.19)$$

It must be noted, though, that a more accurate and slightly different procedure has been followed in computing $\mathbf{r}_{\text{topo}}$, based on proprietary information from ESA (STDM Interface Control Document, Issue 2, May 1992).

- Azimuth and elevation angles are obtained from this last vector:

$$Az = \tan^{-1} \left(\frac{-Y_1}{-Y_2} \right) + \pi \quad (3.20)$$

$$El = \sin^{-1} \left(\frac{Y_3}{\sqrt{Y_1^2 + Y_2^2 + Y_3^2}} \right) \quad (3.21)$$

The EKF uses the Jacobian of these equations. The use of the Hessian matrix has also been considered to compensate for quadratic bias, as suggested in (Mehra 1971), although the small time window seems to make it unnecessary. On the other hand, square root filtering was considered both for the filtering and the Riccati prediction estimation of the error correlation matrix. Namely, Bierman and Thornton algorithms were used, as suggested in (Grewal and Andrews 2001). Finally, the estimation of the noise matrices is considered in more detail in section 5.3.4.2.

If an interferer is present, there will be up to four sets of WSF estimates, as explained in 3.4.2. The processing explained in the previous two sections will be applied to each set. However, before sending the result to the ACU, SARAS system must decide which one corresponds to the desired signal. The simplest way to proceed is to take advantage of available a priori information regarding the satellite orbit. These parameters, suitably transformed to the appropriate coordinate system, can be used to establish an a priori trajectory which will be compared with the ones obtained from all sets of WSF estimates. The desired signal will be assumed to be the one with the least residual error. Schematically:

Figure 3.20 Flowchart of trajectory prediction with interferer

The desired signal parameters are obtained from STDM files. This is a format used by ESA to transmit the predicted spacecraft ephemerides to the ground stations. The parameters will be transformed from their original coordinate frame to the azimuth/elevation or the elevation/cross-elevation one. It will take into account the possible movement of the main dish. Finally, the orbit determination algorithm will be a local interpolation/extrapolation.

M&C and GUI data includes the longitude, latitude, altitude and steering of the main antenna, information (even if not completely accurate) about the satellite trajectory, given by STDM files, and the desired prediction time offset. As an output, the predicted desired signal DoA, and the associated time stamp, are given to the M&C, for plotting and for distribution to the main antenna.

3.5 Monte Carlo Simulations

3.5.1 Introduction

Monte Carlo simulations, performed in a Matlab environment, have proven to be critical to assess the actual performance of the proposed solution, and to take early design decisions. Matlab generates a scenario with a given set of inputs, determined by the type of simulations, and generally produces an estimate of the elevation and cross-elevation offsets (cf. section 3.5.1.1 for a formal definition), obtained from WSF, of the signal under consideration, with respect to the broadside direction of the array. Since the true direction of the signal is known, the total error, for each Monte Carlo iteration, can be computed as:

$$\varepsilon = \sqrt{(\Delta\widehat{el} - \Delta el_T)^2 + (\Delta\widehat{xel} - \Delta xel_T)^2} \quad (3.22)$$

$\Delta\widehat{el}$ and $\Delta\widehat{xel}$ are the estimated values, and Δel_T and Δxel_T are the true ones. This error can be seen as the module of the difference vector between estimated and true steerings, and is represented graphically below:

Figure 3.21 Pointing error definition

The following sections will explore some of the simulations that were carried out, considering a variety of errors and perturbations that can be expected in realistic test scenarios. Before proceeding to the simulations themselves, though, some clarifications regarding the applied flow logic will be given.

3.5.1.1 Coordinate Systems

Coordinate transforms have already been exposed in section 3.4.3, related to the implementation of the Kalman Filter. Azimuth and elevation were seen as the key variables for measurement formulas. These, though, are absolute estimates, whereas it

is more intuitive, for WSF, to operate with relative angular offsets, with respect to the broadside direction of the array.

Thus, instead of absolute elevation and azimuth values, the respective offsets could be considered. The main drawback of this approach is that a fixed pointing error quantified in terms of azimuth offset, for an Az-El antenna mount, will vary with the elevation. One way to solve this is to consider an error coordinate system where one axis coincides with the elevation one, and the other, termed cross-elevation axis, is normal to the beam and contained in the elevation plane. Resulting cross-elevation pointing offsets are related to azimuth offsets with the following formula (Guiar, Lansing, and Riggs 1987):

$$\Delta \widehat{x_{el}} = \Delta \widehat{az} \times \cos(el) \quad (3.23)$$

el is the elevation steering of the array. Elevation and cross-elevation offsets can also be related to the local spherical angles that were used for the definition of the Steering Vector, cf. equation (2.11), with the following equations:

$$\theta = \sqrt{\Delta \widehat{el}^2 + \Delta \widehat{x_{el}}^2} \quad (3.24)$$

$$\varphi = -\tan^{-1} \left(\frac{\Delta \widehat{x_{el}}}{\Delta \widehat{el}} \right) - \pi \quad (3.25)$$

Next figure summarises these relationships:

Figure 3.22 Elevation/cross-elevation coordinate system related to the azimuth/elevation one (Gudim et al. 1999)

The φ definition given in the figure differs from the one in equation (3.25) due to the fact that the el/xel/boresight coordinate system is left-handed, whereas the local spherical one is right-handed. Defining a local vector Y_{local} as:

$$\mathbf{Y}_{local} = [\sin(\theta) \cos(\varphi) \quad \sin(\theta) \sin(\varphi) \quad \cos(\theta)]^T \quad (3.26)$$

The last step is to relate $\mathbf{Y}_{local}$ with the first coordinates of the topocentric state vector $\mathbf{Y}$ mentioned in section 3.4.3. Both coordinate systems are related by a series of rotations, which can be synthesised with the following matrix equation:

$$\mathbf{Y}_{local} = \begin{pmatrix} \sin(Az) \sin(El) & \cos(Az) \sin(El) & -\cos(El) \\ -\cos(Az) & \sin(Az) & 0 \\ \sin(Az) \cos(El) & \cos(Az) \cos(El) & \sin(El) \end{pmatrix} \mathbf{r}_{topo} \quad (3.27)$$

3.5.1.2 Error Statistics

The simulations will compute error statistics derived from the Monte Carlo iterations. The error formula is defined in equation (3.22). Related statistics are sample mean, sample variance or standard deviation, and sample RMSE, defined as:

$$\hat{E}(\varepsilon) = \frac{1}{N_{monte}} \sum_{n=1}^{N_{monte}} (\varepsilon_n) \quad (3.28)$$

$$\hat{\sigma}(\varepsilon) = \sqrt{\frac{1}{N_{monte} - 1} \sum_{n=1}^{N_{monte}} [\varepsilon_n - \hat{E}(\varepsilon)]^2} \quad (3.29)$$

$$\widehat{RMSE}(\varepsilon) = \sqrt{\hat{\sigma}^2(\varepsilon) + \hat{E}^2(\varepsilon)} \quad (3.30)$$

ε_n is the measured error for each Monte Carlo iteration, from a total of N_{monte} . $\widehat{RMSE}$ is the total error, including bias terms. However, when assessing the robustness of the system, it is of interest to compute the error associated with more than one standard deviation. The new formula for $\widehat{RMSE}$ becomes thus:

$$\widehat{RMSE}(\varepsilon) = \sqrt{L \hat{\sigma}^2(\varepsilon) + \hat{E}^2(\varepsilon)} \quad (3.31)$$

L is the number of standard deviations considered in the error assessment. For Gaussian errors, the confidence level p of the error term is related to L with the following formula:

$$p = \text{erf}\left(\frac{L}{\sqrt{2}}\right) \quad (3.32)$$

erf is the error function. For $L = 3.3$, the confidence level is 99.9%. The sample error variance $\hat{\sigma}^2(\varepsilon)$ can be compared to the CRLB, as defined in section 2.4.1. Two precautions must be taken, though.

Firstly, the derivative of the array manifold was defined in equation (2.78) as:

$$\mathbf{D} = \begin{bmatrix} \frac{\partial V}{\partial \varphi} & \frac{\partial V}{\partial \theta} \end{bmatrix} \quad (3.33)$$

In the elevation/cross-elevation coordinate system, it must be redefined as:

$$\mathbf{D} = \begin{bmatrix} \frac{\partial V}{\partial el} & \frac{\partial V}{\partial xel} \end{bmatrix} \quad (3.34)$$

The Δ symbol has been omitted for simplicity of notation. Required derivatives can be easily obtained with the Chain Rule, though:

$$\frac{\partial V}{\partial el} = \frac{\partial V}{\partial \theta} \frac{\partial \theta}{\partial el} + \frac{\partial V}{\partial \varphi} \frac{\partial \varphi}{\partial el} \quad (3.35)$$

$$\frac{\partial V}{\partial xel} = \frac{\partial V}{\partial \theta} \frac{\partial \theta}{\partial xel} + \frac{\partial V}{\partial \varphi} \frac{\partial \varphi}{\partial xel} \quad (3.36)$$

The unknown partial derivatives can be obtained from equations (3.24) and (3.25).

Secondly, the specified approach would compute separate CRLB for each angular estimate, elevation and cross-elevation. For the total error, some modifications must be implemented. Taking the definition of the total error in (3.22), it is clear that it is the root squared quadratic sum of two random variables, the errors in elevation and cross-elevation, which can be approximated by Gaussian density functions with zero mean and equal standard deviation σ . I.e., ε can be computed as:

$$\varepsilon = \sqrt{\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}} \quad (3.37)$$

With the vector form defined as:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{el} \\ \varepsilon_{xel} \end{pmatrix} \quad (3.38)$$

By definition ε follows a Chi distribution (Evans, Hastings, and Peacock 2000), with its mean and variance defined as:

$$E(\varepsilon) = \sigma\sqrt{2} \Gamma\left(\frac{3}{2}\right) \quad (3.39)$$

$$Var(\varepsilon) = 2\sigma^2 \left[1 - \Gamma^2\left(\frac{3}{2}\right) \right] \quad (3.40)$$

$\Gamma(\cdot)$ is the Gamma Function. A CRLB for ε can thus be obtained from the last equation, by replacing σ^2 with the CRLB obtained for the elevation or the cross-elevation angle, which are supposed to be equal.

However, there is one last precaution to be considered, stemming from the fact that the formulas specified for the CRLB were valid only for unbiased estimators, whereas it is clear that ε is not. For the scalar case, the biased CRLB is computed as (Matson and Haji 2006):

$$CRLB_{biased}(\hat{\Psi}) = CRLB_{unbiased}(\hat{\Psi}) \times \left(1 + \frac{db}{d\Psi}\right)^2 \quad (3.41)$$

b is the bias of the estimator:

$$b = E(\hat{\Psi}) - \Psi \quad (3.42)$$

In this special case

$$b = \sigma\sqrt{2} \Gamma\left(\frac{3}{2}\right) \quad (3.43)$$

The derivative of the preceding equation becomes:

$$\frac{db}{d\varepsilon} = \frac{d\sigma}{d\varepsilon} \sqrt{2} \Gamma\left(\frac{3}{2}\right) \quad (3.44)$$

And analytical formula for σ is not available. However, it can be approximated by the CRLB of either the elevation or the cross-elevation estimator. Taking the first one, the equation becomes:

$$\frac{db}{d\varepsilon} = \frac{\Gamma\left(\frac{3}{2}\right)}{\sqrt{2} \text{crlb}_{el}} \frac{d\text{crlb}_{el}}{d\varepsilon} \quad (3.45)$$

The derivative of the CRLB can be easily obtained from the formulas in section 2.4.1. The computation will involve computing second derivatives of the array manifold with respect to elevation. These can be related to the derivatives with respect to spherical angles using Faà di Bruno's formula (Tortolini 1855):

$$\frac{\partial^2 \mathbf{V}}{\partial^2 el} = \frac{\partial^2 \mathbf{V}}{\partial \theta^2} \left(\frac{\partial \theta}{\partial el}\right)^2 + 2 \frac{\partial^2 \mathbf{V}}{\partial \theta \partial \varphi} \frac{\partial \theta}{\partial el} \frac{\partial \varphi}{\partial el} + \frac{\partial^2 \mathbf{V}}{\partial \varphi^2} \left(\frac{\partial \varphi}{\partial el}\right)^2 + \frac{\partial \mathbf{V}}{\partial \theta} \frac{\partial^2 \theta}{\partial^2 el} + \frac{\partial \mathbf{V}}{\partial \varphi} \frac{\partial^2 \varphi}{\partial^2 el} \quad (3.46)$$

In the simulations below, though, it will be shown that the bias contribution is negligible, and that the unbiased CRLB is a valid performance assessment metric.

3.5.1.3 Simulation Parameters

A set of configuration parameters will be used for each Monte Carlo simulation. In order to have synthetic information, the following table will be used each time as a summary:

Parameter	Value
Frequency	MHz
N	2-8
Array radius	m
Array topology	Uniform/non-uniform
Sampling frequency	KHz
Interferer	Yes/no
Interferer power offset	dB
Multipath	Yes/no
Multipath power offset	dB
Carrier modulation	Yes/no
Residual Doppler rate error	%
Number of acquisitions	1-20
K (per acquisition)	1-1000000
Simulation time	ms
Angular excursion (per acquisition)	$^{\circ}$
Phase errors range	$\pm$ $^{\circ}$
Ampl. errors range	$\pm$ dB
Std of position calibration errors	cm
Acquisition range	$\pm$ $^{\circ}$
Spatial resolution	$^{\circ}$
Input C/No	dBHz
Input SNR	dB
Monte Carlo iterations	1-1000
Monte Carlo confidence level	%
Parametric variable	

Table 3.2 Simulation parameters

The rationale behind each configuration parameter is explained below:

- **Frequency:** the RF frequency of the signal received at the antennas. The centre frequency is considered as a valid narrow-band approximation.
- **N:** number of array elements, typically 8.
- **Array radius:** the radius of the main reflector, on the rim of which the array elements are placed. For uniform topologies, it is also the radius of the resulting circular array.
- **Array topology:** non-uniform topologies will be analysed in section 3.5.2.5.
- **Sampling frequency:** the sampling frequency f_s is set as:

$$f_s = 2.5 BW_{low} \quad (3.47)$$

It is slightly greater than the Nyquist frequency, so that a low pass filter can be included to approximate the overall filtering processing of the received signals. A 10th order Chebyshev filter has been considered, with a 50 dB signal suppression beyond the cut-off frequency set to BW_{low} .

On the other hand, BW_{low} is the bandwidth of the low-pass equivalent of a bandpass signal. Typical bandpass signal bandwidth, which is the double of BW_{low} , will be 150 KHz. It is considered a reasonable one to guarantee that the carrier is within it, taking into account the uncertainties in frequency excursions. It is not significant that the desired signal may have wider bandwidths in real cases, especially with suppressed carrier modulations. Indeed, the objective of the system is to estimate spatial statistics, not to perform a full demodulation, so that narrower filters can be used. The advantage, of course, from a simulation point of view, is to reduce the amount of required samples for a given simulation time.

- **Interferer:** it specifies whether an interferer signal is present, within the frequency band of the desired signal.
- **Interferer power offset:** the power offset of the interferer, in dB, with respect to the desired signal.
- **Multipath:** it specifies whether a specular multipath signal is present. It is coherent and with a random phase in its correlation coefficient.
- **Multipath power offset:** the power offset of the multipath component, in dB, with respect to the desired signal.
- **Carrier modulation:** it specifies whether a desired signal with a carrier, or a suppressed carrier modulation are considered. A typical carrier loss of 5.4 dB is considered, with respect to the total power of the desired signal.
- **Residual Doppler rate error:** the percentage of the Doppler rate that cannot be predicted with the orbit propagation algorithms, and will thus remain uncompensated at the moment the acquisition is carried out. This parameter of course affects SARAS system when, prior to spatial estimation, a carrier frequency search is performed, since the power of the desired signal will be smeared across several frequency bins.
- **Number of acquisitions:** number of estimates produced by WSF. More than one estimate will be considered for testing Kalman filters.
- **K (per acquisition):** number of signal samples used for each WSF estimation. Typically, around 100000 samples are considered.
- **Simulation time:** equivalent time, which is established with the following equation:

$$T = \frac{K}{f_s} \quad (3.48)$$

- **Angular excursion (per acquisition):** the amount by which the desired signal moves, in the angular domain, within one acquisition block. It can be established with the following formula:

$$\Delta ang = \frac{\omega_{sat}}{f_s} K \quad (3.49)$$

ω_{sat} is made equal to the pointing rate of the satellite at the considered elevation. In truth, the problem is two-dimensional, but it is implied, without real loss of generality, that the satellite only has a radial movement, in the elevation and cross-elevation coordinate frame, with respect to the broadside direction.

- **Phase errors range:** the range of residual phase calibration errors, with uniform distribution.
- **Ampl. errors range:** the range of residual amplitude calibration errors, with uniform distribution. They are presented in dB.
- **Std of position calibration errors:** the standard deviation of residual position calibration errors, with Gaussian distribution.
- **Acquisition range:** the angular uncertainty window where WSF will search for the desired signal. It has the same value for elevation and cross-elevation axes.
- **Spatial resolution:** the resolution of the search window, for direct optimisation.
- **Input C/No:** the carrier over noise floor ratio. Of course, this parameter is only considered for carrier modulations. It can be derived from the PFD (Power Flux Density) of the signal and the G/T of the system as follows:

$$\frac{C}{N_0} [dBHz] = PFD [dBW/m^2] - M + 228.6 + \frac{G}{T} [(dB/K)] + 20 \log \left(\frac{300}{f [MHz] \sqrt{4\pi}} \right) - L_c \quad (3.50)$$

f is the RF frequency, L_c is the carrier loss, and M is a security margin, to take into account the worst-case scenario.

- **Input SNR:** the signal to noise ratio, in the considered bandwidth, equal to 150 KHz, as already mentioned.

The noise power is determined in dB as:

$$P_N = P_S - SNR [dB] + 10 \log f_s \quad (3.51)$$

Or, alternatively, for carrier signals:

$$P_N = P_S - L_c - \frac{C}{N_0} [dBHz] + 10 \log f_s \quad (3.52)$$

P_S is the signal power of either the phase or quadrature components of the low-pass equivalent signal. In the definition of the preceding equation, the noise power is artificially increased to compensate for the posterior filtering of the overall signal, as already commented.

- **Monte Carlo iterations:** the number of times the same simulation is carried out, to obtain error statistics.
- **Monte Carlo confidence level:** limit to the number of required Monte Carlo iterations, so that the error statistics are within an interval with a given confidence level α . The number of required Monte Carlo iterations N_{monte} is computed as:

$$N_{monte} = \frac{\hat{\sigma}^2}{\alpha \left(\frac{l}{2}\right)^2} \quad (3.53)$$

l is made equal to 0.02° , as a low enough value compared to expected error thresholds. Since the formula uses the estimated variance, and not the true one, N_{monte} will vary as the estimator converges. To give some security margin, at least 30 Monte Carlo iterations will always be considered. The preceding formula is based on Chebyshev's inequality, and it is used in order not to impose a specific distribution function on the estimator.

- **Parametric variable:** the configuration parameter which is changed during the simulation to obtain parametric figures.

3.5.2 One Received Signal

In these first sets of simulation, the influence of white noise will be evaluated, using non-moving signal sources. The parameters of interest here are the SNR and the number of processed samples, and their influence on the performance of the DoA estimation algorithms. The influence of the array topology will also be assessed.

3.5.2.1 White Noise

White noise will be controlled with the SNR parameter. Thus, the first configuration is:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9 °
Spatial resolution	0.05 °
Input C/No	-
Input SNR	Several values
Monte Carlo iterations	1000
Monte Carlo confidence level	95 %
Parametric variable	Input SNR

Table 3.3 **Parameters for white noise simulation**

The results of the simulation are presented below. The first figures show the evolution of several statistics associated with the estimator, as the Monte Carlo simulation progresses:

Figure 3.23 WSF error statistics, as a function of Monte Carlo iterations, for several input SNR. (a) Std. (b) Bias. (c) 3.3 σ RMSE

The estimates corresponding to lower SNR, i.e., those with higher RMSE, have not fully converged, as required by the defined confidence interval, and more Monte Carlo iterations would be needed. In any case, since those SNR values are below the performance threshold, discussed below, the approximation is deemed valid enough. The statistics were obtained using the formulas already explained, and taking into account the error values obtained in each Monte Carlo iteration, as shown below:

Figure 3.24 WSF error, as a function of Monte Carlo iterations, for several input SNR. (a) Full view. (b) Zoom

The dashed line corresponds to the maximum permitted error that still fulfils the corresponding high-level requirement. In the figures it is shown the percentage of time that this requirement is not met, for all iterations and SNR values.

The errors arise due to the appearance of spurious maxima in the WSF spectrum, which in turn are caused by the imperfect estimation of noise and signal subspace. The next figures show some examples of such spatial spectra.

Figure 3.25 WSF spectrum, for an input SNR of -23 dB. (a) 2-D view. (b) Upper view

The presence of the peak clearly marks the position of the desired signal, with respect to broadside. The optimisation function is estimated within the acquisition range ($\pm 0.9^\circ$) with a spatial resolution of 0.05° . The limitation of the acquisition range justifies the fact that the maximum error values in Figure 3.24 are around 1.6° . For a lower SNR, spurious peaks appear:

Figure 3.26 WSF spectrum, for an input SNR of -30 dB

In this case, the maximum does not correspond to a true signal. In fact the desired signal DoA, in the experiment, was -0.015° in elevation, and 0.036° in cross-elevation.

The next figures summarise the statistics of Figure 3.23, for the last Monte Carlo iteration, and compares the standard deviation (std) with the square root of the CRLB:

Figure 3.27 Summary of statistics, for different input SNR. (a) Global view. (b) Zoom

It is clear that the std converges to the CRLB above a given performance threshold, corresponding to -24 dB of input SNR. On the other hand, the high-level requirement ceases to be met for an SNR below -25.5 dB, if the 3.3σ RMSE is taken as a reference.

3.5.2.2 Beyond the CRLB

The preceding sub-section made use of the unbiased CRLB. As already commented, the estimator is not unbiased, due to its positivity constraint, so the CRLB

should be modified to take this into account. The modification is plotted in the next figure:

Figure 3.28 Unbiased and biased CRLB

It is clear that the unbiased CRLB is a good approximation in this case. In light of Figure 3.27, though, it could be argued whether the CRLB is too optimistic, and whether tighter bounds could be found. The problem can be divided in two parts, above and below a given threshold, where the performance of the estimators quickly degrades. Two additional bounds have been considered for these cases, the second-order Bhattacharyya bound (Bhattacharyya 1947) (Abel 1993), and the Barankin bound (Barankin 1949), the last one with the Chapman-Robbins formulation (Chow and Schultheiss 1981) (Boman and Stoica 2001).

The formula for the Bhattacharyya bound, as a function of the FIM, is:

$$BhLB = \frac{1}{KJ_{11}} + \frac{J_{12}^2}{2K^2J_{11}^4} \quad (3.54)$$

On the other hand, the Barankin bound is approximated using a simplified method:

$$BLB = CRLB + \max(\mathbf{T}\mathbf{G}^{-1}\mathbf{T}^T) \quad (3.55)$$

The function is evaluated at a number of test points, $\mathbf{el}_T$ and $\mathbf{xel}_T$, uniformly distributed in the uncertainty window. Besides, defining $\mathbf{el}_S$ and $\mathbf{xel}_S$ as the true DoA:

$$\mathbf{T} = \sqrt{(\mathbf{el}_T - \mathbf{el}_S)^2 + (\mathbf{xel}_T - \mathbf{el}_S)^2} \quad (3.56)$$

$$\mathbf{G} = \left[\frac{|\mathbf{S}_{xS}| |(2\mathbf{S}_{xT}^{-1} - \mathbf{S}_{xS}^{-1})^{-1}|}{|\mathbf{S}_{xT}|^2} \right]^K \quad (3.57)$$

$\mathbf{S}_{xS}$ is the ideal array correlation matrix, in the presence of white noise determined by the input SNR, for the true DoA, and $\mathbf{S}_{xT}$ is the corresponding matrix for the test points under consideration. Finally $|\mathbf{A}|$ is the determinant of matrix $\mathbf{A}$.

