UNIVERSIDAD POLITÉCNICA DE MADRID

**ESCUELA TÉCNICA SUPERIOR
DE INGENIEROS DE TELECOMUNICACIÓN**

**DESIGN AND DEVELOPMENT OF
MULTICHANNEL OPTICAL SYSTEMS FOR
IMAGING AND NON IMAGING APPLICATIONS**

TESIS DOCTORAL

João Rendeiro Marques Mendes Lopes

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ABSTRACT

From the beginning of history of science and technology, humankind has sought inspiration from the wonders created by Nature and natural evolution. In the last century, several key inventions were inspired by biological mechanisms found in diverse organisms in Nature: from the invention of Velcro - inspired by the way burdock burrs get attached to fur - up to the development of new energy technologies. The fins of humpback whales were the inspiration to create more powerful wind turbines, and the wave vortices, produced by schools of swimming fish, inspired a new optimization of spatial disposition of wind farms. Optics is no exception. One of the great challenges of optical design is to achieve optical systems ever smaller, with larger field of view and acceptance. Natural evolution found a solution in the vision system of many invertebrates, such as a fly - the compound eye. A compound eye consists of a large number of extremely small vision systems on a curved macro surface, capturing a large field of view, while maintaining their dimensions small. The concept of compound eye has been adapted for optical design, and in the last quarter of century different kinds of multichannel systems were developed. Similarly to how a compound eye works, light splits and is transmitted through a number of different channels and then recombined, either optically or electronically.

Multichannel systems have met applications in both imaging and nonimaging optics.

In imaging applications, arrays of multi aperture optics have been researched for achieving miniaturization, and for achieving high resolution in specific sectors of the field of view. The segmentation of the field of view has the potential to break the usual trade-off between focal length and field of view, and leads to a reduction of the imaging system's total track length. Multichannel design has been applied in technologies such as vision sensors, head mounted displays and cameras, among others.

Nonimaging optics is a branch of optics that deals with the efficient transfer of light between a source and a receiver. It is the best approach for designing solar concentrators

and illumination systems, among other applications. The name comes from the fact that there is no requirement to create an image of the source, and the only concern is the efficient light transfer, which provides a larger freedom to design systems.

The multichannel approach in nonimaging optics has been used both in illumination as in Concentrated Photovoltaics (CPV) applications by combining it with advanced methods such as the Simultaneous Multiple Surface (SMS) design method and Köhler integration for designing freeform surfaces.

Concentrated Photovoltaics (CPV) is a technology that consists of a concentrator optical system that focuses sunlight onto a photovoltaic cell. The main goal of this technology is to decrease the Levelized Cost of Electricity (LCOE). The strategy to achieve this goal is based on two approaches: (i) by concentrating the sunlight, it is possible to dramatically decrease the amount of photovoltaic cell area, since concentrating optics is used to focus sun light onto a much smaller solar cell (optical materials are much cheaper than solar cells); (ii) as the amount of photovoltaic cell area decreases by a concentrator factor, it is possible to use high efficiency cells, which typically are too expensive to be used without concentration in terrestrial applications.

CPV systems have typically a primary optical element (POE) and a secondary optical element (SOE). On the multichannel approach, the POE divides the incoming bundle of light into multiple ones, and each sector of the SOE has to manage a correspondingly smaller field of view, and provide a smaller magnification. This approach provides both a larger acceptance angle and a potential for an increased magnification as well.

A different type of multichannel optics is diffractive lenses, where the control and generation of wavefronts is achieved by segmenting initial wavefronts and redirecting the segments using interference and phase control. Diffractive optical elements (DOE) have had applications in several different types of optical systems. The first applications were spectroscopic ones. Other applications followed, such as beam splitters, laser-beam shapers, optical interconnectors, etc. The application to imaging systems is more recent. In these configurations, DOEs have the potential to enhance the performance (by controlling chromatic and field curvature aberrations, decreasing size and weight, achieving atomic resolution ($<1\text{nm}$), etc).

The aim of this thesis is to explore the multichannel approach in both imaging and nonimaging applications, using both geometric as well as diffractive optics. The multichannel approach is combined with the use of freeform surfaces to achieve the maximum performance.

This thesis is divided in four chapters:

Chapter 1 establishes the basics by presenting the fundamental concepts required to understand the following chapters. Basic definitions of geometrical, imaging and non imaging optics are covered, as well as an introduction to multichannel systems and freeform surfaces. SMS and Köhler integration design methods are also explained. Each chapter was written to be self-sustainable, so some concepts that are only used in a specific chapter were left out of chapter 1, and they are presented in their respective chapters.

Chapter 2 describes the development of an optical concentrator for a high concentration CPV system, for multijunction photovoltaic cells. Based on a multichannel configuration and Köhler integration, a 9-fold Fresnel-Köhler (FK9) was developed. The motivation for the development of FK9 is to create a system not only capable of overcoming the current limitations of CPV systems, but also to pave the way for systems in the future with four, five and six junction solar cells. FK9 presents a 9-channel configuration, each channel performing Köhler integration, providing a uniform spectral and spatial irradiance distribution on the solar cell and a large acceptance angle. The Chapter establishes the fundamentals of CPV, introduces the FK9's design procedure, and demonstrates, through raytrace simulations, its high performance in key figures of merit: high concentration factor, large tolerances, high optical efficiency, and uniform irradiance on the solar cell. A comparative study between concentrators demonstrates that FK9 can sustain a high performance even at extremely high concentration levels (2000x) and ambient temperatures (45°C). This Chapter also introduces the Fresnel Lens with Variable Focal Point (FL-VFP) as a new design to avoid light crosstalk between different channels, a major issue in multichannel systems. FL-VFP avoids that light impinging in extreme points of the lens crosstalk to other channels of the system, enhancing the performance and acceptance angle of FK9.

Chapter 3 describes an extension of the SMS method, a geometrical optics design method, to design diffractive optical surfaces. This method involves the simultaneous and direct (no optimization) calculation of diffractive and refractive/reflective surfaces. Using the phase-shift properties of diffractive elements as an extra degree of freedom, two rays for each point on each diffractive surface are controlled. The concept of Generalized Diffractive Oval is introduced (in 3D geometry), and the diffractive SMS method (in 2D) is presented as a sequential application of the Generalized Diffractive Oval design procedure. The method can decrease the number of elements of a system, thus decreasing size and weight. Design examples of a 3D freeform diffractive surface coupling two wavefronts and a polychromatic hybrid 2D lens controlling three wavefronts with different wavelengths have been successfully implemented.

Chapter 4 explores the experimental characterization of a double-channel freeform lens, for a head mounted display imaging application. The optical design methods and fabrication

machinery for freeform optics have reached a point where the surfaces can be manufactured with high precision. Yet, accurate measurements of the resultant surfaces are still a standing challenge to the surface metrology community. The discontinuities between channels represent an even larger challenge. In this Chapter, two different cutting-edge freeform metrology technologies are used to measure the lens surfaces. A characterization was made, including roughness and topography measurements. The real and design lenses performance were compared through polynomial interpolation and simulations.

RESUMEN

Desde el comienzo de la historia de la ciencia y la tecnología, la humanidad ha buscado inspiración en las maravillas creadas por la Naturaleza y la evolución natural. A lo largo del último siglo, varias invenciones importantes fueron inspiradas por mecanismos biológicos encontrados en diversos organismos naturales: desde la invención del Velcro – inspirado por la forma en que las rebabas de la planta bardana se agarran al pelo – hasta el desarrollo de nuevas tecnologías energéticas. Las aletas de la ballena jorobada han servido de inspiración a la creación de turbinas de viento de potencia superior, y los vórtices producidos por cardúmenes de peces inspiraron una nueva optimización en la disposición espacial de parques eólicos. La óptica no es una excepción. Uno de los retos más importantes del diseño óptico es lograr sistemas ópticos de dimensiones cada vez más reducidas, y campos de visión y aceptación más amplios. La evolución natural descubrió la solución en sistemas de visión de algunos invertebrados, como por ejemplo las moscas – el ojo compuesto. El ojo compuesto consiste en un número amplio de sistemas de visión de dimensiones reducidas dispuestos en una macro superficie curva, capaces de capturar un campo de visión amplio manteniendo pequeñas dimensiones. El concepto de ojo compuesto ha sido adaptado al campo del diseño óptico, y en el último cuarto de siglo diferentes tipos de sistemas ópticos multicanal han sido desarrollados a partir de él. De forma similar al funcionamiento del ojo compuesto, en los sistemas multicanal la luz es dividida y transmitida a través de diferentes canales y luego recombinada, de forma óptica o electrónica.

Los sistemas multicanal tienen aplicación en óptica de formación de imagen y anidólica (*nonimaging*).

En aplicaciones de formación de imagen, las ópticas con aperturas múltiples han sido investigadas para lograr la miniaturización, y para conseguir alta resolución en sectores específicos del campo de visión. La segmentación del campo de visión soluciona el

compromiso entre una distancia focal corta y un amplio campo de visión, logrando una reducción de la longitud total del sistema óptico. El concepto de multicanal fue aplicado a tecnologías tales como sensores de visión, *Head Mounted Displays* y cámaras entre otros.

La óptica anidólica (*nonimaging*) es una rama de la óptica que se centra en la transferencia eficiente de luz entre una fuente y un receptor. Se trata de la mejor herramienta para el diseño de concentradores solares y sistemas de iluminación, así como otras aplicaciones. El término *nonimaging* proviene del hecho que no existe la necesidad de formar una imagen de la fuente, y su único objetivo es la transferencia eficiente de la luz, lo que crea una mayor libertad para el diseño de sistemas.

En el ámbito de la óptica anidólica, los sistemas multicanal han sido usados en iluminación y en concentración fotovoltaica (CPV), a través de su combinación con métodos avanzados tales como el método de diseño de Superficies Múltiples Simultáneas (SMS) y la integración Köhler para el diseño de superficies *freeform*.

La concentración fotovoltaica (CPV) es una tecnología que consiste en un sistema óptico de concentración que enfoca la luz solar en una célula fotovoltaica. El principal objetivo de esta tecnología es disminuir el coste energético. La estrategia para lograr este objetivo se basa en dos factores: (i) a través de la concentración de la luz solar, se puede reducir sustancialmente el área de la célula fotovoltaica, ya que la óptica de concentración se usa para concentrar la luz en una célula solar mucho más pequeña (los materiales ópticos son mucho más económicos que las células solares); (ii) la reducción del área de la célula fotovoltaica por un factor de concentración permite el uso de células de alto rendimiento, que típicamente son demasiado costosas para la utilización sin concentración en aplicaciones terrestres.

Los sistemas CPV tienen, típicamente, un elemento óptico primario (POE) y un elemento óptico secundario (SOE). En sistemas multicanal, el POE divide el haz incidente de rayos de luz en múltiples haces, y cada sector del SOE debe enfrentarse con un campo de visión proporcionalmente más pequeño, y proporcionar una magnificación menor. De esta forma, se consigue una aceptación más amplia.

Las lentes difractivas son otro tipo de óptica multicanal, para las que el control y la generación de frentes de onda se consigue a través de la segmentación y redirección posterior de los frentes de onda iniciales, usando interferencia y control de fase. Los elementos ópticos difractivos (DOE) han tenido aplicaciones en diferentes tipos de sistemas ópticos: inicialmente, en espectroscopía, y posteriormente, en aplicaciones tales como divisores de haz, formadores de haz láser, interconectores ópticos, etc. El uso en sistemas de formación de imagen es más reciente. En estas configuraciones, los DOEs permiten mejorar el rendimiento (a través del control de la aberración cromática de la curvatura del campo, de la reducción del tamaño y peso, logrando resolución atómica ($<1\text{nm}$), etc).

El objetivo de esta tesis es explorar sistemas multicanal en aplicaciones formadoras de imagen y aplicaciones anidólicas, utilizando óptica geométrica y difractiva. Los sistemas multicanal se combinan con superficies *freeform* para obtener el máximo rendimiento.

Esta tesis está dividida en cuatro capítulos:

En el Capítulo 1 se presentan los conceptos fundamentales necesarios para la comprensión de los capítulos siguientes. En este capítulo, se explican algunas de las definiciones básicas de óptica geométrica, formadora de imagen y anidólica, y se introducen los sistemas multicanal y superficies *freeform*. Adicionalmente, los métodos de diseño óptico SMS e integración Köhler también son explicados. Los capítulos están redactados de forma independiente, por lo que las definiciones específicas y relevantes de cada capítulo sólo se presentan en el mismo capítulo, una vez que los conceptos generales quedan definidos en el Capítulo 1.

En el Capítulo 2, se describe el desarrollo de un sistema CPV de alta concentración para células de múltiples uniones. Un sistema Fresnel-Köhler de nueve canales (FK9) ha sido desarrollado, utilizando una configuración multicanal y la integración Köhler. El FK9 se diseñó con vistas a superar las limitaciones actuales de los sistemas CPV y a potenciar nuevas líneas para el desarrollo de sistemas con células solares de cuatro, cinco y seis uniones. Cada canal del FK9 usa la integración Köhler, consiguiendo una irradiancia con distribución uniforme, tanto espectral como espacial, y un amplio ángulo de aceptación. En este capítulo se definen los conceptos fundamentales de la tecnología CPV, se introduce el procedimiento de diseño del FK9, y se demuestra, a través de simulaciones de trazado de rayos, el buen comportamiento del FK9 en sus principales indicadores de rendimiento: elevado factor de concentración, alta tolerancia de fabricación, alta eficiencia óptica e irradiancia uniforme sobre la superficie de la célula. Se ha realizado un estudio comparativo entre diferentes concentradores, en el cual se demuestra que el FK9 mantiene un alto rendimiento incluso bajo niveles muy altos de concentración (2000x) y de temperatura ambiente (45°C). En este capítulo también se introduce la Lente de Fresnel con punto focal variable (FL-VFP) como un nuevo diseño que viene a evitar la diafonía de la luz entre diferentes canales, es decir, que la luz de un canal acabe siendo transmitida por otro, que es uno de los principales problemas de los sistemas multicanal. La F-VFP impide que la luz incidente en puntos extremos de la lente incida en los demás canales del sistema, mejorando el rendimiento y el ángulo de aceptación del FK9.

El Capítulo 3 se centra en la extensión de método SMS, un método de diseño basado en óptica geométrica, para el diseño de superficies ópticas difractivas. Este método consiste en el cálculo directo y simultáneo (sin procesos de optimización) de superficies difractivas y refractivas/reflectivas. Utilizando las propiedades de cambio de fase de elementos difractivos

como un grado extra de libertad, se controlan dos rayos por cada punto de cada superficie difractiva. Se introduce el concepto de Óvalo Difractivo Generalizado (en geometría 3D) y se presenta el método SMS difractivo (en 2D) como una aplicación secuencial del procedimiento de diseño del Óvalo Difractivo Generalizado. El método permite reducir el número de elementos en un sistema, luego permite disminuir el tamaño y el peso. Se han implementado con éxito algunos ejemplos de una superficie *freeform* difractiva acoplando dos frentes de onda y un lente 2D híbrida controlando tres frentes de onda con diferentes longitudes de onda.

En el Capítulo 4 se explora la caracterización experimental de una lente *freeform* de doble canal, para un *Head Mounted Display*. Actualmente, los métodos de diseño óptico y la tecnología de fabricación de óptica *freeform* ya permiten la fabricación de superficies con alta precisión. Sin embargo, la medición precisa de las superficies fabricadas sigue siendo un reto difícil, y sobre todo en los sistemas multicanal, por las discontinuidades entre canales. En el análisis se han usado dos tecnologías de punta de metrología de superficies *freeform*. La caracterización experimental ha consistido en medidas de topografía de las superficies y de rugosidad. Asimismo, se ha comparado el rendimiento de las lentes reales con la lente de diseño, a través del uso de interpolación polinomial y de simulaciones.

LIST OF MAIN SYMBOLS AND ABBREVIATIONS

List of main symbols

k	Wave number
C_g	Geometrical concentration (the ratio between the areas of the aperture of the system and the receiver)
d	Height of the surface relief of a DOE
H	Hamiltonian function
L	Optical path length
m	order of diffraction
n	Refractive index
$\mathbf{N}$	Normal vector to the surface
n_d	Refractive index of the material after the diffraction of light
n_i	Refractive index of the medium before the interaction with the optical surface
R_a	Average Roughness
$\mathbf{r}_d$	Diffacted ray vector on a surface
$\mathbf{r}_i$	Incident ray vector on a surface
S	Eikonal function
TIS_{av}	Average Total integrated Scattered light
TIS_s	Total Integrated Scattered light weighted with the spectrum of the incident light
W	Spatial grating period
W_i, W_o	Input and output wavefront, respectively, after a transmission through an optical surface

α	Acceptance angle
δ	RMS value of roughness
ζ	Duty cycle of the diffracted field
η_m	Diffraction efficiency
η_{opt}	Optical efficiency of the concentrator
θ_d	Angle between the diffracted ray vector and the normal to the surface
θ_i	Angle between the incident ray vector and the normal to the surface
λ	Wavelength
ϕ	Phase function

List of abbreviations

3J, 4J	Triple and four junction cells, respectively
AFM	Atomic Force Microscope
ASTM	American Society for Testing and Materials
CAP	Concentration-acceptance angle product
CGH	Computer Generated Hologram
CMM	Coordinate Measuring Machine
CPV	Concentrated Photovoltaics
DNI	Direct Normal Irradiance
DOE	Diffraction Optical Element
FF	Fill Factor
FK4	4-Fold Fresnel-Köhler
FK9	9-Fold Fresnel-Köhler
FL-VFP	Fresnel lens with variable focal point
GAaInP	Gallium indium phosphide
GaAs	Gallium arsenide
GCO	Generalized Cartesian Oval
GDO	Generalized Diffraction Optical Element
Ge	Germanium

HMD	Head Mounted Display
ICP	Iterative closest point
Isc	Short circuit current
I-V	Current-Voltage
LCOE	Levelized Cost of Electricity
MJ	Multi-junction
MTF	Modulation Transfer Curve
OCT	Optical Coherence Tomography
PMMA	Polymethylmethacrylate
p-n	positive-negative
POE	Primary Optical Element
PSD	Power Spectral Density
PV	Photovoltaics
RMS	Root Mean Square
Rs	Series resistance
RTP	Refractive truncated pyramid SOE
SILO	SingLe Optical surface
SMR	Spectral Matching Ratio
SMS	Simultaneous Multiple Surface design method
SOE	Secondary Optical Element
SoG	Silicon-on-Glass
STM	Scanning Tunneling Microscopy
TIR	Total Internal Reflection
TIS	Total Integrated scattering
UV	Ultraviolet
Voc	Open Circuit voltage
XTP	Hollow inverted truncated pyramid SOE

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Chapter 1.

BASIC CONCEPTS AND DEFINITIONS

This Chapter introduces basic concepts and definitions that are required for the work developed in this thesis. Topics that are only covered in specific chapters are not introduced here, as they are extensively developed in each chapter.

Section 1.1 presents an introduction to Geometrical Optics, where specific concepts are explained.

Section 1.2 describes the main concepts of nonimaging optics, its applications and a survey of nonimaging optical design methods.

Section 1.3 introduces the fundamental concepts of imaging optics.

Section 1.4 exposes the advantages and challenges of freeform surfaces.

Section 1.5 introduces the concept of multichannel optical systems, and shows some applications in imaging and nonimaging systems.

Section 1.6 describes the Simultaneous Multiple Surface (SMS) design method in geometrical optics. This explanation is useful for Chapter 3, where the SMS method is extended for the design of diffractive surfaces.

Section 1.7 describes the Köhler integration as a concept for nonimaging applications. Köhler integration is the basis of the 9-Fold Fresnel Köhler concentrator developed in Chapter 2.

1.1 Introduction to Geometrical Optics

Light is a perturbation of the electromagnetic field, and its propagation is described by an electromagnetic wave. The basis of classical electrodynamics is Maxwell's equation, which describes all the electromagnetic behavior. When light impinges on a physical feature whose size is sufficiently larger than the wavelength of the electromagnetic wave, then it can be described by a branch of optics known as Geometrical Optics. Under these conditions, the trajectory of light can be described by a set of curvilinear trajectories, known as "light rays", which comply with geometrical laws.

In most practical systems, Maxwell's equations can be simplified to ease the optical modeling. Assuming that:

- i. the medium in which the light propagates is uncharged and non-conducting;
- ii. the material is homogeneous and uniform (the permeability μ and permittivity ϵ do not vary with position in the material or change with time);
- iii. the material is isotropic (μ and ϵ do not change with orientation);
- iv. the material is linear (μ and ϵ do not change in the presence of electric and magnetic fields);

These assumptions can also be used for other branches of optics, such as diffractive optics.

The differential wave equation for the electric field $\mathbf{E}$ of an electromagnetic wave is given by:

$$\nabla^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (1.1.1)$$

where t is the time component.

Equation (1.1.1) may be broken down into three scalar equations and treated separately. A monochromatic wave can in general be described by:

$$E(x, t) = P(x) e^{-i(kx - \omega t + \phi)} \quad (1.1.2)$$

where k is the wave number defined by $k = \omega / c = 2\pi / \lambda$, with c the speed of light, λ is the wavelength, ω is the time frequency, ϕ and is the phase shift and P the absolute value of the electrical field. When modeling an optical system, the spatial behavior of the electric field is of primary concern, and the time-varying part can be ignored for most analysis.

The spatial distribution of the electric field satisfies the Helmholtz equation:

$$\nabla^2 P + k^2 P = 0 \quad (1.1.3)$$

1.1.1 Eikonal Equation, optical rays and wavefronts

If we consider an electromagnetic wave travelling through a medium, the spatial component can be described by

$$\mathbf{E}(\mathbf{r}) = A(\mathbf{r}) e^{ik_0 S(\mathbf{r})} \quad (1.1.4)$$

where k_0 is the wavenumber in vacuum.

The Helmholtz equation can be applied to this distribution, and we get

$$\begin{aligned} \nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \Leftrightarrow -k_0^2 \left[|\nabla S|^2 - \left(\frac{k}{k_0} \right)^2 + \left[\nabla^2 \ln A + |\nabla \ln A|^2 \right] \right] \mathbf{E} \\ + 2ik_0 \left(\frac{\nabla^2 S}{2} + \nabla \ln A \cdot \nabla S \right) \mathbf{E} = 0 \end{aligned} \quad (1.1.5)$$

Considering only the real part of the equation, we have

$$|\nabla S|^2 - \left(\frac{k}{k_0} \right)^2 = \frac{\nabla^2 \ln A + |\nabla \ln A|^2}{k_0^2} \quad (1.1.6)$$

In the geometrical optics approximation, $\lambda \rightarrow 0$ or $k \rightarrow \infty$. So, the second term of Equation (1.1.6) tends to zero, because A varies slowly over the distance λ_0 . Considering that k/k_0 remains finite and defining the refractive index $n = c_0/c$, being c_0 the speed of light in vacuum, we get

$$|\nabla S|^2 = \left(\frac{k}{k_0} \right)^2 = n^2 \quad (1.1.7)$$

Equation (1.1.7) is known as the *Eikonal Function*, and it is one of the fundamental results of Geometrical Optics.

If we rewrite Equation (1.1.7) in terms of coordinates, x , y and z , we get

$$\left(\frac{\partial S^2}{\partial x} + \frac{\partial S^2}{\partial y} + \frac{\partial S^2}{\partial z} \right) = n^2(x, y, z) \quad (1.1.8)$$

And the optical direction cosines can be written as

$$p = \frac{\partial S^2}{\partial x} \quad q = \frac{\partial S^2}{\partial y} \quad r = \frac{\partial S^2}{\partial z} \quad (1.1.9)$$

The Eikonal Function can be rewritten as

$$\nabla S(\mathbf{r}) = n(\mathbf{r}) \cdot \mathbf{t}(\mathbf{r}) \quad (1.1.10)$$

where $\mathbf{t}$ is a unit vector. The meaning of $S(\mathbf{r})$ can be understood considering that it relates with the phase of the electrical field $\phi = k_0 S(\mathbf{r})$. Therefore, for values of $\mathbf{r}$ for which $S(\mathbf{r})$ has the same value, these positions define a surface that has the same phase: the surface is defined as the *wavefront*, and the unit vector $\mathbf{t}$ is always perpendicular to the wavefront at the position $\mathbf{r}$.

If we define an optical ray as a curve that is parameterized by s , in a tridimensional space, then we can rewrite the unit vector $\mathbf{t}$ as $\mathbf{t} = d\mathbf{r}/ds$, and Equation (1.1.10) can be rewritten as

$$\nabla S(\mathbf{r}) = n(\mathbf{r}) \cdot \frac{d\mathbf{r}}{ds} \quad (1.1.11)$$

if we take the first derivative along the ray direction

$$\frac{d}{ds} \nabla S(\mathbf{r}) = \frac{d}{ds} \left[n(\mathbf{r}) \cdot \frac{d\mathbf{r}}{ds} \right] \quad (1.1.12)$$

Through some algebraic manipulation [1], we get to the result

$$\frac{d}{ds} \left(n(\mathbf{r}) \cdot \frac{d\mathbf{r}}{ds} \right) = \nabla n(\mathbf{r}) \quad (1.1.13)$$

The solutions of this equation are curves, each one representing a ray of light. If the media is homogeneous ($\nabla n = 0$), then the solutions of Equation (1.1.13) are straight lines. This is the case for most materials and optical systems.

The optical path length L along a curve C is defined as

$$L = \int_C n ds \quad (1.1.14)$$

When there is an electric discontinuity, and the ray $\mathbf{r}_1$ passes from a media of n_1 to a media of n_2 through a surface, Snell's law can be deduced from the Eikonal function [1], and its vector form is:

$$n_1 \mathbf{N} \times \mathbf{r}_1 = n_2 \mathbf{N} \times \mathbf{r}_2 \quad (1.1.15)$$

Where $\mathbf{N}$ is the normal to the surface and $\mathbf{r}_1$ and $\mathbf{r}_2$ are the ray vectors in media before and after impinging on the surface, respectively. The most well-known form of Snell's law of refraction, equivalent to Equation (1.1.15), is

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (1.1.16)$$

Where θ_1 and θ_2 are the angles of the incident and refracted ray with the normal $\mathbf{N}$, respectively.

Fermat's principle has a direct correspondence with the Eikonal function.

1.1.2 Hamiltonian formulation

Geometrical optics can be described through many formulations. The Eikonal formulation was described in the previous section. Another equivalent formulation is Hamiltonian's [2]. Consider a ray with direction defined by the unit vector $\mathbf{v}$ passing through a generic point (x, y, z) . The ray can be represented as a six-dimensional vector (x, y, z, p, q, r) , where (p, q, r) are the optical direction cosines of the ray, according to (1.1.9). The Hamiltonian formulation states that the trajectories of the rays are given as a solution of the following system of first-order ordinary differential equations:

$$\begin{aligned} \frac{dx}{dt} &= H_p & \frac{dp}{dt} &= -H_x \\ \frac{dy}{dt} &= H_q & \frac{dq}{dt} &= -H_y \\ \frac{dz}{dt} &= H_r & \frac{dr}{dt} &= -H_z \end{aligned} \quad (1.1.17)$$

where the Hamiltonian function is $H = n^2(x, y, z) - p^2 - q^2 - r^2$, t is a parameter defined along the ray, and $H_k = \partial H / \partial k$. The solutions of interest are the ones that are according to the Eikonal function [Equation (1.1.8)], i. e., that fulfill the condition $n^2(x, y, z) = p^2 + q^2 + r^2$. Or, in other words, the Hamiltonian function has to fulfill $H = 0$.

1.1.3 Phase space

$H=0$ implies that a ray can be defined as a five-parameter implicit function, comprising a point in the three-dimensional space and two direction cosines multiplied by the refractive index n . This five-dimension space is known as Extended Phase Space [3]. Each point of the phase space represents the position and the direction of the ray.

A *ray-bundle* $M_{4D}(x, y, p, q)$ is a closed set of points in the extended phase space, where each point represents a different ray. For convenience, a ray-bundle is often defined as its intersection with a reference surface, which defines a four-parameter ray-bundle, usually named Phase Space. Typically, the reference surface Σ_R is defined for a plane $z=\text{constant}$ (or x or y constant), and the phase space is defined by four parameters (x, y, p, q) .

These concepts can be similarly applied in a two-dimensional geometry. The extended phase space is a three-dimensional sub-manifold defined by $n^2(x, y) = p^2 + q^2$ in the four dimensional-space of coordinates (x, y, p, q) . The reference surface becomes a curve, and ray-bundle is a two-parameter entity.

1.1.4 Poincaré invariants and concept of étendue

Étendue is defined as the number of rays included in a bundle or, in other words, the volume in the phase space occupied by a ray bundle. For a ray-bundle crossing the reference surface Σ_R , the étendue is as expressed as [3]

$$E(M_{4D}) = \int_{M_{4D}(\Sigma_R)} dx \, dy \, dp \, dq + dx \, dz \, dp \, dr + dy \, dz \, dq \, dr \quad (1.1.18)$$

If we have two reference surfaces $z = \text{const.}(\Sigma_i \text{ and } \Sigma_o)$, and there is no change in the étendue after propagation, then:

$$\int_{M_{4D}(\Sigma_i)} dx \, dy \, dp \, dq = \int_{M_{4D}(\Sigma_o)} dx' \, dy' \, dp' \, dq' \quad (1.1.19)$$

Where x', y', p' and q' are the local variables at the surface Σ_o .

The theorem of conversation of étendue states that its value is constant during the propagation of a ray bundle through an optical system, i.e. it is independent of the reference surface for which it is calculated. It is considered one of the Poincaré invariants [4].

For a two-dimensional geometry, then the étendue is

$$E(M_{2D}) = \int_{M_{2D}(\Sigma_R)} dx \, dp + dz \, dr + dy \, dq \quad (1.1.20)$$

As the étendue must be conserved for any ray bundle, the differential étendue is also conserved. The étendues of an optical system can be measured to check if the conservation holds, which is generally a good indication on an appropriate optical design. When applied to incoherent sources, the Second Law of Thermodynamics is equivalent to the Étendue Conservation theorem, which states that the étendue of a source cannot increase when going through an optical system. Étendue is a very useful concept in Nonimaging optics.

1.1.5 Symmetry in three-dimensional optical systems

Traditionally, most optical surfaces were spherical or rotationally symmetrical, due to that fact they are much easier to manufacture. The symmetry of the Hamiltonian function [Equation (1.1.17)] makes that the first integral of the system is an invariant along any ray. Therefore, 3D optical systems may be studied as a 2D system.

In rotationally symmetric systems, the convenient coordinate system is cylindrical $(\rho, \theta, z, g, h, r)$, where the symmetry condition $H_\theta = 0$ imposes $h = \text{const.}$ One of the Hamiltonian equations is $H_\theta = -dh/dt$ and it is called *skew invariant*. For this reason, rotationally symmetric systems may be designed in 2D in a meridional plane, with $\theta = \text{const.}$

The reduction of the variables of the system of differential equations corresponds with a 2D optical system of the apparent refractive index [2]:

$$n^*(\rho, z) = \sqrt{n^2(\rho, z) - \left(\frac{h}{\rho}\right)^2} \quad (1.1.21)$$

Rays with $h=0$ are called meridian rays, and rays with $h \neq 0$ are known as skew rays. The ray paths of the latter are not rectilinear in 2D.

1.2 Nonimaging Optics

Nonimaging optics systems have the objective of transferring all the light from a source into a receiver with the highest possible efficiency. Its name comes from the fact that in this branch of optics there is no need to form an image (Figure 1-1), but instead an optical radiative transfer is the main goal.

Figure 1-1 An imaging systems requires a point-to-point coupling from the source to the receiver. A nonimaging system does not have this requirement.

There are two main types of problems in nonimaging optics: the first one involves maximizing the amount of energy that is transferred to a receiver; typical systems where this approach is required are solar concentrators. The second involves to control the distribution of light to form a prescribed irradiance; typical cases are illumination systems for several different types of applications, which may differ on the type of irradiance distribution needed. There are also cases where both problems have to be solved: this is the case for Concentrated Photovoltaics (CPV) systems, where the goal is to achieve a maximum energy transfer for a wide acceptance angle, while keeping a uniform irradiance over the solar cell, to maximize the energy production.

Classical imaging systems are appropriate solutions for some paraxial nonimaging optics. However, in many cases the incident angles on the surfaces can have high values, and the design problem becomes non-paraxial. In this case, the restrictions imposed by image formation are usually inconvenient and unnecessary. Imaging optics requires that the contrast of the light source to be preserved while many nonimaging applications, light illumination, intend a contrast-free distribution upon a surface, even if the source is inhomogeneous.

1.2.1 Design problem in Nonimaging optics

The two already described design problems in nonimaging optics can be formulated in terms of ray bundles. We may define the input bundle (M_i) as the bundle of rays impinging on the entry aperture of the optical system, and the output bundle (M_o) as the one impinging on the receiver, after propagating through the nonimaging device. The optical design starts with the definition of the input and output bundles. The set of rays common to both bundles is called the collected bundle M_c . M_i and M_o are connected to each other through the action of the device. The output bundle M_o is a subset of M_{max} , the bundle formed by all rays that can impinge the receiver.

To maximize the energy transfer from an entry aperture to a source, the bundle M_i should be coupled with M_o , so $M_i=M_c=M_o$. When $M_o = M_{max}$, the concentration is maximized. These type of problems include light injection into an optical fiber, radiation sensors, condenser optics, collimators, solar concentrators, etc.

To prescribe a specific irradiance, it is only required that one bundle must be included in the other, for example M_i in M_o (such that M_i and M_o coincide), but M_c must produce a prescribed irradiance distribution on the receiver surface.

1.2.2 Edge-ray principle

The edge-ray principle is a fundamental principle of nonimaging optics. It states that, for an optical system coupling two ray bundles M_i and M_o , it suffices to couple bundles δM_i and δM_o , where δM_i and δM_o are the edge-ray subsets of bundles M_i and M_o . Therefore, a perfect matching between bundles M_i and M_o implies the coupling of their edge-rays[4]. When a system has a source and a receiver, this principle means that light rays coming from the edges of the source must be deflected onto the edges of the receiver. The edge-ray principle is the design key in most nonimaging systems, and highlights the benefits of removing the condition of imaging formation.

1.2.3 Survey of Nonimaging optical design methods

Since the appearance of nonimaging optics in 1960's, several design methods were developed and applied to different systems. Two dimensional systems include Flow-line method, Tailored edge-ray method. These methods are used to design the optical device in a plane, and then the 3D object is created by rotating or linearly sweeping the 2D design. Other methods were developed to design both 2D and 3D systems, such as Poisson bracket's method, Lorentz geometry method, point source differential equation methods, numerical optimizations and Simultaneous Multiple Surface method (SMS). Others, such as Köhler integration, was developed to provide uniform illumination on a target. Other nonimaging problem includes the Monge-Ampere equation, where the input and output intensity patterns are prescribed. The method provides a surface that accomplishes that transformation but it can be only applied to point sources. [5].

Simultaneous Multiple Surface (SMS) design method is a geometrical optics method that has been developed since the late 1990s for non-imaging applications [6]. Later on, the method was extended to imaging optics, with many successful applications [7]. The SMS method is capable of designing simultaneously several surfaces at once in a direct way without a multiparameter optimization, and it is capable of designing freeform surfaces.

One of the usual objectives of illumination systems is to provide uniform irradiance on a target. There are some methods used to achieve this goal, as Abbe illumination and Köhler integration [1]. Abbe illumination images the light source onto the target area, so it requires that the source has a uniform irradiance. The Köhler integration does not have this restriction to provide uniform illumination of the target, and is quite tolerant to non uniform irradiance of the source. It is a technique with many applications in optics [8], and it was first introduced as a method of providing optimum specimen-illumination in microscopy. Nowadays, it is used as a light homogenizer for projection displays, through the use of lenslet arrays. During recent years, Köhler integration has been used in several nonimaging designs for photovoltaics applications.

SMS method and Köhler integration are used and their application are extended in this thesis, in Chapter 2 and 3, respectively. Both are explained in more detail in Section 1.6 and Section 1.7 , respectively.

1.3 Imaging Optics

The goal of an imaging optics design is to form an image, in which a specified minimum-sized object is resolved over a desired field of view. The field of view is expressed as the

spatial or angular extent in object space, the minimum-sized object is the smallest resolution element which is required to identify or otherwise understand the image. The field of view can be expressed both as an angle or alternatively as a lateral size at a specified distance.

In an imaging system, the physical aperture that limits the amount of light entering the device is called aperture stop. It can be either the physical dimension of the first lens or mirror, or it can be a mechanical device, such as diaphragm.

There are several methods to evaluate the imaging performance of an optical system. The appropriate choice of evaluation depends on the application for which the system is designed. In this thesis, we use the ray spot diagram versus field of view and wavelength, and ray aberration curves and Modulation Transfer Curve (MTF), which are two other main methods used to quantify the quality of image formation.

1.3.1 Ray spot diagram

Spot diagrams are the geometrical image blur formed by the optical system when imaging a point object [9]. A spot diagram is obtained through ray tracing several rays from a point object through the system, and intersecting them with the image plane. An ideal spot diagram is circular, with smallest possible diameter. In real systems, the aberrations caused in the wavefronts cause the spot to have different shapes. The form of the shape is related with the kind of aberration.

A way to evaluate the quality of the image using the spot diagram is to use its root mean square (RMS) distribution. The distance R of the position of intersection of each ray with the image plane is measured, relatively to the centroid, and the RMS spot size is defined as

$$RMS = \sqrt{\frac{\sum R_i^2}{n}} \quad (1.1.22)$$

where n is the number of rays.

1.3.2 Wavefront Aberrations

Any optical system has the goal to transform an impinging wavefront into an output wavefront with a prescribed shape, the *reference wavefront*. In an imaging device, the output wavefront is to be spherical, with its origin point in the image plane. When the shape of the wavefront differs from the reference one, then it is said to be aberrated. Figure 1-2 shows some examples of aberrated wavefronts. In an imaging system, the rays related with an aberrated wavefront impinge on different places in the image plane.

Figure 1-2 Aberrated wavefronts. Source: Ref. [10]

Seidel, in 1857, classified the five primary monochromatic aberrations, as embodiments of departure from the first order theory [11]:

Spherical aberration

Spherical aberration is the typical aberration created by spherical lenses. For on-axis light, rays that are infinitely close to the optical axis come to focus at the paraxial image position. As the ray height above the optical axis at the lens increases, the ray in image space cross the axis or focus closer and closer to the axis.

Coma

Coma aberration affects rays from off-axis point sources, and their spot appears to have a tail. Coma is defined as a variation in magnification with aperture, as rays that transmit through the lens through different portions of the aperture stop cross the image plane at different heights from the optical axis.

Astigmatism

Astigmatism occurs when the rays of the meridional and sagittal plan are not focused at the same distance from the lens.

Field Curvature

A positive lens forms an image on a curved surface. Field curvature aberration happens the image is to be formed on a plane surface, which happens in most cases, and a flat object cannot be brought properly into focus.

Distortion

Distortion is the only aberration that does not result in image blur. Even if an object point is imaged onto a perfect image point, it can be displaced from its paraxial position. Distortion is measured as a lateral displacement of the image point to its paraxial equivalent.

Chromatic aberration is another type of aberration that affects imaging and nonimaging systems. It is the effect of the dispersion of refractive elements. The variation of index of refraction with wavelength creates different bending for different colors. The Abbe number measures the material's dispersion, with values indicating low dispersion, and is defined by [12]

$$V_D = \frac{n_D - 1}{n_F - n_C} \quad (1.1.23)$$

where n_D , n_F and n_C are the refractive indices of the material at the wavelengths of the Fraunhofer D-, F- and C- spectral lines (589.3 nm, 486.1 nm and 656.3 nm respectively).

Chromatic aberrations are distinguished in two different types: Longitudinal chromatic aberration and lateral chromatic aberration. Longitudinal occurs when different wavelengths are focused in different points of the optical axis, while lateral chromatic aberration occurs when the wavelengths are focused in different positions of the focal plane (Figure 1-3).

Figure 1-3 Lateral and longitudinal chromatic aberration

1.3.3 The Modulation Transfer Function (MTF)

The Modulation Transfer Function (MTF) is one of the most used methods to evaluate image quality. MTF describes the contrast, or modulation of the image as a function of spatial

frequency. Defining modulation as [9]

$$\text{Modulation} = \frac{I_{\text{Max}} - I_{\text{Min}}}{I_{\text{Max}} + I_{\text{Min}}} \quad (1.1.24)$$

where I is the intensity, then the MTF is defined as

$$\text{MTF} = \frac{\text{modulation in image}}{\text{modulation in object}} \quad (1.1.25)$$

Figure 1-4 shows several types of MTF curves, for perfect and typical real systems. The MTF is used to evaluate the capability of an optical system of resolving an object at different frequencies.

Figure 1-4 Several types of MTF curves. Source: Ref. [9]

1.4 Freeform surfaces

Freeform optical surfaces belong to a class of surfaces which possess little or no rotational symmetry. Traditionally, most optical surfaces were spherical, due to the fact that they are much easier to manufacture, and thus were the basis of most optical systems during the 20th Century. Computer controlled machining was introduced in optical manufacturing, and enabled the fabrication of rotationally symmetric aspheric surfaces. Now, the recent advances in the optics fabrication community allow the fabrication of freeform surfaces,

which can have arbitrary shapes, regular and irregular surface structure, and whose departure from a sphere can be significant. These new surfaces bring with them new possibilities and degrees of freedom to optical designers that were seldom available with rotationally symmetric surfaces. . These new shapes give rise to new opportunities for the creation of more compact systems, with fewer components and overall higher performance. Freeform surfaces are already having an impact in areas of nonimaging optics such as solar energy, illumination, biomedical engineering and they promise to have a significant impact in other areas of imaging applications [13].

Freeform surface fabrication has seen a great evolution since the introduction of computer numerical control (CNC) by the Rochester Center for Optics Manufacturing in the late 1990s. CNC was originally introduced to automate fabrication of spherical and rotationally symmetric aspheric surfaces. In more recent years, this technology has evolved to the point where it is now used to fabricate surfaces with no rotational symmetry, i. e. freeform surfaces.

The interest on freeform optical surfaces has grown when the limitations of the formulation of rotationally symmetric surfaces were definitely proven by Greg Forbes. A new formulation called Q-polynomial surfaces was introduced by Forbes in 2007 [14], a representation for rotationally symmetric aspheric surfaces with constrained slope. The Q-polynomial formulation is now available for optical design, and it has been introduced for optical surface fabrication and testing equipment. This formulation was successfully used to reduce assembly sensitivity [15] and to achieve higher production manufacturing yield of lenses [16] among other applications. In 2012, Forbes proposed an extension of Q-polynomials to freeform surfaces that enables rms slope control [17]. Recently, Kaya and Rolland suggested partitioning the aperture of the optical surface into small sub apertures, employing local polynomials as surfaces descriptors in each sub aperture [18]. This method allows decreasing the number of terms in the polynomials. Currently, there is an ongoing discussion amongst the optical design, fabrication and testing community on which formulation is the more suitable for design and fabrication of freeform surfaces [19]–[21].

Nonimaging applications have been applying freeform optics for longer than imaging ones, due to the fact that they do not require such high manufacturing precisions. In addition, within the nonimaging field, several direct design methods (i. e., that do not require further multi parametric optimization), were developed, whose solution define a truly accurate freeform shape that takes the light from the source distribution to the desired target distribution. In cases where the source, the target or volumetric constraints have very asymmetric requirements, symmetric optical devices can only offer a partial solution (e. g., if conditions like the efficiency of light transfer or number of elements are relaxed). Freeforms

have more degrees of freedom, thus they can perform multiple functions. The direct methods to design nonimaging freeform devices methods include the 3D tailoring method [22], point-to point mapping [23] and the SMS method [6]. Several examples of freeform application in nonimaging systems can be found in both illumination [24]–[27] and solar energy [28], [29].

Recent years have brought several advances on the application of freeform surfaces to imaging systems, improving compactness, wavefront error, and overall performance. One of the most promising advances was the creation of an aberration theory for the design of systems with Zernike class freeform optical surfaces without rotational symmetry [30], an extension of the nodal aberration theory to freeform surfaces [31]. Freeforms provide also a higher performance for optical systems with a high aspect-ratio [32]. We may observe applications of freeform surfaces to imaging in systems such as long-wave infrared reflective systems [33], compact head mounted displays [34], ultra-compact telephoto lens [35], compact Ofner-Chrip spectrometers [36], and others.

The optical design methods and fabrication machinery for freeform optics have reached a point where the surfaces can be manufactured with high precision. Yet, accurate measurements of the resultant surfaces are still a standing challenge to the surface metrology community, and have proven to be a genuine bottleneck in their usage in real world applications, especially in imaging systems. Metrology is essential, not only to validate the manufactured parts, but also in providing a real-time feedback to the fabrication process. Commercial interferometers are typically used to measure the surface profile of optical components. However, the fringe density created by freeform surfaces exceeds the Nyquist frequency of these instruments, due to high surface departure from the typical reference surfaces, usually flat or spherical surfaces.

A survey of the state of the art of metrology for freeform optics, and describes the main steps required for a complete characterization are presented in Chapter 4.

1.4.1 Generalized Cartesian Oval

A very interesting type of freeform surface is a generalized Cartesian Oval. A generalized Cartesian Oval is a reflective or refractive optical surface that transforms an input wavefront W_i into an output wavefront W_o (Figure 1-5). A generalized Cartesian Oval is based on Fermat's principle [2], and the surface is designed such that the optical path Length L between W_i and W_o is constant. Descartes first solved the problem for spherical wavefronts, and in 1900 Levi-Civita solved the general problem [37].

Figure 1-5 A generalized Cartesian Oval coupling two spherical wavefronts

Eikonal function can be used for designing the surface: considering the eikonal functions of the wavefronts W_i and W_o , S_i and S_o , respectively, a generalized Cartesian oval is designed considering that

$$S_i(\mathbf{p}) = S_o(\mathbf{p}) \quad (1.1.26)$$

where $\mathbf{p}=\mathbf{p}(x,y,z)$ is a generic point of the optical surface.

The resulting optical surfaces are typically freeform, depending on the input and output wavefronts. The Cartesian oval calculation has been applied in both imaging and nonimaging optics, being the basis of the string method, a method used to design many high efficiency CPC concentrators [3].

In imaging optics, the Cartesian oval is a surface that makes all the rays coming from a given point $\mathbf{O}$ focus on another point $\mathbf{O}'$.

The generalized Cartesian Oval is the basis of both SMS method and Köhler integration. Both methods are described in Section 1.6 and 1.7 .

1.5 Multichannel optical systems

The interest in multichannel optical systems has been growing in recent years. This approach has met applications in both imaging and non-imaging devices. The idea behind multichannel systems is to split the input bundle into several optical channels. The output bundle is then reconstructed, either optically or electronically (Figure 1-6). This approach divides the whole field of view, (or acceptance cone), into smaller fields of view, allowing for more compact systems.

Figure 1-6 In multichannel systems, the input bundle is divided into multiple channels. The output bundle gets reconstructed, through optical or electronic stitching

Directing the light into different channels in an optical system is a rather recent solution in technology, but that has been used in nature for millions of years. Several invertebrates use a multichannel approach in their vision - the compound eyes. A compound eye consists on a large number of extremely small vision systems - ommatidia - on a curved macrosurface capturing a large field of view. Multichanneling the light has the potential to break the usual trade-off between focal length and field of view. Each ommatidium has a small field of view, but the sum of all ommatidia that composes the compound eye provides a large field of view (Figure 1-7 left). The compound eye has the advantage of enabling a very large field of view, while the total volume of the eye remains small [38].

The compound eyes are distinguished in two major classes: the apposition compound eyes and the superposition compound eyes.

On the apposition compound eyes, there is only one photoreceptor per ommatidia (channel) (Figure 1-19 center). The photoreceptor of each ommatidium accepts light from a finite angular interval centered on the corresponding optical axis. In these eyes, the intermediate space between ommatidia is blocked by an opaque wall to prevent optical crosstalk between channels. Each photoreceptor forms an image of a small fraction of the field of view, and the brain processes the information of each photoreceptor to form the entire field of view. These eyes can be found in day-active insects such as flies [38].

Figure 1-7 Compound eyes can be found in many invertebrate in nature. (Left) compound eyes of a fly (Center) apposition compound eyes (Right) superposition compound eyes.

The superposition compound eyes use several neighboring ommatidium to combine light on the retina, forming a single real image of the environment. It has the advantage of being much more light sensitive, due to the fact that light bundles from the same point pass through different optical channels. It has also the advantage that the system can be more compact, due to the fact that superposition of field of views from different lenses is allowed. It can be found in many night active insects, due to its increase sensitivity [38].

1.5.1 Multichannel systems in imaging applications

Both apposition and superposition systems have been used for imaging applications, in particular in virtual machine sensors. Their capacity of increasing the field of view of the overall system while using a small focal length lead to miniaturization. The apposition systems uses only one pixel per optical channel, i.e. per lens. This feature, combining the fact that each channel images only a small angular portion on the field of view, the lenses have short focal length, makes that the system have a large depth of field, and the image remains sharp independently of the object image. An example of an apposition microlens array used for an ultra-compact vision system can be found in Ref. [39] (Figure 1-8 left).

The superposition systems use several neighboring lenses to combine light into a single sensor, forming a single real image of the environment. It has the advantage of being much more light sensitive, due to the fact that light bundles from the same point pass through different optical channels. It has also the advantage that the system can be more compact, due to the fact that superposition of field of views from different lenses is allowed. These systems usually require image processing, electronically or by software, to fuse all the distributed image information into a final image of the whole field of view. One of the first superposition devices was the Gabor superlens [40] (Figure 1-8 right).

Figure 1-8 (Left) an apposition multichannel device, an ultra-compact vision system for automotive applications [39] (Right) A superposition device, a Gabor superlens [40]

Microlens arrays can also be used to increase the resolution of a specific portion of the field of view, through different optical channels with different angular resolution and field of view, which allow saving resources and has a number of applications. Belay, Meuret *et al* [41]

propose a three channel multiresolution smart imaging system, where a smaller region of interest of the whole field of view has a higher resolution, and the image is formed through various image processing algorithms, applied at different segments of the image sensor (Figure 1-9).

Figure 1-9 A three channel multiresolution smart imaging system. Each channel has different imaging properties. Source: Ref. [41]

Recently, multichannel approaches have been applied to Head Mounted Displays (HMD). Lanman and Luebke [42] proposed a near-eye light field display, formed by an array of lenslets and a digital display device, where the image to be imaged on the retina is decomposed in small cluster images, one per lenslet. As a superposition system, each lenslet captures a fraction of the field of view. Neighboring lenslets have partially overlapping field of views, imaging overlapping regions of the display. Thus, the appropriate image configuration is obtained with electronic stitching of segments and appropriate display mapping.

Recently, a compact HMD approach using multichannel freeform systems and electronic stitching was presented by Miñano *et al* [43]. The multichannel freeform lenses were measured and characterized in this work, shown in Chapter 4. As such, the system is explained in more detail in the respective chapter.

1.5.2 Multichannel systems in nonimaging applications

Nonimaging devices may use multichannel approaches with more freedom than imaging ones, due to the fact that, as there is no requirement for imaging formation, the split bundles can be optically mixed into the output bundle. This approach has been used to create more compact systems, and with a larger acceptance cone.

A nonimaging system that uses the multichannel approach to become compact is the multichannel ultra-compact RXI collimator [44] (Figure 1-10). The light from the light source is distributed through several channels, and collimated, exiting the device as parallel bundle. The multichannel approach allows the device to achieve the lowest aspect ratio amongst

collimators.

Figure 1-10 Multichannel RXI collimator. The multichannel approach creates a very low aspect ratio.

In Concentrated Photovoltaics applications, the multichannel approach is used to increase the acceptance angle of concentrators. This approach has been extensively used in multifold Köhler concentrators. This family of concentrators is shown and explained in Chapter 2.

1.6 SMS method in geometrical optics

Simultaneous Multiple Surface (SMS) design method is a geometrical optics method that has been developed since the late 1990s for non-imaging applications [6]. Later on, the method was extended to imaging optics, with many successful applications [7].

The SMS method involves the simultaneous calculation of J optical surfaces (refractive or reflective) using J one-parameter wavefronts, for which specific conditions are imposed.

It is a direct design method that may be used to design free-form surfaces. The method is based on a Cartesian Oval, optical surfaces that couple an input wavefront to an output wavefront.

Figure 1-11 A schematic view of SMS method using two freeform surfaces to couple two wavefronts

For imaging applications, the output wavefronts are spherical with origin in points in the image plane. As the wavefronts are perfectly coupled, the RMS spot size for the design fields is 0 (neglecting the Airy disk). Figure 1-12 shows a typical curve RMS spot size curve with field angle of a two surface SMS system coupling two wavefronts.

Figure 1-12 RMS spot size curve of an two surface SMS system for an imaging application Source: Ref. [45]

1.6.1 Generalized Cartesian Oval procedure

A Generalized Cartesian Oval, introduced in Section 1.4.1 , couples an input wavefront with an output wavefront.

SMS optical systems use the same logic of Cartesian Ovals to couple J-wavefronts with J-surfaces, with surfaces being designed simultaneously: each portion of one surface permits the calculation of a new portion of other surface and so on.

In 2D geometry, SMS surfaces are piecewise curves made of several portions of Cartesian ovals. The surfaces are designed gradually and simultaneously: each new portion of a specific surface is a Cartesian Oval that couples the refracted/reflected wavefront on the other surfaces to the output wavefront.

Figure 1-13 provides an example from Ref. [3] of the Cartesian oval procedure to design a two surface lens in 2D geometry.

Figure 1-13 SMS method in 2D geometry, a generalized Cartesian oval procedure Source: Ref. [3]

The goal is to couple the input spherical wavefronts with origin in E_1 and E_2 with the output spherical wavefronts with end points in R_1 and R_2 , respectively. Assume that P_0 , an initial point on the first surface, and the normal of the surface at this point n_0 are given. Let L_1 and L_2 be optical path lengths from point E_1 to R_1 and E_1 to E_2 , respectively. The SMS design procedure based on Cartesian Ovals is as follows

1. We refract a ray r_1 coming from E_2 in P_0 (Figure 1-13a). Knowing L_1 , we can calculate P_1 along the direction of the refracted ray. The ray impinging in P_1 has to go to E_1 , so

we calculate the normal $\mathbf{n}_1$. We trace a ray from $\mathbf{E}_1$ to $\mathbf{P}_0$ and, knowing L_2 , we repeat the previous procedure to calculate $\mathbf{Q}_1$ and its corresponding normal. Now we can choose a curve from $\mathbf{P}_1$ to $\mathbf{Q}_1$ with the previous calculated normals. The curve, as long as it is continuous, can have any shape. In the particular case of Figure 1-13, the curve is defined as circle with center in $\mathbf{C}$.

2. Now we apply the Cartesian Oval procedure (Figure 1-13a). We propagate a spherical wavefront with origin in $\mathbf{R}_2$ and refracted in the curve $\mathbf{P}_1\mathbf{Q}_1$. We calculate the generalized Cartesian Oval $\mathbf{P}_0\mathbf{P}_2$ with origin in $\mathbf{P}_0$ that refracts the incoming refracted wavefront in curve $\mathbf{P}_1\mathbf{Q}_1$ to $\mathbf{E}_1$.
3. Now we can calculate the next portion and continuation of surface $\mathbf{P}_1\mathbf{Q}_1$ (Figure 1-13b). We propagate a spherical wavefront with origin in $\mathbf{E}_2$ and refracted in the curve $\mathbf{P}_0\mathbf{P}_2$. As previously, we calculate the generalized Cartesian Oval $\mathbf{P}_1\mathbf{P}_3$ with origin in $\mathbf{P}_1$ that refracts the incoming refracted wavefront in curve $\mathbf{P}_0\mathbf{Q}_2$ to $\mathbf{R}_1$.
4. We repeat step 2. and 3. iteratively to calculate the remaining extensions of the curve (Figure 1-13c-d).

SMS method based on generalized Cartesian oval can be applied to systems with many surfaces, and so far as been used to design up to 8 aspherical surfaces simultaneously, for imaging applications [45].

1.6.2 SMS chains

The procedure described in Section 1.6.1 enables the construction of continuous optical curves analytically. However, it may lead to complicated generalized Cartesian ovals shapes used for the new portions of the curves. Complex differential equations may be necessary to couple the wavefronts, especially after their deflection on the first surface. The differential equations may be hard to solve and computationally demanding. Therefore, many SMS systems require a *point by point* procedure. Figure 1-14 shows an example for two surfaces. The applied algorithm calculates positions and normal unit vectors of two sets of points that correspond to both surfaces. The procedure is similar to the one previously explained, but now individual rays coming from the wavefronts are traced through the point in an iterative procedure: knowing the constant optical path length between each pair of wavefronts, we trace a ray through point $\mathbf{P}_0$ and calculate $\mathbf{P}_1$ (and its correspondent normal vector $\mathbf{N}_1$); then we trace a ray through $\mathbf{P}_1$ and calculate $\mathbf{P}_2$ and $\mathbf{N}_2$; and we repeat the procedure to obtain a complete SMS chain.

Figure 1-14 The SMS chain calculation for a two refractive surfaces lens

This procedure creates two sets of points (and corresponding normal vectors). To obtain continuous curves, we interpolate each set of points, using the information of the normal vectors to define the curves. The optical path lengths between two pair of wavefronts should be carefully selected so that the chain can be integrated. Otherwise, curves interpolated through a calculated chain will not possess the normal vectors required. Later the SMS surfaces are constructed through a series of SMS chains.

1.7 Köhler Integration

In Concentrated Photovoltaics (CPV) applications, good irradiance uniformity on the solar cell can be achieved using two well known methods of classical optics: a light pipe homogenizer or a Köhler integrator.

The light pipe homogenizer obtains irradiance uniformity after the light bounces one or multiple times in the light pipe walls. The illumination uniformity depends of its length and shape, and the solar cell is encapsulated at its end. The use of a light pipe has some disadvantages. The quality of irradiance uniformity requires a longer light pipe, which increases light absorption and decreases the mechanical stability. It cannot be use with small cells, because the silicon rubber spillage may cause large losses. For high illumination angles, the light pipe walls require metallization, which decreases optical efficiency. Light pipes have been used several times in CPV applications [46]–[48], typically with a length 4 to 5 times

larger than the solar cell size.

1.7.1 Köhler integration concept

A Köhler integrator is formed by two lenses that image one in the other. Consider Figure 1-15. The lens on the top focus the input rays on the bottom lens. The bottom lens images the rays coming from the top lens in the receiver in each point. If the input rays come from a source at a considerable distance, then the irradiance at the entry aperture is almost constant and independent of position errors of the source. As each point of the top lens is distributed throughout the entire receiver, the receiver is by definition uniformly illuminated.

Figure 1-15 Each lens is at a focal distance from the other. If the source is at a considerable distance from the aperture, the illumination of the receiver is uniform

A Köhler integrator usually consists on several lenticulation pairs, displayed in an array (Figure 1-16). Disregarding aberrations, all light impinging on one lenslet array with an arbitrary intensity distribution within the acceptance angle α would be transformed into a constant intensity light beam of $\pm\alpha$, if the lenslets are sufficiently small. If these conditions are met, acceptance angle is defined be $\alpha \approx \arctan\left(\frac{L/2}{T}\right)$.

As such, the input and output bundles are the same, but the spatial and angular features are exchanged.

Figure 1-16 A Köhler integrator array. The integration ensures that the output light is always within an acceptance angle α , both for (a) on-axis situations as for (b) off-axis situations

1.7.2 Köhler integration in two dimensions

Figure 1-17 Design procedure for a curved Köhler integrator

The procedure for designing a Köhler integrator begins from the premise that the rays from the source must be directed to the receiver following two conditions:

1. the rays emitted by one point of the surface must illuminate the entire receiver;
2. the rays arriving one point the receiver must come from the entire source.

Consider Figure 1-17. We want to design the lens consisting in surfaces defined by **I** and **I'** and **O** and **O'** between the source **P_i-P_{i'}** and the receiver **P_o-P_{o'}**. The previous lens was already designed, so **I** and **O** are already defined, while **I'** and **O'** must be calculated. Following the edge-ray principle, we want to design two Cartesian ovals that define both surfaces, following Equation (1.1.26), which can be translated as maintaining the optical path length constant for all the points of the surface. Defining the optical path length between two points **A** and **B** as **[A,B]**, we can establish the following equations that permit to calculate points **I'** and **O'**:

I' belongs to the Cartesian Oval that focus **P_i** in **O'**, with origin in **I**, so:

$$[P_i, I'] + [I', O'] = [P_i, I] + [I, O'] \quad (1.1.27)$$

I' belongs to the Cartesian Oval that focus **P_{i'}** in **O**, with origin in **I**:

$$[P_{i'}, I'] + [I', O] = [P_{i'}, I] + [I, O] \quad (1.1.28)$$

O' belongs to the Cartesian Oval that focus **P_o** in **I'**, with origin in **O**:

$$[P_o, O'] + [O', I'] = [P_o, O] + [O, I'] \quad (1.1.29)$$

O' belongs to the Cartesian Oval that focus **P_{o'}** in **I**, with origin in **O**:

$$[P_{o'}, O'] + [O', I] = [P_{o'}, O] + [O, I] \quad (1.1.30)$$

Solving the four equations above in a equation system, we can calculate points **I'** and

O'. These are conditions are necessary but not enough to ensure a perfecting imaging simultaneously for points **P_i** on **O'**, **P_{i'}** on **O**, **P_o** on **I'** and **P_{o'}** on **I**. For instance, the Cartesian Oval that focus **P_i** on **O'** does not have the same profile as the one that focus **P_i** on **O**, and it is only possible to obtain approximated solutions. But, as the desired effect is a uniform irradiance with a sharp cut-off in points **P_o** and **P_{o'}**, it not necessary to obtain a perfect imaging for all the points.

Figure 1-18 The sharp transition of illuminance in point **T** is obtained by slope discontinuity at **I**.
Source: Ref. [49]

The proposed solution is explained in Figure 1-18. The Cartesian Oval M_{TI} images **I** in **T**. **T** is the final point of the receiver, i.e., the point where the sharp transition of illumination is desired. The source presents a smooth irradiance transition around **S**. The neighboring points of **I** of both surfaces M_{SO} and $M_{SO'}$ are approximately imaged to neighboring points of **T**. The slope discontinuity in **I** and the edge ray principle ensures that an observer placed at point **P** would see a sharp transition of illuminance at **I**: the ray coming from a point slightly above **I** is illuminated by a point far above **S**, and a point slightly below **I** is illuminated by a point far below **S**.

1.7.3 Köhler integration in three dimensions

In three dimensions, the edge points **I**, **I'**, **O** and **O'** become edge curves, and the microlenses curves become microlenses surfaces. Two types of Köhler integration can be defined: one-directional integration, along one preferred direction of a ray bundle, and two directional integration along both directions. The first one only ensures insensibility of the irradiance to source shifts in one direction, the second one provides insensibility to any shift of the source.

Figure 1-19 A free-form integrator lens integrating in one direction

Figure 1-19 illustrates the case for integration in one direction. The edge curves form a family of non-crossing space curves, defining lenticular elements that create several source images along one axis. Figure 1-20 shows the integration in two directions, where the edge curves are formed by crossing two families of space curves, forming a rectangular mesh. Between the adjacent lenticular elements there will be a slope discontinuity. In most cases, the design forms a small step between the surfaces. This step is optically inactive.

Figure 1-20 A free-form integrator lens integrating in one direction integrating in two directions

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Chapter 2.

9-FOLD FRESNEL KÖHLER CONCENTRATOR WITH FRESNEL LENS OF VARIABLE FOCAL POINT

This chapter describes the development of an optical concentrator for a photovoltaic (CPV) system. Based on a multichannel configuration and Köhler integration, the 9-fold Fresnel-Köhler (FK9) with Fresnel lens of variable focal point (FL-VFP) has been developed. The motivation for the development of FK9 is to create a system not only capable of overcoming the current limitations of CPV systems, but also to pave the future for systems with four, five and six junction solar cells.

CPV systems use multi-junction solar cells, due to their high efficiency in energy conversion. Triple-junction cells, the most common on the market, have a record of efficiency of 44.4%, while four-junction cells have been developed recently, already achieving an efficiency of 46%, and they are expected to reach efficiencies over 50% in the incoming years (see Section 2.1.1). These cells are likely to be more expensive, and there is a strong possibility that higher concentration levels will be needed for CPV systems based on these cells to be cost effective.

Non-uniform irradiance patterns over multi-junction cells give rise to power losses, especially when considering spectral irradiance distributions over different junctions (see Sections 2.1.1 and 2.2.2). The optics of conventional CPV systems create spatial and spectral non-uniform irradiance distributions on the cell, which affect the efficiency of the system, and decreases the amount of collected energy. The effects of varying solar spectrum and

irradiance distribution on energy production in 4-junction, 5-junction, and 6-junction terrestrial concentrator cells are shown to be noticeable, and will be increasingly important for future CPV systems.

Interest on Silicone-on-Glass (SoG) lenses for CPV applications has been growing due to combining mass production simplicity and high resistance to external factors, and several companies have adopted this technology. SoG lens are especially suitable when CPV systems are installed in aggressive environments as sand deserts (typically with high DNI), as glass resistance to scratches is significantly higher than PMMA's. However, SoG lenses are quite sensitive to thermal effects, not only due to mechanical contractions, but also due to variations in the refractive index. This limitation has a considerable influence on spectral balance of irradiance distributions, and decreases considerably the performance of SoG based-systems, especially for locations with wide temperature range.

FK9 concentrator design is prepared to overcome these limitations. Based on the arrays of channels of Köhler integrators, the concept is further extended to a larger number of channels. This work shows that FK9 design is prepared to work at very high concentration levels (up to 2000x) and with very high temperature variations (for SoG lenses), maintaining a high irradiance uniformity for the desired spectral range, while keeping a large acceptance angle.

When the number of channels increases, the possibility that light crosstalk between different channels also increases, thus decreasing the acceptance angle. The crosstalk between optical channels becomes the limiting factor for the acceptance angle and consequently, for the concentrator-acceptance angle product (CAP). In this work we explore a new technical solution to overcome the crosstalk issue, and a Fresnel Lens with Variable Focal Point (FL-VFP) is proposed. Maintaining the rotational symmetry of Fresnel lenses teeth, FL-VFP avoids that light impinging in extreme points of the lens crosstalk to other channels of the system. It is shown that this novelty further enhances the acceptance angle.