Figure 3.29 Summary of statistics, for different input SNR. (a) Global view. (b) Zoom

Clearly the Bhattacharyya bound is tighter than the CRLB above the threshold. Below it, the simplified Barankin bound better reflects the behaviour of the sample std, although with a 2 dB offset. It could be argued that a more exhaustive search of Barankin-like bounds could produce better estimates. It is also worth commenting that the sample std is upper-bounded due to the a priori knowledge of the acquisition window. A mathematical basis for bounded CRLB can be found in (Gorman and Hero 1990). In any case, for the subsequent simulations, the CRLB will still be used.

3.5.2.3 Non-Linear Optimisation

In the preceding simulations, the bias term is somewhat higher due to the fact that a direct search optimisation of WSF spectrum is performed, with an angular resolution of 0.05° in both dimensions. This method has been chosen in order to reduce the computational burden of Monte Carlo simulations, while obtaining significant results in terms of variance. For improved performance, though, non-linear optimisation can be considered once the direct search algorithm has been executed.

The following results have been obtained with the same simulation parameters as in section 3.5.2.1, but including a non-linear optimisation stage, using a constrained Newton algorithm limited by the resolution of the initial direct search algorithm.

Figure 3.30 Direct search and non-linear optimisation

Clearly non-linear optimisation improves the bias error below threshold, at the expense of a slightly increased computational cost. However, the additional burden is negligible taken into account that only two optimisation variables, elevation and cross-elevation, are to be considered.

3.5.2.4 Signal Samples

The simulation summarised in Figure 3.27 was carried out with 100000 signal samples. The SNR threshold can be lowered using more samples, at the expense of more computing time. Using the same parameters as in Table 3.3, with 200000 samples instead and, of course, double simulation time, the following results are obtained:

Figure 3.31 Summary of statistics, for different input SNR, and a higher number of signal samples. (a) Global view. (b) Zoom

In this case the SNR threshold is clearly lower than -25.5 dB. The converse simulation, using the number of samples as parametric variable, is thus of interest:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	Several values
Simulation time	Several values
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9 °
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-26 dB
Monte Carlo iterations	1000
Monte Carlo confidence level	95 %
Parametric variable	K

Table 3.4 Parameters for simulation with different number of samples

Figure 3.32 Summary of statistics, for different K. (a) Global view. (b) Zoom

There is clearly a trade-off between K and input SNR. The next two simulations used the same parameters as in Table 3.4, but with input SNR of -20 and -23 dB respectively.

Figure 3.33 Different K parametric simulations. (a) SNR = -20 dB. (b) SNR = -23 dB

In both cases, the performance of the estimator is more or less the same for K = 50000 and 200000 samples, as best appreciated with the Bhattacharyya bound. Since the SNR is halved in the second simulation, this suggests that the trade-off between SNR and K can be approximated as:

$$SNR \leftrightarrow \sqrt{K} \quad (3.58)$$

3.5.2.5 Array Topology

In the preceding simulations, a uniform circular array with radius equal to 7.5 meters was considered. This topology is not random, and arises from physical considerations, taking into account the reflector antenna where the array was to be placed. This information was not available, though, at the start of the project. In fact, an alternative compact topology, where the array antennas were placed in an independent

positioner, was also considered. This sub-section will analyse the impact of the topology in the performance of the DoA estimation algorithms, from three different perspectives:

- The size of the array.
- The number of array elements.
- The geometrical distribution of these elements.

Although compact topologies need not be circular, they will nevertheless be approximated with circular arrays with decreasing radius, for ease of comparison. This yields, for the first analysis, the following simulation parameters:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	Several values
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9 °
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-23 dB
Monte Carlo iterations	350
Monte Carlo confidence level	95 %
Parametric variable	Array radius

Table 3.5 Parameters for simulation with different array radii

Figure 3.34 Summary of statistics, for different array radii. (a) Global view. (b) Zoom

It is not surprising that the performance improves as the array radii increases, since so do the resolution capabilities of the WSF algorithm. From the figures, though, it would seem that beyond 3 meters approximately, scarce improvement can be obtained. Precautions must be taken here, because the SNR was set to a value of -23 dB, above the threshold that was obtained for an array of 7.5 meters. For lower SNR, the array radius beyond which the improvement becomes negligible is larger. The next plots were obtained with an SNR of -26 dB and 500 Monte Carlo iterations:

Figure 3.35 Summary of statistics, for different array radii. (a) Global view. (b) Zoom

In this case, the radius must be greater than 4.5 meters for the 3.3 σ RMSE to be below the maximum error threshold. It can also be seen that above a given threshold the performance worsens. There are two explanations to this phenomenon:

- As the radius increases, so does the likelihood of the presence of grating lobes.
- Not enough Monte Carlo iterations were considered to achieve the required confidence level of 95 %. This is due to computational requirements, since such low SNR levels require approximately 2000 iterations.

In any case, it is clear that distributed topologies are superior to compact ones, at least in some prescribed ranges. Of course, specific array radii values are limited by physical restrictions, taking into account that the array must be attached to an existing parabolic antenna of a prescribed size.

Other reasons for choosing distributed topologies over compact ones were considered during the project that gave rise to the prototype, including economic ones. Indeed, the use of an independent compact array, for example, was deemed too expensive due to the price of electronic antenna positioners. Such reasons, in any case, are beyond the scope of this Thesis.

Figure 3.35 tells us that, beyond hardware restrictions, it is not desirable to increase the array size to arbitrary values, even if the resolution improves. Indeed, the larger the array radius, the more numerous the grating lobes are, thus limiting the maximum acquisition range that can be achieved. This is shown in the next simulations, with the following parameters:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	Several values
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	Several values
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-25 dB
Monte Carlo iterations	500 / 1000
Monte Carlo confidence level	95 %
Parametric variable	Array radius and acquisition range

Table 3.6 Parameters for simulation with different array radii and acquisition ranges

Figure 3.36 WSF std for several acquisition ranges and array radii. (a) Global view. (b) Zoom

It is clear that the threshold in acquisition range increases with lower radii. There is some loss in resolution power, but the performance, for an array with a 4 metres radius, is well below requirements. Bias behaviour is similar:

Figure 3.37 WSF bias for several acquisition ranges and array radii

Thus, distributed arrays outperform compact arrays, but medium-sized arrays would be the optimum choice, at least for SNR in the range of -25 dB. Of course, a 7.5 metre array shall still be considered, due to hardware limitations imposed by the main antenna where the prototype is installed.

The next point in the analysis of array topologies is to consider the required number of array elements, supposing that a distributed implementation has been chosen. Up until now, the number has been set to 8. This supposition shall be tested to see whether it is the optimum choice.

Parameter	Value
Frequency	2300 MHz
N	Several values
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-25 dB
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	N

Table 3.7 Parameters for simulation with different number of array elements

A minimum value of 2 elements has to be considered of course. More elements imply more hardware and installation costs, but also improved acquisition ranges and increased interferer discrimination power. Actual simulation results are shown below:

Figure 3.38 WSF std for several acquisition ranges and array radii. (a) Global view. (b) Zoom

Thus the election of 8 elements is explained, at least for the threshold SNR. It is clear that improvements beyond 8 elements do not justify the extra cost in hardware and installation.

It has been seen that the effective acquisition range of the 8-element uniform array is between $\pm 0.9^\circ$ and $\pm 1^\circ$ (cf. Figure 3.36). While the latter was the actual requirement, the final performance was deemed good enough, at least for the first prototype. Future versions of the system, though, are expected to increase these ranges. In order to overcome the limitation of grating lobes for large-radius arrays, a possible solution is to resort to non-uniform topologies, just as the Nyquist can be violated without aliasing with non-uniform time sampling. This was considered in the following set of simulations:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform – non-redundant
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	Several values
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	2000
Monte Carlo confidence level	95 %
Parametric variable	Acquisition range

Table 3.8 Parameters for simulation with non-redundant topologies

A non-redundant topology has been evaluated against several acquisition range values. Such a topology seeks to implement arrays where baselines among elements are as varied as possible. For the 8-element array, it becomes:

Figure 3.39 Non-redundant circular array with 8 elements

Results are shown below:

Figure 3.40 WSF std for different array topologies

Clearly the non-redundant solution permits to increase the acquisition range well beyond the initial requirement of $\pm 1^\circ$. In contrast, performance for small acquisition ranges is somewhat degraded when compared to the uniform topology, due to the loss in resolution capabilities. In any case, performance is still within acceptable bounds.

3.5.3 Two Signals

3.5.3.1 Uncorrelated Interferer

The presence of interferers modifies the original problem, since WSF must now estimate two different DoA. As commented, it will be up to the Signal Tracker to tell

apart desired from interfering signals, so this section will be restricted to establishing when the signals can be distinguished from one another, i.e., resolved, supposing that the system knows which DoA corresponds to the desired signal. The problem will be limited to one interferer, for simplification and because it is a reasonable scenario for real operations. In any case, the problem may be extended to $N - 2$ interferers, where N is the number of array elements. Furthermore, the interferer will be assumed to be uncorrelated with the desired signal. The mathematical definition of this was given in section 2.3.1. Two situations can be considered:

- One where desired signal and interferer are both within the acquisition window. The interest here is to see the minimum separation between the two of them so as to ensure that WSF can detect both peaks in the spatial spectrum. A second analysis will explore the performance of the modified AIC algorithm to correctly predict the actual number of signals, for different SNR.
- A second one, where the interferer falls outside the acquisition range. This situation is more likely and could severely degrade the performance of the system. Indeed, even if the interferer is not within the acquisition window, the associated grating lobes may appear, even below the required minimum separation. However, the results from the first situation will show that this is not a concern for SARAS, for the current requirements.

The first situation has been simulated using an interferer coming from broadside, and a desired signal component from a DoA with increasing angular separation, but within the acquisition range of the system. The actual parameters are:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	Yes
Interferer power offset	Several values
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	Angular separation and power offset between signals

Table 3.9 Parameters for uncorrelated interferer simulation

Results are summarised below:

Figure 3.41 WSF statistics for several angular separations and power offsets between desired signal and interferer. (a) Std. (b) Bias

It is true that performance worsens as interferer and desired signal approach each other, but this occurs at angular separations low enough to guarantee that the overall system requirements are met. I.e., when WSF can no longer distinguish between both signals, they are much closer than the accuracy and precision requirement of 0.28° . This is achieved through the large separation between array elements and the high resolution performance of WSF. The simulations also show that requirements are still met, even when the interferer is 40 dB stronger than the desired signal.

The following figures show the actual WSF spectrums, for each signal subspace separately, as obtained from the SVE algorithm (cf. section 2.6.2), and for their combination:

Figure 3.42 WSF spectrums with desired signal and interferer. (a) Desired signal. (b) Interferer. (c) Combined subspace. (d) Combined subspace – upper view

Even though the combined signal subspace has a smeared peak, SVE is able to separate both signal and interferer subspaces. It is interesting to note that, while the Rayleigh resolution is approximately 0.6° for a 15 metre aperture at 2.3 GHz, simulations show that, even with a 20 dB power offset, the actual resolution reached by WSF is around $0.2\text{--}0.25^\circ$, which justifies its high-resolution appellation:

Figure 3.43 WSF 3.3σ RMSE with an interferer with 20 dB power offset

It is clear from the preceding results that it is not important whether the interferer is within the acquisition range or not, since if a grating lobe interferes with the desired signal, it will do so only at angular separations well below the requirements of the system. However, if the interferer is allowed to come from a plurality of directions, and not only from broadside, appropriate performance, in terms of 3.3σ RMSE, can only be achieved for an SNR = -22 dB, instead of -24 dB. That is, the presence of an interferer effectively decreases the SNR threshold in 2 dB. This is shown in the next figure, with an SNR = -22 dB and a power offset of 0 dB. The CRLB, as derived in 2.4.1.2, is also specified:

Figure 3.44 WSF statistics for wide angular separations

At -23 dB, the error increases 1.5% approximately, and at -24 dB 3%.

In the preceding simulations the presence of an interferer was supposed to be known a priori by the system. In the general case, a detection technique must be implemented prior to WSF. These were explained in section 2.6.1, and their performance, for different SNR and power offset between signals, are plotted below.

Figure 3.45 Detection of interferers with different algorithms

The figure shows the estimate of the number of signal sources for different algorithms, SNR, and power offset between signals. Since an interferer is present, the correct value is 2. At high SNR, AIC tends to overestimate the number of signals, while MDL underestimates it at low SNR. The modified AIC technique is balanced, with a slight tendency to underestimate it in the threshold values of SNR that have been considered up until now.

3.5.3.2 Multipath

In the preceding section, the interferer was not correlated with the desired signal. The presence of correlation is modelled as:

$$\mathbf{x}(t) = \sigma_s m_s(t) \mathbf{S}\mathbf{o} + \sigma_i \left(\delta^* \times m_s(t) + \sqrt{1 - |\delta|^2} m_i(t) \right) \mathbf{S}\mathbf{i} + \mathbf{n}(t) \quad (3.59)$$

$m_s(t)$ and $m_i(t)$ are normalised signal and interferer sequences. This general case shall not be considered, as correlated interferers usually arise from jamming and are thus unlikely in the envisaged application. Multipath, on the other hand, is a special case of correlation, termed coherence, with $|\delta| = 1$.

The presence of multipath is unavoidable and, as shown in the simulations below, severely degrades WSF performance. This is especially critical when taking into account that SARAS antennas have low gain and have thus reduced spatial discrimination when compared to large reflectors.

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	Yes
Multipath power offset	Several values
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	Power offset between signals

Table 3.10 Parameters for multipath simulation

Figure 3.46 WSF performance with multipath component. (a) Normal view. (b) Zoom

It would seem that with coherent multipath with a power offset below -16 dB with respect to the desired signal, the performance is not affected in a relevant way. Such power losses can be expected due to depolarisation and power absorption during Earth reflections, and to the radiation pattern of the individual elements in the array. The signal loss caused by the latter will be determined in turn by the angular offset between desired signal and multipath. The worst-case scenario for multipath perturbation is a low elevation acquisition, considering the lowest antenna in the array. Angular offset can be calculated using this formula:

$$\Delta\theta = El + \tan^{-1} \left(\frac{h_{ant}(2h_{ant} + r_{sat} \sin El)}{h_{ant}r_{sat} \cos El} \right) \quad (3.60)$$

h_{ant} is the height of the array element above the ground, and r_{sat} is the range of the spacecraft to be acquired. In practice, multipath will limit the minimum elevation that the system is able to cope with. As will be verified in chapter 5, this threshold is approximately 5° , which fulfils the high-level requirement set in 3.2.

These considerations only apply, though, to WSF and DoA estimation, but cannot be extended to calibration, as will be explored in the next chapter. Indeed, during beacon calibration, SNR requirements are more stringent, and this also affects the amount of multipath that can be present.

3.5.4 Calibration Errors

3.5.4.1 Phase Offsets

Uniform phase offset errors can be included in the simulations to represent an imperfect knowledge of the array manifold. The simulation parameters are as follows:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	Several values
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	1000
Monte Carlo confidence level	95 %
Parametric variable	Phase errors range

Table 3.11 Parameters for phase errors simulation

Results are plotted below:

Figure 3.47 WSF performance with phase calibration errors. (a) Normal view. (b) Zoom

There is a clear threshold value for phase errors of $\pm 18^\circ$, which is not too stringent. There is not a perfect match with the Bhattacharyya bound because the latter has been calculated with Gaussian errors, instead of uniform. Gaussian errors give the following performance:

Figure 3.48 WSF performance with Gaussian phase calibration errors. (a) SNR = -24 dB. (b) SNR = -22 dB

Below the threshold, the Bhattacharyya estimate gives a tight bound. The threshold, evidently, is lower than with uniform errors.

3.5.4.2 Amplitude Offsets

Uniform amplitude offsets errors (in dB) can be included in the simulations to represent an imperfect knowledge of the array manifold. The simulation has been set as follows:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	Several values
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	1000
Monte Carlo confidence level	95 %
Parametric variable	Amplitude errors range

Table 3.12 Parameters for amplitude errors simulation

Results are plotted below:

Figure 3.49 WSF performance with amplitude calibration errors. (a) Normal view. (b) Zoom

There is a threshold at ± 0.2 dB of amplitude errors. The requirement can be loosened if a higher SNR is considered. For an SNR of -22 dB:

Figure 3.50 WSF performance with amplitude calibration errors, SNR = -22 dB. (a) Normal view. (b) Zoom

In any case, as mentioned in section 2.5.2.2, amplitude and phase offsets can be decoupled while evaluating the signal subspace, as long as multipath is below a given threshold (cf. section 3.5.3.2 for an empirical estimate). Thus, if precautions are taken to effectively decouple the variables, residual amplitude errors do not affect performance.

3.5.4.3 Position Errors

Gaussian position offsets errors can be included in the simulations to represent an imperfect knowledge of the array manifold. Position errors are the most critical aspect of calibration. The simulation has been set as follows:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	Several values
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	1000
Monte Carlo confidence level	95 %
Parametric variable	Position errors std

Table 3.13 Parameters for position errors simulation

Simulation results are:

Figure 3.51 WSF performance with position calibration errors. (a) Normal view. (b) Zoom

Position calibration requirements are very stringent, in the order of 2 mm. This requirement can be traded with system reliability, and increased up to 1 cm. Simulations also show that, for slightly higher SNR, greater than -23 dB, calibration requirements can also be relaxed:

Figure 3.52 WSF performance with position calibration errors, SNR = -23 dB

3.5.5 Non-Stationarity

3.5.5.1 Moving Signal

The presence of a moving source to be acquired renders more complex the whole operation of SARAS system. Indeed, the movement of the target will have three distinct effects on the overall performance:

- The array manifolds will vary with time.
- The SNR will change, due to the change in the radiation patterns of the individual antennas. Since these have low gain, this effect will be negligible.

- There will be additional bias and variance in WSF estimation, since the DoA needs to be compared with the satellite position at the end of the block of samples used. This is true as long as the angular speed of the satellite is not reliably known.

The CRLB is calculated as in section 2.4.1.1, splitting the signal into blocks where the DoA is approximately constant. For the first block, the FIM $\mathbf{J}$ is computed as usual, using the number of samples in the block. FIM in successive blocks are estimated with the following iterative formula:

$$\mathbf{J}(m) = \mathbf{J}(m-1) + \frac{\mathbf{D}^H \mathbf{D}}{\sigma_n^2(m)} \quad (3.61)$$

$\mathbf{D}$ is the matrix with the derivatives of the array manifold with respect to the angular coordinates. The cumulative CRLB for the m -th block is computed as:

$$\mathbf{C}_{CR}(m) = \left(\sum_{i=1}^m \mathbf{J}(i) \right)^{-1} \quad (3.62)$$

The CRLB must be increased to take into account the additional variance of the results produced by WSF, which are assumed to be uniformly distributed in the angular range where the satellite is moving. For instance, if we consider the elevation coordinate:

$$\sigma_{el}^2(m) = \mathbf{C}_{11}(m) + \frac{(\Delta el_F - \Delta el_M)^2}{3} \quad (3.63)$$

Δel_F is the final elevation position for the satellite in the m -th block, and Δel_M is the mean position. $(\Delta el_F - \Delta el_M)$ is, of course, the added bias.

Non-stationary simulations have been carried out with the following parameters:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	Several values
Simulation time	530 ms
Angular excursion (per acquisition)	0.25°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-22 dB
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	K

Table 3.14 Parameters for non-stationary simulation

An SNR slightly higher than the threshold has been considered to limit the number of required Monte Carlo iterations. The angular excursion corresponds to a satellite angular speed of 0.4688°/s, which in turns, taking into account the sampling rate, corresponds to an angular change of 2.5 µdeg/sample. Results are plotted below, for each angular variable.

Figure 3.53 WSF non-stationary performance. (a), (c), (e) Elevation coordinate. (b), (d), (f) Cross-elevation coordinate. (a)-(b) Standard deviation. (c)-(d) Bias. (e)-(f) 3.3 σ RMSE

The variance component due to the uniform distribution of estimates increases with the number of samples, causing the U-shape of the curves, and setting an optimum value for K , which is in line with the number of samples required for reliable results at the considered SNR. Bias on the other hand is much reduced, and with a zero-mean.

On the other hand, RMSE shows that, in non-stationary scenarios, the number of processed samples must not be too large to limit the effect of angular spread. Simulations have been carried out for angular spreads of 0.375° and 0.533° :

Figure 3.54 WSF non-stationary performance for elevation coordinate. (a) Angular spread of 0.375° . (b) 0.533°

It is clear that beyond 0.533° , the RMSE is always above the required threshold. This value can be set as an approximate limit to the satellite movement. This limit corresponds to a speed of $5.33 \mu\text{deg/sample}$.

This is, however, a stringent restriction. Indeed, as has been discussed in section 3.5.1, the maximum error requirement applies to a combination of elevation and cross-elevation errors, computed as:

$$\varepsilon = \sqrt{(\Delta \hat{el} - \Delta el_F)^2 + (\Delta \hat{xel} - \Delta xel_F)^2} \quad (3.64)$$

As can be appreciated with the following simulations, the non-stationary limit can be increased from the previous value:

Figure 3.55 WSF non-stationary performance for total error. (a) Angular spread of 1° . (b) 1.25°

Angular spreads of 1° achieve the required performance. The results were obtained for an SNR of -22 dB, i.e., above threshold. Decreasing it to -24 dB, angular spread requirements become more stringent:

Figure 3.56 WSF non-stationary performance for total error, SNR = -24 dB. (a) Angular spread of 0.25° . (b) 0.375°

While for an angular spread of 0.25° the resulting error meets the requirement, the contribution from other degradation sources must also be taken into account, as is done in section 3.6.1. For the total error budget to meet the high level requirement, the angular spread must be decreased to 0.125° . Results for this case are shown below:

Figure 3.57 WSF non-stationary performance with an angular spread 0.125° and SNR = -24 dB. (a) Std. (b) Bias. (c) RMSE

3.5.5.2 Doppler and Frequency Offsets

Frequency processing of carriers has not been considered in the preceding simulations. It is clear that the use of an FFT is advantageous with signals with residual carrier, since the power is not uniformly distributed in the spectrum. It is thus desirable to retain only those frequency bins with signal peaks. The robustness of the algorithms can be tested against C/No values. The relationship between this and the PFD was specified in 3.5.1. Introducing the following parameters:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	Yes
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	Several values
Input SNR	-
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	C/No

Table 3.15 Parameters carrier signals simulation

The following results are obtained:

Figure 3.58 WSF performance with carrier processing. (a) Normal view. (b) Zoom

There is a clear threshold for $C/N_0 = 14$ dBHz. As will be explored in chapter 5, when analysing mission link budgets, this threshold implies higher margins when compared to the SNR one.

The C/N_0 threshold is proportional to the bandwidth used to detect the carrier, as with PLL acquisition. In the preceding simulations, the bandwidth was set to 35 Hz. Next figure shows how the threshold changes with BW:

Figure 3.59 WSF performance with carrier processing as a function of BW

It would seem thus advantageous to reduce BW as much as possible. However, in the preceding simulation there was a simplification: no Doppler shift was included. The presence of uncompensated Doppler shift smears the power distribution of the carrier, and thus decreases the effective CNR (Carrier-to-Noise Ratio) of the signal. CNR, instead of SNR, is considered due to the fact that only the frequency bins around the carrier are used for spatial processing. CNR is related to C/N_0 with the following equation, where K is the number of samples, and n_{FFT} is the number of selected bins.

$$CNR = \frac{C}{N_0} - 10 \log(n_{FFT} f_s / K) \quad (3.65)$$

This is shown in the frequency spectra below, obtained from a carrier signal with a C/No of 35 dBHz and a Doppler shift of 500 Hz, prior to compensation and after it.

Figure 3.60 Frequency spectra of carrier. (a) With Doppler shift. (b) After compensation

If perfect compensation could be achieved, Doppler shift would not be an issue. However, there are always uncertainties in the spacecraft trajectories, which are translated into imperfect knowledge of the Doppler shift. Other sources of error may be induced by the satellite PLL. This has been taken into account in the next simulation, where an error of 10% of the original Doppler shift remains after compensation.

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	Yes
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	Several values
Input SNR	-
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	C/No and Doppler shift

Table 3.16 Parameters for Doppler and frequency offsets simulation

Figure 3.61 WSF performance with carrier processing and Doppler shift. (a) Normal view. (b) Zoom

In order to maintain the lowest C/N_0 threshold of 14 dBHz, the error in the estimation of the Doppler rate must be below 10 Hz/s. Each time the error increases in 30 Hz/s (approximately), the required C/N_0 threshold increases in 2 dB.

3.5.6 Kalman Filtering

Simulations have been carried out in Matlab using the EKF structure explained in section 3.4.3. A real LEO satellite trajectory, acquired from an array ground station located in Madrid, is used. Other parameters used for this specific simulation are listed below:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	18
K (per acquisition)	60000
Simulation time	10 s (for all acquisitions). Time is divided between filtering and prediction stages, with 5 seconds each.
Angular excursion (per acquisition)	0° (per acquisition)
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	0 cm
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	100
Monte Carlo confidence level	95 %
Parametric variable	C/No and Doppler shift

Table 3.17 Parameters for Kalman filter simulation

This set of parameters permits to have 18 WSF measurements during the filtering stage. On the other hand, initial position estimates are supposed to be known with a 10% uncertainty. Initial speed and acceleration are obtained from differentiation. The state transition noise matrix is defined as:

$$Q_1 = \{0.3 \times \text{diag}([p_{xi} \quad v_{xi} \quad a_{xi} \quad p_{yi} \quad v_{yi} \quad a_{yi} \quad p_{zi} \quad v_{zi} \quad a_{zi}])\}^2 \quad (3.66)$$

The square is taken element-wise. The measurement noise matrix is:

$$Q_2 = \begin{bmatrix} \left(0.05 \frac{\pi}{180}\right)^2 & 0 \\ 0 & \left(0.08 \frac{\pi}{180}\right)^2 \end{bmatrix} \quad (3.67)$$

Another approach, slightly more involved will be followed in section 5.3.4.2, though. Namely, Q_1 will be derived from STDM state vectors correlated with delayed versions of themselves, and updated in successive Kalman iterations. On the other hand, a less optimistic version of Q_2 will be considered, including cross-correlation components, albeit small ones. In any case, the current configuration is adequate to ascertain the functionality of the EKF filter at the design stage.

Filtering results are shown presently:

Figure 3.62 Kalman filtering. (a) Azimuth. (b) Elevation

Kalman results closely follow azimuth measurements, due to their high accuracy. Elevation results show a smaller oscillation around the true value. The prediction outputs are:

Figure 3.63 Kalman prediction. (a) Azimuth. (b) Elevation

Prediction implies almost no increase in azimuth error, whereas, as was observed, elevation errors were diminished. Previous simulations were obtained in a low elevation scenario, just when the satellite is over the horizon. Similar results are achieved at higher elevations:

Figure 3.64 Kalman prediction at high elevation. (a) Azimuth. (b) Elevation

3.6 Conclusions

3.6.1 DoA Estimation Error Budget

The performance of the system in diverse scenarios has been analysed. From these an overall error budget can be derived and compared with the high-level requirement of $\pm 0.28^\circ$ in acquisition error (cf. section 3.2). Results are summed up in the next table:

Angular error source	Error std ($^\circ$)	Error bias ($^\circ$)
White noise	0.015	0.016
Closely spaced interferer	0.037	0.095
Out of window interferer	0.030	0.004
Multipath	0.008	0.001
Uncalibrated phase offsets	0.010	0.005
Uncalibrated position errors	0.012	0.008
Non-stationary signal	0.017	0.035

Table 3.18 DoA error budget

In the preceding budget, the white noise error contribution was removed from all subsequent error estimations, as noise was present in all simulations. On the other hand, the variance of the closely spaced interferer has been subtracted from the variance caused by the out of window interferer, as only the extra error contribution

when the interferer is received from arbitrary directions must be taken into account in this case. Finally, the bias term is higher due to the direct search optimisation with a 0.05° resolution, since this process adds a uniform error between -0.025 and 0.025° . Thus, the bias term can be reduced by the following factor:

$$\sigma = \frac{0.05}{\sqrt{12}} \quad (3.68)$$

Adding the contributions in Table 3.18, the following error statistics are obtained:

Std ($^\circ$)	0.056
3.3-std ($^\circ$)	0.184
Bias ($^\circ$)	0.164
Total error ($^\circ$)	0.246

Table 3.19 DoA error statistics

It is clear that the initial requirement is met, at least from a simulation perspective, with 99.9% confidence level. The margin that is left will be required to take into account additional error contributions from a hardware perspective, as will be analysed in chapter 5.

3.6.2 Final FoM

In the previous error budget, a number of assumptions were implied, which can be interpreted as the FoM of the simulated system. These are summarised in the next table:

FoM	Simulated value
Threshold SNR	-24 dB (without interferers) -22 dB (with interferers)
Threshold C/No	14 dBHz (for Doppler rates below 10 Hz/s, and carrier detection BW < 500 Hz)
Acquisition range	< +/- 0.9°
Multipath power offset	< -18 dB
Phase calibration error	< 18°
Position calibration error	< 2 mm @ SNR = -24 dB < 6 mm @ SNR = -23 dB
Moving signal angular spread	< 0.125° @ SNR = -24 dB < 1° @ SNR = -22 dB

Table 3.20 SARAS simulation FoM

While the initially required acquisition range figure of $\pm 1^\circ$ is not met, the simulated FoM gets very close. Furthermore, it has also been suggested that a non-uniform topology can improve the range beyond the original requirement (cf. section 3.5.2.5).

These FoM will be compared with real figures in chapter 5, section 5.4. Regarding calibration error requirements, they will be further analysed in the next chapter, which is precisely devoted to array calibration.

CHAPTER 4 CALIBRATION

4.1 Introduction

Chapter 3 has been concerned with the actual estimation of the DoA of the desired signal, which is the main purpose of SARAS system. However it has become apparent, with some of the Monte Carlo simulations, that this process relies on assumptions regarding the phase response model of the array, and that errors in such modelling lead to quick degradation of performance. The case of position errors with respect to the phase centres of the array is especially dramatic (cf. section 3.5.4.3), and must be treated in detail.

Chapter 4 will present a systemic solution to this conundrum. Calibration is one of the most delicate topics in array theory and, as such, a great portion of scientific literature has been devoted to it. Chapter 2 already introduced some statistical calibration algorithms, divided into parametric and non-parametric sub-types. This chapter will expose a solution beyond those, integrating a variety of techniques to obtain the final model of the array response.

4.1.1 State of the Art

Beyond the exposition in chapter 2, two additional topics are explored in this section as the starting point for the design of the calibration subsystem.

4.1.1.1 Photogrammetry

It is clear that the estimation of the relative positions of the phase centres in the array will be one of the critical aspects of the calibration system. Photogrammetry was chosen in the early design stages to contribute to this modelling process. It is a technique which extracts 3-dimensional shapes from a set of 2-dimensional pictures, taken with different orientations. High accuracy can be achieved, even with off-the-shelf cameras.

Photogrammetry is part of an old field of study interested in obtaining distance measures from images of 2-D cameras. Among its numerous applications, one can find such varied ones as cartography, crime scene analysis and body motion. Its principles heavily rely on complex image processing concepts, and benefit from the advent of digital signal systems. Efficient and very precise algorithms can be designed and implemented as a result.

First and foremost, though, the involved cameras, which are to take the pictures, must be calibrated. Position determination, also known as triangulation, cannot be done accurately without knowing their internal physical properties, such as effective focal length, lens distortion characteristics, etc. (Lin 2001). A vast variety of algorithms is available in the literature, but most of them are derived from the one described by Tsai in his paper (Tsai 1987). Commercial software can obtain these

parameters from a set of pictures of a previously calibrated pattern, such as white and black stripped panels or mosaics:

Figure 4.1 Typical camera calibration patterns (a) See (Lin 2001). (b) See (Pappa, Giersch, and Quagliaroli 2001)

The actual processing of the data to produce position estimates includes the determination of the geometrical centre of the targets with sub-pixel interpolation, referencing and 2D to 3D coordinate transform. Algorithms are very complex, and require extensive knowledge of matrix and image processing. Some ideas can be found in (Svoboda 2003). Also, it should be noted that similar systems as the one intended can be found in (Pappa, Giersch, and Quagliaroli 2001) and (Shankar et al. 2009). However, in those references the aim is to determine the whole shape of the antenna or reflector dish. A photogrammetric calibration of a phased array is performed in (McWatters et al. 2004).

4.1.1.2 On-line Calibration

As will become apparent in the sections to come, it is essential to perform on-line calibration, i.e., to obtain an improved calibration model at the same time of DoA estimation, with frequency-multiplexed calibration and received signals. The simultaneous reception of a calibration beacon is similarly performed in radioastronomy, in a technique known as SBI (Same Beam Interferometry) (Thornton and Border 2003) (Thompson, Moran, and Swenson 2001).

Indeed, as stated in (Bagri 2009) and (Majid and Bagri 2008), the use of antenna arrays for signal receiving, instead of large dishes, permits a reduction in the aperture's sizes of the antennas, making it possible to have simultaneous observation of the spacecraft and the calibration source and an improvement in the resulting accuracy.

4.1.2 Contributions

The contributions of this chapter are

- The proposal of a global calibration scheme for the estimation of the array manifold matrix, which is mandatory for the proper performance of the

system presented in chapter 3. The scheme is a combination of algorithms described in chapter 2 with other techniques, to produce a versatile modelling of the array response, and can be applied to a plurality of array systems with different topologies, operating frequencies, etc.

- The characterisation of part of the digital processing of the global calibration scheme, based on Monte Carlo simulations.
- Some initial considerations regarding practical hardware implementation, which will be completed in chapter 5.

4.2 Systemic Analysis Revisited

The structure of the Calibrator subsystem was already explained in section 3.3.1.4. The figure is also included here for clarity's sake:

Figure 4.2 Calibrator architecture (bis)

Calibration processes can be divided into two categories:

- Those that are performed once, or at long intervals, which seek to determine relatively stable parameters of the array model. The information is thus computed and stored for later use. These techniques include:
 - Antenna calibration, which obtains the amplitude and phase response of each antenna using an anechoic chamber.

- Component calibration, which obtains the amplitude and phase response of analogue components, other than the antennas, typically using Vector Analysers.
 - Geometrical calibration, which obtains an initial estimate of the geometrical positions of the antennas, using photogrammetry.
- Dynamic techniques, including test signal injection through couplers, or through the antennas using external probes, calibration towers, or beacon signals from satellites. These techniques are further divided into:
 - Off-line calibration techniques: those that are performed prior to DoA estimation.
 - On-line calibration techniques: those that are coupled with the actual DoA estimation.

The next sections will firstly present a preliminary approach to calibration, which was the initial design conceived for SARAS. Afterwards, an improved calibration scheme that was derived to cope with the instabilities in the array model parameters will be introduced. This is the scheme that was implemented in SARAS prototype to produce reliable results with real satellites.

4.3 Initial Parametric Calibration Scheme

4.3.1 Photogrammetric Calibration

Using photogrammetric techniques, and with the help of a set of retro-reflective targets, the 3-dimensional geometrical shape of the main dish and the array sensors can be extracted from a set of 2-dimensional pictures, thus allowing estimating groups of coordinates and, in the end, calculate a coordinate system which relates the array elements to the steering of the main antenna.

The basic components used in photogrammetry are:

- Retro-reflective targets, the relative distance of which the geometrical calibrator will determine. These targets are placed in the antennas.
- A distance measurement hardware and software. The distance measurement, for this topology, has been performed with the help of a crane, as will be commented in chapter 5.
- A distance reference. Targets are also placed in it. This reference must be robust against changes due to environmental conditions such as temperature.
- A reference coordinate system, in three dimensions. In principle, this coordinate system can be arbitrarily chosen. However, the difference between the coordinate systems from SARAS and the main antenna must be compensated. For this reason it may be advisable to define the coordinate system of SARAS according to that of the main antenna, by placing additional targets if required. Indeed, an additional advantage is

that the coordinate system of the main antenna is usually determined in a more precise way. Finally, it is important to notice that the origin of the coordinate system can be arbitrarily chosen since the DoA information is obtained from relative, and not absolute, distances.

In the photogrammetric process, there are several factors that determine the accuracy of a photogrammetric project and help with the choice of the equipment and required procedure. The main factors are listed below:

- Picture resolution: the higher the resolution of the camera sensor, the better the obtained accuracy, since the shapes of the targets are sharper.
- Camera calibration: in this process, the focal length, format size, principal point and lens distortions are determined and further neutralised. This is one of the main steps to achieve high quality results.
- Angles between pictures: the number of angles at which the same point is photographed determines the accuracy of the results. Photographs with angles close to 90° have much higher accuracy than other angles.
- Picture orientation quality: during bundle adjustment, the location and angle of each camera is computed. The final accuracy increases as the number of points is well positioned and the photo orientation quality increases.
- Picture redundancy: targets are more accurately located when they appear in a large set of different photographs.
- Points: points are better located when their shape is known by the photogrammetric software and its location is easy to spot by improving its brightness against the background.

Array sensors and the main antenna should be aligned with the same broadside steering, but due to multiple causes there may be a slight difference between both coordinate systems. Main factors affecting this misalignment are:

- Mechanical errors when placing the sensors around the main antenna.
- Displacements of the antennas due to changes in the elevation of the main antenna.

This ensuing misalignment is conceptually shown below.

Figure 4.3 Angular error between dish and array planes

Thus, if the array elements are to share the same coordinate system as the main antenna, a dedicated procedure must be implemented, as part of the geometrical calibration, to derive an adequate coordinate system. The flowchart below shows a configuration of this procedure, although variations can easily be considered, as long as the result is an orthonormal coordinate system.

Figure 4.4 Coordinate system alignment procedure

The targets used by the distance measurement software have been placed in several concentric circumferences on the main antenna dish, approximately following its perimeter. A Central Point Estimation is performed for each circumference, taking the average of the Cartesian coordinates of all targets involved, as given by the distance measurement software. After this, a Normal Vectors Estimation is carried out, which determines the normal vectors to the planes defined by the aforementioned circumferences. The planes are a Least Squares approximation using the Cartesian coordinates of the involved targets. With more than one circumference, averages of the normal vectors and of the central points are taken.

The Azimuth Axis Estimation takes the Cartesian coordinates of targets that must be placed in a curve defined by the gradient of the main dish, starting on the central part and ending on the lowest one. Here, lowest must be understood as the part of the dish with the least absolute height, when the main antenna is steered towards 0° of elevation. The azimuth axis is defined as the line which is a Least Squares approximation of the Cartesian coordinates of the targets involved.

The average of the central points will be the origin of the new coordinate system, the average of the normal vectors the z-axis, and the projection of the azimuth axis on a plane obtained from a given circumference will be the x-axis. The y-axis can be obtained directly from this information. The axes are subsequently normalised using a suitably placed distance reference. All this is performed in the Coordinate System Determination stage.

Beyond these considerations, SARAS antennas themselves are subject to perturbations in their positions. Some of these will be stochastic in nature, such as those caused by temperature gradients or wind deformations, and will be ignored or compensated using statistical calibration algorithms. Other perturbations, though, will be deterministic. The main ones will be those provoked by gravitational deformations of the reflector's rim, as a function of elevation. In this case, photogrammetry must be performed for different elevation steerings of the main antenna, and the positions of the array antennas must be stored in all cases. The appropriate set of positions will be used when performing further calibration of the array or DoA estimation, depending on the elevation of the main antenna, and interpolating them if so required.

The estimated points in the model will be geometrical references, whereas in truth it is the phase centres of the antennas that are of interest. In principle, the difference between geometrical references and phase centres will be the same for each antenna, if the targets are placed in identical places. For this to be true, though, the antennas need to be of the same type. Since this may not be the case, it is interesting to complement photogrammetry with anechoic chamber measurements (cf. next section).

As a summary, the following error budget can be derived for the photogrammetric position estimation technique:

Position error source	Error std (mm)
Measurement	5
Elevation-dependent gravitational deformations	1
Stochastic deformations	1
Retro-reflector placement error	2
TOTAL STD	5.6

Tabla 4.1 Position error budget

Regarding the error sources:

- Measurement error is the one specified by the photogrammetric software.
- Gravitational deformation is an estimation of the residual error, due to interpolation between different elevation steerings.
- Stochastic deformations mainly include wind and temperature gradients. Their contribution is typically one order of magnitude below gravitational errors, but this last value is taken as an upper limit.
- Placement error refers to geometrical displacement of the retro-reflectors targets in the array antennas, due to the fact that it is a manual procedure.

The final standard deviation makes it mandatory to consider SNR above -23 dB. There is, however, one caveat, concerning the effect of errors on different spatial coordinates, further developed in section 4.5.5.

4.3.2 Anechoic Chamber Calibration

As commented, the aim of this calibration is to obtain a radiation pattern model of each antenna, in phase and amplitude, as a function of the steering direction, which can be expressed in spherical coordinates θ and ϕ . The number of steering points and the range to be considered will depend on the operation frequency and the required accuracy of the model. Different models should be obtained for all polarisations and frequency schemes to be considered. The number of frequency points will also depend on the required accuracy. The obtained radiation patterns can be expressed as relative to a reference steering, typically broadside, since only relative information is of interest.

The use of an Anechoic Chamber mitigates the effect of reflections. Also, in principle, each antenna is measured separately. If possible, it would be interesting to calibrate them simultaneously in their final geometrical layout, since this might preclude the use of the photogrammetric calibration. However, such a scenario is not possible for highly distributed arrays, as is the case of SARAS and, besides, mechanical deformations could affect the validity of the anechoic chamber model.

The obtained information can be processed in two alternative ways. On one hand, a phase centre model of each antenna can be obtained, relative to the reference one. This model will meet the following equation:

$$\Delta p_n = g_{pol,f}(\theta, \varphi) \quad (4.1)$$

Δp_n is the displacement of one of the n -th antenna coordinates (x , y or z), as a function of the spherical coordinates. Said function g has as configuring parameters the signal's polarisation " pol " and its frequency " f ". Other coordinates, such as elevation and cross-elevation offsets, can be considered in the preceding equation. Also, Δp_n will be 0 for broadside direction.

This information can be combined with the one obtained in the photogrammetric calibration, in order to transform the geometrical positions into phase centres. Indeed, when building the cost function in the DoA estimator, the set of array positions to be used for a given array steering θ_0 and φ_0 is:

$$\mathbf{p}_T = \mathbf{p} + \Delta \mathbf{p}(\theta_0, \varphi_0) \quad (4.2)$$

It is a matrix equation, with $N \times 3$ real matrices, where N is the number of antennas. $\mathbf{p}$ is the set of positions obtained with photogrammetry, and $\Delta \mathbf{p}$ those deduced from the anechoic chamber measurements, resorting to interpolation for the specific values θ_0, φ_0 if so required.

4.3.3 Component Calibration and Test Signal Injection

4.3.3.1 Injection through RF Couplers

The phase centres model of the previous section is not mandatory. Indeed, the amplitude and phase responses from the antenna patterns can be combined with the S-parameter calibration. This calibration seeks to obtain the amplitude $\Delta \mathbf{a}$ and phase offsets $\Delta \boldsymbol{\phi}$ among array branches. The array analogue components can be calibrated separately or partially assembled with a Vector Analyser, for example. Total phase and amplitude offsets, which are $N \times 1$ vectors, are computed as follows:

$$\Delta \mathbf{a} = \Delta \mathbf{a}_1 \times \Delta \mathbf{a}_2 \times \dots \times \Delta \mathbf{a}_m \quad (4.3)$$

$$\Delta \boldsymbol{\phi} = \Delta \boldsymbol{\phi}_1 + \Delta \boldsymbol{\phi}_2 + \dots + \Delta \boldsymbol{\phi}_m \quad (4.4)$$

m components and/or assemblies are considered. Also, vector and matrix operations are carried out element-wise. These offsets are unique, but do not include the contribution from the antennas. It can be added directly with the results of the radiation patterns, with the peculiarity that these will be a function of steering directions and with frequency and polarisation as configuring parameters (not shown in the equations for simplicity's sake):

$$\Delta \mathbf{a}(\theta, \varphi) = \Delta \mathbf{a}_0(\theta, \varphi) \times \Delta \mathbf{a}_1 \times \Delta \mathbf{a}_2 \times \dots \times \Delta \mathbf{a}_m \quad (4.5)$$

$$\Delta \boldsymbol{\phi}(\theta, \varphi) = \Delta \boldsymbol{\phi}_0(\theta, \varphi) + \Delta \boldsymbol{\phi}_1 + \Delta \boldsymbol{\phi}_2 + \dots + \Delta \boldsymbol{\phi}_m \quad (4.6)$$

These offsets can be taken as relative to a reference antenna. Their main disadvantage is that they do not take into account dynamical effects which greatly affect analogue components, such as ageing and temperature drift.

One way to compensate this is to combine the preceding calibration with an updated model, using a test signal injection calibration through a coupler in each array branch. Even if off-line, this injection can be performed shortly before DoA estimation, with the adequate LO frequency.

From the obtained phase and amplitude offsets, though, the contribution of non-desired components must be eliminated; mainly the contribution of the test cables and the transition of the couplers, which are not the same that are followed by the desired signal in the array branches. Thus the equations become:

$$\Delta \mathbf{a}_{desired} = \frac{\Delta \mathbf{a}_{test}}{\Delta \mathbf{a}_{cables} \times \Delta \mathbf{a}_{coupler1}} \quad (4.7)$$

$$\Delta \phi_{desired} = \Delta \phi_{test} - \Delta \phi_{cables} - \Delta \phi_{coupler1} \quad (4.8)$$

The offsets due to the antennas will be added to these results, if no phase centres model is implemented, along with the ones due to the components prior to the couplers, and those of the transition of the couplers. So that:

$$\Delta \mathbf{a}_T = \Delta \mathbf{a}_0(\theta, \varphi) \times \Delta \mathbf{a}_1 \times \Delta \mathbf{a}_{coupler2} \times \Delta \mathbf{a}_{desired} \quad (4.9)$$

$$\Delta \phi_T = \Delta \phi_0(\theta, \varphi) + \Delta \phi_1 + \Delta \phi_{coupler2} + \Delta \phi_{desired} \quad (4.10)$$

Test injection can be performed with other means beyond the use of RF couplers. Three additional systems were considered for SARAS.

4.3.3.2 Injection through a Collimation Tower

The use of an existing collimation tower (cf. section 5.2.4) proved to be ineffective because, due to the low elevation of the tower (4°), shadowing and multipath degradation was observed in some of the antennas, especially those placed in the lower part of the Vil-1 rim. The next figure shows the frequency spectra of the received signals during one of the tests.

Figure 4.5 Frequency spectra of array signals from collimation tower

As plotted in the figure, the frequency peak of the signal from antenna 3, which is the lowest one, is 25 dB below the highest one, due to destructive interference from multipath and shadowing effects.

4.3.3.3 Injection through a Sub-Reflector Probe

In order to overcome the shortcomings of couplers, which do not permit a dynamic calibration of antenna patterns, the use of a probe placed in the sub-reflector of the main antenna was also considered. This method also eliminates the spurious contribution from test cables and couplers, but in turn adds other non-desired error offsets, namely:

- The near-field perturbation of the received signal, since the probe is near the receiving antennas, for the wavelength under consideration.
- The radiation patterns of the antennas, which see the received signal from different directions which do not coincide with the broadside one.
- The different path lengths from the probe to each receiving antenna.

These contributions cannot be obviated, although this calibration method has proven to obtain relative array models robust against changes in the RF frequency. This characteristic, and the absence of multipath, has been critical in the final calibration implementation, as discussed in section 4.4.

4.3.3.4 Injection using the Beacon of a GEO Satellite

In terms of spurious contributions, this is undoubtedly the best method to obtain accurate estimates of the true array phase and amplitude offsets, since the test signal follows the same path as the acquisition one, and the far-field condition holds. The main disadvantages of this calibration scheme are:

- The calibration is limited to available GEO satellites, and to a set of RF frequencies which do not cover the whole S band.
- Accurate estimates require accurate ephemerides of the satellites. Additionally, it is preferable that the satellites are almost motionless, at least during the estimation phase, to facilitate the operation.
- The SNR is lower than with the previous schemes, so that more samples need to be processed. Namely, previous test signal injections require around 100000 samples, whereas this procedure requires from 1 million to 10 million samples.

These drawbacks can be overcome nonetheless, although the limitation in frequency has an impact on the final calibration implementation, as seen in section 4.4. GEO satellites which were deemed adequate for this calibration were Spainsat and XTAR, both with S-band beacons.