FK9 with FL-VFP is not only prepared for the requirements of today's CPV systems, but is also prepared for the requirements of future ones. The capability of high performance at very high concentrations while maintaining spectral balanced irradiance distributions, even at extreme temperature conditions, makes FK9 an excellent candidate for CPV systems with four, five, and six junction cells.

Section 2.1 presents the fundamental concepts of concentrated photovoltaics. Subsection 2.1.1 describes high-efficiency solar cells. A special focus is given to the development state of cells with more than three junctions, and the effects that spectral and spatial non uniform irradiances have on electrical production. Subsection 2.1.2 explains the merit functions used

to evaluate the quality of optics of a CPV system. Subsection 2.1.3 describes the main materials used in optics for CPV. Subsection 2.1.4 shows the main concentrator designs used in CPV systems, while subsection 2.1.5 explores how Köhler integration was used in CPV systems so far.

Section 2.2 focus on the factors that were the motivation to pursue the FK9 development. Subsection 2.2.1 explores some recent studies on the effect of temperature in Silicon on Glass lenses, and its effects on the performance of multi-junction cells. Subsection 2.2.1 describes the effects of spatial and spectral irradiance distribution on the performance of several different CPV architectures, focusing on three recent studies of the effects of different optical systems,

Section 2.3 describes the development of FK9 and FL-VFP. Subsection 2.3.1 provides a technical description of FK9, and subsection 2.3.2 exposes the problem that FL-VFP solves. The design of FK9 and FK9 with FL-VFP is explained in subsection 2.3.3 (design procedure) and 2.3.4 (design optimization).

Section 2.4 shows the ray trace simulation results for FK9 and FK9 with FL-VFP. Optical efficiency (2.4.1), acceptance angle and CAP (2.4.2) and spectral and spatial irradiance on the cell (2.4.3) are shown and assessed. A CAP comparison between different concentrators is shown in subsection 2.4.4

Section 2.5 shows a comparative study on the performance of FK9 and other concentrators for very high concentrations and very high temperatures. The study focuses on the spectral irradiance distributions created on the cell.

2.1 Fundamental concepts of Concentrated Photovoltaics

Concentrated photovoltaics (CPV) is a technology that consists on a concentrator optical system to focus sunlight into a photovoltaic cells. The main goal of this technology to reach the Levelized Cost of Electricity (LCOE), based on two strategies: (i) by concentrating the sunlight, it is possible to dramatically decrease the amount of photovoltaic cell area, since we use concentrating optics (optical materials are much cheaper than solar cells) to focus sun light onto a much smaller solar cell; (ii) as the amount of photovoltaic cell area decreases by a concentrator factor, it is possible to use high efficiency cells, which typically are too expensive to be used without concentration in terrestrial applications.

CPV systems enable a cost-effective use of these cells. These systems are more complex than standard PV, especially high concentration ones, as they require housing, a tracking system, heat dissipation, and may require secondary optical elements.

CPV is of most interest in areas with high levels of Direct Normal Irradiance (DNI), in the order of 2000kWh/m^2 , as these systems are not capable of capturing diffuse radiation.

CPV systems can be classified according to their concentration level. Systems up to 100X concentration levels typically use silicon cells and are primarily single-axis tracked. High concentration systems concentrate the light by factors between 300x to 1000x and use highly efficient multi-junction solar cells based on III-V semiconductors, and usually with dual-axis tracking. More than 90% of the capacity publicly documented to be installed by the end of 2015 is in the form of high concentration PV [1], using III-V triple-junction cells, and this thesis focuses on these systems.

CPV technology has been developing at a considerable speed in the last years. Soitec, in 2015 reported efficiencies of commercially available modules exceeding 30% [2]. There is still a lot of room to increase the solar conversion efficiency of CPV systems, as the first generation of four-junction cells to be developed already achieved an efficiency of 46.0% [3]. Using these cells, in 2015 Soitec demonstrated a CPV module efficiency of 38.9% at Concentrator Standard Test Conditions (CSTC) [2]. In February 2016, Fraunhofer ISE reported a world record efficiency of 43.3% for a new CPV 'mini-module, using four-junction cells [4].

In spite of these significant technological advances in the last decade, CPV technology is facing difficulties to establish itself in the market. Despite the deployment of more than 200MW between 2011 and 2014 [1], CPV industry is currently struggling to compete with PV prices. Leading CPV companies are exiting the market, and others are facing challenges in raising the capital required to reach an economy of scale. Nevertheless, the efficiencies achieved by CPV modules are far above anything possible with traditional PV technology, and the still large room for improvement provide a potential way for a significant cost reduction.

From the optical designer's point of view, a concentrator for PV offers very specific challenges, and nonimaging optics offers the best tools to address these challenges. The optical system must: have high efficiency; deliver high concentration; have a large acceptance angle to accommodate tracking errors, wind movement, inaccuracy in optics manufacture, finite stiffness of the supporting structure, etc; provide spectral and spatial uniform illumination on the cell; tolerate large variations of temperature, among other requirements.

2.1.1 High-efficiency solar cells

So far, triple (3J) junction and four-junction (4J) photovoltaic solar cells hold the record for the highest solar energy conversion efficiency, but only the first ones are commercially available at the moment. Triple-junction cells are composed by three p-n junctions, of

different bandgaps. The three junctions are connected in series and placed one over the other in a single mechanical stack. Each junction is designed to convert a specific band of solar spectrum, such that the maximum spectrum band can be converted into electricity. Figure 2-1 shows the layer structure and quantum efficiency response of each junction of a Ge/GaAs/GaInP triple junction cell, and how each junction captures a band of the solar spectrum can be observed. The green line (Ge), the red line (GaAs) and the blue line (GaInP) correspond, respectively, to the bottom, middle and top junction.

Figure 2-1 (Left) structure of a Ge/GaAs/GaInP triple junction cell. (Right) Quantum efficiency of a Ge/GaAs/GaInP triple junction cell. Source: Ref. [5]

Multi-junction cells reach very high efficiencies, but the series connection nature between the junctions also poses some challenges to achieve a top performance. The short-circuit current density J_{SC} of each junction is determined by their Quantum efficiency, QE , and the spectrum of light incident on the junction Φ_{inc} [6]

$$J_{SC} = e \int QE(\lambda) \Phi_{inc}(\lambda) d\lambda \quad (2.1.1)$$

Where e is the electron charge. For photovoltaic applications, the standard spectrum used is the *ASTM G173 AM1.5D* [7]. To calculate the total value of the photocurrent on each junction, the cell area A_{cell} must be considered

$$I_{SC} = A_{cell} e \int QE(\lambda) \Phi_{inc}(\lambda) d\lambda \quad (2.1.2)$$

As the junctions are series-connected, the photocurrent is limited by the lower photocurrent among the three subcells. For a given cell, the maximum photocurrent is obtained when there is a current match, i.e., when all the subcells generate the same current. If the currents are mismatched, the cell electrical efficiency decreases. Typically, in most advanced commercially available multi-junction cells, the bottom junction produces an excessive photocurrent compared to the middle and top junction. This is the case for the

most commonly used multi-junction cell, based on a Ge/GaAs/GaInP structure (Figure 2-1), where bottom junction (Ge) produces 1.3 to 1.5 times more current than the middle and top junction. So, the photocurrent can only be limited by middle or top junctions. The most common multi-junction cells on the market are designed such that the photocurrents generated by the top and middle junctions are balanced, under the ASTM G173 AM1.5D spectrum. This optimization is accomplished by an appropriate selection of the materials thickness, metal gridlines design and the use of antireflection coating on the cell. In modern and world record efficiency cells using dilute-nitrides (InGaP/InGaAs/InGaAsNSb architecture by Solar Junction [8]) or inverted growth cells (as those InGaP/GaAs/InGaAs structures made by Sharp [9]), the materials are selected so the three junctions generate approximately the same photocurrent under standard spectrum.

The generated concentrated short circuit photocurrent (I_{sc}) is proportional to the luminous flux, but is limited by the minimum photocurrent generated by the cells. The open circuit voltage (V_{oc}) increases logarithmically with photogenerated current. Concentrated sunlight increases solar cell voltages, which implies higher efficiencies as long as the increased series resistance (R_s) (which increase with the excess of current) does not overcompensate them by inducing reduced fill factor (FF). Fill factor, in conjunction with I_{sc} and V_{oc} , indicates the maximum power obtained from a solar cell.

Spectral and spatial irradiance distribution

Most optical systems used in CPV induce non-uniform irradiance on the cell. In addition, if the system has refractive elements, the chromatic aberrations inherent to refraction affect the spectral distribution, which, for multi-junction cells, may unbalance the current match between junctions. Nevertheless, solar cell metallic grids are typically designed for uniform irradiance conditions. This is not because uniform irradiance distribution is optimal in terms of efficiency [10], but because it is simple and it adapts better to arbitrary irradiance distributions.

In the last years, several studies were made to assess the impact of non-uniformity irradiance distributions in solar cells. Experimental evidences showed that non-uniformity affects both current and open circuit voltage. The fill factor of IV curve is especially affected, fact attributed to an increase in series resistance loss [11], [12]. The non-linear diode behavior of multi-junction concentrators was shown to be caused by non-uniform illumination [12]. Non-uniform illumination typically produces degradation in performance since higher intensities occur near the cell center further from the cell bus-bar, producing increased power losses in the cell front grid and front surface diffused regions of conventional planar cells [13]. There were also studies to assess the effects of spectral non-uniformity on multi-junction cells. The spectral variation significantly decreases the current

generated by one of the junctions (Figure 2-2), which consequently decreases the total current passing through the series of junctions [14], [15].

Figure 2-2 Effects of non-uniform illumination with chromatic aberration in a triple junction solar cell. The illumination follow different Gaussian distributions. Source: Ref. [12]

Modeling a multi-junction cell under non-uniform flux plays an important role in understanding the efficiency drop effects. The most commonly applied method to carry out the electrical simulation is to divide the cell into smaller subcircuits representing different parts of the solar cell, and modeling each subcircuit by an electrical circuit. Multi-junction cells can be modeled as a monolithic arrangement of several small sized photovoltaic cells, connected in series to decrease power losses caused by series resistance. Olson [16] modeled the currents spreading behavior under non-uniform irradiance in MJ solar cells. They pointed that the non-uniform illumination can be a major problem for tunnel-junction interconnected III–V MJ cells if the resulting local photocurrent exceeds the peak tunneling current density and can be mitigated via current spreading. But, apart from this situation, if the irradiance distributions corresponding to spectral bands of the different junctions are matched, it is not a severe problem, even if these distributions are not uniform. Espinet-González *et al* [17] developed a three-dimensional distributed model, in which a multi-junction solar cell is divided in several elementary units, and used it to study multi-junction cells under different spatial and spectral non-uniform irradiance patterns [18], [19]. This study also points out for the superior effect of spectral non-uniformity in efficiency drop. Internal currents spreading perpendicular to the main current appear to balance the local photocurrent mismatch between junctions caused by mismatch of irradiance in their bands, and this effect is especially adverse in top and middle junction. A detailed account on the research made on non-uniform irradiance on solar cells is given by Baig *et al* [20].

Section 2.2.2 shows the effects of non-uniform patterns created by different CPV optical systems, for both spatial and spectral irradiance distributions.

Four, five and six junction cells

The theoretical efficiencies of multi-junction cells are over 70%. Although nowadays values are far from it, the effects of those physical limits can be felt in today's cells, and the rate of efficiency increase has begun to lessen. As triple junction concentrator solar cell technology becomes more highly evolved, it has become increasingly evident that the next steps in efficiency will need to come from qualitatively new cell structures, rather than from iterative improvements of the existing 3-junction technology.

A lot of effort is being put in increasing the number of junction in cells to reach efficiencies higher than 50%, and the first four-junction cells have been announced. At the end of 2014, Soitec, CEA-Leti and Fraunhofer ISE announced a world record for solar cell efficiency at 46% using a four-junction (GaInP/GaAs/GaInAsP/GaInAs) cell, at a concentration of 508 suns [21]. Using this cell, Soitec set the record for CPV module efficiency at 38.9% in 2015 [2] and in February 2016, Fraunhofer ISE reported a world record efficiency of 43.3% for a new CPV 'mini-module, using four-junction cells [4]. NREL also announced a four-junction cell, with efficiency of 45.7% [22]. These results are very promising, especially considering that this technology is giving its first steps, and still has a learning curve to accomplish. In the near future, these cells are expected to reach efficiencies above 50%.

The new multi-junction cells deliver new technological challenges. The effects of varying solar spectrum and current balance on energy production in 4-junction, 5-junction, and 6-junction terrestrial concentrator cells are shown to be noticeable, and will be increasingly important for future CPV systems [23]. These cells are likely to be more expensive, and there is a strong possibility that higher concentrations levels will be needed to obtain cost effective CPV systems (although the increase of efficiency can level it up).

2.1.2 Merit functions in optics for CPV

Several functions of merit in Concentrated Photovoltaics (CPV) are used to evaluate the optical concentrator's performance. These section establishes the key figures of merit

Geometrical concentration

Geometrical concentration C_g is defined as the ratio of areas between the concentrator's aperture and the receiver's. In the particular case of CPV, the receiver is the solar cell. Although this rather simple definition, that are some aspects to be clarified on what is considered to be the area of the aperture and the area of the receiver. In most systems, the full area of the aperture is considered. In some others, if there is an element causing shading on the aperture, or a gap, that is considered on the design, the corresponding area can be excluded. For the cell area, it can be useful to consider an area that is slightly smaller than

the entire surface of the cell, to accommodate light losses due to manufacturing alignment errors, as it has been considered in the concentrators shown in this thesis.

Non imaging optics establishes a limit for the maximum concentration that can be achieved on an optical system. This result can also be achieved using thermodynamics [24]. Let us consider a radiation with half-angular aperture θ_A impinging on a concentrator with an aperture of area A_A and refractive index n_A . Let us consider a receiver of area A_R , with impinging radiation of half-angular aperture θ_R . The étendues of both radiations are

$$\begin{aligned} E_A &= \pi n_A^2 A_A \sin^2 \theta_A & (i) \\ E_R &= \pi n_R^2 A_R \sin^2 \theta_R & (ii) \end{aligned} \quad (2.1.3)$$

As the étendue is conserved throughout the optical system, we have $E_A = E_R$. The maximum concentration is obtained when the receiver is illuminated isotropically. In other words, when $\theta_R = \pm \pi/2$. Using the definition of geometrical concentration C_g as $C_g = A_A/A_R$, we can establish the maximum concentration $C_{\max}$

$$C_{\max} = \frac{n_R^2}{n_A^2} \frac{1}{\sin^2 \theta_A} \quad (2.1.4)$$

The F-number ($f/\#$), in an optical concentrator, is defined as the total height by ratio of the aperture (defined by the largest diagonal of the POE).

Acceptance angle

In CPV, the acceptance angle α is defined as half of the angular span within which the concentrator collects more than 90% of the on-axis power [25] (Figure 2-3). The definition of on-axis power may vary, depending on what variable we are interested in analyzing. We may analyze the overall irradiance that reaches the receiver, but a more practical definition of the acceptance angle is the sun's tilting angle for which the limiting junction generated photocurrent is at 90% of its maximum. The maximum value typically occurs at normal incidence. This definition is useful to analyze the optical behavior of the system, since the generated photocurrent only depends on the spectral irradiance level.

Another definition is related with the generated electrical power. In this case, the angle is referred as *effective acceptance angle* α^* . The effective acceptance angle considers both optical and electrical effects, as the solar cell behavior is also taken into account.

In this thesis we use the first definition of acceptance angle.

The acceptance angle is an important figure of merit in CPV systems because the high levels of concentration requires a high tracking accuracy. A high acceptance angle provides a good level of tolerance not only for trackers, but also for shape errors and roughness of optical

surfaces, concentrator module assembly, array installation, structure stiffness and soiling. A good acceptance angle can have a real impact on the overall cost of the system, and on the energy collected throughout the year.

Figure 2-3 The acceptance angle is defined the off-axis angle for which the transmission function is 90%. The image shows the transmission curve of an ideal optical system, vs a real one

Concentration Acceptance Product (CAP)

As CPV has the goal of minimizing energy cost, several aspects have to be taken into account. A key aspect is an efficient and low cost optical design, which is best met when a minimum number of elements are used, combined with relaxed tolerances and high concentrations, which decreases the size of high efficiency solar cells for the same energy production. It is a difficult task to reach high concentration and high tolerance at the same time, so we have to find a trade-off between these two parameters.

One of the merit functions in CPV, which is the basis for our previous statement, is commonly interpreted as the *acceptance-concentration product (CAP)*, and it is given by

$$CAP = \sqrt{C_g} \sin \alpha \leq n \quad (2.1.5)$$

where C_g is the geometric concentration (the ratio between the concentrator entry aperture and the cell active area), and α is the acceptance angle. Equation (2.1.5) is directly related with Equation (2.1.4).

Assuming that n is the refractive index of the material surrounding solar cell, it should be noted that: $CAP \leq n$. This provides the theoretical limit on the capabilities of a system to transfer the flux from a source to the target derived from the conservation of étendue theorem. Most concentrators fail to approach this limit. Surrounding medium is sometimes air ($n \approx 1$), and sometimes a clear silicone ($n \approx 1.4$ to 1.5). CAP is useful because for a given concentrator architecture (the same number and type of optical elements), its value is practically constant.

Optical efficiency

Optical efficiency of the concentrator is defined as the light power transmission efficiency through the concentrator of the light that illuminates the concentrator's entire entry aperture and reaches the target (solar cell). This definition is wavelength dependent. Usually weighted with the *ASTM G173 AM1.5D* spectral distribution, it is used by optics manufacturers as it is cell independent, i.e., the optical system can be analyzed, regardless the cell that it is being used. In a first approximation, the overall module efficiency η_{mod} is the optical efficiency η_{opt} multiplied by the cells efficiency η_{cell} , i.e., $\eta_{\text{mod}} = \eta_{\text{opt}} \times \eta_{\text{cell}}$.

Optical efficiency in concentrators depend on a number of factors: The nature of the deflection (reflection typically has less optical losses than refraction, when using a good mirror), the spectral transmission of the elements, the quality of the materials and the number of elements (more elements typically cause lower efficiency). In addition, if light travels through a medium, there are absorption losses.

Spectral and spatial uniformity on cell illumination

Subsection 2.1.1 described the effects that spatial and spectral non uniformity have in multi-junctions cells. The main reason for the current research on these effects is the fact that optics in CPV systems may create non uniform patterns on the cell, and have chromatic aberrations that affect its spectral distributions in different junctions. Therefore, spatial and spectral uniformity on cell illumination is also an important figure of merit when evaluating the performance of an optical system for CPV. An appropriate optical design may have a huge impact in achieving a proper uniformity on the cell. Other underlying causes may create non-uniformity, such as shape errors in concentrator profiles, improper tracking, misalignment of pair concentrator-cell, impurities in the optical elements that change the reflective/refractive properties, and produce non-uniform flux, that may be different to that expected at the design stage. Also mechanical failures such as loss of transmittance, discoloration of lens, fracture and mechanical fatigue etc might have a negative impact in the system. The spectrum variations throughout a day and a year also create current mismatch between junctions. An optical system that is insensitive to spectral variations may have a real positive impact on the collected energy throughout the year.

As spectral and spatial uniformity on cell illumination was one of the motivations to pursue the development of FK9, the effects of optics of different CPV systems is explored in more detail in Section 2.2.2

2.1.3 Materials used for optics in CPV

Most CPV systems on the market use Fresnel lenses as primary optical element (POE). Fresnel lenses have traditionally been made of Polymethylmethacrylate (PMMA). PMMA lenses are sensitive to oxidative photodegradation [26], are lightweight and are easily manufacturable.

Silicone-On-Glass (SoG) Fresnel lenses have been developed in 1970's [27]. SoG lens is a two component system, in which silicone Fresnel facets are attached to a glass cover. The glass cover, which has almost the same refractive index value as the silicone (hence practically avoids Fresnel losses between both materials), serves as a protection to the external environment and provides a stable support structure. Interest on Silicone-on-Glass (SoG) lenses for CPV applications has been growing due to combining mass production simplicity and high resistance to external factors, and several companies have adopted this technology [28]–[31]. SoG lenses are especially suitable when CPV systems are installed in aggressive environments as sand deserts (typically with high DNI), as glass resistance to scratches is significantly higher than PMMA's. These lenses have higher optical transmission, and a broader spectrum response, compared to PMMA lenses. SoG lenses present a higher resistance to external factors, such as scratching and hail impact, greater chemical stability, but have the disadvantages of potential solarization or corrosion of glass, and a higher weight. These lenses are also more suitable for spectrum-splitting systems (wider high efficiency range compared to PMMA). [29]. SoG lenses are quite sensitive to temperature, and the thermal expansion causes significant variations in the refractive index, affecting the performance of CPV systems. Section 2.2.1 goes into further detail on this subject, as it is one of the reasons to pursue the development of 9-Fold Fresnel-Köhler.

An extended review covering the advantages and disadvantages of SoG and PMMA Fresnel lenses can be found in Ref. [29].

Another option for the POE element is a mirror. Currently, there are not many companies using a mirror as primary optics in their systems. Some companies used mirrors in low concentration systems with silicon cells, but since the cost of standard PV modules dropped in the last years, these silicon-based systems are no longer LCOE competitive. Mirrors can be made of aluminum, with a special coating for surface protection or for higher reflectivity (silver), or glass. Both have a good reflectivity for the majority of the electromagnetic spectrum.

Regarding secondary optical elements (SOE), render glass is the most used material for CPV refractive SOEs, while for reflective surfaces aluminum or silver are commonly used. Plastic materials are not suitable for refractive SOEs, since they would melt with the high

temperatures reached due to the light's concentration created by the POE. B270 glass is typically used for molded parts, while BK7 glass is used for flat surfaces. Recently, Evonik developed a competitor to conventional molded glass, Savosil [32], which consists on a colloidal suspension of silica particles. This material presents excellent UV stability, high optical transmission over the whole solar spectrum wavelength range, lower thermal expansion than glasses, and it can be shaped to freeform surfaces.

2.1.4 Concentrators designs

This section focuses on high concentration systems. Classical high concentration systems use the same concepts as imaging optics. These systems consist of only one component, a reflective parabolic dish or a refractive Fresnel lens (Figure 2-4). For the former, all the Sun rays are sharply imaged on a point, and for a maximum *CAP* their focus must be located at the center of the receiver. Parabolic mirror suffers strongly from coma optical aberration, which limits the *CAP* to the small acceptance angle at the paraboloid rim. This configuration creates a strongly non-uniform illumination on the receiver.

Figure 2-4 Fresnel lenses and parabolic reflectors are the classical concentrators in CPV

Flat Fresnel lenses are very popular in CPV. Since it is essentially an image forming optics, it produces an image of the sun inside of the solar cell, which must be large enough to allow for tolerances (acceptance angle). Moreover, due to its discontinuous nature, its magnification is non-constant, so the image of the sun is blurred and the irradiance distribution is not stepped but bell-shaped. Furthermore, chromatic aberration of Fresnel lenses causes significant differences of the irradiance distributions for the different junction spectral bands and it limits *CAP*. Nevertheless, due to its simplicity, Fresnel lenses without secondary elements are still a competitive solution at moderate concentrations ($C_g \approx 300$ -

500x). The company Soitec, using the technology developed by Concentrix, utilizes this approach on their commercial available modules [33]. With the concentrator system of 500x together with the high-efficient multi-junction solar cells, their module efficiency reaches 31.8% and have more than 70MW of installed power [1] (Figure 2-5).

Figure 2-5 Soitec's 44 MW CPV power plant in Touwsrivier, South Africa

In order to alleviate previously mentioned problems, a dome shaped Fresnel lens instead of flat can be used [34]. The company Daido Steel [35] commercializes this approach. Dome shaped Fresnel lenses have less geometrical and chromatic aberrations, but its manufacturing is a challenging task.

Nonimaging SOEs increase CAP (as n increases) and improve irradiance uniformity. Two current nonimaging solutions present in the market are shown in Figure 2-6. A hollow inverted truncated pyramid reflector (XTP) improves the spectral homogeneity of non-SOE's systems. Company Arzon Solar, using Amonix technology, uses this solution [36]. The solid dielectric version of XTP, refractive truncated pyramid (RTP), by using Total Internal Reflection (TIR), improves both the homogeneity and the CAP. Homogenization is obtained through multiple TIRs on the prism walls. Several companies use this kind of SOE, as Daido Steel [35], Suntrix [37], and recently bankrupted pioneer CPV firm Solfocus used it as well. For instance, with SOG Fresnel lenses and 39.0% efficient MJ cells, Suntrix module shows 28.0% efficiency [37]. Performance of several different SOEs was the subject of previous studies [38].

Figure 2-6 Inverted truncated pyramid secondary concentrators: Left, hollow version (XTP), Right, solid one (RTP).

Freeform concentrators

Breaking the rotational symmetry allows for an optical performance difficult to obtain with classical approaches. One of the best examples of an extremely asymmetric concentrator is the XR free-form technology, developed by company LPI using the SMS 3D design method (Figure 2-7).

Figure 2-7 XR free-form concentrator. (a) CAD design of the concentrator. (b) Free-form lens of the XR free-form concentrator. (c) SMS design method on XR free-form concentrator.

Two free-form surfaces are designed with the SMS 3D method (SMS curves contained in these surfaces, called spines and ribs are shown in Figure 2-7a). The SOE is a highly asymmetric free-form surface (Figure 2-7b), and the homogenization is performed by a very short TIR prism that protrudes from the lens back (its length is similar to the cell side) [Figure 2-7c)]. Simulation results for XR show an acceptance angle of $\alpha = \pm 1.85^\circ$ at $C_g = 1,000\times$, which implies a $CAP \sim 1.0$, the highest ever reported to the authors' knowledge [39].

2.1.5 Köhler integration in CPV

Multi-fold Köhler systems, proprietary technology of LPI [40], represent a family of concentrators developed in recent years. Those concentrators work as the Köhler integration arrays produce a uniform irradiance on the solar cell. In CPV, Sandia Labs was the first company to use this concept in 1989 [41]. This photovoltaic concentrator has one Köhler integrating pair composed of a Fresnel lens as the POE and a single-surface imaging lens as the SOE, which encapsulates the cell. The secondary lens (Single Optical surface - SILO) (Figure 2-8) is placed in the focal plane of the primary lens (Fresnel lens) and it images the Fresnel lens aperture onto the solar cell thus providing uniform irradiance distribution on the solar cell as the Fresnel lens aperture is uniformly illuminated by the sun. If the cell is squared, the primary can be a squared trimmed without losing optical efficiency. The POE images the sun on the SOE aperture. Consequently, the SOE contour defines the acceptance angle.

Figure 2-8 The silo images the Fresnel lens in the cell and achieves uniform irradiance until 300x.

The spatial and spectral uniformity of the SILO is only perfect for perfect tracking, and degrades for off-axis situations.

Concentrations higher than 300x are not adequate because of the incapability of the SILO SOE to image the whole POE properly, whose angular size as seen from the SOE (the SOE “field of view” in imaging terminology) and the required magnification (POE to cell ratio) are too large for a single refractive surface to manage, especially for the wide spectrum required. CAP is limited to approximately 0.43° . In addition, the spectral uniformity between different junctions decreases with concentration. This is further explored in Section 2.2.2

One proposed solution to this limitation is a new technology called multi-fold Köhler

concentrators [42], [43]. The main idea is to concentrate the sunlight through Köhler integrator pairs, divided into multiple channels, each one comprising two optical surfaces. Dividing the incoming light in different channels decreases the angular size of the field of view of the SOE, makes it easy to couple the incoming light and to image each section of the POE. This solution has a great impact in CAP, and the capability to maintain a good irradiance uniformity at high concentrations, even for off-axis situations.

Most explored multi-fold Köhler concentrator is known as the Fresnel-Köhler concentrator [42] that is a system with four channels, each comprising a Fresnel lens as a POE and a free-form refractive surface as the SOE (Figure 2-9). As this Chapter is dedicated to the development of a Fresnel-Köhler of 9 channels, hereafter the 4-fold Fresnel-Köhler is referred to as FK4, while the 9 channel one is referred to as FK9.

Figure 2-9 The FK4 is an advanced 4-fold Fresnel Köhler concentrator. On the left, the optical system is illustrated together with the edge-rays distribution. On the right, actual manufactured Fresnel lens POE and free-form SOE, both 4-fold, are shown. On the bottom-right, actual CCD image of the cell plane under outdoor sun tracking. It is a white square illumination, indicating the excellent spatial and spectral uniformity provided.

The FK4 is four-fold symmetric. A standard version of this optics developed by companies Evonik and LPI, called Ventana™, in its first variant uses PMMA as POE material. It operates at 1024x with an acceptance angle of $\pm 1.1^\circ$ [42], exhibiting a CAP of 0.58. Outdoors electrical efficiency of 32.0% was measured with this Ventana™ when uses a 39.2% efficient Spectrolab C3MJ+ cell [44]. The FK4 prototype with SOG Fresnel lens and Solar Junction A-SLAM™ cell using dilute-nitrides (cell efficiency approx. 42.0%), showed a single-cell concentrator maximum peak efficiency of 36.0% and regressed 35.6% efficiency.

FK4 presents a remarkable spatial and spectral uniformity at high concentrations, even for

off axis situations, which has a real impact in the electrical performance of the system [45]. An example of this benefit is shown in Section 2.2.2 The SOE, although being a freeform with four sectors, does not differ from conventional domes, from the manufacturing point of view, which means that the same manufacturing techniques such as glass molding can be used for the SOEs.

Other types of multi-fold Köhler were designed, for both refractive and reflective POEs. A 4-fold Köhler approach with an asymmetric mirror was developed to address the reflective configuration [46] (Figure 2-10). The system presents a very high CAP (0.73) and a high optical efficiency (89.2%). The reflective nature of the POE means that the system does not have to deal with chromatic aberrations, but the irradiance uniformity, although good and better than conventional CPV systems, is not excellent as other Köhler configurations, due to its asymmetrical nature. An 800x version of the concentrator was experimentally tested and validated in Japan [47].

Figure 2-10 Off axis XR-Köhler concentrator. (Left) Geometry of the concentrator. (Right) Ray tracing configuration

Another approach of a 4-fold Köhler configuration is the Dome-shaped Fresnel- Köhler concentrator [48] (Figure 2-11). The POE is a dome-shaped Fresnel lens developed by Daido Steel. The shape of the POE increases the complexity of the manufacture, but allows for a higher CAP (0.72). The irradiance uniformity is slightly worse than the one of FK4, due to the fact that it is harder to image a POE with a dome shape onto the entire cell than a flat one.

Quite remarkable is Fresnel-RXI Köhler whose POE is a flat Fresnel lens and whose SOE is a four-fold free-form SMS 3D RXI lens which contains two free-form surfaces (Figure 2-11). It leads to a $CAP = 0.85$.

Figure 2-11 (Left) Fresnel RXI Köhler (Right) Domed Fresnel Köhler

2.2 Thermal expansion, refractive index variations and spectral irradiance distributions in CPV systems

The main motivation for pursuing the development of the 9-Fold Fresnel-Köhler (FK9) concentrator was to create a CPV system prepared to achieve a high performance even in extreme conditions, by having a large tolerance to different kinds of external effects.

This section describes some of these effects that affect CPV systems, and that FK9 is prepared to overcome.

2.2.1 Thermal expansion and effects on refractive index in SoG lenses

Thermal expansion changes the material density and thereby alters its optical density. While in reflective materials, this effect can affect slightly the profile of the material, on refractive ones, the refractive index dependence on temperature affects the chromatic aberration effect produced by lenses. These effects are especially high on SoG lenses. For CPV deployment, this can be a major concern, as SoG lenses are especially interesting in locations as high DNI sand deserts, due to its resistance to scratches. These locations typically have very high temperature variation throughout the day, and the variation of refractive index with temperature may affect the spectral balance between different junctions (as

demonstrated in Section 2.2.2 and in Section 2.5).

The effects of temperature in refractive index are well documented [49]–[51]. Schult measured the change of index of refraction with temperature in SoG lenses (Figure 2-12). Schult also pointed out that, in optical systems consisting of only one SoG lens, the short circuit current can drop by more than 10% at a 20°C increase of temperature if thermal effects are not considered when designing the system.

Figure 2-12 Measurement of dependence of index of refraction on temperature in Silicon-on-Glass lenses. Source: Ref. [51]

Askins [49] confirmed that the temperature affects SoG lens due to two primary mechanisms: change of index of refraction, and thermal expansion induced deformation of the facets. He showed that a variation of temperature produces a change in the focal distance of the lenses that is approximately linear and related only with the change in index of refraction. On the other hand, the coefficient of thermal expansion mismatch between silicon and glass produces a widening of the flux profile, at any temperature other than the temperature of cure. Both effects should be taken into consideration when designing the optical system.

Hornung [50] estimated the influence of Fresnel lens temperature on energy generation of a concentrator photovoltaic system on different locations, comparing systems with only one optical element, a Fresnel lens made of PMMA and SoG. The simulations revealed a higher a priori efficiency for SOG Fresnel lenses than for PMMA Fresnel lenses, but when accounting variation of lens temperatures, the reduction of optical efficiency of SoG lenses is roughly

twice as high compared to the reduction for PMMA lenses.