The obtained phase offsets from a GEO satellite must be corrected to eliminate the spurious contribution due to the different z-coordinates of the array antennas. The following equation applies:

$$\Delta\phi_T = \Delta\phi_{\text{GEO}} - \frac{2\pi}{\lambda} \mathbf{z} \quad (4.11)$$

λ is the wavelength of the GEO satellite signal, and $\mathbf{z}$ the z-coordinate vector of the array elements. The preceding equation is true for signals received from broadside. If a different known DoA is involved, the correction must bear in mind the complete array manifold formula.

4.3.4 Array Model Generation

The positions, amplitude and phase offsets are combined to obtain an a priori parametric model. This model follows the equation below, which defines the array SV, already introduced in chapter 2:

$$\mathbf{v}_T = \Delta\mathbf{a}_T \exp \left[j \frac{2\pi}{\lambda} \mathbf{p}_T \mathbf{u} + j \Delta\phi_T \right] \quad (4.12)$$

This model can be improved with additional statistical calibrations, which process the received digital signal samples. This stage may be mandatory if no test signal injection is carried out, in order to compensate dynamical errors. Also, in some scenarios, such as high operating frequencies, the position estimation will have very stringent requirements. Thus, improved modelling may be critical. Statistical parametric algorithms were already discussed in section 2.5.2. Least Squares will be reviewed in this section.

The processing starts with an estimation of the $N \times 1$ signal subspace vector $\mathbf{v}_S$, as in DoA estimation, for different steering directions, so that the resulting matrix $\mathbf{V}_{\text{err}}$, of size $M \times N$, where M is the number of measurements, is defined as:

$$\mathbf{V}_{err} = \begin{bmatrix} \mathbf{v}_{S1}^T / \mathbf{v}_{T1}^T \\ \mathbf{v}_{S2}^T / \mathbf{v}_{T2}^T \\ \vdots \\ \mathbf{v}_{SM}^T / \mathbf{v}_{TM}^T \end{bmatrix} \quad (4.13)$$

In the equation $\mathbf{v}_{Tm}$ is the a priori calibration model for the steering corresponding to the m-th measurement. The vector division is performed element-wise. In this way $\mathbf{V}_{err}$ has only the effects of modelling errors from the a priori parametric model.

The residual amplitude offsets $\Delta \mathbf{a}_{err}$ are computed taking the mean of the absolute value of $\mathbf{V}_{err}$ for all steerings. Regarding positions, defined as x, y and z coordinates, and phase offsets, the Least Squares equation is then computed as:

$$\underbrace{\begin{pmatrix} \frac{2\pi}{\lambda} \times \sin \theta_1 \cos \varphi_1 & \frac{2\pi}{\lambda} \times \sin \theta_1 \sin \varphi_1 & \frac{2\pi}{\lambda} \times \cos \theta_1 & 1 \\ \frac{2\pi}{\lambda} \times \sin \theta_2 \cos \varphi_2 & \frac{2\pi}{\lambda} \times \sin \theta_2 \sin \varphi_2 & \frac{2\pi}{\lambda} \times \cos \theta_2 & 1 \\ \frac{2\pi}{\lambda} \times \sin \theta_3 \cos \varphi_3 & \frac{2\pi}{\lambda} \times \sin \theta_3 \sin \varphi_3 & \frac{2\pi}{\lambda} \times \cos \theta_3 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{2\pi}{\lambda} \times \sin \theta_M \cos \varphi_M & \frac{2\pi}{\lambda} \times \sin \theta_M \sin \varphi_M & \frac{2\pi}{\lambda} \times \cos \theta_M & 1 \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} x_1 & x_2 & \dots & x_N \\ y_1 & y_2 & \dots & y_N \\ z_1 & z_2 & \dots & z_N \\ \varphi_1 & \varphi_2 & \dots & \varphi_N \end{pmatrix}}_{\mathbf{X}} = \frac{\angle \mathbf{V}_{err}}{\mathbf{B}} \quad (4.14)$$

Matrix $\mathbf{X}$ includes the unknowns to be estimated to form the residual terms $\mathbf{p}_{err}$ and $\Delta \phi_{err}$. Finally, the improved calibration model is calculated using the following equations:

$$\Delta \mathbf{a}_{improved} = \Delta \mathbf{a}_T \times \Delta \mathbf{a}_{err} \quad (4.15)$$

$$\Delta \phi_{improved} = \Delta \phi_T + \Delta \phi_{err} \quad (4.16)$$

$$\mathbf{p}_{improved} = \mathbf{p}_T + \mathbf{p}_{err} \quad (4.17)$$

Since matrix $\mathbf{B}$ contains angular information, there is a modulo 2π uncertainty, so that residual phase and amplitude errors in $\mathbf{V}_{err}$ must be small enough to guarantee that said uncertainty does not affect the calibration model or, conversely, the a priori calibration model must be good enough to avoid such uncertainties. This restriction can be somewhat relaxed if position and phase errors are calculated separately. Indeed, the column of $\mathbf{A}$ corresponding to the phase offsets has identical values, so that position can be firstly calibrated using differential measures $d\mathbf{V}_{err}$:

$$d\mathbf{V}_{err} = \begin{bmatrix} \frac{\mathbf{v}_{err2}}{\mathbf{v}_{err1}} \\ \frac{\mathbf{v}_{err3}}{\mathbf{v}_{err2}} \\ \vdots \\ \frac{\mathbf{v}_{errM}}{\mathbf{v}_{errM-1}} \end{bmatrix} \quad (4.18)$$

v_{errm} is the m -th row of V_{err} , and vector division is performed element-wise. Phase offsets can be obtained afterwards taking advantage of the updated position estimation.

4.4 SARAS Calibration Scheme

4.4.1 On-line Calibration

All the previous methods perform the array calibration before the acquisition of the desired satellite. Thus, the obtained calibration model is different from the one which applies to the acquisition scenario, mainly due to dynamic changes in the analogue components of the array. At first, it was thought that these changes were small enough to be obviated, especially if acquisition was performed some minutes or seconds after calibration. Test campaigns showed, though, that this was not true, and that the calibration model degraded quickly, as shown below:

Figure 4.6 Integral WSF spectrum obtained with off-line calibration

The plot shows the WSF spatial spectrum of Integral satellite, using an off-line calibration model which was estimated one minute prior to the acquisition (cf. section 5.3 for more details on the tests with real satellites that were carried out with SARAS prototype). The highest peak corresponds to the desired DoA of the satellite, although the presence of a spurious peak can already be appreciated, even after only one minute lapse.

The observed instabilities are mainly provoked by the RF/IF mixers, which have numerous amplifiers, and by the long downlink RF cables (cf. chapter 5 for more hardware details), and have motivated the implementation of an on-line calibration scheme that can overcome them. The idea is to inject a test signal as the actual acquisition is carried out, so that environmental conditions are the same. In order to

avoid interferences, the calibration signal is frequency-multiplexed with the one from satellite.

Figure 4.7 Frequency spectrum of array signals from Cryosat and Vil-1 sub-reflector probe

In the figure, the calibration signal is injected with the sub-reflector probe, as commented before, and has a 100 KHz frequency offset with respect to the desired Cryosat signal. The calibration + acquisition procedure is straightforward, and follows the steps below:

- An FFT of the incoming samples is performed, and the frequency bins around the highest peak are selected. These correspond to the calibration signal, since it has more power than the signal from the satellite.
- The amplitude and phase offsets of the calibration signal are estimated using the Eigen algorithm.
- The calibration signal is filtered out from the incoming samples.
- The filtered samples are processed as usual, using the WSF algorithm and the estimated phase and amplitude offsets from the preceding steps.

The filtered spectrum is shown below, with only the signal from Cryosat visible:

Figure 4.8 Frequency spectrum of array signals from Cryosat, with filtered probe signal

On-line calibration clearly overcomes dynamic degradation, although it can only be performed with test signals from the sub-reflector or injected via couplers. Thus a problem remains, i.e., the compensation of spurious components which are not part of the true array manifold. In principle, S-parameter measurements can be used for the coupler injection technique, but this is no longer true for on-line calibration with the probe. And since this last solution has proven to be the most reliable one, an additional step is mandatory to eliminate near-field and other distortions, and will be analysed in the next section.

4.4.2 Off-line + On-line Calibration

The idea is to combine two calibration stages, off- and on-line, so as to guarantee robustness against dynamic effects and eliminate spurious contributions inherent to the sub-reflector probe injection method.

4.4.2.1 Off-line Substage

This calibration substage is charged with the estimation of spurious components. In order to do this it performs a calibration with frequency multiplexed signals, one coming from a GEO satellite, and the other one from the sub-reflector probe. The frequency multiplexed signals follow the same logic as in section 4.4.1.

This calibration only seeks to model differences between the propagation paths of the auxiliary calibration signals (proxy model) and the ones from a satellite signal coming from broadside (true model). These differences are computed as follows:

$$\Delta \mathbf{a}_{\text{delta}} = \frac{\Delta \mathbf{a}_{\text{GEO}}}{\Delta \mathbf{a}_{\text{probe}}} \quad (4.19)$$

$$\Delta\phi_{\text{delta}} = \Delta\phi_{\text{GEO}} - \frac{2\pi}{\lambda}\mathbf{z} - \Delta\phi_{\text{probe}} \quad (4.20)$$

Element-wise division is performed in the first equation. The *delta* components are stored for later use, with the assumption that they include the spurious contribution of the on-line calibration model, and that this contribution is stable over time, or at least for the time elapsed between off-line calibration and acquisition. The phase and amplitude offsets are obtained using the Eigen algorithm with each signal in turn; filtering out the first one once it has been processed, as in section 4.4.1. For this processing to be viable, carrier signals must be considered.

In principle, the proxy model can be obtained either with test injection via couplers or with the sub-reflector probe. However, since the estimation in this substage is limited to the frequency of the GEO beacon signal, frequency-dependent perturbations will not be calibrated. In this regard, test campaigns have shown that the sub-reflector probe is robust against changes in frequency. Indeed, satellites have been correctly acquired with frequencies ranging from 2201 to 2260 MHz, using a GEO satellite centred at 2232 MHz, as discussed in chapter 5. This was not the case when using injection via couplers, so in this situation a full frequency-dependent model is still pending. These considerations justify the fact that only the sub-reflector probe was used in the final calibration implementation.

4.4.2.2 On-line Substage

This substage proceeds as in section 4.4.1, using as an auxiliary calibration signal the sub-reflector probe, as discussed in the preceding subsection. The frequency offset between satellite and calibration signals should be consistent with the one used in the off-line calibration. Also, if *delta* values from the off-line substage are more than one day old, a new off-line calibration is recommended, prior to the acquisition.

4.4.2.3 Combination Substage

This substage gives the final amplitude and phase offsets estimates to the WSF algorithm. These are obtained with the following equations:

$$\Delta\mathbf{a}_{\text{final}} = \Delta\mathbf{a}_{\text{delta}} \times \Delta\mathbf{a}_{\text{on-line}} = \frac{\Delta\mathbf{a}_{\text{GEO}}}{\Delta\mathbf{a}_{\text{probe}}} \times \Delta\mathbf{a}_{\text{on-line}} \quad (4.21)$$

$$\Delta\phi_{\text{final}} = \Delta\phi_{\text{delta}} + \Delta\phi_{\text{on-line}} = \Delta\phi_{\text{GEO}} - \frac{2\pi}{\lambda}\mathbf{z} - \Delta\phi_{\text{probe}} + \Delta\phi_{\text{on-line}} \quad (4.22)$$

The *on-line* estimates are the ones obtained in the on-line substage. Ideally, *on-line* and *probe* components should be equal, if no dynamic errors were present. As it is, the calibration procedure is able to take them into account.

Thus the system uses a combination of the array model obtained in the two preceding calibration stages. From the first substage, it obtains the amplitude and phase compensation values so that the final calibration model adapts its response to

the RF pathway of a satellite signal. From the second substage it obtains the actual amplitude and phase offsets, so that drifts in time are included in the model.

4.4.3 Calibration Scheme Summary

The final calibration scheme is summarised in the flowchart below:

Figure 4.9 Calibration procedure in SARAS

Photogrammetric calibration provides a set of Cartesian coordinates for different elevations, which are improved with interpolation, taking into account the elevation where the main antenna is steered during acquisition. This information is given by the ACU of the main antenna. The coordinates are part of the array manifold model used in the WSF spatial scan, and permit to correct the off-line calibration offsets obtained from the GEO satellite, as shown in equation (4.20).

It is important to mention that, although the calibration scheme gives as output both amplitude and phase offsets, only the last ones are used. Indeed, the information of the signal's DoA is stored in the phase component of the array manifold. Amplitude components have been maintained, though, for future implementations of WSF, namely, for multipath resilient ones.

Finally, it must be pointed out that no LS calibration was used (cf. section 4.3.4). LS calibration can be carried out with GEO or LEO satellites, or with a collimation tower. GEO satellites are not abundant enough, though, to obtain a large number of independent steerings so as to avoid the effects of multicollinearity. LEO satellites, on the other side, have low SNR. The best option would be a collimation tower. However, as already discussed, multipath degradation is significant and no reliable estimates can

be obtained in this case. A future implementation of SARAS system may rely on a multipath resilient LS calibration. Indeed, this calibration can improve the overall array manifold, which is mandatory for higher frequencies.

4.4.4 Calibration ConOps

SARAS calibration is modelled with the following ConOps:

Figure 4.10 Calibration mode ConOps

The preceding ConOps does not take into account the initialisation of the external systems, i.e., ephemerides repository, main antenna and its sub-reflector probe, which are considered as independent. Additionally, photogrammetry is kept outside the normal operation, since it is only performed once, or within large time intervals. On the other hand, on-line calibration, as expected, is part of the Acquisition Mode, defined in chapter 3. Finally, the configuration parameters mentioned in the figure include the processing bandwidth and calibration time lapse.

4.5 Monte Carlo Simulations

In the preceding discussions some of the points have been illustrated with real tests results, rather than simulations as was done in chapter 3, even though test campaigns will be formally introduced in chapter 5. The reason for this is that, as commented, the issues with the first calibration scheme were discovered during test campaigns and not during the design phase, and the new calibration method was directly tested with the actual prototype. In this section, though, some of the properties and FoM of this method will be analysed through simulation, to include quantitative information about its performance.

The performance will be limited by the accuracy and precision of the phase offsets estimates since, as already discussed, they contain the complete DoA information required by WSF. Besides, the performance of WSF as a function of phase errors has already been analysed in section 3.5.4.1, so it needs not be repeated here. The following aspects will be analysed:

- The effect of white thermal noise.
- The effect of multipath.
- The effect of a moving GEO satellite.
- The effect of frequency-dependent errors.
- A more detailed analysis of position errors.

4.5.1 C/No Threshold

The effect of noise on the ability of Eigen algorithm to estimate phase offsets in the array manifold has been simulated using a non-moving source. The idea is to reproduce conditions of the off-line calibration stage, where a GEO satellite is considered; since this is the case where the lowest C/No values will be encountered, when compared to the power levels of the sub-reflector probe beacon. Simulation parameters are as follows:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	Yes
Residual Doppler rate error	0%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	-
Ampl. errors range	-
Std of position calibration errors	-
Acquisition range	-
Spatial resolution	-
Input C/No	Several values
Input SNR	-
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	C/No

Tabla 4.2 Calibration parameters for C/No simulation

No Doppler shift is considered due to the fact that GEO satellites have very low radial speeds with respect to the ground stations. Results are shown below:

Figure 4.11 Eigen performance in the presence of white noise

It is clear that acceptable levels of residual errors, below 18° as determined in section 3.5.4.1, require larger C/No levels, around 26 dBHz, than those considered for DoA estimation (cf. section 3.5.5.2). This implies that, for a given noise level, more samples are required. However, since this is an off-line stage, this extra amount of processing time is not an issue. On the other hand, though, the requirement of a stable beacon, from a non-moving object, becomes more stringent, as explored below.

Results have been obtained, as commented, using a non-moving satellite as baseline and, therefore, a single FFT bin, of 1.875 Hz, has been processed, thus minimising the noise power for the given sampling frequency, set to 187.5 KHz as in chapter 3. For simplicity, phase error has been computed for the second array branch (the first one being the reference one), since the statistical distribution is supposed to be the same in all of them. Lastly, in computing the CRLB, the following equation is used for the derivative of the array manifold as a function of the phase error in the second branch:

$$dV = \begin{bmatrix} 0 \\ jV(2) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (4.23)$$

Results with real GEO satellites have shown that at least 1 million samples are mandatory. Taking into account the relationship in equation (3.58), and the fact that, in section 3.5.5.2, a threshold of 14 dBHz was obtained with 100000 samples, increasing the number of samples 10 times implies an increase in the C/No threshold of 5. However, in Figure 4.11, the Eigen algorithm does not take advantage of the implicit array gain, which comes into play in WSF estimations, i.e., WSF makes use of statistical information from all array branches combined, whereas in this simulation only the phase error from the second array branch is considered.

The array gain, as defined in (2.27), amounts to 9 dB for an 8-element array. Thus the effective C/No should be around 28 dBHz. Figure 4.11 shows a slightly optimistic performance. It must be noted, too, that in order to obtain this C/No

threshold, a carrier detection bandwidth of 1.875 Hz was used, whereas in section 3.5.5.2, this bandwidth was initially set to 35 Hz.

4.5.2 Multipath Degradation

The effect of multipath has also been explored. Multipath does not affect the off-line calibration stage when using GEO satellites, due to their high elevations ($> 5^\circ$), but this is no longer the case should a collimation tower be required. This is an upgrade which may be considered in future implementations of the system, as discussed in chapter 6, so the actual degradation must be quantified.

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	Yes
Multipath power offset	Several values
Carrier modulation	Yes
Residual Doppler rate error	0%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	-
Ampl. errors range	-
Std of position calibration errors	-
Acquisition range	-
Spatial resolution	-
Input C/No	25 dBHz
Input SNR	-
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	Multipath power offset

Tabla 4.3 Calibration parameters for multipath simulation

The performance is shown in the next figure:

Figure 4.12 Eigen performance in the presence of multipath

Once more, requirements become more stringent than when dealing with the DoA estimation stage (cf. section 3.5.3.2). The calibration problem in the presence of multipath is a very complex one and goes beyond the scope of this Thesis.

4.5.3 Moving GEO satellite

Even though in section 4.5.1 it has been assumed that Doppler shift is very low with GEO satellites, even a small angular offset can affect the calibration scheme, especially when processing a large amount of samples. The next simulations establish, for a given C/No, what is the maximum angular speed that a calibration GEO satellite can have.

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	Yes
Residual Doppler rate error	0%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	Several values
Phase errors range	-
Ampl. errors range	-
Std of position calibration errors	-
Acquisition range	-
Spatial resolution	-
Input C/No	25 dBHz
Input SNR	-
Monte Carlo iterations	500
Monte Carlo confidence level	95 %
Parametric variable	Satellite angular speed

Tabla 4.4 Calibration parameters for GEO satellite simulation

Figure 4.13 Eigen performance in the presence of angular drift

The CRLB is set for the non-moving case. As the angular drift increases, so does the phase standard deviation, since phase and angular offsets are related through the array manifold formula.

4.5.4 Frequency-Dependent Errors

Phase estimates can be divided into two components:

- GEO satellite estimates at a given frequency.
- Dynamic errors at the desired signal frequency.

The sum of both components gives the actual calibration estimate. It is clear that large frequency offsets will produce uncompensated errors. Evaluating these is not evident, though, since the array phase response as a function of frequency is not known. Indeed, there is a known dependence of the form:

$$\Delta\phi \sim \frac{2\pi}{c} \Delta f \times g(\Delta p, \theta, \varphi) \quad (4.24)$$

$g(p, \theta, \varphi)$ is a function of the DoA of the satellite, derived from the spherical coordinate frame, and the Cartesian coordinates of the array elements, as already commented. However, the analogue components of the array will have additional phase changes with frequency, which may even be non-linear for sufficiently large values:

$$\Delta\phi = \frac{2\pi}{c} \Delta f \times g(\Delta p, \theta, \varphi) + h(f) \quad (4.25)$$

$h(f)$ is the phase dependence of analogue components with frequency. However, in the case of broadside or near broadside signals, the fact that phase offsets from the probe are subtracted from those obtained with the GEO beacon implies that frequency-dependence is reduced to:

- Antenna patterns.

- Different RF paths from the probe to the antennas.
- Frequency-dependent errors due to near-field considerations.

Supposing that the frequency dependence is linear, the following restriction results:

$$\Delta f \ll \frac{\Delta\phi_T}{K} \quad (4.26)$$

K is a parameter representing the afore-mentioned frequency linear dependence. The next plot shows this dependence, with $\Delta\phi_T = 18^\circ$:

Figure 4.14 Frequency errors in array calibration

For example, for frequency offsets of 10 MHz between GEO and desired signals, the phase dependence parameter K should be below 1.8 °/MHz in order to obtain a residual phase error lower than the threshold of 18° .

As commented, the probe technique has proven to be effective in SARAS prototype, which implies that the K parameter is low enough for the considered frequency offsets. On the other hand, this is not the case when using test signal injection via couplers. Indeed, in this case, frequency dependence is dominated by test signal cables, which are not calibrated with the GEO signal. As can be seen in Figure 5.15, for some test cables K reaches values of up to 10 °/MHz. This places a very stringent restriction on frequency offsets between GEO satellite and the spacecraft to be acquired.

4.5.5 Position Errors Revisited

The effect of calibration errors in DoA estimation performance was exposed in section 3.5.4.3. The 2 mm requirement at SNR of -24 dB may seem too stringent. Even if carrier signals are used, with the effective increase in SNR that this entails, practical considerations may limit the final precision of the photogrammetric routines, at least when using COTS components, and random deformations may further increase the error contribution. Of course, the effect is heightened at higher frequencies, due to the

fact that the important parameter is the electrical length, as a function of the wavelength.

However, while all spatial coordinates have been treated identically, it must be noticed that errors in z-coordinate will have a greater impact than those in the other two Cartesian coordinates. This becomes apparent when analysing the array manifold formula, taking into account that small offsets from broadside are considered in general:

$$\alpha_n(\theta, \varphi) = -k [\sin(\theta) \cos(\varphi) x_n + \sin(\theta) \sin(\varphi) y_n + \cos(\theta) z_n] \quad (4.27)$$

For small θ , the contribution from x_n and y_n errors is minimised. This is proved in the next simulations:

Parameter	Value
Frequency	2300 MHz
N	8
Array radius	7.5 m
Array topology	Uniform
Sampling frequency	187.5 KHz
Interferer	No
Interferer power offset	-
Multipath	No
Multipath power offset	-
Carrier modulation	No
Residual Doppler rate error	10%
Number of acquisitions	1
K (per acquisition)	100000
Simulation time	530 ms
Angular excursion (per acquisition)	0°
Phase errors range	0°
Ampl. errors range	0 dB
Std of position calibration errors	Several values
Acquisition range	+/- 0.9°
Spatial resolution	0.05 °
Input C/No	-
Input SNR	-24 dB
Monte Carlo iterations	2000
Monte Carlo confidence level	95 %
Parametric variable	Position errors std

Tabla 4.5 Calibration parameters for position errors simulation

Figure 4.15 DoA estimation with position errors. (a) x/y coordinates. (b) z coordinate

It is clear that z is the critical coordinate. Thus, random deformations of the main dish have a lower impact on SARAS if they only perturb the x or y coordinates of the array system. In practical terms, this can effectively decrease the threshold SNR, even in the presence of strong deformations.

4.6 Conclusions

An integral calibration scheme has been presented, which combines a variety of techniques and calibration sources in order to obtain the required data for the DoA estimation stage. Parametric modelling is at the core of the photogrammetric calibration, which produces the Cartesian coordinates of the array, but this model is coupled with a more robust dynamic one, divided into off-line and on-line stages. With the use of a sub-reflector probe along with a beacon from a GEO satellite, dynamic phase offsets due to uncompensated calibration errors, mainly in the analogue components of the array branches, can be minimised, yielding a robust scheme overall. Some of the critical aspects of the calibration subsystem are the following:

- Position precision remains a stringent requirement, although it has been verified that the critical coordinate is the z - one.
- The availability of a GEO satellite and a sub-reflector probe are mandatory. Frequency dependent errors must be considered, so that the GEO satellite must have a similar frequency as the one from the spacecraft to be acquired.
- The use of a collimation tower is precluded for the moment, due to the influence of multipath.
- Drift in GEO satellites must be minimised, or pre-compensated if it is known a priori, so that the off-line stage produce reliable results.

Some of these aspects will be further addressed in chapter 6.

CHAPTER 5 TEST CAMPAIGNS AND VALIDATION

5.1 Introduction and Contributions

After the presentation of SARAS system and the analysis of its performance through Monte Carlo simulations, both for the Acquisition and Calibration modes, this chapter is devoted to the exposition of the actual hardware implementation of the first prototype, and the results obtained from real satellites.

5.1.1 State of the Art

Hardware elements were selected following a COTS criteria, so that the budget was kept under control. The most innovative components are the down-converters and the MDP (Multichannel Digital Processor). Indeed, multichannel RF/IF and FPGA processors are not commonly used, even in S-band, when the number of array channels is greater than 3. This is due to the fact that one of the main applications of these multichannel systems is monopulse tracking, where just one sum and two delta channels are considered.

On the other hand, the main antenna where the prototype was installed is the Vil-1 antenna, part of the infrastructure of ESAC station, a 15 metre diameter dish which operates in the S-band radio frequency (1.8 - 2.7 GHz). The antenna, built in 1974, is capable of automatic tracking, ranging, frequency and timing measurements, monitoring, control and two-way communications. The antenna was mothballed in 2012, and was passed to the Cooperation through Education in Science and Astronomy Research project in order to be used as a radio telescope for educational purposes.

5.1.2 Contributions

The contributions of this chapter are

- The definition of the test plan that was performed with real satellite signals, in order to obtain the FoM of a prototype of SARAS system.
- The analysis of the results from these tests, and their comparison and validation using the simulation results from chapters 3.

5.2 Hardware Elements

5.2.1 Antennas and RF/IF Elements

5.2.1.1 Hardware

The antenna for each of the 8 sensors of the array is a RHCP helix antenna from European Antennas, AMHS1.7-25R"D1 model ⁹:

Figure 5.1 Helix antenna used as SARAS sensors

Its main characteristics are:

- Weight (including radome): 1.5 Kg
- Polarisation: RCHP (selected for simplicity's sake, although the next version of the system shall have linearly polarised antennas, to cope with both circular polarisations).
- Gain: 14 dB

Each antenna is followed by an outdoor housing box which includes a coupler, for test signal injection calibration, an RF filter and an LNA. The hardware is shown in the next photographs:

(a)

(b)

Figure 5.2 (a) Mechanized box with all the elements installed. (b) Antenna and housing box

The next table shows a summary of some parameters of the RF elements:

⁹ Cobhan Antennas, AMHS1.7-25R"D1 Datasheet

Element	Coupler	RF Filter	LNA
Supplier	MLCI	Anatech Electronics	Miteq
IL / NF (dB)	0.45 (IL)	0.8 (IL)	0.6 (NF)
Coupling (dB)	30	-	-
Rejection (dB)	-	> 95 (@ DC-2120 and 2400-4000 MHz)	-
Gain (dB)	-	-	40

Table 5.1 First RF elements parameters

In order to reduce the number of cables, the LNAs have been fed using Bias Tees. Prior to the LNA, a Transmission Reject Filter has been installed, so as to prevent saturation from the main antenna transmission. It also discriminates RF interferers outside the band of interest, and serves as a first image band suppressor. The functionality of couplers has been discussed in the Calibration chapter.

Long RF cables are required to link the LNA outputs and the DC (Down-Converter) on one hand, and the coupler and the test generator on the other, since both the DC units and the test generator are located in a cabin, which is solidary with the whole Vil-1 antenna in its azimuth movement, as shown below:

(a)

(b)

Figure 5.3 (a) Vil-1 antenna and cabin. (b) Rack with equipment inside the cabin

The following lengths have been set for each array branch, taking into account the required slack for the elevation movement of the antenna:

Ant 1	Ant 2	Ant 3	Ant 4	Ant 5	Ant 6	Ant 7	Ant 8
19 m	19 m	22 m	25.5 m	29 m	29 m	26 m	22.5 m

Table 5.2 RF path lengths in Vil-1 prototype

Even though differences between RF path lengths can be calibrated with on-line or off-line schemes, it should be kept within bounds so as not to preclude the narrowband model assumption (cf. section 2.1). In this case, the maximum length difference is 10.4 metres. Supposing that the speed of propagation is $0.8c$, and that the maximum bandwidth is 6 MHz, we obtain:

$$BW \times \Delta T_{max} = 6 \text{ MHz} \times \frac{10}{0.8c} = 0.25 \quad (5.1)$$

The result is below 1, as required, although smaller values are desirable. Thus, it can be expected that signals which occupy the full bandwidth may have some wideband distortion. For the RF downlink cables, Huber&Suhner S_04272_B model was selected. For the test injection cables, Phasetrack II 200 from Times Microwave Systems was chosen due to its high phase stability and flexibility¹⁰.

The frequency conversion circuit, from RF frequencies between 2.2 and 2.3 GHZ to an IF frequency of 70 MHz, was constructed by the UPM (Universidad Politécnica de Madrid) and has eight receiver blocks containing eight conversion circuits, and RF/IF amplification and filtering chains.

The conversion gain, around 80 dB, has been designed to obtain an appropriate power level at the ADC input, taking into account the reduced G/T of each radiating element (cf. next sub-section). Another critical aspect is that the LO signal must be the same for all the frequency conversion circuits and be obtained from a single PLL circuit, so that phase drifts among array branches are eliminated. Finally, filtering stages, both in RF and IF, ensure that the contribution of out-of-band interferers and of image band signals is minimised. The mixers are preceded by a set of bias tees to provide a DC (Direct Current) supply to the remotely located LNA.

¹⁰ <http://www.timesmicrowave.com/cms/products/assemblies/phase-critical-assemblies/phase-track-cable-assemblies/index.shtml>

(a)

(b)

Figure 5.4 (a) UPM down-converters (b) with laboratory tests showing the circuit gain for different frequencies

5.2.1.2 Station Budgets

With the specifications of the previous sub-section and the help of Agilent's Genesys software, the G/T budgets shown below have been obtained.

G/T budget for EI < 10°				
	Units	Nominal Case	Adverse case (with rain)	Favourable Case
Antenna gain	dB	14.3	14.6	14
RF/IF noise figure	dB	2.8	3	2.2
Antenna temperature	K	37.1	80.1	37.1
Total noise temperature	K	317.8	388.6	241.6
G/T	dB/K	-10.7	-11.9	-9.2

Table 5.3 G/T analysis for elevations lower than 10°

G/T budget for EI > 10°				
	Units	Nominal Case	Adverse case (with rain)	Favourable Case
Antenna Gain	dB	12	10	13
System noise figure	dB	2.8	3	2.2
Antenna Temperature	K	8.3	17.4	8.3
Total noise temperature	K	289	325.9	212.8
G/T	dB/K	-10.3	-11.2	-8.7

Table 5.4 G/T analysis for elevations greater than 10°

The total noise temperature is calculated as:

$$T_{tot} = \eta T_{sky} + 0.5 \times (1 - \eta) \times (T_{sky} + T_{earth}) + T_{sys} \quad (5.2)$$

η is the radiation efficiency of each antenna, with an assumed value of 0.6. T_{sky} and T_{earth} are the noise temperatures of sky and Earth. For low elevations, they are equal to 35K and 50K respectively. For high elevations, 6K and 10K values are used. The presence of rain is not considered. Finally, T_{sys} is the noise temperature of the RF/IF receiver, which is related to the noise figure F (dB) with the following formula:

$$T_{sys} = T_0 \left(10^{F/10} - 1 \right) \quad (5.3)$$

T_0 is the reference temperature, made equal to 310K, and F is set to 3 dB as a worst-case value. A high reference temperature is considered to take into account the heating of outdoor, non-refrigerated SARAS RF components.

Unfortunately, due to the small values involved, the G/T of the prototype could not be directly calculated. In order to meet the requirement of an input SNR greater than -24 dB (cf. section 3.5.2.1), the minimum required nominal PFD at reception is approximately -160.5 dBW/m², for low elevations. This threshold will be considered when analysing the link budget of each spacecraft in the next subsection, where these considerations will also be refined taking into account the presence of FFT processing, which can give a variable extra margin, depending on the mission and the scenario (typically around 4.5 dB).

Extensive measurements with test signal injection were carried out, though, to establish the power level profile for each of the array branches. Next table summarises the results, in an approximate way, for the nominal case. The first two columns are obtained from top to bottom, setting the minimum power level corresponding to the threshold PFD. The third is set in the opposed order, starting from the maximum power level that the ADC is able to process (cf. section 5.2.2.2).

Power level profile				
	Units	Minimum level (with FFT)	Minimum level (no FFT)	Maximum level
Antenna output	dBm	-149.5	-145	-102.5
LNA output	dBm	-109.5	-105	-62.5
RF cable output	dBm	-129.5	-125	-82.5
DC output	dBm	-49.5	-45	-2.5
ADC input	dBm	-29.5	-25	-2.5

Table 5.5 Approximate power level profile for SARAS system

An average loss in the RF cables of 20 dB has been assumed, taking into account real measurements. Also, a programmable gain prior to the ADC can be set between 0 and 20 dB. With this, as can be appreciated, the dynamic range is

approximately 47 dB, and acceptable PFD range from -165 to -118 dBW/m². Due to safety considerations, though, the maximum PFD is usually limited to -154 dBW/m².

5.2.1.3 *Link Budgets*

Link budgets have been elaborated for some of the target missions of SARAS, namely, Cryosat 2, GOCE, INTEGRAL and XMM, following CCSDS (Consultative Committee for Space Data Systems) standards (CCSDS 2011). They are included below, for an elevation of 5°.

Downlink BUDGET for Space-to Ground Communications

Propagation Downlink							
	Units	Nom	Adv	Fav	Mean	Var	PDF
Rain Present	Rain - Clear	Clear	Rain	Clear			
Availability	%		99.990				
Temperature ground height	K	283.431	283.431	283.431			
Rain intensity for availability	mm/h		20.692				
Corefraction index Nwet	A/m		34.404				
Rain height	Km		3.188				
Surface water vapour density	g/m ³	2.013	2.013	2.013			
Integrated water vapour content	Kg/m ²		23.122				
Total Columnar content liquid water	Kg/m ²		0.306				
Atmospheric Gaseous Attenuation	dB	0.424	0.424	0.424			
Rain Attenuation	dB	0	0.053	0			
Exceeded Atm. Gaseous Attenuation	dB	0	0.423	0			
Clouds Attenuation	dB	0	0.016	0			
Total Atmospheric loss	dB	0.424	1.924	0.424	1.174	0.062	GAUSS
Scintillation	dB	0	1.499	0			
Polarization Mismatch	dB	0.129	0.228	0.057	0.143	0.002	UNI
Spreading Loss	dB	139.298	139.298	139.298	139.298		
Free Space Loss	dB	167.601	167.601	167.601	167.601		
Total Attenuations	dB	0.553	2.152	0.481	1.317		
Total Propagation Loss	dB	168.153	169.752	168.082	168.917	0.065	

Downlink Budget							
	Units	Nom	Adv	Fav	Mean	Var	PDF
XPD	dB	21.297	18.815	24.806			
Flux at Reception	dBW/sqm	-159.602	-163.192	-157.322	-160.291	0.408	
Maxim. Flux Density	dBW/sqm	-154.000	-154.000	-154.000			
Flux Margin	dB	5.602	9.192	3.322			
RF/IF gain	dB	80.000	74.000	84.000			
Maximum Flux Density for ADC	dBW/sqm	-118.488	-112.168	-122.798			
Minimum Flux Density for ADC	dBW/sqm	-178.048	-171.728	-182.358			
Flux within ADC Range	Yes/No	Yes	Yes	Yes			
SARAS Gain	dB	23.321	23.001	23.631	23.316	0.033	UNI
Total Noise Temperature	dB	24.767	25.667	23.587	24.627	0.120	GAUSS
G/T SARAS Array	dB/K	-1.446	-2.666	0.044	-1.311	0.153	
Rx Power	dBm	-143.743	-147.752	-141.082	-144.451		
Rx S/No at Downlink	dBHz	39.121	34.212	42.962	38.554	0.563	
WSF filtering	MHz	0.200	0.200	0.200	0.200		
WSF filtering suppression	dB	0.260	0.302	0.255	0.272	0.00	TRI
Input SNR	dB	-23.18	-28.13	-19.33	-23.76		
Req. input SNR	dB	-24.03	-24.03	-24.03	-24.03		
SNR Margin	dB	0.850	-4.100	4.697	0.271	0.564	
Mean-3 sigma	dB	-1.981					
Margin -w.c RSS	dB	-1.845					

FFT + WSF							
Carrier Suppression	dB	4.715	5.927	3.696	4.779	0.208	TRI
Input C/No	dBHz	25.374	19.253	30.235	24.743	0.771	
Req. input C/No	dBHz	18.000	20.000	14.000	17.000		
FFT + WSF	dB	7.374	-0.747	16.235	7.743	0.771	
Mean-3 sigma	dB	5.108					
Margin -w.c RSS	dB	4.419					

Table 5.6 Link budget for Cryosat 2

In this case the simple SNR budget is not fully met, taking into account the worst case Root Sum Square and the 3σ requirements. However, the budget when using FFT prior to WSF increases in approximately 7 dB in the nominal case, which is more than enough for this mission. Of course, the specific C/No threshold to be considered depends on the final uncertainty in the estimation of the Doppler rate, as explored in section 3.5.5.2.

Downlink BUDGET for Space-to Ground Communications

Propagation Downlink							
	Units	Nom	Adv	Fav	Mean	Var	PDF
Rain Present	Rain - Clear	Clear	Rain	Clear			
Availability	%		99.990				
Temperature ground height	K	283.431	283.431	283.431			
Rain intensity for availability	mm/h		20.692				
Corefraction index Nwet	A/m		34.404				
Rain height	Km		3.188				
Surface water vapour density	g/m ³	2.013	2.013	2.013			
Integrated water vapour content	Kg/m ²		23.122				
Total Columnar content liquid water	Kg/m ²		0.306				
Atmospheric Gaseous Attenuation	dB	0.425	0.425	0.425			
Rain Attenuation	dB	0	0.056	0			
Exceeded Atm. Gaseous Attenuation	dB	0	0.425	0			
Clouds Attenuation	dB	0	0.017	0			
Total Atmospheric loss	dB	0.425	1.942	0.425	1.184	0.064	GAUSS
Scintillation	dB	0	1.516	0			
Polarization Mismatch	dB	0.421	0.566	0.000	0.283	0.027	UNI
Spreading Loss	dB	133.656	133.656	133.656	133.656		
Free Space Loss	dB	162.130	162.130	162.130	162.130		
Total Attenuations	dB	0.847	2.508	0.425	1.467		
Total Propagation Loss	dB	162.977	164.638	162.555	163.597	0.091	

Downlink Budget							
	Units	Nom	Adv	Fav	Mean	Var	PDF
XPD	dB	21.297	18.815	24.806			
Flux at Reception	dBW/sqm	-159.741	-159.329	-154.941	-157.218	0.214	
Maxim. Flux Density	dBW/sqm	-154.000	-154.000	-154.000			
Flux Margin	dB	5.741	5.329	0.941			
RF/IF gain	dB	80.000	74.000	84.000			
Maximum Flux Density for ADC	dBW/sqm	-118.316	-111.996	-122.626			
Minimum Flux Density for ADC	dBW/sqm	-177.876	-171.556	-182.186			
Flux within ADC Range	Yes/No	Yes	Yes	Yes			
SARAS Gain	dB	23.321	23.001	23.631	23.316	0.033	UNI
Total Noise Temperature	dB	24.767	25.664	23.587	24.625	0.120	GAUSS
G/T SARAS Array	dB/K	-1.446	-2.663	0.044	-1.310	0.153	
Rx Power	dBm	-144.347	-144.398	-138.815	-141.690		
Rx S/No at Downlink	dBHz	38.517	37.569	45.228	41.315	0.394	
WSF filtering	MHz	0.200	0.200	0.200	0.200		
WSF filtering suppression	dB	0.260	0.302	0.255	0.272	0.00	TRI
Input SNR	dB	-23.78	-24.77	-17.07	-21.00		
Req. input SNR	dB	-24.03	-24.03	-24.03	-24.03		
SNR Margin	dB	0.247	-0.743	6.963	3.033	0.394	
Mean-3 sigma	dB	1.150					
Margin -w.c RSS	dB	-3.576					

FFT + WSF							
Carrier Suppression	dB	12.254	13.078	11.211	12.181	0.146	TRI
Input C/No	dBHz	17.233	15.460	24.987	20.104	0.540	
Req. input C/No	dBHz	18.000	20.000	14.000	17.000		
FFT + WSF	dB	-0.767	-4.540	10.987	3.104	0.540	
Mean-3 sigma	dB	0.900					
Margin -w.c RSS	dB	-4.678					

Table 5.7 Link budget for GOCE

In this case the improvement using FFT processing greatly depends on the effective compensation of the Doppler shift, due to the high carrier suppression losses. In any case, while the worst case Root Sum Square requirement is not met, the 3σ one is. There is great variability between nominal and adverse cases, though.

For INTEGRAL and XMM a different casuistry arises, due to the fact that their orbits are highly elliptical. The link budgets below have been calculated for the

maximum elevations where requirements are met, where a linear increase in altitude with elevation has been implemented as an approximation to the elliptical orbit.

Downlink BUDGET for Space-to Ground Communications

Propagation Downlink							
	Units	Nom	Adv	Fav	Mean	Var	PDF
Rain Present	Rain - Clear	Clear	Rain	Clear			
Availability	%		99.990				
Temperature ground height	K	283.431	283.431	283.431			
Rain intensity for availability	mm/h		20.692				
Corefraction index Nwet	A/m		34.404				
Rain height	Km		3.188				
Surface water vapour density	g/m ³	2.013	2.013	2.013			
Integrated water vapour content	Kg/m ²		23.122				
Total Columnar content liquid water	Kg/m ²		0.306				
Atmospheric Gaseous Attenuation	dB	0.134	0.134	0.134			
Rain Attenuation	dB	0	0.018	0			
Exceeded Atm. Gaseous Attenuation	dB	0	0.134	0			
Clouds Attenuation	dB	0	0.005	0			
Total Atmospheric loss	dB	0.134	0.512	0.134	0.323	0.004	GAUSS
Scintillation	dB	0	0.377	0			
Polarization Mismatch	dB	0.171	0.256	0.114	0.185	0.002	UNI
Spreading Loss	dB	162.759	162.759	162.759	162.759		
Free Space Loss	dB	191.117	191.117	191.117	191.117		
Total Attenuations	dB	0.305	0.768	0.248	0.508		
Total Propagation Loss	dB	191.422	191.885	191.365	191.625	0.006	

Downlink Budget							
	Units	Nom	Adv	Fav	Mean	Var	PDF
XPD	dB	21.297	18.815	24.806			
Flux at Reception	dBW/sqm	-159.554	-162.982	-157.934	-160.208	0.427	
Maxim. Flux Density	dBW/sqm	-148.500	-148.500	-148.500			
Flux Margin	dB	11.054	14.482	9.434			
RF/IF gain	dB	80.000	74.000	84.000			
Maximum Flux Density for ADC	dBW/sqm	-118.433	-112.113	-122.743			
Minimum Flux Density for ADC	dBW/sqm	-177.993	-171.673	-182.303			
Flux within ADC Range	Yes/No	Yes	Yes	Yes			
SARAS Gain	dB	23.321	23.001	23.631	23.316	0.033	UNI
Total Noise Temperature	dB	24.410	24.938	23.112	24.025	0.093	GAUSS
G/T SARAS Array	dB/K	-1.089	-1.937	0.519	-0.709	0.126	
Rx Power	dBm	-143.792	-147.625	-141.805	-144.465		
Rx S/No at Downlink	dBHz	39.429	35.068	42.714	39.141	0.554	
WSF filtering	MHz	0.200	0.200	0.200	0.200		
WSF filtering suppression	dB	0.260	0.302	0.255	0.272	0.00	TRI
Input SNR	dB	-22.87	-27.27	-19.58	-23.17		
Req. input SNR	dB	-24.03	-24.03	-24.03	-24.03		
SNR Margin	dB	1.159	-3.244	4.449	0.859	0.554	
Mean-3 sigma	dB	-1.375					
Margin -w.c RSS	dB	-1.141					

FFT + WSF							
Carrier Suppression	dB	10.825	8.280	14.865	11.323	1.838	TRI
Input C/No	dBHz	19.573	17.758	18.818	18.787	2.392	
Req. input C/No	dBHz	14.000	14.000	14.000	14.000		
FFT + WSF	dB	5.573	3.758	4.818	4.787	2.392	
Mean-3 sigma	dB	0.148					
Margin -w.c RSS	dB	2.143					

Table 5.8 Link budget for INTEGRAL

A common C/No threshold of 14 dBHz can be considered due to the slow motion of the satellite. The achieved maximum elevation is 16°, although, as will be verified with real measurements, this is a pessimistic limit. Indeed, the mentioned slow motion permits to increase the number of processed samples without Doppler impact,

thus decreasing the effective C/No thresholds. Also, the linear approximation of the altitude variation does not truly reflect the elliptical orbit.

Downlink BUDGET for Space-to Ground Communications

Propagation Downlink							
	Units	Nom	Adv	Fav	Mean	Var	PDF
Rain Present	Rain - Clear	Clear	Rain	Clear			
Availability	%		99.990				
Temperature ground height	K	283.431	283.431	283.431			
Rain intensity for availability	mm/h		20.692				
Corefraction index Nwet	A/m		34.404				
Rain height	Km		3.188				
Surface water vapour density	g/m ³	2.013	2.013	2.013			
Integrated water vapour content	Kg/m ²		23.122				
Total Columnar content liquid water	Kg/m ²		0.306				
Atmospheric Gaseous Attenuation	dB	0.058	0.058	0.058			
Rain Attenuation	dB	0	0.009	0			
Exceeded Atm. Gaseous Attenuation	dB	0	0.057	0			
Clouds Attenuation	dB	0	0.002	0			
Total Atmospheric loss	dB	0.058	0.195	0.058	0.126	0.001	GAUSS
Scintillation	dB	0	0.137	0			
Polarization Mismatch	dB	0.163	0.256	0.089	0.172	0.002	UNI
Spreading Loss	dB	166.054	166.054	166.054	166.054		
Free Space Loss	dB	194.450	194.450	194.450	194.450		
Total Attenuations	dB	0.220	0.451	0.146	0.298		
Total Propagation Loss	dB	194.671	194.901	194.597	194.749	0.003	

Downlink Budget							
	Units	Nom	Adv	Fav	Mean	Var	PDF
XPD	dB	21.297	18.815	24.806			
Flux at Reception	dBW/sqm	-164.412	-165.549	-162.112	-164.047	0.478	
Maxim. Flux Density	dBW/sqm	-144.000	-144.000	-144.000			
Flux Margin	dB	20.412	21.549	18.112			
RF/IF gain	dB	80.000	74.000	84.000			
Maximum Flux Density for ADC	dBW/sqm	-118.394	-112.074	-122.704			
Minimum Flux Density for ADC	dBW/sqm	-177.954	-171.634	-182.264			
Flux within ADC Range	Yes/No	Yes	Yes	Yes			
SARAS Gain	dB	23.321	23.001	23.631	23.316	0.033	UNI
Total Noise Temperature	dB	24.341	24.871	23.019	23.945	0.095	GAUSS
G/T SARAS Array	dB/K	-1.020	-1.870	0.611	-0.629	0.128	
Rx Power	dBm	-148.681	-150.231	-145.997	-148.331		
Rx S/No at Downlink	dBHz	34.609	32.529	38.615	35.355	0.608	
WSF filtering	MHz	0.200	0.200	0.200	0.200		
WSF filtering suppression	dB	0.260	0.302	0.255	0.272	0.00	TRI
Input SNR	dB	-27.69	-29.81	-23.68	-26.96		
Req. input SNR	dB	-24.03	-24.03	-24.03	-24.03		
SNR Margin	dB	-3.661	-5.783	0.350	-2.927	0.608	
Mean-3 sigma	dB	-5.268					
Margin -w.c RSS	dB	-4.318					

FFT + WSF							
Carrier Suppression	dB	8.198	10.998	6.269	8.488	0.942	TRI
Input C/No	dBHz	17.380	12.500	23.315	17.836	1.551	
Req. input C/No	dBHz	14.000	14.000	14.000	14.000		
FFT + WSF	dB	3.380	-1.500	9.315	3.836	1.551	
Mean-3 sigma	dB	0.100					
Margin -w.c RSS	dB	0.505					

Table 5.9 Link budget for XMM

In the case of XMM, the maximum achievable elevation is 40°.