2.2.2 Effects of spatial and spectral irradiance distribution on the performance of several CPV architectures

The effects of non-uniform irradiance distributions on multi-junction cells was explained in Section 2.1.1. Different CPV optical systems create different kinds of irradiance, both spectral and spatial. This section describes recent studies made on how irradiance patterns created by different CPV architectures may affect the performance of multi-junction cells. Several studies were made to assess the effect of non-uniformity in cells [52], [53]. We highlight two of the most recent studies, as both take into account the spectral irradiance patterns of real CPV architectures.

Figure 2-13 Electrical measurements for Fresnel + bare cell (left), Fresnel + pyramid SOE (center) and Fresnel + dome SOE (right) at different primary-to-cell distance. Design position is 250mm. Plots at the bottom represent ratio between top and middle photogenerated current for each measurement positions at 1000W/m² (SMR=0.96). Ratios higher than 1 (red) indicate middle-limited zones, whereas ratios lower than 1 (blue) indicate top-limited zones. In green areas, top and middle

photogenerated current is matched. Source: Ref. [15]

Victoria *et al* [15] studied the effects of spatial distribution of irradiance and spectrum in multi-junction cells for three different CPV optical systems. Using the Helios 3198 solar simulator, an experimental characterization (based on electrical measurements) was performed for three Fresnel based-systems: a RTP (pyramid) type concentrator, a silo (dome) and a single Fresnel lens with no SOE, just a bare cell (Figure 2-13).

The three evaluated systems were designed for the same concentration (625X) at different cell-to-lens distances, to assess the different irradiance and spectral distributions that the three CPV systems can create. The systems were evaluated for different irradiance levels, each with a different spectrum characterized by the *Spectral Matching Ratio (SMR)*, defined as the ratio between the top and middle subcell photogenerated currents under the spectrum in study, divided by that ratio under the reference spectrum. SMR is an indicator of how blue-shifted ($SMR > 1$) or red-shifted ($SMR < 1$) the light is with respect to the reference spectrum.

The results of the study are presented in Figure 2-13. The plots at the bottom represent the ratio between top and middle photogenerated current for each measurement position at 1000W/m^2 ($SMR = 0.96$), and show how the balance between spectral irradiance in these two junctions is. The first conclusion of this study is the fact that the addition of an SOE has a real impact on the spectral distributions, and is reflected on the electrical measurements: the Fresnel + bare cell presents a much lower fill factor value when the short circuit current I_{sc} has high values than the systems with SOE, and it is much more sensitive to spectrum fluctuations. For both systems with SOE there is a clear increase in fill factor when the spectral distribution becomes more uniform, even if the increase in uniformity is not that significant (especially for the pyramid SOE, which has a quite uniform distribution).

Espinet-González *et al* [45], using the three-dimensional distributed model (described in Section 2.1.1), in which a multi-junction solar cell is divided in several elementary units, studied the triple-junction cell performance under Fresnel-based concentrators, accounting for chromatic aberrations. The Fresnel-based concentrators were the SILO, the XTP, the RTP and the 4-fold Fresnel Köhler, all designed for $C_g = 850x$. Although the SILO and RTP have typically good irradiance uniformity for specific concentration levels, the uniformity degrades as concentration goes higher, and at 850x, some spectral uniformities appear for this configuration. The spectral distributions and electrical performance simulations can be seen in Figure 2-14, and the results shows that spectral non-uniformity affects considerably the fill factor and efficiency of the system. Notice that the RTP achieves decent irradiance uniformity but slightly unbalanced between the Top cell, Middle cell and Bottom wavelengths, and still an efficiency loss of more than 2% due to spectral irradiances is

observed. The RTP is widely used in CPV systems, and this loss due to spectral unbalance is considerable.

Figure 2-14 triple-junction cell performance under Fresnel-based concentrators, accounting for chromatic aberrations. (Top) Irradiance maps/Normalized photocurrent densities at normal incidence, and for the top cell (TC), middle cell (MC) and bottom cell (BC). (Middle) IV curves at normal incidence (Bottom) Efficiency and fill factor. Source: Ref. [45]

Recently, Besson *et al* [54] developed a methodology that measures spectrally resolved irradiance distribution for CPV optical systems, In conjunction with electrical I-V curves, this method is a mean to visualize and characterize the effects of chromatic aberration and nonuniform flux profiles under controllable testing conditions. The characterization test bench decouples the temperatures of the primary optical element (POE) and the cell allowing their respective effects on optical and electrical performance to be analyzed. The

method was used to assess both effects that are described in this section, and for which the FK9 was developed: in a 500x CPV system with a SoG lens as a POE and no SOE, the temperature of the POE was changed, and the variation of spectral irradiance distributions was measured, as well as their effect on the overall electrical performance. The results are shown in Figure 2-15.

Figure 2-15 (Left) $J_{\text{top}} / J_{\text{mid}}$ ratio and the current density normalized by the average value on the cell for a centered cross-section of the spot for a single optical element concentrator (SoG lens). (Right) IV curve of the concentrator. The concentrator is evaluated for two different SoG temperatures. Source: Ref. [54]

The variations of temperature cause notable chromatic aberrations, and the normalized current densities of top and middle cell change considerably. The electrical performance changes as well, with a decrease of fill factor and an increase of I_{sc} , due to the fact that there was an initial current mismatch between junctions.

The same work compares this system with a 500x FK4 with a PMMA lens (Figure 2-16). The system is insensitive to the chromatic variations of the PMMA lens, and both the spectral irradiance distributions as the electrical performance remain unchanged, experimentally proving that an appropriate optical design can compensate the chromatic variations of the optical components. FK9 further explores this idea, and Section 2.5 shows the spectral irradiance distributions of different concentrators at different temperatures and concentrations. FK9 outperforms the other concentrators, maintaining the uniformity.

Figure 2-16 $J_{\text{top}}/J_{\text{mid}}$ ratio and the current density normalized by the average value on the cell for a centered cross-section of the spot for a FK4 (PMMA lens). (Right) IV curve of the concentrator. The concentrator is evaluated for two different PMMA temperatures. Source: Ref. [54]

2.3 Optical design of 9-Fold Fresnel Köhler with Fresnel lens with variable focal point

This section describes the optical design of the 9-fold Fresnel-Köhler (FK9).

Based on the arrays of Köhler integrators, the concept is further extended to a larger number of channels. 9-fold Fresnel-Köhler (FK9) concentrator design is presented, and realistic simulations results are shown. This work shows that FK9 design is prepared to work at very high concentration levels (up to 2000x), maintaining a high irradiance uniformity on the solar cell for the desired spectral range. When the number of channels increases, the possibility that light crosstalk between different channels also increases, thus decreasing the acceptance angle. In this work we explore a new technical solution to overcome the crosstalk issue, and a Fresnel Lens with Variable Focal Point (FL-VFP) is proposed, which prevents light impinging in extreme points of the POE from crosstalking to other channels of the system. It is shown that this novelty further enhances the acceptance angle.

Figure 2-17 FK9 ray tracing simulation, where the 9 focus on SOE sectors are visible, as well as the image on the cell

2.3.1 Technical description of FK9

FK9 consists on a Primary Optical Element (POE), a flat Fresnel lens divided in 9 sectors, and a Secondary Optical Element (SOE) also divided in 9 sectors, all of them free form, and coupled with the respective sector of the POE, as seen in Figure 2-17. The POE consists in 1 symmetric central sub Fresnel lens, four lateral and four diagonal sub Fresnel lenses, each of them symmetric with each other relative to the Fresnel lens center (Figure 2-18). Each one of the four lateral and four diagonal sub-lenses may be seen as off-center square pieces of a symmetric Fresnel lens, each one with their axis of symmetry.

Figure 2-18 Schematic view of FK9 concentrator. Three different fold types, central, lateral and diagonal (in the picture, orange, green and blue, respectively) form a 9 fold system. Each sector of the SOE images the corresponding sector of the POE in the cell.

The SOE has the same structure: 1 central sub-lens, four lateral and four diagonal sub-lenses, all symmetric with each other relative to the center axis, all of them free form. Each sub-lens is coupled with a sector of the POE, imaging each square sector of the Fresnel lens into the cell, thus performing Köhler integration. This integration allows that, even when the concentrator is misaligned with an angle within the acceptance, each surface of the SOE can still control the incoming bundle from each sector of the POE and redirecting and redirect it towards cell surface, allowing an excellent irradiance uniformity, even for off-axis angles. The fact that the input bundle is divided into 9 different bundles enables a smaller beam angle of each bundle coming from the different sectors of the SOE. Also, allows that the thermal stress of each light spot inside the SOE is reduced by the ninth. The optical design is optimized such that the extreme rays that impinge in the POE are directed into the extremity of the cells, both for an aligned angle as for a design acceptance angle.

The integration allows for an excellent spectrum and spatial irradiance on the cell, even at very high concentrations and extreme thermal conditions, as presented in Section 2.4.3. The

fact that the incoming bundle of Sun rays is divided into nine channels makes that each SOE sector will need to manage a correspondingly smaller field of view and also provide a smaller magnification.

This eases the goal to obtain a very high acceptance for rays coming from any direction, thus allowing a high CAP. Moreover, this incoming light split reduces the risk of glass solarization in the SOE if compared with other conventional Fresnel-based concentrators, where a single light focus appears inside the SOE.

FK9 optical surfaces are very similar to those in other conventional concentrators, from a manufacture point of view. The POE of the FK9 is similar to conventional flat Fresnel lenses, while the SOE is equivalent to typical dome-shaped SOEs used in other concentrators. This implies that the same low-cost and already mature manufacturing techniques, such as compression molding, hot embossing and glass molding, can be applied to this concentrator, enabling a production cost similar to those conventional concentrators.. Although the fabrication of the mold of the POE is more demanding than a conventional Fresnel lens due to segmentation and assembly, for volume manufacturing the potential for cost reduction is increased due to loose assembly/alignment tolerances (through high acceptance angle) [55].

2.3.2 Fresnel lens with Variable Focal Point

Multichannel systems design always have to struggle with optical crosstalk between different channels. For solar concentrators, where it is intended the acceptance cone to be as large as possible for each channel, it can result in a major issue, which limits the CAP of the system.

In FK9, although the concentrator has already a high CAP (as presented in Section 2.4.2), the major limitation to further increase its value is the crosstalk between different channels that occurs when the system is at a considerable off-axis angle. Crosstalk happens between sublenses of the SOE, especially for rays that are refracted in areas of the Fresnel lens which are farther away from the optical axis. The main limiting errors for this situation are the chromatic aberrations of Fresnel lens. In this section, a Fresnel lens with variable focal point is proposed, which has the ability to control the rays coming from extreme points, i.e., far from the optical axis and increase the CAP of the system.

Figure 2-19 Schematic view of FK9 with Fresnel Lens with Variable Focal Point (FL-VFP). On the left - Section view of the concentrator on the optical axis plane. On the right - Isometric schematic view of the system

Consider Figure 2-19, where both problem and proposed solution are explained. In this case, only the Fresnel lens of the lateral sector of FK9 is considered, although the same considerations can be made to the diagonal sector. To fulfill the requirement that the whole cell area is uniformly illuminated, Köhler integration has to be achieved, and ray a and b , coming from the optical axis boundaries of the Fresnel lens in an on-axis case, have to impinge on the extremes of the solar cell R and L . While the first condition is ensured by the free-form surface of the SOE and direction of ray b , for the second one, it is required that the “virtual” focal point of the Fresnel lens is in position f_1 (virtual because the focal point actually does not exist, due to the refraction of rays in the SOE surface). The fact that the focal point is at a considerable distance from the surface of the SOE makes that, for an off-axis angle, rays refracted in points far from the optical axis k such as points B' and C' , will refract in SOE in points far from the optical axis vertical plane k' in the y direction. This implies that, from a specific off-axis angle, these rays will crosstalk with the facet of the SOE's diagonal sector. The same crosstalk situation occurs for the diagonal sector.

The proposed solution to avoid optical crosstalk and further enhance the concentrator performance is a Fresnel Lens with Variable Focal Point (FL-VFP). FL-VFP intends to add an extra degree of freedom to the Fresnel lens, such that rays refracted in points distant from the optical axis can be further controlled, without perturbing the Fresnel lens rotational symmetry and maintaining Köhler integration. Once again, consider Figure 2-19. In FL-VFP, while you move farther away from point A to point B in the x direction, the focal point of the corresponding Fresnel lens tooth changes along line i from f_1 to f_3 , always on the optical axis

plane k' . This arrangement allows that tooth 1 (and its corresponding points C and extreme point C') and tooth 2 (and its corresponding point B and extreme points B') focus in points f_2 and f_3 , respectively. Therefore, rays coming from points in POE far from the optical axis k have a focal point closer to the SOE's surface, and will refract on the SOE closer to symmetry plane k' . Consequently, the off-axis angle to which crosstalk is avoided further increases, increasing the acceptance angle of the system.

To ensure that point f_3 , focal point of tooth 1 is close enough to the surface, the function r that relates the distance t , the distance between a generic tooth o and point A and the distance i , distance between f_1 and f_o , focal point of tooth o , is an hyperbolic function, due to its rapid growth.

Figure 2-20 The projection of the acceptance cone of different points of the POE creates "ellipses" on the SOE's surface. The blue acceptance cone is on the optical axis and its projection ellipse impinges only on the SOE's lateral sector. The yellow acceptance cone is on a point far away from the optical axis. Using the normal Fresnel lens, the projection ellipses impinges on two sectors of the SOE, creating crosstalk between optical channels. Using the Fresnel lens with variable focal point, the position and orientation of the ellipse is corrected, and it only impinges on one channel

Figure 2-20 displays a graphical visualization of the impact of FL-VFP, where the acceptance cones impinging in two points of the lateral sector of the POE are represented, for a point on the optical axis (blue acceptance cone) and a point outside of the optical axis (the yellow acceptance cone). The rays within an acceptance cone, when impinging on the SOE, form a "projection ellipse", as the ones shown in the image. The projection ellipse created by the blue acceptance cone impinges only on the lateral sector, and all its rays are refracted to design point on the cell. As the yellow acceptance cone impinges on a point far away from the optical axis, its projection ellipse impinges on the SOE in two sectors, creating crosstalk between optical channels. The rays from the yellow acceptance cone that impinge on the diagonal sector are not refracted to the cell, affecting both acceptance angle and the irradiance uniformity for off-axis situations. Using FL-VFP, this situation is corrected, and the orientation and position of the yellow projection ellipse is corrected, impinging only on the lateral sector. The cross talk is avoided and the rays are refracted toward the solar cell.

Figure 2-21 shows the positive effect of FL-VFP. For an off-axis situation of 1.2° , it is noticeable the capability of FL-VFP to prevent an optical crosstalk situation for a 1000x FK9.

Figure 2-21 Ray tracing of 1000x FK9 and FK9 with FL-VFP for an off-axis angle of 1.2° . On the left, it is visible the optical crosstalk situation on the SOE while on the right, with FK9 with FL-VFP, the crosstalk is prevented, further improving the acceptance angle

2.3.3 Design procedure

Figure 2-22 Design procedure of FK9 concentrator

The design procedure of an FK9 concentrator involves the design of three sectors, the central, the lateral and the diagonal sector. The lateral and diagonal are then replicated three times each around the z axis to obtain the final architecture. The design procedure guaranties the maximum uniformity on the solar cell: the maximization of CAP is obtained by an optimization procedure, where the key factor is decreasing the crosstalk between channels.

The FK9 concentrator can be design with and without FL-VFP. First, we present the design procedure without the FL-VFP, and then, we integrate it in the procedure. As each sector is designed individually, we present the design for the central, lateral and diagonal sector separately. The procedure starts by defining the concentration value (which defines the size of the POE and SOE) the $f/\#$ of the whole system, and the acceptance angle θ for which the system is designed. This acceptance angle is not necessarily the final acceptance angle of the

system, and it is used as an optimization parameter to achieve the maximum CAP.

Central sector

Consider Figure 2-22 (left). The central sector is designed considering the diagonal of cell (plane j'), such that the diagonal points **LD** and **RD** are illuminated. The focal point f_c is necessarily on the z axis for symmetry reasons. Having these values defined:

- We start by designing a Cartesian oval coupling spherical wavefronts with origin in **M**, the central point of the Fresnel lens, and **S**, the central point of the solar cell. We calculate the height of the Cartesian oval and the z coordinate of focal position f_c such that the ray a and b , the normal ray and extreme ray with incidence angle θ that are refracted in **C'**, respectively, are refracted in the SOE towards **LD**.
- The second step defines the limits of the central sector of the SOE (points **Q** and **R**). Point **Q** is calculated as the intersection of extremes ray c (from **M**) or b (from **C'**) with the SOE, depending on which intersects the surface at the largest distance from the z axis. In Figure 2-22 (left), we show the case in which b is used for this purpose. **R** is symmetric to **Q**.

This first step ensure us that all the rays impinging in **M** within the acceptance angle θ are impinging in the center of the cell **S** and that, due to the edge ray principle and the casi-image creation of **C'** in **LD**, there is a sharp cut off in **LD**. The other rays impinging in **C'**, although are not imaged in **LD**, impinge in points inside the cell very close to it. Furthermore, the correlation between positions of normal incidence rays impinging on **M** to **C'** and rays impinging on **LD** to **S** is linear, producing the desired uniform irradiance distribution on the cell surface. This property is also fulfilled when considering 3D design.

Lateral sector

Consider Figure 2-22 (center).The lateral sector is designed considering the side view of the cell (plane k'). The procedure starts by setting β , which is the angle between the Fresnel lens and ray d , the normal ray impinging on **B** after refraction. β determines the angle of refraction and it will be used in the optimization process, and the focal point f_1 of the Fresnel lens is along ray d .

We want to design the surface of the SOE's lateral sector as the Cartesian oval that couples the spherical wavefronts with origin in **B** and **L**, the lateral left point of the cell. As we have two degrees of freedom (the height of the Cartesian oval and the position of focal point f_1 along ray d), we may use them to fulfill two conditions: that rays e and f , the normal ray and extreme ray with incidence angle θ that are refracted in **A**, respectively, are refracted in the Cartesian Oval in the direction of **R**, the lateral right point of the cell.

Similarly to the central sector, the correlation between positions of normal incidence rays impinging on **A** to **B** and rays impinging on **L** to **R** is linear, producing the desired uniform irradiance distribution on the cell surface.

Diagonal sector

Consider Figure 2-22 (right). The diagonal sector is designed considering the diagonal view of the cell (plane j'), and the extreme diagonal points of the cell **LD** and **RD**. The design of the diagonal sector is equivalent to the design of the lateral sector: as in the previous sector, the procedure starts by setting ω , which is the angle between the Fresnel lens and ray h , the normal ray impinging on **D** after refraction. ω determines the angle of refraction and it will be used in the optimization process, and the focal point $\mathbf{f}_4$ of the Fresnel lens is along ray h .

We want to design the surface of the SOE's diagonal sector as the Cartesian oval that couples the spherical wavefronts with origin in **D** and **LD**, the diagonal left point of the cell. As we have two degrees of freedom (the height of the Cartesian oval and the position of focal point $\mathbf{f}_4$ along ray h), we may use them to fulfill two conditions: that rays i and j , the normal ray and extreme ray with incidence angle θ that are refracted in **C'**, respectively, are refracted in the Cartesian Oval in the direction of **ERD**, the extreme diagonal right point of the cell.

Variables for optimization of FK9: θ , β and ω

Design procedure of FK9 with Fresnel lens of variable focal point

When adding the FL-VFP, we get a number of new variables for optimization. Now, the focal point of each tooth of the lateral and diagonal Fresnel lenses is variable.

We explain this change by considering the lateral sector. Consider Figure 2-22 (center). The normal ray impinging on the Fresnel lens in **A** is directed towards the focal point $\mathbf{f}_1$, while the normal ray impinging in **B** is directed towards $\mathbf{f}_3$. As we intend to maintain the direction of d the same as in the design of the FK9 without the FL-VFP, $\mathbf{f}_1$ and $\mathbf{f}_3$ are in the direction of d , and the focal points of the remaining teeth are in the line that joins $\mathbf{f}_1$ and $\mathbf{f}_3$. Thus, we need to define the distribution of focal points along this line. $\mathbf{f}_1$ and $\mathbf{f}_3$ are at a distance f_{13} from each other. We main define a focal point $\mathbf{f}_0$ between $\mathbf{f}_1$ and $\mathbf{f}_3$ such that:

$$\mathbf{f}_0(t) = \mathbf{f}_1 + T_L(t)(\mathbf{f}_3 - \mathbf{f}_1) \quad (2.1.6)$$

where $t \in [0,1]$ and $T_L \in [0,1]$. Function T has to have a fast growth such that, when going from **A** to **B**, the focal point $\mathbf{f}_0$ approaches rapidly the SOE's surface. This way, the majority of the teeth between **A** and **B** have their focal points close to the surface. To ensure a rapid growth, T_L is an hyperbolical function:

$$T_L(t) = k_L + \frac{r_L}{u_L t + m_L} \quad (2.1.7)$$

where $k_L = u_L + \frac{\sqrt{u_L^2 - 4r_L u_L}}{2u_L}$ and $m_L = \frac{\sqrt{u_L^2 - 4r_L u_L} - u_L}{2}$. r_L and u_L are the variables that determine the growth rate of the function, and are parameters for the optimization. Note that $T(0)=0$ and $T(1)=1$. Figure 2-23 shows a graphical example of function $T_L(t)$.

Figure 2-23 Variation of focal points between f_1 and f_3 . The hyperbolic function ensures a rapid growth at the beginning.

The diagonal sector follows the same logic applied to the lateral sector: the focal points are in the line between f_4 and f_5 , being f_5 the focal point of point **D**, and at a distance f_{45} from f_4 (Figure 2-22 right). The focal points follow a function $T_D(t)$, equivalent to function $T_L(t)$ of the lateral sector, with variables for optimization r_D and u_D . So, FL-VFP applied to both lateral and diagonal sector adds f_3 , f_5 , r_L , u_L , r_D and u_D to the optimization.

Variables for optimization of FK9 with FL-VFP: θ , β , ω , f_{13} , f_{45} , r_L , u_L , r_D and u_D

2.3.4 Design optimization

The variables for optimization identified in the design procedure allow for a vast family of possible FK9 designs. As, by design, the irradiance uniformity is achieved in all of them, the optimization intends to maximize the concentrator's CAP.

As shown in previous sections, refractive elements induce chromatic aberrations, which affect the position of the POE focal point for different wavelengths. In CPV systems, the variation of the position of focal point with the wavelength creates the effect that the short circuit current created by each junction presents a different acceptance angle. The acceptance angle of the whole system is defined as the smallest one of the junctions. The geometry of the system implies that the acceptance angle should be measured in two

different directions, horizontally and diagonally, so we may define two acceptance angles for each junction. Therefore, when working with triple junction cells, we may consider six different acceptance angles. Table 2-1 identifies the acceptance angles for each junction.

Table 2-1 Each junction has two different acceptance angles, an horizontal and a diagonal one. The table identifies each of the acceptance angles

	Horizontal acceptance angle	Diagonal acceptance angle
Top junction	α_{TH}	α_{TD}
Middle junction	α_{MH}	α_{MD}
Bottom junction	α_{BH}	α_{BD}

The global acceptance angle of the concentrator is the minimum acceptance angle of these six, and the optimization's goal is to maximize the limiting acceptance angle, maximizing the CAP as well.

The variables for optimization were identified in the previous section. For FK9, the variables are the design acceptance angle θ , one coordinate of the lateral Fresnel lens focal point $\mathbf{f}_1$, β , and one coordinate of the diagonal Fresnel lens focal point $\mathbf{f}_4$, ω . Considering the design with the FL-VFP, we need to add six more variables, f_{13} , f_{45} , r_L , u_L , r_D and u_D . Hence, the global acceptance angle of the system will be a function of these variables, $\alpha(\theta, \beta, \omega)$, and $\alpha(\theta, \beta, \omega, f_{13}, f_{45}, r_L, u_L, r_D, u_D)$, for the design with FL-VFP. The number of variables of FL-VFP increase the optimization's complexity, but it has a significant impact on the acceptance angle, as shown in Section 2.4.2 The optimization procedure tries to change the variables in order to maximize the limiting acceptance angle. As the main limiting factor of the acceptance angle is the crosstalk between different optical channels, in actuality, the maximization of CAP is achieved when the maximum crosstalk is avoided, i.e. when the number of projection ellipses of different acceptance cones (Figure 2-20) and different wavelengths within its respective optical channel is maximized.

2.4 Raytrace Simulation Results of FK9 with FL-VFP

Simulation results are presented for both FK9 and FK9 with FL-VFP, designed for a geometrical concentration of $\mathbf{C_g = 1000x}$ (over cell illuminated area) and a F-number of 1. All the following simulation results correspond to raytracing simulations carried out under the same realistic conditions: AM1.5D sunlight spectrum, finite sun ($\pm 0.265^\circ$) and Fresnel and absorption losses. POE is made on PMMA, and SOE is made on Schott B270 glass. Moreover,

no anti reflecting coating has been applied on SOE. POE teeth rounded edges were not considered. The photovoltaic cells high efficiency triple junction cells of approximately 40% efficiency, and the quantum efficiencies provided by the manufacturer were used for the optical efficiency calculations.

Figure 2-17 presents a ray tracing simulation of FK9 concentrator. The optical behavior at normal incidence of the system can be observed: each focus of the nine POE sectors impinging in the correspondent SOE sector surface are integrated, and equally distributed over the entire cell surface.

2.4.1 Optical efficiency

The optical efficiency is calculated considering the photocurrents generated by each junction, as

$$\eta_{opt} = \frac{I_{SC}^{conc}}{C_g I_{SC}^{1sun}} = \frac{\min\{I_{SC,top}^{conc}, I_{SC,middle}^{conc}, I_{SC,bottom}^{conc}\}}{C_g \min\{I_{SC,top}^{1sun}, I_{SC,middle}^{1sun}, I_{SC,bottom}^{1sun}\}} \quad (2.1.8)$$

The currents of each junction are calculated using AM1.5D sunlight spectrum and the quantum efficiencies provided by the manufacturer. The results are shown in Table 2-2.

Table 2-2 FK9 AND FK9 FL-VFP concentrator simulation results for $C_g = 1000x$

$C_g=1000x$	FK9	FK9 FL-VFP
Optical Efficiency	83.9%	83.8%

There are no significant differences in the optical efficiency of both concentrators. Based in previous works, a global efficiency increase of 2% can be expected if a proper anti reflecting coating is applied [56].

2.4.2 Acceptance angle, angular transmission curve and CAP

As seen in the previous sector, the acceptance angle is the minimum between the six of Table 2-2. In reality, as the Ge bottom junction of typical multi-junction cells produces an excess of photocurrent, only the acceptance angles of middle and top cells can be considered.

The CAP values are calculated using the concentration and acceptance angle values, according to Equation (2.1.5). Figure 2-24 shows the acceptance curves for both FK9 and FK9 FL-VFP.

Figure 2-24 FK9 and FK9 FL-VFP acceptance curves for $C_g = 1000x$. For the FK9, acceptance angle is $\pm 1.12^\circ$, while for FK9 with FL-VFP, the acceptance angle is $\pm 1.2^\circ$

1000x FK9 has an acceptance angle of $\pm 1.12^\circ$. Also, the curve almost maintains its maximum efficiency until it reaches 1° of deviation, ensuring a very high efficiency within the acceptance angle. The concentration level of $C_g = 1000x$ and an acceptance angle of $\pm 1.12^\circ$ represents a CAP of 0.62, a considerable improvement due to a higher number of channels. The losses that can be observed after 1° of deviation are mostly due to Fresnel reflection effects on the SOE surface. This implies that, if an optimized anti reflecting coating for the appropriated incidence angles is applied on the SOE optical surface, the acceptance angle and the CAP can be further increased.

When the Fresnel Lens with Variable Focal Point (FL-VFP) is introduced in FK9 design, although the complexity of the system increases, it has the advantage that the acceptance angle rises to $\pm 1.2^\circ$, which represents a CAP of 0.66. This considerable increase is due to the fact that FL-VFP can prevent optical crosstalk for a higher off-axis angle.

Table 2-3 summarizes the simulation results for both concentrators.

Table 2-3 FK9 and FK9 FL-VFP concentrator simulation results for $C_g=1000x$

$C_g=1000x$	FK9	FK9 FL-VFP
Optical Efficiency	83.9%	83.8%
Acceptance angle	$\pm 1.12^\circ$	$\pm 1.2^\circ$
CAP	0.62	0.66
f-number	1	

2.4.3 Irradiance and intensity on the cell

The importance of spectral and spatial irradiance uniformity on solar cells has been highlighted throughout this Chapter, and one of the motivations to pursue the FK9 design was the quality of irradiance.

Figure 2-25 1000x FK9 spectral irradiance distributions for Top, Middle, and Bottom junction of a Triple MJ cell, for normal incidence and for a misalignment angle of 0.8° . The irradiance distribution is analyzed throughout the full AM 1.5D Sun light spectrum

The irradiances distributions were considered for different bands of the ASTM G173 AM1.5D Solar spectrum, correspondent to part of the spectrum absorbed by each junction. So, the irradiance on the cell is shown for the spectrum band 360-675 nm (top junction), for the band 675-935 nm (middle junction) and bottom junction (935-1800). Figure 2-25 shows the spectral irradiance distributions on each junction, for normal incidence, and for an off-axis angle of 0.8° that FK9 produces. FL-VFP does not affect the irradiance distributions, leading to similar results when the Variable Focal Point is applied. At normal incidence, spatial irradiance is almost perfectly uniform for all junctions. This uniform spatial irradiance is kept for the full Sun light spectrum. This is especially important to avoid current mismatches between junctions and high series resistant losses, and even more if one considers cells with 4 and 5 junctions, as they are more likely to be sensitive to spectrum variations, thus it is more difficult to achieve current match.

For a misalignment of 0.8° , the spatial irradiance is still very uniform, especially in the Top and Middle junctions. When dealing with the widely used Ge-bottom MJ cells, the current is

typically limited by the Top and Middle junction, thus is likely that this less uniform distribution on the Bottom Cell does not have a considerable effect on the overall performance. Köhler integration on every surface of the SOE enables that the spectral irradiance is essentially constant within the acceptance angle.

2.4.4 Comparison of CAP between different Fresnel lens-based systems

This section provides a comparison of CAP between different Fresnel lens-based systems. The selected architectures are: the Fresnel lens with RTP as an SOE (that acts as a light pipe homogenizer), the SILO, the Fresnel- Köhler of four sectors (FK4), and the two FK9 concentrators presented above (FK9 and FK9 FL-VFP) (Figure 2-26). These CPV architectures are well known for providing rather good to excellent uniformity at specific concentrations.

Figure 2-26 The RTP, silo, FK4 were selected for comparison with FK9 and FK9-FL-VFP due to the fact that these systems provide rather good to excellent uniformity on the solar cell

Figure 2-27 presents the CAP comparison results, where CAP values for the first three systems were taken from the literature [57]. While FK4 and FK9 are the systems with highest performance in this field, FK9 still outperforms the FK4 concentrator, presenting a higher CAP of **0.62** and higher acceptance angle, therefore reaching a higher tolerance value. Using FK9 design with the FL-VFP, this value further increases to **0.66**, being the highest value amongst the Fresnel-based systems designed for excellent irradiance uniformity.

Figure 2-27 Comparison of CAP between different Fresnel lens-based systems with rather good to excellent irradiance uniformity

2.5 Spectral irradiance distributions at high concentrations and high temperatures

Section 2.1.3 described how SoG lenses are being chosen by CPV companies as a POE for their system, due to the fact that they are especially suitable when CPV systems are installed in aggressive environments as sand deserts (typically with high DNI), as glass resistance to scratches is significant higher than PMMA's. This section also showed the effects of temperature on the index of refraction of SoG lenses, and that this may have a real impact on the spectral irradiance distributions and thus, on the solar cell performance. Considering that sand deserts are typically regions with large temperatures variations, this has to be taken into account in the CPV system.

Section 2.1.1 showed the advances of four junctions cells, that currently hold the world record for efficiency (46%) for photovoltaic cells, and stated the fact that is likely that the spectral balance between different junctions will be increasingly important for future CPV systems, as current matching is harder for a higher number of junctions. This section also stated that there is a strong possibility that higher concentration levels will be needed.

This section studies how different CPV architectures are prepared to benefit the advantages of SoG lenses, and meet the expected requirements of four and five multi-junction cell, through variation of concentration and temperature

To analyze the effects of high concentrations and thermal variations in SoG lens based CPV systems on the spectrum irradiance distributions, a comparative study between three different SoG lens-based systems was made to evaluate their irradiance and spectral maps when designed for 1000x and 2000x concentration ratio, at two different POE temperatures, 20°C and 45°C.

The analyzed systems were the SILO, FK4 and FK9 (Figure 2-28). These systems were chosen for their capability of creating uniform spectral irradiance distributions at a specific concentration level. All three systems were designed with a SoG lens as POE, and an SOE made of b270 glass. The systems were designed for a POE temperature of 20°C.

As explained and referenced in Section 2.1.3 , the variation of index of refraction affects only the focal distance of the lenses, while the thermal expansion of the lens facets produces a widening of the flux profile. Due to the design procedure of FK, the latter affects mostly the acceptance of the system, while the change of the focal distance affects the spectral irradiance. In this work, only this effect is studied, so no thermal expansion was taken into account. The index of refraction dependence with temperature of SoG lenses was defined according to the measurements made by Schult [51], and set according to the graphic of Figure 2-12.

For a difference of 25°C, a decrease of -0.009 is considered for SoG lens refractive index. Only Bottom and Top cell irradiance distributions are presented to represent the most extreme cases. Although most cells in the market produce an excess of current in the Bottom cell, the most efficient triple-junction cells in the market have a current match between the three junctions (like the dilute-nitrides from Solar Junction or the inverted growth cells from Sharp, as mentioned in Section 2.1.1). Also, for four, five and six multijunction cells, the spectrum captured by the Bottom cell of a triple junction cell is divided among more cells, and the current balance between this junctions in this spectrum is likely to be extremely important. As FK9 is thought for these high-efficiency cases, the extreme cases of the Top and Bottom cell must be evaluated.

Figure 2-28 Spectral Irradiance distributions on Top and Bottom cell for three different CPV systems well known for their excellent irradiance uniformity at specific concentrations - $C(x)$ gives the irradiance on the point x over the irradiance at the concentrator's aperture (Fresnel lens)

All results are presented in Figure 2-28. It can be seen, that for 1000x at $T_{POE} = 20^\circ\text{C}$, the SILO already presents a non-uniform irradiance, especially at top cell. For this same concentration, when T_{POE} increases to 45°C , the refractive index variation causes a

considerable change in bottom cell irradiance distribution. Considering $C_g=2000X$, this effect is even clearer: for $T_{POE} = 20^{\circ}C$, the non-uniformity has increased considerably in both junctions, only due to the concentration increase. If we analyze an even more extreme case of $C_g = 2000x$ and $T_{POE} = 45^{\circ}C$, the non-uniformity in bottom cell irradiance has a dramatic change.

FK4 and FK9, as expected and presented previously, have an excellent spectral irradiance distribution for both cells, at $1000x$ and $T_{POE} = 20^{\circ}C$. When temperature increases to $45^{\circ}C$, FK4 bottom cell irradiance uniformity suffers a slight decrease, while the FK9 maintains the irradiance uniformity in both junctions. Still, these results are also supported by previous work [58], where FK4 presents high tolerance to color separation for a vertical misalignment of the SOE.

When $C_g = 2000x$ is considered, while both FK4 and FK9 still maintain an almost excellent uniformity for both junction at $T_{POE} = 20^{\circ}C$, for $T_{POE} = 45^{\circ}C$, FK4 suffers a significant non uniform variation for the Bottom cell irradiance, while FK9 uniformity variation in both junctions is almost negligible, maintaining an excellent uniformity.

This work shows that FK9 is designed to work under extreme conditions: its tolerance to temperature changes provides the possibility of selecting the used materials, disregarding their effects on spectral irradiances. The fact that the spectral irradiances are still uniform even at very high concentration and temperature show that FK9 is prepared to meet the expected requirements of four, five and six junction cells.

2.6 Conclusions

CPV systems still have to strive to reach a Levelized Cost of Electricity able to compete with other energy sources. To accomplish it, it is essential to decrease the production cost and increase the energy collection and power production. The recent advances in 4 junction cells are a promising start to reach a higher power production, having recently reached an efficiency of 46%. 4 Junction cells surpassing 50% efficiency are expected in the incoming years. The effects of varying solar spectrum and irradiance distribution on energy production in 4-junction, 5-junction, and 6-junction terrestrial concentrator cells are shown to be noticeable, and will be increasingly important for future CPV systems, and they are likely to require larger concentration.

CPV systems are especially interesting in regions with high DNI, such as sand deserts, which can be really aggressive for the optical elements due to scratching. Silicon on Glass (SoG) lenses have been adopted by several CPV companies due to combining mass production simplicity and high resistance to external factors.

This Chapter described the development of 9-Fold Fresnel-Köhler (FK9), a multichannel CPV system. Splitting the incident light into multiple channels combined with the Köhler integration creates a system capable to achieve very high performance in the merit functions of optics for CPV. FK9 combines a large CAP with an excellent spectral and spatial irradiance uniformity over the solar cell.

In this Chapter, a Fresnel Lens with Variable Focal Point (FL-VFP) is also proposed. Multichannel systems always have to struggle with optical crosstalk between channels. The fact that the incoming bundle of light is divided into multiples channels has the potential to increase the acceptance of the system but, when using a larger number of channels, the optical crosstalk between channels becomes the limiting factor for a even higher concentration-acceptance angle product (CAP). In FK9, although the concentrator has already a high CAP, the major limitation to further increase this quantity is the crosstalk between different channels that occurs when the system is at a considerable off-axis angle. Crosstalk happens between sublenses of the SOE, especially for rays that are refracted in areas of the Fresnel lens which are farther away from the optical axis. The main limiting errors for this situation are the chromatic aberrations of Fresnel lens. In this section, a FL-VFP has the ability to control the rays coming from extreme points, i.e., apart from the optical axis and increases the CAP of the system. Although the focal point is variable, it varies along a line, such that each teeth of the lens is still rotationally symmetric.

FK9 presents an optical efficiency of 83.9%, a CAP of 0.62 and excellent spectral and spatial irradiance uniformities, even for off-axis angles. FK9 with FL-VFP further extends the CAP to 0.66, with a optical efficiency of 83.8%. The irradiance uniformities are the same in both systems. The irradiance distribution on the different junctions of a Triple MJ cell is excellent for normal incidence, and is essentially constant within the acceptance angle, enabling that no losses due to current mismatches occur, even for misaligned angles.

Table 2-4 FK9 and FK9 FL-VFP optical merits for $C_g=1000\times$

$C_g=1000\times$	FK9	FK9 FL-VFP
Optical Efficiency	83.9%	83.8%
Acceptance angle	$\pm 1.12^\circ$	$\pm 1.2^\circ$
CAP	0.62	0.66
f-number	1	

This Chapter explored recent research on the effects of non-uniform spectral and spatial irradiance distributions on the solar cell. In particular, the effects that irradiance distributions created by different CPV optical systems may have on the power production. The efficiency losses can be quite significant. The spectral balance is expected to be even more important in 4, 5 and 6 junction cells. A comparative study on the spectral irradiance distributions was

made between three different CPV configurations. The chosen configurations, SILO, FK4, and FK9, have excellent irradiance uniformity at specific concentrations. Using SoG lenses in these three systems, the spectral distributions were compared for different temperatures (25°C and 45°C) and different concentration values (1000X and 2000X). The temperature affects the refractive index of the lens, changing the spectral distributions. FK9 is the only system capable of maintaining uniform spectral and spatial irradiance, even at extreme conditions. This capability combined with the high CAP of this system ensures that FK9 is able to maintain a high performance even at high concentration and temperature values.

FK9 with FL-VFP is not only prepared for the requirements of today's CPV systems, but is also prepared for the requirements of future ones. The capability of high performance at very high concentrations while maintaining spectral balanced irradiance distributions, even at extreme temperature conditions, makes FK9 an excellent candidate for CPV systems with four, five, and six junction cells.

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Chapter 3.

SMS DESIGN METHOD FOR DIFFRACTIVE SURFACES

The Simultaneous Multiple Surface design method is extended to include design of diffractive optical surfaces besides refractive and reflective ones. This method involves the simultaneous and direct (no optimization) calculation of diffractive and refractive/reflective surfaces. Using the phase-shift properties of diffractive elements as an extra degree of freedom, two rays for each point on each diffractive surface are controlled. Representative diffractive systems designed by the SMS method are shown.

Section 3.1 presents an introduction, where the fundamentals of diffractive lenses are explained: the way to classify diffractive elements, an historical introduction and the state of the technique. The scalar theory of diffraction and the geometrical model for diffractive optics are also introduced, as the basis for the diffractive SMS design method.

Section 3.2 explains in further detail the Generalized Grating Model, where relevant mathematical deductions to the diffractive SMS method are explained in detail.

Section 3.4 presents three selected examples of optical systems designed with the diffractive SMS method. An analysis on the diffractive efficiencies on these SMS-based diffractive systems is also shown.

3.1 Introduction to Diffractive Lenses

The use of diffractive optical elements (DOE) has been steadily growing in the last 30 years.

New fabrication methods (such as electro-beam lithography applied to optical elements) and new design tools turn DOEs a viable new solution to optical designers. Several different types of optical systems currently use DOEs. The first applications were spectroscopic ones, where the superior dispersion of diffractive gratings relative to prisms is quite well known. Other applications followed, such as beam splitters, laser-beam shapers, optical interconnectors, etc. The application to imaging systems is more recent. In these configurations, DOEs have the potential to enhance the performance (controlling chromatic and field curvature aberrations, decreasing size and weight, achieving atomic resolution ($<1\text{nm}$), etc).

Diffractive optics differs from physical optics, in the sense that describes the control and generation of wavefronts by segmenting initial wavefronts and redirecting the segments through the use of interference and phase control. As such, it is a subfield of physical optics, which deals with all forms of propagation of electromagnetic waves. DOEs use a thin micro structure pattern to change the phase of the light. The phase pattern, once properly designed, can manipulate the light propagation and the diffracted wavefront to the desired goal [1]. The light propagating through a DOE is diffracted to several orders of diffraction. The amount of light that go into each order is a parameter that can be controlled to a certain extent with an appropriate design.

A key parameter to evaluate a DOE is the *diffraction efficiency* η_m which is defined as the ratio between the light intensity I_m that is distributed to an order of diffraction m and the light intensity I of the impinging wave. According to the scalar theory of diffraction, the diffraction efficiency can reach a value of 100% in specific types of DOE, but only for a specific wavelength. The diffraction efficiency is discussed in more detail in Section 3.1.6 .

3.1.1 Diffractive lens

Figure 3-1 A Fresnel Zone Plate is a diffractive lens that uses constructive interference of waves from adjacent zones to form a focus *Source: Ref. [2]*

A diffractive lens is a type of DOE that, through the use of constructive interference from adjacent zones, can focus the incoming wavefront in a point for a specific order of diffraction, typically ± 1 . The simplest type of diffractive lens is a *Fresnel zone plate* (Figure 3-1), that consists of alternate transparent and opaque rings, the *diffractive zones*. The rings are designed such that the phase difference between the limit of two consecutive transparent zones and the focal point for the order of diffraction 1 is 2π [3]. The Fresnel zone plate creates an infinite number of foci, as the incoming light distributes through different orders of diffraction, and each contributes for a different focal point. The diffraction efficiency for each order is lower than 10%.

A type of diffractive lens that can achieve 100% efficiency for the design wavelength is a *kinoform* (Figure 3-2). Unlike the Fresnel zone plate, all the diffractive zones transmit, and the diffractive zones are defined by a surface relief. The light is modulated only by affecting the phase of the incoming wave. The main characteristic of a kinoform is the "introduction of a progressive phase shifting of 2π in each zone", which is achieved "by changing smoothly the thickness in each section of the device", that can direct all the light into a specific order of diffraction [4].

Figure 3-2 Kinoform lens converting a plane wavefront into a spherical one through interference. All the monochromatic light is directed into a defined diffraction order. Source: Ref. [1]

3.1.2 Classification of DOEs

The methods developed in this thesis have the potential to design several different types of DOEs, but they were initially thought for designing diffractive lens. These elements, as well as the other DOEs, can be classified attending to their operation mode, type of wave modulation, manufacture mode and periodicity. Herrera [5] provides a useful summary for the purposes of this thesis, shown in Figure 3-3.

Operation mode

DOEs can operate either through transmission or reflection, and the choice between one and the other depends on the application and the requirements for fabrication.

Figure 3-3 Classification of DOEs according to the mode of operation, discretization, modulation and periodicity. Source: Ref. [5]

Discretization

The discretization of DOEs is related with the manufacture procedure. The scalar theory of diffraction, explained in Section 3.1.5 establishes that a smooth DOE, i.e., a diffractive element in which each diffractive zone is a single continuous surface (such as a kinoform, or a blazed grating), has a 100% diffraction efficiency. The fabrication of smooth DOE can be difficult and quite expensive. A solution is to approximate it through a series of discrete levels. The discretization can be of two levels (binary optics), which introduces phase difference of 0 or π in the impinging wavefront, or it can be of N levels (multilevel). A higher number of levels approximates the profile to the smooth case, and increases the amount of light going to the desired order of diffraction. For example, while for the binary case, the diffraction efficiency is 40.5% for orders ± 1 , for a 8 level structure, this value increases to 95% [1]. Figure 3-4 shows an example of a discretization process, from a Fresnel zone plate into a kinoform.

Modulation

An electromagnetic field is characterized by its amplitude and phase. A diffractive element can modulate the incoming field by either changing its amplitude or its phase, creating a different field at the exit. An element that modifies the amplitude is known as an amplitude DOE, while an element that modifies the phase is known as a phase DOE. A DOE can be a combination of both, affecting both phase and amplitude [6]. An amplitude DOE is obtained through alternating opaque and transparent regions. An alternative is to use a photosensitive substrate, and to create a mask that changes the transmission according to a sinusoidal function (a device known as sinusoidal zone plate or Gabor zone plate [7]). A phase DOE is obtained by changing the thickness of the substrate, such that the limit between different diffractive zones is defined by a jump on the height of the zone. A surface relief is created, which modulates the phase of the incoming wavefront. For this reason, a phase DOE is also known as a volume DOE. A phase DOE has the ability to transmit all impinging light (neglecting absorption).

Figure 3-4 Profiles of a diffractive lens ranging from a two-level Fresnel phase plate (top) and four-level and eight-level structures (center) to a kinoform profile. It shows the discretization of the profile to generate the transitions for the multilevel structures. *Source: Ref. [1]*

Periodicity

A DOE can be classified as periodic or aperiodic, whether there is an even repetition of its structure, or if the structure changes throughout the profile, respectively. A periodic DOE only changes the direction of the impinging wavefront, maintaining its shape (i.e., if the wavefront is plane, it continues to be plane, only with a different direction). Examples of

periodic DOEs are diffraction gratings and beam splitters. Aperiodic DOEs may change the shape of the wavefront, and can be designed to change them according to different requirements of the system. Examples of aperiodic DOEs are kinoforms, wavefront correctors, beam shapers and diffusers.

3.1.3 Historical introduction

The control of light through diffraction offers a different set of characteristics that Geometrical Optics cannot offer. Sommerfeld, in 1896, defined diffraction as "any deviation of light rays from rectilinear paths which cannot be interpreted as reflection or refraction" [8]. Diffraction occurs in any wave phenomenon, whether it is mechanical, electromagnetic, etc. The closer the dimension of the object in which the wave impinges is to the wavelength, the larger the diffractive effects will be, and larger the deviation angles at the output.

The interest in diffractive optics elements (DOE) grew throughout the XX century, and had a peak during the 90s, with the development of micro fabrication and design techniques, that allowed high efficiency DOEs. Nowadays, DOEs are extensively used in systems operating in infrared wavelengths, as well as in laser beam shaping. Combined with refractive/reflective elements, DOEs also paved their way into imaging systems, emerging as novel optical elements that can significantly improve performance, size and weight of conventional systems.

Hazra [9] and Herrera [5] provide an interesting hindsight on the historical evolution of DOEs. The first accurate scientific experiments with diffraction are attributed to Grimaldi in the XVII century [10]. Using a very small aperture in a box and a source of light, Grimaldi observed bands of light in the zone where the corpuscular theory of light only predicted shadow. The laws of diffraction had to wait for the development of the wave theory to be clearly stated. Hooke [11] and Huygens [12], (who proposed that the points of an initial wavefront can be considered as sources of secondary spherical wavefronts – The Huygens principle), made some early attempts in wave theory in XVII century, but only in the XIX century the foundations of wave theory were set. Young established the revolutionary concept of interference in 1802 [13]. Fresnel, in 1818, modified Huygens principle, and proposed that the secondary spherical wavefronts interfere between each other. The new Huygens-Fresnel principle allowed to calculate the distribution of light in diffraction with great accuracy [14]. Maxwell established light as an electromagnetic wave in his theory of classic electrodynamics [15]. In 1882, Kirchhoff initiated the rigorous mathematical description of the Huygens-Fresnel principle [16]; Sommerfeld and Lord Rayleigh further developed the theory in 1897, establishing the deduction of the electromagnetic field on the diffractive aperture, creating what is now known as the Rayleigh-Sommerfeld diffraction theory [8].

The first DOEs date from the end of the XVIII century. David Rittenhouse, an American astronomer, manufactured the first diffraction grating, 53 hairs between two finely threaded screws, with an approximate spacing of about 100 lines per inch [17]. During the XIX century, diffraction gratings keep improving: in 1821, using a similar structure as Rittenhouse, Fraunhofer deduced the grating equation and quantified the dependence of diffraction with wavelength [18]. Rutherford, in 1870, matched the prisms resolution, with a 35.000 fringes carved on a two-inch mirror [19]. Since then, many remarkable developments in both theory and manufacture of diffractive gratings occurred. Rowland [20] developed the concave grating, which can perform both imaging as well as dispersion. Also, Rowland and Anderson increased the gratings resolution to values up to 350.000 [21]. Wood made a fundamental contribution to development of gratings, by inventing the blaze grating, gratings with controlled groove shape that can perform phase modulation and thus, increase diffraction efficiency [22]. Diffraction gratings are well-established and have been shaping modern science and technology, with applications in spectroscopy, spatial filtering, wavelength selectors for tuneable lasers, masks for photolithography, etc. An excellent overview of diffraction gratings is given by Singh [23].

The first diffractive element with imaging properties was the Fresnel zone plate, which can focus light. This structure was developed by Lord Rayleigh in 1871 [3], and consists of alternating transparent and opaque annuli, with radii chosen such that the distance from the edge of each annulus to the focal point is an integral multiple of half wavelength longer than the axial focal length. However, the Fresnel zone plate creates several focal lengths for different orders of diffraction, and the intensity of each order as a fraction of the incident intensity, known as diffraction efficiency, is lower than 10%. The low efficiency, presence of several focal points and the absorption zones limits the practical use of Fresnel Zone Plates. However, by replacing the opaque zones with transmitting material with different height, such that a half wavelength of phase delay was introduced, the amplitude at the focal point increases by a factor of two and the diffraction efficiency in the first orders of diffraction reaches 40%. This was the observation of Lord Rayleigh, and resulted in phase reversal zone plates, manufactured by Wood [24].

During the first half of XX century there were several theoretical works on diffractive optics, but the next big step for DOEs only came in 1948, with the discovery of holography, or wavefront reconstruction by Gabor [25]. This technique enables to store the information of an object, and later to reconstruct a tridimensional image of the same object. The information is encoded in a photographic plate as the result of the interference of a reference coherent beam and the reflection of an illumination beam in an object. The interference pattern is very similar to a zone plate, and holograms can be considered as

generalized zone plates [26]. The invention of the technique of off axis holography using a laser source in the sixties boosted the production of holograms, and consequently, of zones plates to create specific diffraction patterns [27]–[29]. However, a drawback of optically recorded holographic optical elements (HOE) is the difficulty of producing arbitrary phase function for the optical element. So, in 1967, Lohmann and Paris developed the Computer Generated Holograms (CGH): the diffraction figure is generated digitally and printed [30], which increases tremendously the possibilities for different DOEs. The new computational capabilities drove the research on high-efficient holograms, and in 1971 the kinoform was developed, a phase only hologram in which the phase modulation is achieved by a surface relief profile [4]. A kinoform can reach a theoretical 100% diffraction efficiency, and soon high efficient kinoforms were manufactured and demonstrated [31]. Kinoforms spread through different areas, not only as diffractive imaging lens, but also for emerging areas such as signal processing, optics for computing, laser beam integrated optics [32–34].

In the following two decades the fabrication capabilities of DOEs increased considerably: in the 70's photolithography was adapted to manufacture diffractive elements, which made possible the fabrication of multilevel Fresnel lens through surface relief [35]. The 80's was the beginning of micro-optics, with the development of techniques such as electro-beam lithography [36].

3.1.4 State of the technique

The state of the technique reported in this section is focused mostly on diffractive lenses and the different fields and technologies in which they are applied. The holographic applications of DOE's are not considered. A special focus is given to applications in which kinoform (or multilevel) lenses are used.

DOEs enabled recent advancements in photonic systems and consumer products, such as augmented reality devices and heads-up displays, high power lasers, high resolution surveillance systems as well as instrumentation for exoplanet research [37]. DOEs are attractive solutions for adjusting the properties of optical systems [38], [39], their dispersive properties allow them to correct chromatic aberrations in imaging optical systems [40]. Their opto-thermal characteristics, ability to transmit almost all incident light and their ability to locally alter wavefront curvature provide additional flexibility in controlling the propagation characteristics of light [41], [42].

Diffractive lenses can reach high diffraction efficiencies, but the efficiency is highly dependent on the wavelength of the incoming light. Figure 3-14, in Section 3.1.6 displays the dependence of diffraction efficiency with wavelength for different orders of diffraction for an ideal kinoform. For this reason, diffractive lenses met their first applications in systems

working on a monochromatic or narrowband regime, or applications in which the diffraction efficiency was not that important or that it was interesting to distribute the light in different orders, to create, for example, multifocal lenses. DOEs are increasingly important in systems working on spectral regions where optical-quality vitreous materials are sparse or not readily attainable. This especially applies to long-range wavelengths, from mid-wave infrared (MWIR) to terahertz (THz), a range in which DOEs perform exceptionally well [37].

DOEs have negative dispersion [1]. When combined with refractive elements (with positive dispersion), DOEs can correct for chromatic aberrations. This is one of the most appealing characteristics of DOEs for imaging systems, and it has met several applications in IR systems. As an example, Figure 3-5 shows a detector with a high resolution focal plane array that operates in the long wave infrared (LWIR) range, which requires imaging objectives to match their performance. The wide angle ($\pm 30^\circ$) objective lens assembly contains two DOE components for correction of both axial and chromatic aberration [37].

Figure 3-5 Wide angle LWIR objective incorporating two DOE components. The first DOE corrects the axial chromatic aberration and the second corrects the lateral chromatic aberration. Source: Ref. [37]

Kinoform lenses have been used in X-rays spectrum has a method to achieve efficient nano-focusing. Synchrotron-based chemical imaging techniques, like scanning x-ray fluorescence microscopy (SXFM), x-ray absorption spectromicroscopy (microXAS) and micro x-ray diffraction (microXRD), all require highly efficient focusing of hard x-rays. The desired spot size has moved from micro to nano-regime. Kinoforms are attractive to x-ray nanofocusing do to the fact that they can bend all incoming photons into one single focus. Moreover, the absorption losses are very small, because the height of the surface relief is the order of the wavelength, making these devices very efficient in the x-ray range (which is the main drawback for refractive lenses) [43–45].

Figure 3-6 microscopic view of a 2 level diffractive lens for x-ray nano-focusing *Source: Ref. [46]*

A recent solution to use DOEs in broadband applications is to use a diffractive doublet. This element allows extending the diffraction efficiency. The diffractive doublet maintains a high diffraction efficiency for a much wider spectrum, making it a viable optical element for broadband applications [37] (Figure 3-7).

Figure 3-7 Comparison between diffraction efficiencies of a single kinoform and diffractive doublet. **The diffractive doublet maintains a high diffraction efficiency for a much wider spectral band, making it a viable optical element for broadband applications.** *Source: Ref. [47]*

The first applications of a diffractive doublet were also in infrared, but in a broadband regime. Soskind provides a us a good comparison to assess the impact in size, weight and number of elements [37]. Figure 3-8(a) shows an example of a dual-band IR object, with six components, two of them of gallium arsenide (GaAs). The system is designed to operate within the spectral ranges of 3-5 and 8-12 μm, and weights 2.82kg. Figure 3-8(b) presents a system designed for the same optical requirements, which uses a diffractive doublet. The number of components is reduced from six to three, the weight decreases to 0.99kg and the axial dimension decreases 10%. Also, eliminates the need to use GaAs, a highly toxic material.

Figure 3-8 (a) Example of a dual-band IR design using a six-element Petzval objective lens containing two lens groups, and **(b)** a similar setup incorporating a diffractive optical element (DOE), a diffractive doublet. GaAs: Gallium arsenide. *Source: Ref. [47]*

The diffractive doublet was recently introduced in products working in the visible spectrum, already in the market. In 2012, Nikon introduced the CFI60-2 objective lens for industrial microscopes, with phase Fresnel lens, for chromatic aberration, increasing the microscope working distance and decreasing weight. Canon introduced the Multi-layer DOE (their patent diffractive doublet) in telephoto cameras and ultra-wide-angle interchangeable zoom lens, decreasing considerably size and weight and increasing resolution of cameras [48].

Figure 3-9 In broadband systems, a single DOE has a low diffraction efficiency, and the light in other orders affect the contrast (left). By using a diffractive doublet, diffraction efficiency can increase considerably. On the right, a Multi-layer diffractive optical element is displayed, the diffractive doublet developed and patented by Canon. This DOE has been successfully introduced in Canon's photographic cameras, decreasing considerably size and weight and increasing resolution. *Source: Ref. [49]*

A kinoform that has its surface relief adjusted to equally distribute the light in two different foci can have interesting applications. This approach is used in diffractive multifocal intraocular lens (Figure 3-10). Using a kinoform, a near and a far foci are created to accommodate the eyes needs, and it's a technique used after cataract surgery [50].

Figure 3-10 A diffractive multifocal intraocular lens uses a kinoform adjusted to create two foci with the same optical efficiency. *Source: Ref. [51]*

Freeform DOEs are interesting devices that can combine the additional degrees of freedom provided by freeform surfaces with the ones provided by diffractive surfaces. However, their fabrication is difficult and may be expensive, as the available manufacturing techniques as lithography, e-beam writing, are typically applied on simple shapes, such as flat or spherical surfaces. Consequently, freeform DOEs have been sparsely noted in the literature. Recently, some attempts were made to develop of these devices. Johnson *et al* [52] proposed using an i-line maskless lithography as a new method of freeform printing. De Clercq *et al* [53] presented an spectrometer for spaceborn hyper spectral applications with a free-form grating, manufactured by Single Point Diamond Turning.

Recently, the application of DOEs to non-imaging systems has been studied [54], [55].

3.1.5 Scalar Theory of Diffraction

The design of diffractive elements can be categorized according to the optical theory used to model the element: geometrical theory of diffraction, scalar theory of diffraction, or vector theory of diffraction. Geometrical theory is the simplest one, and rays are traced through the element using the grating equation, but it does not consider the field propagation and energy distribution. The geometrical theory can be complemented with the scalar theory, where the diffractive elements are modeled as infinitely thin phase plates and wavefronts are propagated from one surface to another by the scalar diffraction integrals. This model neglects secondary reflections, polarization effects and the thickness of the element. Finally,

the vector approach is the most complete, as it is a numerical solution to Maxwell's equations, and all effects predicted by Maxwell's equations, including those mentioned above are included. However, the vector theory demands intense numeric computation, and in most cases where the feature is much larger than the wavelength, the geometrical and scalar theory predict rather well the behavior of light.

The three theories can be correlated with the size of the features. As a rule of thumb, it can be considered that: if the feature is much larger than the wavelength, geometrical optics is used. If the feature is in the order of wavelength or smaller, the vector theory is used, and in intermediate cases, the scalar theory is appropriate. This thesis uses the geometrical model of diffraction, and complements it with the scalar theory, for diffraction efficiency calculations.

Scalar Theory of Diffraction is an approximation for light propagating through periodic structures, and is extremely helpful and useful for understanding the behavior of many DOEs. It provides a set of simple equations that describe the propagation of light between two planes within an optical system. The light distribution of the diffractive element can be calculated if we are given the transmission function. The Rayleigh-Sommerfeld (RS) integral can be used to calculate the transmission function of the diffractive element. The integral is based on the scalar theory of diffraction.

Huygens principle states how a wave propagates through a narrow slit, by using the definition of a point source. The propagating field at the aperture is considered as a superposition of several secondary point sources, all emitting spherical wavefronts, all in phase with one another. The resulting envelope of the superposition of the wavefronts provides the propagating field [56]. Fresnel stated that intensity distribution after the aperture is the result of interferometric interaction between the Huygens point sources, creating Fresnel zones.

Figure 3-11 shows the case of light travelling in the positive z direction through an aperture located in the x - y plane. The diffracted field P is evaluated on a parallel plane to the aperture at a distance Z , with local coordinates (X,Y,Z) . All points inside the aperture contribute to each point of the diffracted field, according to the Huygens principle.

Figure 3-11 The distribution at the Evaluation Plane of a monochromatic field propagating through an aperture is described by the Kirchhoff's diffraction integral. source: Ref. [1]

If the temporal component of the field is neglected, the beam size and the distance from the aperture to the evaluation plane are large compared to the wavelength, the Helmholtz equation that describes the spatial distribution of the electric field, Equation (1.1.3), can be reduced to the Rayleigh-Sommerfeld integral [1]:

$$P(X, Y, Z) = \frac{1}{2\pi} \iint P(x, y, 0) \frac{e^{-ikr}}{r} \frac{Z}{r} \left(ik + \frac{1}{r} \right) dx dy \quad (3.1.1)$$

where r is the distance from a point $P(x, y, 0)$ at the aperture plane to a point $P(X, Y, Z)$ on the evaluation plane.

When the distance between the evaluation and aperture plane is much larger than the size of the aperture, it is useful to use the Fraunhofer (or far-field) approximation. For transmitting diffractive optics, it is common that phase variations are induced by the aperture, so phase should be included in the integral. Under this approximation and after some algebraic manipulation, Equation (3.1.1) is reduced to a two-dimensional Fourier transform [1].

$$P(X, Y, Z) = \iint u_0(x, y) \exp \{ i\phi_0(x, y) \exp[-ik(xX/R)] \exp[(-ik(yY/R))] \} dx dy \quad (3.1.2)$$

Where $u_0(x, y)$ and $\phi_0(x, y)$ is the amplitude and phase variation of the field in the aperture plane, respectively. The far-field diffraction pattern is the Fourier transform of the aperture function. Equation (3.1.2) is the foundation for calculating diffraction efficiencies of surface relief diffractive elements in the scalar regime. For a periodic structure, the amplitudes of the various diffraction orders can be determined by a Fourier transformation of the DOE transmission function.

3.1.6 Transmission function and diffraction efficiency

Optical elements can be described by a mathematical function called the transmission function. The transmission function describes how the incoming light (amplitude and phase) is changed when travels through the device. In the case of a DOE with a periodic structure, the transmission function is also periodic and therefore, can be expressed as a Fourier sum. The scalar theory of diffraction considers the thin phase lens approximation.

Figure 3-12 A blazed grating directs all the monochromatic light into a specific order of diffraction

The blaze grating is a phase only periodic DOE that can distribute all the incoming monochromatic light into a single order. Consider a plane wave impinging on a blazed grating of spatial period W , height d . The wave is diffracted from a medium with refractive index n_i into another one with refractive index n_d (Figure 3-12). Under the thin phase lens approximation, then $W \gg d$. Under this approximation, the phase delay in each module of the periodic structure is

$$\phi(x) = \frac{2\pi}{\lambda} d (n_i - n_d) \frac{x}{W}, \quad 0 \leq x \leq W \quad (3.1.3)$$

$\phi(x)$ is maximum when $x = W$, $\phi(W) = \frac{2\pi}{\lambda} d (n_i - n_d)$. If we define $\phi_0 = \frac{2\pi}{\lambda} d (n_i - n_d)$, the maximum value of ϕ , we can rewrite Equation (3.1.3) as

$$\phi(x) = \phi_0 \frac{x}{W}, \quad 0 \leq x \leq W \quad (3.1.4)$$

The transmission function, within a period 0 to L , is given by:

$$t(x) = \exp\left(i \phi_0 \frac{x}{W}\right) \quad (3.1.5)$$

As the transmission function is periodic, it can be described as a Fourier series, which for a general function $f(x)$ takes the form [57]:

$$f(x) = \sum_{m=-\infty}^{+\infty} C_m \exp\left(i \frac{2\pi m}{W} x\right) \quad (3.1.6)$$

where the coefficients C_m are

$$C_m = \frac{1}{W} \int_0^W f(t) \exp\left(-i \frac{2\pi m}{W} t\right) dt \quad (3.1.7)$$

where W is the period. If we apply the Fourier development to the transmission function $t(x)$ of (3.1.5), with period W , the Fourier coefficients have the form:

$$C_m = \frac{1}{W} \int_0^L \exp\left(i \phi_0 \frac{x}{W}\right) \exp\left(-i \frac{2\pi m}{W} x\right) dx \quad (3.1.8)$$

Considering that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$ and the function $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, the Fourier coefficient of the transmission function takes the form

$$C_m = \exp\left(i \left[\frac{\phi_0}{2\pi} - m \right]\right) \text{sinc}\left(\frac{\phi_0}{2\pi} - m\right) \quad (3.1.9)$$

Inserting Equation (3.1.9) into Equation (3.1.6), we obtained the transmitted light field after the diffraction on the blazed grating. m is the coefficient of Fourier series, and as such, can only have integer values. Physically, the fact that m can only assume integer values means that the field propagates in m discrete direction. In other words, the field propagates into m orders of diffraction. A key parameter is the diffractive efficiency η_m for each order of diffraction. This is defined as the light intensity (I_m) at each order of diffraction m , normalized with respect to incident beam. The Fourier series exactly defines the transmission function, which has amplitude of unity, therefore the intensity of the incident beam is also unity. The intensity of each order will be squared modulus of its coefficient in the Fourier series of the transmission function [58], thus the diffraction efficiency η_m is given by:

$$\eta_m = \text{sinc}^2\left(\frac{\phi_0}{2\pi} - m\right) \quad (3.1.10)$$

Considering that $\text{sinc}(0)=1$, the diffraction efficiency in the m^{th} order is 100% if the maximum phase delay of each zone is $\phi_0 = 2\pi m$, for the blazed grating structure. If we rewrite the diffraction efficiency as a function of the optical path difference, using Equation (3.1.3) and (3.1.4) we get:

$$\eta_m = \text{sinc}^2\left(\frac{n_i - n_d}{\lambda} d - m\right) \quad (3.1.11)$$

and the ideal height d for which the diffraction efficiency is 100% is

$$d = \frac{m\lambda}{n_i - n_d} \quad (3.1.12)$$

Equations (3.1.10) and (3.1.11) show that, on phase DOEs, the diffraction efficiencies are dependent of the phase modulation introduced by the DOE in the incoming wave. Equation (3.1.12) also shows why the thin phase lens approximation ($W \gg d$) holds: the scalar theory can be applied when the slit's dimension is much larger than the wavelength. Equation (3.1.12) shows that d is in the order of magnitude of λ , so we have immediately $L \gg d$. Generally the scalar diffraction approximation is applicable when the grating period L exceeds five times the wavelength λ [59].