5.2.2 Digital Processor

5.2.2.1 Hardware

The Digital Processor's functionality is shared between two components. The first one is a COTS (Commercial Off-The-Shelf) platform from Nutaq company embedded in a cPCI (Compact Peripheral Component Interconnect) chassis and based on VHS-ADC Virtex 4 LX160 FPGAs with four ADC modules, each one containing sixteen channels, and one DAC module. The ADC maximum sampling rate is 105 Msps, with 14 bits per sample. It supports a GPS disciplined clock module for inter-board synchronisation and for time stamp reception, which is considered part of the F&T subsystem.

Figure 5.5 Digital Processor test setup

From the four available ADC modules, one of them is used to obtain the timestamp (cf. section 5.2.3.1), and the second one is used to process the input samples from the 8 array branches. The other two ADC and the DAC are not used in SARAS application, except for the triggering mechanism from the latter (cf. section 5.2.3.1).

The conceptual diagram of the processing performed in the FPGA boards in this second ADC module is shown below. It can be seen that it corresponds to the Digital Conditioner presented in section 3.4.1:

Figure 5.6 Digital Conditioner conceptual diagram

As already commented, digital processing in this platform includes signal conditioning with CIC and CFIR (Compensated Finite Impulse Filter) filters. Three configuration modes are considered, which depend on the received signal's characteristics: wideband (6 MHz, decimation factor of 8), narrowband 1 (200 KHz, decimation factor of 200) and narrowband 2 (400 KHz, decimation factor of 100) modes. The first one is oriented to capture the whole signal bandwidth with an IF bandwidth filter, whereas narrowband modes are applied for modulations with a significant amount of power in the carrier; the idea being to filter the carrier at each sensor with a narrowband digital filter to get rid of as much noise as possible and increase the sensitivity of the system. The second narrowband mode is useful when uncompensated Doppler shift is high.

The second hardware element of the Digital Processor, a PC termed SARAS Computer (SC), encompasses the Statistical Processor, the Time Stamping and the Signal Tracker and Discriminator subsystems (cf. section 3.3.1.2). The specifications of the SC are:

- Processor: Un Intel® Xeon® E5645 (Six Core, 2.40GHz, 12MB Cache, 5.86GT/s Intel® QPI).
- O.S: Windows® 7 Professional original (64Bit OS).
- Memory: 32GB (4x4GB + 2x8GB) 1333MHz DDR3 ECC RDIMM requires OS 64 bits.
- Hard Disk: SATA 1.5 TB (7200 rpm) 3,0 Gb/s NCQ and 16 MB DataBurst.
- Graphical: 2GB GDDR5 NVIDIA Quadro 4000 - 2DP, 1DVI (1DP-DVI, one adaptador DVI-VGA).
- GPU: PCIe x16 C2075 de NVIDIA Tesla.
- Monitor: Dell Professional P2312H 58 cm (23").

In order to communicate both hardware elements, it is required to introduce a host-PC. For SARAS prototype, it is a CPU card, fully integrated in the chassis, which configures and controls all the VHS-ADC modules through the cPCI backplane. Samples processed in the Digital Conditioner are stored in a RAM (Random Access Memory) and accessed by the Host-PC, which in turns sends them to the SC through a LAN (Local Area Network) link.

Statistical processing algorithms, as analysed in chapters 2 and 3, are integrated in the GUI (cf. section 5.2.5). Next figure shows a flowchart, including user selection, based on the processing presented in sections 3.4.2 and 3.4.3.

Figure 5.7 Flowchart for processing in SC

5.2.2.2 Signal Level and Quantisation Noise

The interface between the RF/IF and the ADC deserves special attention, to guarantee that quantisation noise does not degrade the performance of WSF. We will consider a quantiser with an X_m full scale, or $2X_m$ range, and Δ resolution. For a **(B+1)-bits ADC**, we get:

$$\Delta = \frac{2X_m}{2^{B+1} - 1} = \frac{X_m}{2^B - 1/2} \approx \frac{X_m}{2^B} \quad (5.4)$$

The approximation implies that B is high enough. A uniform quantisation error has then a standard deviation equal to:

$$q_e = \frac{\Delta}{\sqrt{12}} = \frac{X_m}{2^B \sqrt{12}} \quad (5.5)$$

Quantisation noise is:

$$\sigma_e^2 = \frac{q_e^2}{R(\Omega)} = \frac{q_e^2}{50} \quad (5.6)$$

The quantisation SNR is thus:

$$\text{SNR}_q = 10 \log \left(\frac{\sigma_x^2}{\sigma_e^2} \right) = 6.02 \times B + 27.78 - 20 \log \left(\frac{X_m}{\sigma_x} \right) \quad (5.7)$$

For a sinusoidal signal with X_p as peak value we have:

$$q_x = \frac{X_p}{\sqrt{2}} \Leftrightarrow \sigma_x^2 = \frac{q_x^2}{50} \quad (5.8)$$

For a full-scale signal, $X_p = X_m$ and:

$$\text{SNR}_q = 6.02 \times B + 7.78 \quad (5.9)$$

Minimum signal power will be determined by the noise floor:

$$\text{SNR}_q = 6.02 \times B + 27.78 - 20 \log \left(\frac{X_m}{\sigma_x} \right) \geq \text{SNR}_{q_{\min}} \quad (5.10)$$

$$\Leftrightarrow \sigma_x \geq 10^{(\text{SNR}_{q_{\min}} - 27.78 - 6.02B)/20} X_m \quad (5.11)$$

$\text{SNR}_{q_{\min}}$ is the quantisation SNR threshold that the system can tolerate. The threshold may be determined evaluating its influence on the overall SNR degradation, bearing in mind that the total SNR is the harmonic mean of thermal and quantisation SNRs:

$$\text{snr}_T = \frac{1}{\frac{1}{\text{snr}_{\text{thermal}}} + \frac{1}{\text{snr}_q}} \quad (5.12)$$

Taking into account that the system has been designed to operate at very negative SNR, in the range of -24 dB, it is clear that for positive SNR_q , the effect of quantisation noise is negligible. However, under certain conditions where the sampling clock and the signal are harmonically related, the quantisation noise becomes correlated and the energy is concentrated in the harmonics of the signal (Kester, 2009). The degradation caused by correlated noise was already explored with multipath, and is especially critical during array calibration. Therefore, it is desirable to have substantially high SNR_q values, and so $\text{SNR}_{q_{\min}}$ is set to 40 dB.

Maximum signal power on the other hand will be limited by the probability of saturation/clipping. For random signals, the probability density functions of which can be approximated by Gaussian ones, the maximum power may be fixed so that the saturation probability is low enough. For example, taking a 3σ dispersion and using Tchebycheff inequality:

$$\Pr(X \geq 3\sigma) = 0.27\% \quad \Leftrightarrow \quad \sigma \leq \frac{X_m}{3\sqrt{50}} \quad (5.13)$$

So that the maximum admissible σ is $\frac{X_m}{4\sqrt{50}}$ in order to get a saturation probability of 0.27%. This σ parameter is related to the desired signal power as shown below:

$$\sigma_x^2 = \frac{\sigma^2}{1 + \frac{1}{\text{snr}_T}} \quad (5.14)$$

This stems from the fact that saturation restriction must apply to desired signal plus noise components, and it implies that both are uncorrelated. In any case, it is clear that saturation scenarios occur with strong signals, with high SNR, so that the preceding equation can be approximated by $\sigma_x^2 = \sigma^2$ without loss of generality. Thus, the acceptable range of signal power at ADC input is:

$$\sigma_x^2 \in \left[10^{(12.22-6.02B)/10} X_m^2, \frac{X_m^2}{450} \right] \quad (5.15)$$

Taking $B+1=14$ bits, an ADC range of 1 Vpp, i.e., $X_m = 0.5$ V, the range becomes:

$$\sigma_x^2 \in [-42.06 \text{ dBm} \quad -2.55 \text{ dBm}] \quad (5.16)$$

A different approach is considered in (Taggart et al. 2007). For a 14-bit ADC, an optimum Loading Factor (LF) of -14 dB is established, both for 8-PSK and white Gaussian noise signals. The Loading Factor is defined as:

$$\text{LF} = 10 \log \frac{\sigma_x^2}{X_m^2} \quad (5.17)$$

With this definition, the maximum signal level is 10 dBm, which is less stringent than the prior specification. The minimum PFD specified in 5.2.1.2 fulfils the lower bound, so that quantisation noise is not an issue.

5.2.3 F&T

5.2.3.1 Hardware

Once more, hardware is divided into two components:

- A frequency generator, which sends the LO signal to the DC. This is a standard RF signal generator from Agilent, N9310A model.
- A synchronisation card, which is part of the same hardware element from Nutaq constituted by the FPGA boards and the host-PC (cf. 5.2.2.1).

The synchronisation card, called ADAC Sync ¹¹, has the following characteristics:

- A standard GPIO-32 (General Purpose Input/Output) connector is used to transmit timestamps (time and position information) to other boards equipped with a GPIO connector.
- GPS receiver module. It is used to supply UTC (Universal Time Coordinated) time, as well as a 1-PPS (Pulse Per Second) clock output used in adjusting the 10-MHz, on-board reference clock to a precision of 0.1 ppm.
- Additionally, PPS and clock MMCX (Micro-Miniature Coaxial) outputs are available to supply these signals.

Synchronisation amongst the two ADC modules is performed with a triggering mechanism from the DAC module, as shown in the next diagram:

¹¹ <http://nutaq.com/en/products/adacsync>

Figure 5.8 Trigger mechanism concept

On the other hand, the timestamp functionality is carried out with the aid of a GPS antenna, located on the roof of VIL-1 cabinet and connected to the rack through a low loss RF cable.

Figure 5.9 GPS antenna for SARAS system

5.2.3.2 Frequency and Time Synchronisation

Requirements for synchronisation amongst the array branches are critical for the proper performance of the system, and will be considered in more detail in this section, in their two aspects: synchronisation of LO frequency signals, and of ADC sampling.

Starting with the LO signal generation, a given noisy carrier with desired frequency f_0 can be modelled as (Riley 2007):

$$V(t) = [1 + \varepsilon(t)] \sin\{2\pi[f_0 + f_{err}(t)]t + \phi(t)\} \quad (5.18)$$

$\varepsilon(t)$ is the amplitude noise, $\phi(t)$ is the phase noise, and $f_{err}(t)$ is a bias in the realisation of the desired frequency. These error contributions will be considered separately.

Starting with the frequency bias term, it includes fixed and slow-changing biases in frequency, due to external effects such as temperature variation or ageing of components. These effects are not critical due to two reasons:

- The bias tends to be small in magnitude. Thus, even if it translates into a frequency shifting of the IF signal, this does not imply a great loss of the desired signal's power as long as some frequency guard bands are present in the narrowest filter. For those cases where accumulated errors, mainly due to ageing, cause large variations, they can be easily corrected recalibrating the frequency source or replacing it.

- The error, even though stochastic, is slow-changing in nature. In terms of WSF performance, thus, it can be compensated with calibration schemes as discussed in chapter 4, even with those which are not on-line. It is also important to notice that if the frequency generator creates a single LO which is distributed to all mixers in the DC, the need for calibration in this instance is completely eliminated as the bias is the same in all branches in the array. This would not be the case, though, if, instead of a common LO, a 10 MHz or similar reference was propagated, to minimise RF losses, leaving for each DC chain the task of up-converting this frequency to the final LO signal.

Amplitude and phase noise require another approach, though, because their stochastic nature is highly dynamic. On-line calibration schemes can compensate their degradation effects, as long as the frequency difference between calibration and desired signals is not significant for the prediction of both noises. However, for SARAS, resorting to a common LO, as discussed in the preceding paragraph, has been the preferred solution. Indeed, once more, amplitude and phase noise will be the same amongst all array branches and will not have any effect on WSF performance.

Synchronisation of the ADC samplers is the other main topic in the performance analysis of the F&T system. Time jitter in the sampling process causes time-shifts in the incoming samples, as illustrated below:

Figure 5.10 Time jitter in array ADC

Time-shifted versions imply that the modulation signal is different, and a phase-shift ensues. Considering two array branches:

$$\begin{cases} x_1(t) = \tilde{s}(t) \\ x_2(t) = \tilde{s}(t)e^{j\varphi(t)} \end{cases} \quad (5.19)$$

$\tilde{s}(t)$ is the complex envelope of the modulated signal. If the signals are sampled at the same time, it yields:

$$\begin{cases} x_1[nT] = \tilde{s}[nT] \\ x_2[nT] = \tilde{s}[nT]e^{j\varphi[nT]} \end{cases} \quad (5.20)$$

With synchronisation errors, the equations become:

$$\begin{cases} x_1[nT] = \tilde{s}[nT] \\ x_2[nT + \varepsilon_t] = \tilde{s}[nT + \varepsilon_t] e^{j\varphi[nT + \varepsilon_t]} \end{cases} \quad (5.21)$$

Of course, the idea is to guarantee that:

$$\begin{cases} \tilde{s}[nT] \approx \tilde{s}[nT + \varepsilon_t] \\ e^{j\varphi[nT]} \approx e^{j\varphi[nT + \varepsilon_t]} \end{cases} \quad (5.22)$$

The second condition is much less stringent, so the attention will be centred in the first one. Considering for simplicity a carrier signal in the IF frequency, it yields:

$$\tilde{s}[nT] = \exp[j2\pi f_c(nT + \varepsilon_t)] = \exp\left[j2\pi \frac{f_c}{f_s}(n + \varepsilon_t f_s)\right] \quad (5.23)$$

It can be appreciated that the error adds a spurious phase offset. Limiting its rms (root mean square) value to 1° , which is a negligible degradation taking into account the discussions from 3.5.4.1, and bearing in mind that $f_c = 70\text{MHz}$, then:

$$2\pi f_c \sigma_t \leq \frac{\pi}{180} \Leftrightarrow \sigma_t \leq 40\text{ps} \quad (5.24)$$

σ_t is the time jitter rms of the sampling clock. This requirement translates in turn into a maximum total phase noise, as well as a threshold SNR for the clock (Smith 2004):

$$\sigma_\theta = 2\pi f_s \sigma_t \leq 1.5^\circ \quad (5.25)$$

$$\text{SNR}_{\text{clk}} = -20 \log \sigma_\theta \geq 31.7 \text{ dB} \quad (5.26)$$

f_s is equal to 105 MHz. It can be appreciated that the phase jitter in the clock can be larger than the one set in the signal, due to the fact that both SNR are related by the following formula:

$$\text{SNR}_{\text{sig}} = \text{SNR}_{\text{clk}} + 20 \log \frac{f_s}{f_c} \quad (5.27)$$

Of course, SNR_{sig} refers only to the noise component due to phase noise in the clock, not to additional contributions from thermal noise. The phase noise of the 10 MHz reference from ADAC Sync has the following specifications:

- - 83 dBc/Hz @ 10 Hz.
- - 115 dBc/Hz @ 100 Hz.
- - 135 dBc/Hz @ 1 KHz.
- - 140 dBc/Hz @ 10 KHz.

This noise can be integrated with linear approximations, yielding (Brannon 2004):

$$n_{\text{clk}} = \sum_{i=1}^n 10^{\frac{N_o^{(i)} + N_o^{(i+1)}}{20}} (f^{(i)} - f^{(i+1)}) + 10^{\frac{N_o^{(i+1)}}{10}} (BW_{\text{clk}} - f^{(i+1)}) \quad (5.28)$$

$N_o^{(i)}$ represents the noise spectral density at frequency $f^{(i)}$, and BW_{clk} represents the maximum bandwidth of the clock generator, which, for ADAC Sync, is 350 MHz. Using the specifications, total noise equals to -54.5 dBc. This value in turn translates into a phase jitter of:

$$\sigma_\theta = \sqrt{2 \times n_{clk}} = 0.15^\circ \quad (5.29)$$

This meets the requirement set in equation (5.21).

5.2.3.3 *Timestamp Accuracy*

The last aspect of the F&T is its timestamp functionality. The timestamp is used for several reasons:

- Combined with the STDM file, it is one of the ways to obtain the a priori DoA estimation.
- Combined with the STDM file, it is used to compute a nominal trajectory to be compared with Kalman filtered outputs such as to discriminate potential in-band interfering signals.
- Combined with the STDM file, it is used for results plotting and comparison between estimated and nominal trajectories.
- Combined with the MDP samples, it serves as a time stamp for moving satellites used in beacon and autocalibration routines.
- Combined with the WSF output, it is given to the Kalman filter as a time stamp for proper trajectory estimation.
- Combined with the output of the Kalman filter, it is given to the main antenna so that it knows when to correct its steering.
- Combined with the steering information from the main antenna, it is used to perform proper coordinate frame transformations. The main ones are relative (local spherical coordinates or el/xel offsets) and absolute (azimuth/elevation or ECI coordinates).

Clearly, it is the last three applications which require the greatest accuracy, and thus the ones which will determine the timestamp accuracy requirement in SARAS. The idea is that accuracy should be such as to guarantee that the satellite angular movement does not imply a great bias in DoA estimation. Taking a satellite with $1^\circ/\text{s}$ speed, and fixing the maximum allowed bias to 0.01° , the accuracy requirement becomes:

$$\varepsilon_{UTC} \leq \frac{0.01}{1} = 10 \text{ ms} \quad (5.30)$$

This is not too stringent, even taking into account latencies in the communications between subsystems.

5.2.4 *Calibrator*

As explored in chapter 4, calibration routines include:

- Photogrammetry.
- S-parameter estimation.
- Test signal injection:
 - Through RF couplers.
 - Through a collimation tower.
 - Through a sub-reflector probe.

Photogrammetric hardware is summarised below:

Element	Model	Comments
Camera	Canon PowerShot SX110 IS	COTS
Extended flash		Required at night to compensate the distance to the retro-reflective targets
Retro-reflective targets		-
Distance scale bar	Brunson 803-MCP	<p>Length tolerance of ± 0.003 mm. with a total stack up error of ± 0.013 mm.</p> <p>The thermal expansion coefficient is as low as $1.26 \cdot 10^{-6}$ mm/°C for a temperature range of -18°C to 93°C.</p>

Crane		-
Processing software		-

Table 5.10 Hardware for photogrammetric calibration

The first stage in the photogrammetric procedure is the target installation. During this stage a set of retro-reflective targets is placed on the main antenna and the array elements. Only the coordinates of the point where a target is located are calculated. The number and positions of the targets generally depends on the information to be extracted from the object. In the case of SARAS two circumferences consisting of 48 targets and 1 target in each array sensor have been placed to calculate the plane formed by the array sensors and the one formed by the main antenna, and thus correct the residual misalignment that was commented in chapter 4.

Targets are made of a retro-reflective material. In this way, it is possible to achieve a high contrast between the background and the target, so their location is easier to estimate and much more precise because the edge is clearly recognized. In addition to the targets, two scale bars have also been attached in random positions. Those bars are used to provide a robust length reference and to properly define the quality of the photogrammetric measurement. The accuracy will be directly affected by the accuracy in the length of the scale bars so the use of a well determined bar with low a thermal expansion coefficient was preferred.

The disposition of retro-reflective targets and scale bars in Vil-1 antenna is shown below:

Figure 5.11 (a) Overview of photogrammetric targets. (b) Scale bar

The next steps are the photo-shoot campaigns. During these a set of pictures of the object have been taken. Although pictures can be taken either during the day or during night, poor contrast was obtained from targets because of sunlight. Thus, only night-time pictures have been finally taken into account.

Figure 5.12 Ambient light effects (from left to right): low target reflection, excess of reflection and correct contrast during the night

In order to be able to take pictures from different locations a crane was required. As the whole Vil-1 antenna had to fit into a single picture, a critical aspect especially for higher elevations, the crane range needed to be large enough. During VIL-1 photo-shoot campaigns a 32 metre crane was used to obtain the pictures.

Figure 5.13 Different pictures of VIL-1 from the crane and of the crane itself

With the pictures the software processing can be carried out. During this stage several steps are required. The first of them is the point marking, i.e., indicating to the software where the targets are located. Once the points are marked the next step is called “Target Referencing”, a process in which a target is related to itself through different pictures. After this step the software is able to trace lines of sight that intersect at the location of the target and thus, a fairly accurate provisional position can be assessed.

Finally, after all the targets have been marked and referenced, the measurement project can be processed to solve the coordinates of the targets and the positions and orientations of the cameras in a single process called “Bundle Adjustment”. The model derived can be appreciated in the next figures:

Figure 5.14 (a) Picture at night. (b) Output from PhotoModeler

The most sensitive coordinate when calculating the location of the array sensors is the one that lies within the pointing axis of the antenna, as shown in chapter 4. The following table summarizes the accuracies obtained when evaluating the location of the antennas with respect to this coordinate. The accuracies are given by the location estimation software.

Elevation (degrees)	Accuracy (mm)
30	0.361
40	0.419
50	0.418
60	0.511
70	0.471
80	0.564

Table 5.11 Photogrammetric accuracies obtained during SARAS campaigns

Measurements have been taken at different elevations to account for the gravitational deformations of the main dish. After all the previous steps, the coordinates of the array sensors are obtained, along with their variation with elevation when compared to a reference position.

Table 5.12 Z-coordinate variation for each elevation and each antenna

Regarding S-parameter estimation of analogue components, a Vector Analyser was used. Some of the S-parameters phase and amplitude offsets are shown below:

Figure 5.15 Amplitude – (a) and (c) – and phase – (b) and (d) – offsets obtained from S-parameters for RF cables between antennas and LNA and for high-stability test cables

The use of on-line calibration schemes, though, as discussed in chapter 4, implies that S-parameters are no longer required in SARAS.

Test injection comes in three different formats. The hardware required for test signal injection through couplers was already discussed in section 5.2.1.1. The sub-reflector probe was part of the existing Vil-1 antenna, and is controlled independently from SARAS subsystems. The same applies for the collimation tower, a picture of which is shown below:

Figure 5.16 Collimation tower from ESAC station

Calibration algorithms, except those performed by PhotoModeler, are carried out in the SC, and are accessible through the GUI, detailed in the next section.

5.2.5 M&C and GUI

The functionalities associated with the M&C and the GUI are allocated to the SC, and implemented in a Matlab environment. The next figures present some snapshots of the GUI:

Figure 5.17 GUI main page

Figure 5.18 GUI mode selection

Figure 5.19 GUI results analysis

The main modes of operation are:

- M&C, where some of the subsystems and associated parameters can be monitored and controlled.
- Acquisition, where the WSF and Kalman functionalities are implemented, along with on-line calibration.
- Calibration, where off-line routines are implemented.

Additional functionalities include:

- Mission and ground station configuration, where parameters associated with the desired spacecraft and ground station can be set.
- Programming of acquisitions at defined times.
- Logging information.
- Plotting of stored results and statistics from real acquisitions.

The M&C is carried out through several interconnections between devices, centred in the SC, and summarised in the next figure:

Figure 5.20 Hardware architecture

As can be appreciated, the devices are collocated in the Vil-1 cabinet, where SARAS rack is placed, and communicate with the SC, placed in the MER (Main Equipment Room), through a fibre optic.

5.2.6 Summary of Physical Allocation

The next table summarises the allocation between functionalities and hardware elements that has been discussed in the previous subsections:

SARAS PHYSICAL ALLOCATION	
Subsystem	Hardware elements
RF/IF	Antennas LNA, filters, couplers, mixers and RF cables
F&T	Signal generator ADAC Sync GPS antenna
Digital Processor	ADC and FPGA boards Host-PC SC
Calibrator	Photogrammetric equipment Vector Analyser Signal generator Sub-reflector probe Collimation tower SC
M&C and GUI	Switch USB hub SC

Table 5.13 Summary of SARAS physical allocation

5.3 Tests

Tests using SARAS prototype were carried out incrementally, with the following progression:

- Injection of test signals through the couplers to verify the status of all RF/IF chains.
- Use of the collimation tower in ESAC and the subreflector probe to include the antennas in the loop.
- Reception of signals from GEO satellites, firstly considered due to their high availability.
- Reception of signals from target missions, slow and fast-moving, combined with off-line and on-line calibration of the array.
- Testing of advanced scenarios such as the presence of interferers and the use of Kalman filters.

Details of the setup as well as the results are given in the subsequent sections.

5.3.1 Test Signal Injection and Collimation Tower Tests

The first tests were performed, as mentioned, using the test signal generator and the couplers in the RF/IF chains, to verify their integrity. Received signals, prior to digitisation, were monitored in a Spectrum Analyser, as shown below:

Figure 5.21 Spectrum of test signal in one of the array branches

Signal injection was also used to establish the power level profile of each array branch, as approximated in Table 5.5. On the other hand, the Collimation Tower was initially considered as a way to perform off-line LS calibration of the residual errors in the array, especially in the positions of the radiating elements. Unfortunately, it was found out that the elevation of the tower with respect to the Vil-1 antenna, of 4° approximately, was below the threshold where multipath starts to severely degrade the estimates. This situation motivated the development of the combined off/on-line calibration scheme that was discussed in chapter 4.

5.3.2 GEO Signal Reception

Two GEO satellites were used in this set of tests, XTAR and Spainsat. While initially the aim was to test all the analogue equipment, including the antennas, as discussed in chapter 4, these satellites were also used to perform off-line calibration of the array, combining their signals with a frequency multiplexed test one from the subreflector probe in Vil-1.

This off-line calibration model, obtained with either GEO satellite, enabled afterwards to estimate the DoA of each GEO with on-line calibration. Next figures show WSF spectrums of both GEO, located in the broadside direction of the array.

Figure 5.22 WSF spectra of GEO satellites. (a) XTAR. (b) Spainsat

Both spectra were obtained using an off-line calibration model from XTAR. Clearly, this model is better suited for the very satellite whence it was obtained in the first place. However, it is still valid for Spainsat, even if the peak in WSF is less pronounced.

Frequency spectra of XTAR, combined with the probe test signal prior to and after filtering are also included below:

Figure 5.23 Frequency spectra of XTAR. (a) XTAR + probe. (b) After probe filtering

5.3.3 Target Missions Acquisition Tests

This section includes the main results of the system, as it describes the tests that were performed with some of the target missions, and some additional ones, and which enabled to demonstrate the validity of SARAS prototype. The statistical processing scheme applied to obtain the DoA is the one discussed in chapters 3 and 4, based on the algorithms from chapter 2. Namely, calibration is divided into off-line and on-line stages, and the statistical information from the resulting array manifolds is extracted using a combination of Eigen and WSF algorithms. In the presence of a carrier signal, a pre-FFT filtering is also considered.

5.3.3.1 Broadside Acquisition of Cryosat-2

Cryosat-2 was one of the first satellites to be acquired. As was commented in Table 5.6, the use of FFT is required to close the link budget with sufficient reliability. On the other hand, the probe signal was injected with a 100 KHz offset. The next figures show the WSF spectrum of the received signal, while impinging from broadside direction:

Figure 5.24 WSF spectrum for Cryosat-2- (a) Upper view. (b) Normal view

As can be appreciated, the true elevation and cross-elevation offsets are estimated with great precision and accuracy, well below the threshold of 0.28° . For this satellite, a 10.25 dB CNR threshold was established, so that estimations performed above it were always reliably performed. SNR, or CNR, estimations are carried out with the help of the eigenvalues from the correlation matrix. The output SNR for one signal can be estimated as:

$$SNR_{out} = \frac{\hat{\lambda}_1 - \frac{1}{N-1} \sum_{i=2}^N \hat{\lambda}_i}{\frac{1}{N-1} \sum_{i=2}^N \hat{\lambda}_i} \quad (5.31)$$

$\hat{\lambda}_1$ is the estimated dominant eigenvalue. This is true if no interferer is present, and will be considered as the baseline result. In any case, a more detailed analysis of SNR and C/No thresholds will be performed in section 5.4.

5.3.3.2 Low Elevation Acquisition of Metop-B

One of the objectives of SARAS system is to acquire launchers. Due to the criticality of this scenario, the acquisition must be carried out as soon as possible, with low elevations. At these, degradation due to multipath and correlated noise coming from the ground may limit proper performance. During the test campaigns, several acquisitions were done at low elevations. The figures below show the WSF spectrum of Metop-B for high elevation, above 15° , and at 5.4° :

Figure 5.25 WSF spectrum for Metop-B (a) High elevation. (b) 5.4°

Although the sharpness of the peak decreases in the second case, the estimation is still valid, as desired. CNR threshold was found to be around 16.5 dB. Next figure shows the actual WSF performance, as a function of the actual elevation of Vil-1 antenna, for the same satellite:

Figure 5.26 WSF performance as a function of Vil-1 elevation

In the figure the acquisition limit is approximately 3°, although the actual limit was found closer to 5°, as was specified in SARAS requirements, and as was observed with collimation tower tests.

5.3.3.3 Acquisition of *INTEGRAL* at the Edge of the Acquisition Window

Acquisition cannot be limited to broadside direction. Indeed, in real scenarios, the uncertainty that must be eliminated may imply that the steering of the main antenna is incorrect and that the signal is close to the uncertainty window's edge. It is thus vital to know the acquisition range of the algorithms. This was established using Integral, among others:

Figure 5.27 Figure 1. WSF spectrum for Integral. (a) Broadside acquisition. (b) Acquisition with 0.9° of radial offset

As can be seen, WSF correctly predicts the offsets of the received signal, for uncertainty windows of up to $\pm 0.9^\circ$ in elevation and cross-elevation angles. Larger windows produce incorrect estimates for some acquisitions, due to the presence of grating lobes. These originate from the fact that the inter-element distance in the array is greater than half the wavelength, as discussed in chapter 2.

A possible improvement of the system is to rely on non-uniform distributions of the array elements, so that the influence of grating lobes is minimised and the acquisition range can be increased (cf. section 3.5.2.5). A trade-off ensues, though, between grating lobe and white noise robustness, which must be carefully modelled. A possible configuration in this case is a non-redundant array.

The next figure shows the frequency spectrum of the received Integral signal, and gives an idea of the low SNR that are implied in these scenarios and, thus, demonstrates the robustness of the current system against white noise:

Figure 5.28 Frequency spectrum of probe + Integral signal

The peak on the left side of the spectrum is the carrier of the sub-reflector probe, whereas the small peak on the right side is the one from Integral. This satellite has been correctly acquired with ranges above 100 000 km.

5.3.3.4 Doppler Compensation with Metop-A

The use of FFT processing and carrier acquisition is limited by frequency shifts of said carrier as the samples of the signal are acquired. One of the main contributions comes from Doppler shift, and can thus be compensated prior to frequency processing using trajectory information derived from STDN files. This translates into improved performed in low signal level scenarios. The next figures show the frequency spectrum of the signal from Metop-A satellite, prior to and after Doppler compensation:

Figure 5.29 Metop-A frequency spectrum (a) Prior to Doppler compensation. (b) After Doppler compensation

In the second case the frequency dispersion is greatly reduced, and less frequency bins can be selected. In real scenarios, where the orbits of satellites are not perfectly known, residual errors in Doppler estimation may be around 10%.

5.3.3.5 Acquisition of Swarm 1 in LEOP

The system was tested with Swarm 1 satellite, shortly after its launch, on Friday 22nd of November, 2013, while the orbit was not stable enough, which is precisely the main scenario that SARAS was designed for. The next figures show the frequency and WSF spectrums that were obtained:

Figure 5.30 Swarm 1 spectra. (a) Frequency spectrum. (b) WSF spectrum

5.3.4 Tracking Tests

While SARAS is mainly conceived as an acquisition aid, it has been designed to include tracking and prediction features, which can be useful in the first stages prior to the hand-out to the main antenna. Tests were thus carried out in this sense, and are presented in the next subsections.

5.3.4.1 Acquisition of XMM with a Motionless Antenna

Prior tests were performed while Vil-1 antenna was tracking the desired satellite. However, in real launcher acquisitions, it may be that the main antenna is in park mode, motionless, while waiting for an improved estimate of the launcher's DoA. It is thus mandatory to know how far away from broadside can a given satellite be, with SARAS still being able to perform correct estimates. This parameter has been termed SARAS FoV (Field of View). It must not be confused with the acquisition window range, though. The next figure, not to scale, shows the difference between these two FoM:

Figure 5.31 Acquisition range and FoV

In order to estimate the FoV of SARAS, tests were carried out with XMM, with a motionless Vil-1 antenna. The next figures show the true elevation and azimuth trajectories, in green, as derived from the STDM file, and the ones estimated with WSF, in blue:

Figure 5.32 XMM DoA estimation with a motionless Vil-1 antenna. (a) Elevation. (b) Azimuth

As the satellite moves away from broadside, the system is able to correctly track it. In these tests it was established that SARAS FoV was above 15° , and met the specified requirement derived from a priori simulations. It is important to note that this functionality is unique to array system, and stems from the low directivity of each of the radiating elements. Its main advantage is that it gives the system further reliability, as it extends the time period where a valid acquisition can be performed. Indeed, for single-

antenna acquisition aid systems, there is a very small time opportunity to correctly estimate the DoA, which renders the overall process all the more critical.

As a side note, while it has been pointed out in chapter 4 that degradation due to position calibration errors is mainly due to the z-coordinate, this is no longer true when the satellite is far away from the broadside direction of the array. In any case, as proven by the test campaign, wide FoV are achievable with real satellite signals.

5.3.4.2 Kalman Tracking

The Kalman filter has been tested with the same set of measurements as in the previous section, taking advantage of STDM files with very high resolution. The design of the Kalman filter is based on the one presented in section 3.4.3, with uniform acceleration in the ECI coordinates state vector. The measurement error matrix, Q_2 , is set to:

$$Q_2 = \begin{bmatrix} \left(0.5 \frac{\pi}{180}\right)^2 & \left(0.05 \frac{\pi}{180}\right)^2 \\ \left(0.05 \frac{\pi}{180}\right)^2 & \left(0.5 \frac{\pi}{180}\right)^2 \end{bmatrix} \quad (5.32)$$

Diagonal elements give some margin to the threshold set to SARAS, of 0.28° . Non-diagonal elements take into account the non-linear nature of the measurement equations. Q_1 has been estimated by correlating the STDM state vector with a delayed version of itself. As it is, this is a realistic scenario for launchers, where the uncertainty sometimes is more related to delays in the time of launch than to the trajectory itself. This justifies the use of high resolution STDM files for this test.

However, since XMM is a slow-moving satellite, the simulation must be adapted to test what would happen in a fast-moving satellite scenario, as is the case with launchers. Namely, in the test, the XMM speed was around $0.015^\circ/\text{s}$, whereas in launchers at low elevations speed can go up to $1^\circ/\text{s}$, i.e., multiplied by a factor of 70. Therefore, to reproduce the launcher uncertainty window in ECI coordinates, the typical uncertainty of 1 second in the time of launch has been multiplied by 70 in these tests, yielding a Q_1 matrix which expresses the correlation of the STDM state vector with a version of itself delayed 70 seconds. Results are shown below for elevation and azimuth angles:

Figure 5.33 XMM DoA estimation with Kalman filter. (a) Elevation. (b) Azimuth. (c) Elevation error. (d) Azimuth error

The plots clearly show that the behaviour of elevation and azimuth filtered outputs is very different, which stems from the fact that their measurement equations differ. One is based on the $\tan^{-1}(\quad)$ function, the other one on $\sin^{-1}(\quad)$. The elevation Kalman trajectory starts with an erroneous estimate, due to the use of a delayed STDM file for the computation of the ECI coordinates, but quickly corrects itself and converges to the measurement estimates. On the other hand, azimuth takes longer to converge, but seems to achieve smoother estimates after a certain number of iterations.

The results suggest an improved method for elevation filtering. Indeed, the convergence to measurement estimates indicates that the Q_1 is too large compared to the measurement error. In order to correct this, it is proposed that once elevation starts to converge, after the 5th iteration, the Q_1 matrix be updated with the following equation, where k is the Kalman iteration:

$$Q_1^{(k)} = Q_1^{(k)} \frac{\left(el_{kalman}^{(k-1)} - el_{WSF}^{(k-1)} \right)^2}{\left(el_{kalman}^{(1)} - el_{WSF}^{(1)} \right)^2} \quad (5.33)$$

The new results are:

Figure 5.34 XMM DoA estimation with Kalman filter and modified Q_1 matrix. (a) Elevation. (b) Elevation error

The new elevation trajectory further reduces its oscillations. The preceding tests took only into consideration the filtering capabilities of Kalman algorithm. However, the filter is also used for prediction. In this case it is interesting to know how far can the filter produce predictions which are below the 0.28° threshold:

Figure 5.35 XMM DoA prediction

The limiting angle is the elevation one, and the maximum time offset is 70 seconds which, extrapolated to a faster moving satellite, implies a maximum time offset of around 1 second. The results show that there is a trade-off between Kalman convergence speed and its predicting capabilities, as in this case the azimuth trajectory is much better behaved. In any case the iterative modification of the Q_1 matrix improves the behaviour of the elevation parameter:

Figure 5.36 XMM elevation prediction comparison

The preceding conclusion is true as long as 20 estimates are used in the Kalman filtering stage, but the situation is reversed when only 10 are considered, for example. This stems from the fact that the prediction time margin depends on how many estimates are used in the filtering stage, and that there is a minimum number of estimates required for an optimised prediction behaviour. The next figure shows the elevation prediction errors for different sets of filtering estimates:

Figure 5.37 XMM elevation prediction with several filtering estimates

For 5 estimates, the maximum time offset is 15 seconds, i.e., 20% of the one allowed with 20 estimates. For 10 estimates it is 35 seconds, and for 15 it is 70 seconds, as with 20. It seems that in this scenario, the maximum prediction capabilities are attained when considering 15 estimates in the filtering stage. Tests also show that at this point the modification of the Q_1 matrix starts improving the prediction stage:

Figure 5.38 XMM Kalman prediction with several filtering estimates and Q_1 estimation procedures

5.3.4.3 Tracking over Long Time Periods

INTEGRAL spacecraft was tracked during more than three hours, taking advantage of its highly eccentric orbit, yielding 600 acquisition estimates. Next figures show the trajectory in elevation and azimuth angles, as estimated by SARAS, compared to the true one.

Figure 5.39 INTEGRAL tracking over a long time period. (a) Elevation. (b) Azimuth

Corresponding tracking errors are shown below in both coordinate references, elevation/azimuth and elevation/cross-elevation:

Figure 5.40 Tracking errors. (a) EI/Az. (b) EI/Xel

The maximum allowed error is shown in figure (b). Although it is the total radial error that must be compared with this baseline in order to evaluate the performance of the system, as in the next figure:

Figure 5.41 Total tracking error

It is clear that most of the time, around 96.5% of the 600 estimates, the tracking error is within bounds. However, it may be desirable to increase the reliability of the system even further. One way to achieve this is to discard estimates where the C/No, obtained as a function of the CNR, is below a given threshold. The figure below shows the error percentage that can be achieved, as a function of the C/No threshold chosen by the end user:

Figure 5.42 Error percentage as a function of C/No threshold

It is clear that, as the threshold increases, so does the reliability, i.e., the error percentage decreases. A reliability of 99.7% corresponds to a C/No threshold of 17.2 dBHz. The price to pay for this increased robustness is a lower availability, since some of the estimates cannot be used for the acquisition process. This availability percentage is plotted below, as a function of the threshold:

Figure 5.43 Availability percentage as a function of C/No threshold

For a reliability of 99.7%, the maximum availability is almost 50%. Reduced availability translates into increased acquisition time, on average, with the following formula:

$$t_{eff} = 100 \times \frac{t_{single}}{availability (\%)} \quad (5.34)$$

t_{single} is the acquisition time of a single estimate, and t_{eff} the effective time for the required reliability.

5.3.5 Interferer Discrimination

The robustness against interfering signals is a very valuable characteristic of the system, and a clear improvement over traditional solutions. A test was carried out using the signal from the GEO satellite XTAR, and an internally generated interferer using the test injection via couplers. The resulting frequency spectrum is shown below:

Figure 5.44 Frequency spectrum of XTAR and test signal injected via couplers

The highest peak corresponds to the interfering signal and, as can be appreciated, they have been made to almost coincide in frequency so that no FFT filtering can be applied. Even in this case, the system provides several WSF spectrums, one of which corresponds to the desired signal:

Figure 5.45 Figure 2. WSF spectrums. (a) Interferer spectrum. (b) XTAR spectrum

The right hand spectrum correctly shows the true DoA of the signal. The system a priori cannot tell which spectrum corresponds to the interferer and which one to XTAR, but this can be established at a later point, using EKF and trajectory discrimination based on STDM files, or by determining which spectrum has the sharpest peak, for example. The important conclusion of this test, though, is that accurate DoA can be estimated, even in the presence of interferers with a higher power level than the desired signal's, and with the same frequency.

The performance of the modified AIC algorithm using XTAR has been compared with simulation data, as in section 5.4. The figure below shows that good fitting is achieved:

Figure 5.46 Modified AIC performance. (a) Estimator statistics. (b) Comparison with simulation data

The improvement obtained with XTAR with respect to simulations is due to the large power offset between desired and interferer signals, greater than 20 dB.

5.3.6 Calibration Results

5.3.6.1 Calibration Stability

This section shows some more extensive results concerning the stability of the off-line phase offsets calibration using GEO satellites, with samples taken several days apart. The first figure shows the phase stability of the difference between the XTAR GEO phase response and the one with the probe with a 100 KHz frequency offset (cf. chapter 4 for more details on the calibration scheme).

Figure 5.47 Phase stability of XTAR – probe scheme

From the plots it is clear that all array branches show instabilities in the analysed time frame, except for the 6th one. Instabilities take days to appear, but they usually imply huge changes in phase response. The next figures show both the contributions from XTAR and from the probe, to determine which one is more stable:

Figure 5.48 Phase stability. (a) XTAR. (b) Probe

Both signals show instabilities. There is a drift contribution which tends to disappear when their difference is taken, which indicates that it is the same for both signals. Measurements have also been taken in the short term:

Figure 5.49 Phase stability in the short term for the XTAR – probe scheme

Stability is clearly guaranteed at least during one hour, which is acceptable taking into account that this calibration can be performed prior to acquisition. These results justify the fact that the system warns the end user when off-line calibration files are older than one day. In any case, future implementations of SARAS will be more stringent in terms of phase stability.

5.3.6.2 Calibration as a Function of Frequency

As discussed in chapter 4, calibration models are frequency dependent, so that off-line calibration using a given GEO satellite may not be directly extropolable to other frequencies. This was verified acquiring XTAR and Spainsat satellites, which are up to 10 MHz apart. Resulting errors in estimations were in the order of 0.1° , which is not dramatic, but may be so for satellites with increasing distant frequencies.

The frequency dependence has been estimated using a test setup which combines both test signal injection routines simultaneously, using the signal via couplers as a calibration model, and the signal from the probe as a satellite proxy. Elevation and cross-elevation misalignments, as a function of frequency, are plotted below, taking 2250 MHz as a reference:

Figure 5.50 Elevation and cross-elevation misalignments

From these measurements three conclusions are drawn:

- There is, indeed, a strong frequency-dependence which needs compensation, if a GEO calibration is to be used for LEO satellites with differing frequencies. This is especially true for the cross-elevation angle.
- The dependence is approximately linear, with the exception of the 2300 MHz frequency. The band limit may imply a group delay perturbation which is affecting the estimation at that frequency.
- The frequency-dependence, from a relative point of view, is stable for at least one day.

Frequency misalignment, though, as already commented, is partially compensated completing the calibration with the on-line phase. In any case, residual errors are acceptable for the considered RF frequency band, between 2.2 and 2.3 GHz. Migration to other bands may need an upgrade in the calibration scheme, as briefly discussed in chapter 6.

5.4 Comparison with Simulation Data

The objective of this final section is to compare some of the simulation results exposed in chapter 3 with actual system performance using satellite signals and the prototype installed in Vil-1 antenna. Of course, in order to obtain significant statistical results from real satellites, measurements in the same conditions ought to be performed for a number of iterations. Since this is not easily accomplished, it is important to mention that, while real measurements were obtained for a given SNR, extra results have been derived with artificially increased noise or other type of degradation errors using Matlab routines and Monte Carlo simulations. Unless

otherwise stated, scenarios will be set out for an 8-element array, operating in S-band, and 100000 samples will be processed.

5.4.1 SNR Threshold Analysis

The simulated performance of the system in the presence of Gaussian white noise was explored in section 3.5.2.1. The analysis is expanded in the next figures, including several statistical bounds: the CRLB, the second-order Bhattacharyya bound, and the Barankin bound with the Chapman-Robbins formulation, as well as real measurements obtained from CALIPSO and Pleiades spacecraft.

Figure 5.51 WSF estimator standard deviation in the presence of white noise

The results were obtained with an acquisition window of $\pm 0.9^\circ$ in elevation and cross-elevation. As already commented in chapter 3, there is a clear threshold at -24 dB of input SNR, with pronounced degradation of WSF simulated performance below it. This threshold effect is partly captured by the Barankin bound, with a 1 dB offset, due to the fact that the bound requires the definition of test points which do not fully contain all statistical information of extreme events. Regarding real measurements, those corresponding to CALIPSO are close to the simulated results, especially for SNR above -22 dB. Pleiades, on the other hand, is slightly degraded. For CALIPSO, the signal was coming from the broadside direction, whereas in Pleiades an offset (0.2° in elevation and 0.3° in azimuth) was introduced in the main antenna, which may account for the degradation.

Detailed statistical information from both spacecraft is given in the figures below, further demonstrating the improvement when considering broadside steering:

Figure 5.52 CALIPSO SNR statistics. (a) Normal view. (b) Zoom

Figure 5.53 Pleiades SNR statistics. (a) Angular offset statistics. (b) Comparison with broadside

As explained, real measurements were obtained for a given SNR, and additional results have been derived with artificially increased noise using Matlab routines and Monte Carlo simulations.

5.4.2 Array Topology

The use of distributed topologies was justified by the increased resolution and white noise robustness. However, it was found that increasing the size of the array comes at the expense of the presence of grating lobes, thus limiting the maximum acquisition range that can be achieved. This is summarised in the next figure, using as a comparison real results, with an SNR of -21 dB, from Pleiades spacecraft:

Figure 5.54 Acquisition range for different array radii

Good fitting between simulated and real results has been verified, with a slight improvement for the data from Pleiades, due to the fact the spacecraft was not located at the edge of the acquisition window, but with an offset of 0.2° in elevation and 0.3° in azimuth.

Complete statistics are provided below for Pleiades spacecraft, taking also into account the improvement achieved at higher SNR:

Figure 5.55 Pleiades acquisition range statistics. (a) Fixed SNR. (b) Several SNR

Pleiades has also been used, at an SNR of -21 dB, to analyse the effect of eliminating some of the radiating elements of the array, as shown below:

Figure 5.56 Pleiades statistics as a function of array size

As desired, there is some robustness in the system, which can still operate at the edge of performance with 6 or 7 elements. In any case, it is justified that 8 elements were a good compromise between reliability and cost considerations.

5.4.3 Calibration Errors

SARAS is sensitive to calibration errors in the modelling of the SV, namely, to uncompensated phase offsets in the array branches, and to errors in the determination of the position of each radiating element's phase centre. Unbalances in amplitude response, on the other hand, do not affect performance, since it is the phase of the SV which contains the DoA information that needs to be extracted. This is not the case in more complex scenarios, such as in the presence of strong multipath, but, as verified during the test campaigns, multipath levels are low for elevations above 5° .

Next figures show the performance of WSF in the presence of calibration errors, both simulated and with real data from CALIPSO spacecraft, at an SNR of -22 dB. In both scenarios, calibration errors have been added to the models used for the SV.

Figure 5.57 WSF performance with phase calibration errors

While the threshold for the simulated performance is around 18° , it decreases to $10\text{-}15^\circ$ for CALIPSO. Simulated results are slightly below the Lower Bounds because they considered uniform random errors, whereas the bounds were specified for Gaussian ones.

The figure below clearly underpins the importance of proper position calibration, as errors beyond 2 mm can cause severe degradation.