In the scalar theory, the diffraction efficiency of an arbitrary DOE can be directly related to the diffraction efficiency of a grating [60]. In a sufficiently local area, the following approximations are true in the light of the validity of the scalar theory:

- (i) an arbitrary wave impinging on a freeform DOE can be approximated to a plane wave for each diffractive zone;
- (ii) the radius of curvature of a curved DOE (i.e. with a curved substrate) is much larger than the grating period, so it can be approximated to a plane substrate for each diffractive zone;
- (iii) even for an aperiodic DOE, where the grating period changes throughout the surface, the thin phase lens approximation is still valid (as long as the condition $L > 5\lambda$ is kept). Therefore, it is only necessary to determine the diffraction efficiency of a grating structure, even for a DOE such as freeform kinoform. This approximation is typically valid, although does not describe the ideal surface relief of a kinoform.

Hence, in the scalar theory, the diffraction efficiency of an arbitrary DOE can be directly related to the diffraction efficiency of a grating [60].

The condition for having maximum efficiency is such that the phase delay ϕ between the incident and transmitted wavefront in each diffractive zone is constant, and the difference to each consecutive zone has to be $2\pi m$. If the phase delay in each zone is constant, this means that each zone can be thought as a micro-lens that refracts the light, and the condition for maximum efficiency is such that the light is refracted in the same direction as the one going to the m^{th} order (Figure 3-13). In other words, the ideal surface relief consists of a set of Cartesian ovals, each one coupling the input wavefront into an output wavefront. When a kinoform is coupling a plane wavefront into a spherical one, the maximum diffraction efficiency is obtained when the surface relief is a truncated paraboloid, with paraboloid fragments in each zone [6]. This means that the ideal maximum height d of each zone is not

necessarily constant throughout the surface. However, even in the non-ideal case of a constant d , for most practical applications the effects on the diffraction efficiency are minimal and can be neglected. Golub [61], still using the thin phase lens approximation, presented a method to calculate a surface relief with a variable d for curved surfaces with a variable grating period. However, he also shows that this is only relevant in very specific conditions.

Figure 3-13 In a phase DOE as the curved kinoform in the image, each diffractive zone can be thought as a micro-lens that refracts the light (the blaze "ray"). The diffraction efficiency is maximum when the direction of the blaze "ray" coincides with the one going to the m^{th} order. Source: Ref. [62]

Equation (3.1.11) shows that the diffraction efficiency is highly dependent of λ , and Figure 3-14 shows a plot of the diffraction efficiency as a function of wavelength.

Figure 3-14 Diffraction efficiency as a function of wavelength. Source: Ref. [59]

As mentioned previously, the wavelength dependence is a concern when the DOE is to be used in an optical system operating over a broad wavelength band. For imaging systems, the residual light diffracted into different orders may decrease the contrast, affecting the MTF curve for low frequencies. Also, there is the possibility of creating double images in the other diffracting orders.

Figure 3-15 A blazed grating with a wave impinging with an incidence angle θ_i

Equation (3.1.11) is valid for plane waves with normal incidence. For waves impinging with an incidence angle θ_i to the normal of the substrate (Figure 3-15), Equation (3.1.11) can be rewritten as [60]

$$\eta_m = \text{sinc}^2 \left(\frac{d}{\lambda} \left[\sqrt{n_i^2 - \sin^2(\theta_i)} - n_d \cos(\theta_i) \right] - m \right) \quad (3.1.13)$$

and the ideal height d for which the diffraction efficiency is 100%, considering an incidence angle θ_i is

$$d = \frac{m\lambda}{\sqrt{n_i^2 - \sin^2(\theta_i)} - n_d \cos(\theta_i)} \quad (3.1.14)$$

Figure 3-16 shows a plot of Equation (3.1.13), for different substrates materials (i.e. different refractive indexes n_i), and considering that the DOE is contact with air ($n_d=1$). For typical materials used in optical systems, with refractive indexes between 1.4 and 1.5, the diffraction efficiency is almost constant until a high incidence angle, approximately 50°. For most practical applications, the incidence angle effect on the diffraction efficiency can be neglected.

Figure 3-16 Diffraction efficiency as a function of the incidence angle, for substrates with different refractive index n_i .

Most diffractive structures with surface relief are manufactured by lithography, using a multilevel approach, as shown in Figure 3-4. The phase differences between levels must be accounted for diffraction efficiency calculations. For a DOE with a multilevel structure of N levels, illuminated at normal incidence, the diffraction efficiency is given by [60]

$$\eta_m^N = \left[\text{sinc}(\phi_0 - m) \frac{\sin(\pi m / N)}{\sin(\pi(\phi_0 - m) / N)} \right]^2 \quad (3.1.15)$$

3.1.7 Extended theory of diffraction

Diffraction efficiency calculations based on scalar theory are completely independent of wavelength-to-period ratio, largely due to the assumption that the phase delay occurs in an infinitely thin boundary of the substrate. This assumption begins to fail when the wavelength-to-period ratio increases. The extended theory of diffraction tries to fill the gap, when the scalar theory begins to fail, but before passing to the vector theory calculations, more demanding computationally.

The first visible effect of the wavelength-to-period ratio increase on predictions of scalar theory is the shadowing, which decreases the fill factor of the propagating field after diffraction, an effect known as the duty cycle ζ of the diffracted field. Figure 3-17 shows the appearance of gaps in the propagating wavefront the reduction of the duty cycle after diffraction.

Figure 3-17 The shadowing of the gaps between diffractive zones affects the duty cycle of the diffracted field, decreasing diffraction efficiency. *Source: Ref. [63]*

Changes in diffraction efficiency due to shadowing are accounted by introducing the duty cycle ζ into the diffraction efficiency calculations:

$$\eta_{\text{ext}} \approx \zeta \eta_{\text{scalar}} \quad (3.1.16)$$

The duty cycle of a transmission grating is found by [63]

$$\begin{aligned} \zeta = \frac{W_m}{W} = 1 - \frac{1}{\sqrt{n_i^2 - (n_d \sin \theta_i + m\lambda/W)^2}} \\ \times \frac{[n_d \sin \theta_i + m\lambda/L_g](m\lambda/W)}{\left(\left| n_d \cos \theta_i - \sqrt{n_i^2 - [n_d \sin \theta_i + m\lambda/W]^2} \right| \right)} \end{aligned} \quad (3.1.17)$$

Note that, accounting with the duty cycle, the diffraction efficiency depends on the period, which means that, in a DOE with a variable period (or a variable phase function), the diffraction efficiency changes throughout the structure.

3.1.8 Ray tracing in diffractive surfaces

The simplest model for diffractive surfaces is the geometrical diffractive optics model. Unlike the scalar theory, it does not take into consideration diffraction efficiencies, but is a powerful method due to its simplicity, especially when used in conjugation with the scalar theory. The most interesting aspect about this model is the fact that, if we consider a phase function ϕ of the wavefront, we can use ray tracing to model an optical system that includes a diffractive element, using the grating equation. The use of this model depends on the application. Typical applications of this method include the design of most optical systems. With this approach, we can model diffractive surfaces in ray trace softwares and optimize the

optical system together with other refractive/reflective surfaces. Softwares like *Code V*® and *Zemax*® are prepared to model diffractive surfaces, and use them in their optimization routines.

In cases where ray tracing is adequate, two methods can be used: the generalized grating model and the Sweatt model.

Sweatt model

In the Sweatt model, the diffractive element is replaced by an equivalent refractive element [64]. However, the equivalent refractive lens is made of a very high refractive index (for example $n=10,001$). Thus, its thickness is reduced down to its underlying surface (planar or any other curvature), resembling a planar diffractive lens, and thus allowing ray trace through the lens according to the Snell Descartes law. It is a useful model because the results derived for refractive optics can be directly applied to diffractive surfaces, thus it can be used on software that do not support the grating model.

Generalized grating model

A DOE can be ray traced by using the local grating concept applying the grating equation. If we define a phase function ϕ between the input wavefront and the m diffracted wavefronts of a grating, then we can define the vector form of the grating equation as [65]

$$n_d(\mathbf{N} \times \mathbf{r}_d) = n_i(\mathbf{N} \times \mathbf{r}_i) + \frac{m\lambda}{2\pi}(\mathbf{N} \times \nabla\phi) \quad (3.1.18)$$

where λ is the wavelength in the vacuum, m is the order of diffraction, $\mathbf{r}_i, \mathbf{r}_d$ and n_i, n_d are the incident, diffracted ray vectors and the refractive index of the incident and diffracted side, respectively. $\mathbf{N}$ is the normal vector to the surface, and $\nabla\phi$ is the gradient of the phase function (Figure 3-18). Comparing Equation (3.1.18) with Snell's law, the difference is the term dependent on ϕ . For the light diffracted into the order of diffraction $m=0$, the DOE behaves as geometric optics element.

Figure 3-18 The diffracted ray $\mathbf{r}_d$ on a diffracted surface can be calculated as function of the incident

ray r_i , refractive indexes n_i, n_d , the normal to the surface $\mathbf{N}$ and the gradient of the phase function $\nabla\phi$

For the case of a linear grating, as the one of Figure 3-12, as the grating period L is constant throughout the surface, the gradient of the phase function is a constant, defined by $\nabla\phi = \phi'(x) = 2\pi/L$. Thus, for this example, Equation (3.1.18) can be rewritten as

$$n_i \sin(\theta_i) = n_d \sin(\theta_d) + \frac{m\lambda}{L} \quad (3.1.19)$$

Equation (3.1.19) is the most well-known form of the grating equation, for linear gratings with a constant period L .

The generalized grating model, its properties, and how can be used for optical design is further developed and explained in Section 3.2 .

3.2 Generalized Grating Model

The generalized grating model is based upon the premise that a DOE with a phase profile $\phi(x, y)$, where z is taken as the optical axis and the element is assumed to lie on a substrate, may be locally modeled as a grating of spatial frequency [65]

$$\mathbf{f}(x, y) = \frac{1}{2\pi} \nabla\phi \quad (3.1.20)$$

where $\nabla\phi$ is the gradient of the phase function. The magnitude of $\mathbf{f}$ determines the period, which is explicitly given by

$$T(x, y) = \frac{2\pi}{|\nabla\phi|} \quad (3.1.21)$$

The phase function $\phi(x, y)$ is defined on the surface substrate where the grating structure lies upon. It is defined as the phase difference between an incident wavefront and a diffracted wavefront. It can be related with the optical path length L between a point on the surface and the incidence and diffracted wavefront by

$$\phi = \frac{2\pi}{\lambda} L \quad (3.1.22)$$

The phase function can be expressed in any way, as long as it is defined at each point on the surface. Typical ways to define it are decomposition in polynomials, or explicit analytical expressions for simple elements. The decomposition in polynomials can be made using a radial coordinate, for rotationally symmetric cases, or (x, y) coordinates for freeform surfaces. Other common polynomial representations are the ones using other basis functions, such as the Zernicke polynomials. The polynomial representations are commonly used in ray-tracing

software.

The methods to design diffractive surfaces developed in this thesis are thought for curved and freeform DOE substrates, thus the description of the profile of the substrate is fundamental. Hereafter, we refer to the shape of the substrate as the *macro surface* of the DOE.

3.2.1 Eikonal function in diffractive surfaces

The Eikonal function may be used to design diffractive surfaces, and a parallel can be established with the method used to design conventional geometric optics. The relation between the Eikonal and phase function of the diffractive surface may be derived from the generalized grating equation, presented in Equation (3.1.18). We presented here again, for convenience:

When a ray vector $\mathbf{r}_i$ impinges a diffractive surface with a phase function ϕ , the diffracted ray vector $\mathbf{r}_d$ the generalized grating equation is

$$n_d (\mathbf{N} \times \mathbf{r}_d) = n_i (\mathbf{N} \times \mathbf{r}_i) + \frac{m\lambda}{2\pi} (\mathbf{N} \times \nabla \phi) \quad (3.1.23)$$

where n_i and n_d are the refractive index of the incident and diffracted side, respectively, λ is the wavelength in the vacuum, $\mathbf{N}$ is the normal vector to the surface, m is the order of diffraction and $\nabla \phi$ is the gradient of the phase function.

Equation (3.1.23) may also be expressed in terms of the Eikonal functions of the incident and diffracted wavefronts, S_i and S_d .

Let $\mathbf{p}=\mathbf{p}(x,y,z)$ be a point on the macro diffractive surface. Let $\mathbf{t}_1$ and $\mathbf{t}_2$ be orthogonal tangent vectors to the surface at $\mathbf{p}$, that define the tangent plane. We may define a local coordinate system (x_1, x_2, x_3) , in which $\mathbf{t}_1$, $\mathbf{t}_2$ and $\mathbf{N}$ are in the direction of x_1 , x_2 and x_3 axes, respectively. Locally, the surface can be represented by a parametrization $\mathbf{d}(x_1, x_2)=(x,y,z)$. Then, Equation (3.1.23) can be rewritten as

$$n_d [(\mathbf{t}_1 \times \mathbf{t}_2) \times \mathbf{r}_d] = n_i [(\mathbf{t}_1 \times \mathbf{t}_2) \times \mathbf{r}_i] + \frac{m\lambda}{2\pi} \left(\frac{\partial \phi}{\partial x_1} \mathbf{t}_2 - \frac{\partial \phi}{\partial x_2} \mathbf{t}_1 \right) \quad (3.1.24)$$

Using the vector triple product $[(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}] = -(\mathbf{c} \cdot \mathbf{b})\mathbf{a} + (\mathbf{c} \cdot \mathbf{a})\mathbf{b}$ and the wave number $k=2\pi/\lambda$, equation (3.1.24) can be rearranged as

$$\left(-n_d \mathbf{r}_d \cdot \mathbf{t}_2 + n_i \mathbf{r}_i \cdot \mathbf{t}_2 + \frac{m}{k} \frac{\partial \phi}{\partial x_2} \right) \mathbf{t}_1 + \left(n_d \mathbf{r}_d \cdot \mathbf{t}_1 - n_i \mathbf{r}_i \cdot \mathbf{t}_1 - \frac{m}{k} \frac{\partial \phi}{\partial x_1} \right) \mathbf{t}_2 = 0 \quad (3.1.25)$$

so, the scalar equations on each axis are

$$\begin{aligned}
n_d \mathbf{r}_d \cdot \mathbf{t}_1 &= n_i \mathbf{r}_i \cdot \mathbf{t}_1 + \frac{m}{k} \frac{\partial \phi}{\partial x_1} \\
n_d \mathbf{r}_d \cdot \mathbf{t}_2 &= n_i \mathbf{r}_i \cdot \mathbf{t}_2 + \frac{m}{k} \frac{\partial \phi}{\partial x_2}
\end{aligned} \tag{3.1.26}$$

Considering that $\mathbf{t}_1 = \frac{\partial \mathbf{d}}{\partial x_1}$ and $\mathbf{t}_2 = \frac{\partial \mathbf{d}}{\partial x_2}$, we can integrate locally both expressions of Equation (3.1.26).

$$\begin{aligned}
\int n_d \mathbf{r}_d \cdot \frac{\partial \mathbf{d}}{\partial x_1} dx_1 &= \int n_i \mathbf{r}_i \cdot \frac{\partial \mathbf{d}}{\partial x_1} dx_1 + \int \frac{m}{k} \frac{\partial \phi}{\partial x_1} dx_1 \\
\int n_d \mathbf{r}_d \cdot \frac{\partial \mathbf{d}}{\partial x_2} dx_2 &= \int n_i \mathbf{r}_i \cdot \frac{\partial \mathbf{d}}{\partial x_2} dx_2 + \int \frac{m}{k} \frac{\partial \phi}{\partial x_2} dx_2
\end{aligned} \tag{3.1.27}$$

Using Equation (1.1.11), we can integrate both equations in function of the incident S_i and diffracted wavefront S_d , as

$$\begin{aligned}
S_d(\mathbf{p}) &= S_i(\mathbf{p}) + \frac{m\lambda}{2\pi} \phi(\mathbf{p}) + c_1(x_1) \\
S_d(\mathbf{p}) &= S_i(\mathbf{p}) + \frac{m\lambda}{2\pi} \phi(\mathbf{p}) + c_2(x_2)
\end{aligned} \tag{3.1.28}$$

Where c_1 and c_2 are constants. Comparing both expressions of Equation (3.1.28), we finally get the relation between the incident S_i and diffracted S_d Eikonal functions and phase function ϕ :

$$S_d(\mathbf{p}) = S_i(\mathbf{p}) + \frac{m\lambda}{2\pi} \phi(\mathbf{p}) \tag{3.1.29}$$

Equation (3.1.29) may be used to design diffractive surfaces, as it is the base of the diffractive SMS method presented in Section 3.3.

3.2.2 Vector calculation of generalized grating equation

The generalized grating equation, Equation (3.1.23), may be used for ray tracing, but the direction of the diffracted ray $\mathbf{r}_d$ has to be derived from this equation.

If we define the surface gradient as $\nabla_t \phi = \nabla \phi - \mathbf{N}(\mathbf{N} \cdot \nabla \phi)$, the diffracted ray vector $\mathbf{r}_d$ can be described as a linear combination of $\mathbf{r}_i$, $\mathbf{N}$ and $\nabla_t \phi$:

$$\mathbf{r}_d = \sigma \mathbf{r}_i + \mu \mathbf{N} + \gamma \nabla_t \phi \tag{3.1.30}$$

If we apply the cross vector product of $\mathbf{N}$ to Equation (3.1.30), and considering that $\mathbf{N} \times \nabla_t \phi = \mathbf{N} \times (\nabla \phi - \mathbf{N}(\mathbf{N} \cdot \nabla \phi)) = \mathbf{N} \times \nabla \phi$, we get

$$\mathbf{N} \times \mathbf{r}_d = \sigma (\mathbf{N} \times \mathbf{r}_i) + \gamma (\mathbf{N} \times \nabla \phi) \tag{3.1.31}$$

Comparing Equation (3.1.31) with Equation (3.1.23), we immediately get

$$\sigma = \frac{n_i}{n_d} \quad \gamma = \frac{m}{k n_d} \quad (3.1.32)$$

Where $k = 2\pi/\lambda$ and λ is calculated in the vacuum. We still need to calculate μ . Squaring both sides of Equation (3.1.30) and considering that $\mathbf{r}_d \cdot \mathbf{r}_d = \mathbf{r}_i \cdot \mathbf{r}_i = \mathbf{N} \cdot \mathbf{N} = 1$, and that $\mathbf{N}$ is perpendicular to $\nabla_t \phi$ ($\mathbf{N} \cdot \nabla_t \phi = 0$)

$$\begin{aligned} \mathbf{r}_d \cdot \mathbf{r}_d = 1 &= \left(\frac{n_i}{n_d} \mathbf{r}_i + \mu \mathbf{N} + \frac{m}{n_d k} \nabla_t \phi \right) \cdot \left(\frac{n_i}{n_d} \mathbf{r}_i + \mu \mathbf{N} + \frac{m}{n_d k} \nabla_t \phi \right) \Leftrightarrow \\ &\Leftrightarrow \mu^2 + \mu \left(2 \frac{n_i}{n_d} \mathbf{r}_i \cdot \mathbf{N} \right) + \left(2 \frac{n_i}{n_d^2} \frac{m}{k} \mathbf{r}_i \cdot \nabla_t \phi + \left(\frac{m}{n_d k} \right)^2 \nabla_t \phi \cdot \nabla_t \phi - 1 + \frac{n_i^2}{n_d^2} \right) = 0 \end{aligned} \quad (3.1.33)$$

Equation (3.1.33) of second degree can be solved for μ and, after some algebraic manipulation, results in

$$\mu = -\frac{n_i}{n_d} \mathbf{r}_i \cdot \mathbf{N} \pm \sqrt{1 - \frac{n_i^2}{n_d^2} (1 - (\mathbf{r}_i \cdot \mathbf{N})^2) - \frac{m}{k n_d^2} \left(\frac{m}{k} \nabla_t \phi \cdot \nabla_t \phi + 2 n_i \mathbf{r}_i \cdot \nabla_t \phi \right)} \quad (3.1.34)$$

Choosing the positive sign solution of Equation (3.1.34), and using the results of Equation (3.1.32) in Equation (3.1.30), we get the direction of the diffracted ray and a function of the direction of the incidence ray, the normal to the surface and the phase function

$$\begin{aligned} \mathbf{r}_d &= \frac{n_i}{n_d} \mathbf{r}_i + \frac{m}{k n_d} \nabla_t \phi + \\ &+ \left(-\frac{n_i}{n_d} \mathbf{r}_i \cdot \mathbf{N} + \sqrt{1 - \frac{n_i^2}{n_d^2} (1 - (\mathbf{r}_i \cdot \mathbf{N})^2) - \frac{m}{k n_d^2} \left(\frac{m}{k} \nabla_t \phi \cdot \nabla_t \phi + 2 n_i \mathbf{r}_i \cdot \nabla_t \phi \right)} \right) \mathbf{N} \end{aligned} \quad (3.1.35)$$

Equation (3.1.35) is used to diffract individual rays on diffractive surfaces.

3.3 Diffractive SMS method

This section describes an extension of the SMS method to design diffractive surfaces. Diffractive surfaces are calculated together with the remaining surfaces of the optical system and through a direct method, i. e. with no multi-parametric optimization.

In the traditional SMS method, the relationship between degrees of freedom (J surfaces) and the number of wavefronts is not exactly one to one [66]. An optical system can control more than J wavefronts with J surfaces, provided the footprint of the rays of the wavefronts

on the optical surfaces is not covering 100% of those optical surfaces [67]. This footprint can be controlled with the pupil definition. In the development of the diffractive SMS method, we only consider 100% footprint coverage. In typical SMS imaging applications, the wavefronts to couple in the design are spheres centered at J points of the object plane. The J design conditions are such that the wavefronts are perfectly focused at J image points [68].

The diffractive SMS method uses the phase-shift properties of DOEs as an extra degree of freedom, which enables the control of two rays per each point of the surface (instead of one ray as is the case of non-diffractive surfaces). For a single diffractive surface (as in Section 3.3.2), in which the controlled rays footprint coverage of the surfaces is 100%, the set of extra rays form, in general, a wavefront, which means two wavefronts can be controlled with a single surface. Wavefronts of different wavelengths can also be coupled (with obvious applications in achromatic designs).

Subsection 3.3.1 describes how a freeform refractive surface, with a predefined macro surface, can be altered to become a diffractive surface that couples one wavefront, and establishes some basis for the description of the diffractive SMS method.

Subsection 3.3.2 introduces the concept of generalized diffractive Oval, a diffractive surface that couples two wavefronts.

Subsection 3.3.3 describes the diffractive SMS method to design simultaneously a hybrid system, with one refractive and a diffractive surfaces.

3.3.1 Example of a DOE on a generalized surface

One interesting consequence of the relation between the Eikonal and phase function [Equation (3.1.29)] is the fact that any continuous predefined surface, independently of its shape, can be transformed in a DOE that couples one wavefront into another wavefront. This is achieved by introducing a modulation on the surface according to the phase function. It can be a phase modulation (if the diffractive zones are defined by a surface relief) or an amplitude modulation (if the zones are defined by changing the transparency of the substrate). In particular, the transformation of a generalized surface into a kinoform that couples one wavefront can be achieved in two steps: first, using the pre-defined macro surface, one calculates the phase function on the surface using Equation (3.1.29). This step only considers the order of diffraction m for which the system is designed, typically $m = \pm 1$. The phase function on the macro surface is used to calculate the loci where the 2π phase shifts happen, i.e., the transition points between each diffractive zone. In the second step, the depth and shape of each diffractive zone is designed to optimize the diffraction efficiency, i.e., to ensure that the highest amount of light is diffracted into the desired order of

diffraction.

Consider Figure 3-19, which serves as an example of the creation of a diffractive surface that couples one wavefront from a refractive surface with an arbitrary shape. A two-dimensional situation is shown for convenience of explanation, but the case is general for three dimensions. Figure 3-19a) shows a predefined refractive surface of arbitrary shape, defined by a parametric function $F(x,y,z) = 0$. We intend to couple the input monochromatic wavefront WF_i with a wavelength λ , in a medium with refractive index n_i , to another output wavefront WF_d , immersed in a medium with refractive index n_d . The macro surface of the DOE is already fixed (i.e., the predefined surface), therefore, we may use Equation (3.1.29) to calculate the locations of the transition points between discontinuities. Consider Figure 3-19b), where $\mathbf{p}$ is a arbitrary point on the macro surface, and $\mathbf{a}$ and $\mathbf{b}$ are points on WF_i and WF_d such that the correspondent rays perpendicular to the wavefronts impinge on $\mathbf{p}$, respectively. S_i and S_d can be written as

$$\begin{aligned} S_i(\mathbf{p}) &= n_i |\mathbf{p} - \mathbf{a}| \\ S_d(\mathbf{p}) &= n_d |\mathbf{p} - \mathbf{b}| \end{aligned} \quad (3.1.36)$$

As S_i and S_d are already fixed for each point $\mathbf{p}$ (as the macro surface is previously defined), we can directly use Equation (3.1.29) to calculate ϕ

$$\phi(\mathbf{p}) = \frac{2\pi}{m\lambda} (n_d |\mathbf{p} - \mathbf{b}| - n_i |\mathbf{p} - \mathbf{a}|) \quad (3.1.37)$$

The transition points $\mathbf{p}_j$ of each diffractive zone j are calculated using Equation (3.1.37), such that $\phi(\mathbf{p}_j) = 2\pi mj$, j being a natural number.

The surface relief is constructed according to the calculated transition points, and designed to optimize the diffraction efficiency of the order of diffraction m [Figure 3-19c)], according to the methods of the scalar theory described in Section 3.1.6 .

Figure 3-19 a) An arbitrary predefined surface can be transformed in a diffractive surface that couples an input wavefront WF_i with an output wavefront WF_j . b) The transition points p_j are calculated on the predefined surface such that, for an order of diffraction m , in each point the phase ϕ is a multiple of $2\pi m$ c) The surface relief is designed to optimize the diffraction efficiency

3.3.2 Generalized Diffractive Oval

The previous example showed that, even with a predefined macro surface, a diffractive surface can couple one wavefront, as long as the phase shift in the discontinuities is a multiple of 2π . This section shows that a diffractive surface may couple two wavefronts, by having the freedom to design the macro surface. The phase function grants us an extra degree of freedom, compared to refractive/reflective surfaces.

Let Wa_i and Wb_i be two wavefronts impinging on a diffractive surface of wavelength λ_a and λ_b , respectively. Let the diffracted wavefronts be respectively, Wa_d and Wb_d . The phase function ϕ jumps are determined by the surface discontinuities so we will consider that ϕ as well as m are equal for both couples of wavefronts. For a point p on the macro diffractive surface, we have two equations like Equation (3.1.29)

$$S_{a_d}(\mathbf{p}) = S_{a_i}(\mathbf{p}) + \frac{m\lambda_o}{2\pi}\phi(\mathbf{p}) \quad S_{b_d}(\mathbf{p}) = S_{b_i}(\mathbf{p}) + \frac{m\lambda_o}{2\pi}\phi(\mathbf{p}) \quad (3.1.38)$$

by subtracting both expressions we get

$$\frac{1}{\lambda_a} [S_{a_d}(\mathbf{p}) - S_{a_i}(\mathbf{p})] = \frac{1}{\lambda_b} [S_{b_d}(\mathbf{p}) - S_{b_i}(\mathbf{p})] \quad (3.1.39)$$

Equation (3.1.39) is the implicit equation of the macro diffractive surface. When $\lambda_a = \lambda_b$, this equation is the same obtained for the macro surface profile on the design of microstructured surfaces [69], [70], which are not necessarily diffractive. The phase shift in the discontinuities of these microstructured surfaces is not necessarily a multiple of 2π as it is in the diffractive surfaces. We shall call a surface like the one given by Equation (3.1.39) a Generalized Diffractive Oval (GDO) for wavefronts Wa_i and Wb_i . Once the macro surface is calculated, ϕ can be calculated using any of the expressions in Equation (3.1.38). The discontinuity curves on the macro surface separating diffractive zones are a family of curves $\phi = 2\pi l + A$, where A is a family constant that can be freely chosen and the integer number l is the parameter of the family.

Figure 3-20 A Diffractive Oval, a freeform diffractive surface imaging two points

As an example, Figure 3-20 shows the design of a diffractive surface for imaging two points, **a** and **b** into points **a'** and **b'**. For a generic point **p** at the Diffractive Oval, $S_{a_i} = n_i |\mathbf{p} - \mathbf{a}| + A_i$ and $S_{a_d} = -n_d |\mathbf{a}' - \mathbf{p}| + A_d$, where A_i and A_d are constants. The same equivalent expressions are valid for points **b** and **b'**. If we consider the optical path length $L_{\mathbf{a-p-a'}}$ of the path **a-p-a'** as $L_{\mathbf{a-p-a'}} = n_i |\mathbf{p} - \mathbf{a}| + n_d |\mathbf{p} - \mathbf{a}'|$, and the equivalent case for $L_{\mathbf{b-p-b'}}$, we can rewrite Equation (3.1.39), as

$$2\pi \frac{L_{\mathbf{a-p-a'}}}{\lambda_a} - 2\pi \frac{L_{\mathbf{b-p-b'}}}{\lambda_b} = \text{const} \quad (3.1.40)$$

Where the first $(2\pi L_{\mathbf{a-p-a'}} / \lambda_a)$ and second $(2\pi L_{\mathbf{b-p-b'}} / \lambda_b)$ term are, respectively, the phase along the path **a-p-b'** and along **b-p-b'**. Equation (3.1.40) shows that the diffractive macrosurface is such that the difference between phase lengths is a constant. This calculation is similar to that of a Cartesian Oval [71], in which a refractive/reflective surface

focuses the rays from a point into another point . For this reason we will call the surface of Equation (3.1.40) a *Diffractive Oval* (which is the same as the GDO of Equation (3.1.39) but restricted to spherical wavefronts). In summary, a Diffractive Oval focuses 2 points into another 2 points.

3.3.3 Diffractive SMS method design for a hybrid system

The theory described is applicable to 2D and 3D geometries. In this section, we restrict the analysis to 2D. This section shows the diffractive SMS design method applied to a hybrid system, with one refractive and one diffractive surface. The new degree of freedom allows a perfect coupling of three wavefronts.

The SMS method calculates different portions of the deflecting surfaces by a sequential application of the Generalized Cartesian Oval (GCO) procedure. The diffractive SMS method uses a similar strategy, except that the GCO is replaced by the GDO procedure of Section 3.3.2 when portions of the diffractive surface are calculated.

Figure 3-21 Procedure to design simultaneously a refractive and a diffractive surface

Consider Figure 3-21. Let **a**, **b** and **c**, be the origin points of the spherical wavefronts (with wavelengths λ_a , λ_b and λ_c) to be coupled into other wavefronts with end points in **a'**, **b'** and **c'**, respectively. Assume that **d**₀, a initial point on the diffractive surface *d*, the normal of the macro surface at this point **N**₀ and the gradient $(\nabla\phi)_0$ also at this point are given. Let's take point **d**₀ as the origin of the phase function $\phi(\mathbf{d}_0) = 0$. The lens refractive index relative to the outside medium is *n*. Let *L*_a, *L*_b and *L*_c be optical path lengths from point **a** to **a'**, **b** to **b'** and **c** to **c'** respectively. The optical path length along a ray is calculated as in a non-diffractive configuration plus the term $m\lambda \phi(\mathbf{p})/(2\pi)$ of the point **p** where the ray crosses the diffractive surface. The design procedure is as follows.

1. Since **d**₀, **N**₀ and $\nabla\phi(\mathbf{d}_0)$ are known we can trace back 3 rays from **a'**, **b'** and **c'**

passing through $\mathbf{d}_0$ and diffract them using Equation (3.1.35) (Figure 3-21a). Along the ray from $\mathbf{a}'$ we can calculate the point $\mathbf{r}_{a0}$ where the ray must refract to get an optical path length $\mathbf{a}-\mathbf{r}_{a0}-\mathbf{d}_0-\mathbf{a}'$ equal to L_a . In a similar way we can calculate the points $\mathbf{r}_{b0}$ and $\mathbf{r}_{c0}$ on the trajectories of the rays coming from $\mathbf{b}'$ and $\mathbf{c}'$. As we know the deflection of the rays at $\mathbf{r}_{a0}$, $\mathbf{r}_{b0}$ and $\mathbf{r}_{c0}$, we can calculate the normal vectors to the refractive surface r at these 3 points. Now we can choose as first portion of the surface r to any surface passing through the points $\mathbf{r}_{a0}$, $\mathbf{r}_{b0}$ and $\mathbf{r}_{c0}$, whose normals fit the preceding calculation. As in the conventional SMS method it is advisable to choose this surface as well as L_a , L_b and L_c so this portion of r is smooth. The design procedure works also with more “wild” surfaces, but the final design, although mathematically correct, could be physically undoable.

2. We can now refract the rays with origin in $\mathbf{b}$ and $\mathbf{c}$ on the previously calculated portion of the surface r , and calculate the GDO that focuses that refracted rays into the points $\mathbf{b}'$ and $\mathbf{c}'$. The constant of the GDO calculation (that appears in Equation (3.1.40)) has to be selected so the GDO passes through $\mathbf{d}_0$. Once the GDO is calculated we can calculate the phase function ϕ as well as the tangential component of $\nabla\phi$ (i.e. $\mathbf{N}\times\nabla\phi$) along this portion of the diffractive surface d (using any of the equations of Equation (3.1.38)). The first section of d can be calculated until point $\mathbf{d}_1$ (Figure 3-21b).
3. Now we can diffract back rays from $\mathbf{a}'$ through this recently calculated portion of surface d using Equation (3.1.35) and calculate a new portion of r as the GCO that passes through $\mathbf{r}_{a0}$ and focuses these rays into the point $\mathbf{a}$. The new portion can be calculated until point $\mathbf{r}_{a1}$ (Figure 3-21c).
4. Steps 2 and 3 are repeated iteratively to calculate the remaining extensions of d and r .

Diffractive SMS point-by-point

The method described above allows the complete calculation of both the refractive and diffractive surfaces, by using the GCO and GDO procedures to couple the incoming wavefronts. However, after travelling through the first surface, the wavefronts can become extremely aberrated, and may have complex shapes. The mathematical definition of the Ovals required to couple the aberrated wavefronts may lead to complex differential equations, hard to compute and to include in a design algorithm. A solution to avoid complex differential equations is to calculate the surfaces *point-by-point*. Although this alternative does not completely describe the surfaces and the phase function (i.e. it calculates sets of points instead of continuous functions), it provides a good solution for a simpler and more

flexible algorithm. The continuous surfaces and phase function are then calculated by interpolation, using the location of the points, information of the normals to the surfaces and gradient of the phase function.

To use the point-by-point algorithm, we have to rewrite Equation (3.1.38), considering again Wa_i and Wb_i , two wavefronts impinging on a diffractive surface of wavelength λ_a and λ_b , respectively. Let $\mathbf{p}_j$ be a point on the macro surface and one of the discrete points the we intend to calculate. Let $\mathbf{p}_0$ be the initial point of the macro surface. Then, from Equation (3.1.38) for wavefront Wa_i , we have that $S_{a_d}(\mathbf{p}_j) = S_{a_i}(\mathbf{p}_j) + \frac{m\lambda_a}{2\pi}\phi(\mathbf{p}_j)$ for a generic point $\mathbf{p}_j$, and $S_{a_d}(\mathbf{p}_0) = S_{a_i}(\mathbf{p}_0) + \frac{m\lambda_a}{2\pi}\phi(\mathbf{p}_0)$, for an initial point $\mathbf{p}_0$. We may sum both equations. Performing the same operation for the Eikonals of wavefront Wb_i , we the following equation system

$$\begin{cases} [S_{a_d}(\mathbf{p}_j) - S_{a_i}(\mathbf{p}_j)] - [S_{a_d}(\mathbf{p}_0) - S_{a_i}(\mathbf{p}_0)] = \frac{m\lambda_a}{2\pi} [\phi(\mathbf{p}_j) - \phi(\mathbf{p}_0)] & (i) \\ [S_{b_d}(\mathbf{p}_j) - S_{b_i}(\mathbf{p}_j)] - [S_{b_d}(\mathbf{p}_0) - S_{b_i}(\mathbf{p}_0)] = \frac{m\lambda_b}{2\pi} [\phi(\mathbf{p}_j) - \phi(\mathbf{p}_0)] & (ii) \end{cases} \quad (3.1.41)$$

If we define $\Delta\phi_1 = \phi(\mathbf{p}_1) - \phi(\mathbf{p}_0)$, we may state that $\phi(\mathbf{p}_j) - \phi(\mathbf{p}_0) = j \Delta\phi_1$, j being a natural number. Then, we may rewrite the Equation System (3.1.41) as

$$\begin{cases} \frac{[S_{a_d}(\mathbf{p}_j) - S_{a_i}(\mathbf{p}_j)] - [S_{a_d}(\mathbf{p}_0) - S_{a_i}(\mathbf{p}_0)]}{\lambda_a} = \frac{m j \Delta\phi_1}{2\pi} & (i) \\ \frac{[S_{b_d}(\mathbf{p}_j) - S_{b_i}(\mathbf{p}_j)] - [S_{b_d}(\mathbf{p}_0) - S_{b_i}(\mathbf{p}_0)]}{\lambda_b} = \frac{m j \Delta\phi_1}{2\pi} & (ii) \end{cases} \quad (3.1.42)$$

Solving the Equation system (3.1.42) for any real value of $\Delta\phi_1$, we can calculate the point $\mathbf{p}_j$. Changing the value of j , we get a discrete set of points that belong to the diffractive macro surface. The value of $\Delta\phi_1$ may be decided and fixed, depending on how dense does the designer wants the set of points to be. Setting $\Delta\phi_1 = 2\pi$, then the set of points $\mathbf{p}_j$ correspond to the transition points between diffractive zones.

The macro surface may then be calculated by interpolating points $\mathbf{p}_j$.

Subtracting (i)-(ii) from Equation system (3.1.42), we get Equation (3.1.40), the implicit equation of the macro surface, the GDO.

Figure 3-22 Procedure to design simultaneously a refractive and a diffractive surface point-by-point

Equation system (3.1.42) may now be used to calculate a hybrid system point-by-point. The procedure is, naturally, quite similar to one previously presented in Figure 3-21. Consider Figure 3-22, where we have the same exact initials conditions as the ones previously presented, and step 1 is the same. So, we describe the point-by-point method from step 2:

2. Consider Figure 3-22b). We may now refract the rays with origin in b and c on the previously calculated portion of the surface r , but now, considering the optical path difference between $b-r-d_1-b'$ and $b-r-d_0-b'$, and between $c-r-d_1-c'$ and $c-r-d_0-c'$ we use Equation system (3.1.42) to calculate point d_1 , as well as the points in surface r where the rays coming from b and c are refracted in the direction of d_1 . Consequently, we define the input ray vectors r_{b1n} and r_{c1n} that come from r to d_1 and the output rays vectors r_{b1d} and r_{c1d} that are diffracted from d_1 to b and c ,

respectively. Ray vectors $\mathbf{r}_{b1n}$, $\mathbf{r}_{c1n}$, $\mathbf{r}_{b1d}$ and $\mathbf{r}_{c1d}$ are used in Equation (3.1.35) to calculate the normal to the surface $\mathbf{N}_{d1}$ and $\nabla\phi$.

3. The following step is to calculate the point $\mathbf{r}_1$ and the consequent extension of surface r (Figure 3-22c). Having calculated $\mathbf{N}_{d1}$ and $\nabla\phi$, we can diffract the ray with the vector $\mathbf{r}_{a1d}$ and calculate the vector $\mathbf{r}_{a1n}$. As we now know the direction of the diffracted ray, we know that $\mathbf{r}_1$ has to be in the line created by $\mathbf{r}_{a1n}$ and $\mathbf{d}_1$. Therefore, we apply Equation (3.1.42)(i) the the path $\mathbf{a}-\mathbf{r}_1-\mathbf{d}_1-\mathbf{a}'$ and $\mathbf{a}-\mathbf{r}_0-\mathbf{d}_0-\mathbf{a}'$ to calculate $\mathbf{r}_1$. Once we have $\mathbf{r}_1$, we may use $\mathbf{r}_{a1n}$ and $\mathbf{r}_{a1}$ to calculate the normal $\mathbf{N}_{r1}$ in $\mathbf{r}_1$, which is used to interpolate the surface r to $\mathbf{r}_1$.
4. We repeat step 2. and 3. to calculate the remaining points $\mathbf{d}_j$ of surface d and extensions of surface r .
5. Using points $\mathbf{d}_j$ and respectively normals $\mathbf{N}_{dj}$ we can perform an interpolation and calculate the profile of the diffractive surface d .

The point-by-point procedure, although is not an analytical method as the procedure based on the generalized Diffractive Ovals, as the advantage of being computationally simpler to implement. Is up to the optical designer to select which one to use, depending on the application and on the complexity of the aberrated wavefronts.

3.4 Design Examples

This section describes three examples of systems designed using the diffractive SMS method. Since the method is based on perfect coupling of wavefronts, our performance evaluation is focused on the RMS spot size over a range of fields and for $m=1$. The systems are designed for acrylic (PMMA) refractive index.

The systems were analyzed using Code V[®]. The sets of points calculated for each surfaces are interpolated using aspheres or XY polynomials, depending if the system was designed in two or three dimensional geometry. In XY polynomials, the sag of the surface z parallel to the z -axis is defined by:

$$z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1+k)c^2(x^2 + y^2)}} + \sum_{j=2}^{66} \mathbf{C}_j x^m y^n \quad (3.1.43)$$

with $j = \frac{(m+n)^2 + m + 3n}{2} + 1$, c is the vertex coordinate, k is the conic constant and $\mathbf{C}_j$ is the coefficient of the monomial $x^m y^n$.

The phase polynomial function is the

$$\phi(x,y) = \frac{2\pi}{\lambda} \sum_{j=2}^{66} C_j x^m y^n \quad (3.1.44)$$

where j has the same definition as in the XY polynomials.

In SMS-based diffractive systems, the input wavefront may impinge on the diffractive surface with a large incidence angle. Additionally, the phase function may change rapidly throughout the surface, which leads to a rapid decrease of the spatial period, and affects the duty cycle of the diffracted field. So, it is necessary to perform an analysis of the diffraction efficiency of these systems, using the extended scalar theory of diffraction. Considering that SMS-based diffractive systems are designed to couple different fields, for each one of the presented examples, an analysis was on the changes of diffraction efficiency for each field.

3.4.1 Freeform monochromatic Diffractive Oval coupling 2 wavefronts

Figure 3-23 Single freeform monochromatic diffractive surface coupling 2 spherical wavefronts. (top) 2D and 3D representation of the system. (Bottom) RMS spot size over object plane

This example shows a freeform monochromatic diffractive surface coupling 2 spherical wavefronts (a Diffractive Oval) (Figure 3-23). Both origin points in the object plane **a** and **b** are embedded in PMMA ($n_i = 1.4914$, $n_d=1$ at $\lambda=587.6\text{nm}$). **a** and **b** are 8 mm away and **a'** and **b'** are 22 mm away. The lens vertex is at 49.9 and 50 mm from the object and image plane, respectively, with a lens aperture of 20mm diameter. RMS spot size field map is displayed in Figure 3-23). The two design wavefronts are perfectly coupled (RMS=0) and the RMS spot size curve over the y axis Figure 3-23 Bottom right) has the typical shape of a two

surface non-diffractive SMS design (Figure 1-12), the difference being only one surface is required in the diffractive case.

Diffraction efficiency

This system is designed to two external points in the object plane, **a** and **b**. The diffraction efficiencies were evaluated for both points and also for a central point. To evaluate if the scalar theory of diffraction is appropriate for the efficiency calculation, the variation of the spatial period L in the y axis was plotted (Figure 3-24). In the border of the lens, the spatial period is only slightly above 5λ . For this reason, the extended theory of diffraction was used for the calculations. The maximum incidence angle on the surface is 6.1° so, according to Figure 3-16, this variable should not have much influence on the overall diffraction efficiency.

Figure 3-24 Variation of the spatial period L . The red line represents 5λ , the limit for which the scalar theory of diffraction is valid.

The surface relief was designed such that the maximum height d to the macro profile of each zone is constant throughout the surface (the normal case for diffractive surfaces due to fabrication constraints). d was calculated such that diffraction efficiency is optimized in the center of lens, for the wavefront with origin in point **a**, according to Equation (3.1.14).

The system is monochromatic, for $\lambda=587.6$ nm, so the surface relief was optimized for this wavelength. The diffraction efficiencies were calculated according to Equation (3.1.16), for wavefronts coming from point **a**, **b**, and a central point of the object plane, and their variation on the y axis was plotted (Figure 3-25).

Figure 3-25 Diffraction efficiencies of the monochromatic diffractive oval, for three points of the object plane, for different wavelengths. The duty cycle causes a high decrease on the diffraction efficiency on the outer regions of the lens, especially for non-wavelengths

The first observation is related with the design wavelength, 587.6 nm (blue curve). As expected, the diffraction efficiency is 100% in the center of the lens for point **a**. However, it is also maximum for the other two analyzed points. Also, the blue curve only has a slight asymmetry, and almost does not differ for different points, which is related with the low sensitivity to incident angles, especially for systems with incident angles with such small values as this one. The low wavelength-to-period ratio in the outer regions of the lens affects the diffraction efficiency, especially for non-design wavelengths. For these last ones, the effect of the incidence angle is slightly higher, and the asymmetry of the curves is larger. Figure 3-26 shows the non dependence of the system with the incidence angle: although the surface relief is optimized for point **a**, the average diffraction efficiency variation with λ

curves completely overlap for the three points. The effect of shadowing in the outer regions causes an overall decrease in the diffraction efficiency, with larger impact for higher wavelengths. For the design wavelength, the efficiency is 97%.

Figure 3-26 Diffraction efficiency dependence with wavelength, for three fields. The curves completely overlap.

3.4.2 Freeform bichromatic Diffractive Oval

Figure 3-27 Freeform bichromatic Diffractive Oval surface. (left) 2D representation of the system. (center and right) RMS spot size over object height and wavelength. Although only two wavelengths are being controlled (RMS=0), the design has almost RMS=0 for all wavelengths in between.

DOE's are typically used for correcting chromatic aberration, so it is interesting to design a system for two spherical wavefronts originated in the same point, but with two different wavelengths, 670.4 and 403.2nm (Figure 3-27). The lens vertex is at 72 and 28 mm from the object and image plane, respectively, with a lens aperture of 25mm. RMS spot size map is

also used but, unlike Figure 3-23 center, now one of the axis represents the wavelength, to evaluate chromatic behavior. The *V-shaped* profile of the RMS spot size over the object height (Figure 3-27 right) is typical from devices that perfectly image only one wavefront. It is noticeable that the RMS line is exactly the same for both wavelengths. The systems perfectly images the wavefront on the design wavelengths. Although only two wavelengths are being controlled, RMS spot size values are very low on the wavelengths between the two design ones, which emphasizes the reason why diffractive devices are so successfully used for chromatic corrections.

Diffraction efficiency

Unlike the previous system, the bichromatic Diffractive Oval is designed for the central point of the object plane, and optimized for two different wavelengths. The diffraction efficiency is assessed only for the central point, and two different profile reliefs are studied, each one optimized for one of the design wavelengths, 670.4 and 403.2nm. The system is well within the boundaries required by scalar theory (Figure 3-28).

Figure 3-28 Variation of the spatial period L . The red line represents 5λ , the limit for which the scalar theory of diffraction is valid.

The incidence angle on this system is higher, the maximum being 40° in the outer regions of the lens. To assess the effect of the incidence angle, both scalar and extended theories were used to calculate the diffraction efficiencies, which were optimized for both design wavelengths, 670.4 nm (Figure 3-29 top) and 403.2nm (Figure 3-29 bottom).

Figure 3-29 Diffraction efficiencies calculated using both the scalar and extended scalar theory. (*top*) Surface relief optimized for $\lambda = 670.4\text{nm}$. (*bottom*) Surface relief optimized for $\lambda = 403.2\text{nm}$

The plot of diffraction efficiencies of scalar and extended theories are very similar, which is consistent with the fact that the ratio wavelength-to-period is small (Figure 3-28). It also shows that the main differences in the outer regions of the lens are due to the high incidence angles, and not due to shadowing. Nevertheless, the surface relief optimized for $\lambda = 403.2\text{nm}$ is rather insensitive to incidence angles, and the diffraction efficiency value for each selected wavelength is kept rather constant throughout the lens. The effect of the high incidence angles decreases the maximum diffraction efficiency to 98%.

Figure 3-30 Diffraction efficiency dependence with wavelength, for surfaces reliefs optimized for two different wavelengths

3.4.3 One refractive + one diffractive surface coupling three wavefronts

The third example is a hybrid system, with a refractive and a diffractive surface, and is designed using the diffractive SMS method in section 2.2. This is a 2D design and only rays contained in the lens plane are considered in the ray tracing. The system images 3 points, 2 of them at one wavelength and the third point at another wavelength (Figure 3-31).

Figure 3-31 Hybrid system coupling three wavefronts. (top) 2D representation of the system. (bottom) RMS spot size over the object height and wavelength. The control of only three wavefronts allows that a set of combination of wavefronts with specific wavelengths

The central design object point (0 mm) has $\lambda=403.2\text{nm}$ and the two outer design points (4 and -4mm) have $\lambda=670.4\text{nm}$. The vertex of the first surface is at 56.7mm from the object plane, while the second surface vertex is at 49mm from the image plane. The lens aperture is 23mm. The results are, again, represented by a RMS spot size map with one axis representing the object height, and the other axis representing the wavelength. We can see, for both wavelengths, that the design points are perfectly imaged. For $\lambda=403.2\text{nm}$, where only one field is being controlled, we can observe that the RMS curve presents a much smoother behavior than the previous example (Figure 3-27). The shape of the RMS curve is typical of an aplanatic system [72]. The most interesting result is the *ring-shaped* RMS spot size map, which shows a set of combination of wavefronts with specific wavelengths that, although not perfect solutions as in the case of the design points, still have a very low RMS spot size value.

Diffraction efficiency

The system is designed to image two outer object points at one wavelength, symmetric to

the optical axis, and one central point at another wavelength. As the outer points (and the system) are symmetric, the diffraction efficiency is optimized to the central point, and only one of the outer points is analyzed. As it happened in the last example, the system is well within the boundaries required by scalar theory (Figure 3-32).

Figure 3-32 Variation of the spatial period L . The red line represents 5λ , the limit for which the scalar theory of diffraction is valid.

The maximum incidence angle of this system is approximately 40° , for the external design point, and the ratio wavelength-to-period is small, so the shadowing does not have a major impact on the diffraction efficiency. Figure 3-33 shows the variation of diffraction efficiency throughout the surface.

Figure 3-33 Diffraction efficiencies of hybrid system, for two points of the object plane, for different wavelengths.

As the previous one, the diffraction efficiency is kept rather constant along the surface for the non-design wavelengths, as a slight decrease for the design wavelength, which causes the maximum value of diffraction efficiency to decrease to 98%. Although the efficiency decrease is caused by the effect of the incidence angle, there are no apparent differences for the central and the external design points, both in Figure 3-33 and Figure 3-34, where the curves of diffraction efficiency for both design points completely overlap.

Figure 3-34 Diffraction efficiency dependence with wavelength, for two design points. The curves completely overlap.

3.5 Conclusions

This chapter described an extension of the Simultaneous Multiple Surface (SMS) to design diffractive surfaces. The diffractive SMS method constructs the optical system by designing simultaneously diffractive and refractive/reflective surfaces. Using the phase-shift properties of diffractive elements as an extra degree of freedom, two rays for each point on each diffractive surface are controlled, in opposition to refractive/reflective surfaces, where only one ray for each point can be controlled. The diffractive SMS method uses the extra degree of freedom provided by the phase shifting properties of DOEs to control two rays on each point of each diffractive surface.

The concept of generalized diffractive oval (GDO) is introduced, as a freeform diffractive surface with the ability to couple two incident wavefronts. A complete analytical description of the surface is obtained. Different design examples are provided, such as freeform monochromatic diffractive oval coupling two wavefronts, and a freeform bichromatic diffractive oval, coupling two wavefronts with different wavelengths. For these examples, all the design wavefronts are perfectly coupled, creating perfect imaging (neglecting the Airy disk diffraction limit). An interesting result of the freeform bichromatic diffractive oval is the fact, not only the design wavelengths present a perfect imaging, the spectrum band between them also presents very low RMS spot sizes.

The diffractive SMS method calculates different portions of the diffractive surfaces by a sequential application of the GDO procedure. To design simultaneously diffractive and deflective surfaces, GDO (for diffractive surfaces) and Generalized Cartesian oval (GCO) (for

deflective surfaces) are calculated simultaneously and sequentially. A 2D example of a hybrid (1 diffractive and 1 refractive surfaces) lens designed with the diffractive SMS is provided, coupling three wavefronts of two different wavelengths. The design points were perfectly imaged, and the RMS curve of the design points is typical from aplanatic systems. The most interesting result is a set of combinations of field angle/wavelength that, even though they were not controlled, present a very low RMS spot size value, forming *ring-shaped* RMS spot size map (with one axis representing the field and the other the wavelength).

An analysis of the diffraction efficiency of the design examples was made, based on the extended scalar theory of diffraction. The study assessed the effect of the variation of the grating period throughout the lens, as well as the effect of the incidence angle of light. The last is important for SMS-based systems, because the systems are designed for multiple wavefronts. The analysis did not reveal any relevant difference on the diffraction efficiency for the different wavefronts, and the SMS-based diffractive systems must have the same considerations than other diffractive designs to account for diffractive efficiency effects.

The diffractive SMS method proved to be reliable, and with the potential to be used both as a direct method for a final design, or to create a good starting point for further optimization. Its potential to design freeform diffractive surfaces may have an impact in imaging and spectroscopy applications, among others, by decreasing size, weight, number of elements of the optical system, and correcting for chromatic aberrations.

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Chapter 4.

EXPERIMENTAL CHARACTERIZATION OF FREEFORM SURFACES

The optical design methods and fabrication machinery for freeform optics have reached a point where the surfaces can be manufactured with high precision. Yet, accurate measurements of the resultant surfaces are still a standing challenge to the surface metrology community, and have proven to be a genuine bottleneck in their usage in real world applications, especially in imaging systems. Metrology is essential, not only to validate the manufactured parts, but also in providing a real-time feedback to the fabrication process. Commercial interferometers are typically used to measure the surface profile of optical components. However, the fringe density created by freeform surfaces exceeds the Nyquist frequency of these instruments, due to high surface departure from the typical reference surfaces, usually flat or spherical surfaces.

This chapter is dedicated to the experimental characterization of a double-channel freeform lens, using freeform metrology. The lens is a near-eye imaging device of a head mounted display (HMD). The fact that the lens is double-channel and both surfaces of each channel are freeform pose some challenges for freeform optics metrology. Different samples of the lens were manufactured using two different techniques, injection molding and diamond-turned. Both were measured and evaluated.

Section 4.1 gives an overview of metrology for freeform optics, where the technologies and steps required for a complete characterization are explained

Section 4.2 describes the optical system containing the double-channel freeform lenses.

Section 4.3 offers a small technical description of the two freeform Coordinate Measuring Machines (CMMs) used to measure the lenses.

Section 4.4 shows the measurements on the lenses, with two different CMM.

In Section 4.5 , the roughness of the lenses was measured.

Section 4.6 offers a raytrace performance evaluation, comparing the design lens with the manufactured ones, using data obtained from the freeform measurement.

4.1 Overview of metrology for freeform optics

The complete characterization of a freeform surface involves: Measurement of the surface, where typically multiple profiles or a point cloud data file is generated from the measured surface; Registration (or alignment), where the measured points are aligned with the design data (typically a CAD- model), such that it is possible to evaluate the deviation between the experimental and design data; Filtering, a technique used to separate the effects of roughness, waviness and real form deviation. In some cases, characterization also requires stitching multiple images, acquired from a set of sub-aperture acquisitions of the surface. Finally, the surface roughness and waviness should also be measured. A survey on general freeform surfaces, from car body parts to optics, was made by Savio et al. [1], and Fang et al. in 2013 presented a more specific one about manufacturing and measurement of freeform optics [2].

This section surveys the state of the art of metrology for freeform optics, and describes the main steps required for a complete characterization.

4.1.1 Measurement of optical surface

Metrology requirements for freeform optical surfaces vary, depending on the application. For non-imaging applications such as illumination and solar concentration, the requirements can be in the micrometer scale, while for imaging applications they extend down to the nanometer scale. Both contact and non-contact measurement techniques are used. A way to discriminate between different measurement methods is achieved by dividing them into scanning (using Coordinate Measuring Machines-CMM) and non-scanning techniques, with imaging and full-field interferometric methods. While the last ones are very fast and have a very low uncertainty, they are sensitive to environmental influences and the lateral resolution is limited by the Rayleigh criterion. Other limitations can arise from specific surface structures and high surface curvatures that many freeforms have that can lead to

phase jumps. CMM based-methods can overcome these problems, but are typically very slow. Also, depending on the probe, CMM can be less precise and if it is a contact probe, there is always the risk of damaging the surface. Recently, both techniques have been combined, with interferometric and other imaging probes integrated in CMMs.

Interferometry is widely used on metrology of optical parts, due to the fact that it provides a fast measurement with sub-nanometric resolution in the direction of beam propagation. The high precision allows for interferometry to also measure surface roughness. A typical configuration is a Fizeau's (Figure 4-1), which uses the wave properties of light to compare the optical path difference between a test surface and a reference surface. Inside an interferometer profiler, a light beam is split, reflecting half the beam from a test material which is passed through the focal plane of a microscope objective, and the other half of the split beam is reflected from the reference mirror. When the distance from the beam splitter to the reference mirror is the same distance as the beam splitter is from the test surface and the split beams are recombined, constructive and destructive interference occurs in the combined beam wherever the length of the light beams vary. This creates the light and dark bands known as interference fringes. Since the reference mirror profile is well known (it can be flat or spherical) the optical path differences are due to height variances in the test surface. The entire surface is measured in one single shot, without contact, and it has been on the market since the 1970s [3].

Although interferometers have a high precision for near-flat or spherical parts, they struggle when the departure from the reference surface increases, because the number of fringes also increases, which limits the resolution. A way explored by manufacturers as QED and others is to use sub aperture stitching interferometry [4]. Decreasing the size of the aperture decreases the number of fringes in the detector. Thus, by imaging several sub-apertures over the surface and then stitching them together by post-processing the images, the entire surface can be measured.

Figure 4-1 Fizeau Interferometer is a typical instrument to measure the topography of optical surfaces, but has limitations when the surface has high departure from a flat or spherical surface

Another technique used to overcome the limitation caused by high departures is to match the reference and the surface under test by using a correction element such as a null lens or

a Computer Generated Hologram (CGH), typically a diffractive optical element manufactured by high precision lithographic methods [5]. Null methods are low cost but require setup time, while CGH can be considerably expensive, as each reference has to be specifically designed and manufactured for each specific surface to be measured, and are not practicable for all freeforms [6]. Figure 1.2 illustrates both techniques.

Figure 4-2 Techniques to overcome limitations of interferometry on large departures from flat or spherical surfaces. (Left) QED images multiple sub-apertures and stitch them together to obtain an entire measurement of the surface [28]. (Right) CGH are used to match the reference and the surface under test [6]

The most common instrument used to measure freeforms is a Coordinate Measuring Machine (CMM). CMM can use up to six-axes of motion to scan the surface with a probe to measure individual points. Depending on the type of probe, CMM can be either contact or non-contact. The measurement speed of these machines is slow, and a measurement of a small part can take up to several minutes. Some CMMs use data from the part (a CAD file, for example) to “follow” the surface, ensuring that the probe does not collide with any section of the part. The probe can also be kept approximately perpendicular to the surface, to incorporate Abbe's principle [7], [8].

The probe is the essential sensor of a CMM, and the type of technology that it uses defines the amplitude precision (while the lateral spacing precision is defined by the CMM itself). Contact probes are typically called profilometers, and they use a stylus-based tip, with a conical or pyramidal shape made of ruby or diamond [9]. If the surface hardness is not sufficiently high, the pressure exerted by the probe can damage the surface. Special types of micro-probes with higher resolution were developed to measure freeform optics [10].

Several non-contact techniques have been integrated in CMMs by a variety of companies and research institutes. Scanning Tunneling Microscopy (STM) and Atomic Force Microscope (AFM) are some of the important techniques that have been already implemented. STM uses an electrical near-field interaction with a conductive surface, and the probe is a set of wire

tips with a radius of single nanometers [11]. AFM uses a sharp cantilever tip that is kept at a distance of a few nanometers from the surface, being attracted by surface forces [12]. Both techniques have a lateral resolution of nanometers and a vertical one of sub-nanometers, but the scanning is very time-consuming.

Figure 4-3 (Left) Scheme of a CMM. (Right) Contact micro Probe from ISARA 400 CMM of IBS measuring the surface of an aspheric lens [13]

Another technique used to measure freeform optics is an auto focus laser probe mounted on a CMM. The laser beam is focused on a surface, and the reflected light is detected by a Four quadrant Detector. The vertical axis surface position is detected by an AutoFocus sensor. White light confocal chromatic imaging microscopy in CMMs has also been suggested to measure freeforms. This technique is used on UltraSurf from Optipro [6], which also has the property of ensuring that the probe is also perpendicular to the surface, through data from the CAD file of the design.

Recently, Optical Coherence Tomography (OCT) has been suggested to measure freeform surfaces, using interferometry in the Fourier-Domain [14].

The maximum slope that optical probes can measure is limited, and significantly lower than the maximum slope which contact probes can measure. This limitation can be a major drawback for more demanding freeform surfaces. Gómez-Pedrero *et al* [15] proposed recently a five-axes CMM using a dual mechanical/optical probe that allows fast and precise measurements, even for high slopes regions of the optical surfaces.

4.1.2 Characterization

The full characterization of a freeform surface is completed with registration (also called matching or alignment) and filtration.

Registration is defined as the alignment of measured points with the design points such that an appropriate quantification of the deviation can be made. Two main registration methods

maybe used. One uses fiducials - reference points in the freeform to align the coordinate systems of the experimental data and design data. This is the preferred method, but it is not always possible, since frequently freeform surfaces do not have fiducials that can be used. The other method makes use of algorithms such as the iterative closest point (ICP) coupled reference data method [16], that use all data points and try to decrease the minimum distance between surfaces,. Softwares like Geomagic have some of these methods implemented. For some surfaces and some noisy point clouds, fitting then into an analytical surface is a complex challenge, demanding more powerful algorithms than the ICP. One approach is to fit point cloud points to Zernike circle polynomials [17], [18]. Recently, Ibañez *et al* [19] proposed a method to fit Zernike polynomials to arbitrary shapes (such as freeform lenses).

Filtration is a method that separates form and systematic errors such as roughness, waviness or form deviation such that the effects of the three components are evaluated. By applying Fourier transforms over spatial frequency, with Gaussian or 2RC filtering, these components can be properly separated and the real figure error data can be obtained.

4.1.3 Surface roughness, waviness and scattering

Roughness is a measure of the topography relief of a surface. In optical surfaces, it affects the amount of scattering on an optical system. Roughness features that produce light scattering are typically separated by submicrometers to fractions of a millimeter. Features separated by larger distances, from hundreds of micrometers to several millimeters, are usually termed *waviness* and contribute to small-angle scattering. Features whose separations are still larger define the deviation from the ideal geometrical optics surface. The separation between similar surface features along the surface is referred as *surface spatial wavelength*. To distinguish between roughness and waviness, one should define spatial wavelengths bands and limits to each the topography relief of the surface is measured.

Roughness can be created through a number of reasons. Each reason creates different features. Scratches are present on most surfaces, and are generally produced by finishing operation and can have widths as large as 10 μ m or as small as a few tenths of a micrometer on optical components (disregarding improper handling). Microirregularities are typically distributed over the entire surface. These can be tiny scratches (known as sleeks), caused by high-quality optical polishing processes, granularity or crystallites in films deposited on surfaces, or undulations lefts by chemical etching or electropolishing. Another form of roughness is grating grooves, typically caused by machinery used for manufacture. They are more pronounced when diamond turning is used, and may consists of cusped grooves formed by the diamond tool, chatter marks in the grooves, longer irregular-spaced grooves

caused by machine-tool-chip interactions, and structure in the material such as second-phase particles and inclusions.

Upon selecting the appropriate spatial wavelength band, roughness R_a is the arithmetic average of the absolute values of the profile height deviations from the mean line, recorded within the evaluation length, given by [20]

$$R_a = \frac{1}{A} \int_A |z(x, y)| dx dy \quad (4.2.1)$$

Where z is the height function from the midline, and A is the evaluated area.

The RMS value of roughness is given by

$$\delta = \sqrt{\frac{1}{A} \int_A |z(x, y)|^2 dx dy} \quad (4.2.2)$$

RMS roughness is generally used to describe the finish of optical surfaces, while R_a is normally used for roughness of machined surfaces.

The Power Spectral Density (*PSD*) function is defined as the squared modulus of the Fourier Transform of the surface topography:

$$PSD(f) = \lim_{L \rightarrow \infty} \left| \int z(x) e^{-2\pi i f x} dx \right|^2 \quad (4.2.3)$$

PSD expresses the power of different roughness components in terms of the lateral surface spatial frequencies, being very useful to identify the type of surface error induced by the manufacture.