Figure 5.58 WSF performance with position calibration errors

Detailed statistics for CALIPSO are provided below:

Figure 5.59 CALIPSO calibration statistics. (a) Phase errors. (b) Position errors

5.4.4 Moving Signal and Frequency Processing

In the presence of signals with non-suppressed carrier, as already discussed, great improvements may be achieved with pre-FFT processing, as those frequency bins close to the carrier have higher signal power.

The next figure shows the performance of the combined FFT and WSF processing, as a function of C/No, with simulated and real data from Cryosat-2 and INTEGRAL satellites. In all cases, frequency processing down to a bandwidth of 35 Hz has been performed, even if narrower processing may be optimum for perfect knowledge of Doppler shift.

Figure 5.60 WSF performance with FFT processing as a function of C/No

The figure shows the 3.3 σ RMSE (Root Mean Square Error). For INTEGRAL, the compensation of the Doppler shift does not translate into a meaningful improvement. Indeed, the spacecraft presents a highly eccentric orbit and, during the acquisition, it had a very small angular speed. The effect of Doppler compensation is more clearly seen for Cryosat-2. Of course, perfect compensation of Doppler shift is not

achievable for spacecraft with unstable orbits, so that the final performance will be in an intermediate position. The differences between Cryosat-2 and INTEGRAL, with Doppler compensation, are due to some imprecision in the estimation of the actual C/No, taking into account the small number of processed samples.

In fact, it is interesting to note, though, that while the C/No threshold for INTEGRAL in this figure is 15 dBHz, it was found in section 5.3.4.3 that it ought to be increased to 17.2 dBHz for reliabilities of 99.7%. This further confirms that there may be some inaccuracies in the estimation of the C/No, which give a slightly optimistic performance in this case. Indeed, in the previous results, a threshold of 15 dBHz corresponded to a reliability of 99% approximately.

Complete statistics for Cryosat-2 whence the previous figure was obtained are provided below:

Figure 5.61 Cryosat-2 C/No statistics. (a) Without Doppler compensation. (b) With Doppler compensation

The degradation caused by the introduction of offsets can also be appreciated here, comparing previous results, which had a 0.4° offset in both elevation and azimuth, with those obtained for the broadside direction:

Figure 5.62 Cryosat-2 C/N₀ statistics with and without offsets

Doppler compensation does not yield the same results when angular offsets are introduced. On the other hand, in the previous figure, the broadside estimation error is not affected by Doppler compensation due to the fact that, at the moment of acquisition, the Doppler shift of Cryosat-2 was very small. Finally, Cryosat-2 broadside scenario achieves the same performance as with INTEGRAL. Statistics for INTEGRAL are also included:

Figure 5.63 INTEGRAL C/N₀ statistics. (a) Without Doppler compensation. (b) With Doppler compensation

5.5 Conclusions

An S-band array-based acquisition system, SARAS, has been designed and implemented in the frame of a contract with the European Space Agency. The system relies on a distributed topology, with radiating elements placed on the rim of a main reflector antenna for TT&C communications, with improved resolution capabilities. Electronic search coupled with robustness against thermal noise makes SARAS a solid alternative for state-of-the-art solutions in scenarios where low power level and fast

moving signals can be expected, and where reliable acquisition is critical, such as in LEOP.

This chapter has presented the hardware elements of the prototype, with the physical allocation of requirements and functionalities identified in chapter 3. Tests procedures to evaluate the final performance of the system have been discussed, and the main results associated with them have been evaluated, separately and compared with the simulation data from chapter 3. The next table summarises some of the FoM previously discussed, both for simulated and real scenarios.

FoM	Simulated value	Real value
Threshold SNR	-24 dB (without interferers) -22 dB (with interferers)	Between -22 and -20 dB
Threshold C/No	14 dBHz (for Doppler rates below 10 Hz/s, 35 Hz BW)	Between 15 and 17 dBHz (for compensated Doppler, 35 Hz BW)
Acquisition range	< +/- 0.9°	< +/- 1.1° ¹²
Phase calibration error	< 18°	< 15°
Position calibration error	< 2 mm @ SNR = -24 dB < 6 mm @ SNR = -23 dB	< 2mm

Table 5.14 SARAS FoM

From the table it is clear that there is a strong correlation between simulated and real values. Simulations, as well as the overall design of the system, are thus validated.

¹² Probably over-estimated due to limited amount of statistical information.

CHAPTER 6 CONCLUSIONS

6.1 Summary of Contributions and Conclusions

During the preceding chapters, the following contributions have been presented:

- The introduction of a novel acquisition aid system for satellites and launchers.
- The description of DoA estimation algorithms and related techniques, along with Monte Carlo simulations and associated error budgets.
- The description of a global calibration scheme, along with Monte Carlo simulations.
- The description and analysis of tests performed with a prototype of the system and real satellite signals, and the comparison of the results obtained therein with the former simulations.

SARAS project has sought to demonstrate the advantages of antenna arrays and statistical digital signal processing to obtain accurate estimates of the DoA of TT&C signals. Regarding the technological competitiveness of SARAS, it has been shown that, compared to other acquisition systems, its capacities are between the best that an aid system can offer. The advantages that have been explored are listed below:

- Increased white noise robustness, taking advantage of super-resolution algorithm and a large separation between array elements. This translates for example into a less stringent G/T requirement for each array RF/IF branch.
- Diminished mechanical impact, since each element is distributed in the whole rim of the main antenna, and their total weight is decreased.
- Increased angular coverage, due to the smaller physical extent of each one of SARAS antennas.
- Ability to simultaneously detect various angular separated sources. If more than one signal is present, the system can find their DoA in the angular spectrum. As a matter of fact, this is a way to discriminate sources of interference.
- No need to move the main dish during the acquisition process. The search procedure is electronic, allowing the main antenna to go on with its program track or stay motionless whilst the acquisition is done.
- Improved acquisition time as the whole uncertainty window is swept at once electronically, whereas with conical scan acquisition times of up to 1 minute are required.

The table below summarises some of the FoM of the system, derived from test campaigns with real signals:

FoM	Value
Tracking error	< 200 mdeg
Acquisition range	+/- 0.9°
FoV	> 15°
Threshold SNR	-22 dB
Threshold C/No	15 dBHz
Acquisition time	< 12 s
Elevation range	5 to 90°

Table 6.1 SARAS FoM

With the successful acquisition of real satellite signals, a TRL of 6 has been achieved with the current prototype. Despite this, some challenges are still present for the final implementation of a TRL 9 system. These will be presented in the next section.

6.2 Main Challenges and Proposed Future Developments

The main improvement for SARAS system is to consider its migration to X-band (SARAX), as is the case of XAA. Indeed, the X-band market is growing up and is mature enough in TT&C and satellite communications. The migration to X-band is a possible improvement of SARAS system in a scenario where its superior performance is critical. In making it fully operational, though, a number of challenges have been identified.

6.2.1 Reduction in Acquisition Time

The main challenge is the reduction of acquisition time. The current prototype performs successful acquisition in less than 12 seconds, but future scenarios will require an improvement in one order of magnitude, below the current FoM of XAA, around 10 seconds. To accomplish this, a migration of the signal processing stage from the Matlab platform to FPGA boards has been planned, combined with an optimisation of the M&C subsystem.

The FPGA implementation concept, along with the associated interfaces, is shown in the next figure.

Figura 6.1 Improved digital processing

6.2.2 Global Calibration with Multipath

As commented in chapter 4, one of the main problems of the current calibration scheme is that it relies on a GEO satellite which must have a similar frequency compared to the spacecraft to be acquired. While in S-band this is not a major issue, the presence of GEO satellites with X-band beacons for arbitrary geographical locations may not be guaranteed.

In order to eliminate this dependence, a new calibration scheme has been conceived, which makes use of a calibration signal from the sub-reflector probe, as in SARAS, and another one sent by a collimation tower in the far-field of the system. The use of collimation towers is limited by the degradation caused by multipath, so the new processing algorithm must combine statistical techniques such as Least Squares with spatial filtering to eliminate multipath components, while guaranteeing some robustness against white noise.

Another alternative to mitigate multipath would be to decrease the beamwidth of the individual array elements. This would cause in turn, though, a limitation in the FoV of SARAS, and an increase in the mechanical impact of the system.

6.2.3 Topological Analysis

The acquisition range of the system is currently $\pm 0.9^\circ$ in elevation and cross-elevation, with respect to broadside steering. The FoM is limited by the presence of grating lobes, and could be greatly improved by using non-uniform topologies, as already outlined in chapter 3. An exhaustive topological analysis should thus be carried out so as to increase the acquisition range at least up to $\pm 1.5^\circ$, while maintaining the current robustness of the system against thermal noise.

6.2.4 Kalman Filter

The prototype has a first version of the Kalman Filter, which can be improved with the following functionalities:

- Improved estimation of the error covariance matrices, so as to grant upon the filter a greater robustness against a variety of statistical scenarios.
- Improved non-linear equations system, for the transition matrix, with special emphasis on the modelling of non-uniform acceleration, based on the force system acting upon the spacecraft.
- Evaluation of alternative architectures, such as UKF or particle filters.

6.2.5 Launcher Acquisition

A special case is the acquisition of launchers, since they usually emit more than one signal at different polarisations and/or frequencies. In this case, a simultaneous acquisition can be performed with the received digital signal samples, if they are divided in different time slots. For the case of two desired signals, half of them will be processed in the RF/IF subsystem and the MDP with a given LO frequency and the other half with the other value. The remaining statistical processing will be performed for both kinds of time slots separately. This concept is shown in the next figure:

Figura 6.2 Launcher signal processing

The discrimination using the Kalman Filter is simpler, since the change in LO frequency renders one of the signals very weak in the time slots which have the parameters defined for the other signal, and vice versa. This is not true, though, if the frequencies are closely spaced, which is not usual, or if two signals are sent with the same frequency and orthogonal polarisations, while the antennas use a single polarisation scheme. In this case, a complete discrimination can be performed, as already discussed in chapter 3 when considering the presence of interferers.

It may be, for rotating launchers for example, that only one of two emitted signals can be received, but the system does not know a priori which one will be visible when the acquisition is started by the end user. In this case, signal discrimination must include an SNR estimator, which, as the WSF algorithm, uses the statistics of the received digital samples. Those time slots with the lowest SNR will correspond to the signal which is not visible, and thus with less reliable, if at all, DoA calculations. The others will be those to be considered valid. For this to work properly, the rotation of the

launcher must be slow enough so that the visibility or non-visibility of the signals does not change during the digital samples' acquisition time in the MDP.

6.2.6 Subsystem Improvement

Other improvements include:

- Use of linear instead of circular polarisation antennas. This enables the acquisition of both RHCP and LHCP signals, without the need to double the RF/IF circuits, at the expense of an extra 3 dB polarisation mismatch loss.
- Use of fibre optics for distribution of F&T signals, or replacement of RF cables. The main point here would be to carry out a cost-benefit analysis.
- M&C improvement using commercial software packages based on client-server architectures. This improvement must be coupled with the implementation of a bi-directional interface with the ACU of the main antenna, in order to be able to correct its steering with minimum delay.

6.3 Commercial Analysis

6.3.1 Market Applications

The main market for SARAS system is **ground segment stations devoted to TT&C, including LEOP stages**. Therefore, there are two major applications:

- **Fast acquisition of S-band satellites and launchers in LEOP stages**, where angular uncertainty may represent an issue even in S-band. The main application within ESA programmes is the acquisition of the Ariane and Vega rockets during LEOP. Due to uncertainties in the prediction of the ephemerides of the rocket the traditional conical scan made with a large dish takes a long time to converge. This limits the program track accuracy and the high angular speed makes it difficult to keep the launcher within the narrow beam of the large reflector antenna.
- **Fast acquisition in operational stages of X-band satellites**, when orbits are not stable enough. This may be the case the first days after LEOP, and also in case of malfunction or orbital manoeuvres.

Beyond these, due to the wider angular window of exploration, it becomes possible to **obtain the position of closely spaced satellites**. There are three cases that can benefit from it:

- **Clusters of satellites**. A lot of proposals are being made about distributed payloads (for remote sensing, communications) among closely spaced collaborative satellites. Angular determination and tracking can be made simultaneously for all these satellites from a single antenna coupled with SARAS acquisition aid system.

- **Collocated GEO satellites.** Simultaneous angular position determination of all satellites can be made with SARAS. In fact, even the acquisition of satellites in adjacent orbital slots can be made at the same time.
- **Signal discrimination when several satellites passes are concentrated at the same time,** by distinguishing between multiple satellites. When two or more satellites are in the antenna's FoV, the algorithm can easily distinguish and locate the satellites, which is not possible using the monopulse or conical scan methods.

Taking into account its signal discrimination capabilities, a possible spin-off application of SARAS is as an interference reduction system. As it is described in (ITU-R 2009), a distributed array in a reflector antenna can be used to improve FSS (Fixed Satellite Service) link performance when sharing the same frequency band with other services. I.e., an interference reduction technique can be implemented by using small adaptive antennas to work as a side-lobe canceller. The next figure illustrates this concept:

Figura 6.3 Interference Rejection Technique (ITU-R 2009)

6.3.2 Customers

The competitive and evolving nature of the satellite market, combined with the rapidly expanding customer needs, have led to a steady increase in the performance of ground stations over the years. Additionally, with the increasing complexity of satellite technology, as well as the rise in the number of satellites, the art of providing ground support is becoming highly challenging. End customers can be:

- **Ground stations owners** (commercial firms providing TT&C services as well as space agencies) that have to upgrade existing TT&C stations.

Owning a ground station or a complex of ground stations cannot really be separated from operating the facilities. If an owner does not carry out the operation of the ground facilities within his organisation, he will arrange with an outside company to operate the facility on his behalf. Although at this moment ground segment stations devoted to TT&C, including LEOP stages, communicate with the spacecraft in S-band, the ongoing trend to migrate TT&C to X-band will make critical the presence of acquisition aid systems in the near future in these stations. The owners of ground stations can be considered as being divided between the following categories:

- **Spacecraft Owners:** owners who are principally satellite operating organizations such as Intelsat, Eutelsat, Inmarsat, SES-Astra and ESA. These owners certainly have TT&C ground stations to control the satellites but will also have in most cases uplink stations related to the satellite payloads. For example, Inmarsat has shore station complexes that include several ground stations.

Likewise Eutelsat has broadcast uplink ground stations and communication stations to access the payloads. In the area of Earth Observation an organisation such as ESA has TT&C stations and data collection complexes within its direct ownership and control. ESA also owns and controls the ground stations for science missions.

- **Teleport Owners:** owners who lease satellite capacity. There is a large category of ground station operators who do not control satellites but lease satellite capacity from the satellite operating organisations. These ground station owners are principally concerned with communications traffic. They represent the majority of the ground segment operators in Europe, such as Deutsche Telekom and British Telecom, Individual VSAT (Very Small Aperture Terminal) Owners, VSAT Network Owners (e.g. Ford Europe) and small terminal users such as those for Inmarsat "Standard M".
- **Data Collectors and Distributors:** within the field of Earth Observation and Scientific Satellites there are several Ground Station sites that are not owned by the satellite providers such as ESA but collect and process data on behalf of specific groups of users. These are designated Data Collectors and Distributors such as SCC (Swedish Space Corporation) or Eumetsat.
- **Ground segment engineering companies and antenna manufacturers** in charge of building new stations. The first case will provide a peak of demand in the first years, while the second case will mean a steady need throughout time. Antenna manufactures such as ASC, Harris, HITEC, Vertex, INDRA, Orbital, Intertronics, Mechatronics, Thales, etc. may be interested on adding SARAS system in their antennas.

For instance, Orbital Systems currently designs and manufactures 1.8m - 3.0m tracking antenna systems including multi-band models that cover X-S-L band. Currently, they are releasing a new 2.4m/3.0m TT&C antenna system that downlinks a high data rate signal on X-band while up-linking in S-band, although a variation that supports a downlink in S-band is also possible. This configuration is useful for satellite control in S-band while obtaining a high rate signal for the payload in X-band. The 2.4m antenna is not an auto-tracking antenna, but Orbital Systems performs LEOP support using their built ACU for some applications^{13 14}. This works by allowing rapid real-time adjustment to the time offset of the ephemeris. The orbit is predicted for the launch, but the exact time of the launch is variable, so the initial ephemeris is loaded and then the tracking time is offset by the time offset of the lift-off itself. Once the satellite is in view, the operator of the antenna can also make more fine adjustments in time to maximise the signal.

Since their aperture size is relatively small, there is not a great need for auto-tracking. Indeed an X-band antenna with 3m diameter has about 1 degree -3dB beamwidth. In addition, if the downlink is performed in S-Band, then the -3dB point of the corresponding 2.4m antenna would widen out even more to about 3.8 degrees. Besides it is assumed that before the second pass updated ephemerides are available. This has worked successfully for several satellite launches they have done at KSAT up on Svalbard and at Tromso, Norway.

Obviously if the antenna aperture was much larger, 6m or more, this scheme would not work because the beam width would be too narrow. They have also resorted to the idea of doing the first pass acquisition with a large auto-tracking antenna such as the ones owned by KSAT. Therefore SARAS system might make sense in case they would build antennas with higher diameter.

- **Launchers companies,** There are also special needs in S-band in the field of launchers, where the dispersion window is wider. Companies such as Arianespace, ISC Kosmotras, among others, may be interested in having SARAS for launcher support.

On the other hand, it must not be forgotten that a lot of countries have **military satellites**, with their own ground segments, which are kept confidential. The number of antennas is considerable in this sector, and it provides another good market opportunity which can significantly increase the customer base. Here the main application will not be acquisition in LEOP stages as before, but fast acquisition during nominal operations should a problem arise. For example, quick access to space to give communications support for troops and strategic battle zone observation.

¹³ <http://www.orbitalsystems.com/wp-content/uploads/2012/09/2.4AEBP-2.4m-Data-Sheet-B.04.pdf>

¹⁴ <http://www.orbitalsystems.com/wp-content/uploads/2012/09/ACU-2-Data-Sheet-vB.02.pdf>

6.3.3 Competing Systems

To finalise this section, a table is included where some of the FoM of SARAS are compared with the two main precursors in terms of acquisition aid systems, XAA and Conical Scan. While it is true that XAA operates in X-band, and thus the conditions are not the same, the table must be understood from a broad perspective, and as a milestone for future developments.

FoM	SARAS	Conical Scan	XAA
Threshold PFD	-165 dBW/m ² (with FFT)	N/A	-165 dBW/m ²
G/T	-1.5 dB/K (for the whole array, estimated)	> 25 dB/K (estimated)	> 14.5 dB/K
Acquisition time	< 12 s	1 min	< 10 s
Acquisition range	+/- 0.9°	+/- 1° (estimated)	+/- 1° (estimated)
System weight	< 50 kg	N/A	> 200 kg (estimated)
FoV	> 15°	Same as acquisition range	Same as acquisition range

Table 6.2 Comparison between SARAS, Conical Scan and XAA

It is clear that SARAS postulates itself as an eclectic combination of Conical Scan and XAA advantages. With the proposed improvements and its migration to X-band, its performance will outmatch both Conical Scan and XAA.

6.4 Publications

In this section are listed the publications that have been derived from this Thesis, or are related to it indirectly.

6.4.1 Journals

- [1] A. Anton, I. Garcia-Rojo, A. Giron, E. Morales, and R. Martínez, "Analysis of a Distributed Array System for Satellite Acquisition", in *IEEE Transactions on Aerospace and Electronic Systems*, 2015. **UNDER REVIEW.**
- [2] A. Anton, I. Garcia-Rojo, A. Giron, E. Morales, and R. Martínez, "Phase Response Calibration of a Distributed Antenna Array for Satellite Acquisition", in *IEEE Antennas and Wireless Propagation Letters*, 2015-2016. **FORMALLY ACCEPTED, PENDING FINAL PUBLICATION.**

6.4.2 Conferences

- [1] I. Garcia-Rojo, A. Anton A. Giron, E. Morales, and R. Martínez, “SARAS: New Concept of Acquisition Aid System for the Fast Localization and Tracking of Satellites and Launchers”, in the 65th International Astronautical Congress (IAC2014), 2014.
- [2] A. Anton, I. Garcia-Rojo, A. Giron, and E. Morales, “Advanced array signal processing algorithms for DoA estimation of satellites and launchers”, in *Advanced Satellite Multimedia Systems Conference and the 13th Signal Processing for Space Communications Workshop (ASMS/SPSC), 2014 7th*, 2014, pp. 78–85.
- [3] I. Garcia-Rojo, A. Anton, et al., “SARAS: Arrays de antenas distribuidas como soporte a la adquisición y seguimiento de satélites desde estaciones terrenas de TT&C”, in *Congreso Nacional de I+D en Defensa y Seguridad DESEi+d*, 2013.
- [4] M. Sierra, J. Fernandez, I. Garcia-Rojo, A. Giron, M. Inclan, A. Anton, et al., “GEODA: Una antena inteligente en array conformado para comunicaciones simultáneas con varios satélites LEO y MEO”, in *Congreso Nacional de I+D en Defensa y Seguridad DESEi+d*, 2013.
- [5] M. Sierra, J. Fernandez, R. Martinez, M. Inclan, J. Garcia-Gasco, I. Garcia-Rojo, A. Anton, et al., “GEODA: An S-band Active Electronically Scanned Array For multiple TT&C Satellite Communications”, in *COST IC1102 (VISTA) – IC1004 Joint Workshop*, 2013.
- [6] I. Garcia-Rojo, A. Anton, et al., “Distributed Arraying Techniques as a Fast and Robust Acquisition and Tracking Aid of TT&C Ground Station Dishes”, in *the 6th ESA International Workshop on Tracking, Telemetry and Command Systems for Space Applications*, 2013.
- [7] I. Garcia-Rojo, A. Anton, et al., “GEODA: An S-Band Active Electronically Scanned Array for Transmission and Reception of Multiple TT&C Signals from LEO and MEO”, in *the 6th ESA International Workshop on Tracking, Telemetry and Command Systems for Space Applications*, 2013.
- [8] A. Antón, A. Torre, I. García-Rojo, I. Albarran, and R. Hernández, “High resolution techniques for fast acquisition of satellite signals”, in *2010 IEEE International Symposium on Phased Array Systems and Technology (ARRAY)*, 2010, pp. 1017–1024.
- [9] A. Antón, R. Martinez, A. Torre, I. García-Rojo, R. Hernández, and I. Albarrán, “SAGE and Maximum Entropy Algorithmic Comparison for adaptive antenna array calibration”, in *2010 IEEE International Symposium on Phased Array Systems and Technology (ARRAY)*, 2010, pp. 447–452.
- [10] A. Torre, I. Garcia-Rojo, A. Anton, I. Albarran, and R. Hernandez, “All Digital Geodesic Dome Array Antenna for TTC”, in *the 5th ESA International Workshop on Tracking, Telemetry and Command Systems for Space Applications*, 2010.

- [11] A. Antón, R. Martínez, A. Torre y M. A. Salas, “Calibración de Arrays de Antenas Adaptativas mediante Máxima Entropía”, in *XXIV Simposium Nacional de la Unión Científica Internacional de Radio*, 2009.
- [12] A. Antón, R. Martínez, M. A. Salas, “Beamforming and Signal Tracking Issues in Adaptive Antenna Arrays’ Signal Processing”, in *27th AIAA International Communications Satellite Systems Conference*, 2009.
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6.4.3 Patents

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- [3] A. Torre, J. Gonzalo, “Patente Española: ES2361102 B1- Sistema y Método de Adquisición Rápida de Satélites o Lanzadores”, OEPM ES2361102 B1, 2009. **PRECURSOR OF THE SYSTEM.**

6.4.4 Technical Reports

During the course of SARAS project a number of Technical Reports were produced for ESA’s review, the titles of which are reproduced below:

- [1] SARAS Requirements Specification
- [2] SARAS System Design
- [3] Interface Control Document

- [4] Multichannel Digital Processor
- [5] MDP Test Procedure and Test Report
- [6] SARAS System Calibration
- [7] SARAS Computer
- [8] SC Test Procedure and Test Report
- [9] SARAS User Manual
- [10] Frequency and Time Subsystem
- [11] Ancillary Functions
- [12] SARAS AIV
- [13] SARAS Test Plan
- [14] SARAS Test Procedure and Dry-Run Test Results
- [15] SARAS Test Procedure and OSAT Test Results
- [16] Commercial Feasibility Analysis
- [17] Executive Summary

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