Roughness affects the amount of light scattering. In imaging systems, scattering affects the contrast and quality of image formation, while in non-imaging devices affects the efficiency of the system. A useful parameter that evaluates the amount of light scattering produced by roughness is the Total Integrated Scatter (TIS), defined as the integrated scattered light normalized by the specular reflected light. In a first order-approximation, and for surfaces with low level of scattering, TIS can be simply related with the roughness. For reflective surfaces, TIS is defined as the ratio between specular reflectance R_s and total reflectance R_o ,

$$TIS_{refl} \approx \left(\frac{4\pi\delta \cos \theta_i}{\lambda} \right)^2 \quad (4.2.4)$$

Where δ is the RMS roughness, θ_i is incidence angle and λ is the wavelength.

In transmission, TIS, for normal incidence, is given by:

$$TIS_{trans}(\lambda, \delta) = \left(\frac{2\pi\delta}{\lambda} (n(\lambda) - 1) \right)^2 \quad (4.2.5)$$

Where n is the refractive index.

Optical profilers for roughness measurement

There are several methods for measuring roughness, non-contact and contact methods. The contact methods uses diamond stylus probe that touches the surface. Non-contact methods are preferred due to simplicity and because there is no risk of damage to the surface. In this thesis, we used and focus on the last ones.

Optical profilers are interference microscopes, and are used to measure height variations – such as surface roughness – on surfaces with great precision using the wavelength of light as the ruler. Optical interference profiling is a well-established method of obtaining accurate surface measurements.

Optical profiling uses the wave properties of light to compare the optical path difference between a test surface and a reference surface. Inside an optical interference profiler, a light beam is split, reflecting half the beam from a test material which is passed through the focal plane of a microscope objective, and the other half of the split beam is reflected from the reference mirror (Figure 4-4).

Figure 4-4 Optical profiler uses interference patterns between the test and a reference surface to measure roughness. Source: Ref. [21]

When the distance from the beam splitter to the reference mirror is the same distance as the beam splitter is from the test surface and the split beams are recombined, constructive and destructive interference occurs in the combined beam wherever the length of the light beams vary. This creates the light and dark bands known as interference fringes.

Since the reference mirror is of a known flatness – that is, it is as close to perfect flatness as possible – the optical path differences are due to height variances in the test surface.

This interference beam is focused into a digital camera, which sees the constructive interference areas as lighter, and the destructive interference areas as darker.

In the interference image (an "interferogram") of Figure 4-4, each transition from light to dark represents one-half a wavelength of difference between the reference path and the test path.

If the wavelength is known, it is possible to calculate height differences across a surface, in fractions of a wave. From these height differences, a surface measurement is obtained.

4.2 Description of optical device: double channel double-sided freeform lens for HMD

Head mounted display (HMD) technology is a rapidly developing area. One aspect of HMD technology provides a full immersive visual environment (which can be described as virtual reality), such that the user only observes the images provided by one or more displays, while the outside environment is visually blocked. These devices have application in areas such as entertainment, gaming, military, medicine and industry. In recent years, there has been large interest in this technology for entertainment purposes, and companies such as Samsung, Facebook, Google, are making serious investments in the development and commercialization of HMDs.

A head mounted display consists typically of one or two displays, their corresponding optical systems, which images the displays onto a virtual screen to be visualized by the user's eye, and a helmet that visually blocks the external environment and provides structural support to the mentioned components. The display may also have a pupil tracker and/or a head tracker, such that the image provided by the display changes according to the user's movement.

4.2.1 Description of optical system

The lens that was experimentally characterized in this Chapter is part of a Head Mounted display, recently presented in Ref. [22].

The system consists of in a near-eye system, with a digital display and double channel freeform lens, with channels that are identical and symmetrical to each other, as shown in Figure 4-5. The fact that the lens is double-channel allows that the focal length of each channel to be smaller, thus the display can be much closer to the lens, making the whole system much more compact. The system creates a far-field image, so the lens is designed such that the pixels on the display are transmitted to the eye as plane wavefronts. The situation displayed in Figure 4-5 right shows the case when the eye is "looking" parallel to the optical axis. The plane wavefront b_1 is the result of two different wavefronts, a_{11} and a_{12} ,

with origins in pixels of the display op_{11} and op_{22} , respectively. This situation creates an images overlapping, and the eye sees pixels op_{11} and op_{22} overlapped. The solution is providing the same information to op_{11} and op_{22} , i.e., the regions of the digital display that the eye sees overlapped must have the same image.

Figure 4-5 Near-eye system with a double channel freeform lens

Figure 4-6 explains the image overlapping with a simple example. On the bottom right, the digital display shows two images, in which a portion of the image is repeated (the images are shifted). Each half of the display is in the field of view of each optical channel. On the left, the image shows the virtual image as seen by the eye. For clarity, we refer to the pixels of the display as *opixels* and the pixels of the virtual image as *ipixels*. In order to explain why this is necessary, a two-dimensional schematic drawing has been added on top of the figure. It shows the relevant rays to define the edges of the mapping between opixels and ipixels. In this drawing, $r0a$, $r1a$, $r2a$ and $r3a$ indicates the ipixel positions on the image seen by the eye.

As there is a partial duplication of the information on the display, ipixel $ip1$ is formed by the projection of two opixels $op11$ and $op12$. Each one of these opixels are projected into the eye through different optical channels, and form the same parallel wavefront, thus they share the same information such the eye can create a complete image from the segmented image on the screen. Ipixel $ip2$ is outside the overlapping region, thus it is only created by opixel $op21$. The correct mapping between opixels and ipixels is a function of the focal length of each channel of the lenses.

Figure 4-6 The lenses focus different pixels of the screen onto the same parallel wavefront into the eye. This pixels have the same information to avoid image overlapping, thus the image on the display is partially repeated and shifted.

Table 4-1 Summary of optical properties of head mounted display

Horizontal Field of view	Vertical Field of view	Focal length	Eye box	Spectrum range
100°	75	23mm	15 mm	390-700nm (visible spectrum)

4.2.2 Description of freeform lens and pre-measurement analysis

The lens to be measured is a 2-channel imaging lens made of Zeonex 48R (Figure 4-7). The lens comprises two identical and symmetric folds, each of them with two freeform surfaces. Each fold possesses plane symmetry on the yz plane.

Figure 4-7 Free form lens consists of two folds. Both folds A and B are identical (symmetric respect the union plane). Each fold has two surfaces, bottom surface S1 and top surface S2.

Table 4-2 Macroscopic dimensions of double channel lens

Center thickness (mm)	Width (mm)	Length (mm)
8.8	25	33.8

Two different versions of the double-channel lens were manufactured, using two different fabrication techniques: the first one was made using injection molding, and the second one was made using a multi-axis diamond turning technique. While the first was manufactured as "full lens", the second one was manufactured as a half lens, and the two folds were produced separately. The half lenses were coupled together in the opto-mechanical structure. The surfaces of the half-lens are the same as one of the folds of the full lens. The main difference between the parts is the position of the flanges and the existence of a sprue on the full lens, required by the injection molding technique (Figure 4-8). The topography and roughness of both lens was measured and presented in the next sections.

Figure 4-8 Two different versions of the lens were manufactured: an injection molded lens (Full lens) and a diamond turned lens (half lens). The diamond turned lens consists of only one of the folds of the injection molded lens

The part were pre-analyzed to assess the kind of measurement technique that would be appropriate for full characterization. The simplest technique would be a one-shot imaging interferometer, but it requires that the departure from a flat or from a sphere and the slope is small. Thus, the CAD data was used to evaluate the slope and the departure from a best fit-sphere. The best fit-sphere was obtained to find the best adjust of the following spherical surface description to a point cloud data of both surfaces:

$$z(x,y) = \left[\frac{c((x+\Delta x)^2 + (y+\Delta y)^2)}{1 + \sqrt{1 - c^2((x+\Delta x)^2 + (y+\Delta y)^2)}} + \Delta z \right] \cos \theta_x + (y+\Delta y) \sin \theta_x \quad (4.2.6)$$

where z, x, y are the point coordinates of the surface, Δx , Δy and Δz is the displacement of both coordinates relatively to the origin, c is the curvature of the spherical fitting and θ_x is the rotated angle of the surface around the x axis.

Figure 4-9 shows the results obtained from the measurements.

The maximum slope on the freeform is slightly above 7° , which would be acceptable for the interferometric measurement. The main limitation arises from the high departure of the measured surface profile from a sphere, which has a maximum value of 2mm for a 12.5mm. The one-shot interferometer technique requires that the surface is compared to a flat or a

spherical surface. The high departure from as sphere suggests that more freeform oriented-techniques are required for the measurement, as it is too high to create interferometric patterns with the reference surface.

Figure 4-9 (Top) Slope of both surfaces. (Middle) Best-fit sphere to a cloud of points obtained from the CAD file (Bottom) Departure from best fit-sphere

4.3 Freeform measurement devices

Two different techniques were used to measure the lenses. A 5 axes CMM non-contact system, with a low-coherence interferometry probe, UltraSurf from OptiPro, and a 3 axes CMM contact system, UPMC ACC 850 Carat from Zeiss. This section describes both systems.

4.3.1 UltraSurf technical description

UltraSurf is a non-contact measuring system, developed by OptiPro, based on a 5 axes CMM,

with the possibility of using two different probes, a low-coherence interferometry with a near infrared laser and a white light confocal chromatic probe [7].

UltraSurf uses five air bearing axes and high-resolution positioning encoders to scan with a non-contact probe over the part under test (Figure 4-10). The five axes of motion are to keep the non-contact probe perpendicular to the surface of the freeform (Table 4-3). The probe “follows” the freeform surface by using the data from the CAD model (using either equations of the surface, or a point cloud data file).

UltraSurf uses two different non-contact probes. The first probe uses low-coherence interferometry, with a near infrared laser and can measure distance when coupled with an internal reference plate. It is able to read both surfaces and thickness of an optical window at a single point. The second probe uses confocal chromatic sensing with white light to measure distance at a single point. It is more accurate than the first probe, but can only measure one surface at a time.

Figure 4-10 UltraSurf technical details. Axes X, Y, and C move the part; axes Z, B move the probe.
Source: Ref. [7]

Table 4-3 UltraSurf with labeled axes of motion. Axes X,Y, and C move the part; and axes Z, B move the probe

Axis:	X,Y,Z	B	C
Travel	200 mm	360°	360°
Resolution Max.	5 nm	0.02 second	0.01 second
Velocity	20 mm/s	6 RPM	6 RPM

*Typical measurement accuracy is currently +/- 200 nm

4.3.2 UPMC ACC 850 CARAT technical description

UPMC ACC 850 Carat is a 3 axes CMM contact system, with highly dynamic servos and a tactile high speed scanning measuring head, with capability to measure up to 80.000 points. It also uses a optic Laser sensor. The probe uses a rubin/ruby stylus, with 1mm radius and a contact force of 100mN. The reported accuracy is 200 nm.

Figure 4-11 UPMC ACC 850 Carat of Zeiss, a contact CMM system

4.4 Measurement of freeform lens

This section describes a freeform surface measurement of an imaging lens. Two different techniques were used for the measurement: the first measurement was made in collaboration with the company OptiPro, using UltraSurf. The second measurement was made with the company Zeiss, using the UPMC ACC 850 carat.

4.4.1 Measurement on Optipro

Only surface S1 of the diamond turned part was measured on the Optipro. To secure the lens in a “horizontal” position, a holder was designed and printed on a 3D plastic printer, with a precision of less than 100 μm . Fiducials were added to the holder, for alignment. Given that UltraSurf uses CAD data to keep the probe always perpendicular to the surface, fiducials are an essential feature for registration. Fiducials are dome-shaped, to ease the alignment - the user of UltraSurf, when performing the alignment, searches for a variation in height. Thus, the center of the fiducial corresponds to the highest point. Four fiducials were added to the holder, and their position relative to the part was properly documented (Figure 4-12).

Figure 4-12 Lens secured by a 3D printed holder, with surface S1 facing up. The holder has four dome-shaped fiducials for alignment. Although the holder was made in a high precision 3D printer, the non-ideal dome still affected the alignment. The holder is secured with double sided sticky tape on the feet. Low thermal expansion clay was added around the feet to provide further support. The low-coherence interferometer probe is always perpendicular to the surface

Although the fiducial locations were well documented, finding their apex was still difficult. The most likely reason is the “flat” areas on top of the domes created by the 3D printed contour. The holder is secured in OptiPro with double sided sticky tape on the feet ((Figure 4-12). Low thermal expansion clay was added around the feet to provide further support. A step and hold measurement was made, averaging 5 points at each location.

The sag table obtained from the design data was used as an input to create the measurement path and to analyze the collected data. The lateral resolution was 0.25mm by 0.25mm, and the measurement took 2hours, with 15 minute setup time. A total number of 5662 points were attempted, but only 4429 points were measured with a point coverage of 78%. UltraSurf was not able to measure the region of the lens where the slope was high; as a consequence of not being possible to maintain a perpendicular orientation of the probe in this region.

Figure 4-13 Measurement results of surface S1 measured in OptiSurf. The area of the surface with highest departure could not be measurement, due to the fact that it was not possible to maintain the probe perpendicular to the surface

Figure 4-13 shows the results of freeform lens measurement with OptiSurf. The results are shown as an error map over the surface of the lens. It is clear in the Figure which areas of the lens were not measured due to non-perpendicular metrology - probe must stay ± 2 degrees parallel to the surface normal, which was not possible due to the lens geometry. The color scale is between $\pm 5 \mu\text{m}$, and most error falls between $\pm 2 \mu\text{m}$.

Figure 4-14 shows another representation of the measurement made in OptiSurf, for comparison with measurements made in UPMC ACC 850 Carat.

Figure 4-14 Measurement of surface S1 of half lens. Measured in OptiSurf

Freeform lens reveals a larger error in the center of the surface, around $4 \mu\text{m}$. The

measurement also shows a waviness effect of $\pm 1 \mu\text{m}$, a magnitude higher than UltraSurf system noise, and this effect is visible in Figure 4-13 - a “grid line” effect. To assess if this effect was an irregularity of the surface, a laser pointer test was made - a green Laser was pointed through the lens, and pointed to surface 1 an in tangential position. The transmitted and reflected laser pattern was projected onto a white wall at a distance of 1 meter (Figure 4-15). The test reveals linear artifacts on the surface of the lens, in accordance to the measurement performed in OptiPro. These linear artifacts can have an impact on the image formation of the lens. To assess the effects of the waviness, a roughness analysis was made on the freeform lens, and presented Section 4.5 .

Figure 4-15 Green laser pointer test reveals linear artifacts on the surface with glancing incident and transmitted laser beams

4.4.2 Measurement on the UPMC ACC 850 Carat

Both diamond-turned (Full lens) and injected molded (half lens) parts were measured on UPMC ACC 850 Carat. In this case, there was no need to use the 3D printed holder, as both lenses were positioned using holders from Carat, which grabbed the flanges. The flanges and sprue of both parts were used as fiducials for the measurements and registration (Figure 4-16).

The lateral resolution was 0.25mm by 0.25mm, and a contact strategy was followed, using a ruby stylus. The contact force of the stylus in each point was 100 mN.

Figure 4-16 The flanges and sprues of both parts were used as fiducials for the measurement. (Left) full lens (injection molded). (Right) half lens (Diamond turned)

Half lens measurement

Figure 4-17 shows the measurement of surface deviation, for a range of $[-65, 65 \mu\text{m}]$.

Figure 4-17 Measurement results of topography difference between the experimental measurement of the half lens and the design lens. Measured in UPMC ACC 850 Carat, for a surface deviation range of $[-65, 65 \mu\text{m}]$

Carat was able to measure the areas with higher slope, and it reveals large surface deviation in these areas, in the order of $50 \mu\text{m}$. The center regions of the lens show small deviations, so a higher resolution graphic is required to analyze this region (Figure 4-18).

Figure 4-18 Measurement results of topography difference between the experimental measurement of the half lens and the design lens. Measured in UPMC ACC 850 Carat, for a surface deviation range of $[-15, 15] \mu\text{m}$

The peripheral regions shows an error higher than $15 \mu\text{m}$. The errors in the center regions mostly fall in interval $\pm 2 \mu\text{m}$. Near the $(0,0)$ mm point, the error is slightly larger, approximately $5 \mu\text{m}$. This deviation, although small, has an impact on the performance of the system, due to the fact that it is in the center of the lens, region where the image formation is more demanding, as shown in Section 4.6 .

Full lens measurement

Figure 4-19 and Figure 4-20 shows the measurements of the injection molded full lens, of both folds. As the range of surface errors is large, Figure 4-19 shows the results in the range $[-65, 65] \mu\text{m}$, while Figure 4-20 shows the results in the range $[-15, 15] \mu\text{m}$.

Figure 4-19 Measurement results of topography difference between the experimental measurement of the full lens and the design lens. (Right) Surface S1. (Left) Surface S2. Measured in UPMC ACC 850 Carat, for a surface deviation range of $[-65, 65 \mu\text{m}]$

The results of Figure 4-19 reveal large errors in the extremities on the lens, in the order of $45 \mu\text{m}$. The regions of surface S1 near the union between the folds were not measured, due to the fact that the CMM was not able to measure a pronounced kink as the one presented in the lens. This reveals some of the challenges that the metrology still have for the characterization of complex optical lens, such as multichannels.

Figure 4-20 Measurement results of topography difference between the experimental measurement of the full lens and the design lens. (Right) Surface S1. (Left) Surface S2. Measured in UPMC ACC 850 Carat, for a surface deviation range of $[-15, 15 \mu\text{m}]$

Figure 4-20 confirms the large errors in the periphery of the lens, in both surfaces. The folds are quite symmetrical, and in the center region of both folds, most errors fall in the range $\pm 3 \mu\text{m}$. Surface S2 shows an average error of $-7 \mu\text{m}$. As both surfaces were measured relatively to the defined fiducials, this average error reveals that surface S2 is displaced approximately $7 \mu\text{m}$ from the design position. Although the center region is within the error margin typical in imaging systems with these characteristics, the regions with large deviation still occupy a large area of the lens. However, as these regions are in periphery of the lens, the imaging performance is only slightly affected, as shown in Section 4.6

4.5 Roughness measurement and analysis

The topography measurements detected a waviness throughout the surface of the lenses. This effect was especially pronounced on the diamond turned lens. This section describes the roughness measurement and results. The measurements were made in the Institute of Optics of University of Rochester, NY.

4.5.1 Experimental procedure

The roughness measurements were made using an interferometry optical profiler, NewView 600 from Zygo [23]. The optical profiler evaluates a specific area (approximately 140x 110 μm) over the surface. Using interferometry, the optical profiler evaluates the surface topography on a nanometric scale.

The average roughness R_a and RMS roughness δ are calculated using Equations (4.2.1) and (4.2.2), respectively. The height function of the deviations from the midline of the surface are easier to apply to a plane surface. As the surface substrate is curved, a 4th polynomial function is fitted to the profile over the measured area, and then subtracted from the original function, to obtain a plane substrate. Although the surface is freeform, for such a small region of interest, a 4th degree polynomial provides an appropriate fitting. The values remaining are the profile deviations relatively to a plane surface.

The lenses is placed on a 3D printed holder (Figure 4-21). The lens should be as perpendicular as possible to the optical profiler, and the focus position of the probe should be such that the interference fringes are seen by the CCD camera. Table 1 shows the magnification and the wavelength band. The roughness values were calculated for scale lengths from 1 micron to 50 microns, the lower and upper cutoff spatial wavelength being 1 and 50 microns.

Table 4-4 Specifications for roughness measurement in the optical profiler

Magnification	20x
Wavelength band	475 +- 125nm
Lower Cutoff Spatial wavelength	1 μm
Higher Cutoff Spatial wavelength	50 μm

The measured RMS roughness δ values were used to calculate the Total Integrated Scatter (TIS) of the lenses. As a first order approximation, these calculations were made to provide an estimation of the diffuse light in the optical system of the HMD. The paraxial approximation is used, so TIS is calculated using Equation (4.2.5).

TIS value was calculated for each surface of the samples, and it was estimated the total amount of diffuse light produced by each sample.

The TIS calculation was made using both the dispersion data from the lens material (Zeonex-E48R) and the real spectrum of the light emitted by the display.

The dispersion formula of the Zeonex-E48R is given by [24]

Figure 4-21 Lens prepared to be measured on the optical profiler NewView 600 from Zygo, mounted on 3D printed holder.

$$n^2 - 1 = \frac{1.2969\lambda^2}{\lambda^2 - 0.011721} \quad (4.2.7)$$

The spectrum of the display, according to its manufacturers, has a strong green component. The normalized spectrum $S_n(\lambda)$ can be seen in Figure 4-22.

Figure 4-22 Light spectrum of the display. The spectrum is normalized to its maximum

Using Equation (4.2.5), TIS is weighted with the display spectrum TIS_s is given by:

$$TIS_s(\lambda, \delta) = TIS_{trans}(\lambda, \delta) S_n(\lambda) \quad (4.2.8)$$

The average TIS_{av} is obtained integrating across the spectrum of the display:

$$TIS_{av}(\delta) = \frac{\int_{\lambda_{min}}^{\lambda_{max}} TIS_s(\lambda, \delta) d\lambda}{\int_{\lambda_{min}}^{\lambda_{max}} S_n(\lambda, \delta) d\lambda} \quad (4.2.9)$$

4.5.2 Experimental results

Diamond turned lens (Half-lens)

The roughness effects in the diamond turned lens could be seen not only in the topography measurements presented in Section 4.3 , but also through visual inspection. When using the lenses in the optical system described in Section 4.2 , the image formation was affected by some diffractive effects on the lenses surface, visible to the user. The diffraction effects were different in different samples, so three samples were selected: A "Good" one (no diffraction effects), an "average" one, and a "bad" one. The roughness value of the three lenses was measured across the area of one of the surfaces of the lens.

10 measurements were made over a considerable area of the surfaces. Figure 4-23 shows examples of the 3D topography map obtained by the optical profiler, for each sample. The topography maps show pronounced periodic grating grooves, with a period of 15 μm . The grooves are typical from machinery used for manufacture, created by the diamond tool. The period is the same in the three samples, only the height changes.

Figure 4-23 3D topography map of diamond turned lens samples. (Top) "Good lens" (Bottom left) "Average lens" (Bottom right) "Bad lens"

Table 4-5 shows the summary results for three lenses, showing also the RMS value of the

height variation over the midline.

Table 4-5 Average and RMS roughness values of three diamond turned lenses

	R_a (nm)	Standard Deviation	RMS (nm)	Standard deviation
"Good" lens	18.231	3.012	23.745	5.421
"Average" lens	26.334	2.912	33.251	7.221
"Bad" lens	84.784	2.951	102.241	4.694

The results are consistent with the visual inspection. The "Good" lens presents a $R_a = 18.231 \pm 3.012$ nm, the "Average" one an $R_a = 26.334 \pm 2.912$ nm and the "Bad" one a much higher value, of 84.784 ± 2951 nm. As the standard deviation value reveals, the roughness measurements over the surface were consistent, and is approximately constant over the entire surface. Both surfaces of the presented shown the same results.

Figure 4-24 shows the power density functions (PSD) of the three samples.

Figure 4-24 Power density function of diamond turned lens, in both directions

The peaks in frequency of the three lenses agree. Analyzing the contribution of X and Y directions, we calculate the period of the grating of approximately $15 \mu\text{m}$, which is consistent with Figure 4-23. The "Good" lens also shows a peak around 25 mm^{-1} .

Figure 4-25 displays the nominal TIS and the TIS_s of surface S1 of the three lenses, weighted with the spectrum of the display.

Figure 4-25 Total integrated scatter of surface S1 of the three diamond turned lenses. (*Left*) Nominal TIS (*Right*) TIS weighted with the spectrum of the display S_n . The “Bad” lens presents a very high TIS value, which means that the first order approximation for the TIS calculation is not valid. Still, the graphic is presented to show the difference in order of magnitude between the three lenses

The TIS value of the “Bad” lens is very high. The fact that the value is so large means that the first order approximation of the scatter theory is not valid, because this theory is only valid for low scattering values. Still, the graphics are displayed to show the difference in order of magnitude between the three lenses.

Table 4-6 TIS and TIS_s values diamond turned lenses for surface S1

	TIS (%)			TIS_s (%)		
"Good" lens	4.2	±	1.9	1.9	±	0.8
"Average" lens	8.2	±	3.5	3.8	±	1.7

Table 4-6 shows the values of TIS and TIS weighted with the spectrum of the “Good” and “Average” lenses. The “Bad” lens is not considered, since its high scattering makes the first order approximation inappropriate. The table shows that both the lenses present a rather high value when the nominal TIS is considered, although the “Good” lens presents a 2% TIS when weighted with the spectrum of the display, which makes it more viable for the desired application.

Injection molded lens (Full-lens)

Two samples of the injection molded lens were selected for the measurement. There was no visible diffraction effects on the image formation. The roughness was measured in both surfaces of both samples.

10 measurements were made over a considerable area of the surfaces. Figure 4-26 shows examples of the topography map obtained by the optical profiler, for surface S1 and S2. Both samples displayed similar topography maps for each surface, both the surfaces exhibit a different kind of roughness. While surface S1 was a mixture of scratches and casi-random

microirregularities, probably caused by polishing, surface S2 has a grating grooves, which the period is not as well defined as for the diamond-turned case.

Figure 4-26 Topography map of injection molded samples. (Left) Surface S1 (Right) Surface S2. Both samples displayed similar topography maps for each surface. The black dots on the right correspond to noise data that was removed from the measurement

Table 4-7 Average and RMS roughness values of two injection molded lenses

		R _a (nm)	Standard Deviation	RMS (nm)	Standard deviation
Sample 1	Surface S1	9.212	0.394	12.717	0.912
	Surface S2	6.535	0.748	9.521	1.262
Sample 2	Surface S1	8.988	0.731	13.273	1.053
	Surface S2	7.706	0.019	12.302	0.823

All roughness values are below the 10nm, value typical and usually considered acceptable for this kind of imaging applications. Surface S2 has a slightly lower Ra value than Surface S1, although having the grooves.

Figure 4-27 shows the power density of both samples in both directions. (PSD) agrees with the topography maps. While in surface S1, the roughness is randomly spread throughout the surface, surface S2 presents some mild peaks in low frequencies, from the grooves identified previously.

Figure 4-27 Power density function of injection molded lens, in both directions. (Top) Surface S1. (Bottom) Surface S2

Figure 4-28 presents the TIS values for both lenses.

Figure 4-28 Total integrated scatter of both surfaces of the two injection molded lenses. (Left) Nominal TIS (Right) TIS weighted with the spectrum of the display S_n . (Top) Sample 1 (Bottom) Sample 2

The lenses have a TIS value below 0.7% across the entire spectrum, for both surfaces. Surface S1 has a higher TIS value for both lenses, caused by the microirregularities visible in

Figure 4-26.

Table 4-8 Average TIS values for both surfaces of diamond turned samples

		TIS (%)			TIS _s (%)		
Sample 1	S1	1.21	±	0.17	0.55	±	0.08
	S2	0.67	±	0.17	0.31	±	0.08
Sample 2	S1	1.31	±	0.17	0.61	±	0.09
	S2	1.12	±	0.21	0.52	±	0.09

Table 4-8 presents the average TIS values. The weight values with the spectrum of the display, TIS_s presents values below 0.7% for both surfaces. The injection molded lens present much lower values than the direct cut ones, and less diffuse light can be expected in the optical system.

In a HMD system, the amount of diffuse light can cause discomfort to the user and affect the immersive experience. To assess the amount of diffuse light one can expect in the optical system, the TIS values of both surfaces were considered to estimate the global contribution for diffuse light TIS_{total}, through

$$TIS_{Total}(\lambda) = 1 - [1 - TIS_{S_{S1}}(\lambda)][1 - TIS_{S_{S2}}(\lambda)] \quad (4.2.10)$$

Where $TIS_{S_{S1}}$ and $TIS_{S_{S2}}$ are the total integrated scatter weighted with the spectrum of surface S1 and S2, respectively. The average TIS is obtained using, again, Equation (4.2.9), and the results are presented in Table 4-9.

Table 4-9 Total TIS of injection molded lens

		TIS _{Total} (%)		
Sample 1		0.86	±	0.35
Sample 2		1.12	±	0.36

The total integrated scattered light is around 1% of the total light of the display. Although the calculations were made through a first order approximation, these values provide good estimation of the magnitude of diffuse light in the optical system.

4.6 Performance comparison of manufactured lenses

The measurement of surface topography described in Section 4.4 provides data on the surface deviations for lenses manufactured with two different techniques, injection molding

and diamond turn. This section describes the evaluation of both lenses in the optical system, comparing its performance with the design lens. The performance comparison is made through raytrace, using Code V®. Roughness values were not considered.

4.6.1 Polynomial interpolation of experimental measured points from surfaces

In Code V®, surfaces are described by polynomials, so the point cloud data obtained from the measurements is interpolated using polynomials. The chosen polynomials are XY. In XY polynomials, the sag of the surface z parallel to the z -axis is defined by:

$$z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1+k)c^2(x^2 + y^2)}} + \sum_{j=2}^{66} \mathbf{C}_j x^m y^n \quad (4.2.11)$$

with $j = \frac{(m+n)^2 + m + 3n}{2} + 1$, c is the vertex coordinate, k is the conic constant and $\mathbf{C}_j$ is the coefficient of the monomial $x^m y^n$.

Some peripheral regions of both injected and diamond turned lenses were not possible to interpolate and keep a low error to the real points, so they were removed from the interpolation, in such a way that the remaining regions of the lens can be analyzed.

The interpolation quality was evaluated through the calculation of the RMS value of the difference between the interpolated function to the experimental points. All the interpolations of the surfaces from both lenses had an RMS value of less than $0.2\mu\text{m}$.

The coefficient values of the interpolated surfaces from the injection molded and diamond turned lenses were inputted into Code V. The displacement of surface S2 that was identified in Section 4.4.2 was taken into account, and was also inserted in Code V.

4.6.2 Figure of merit of performance comparison

Section 4.4 showed that the surface error is different for different regions of the lens. This is naturally more common for freeform surfaces than for rotational symmetric ones, and it is interesting to evaluate the performance across the lens, such that the contribution of different regions of the lens to overall performance of the system can be measured.

The configuration of the optical system under evaluation (Section 4.2 allows for such evaluation. Consider Figure 4-29, where it is shown a ray trace of the optical system in Code V, for one fold of the multichannel lens.

Figure 4-29 Ray trace of the optical system in Code V. The fields can be selected such that they intersects the first surface in separated regions that do not have common area. This configuration can be used to evaluate different regions of the separately

The system aperture is defined by the size of the eye pupil, and quasi-parallel fields are traced through the lens and focused on the display, to address to the far field. Due to the relative size and position of the aperture to the lens, each field travels through the lens in very specific regions. This situation is clear in Figure 4-29, where each field intersects the first surface in separated regions that do not have common area. So, by analyzing and comparing the image formation capability of properly selected fields, we may create a performance map of the lens across the surface.

The selected figure of merit is also related to the optical system. As previously explained, the system intends to couple quasi parallel wavefronts into spherical one, with origin in the pixels of display (opixels, as explained in Section 4.2). In reality, the fact that the opixels are not singular points and have a defined size means that the required image formation quality can be related with the opixel diameter d_{op} . The figure of merit is given by:

$$p = \begin{cases} \frac{d_{op}}{RMS_{spot}}, & \text{if } d_{op} < RMS_{spot} \\ 1, & \text{if } d_{op} \geq RMS_{spot} \end{cases} \quad (4.2.12)$$

where RMS_{spot} is the RMS spot size. We call p

A Code V macro was written to trace fields all across the lens aperture (only one of the folds), and a RMS spot size map is create. The opixel diameter is $46.5 \mu\text{m}$, and a monochromatic spectrum of 620nm is used. The injection molded and diamond turned lens are compared to the design lens, and the results are shown in Figure 4-30.

Figure 4-30 Resolution across the field of view of design and manufactured lenses. The resolution is calculated as the ratio between the opixel size of the display and the RMS spot size. When the RMS spot size is equal or smaller than the opixel size, the value is 1

The lens was designed to have a larger resolution for the center fields, due to the fact that the human eye's resolution in the mid and far peripheral vision is much lower than in center and near peripheral vision. The difference is particularly large for high tangential field angles, as the horizontal field of view of the optical system is around 100°, and the resolution in the limits of the field of view is not as crucial. RMS spot size map of the injection molded lens is similar to the one of the design lens: the center fields have a high resolution, but it decreases faster along the tangential axis, and the region of high resolution is smaller. The image formation is slightly affected for the center fields. Figure 4-20 shows that the surface

deviation on the peripheral regions of the lens is large, reaching several microns of deviation. However, in this system, this deviation does not affect significantly the system, as the resolution requirements are not as demanding as for the center regions. The resolution of the injection molded lens slightly changes in the peripheral regions, compared to design lens. The displacement of the surfaces does not affect significantly the performance of the lens.

The diamond turned lens presents a significantly different RMS spot size map. Although globally better, the center fields present a lower resolution than the ones in the design lens, up to a 30% decrease in resolution. This is interesting because although the surface deviation of the diamond turned lenses is globally better than the injection molded lens (Section 4.4), the deviations in the center regions of the lens affect significantly the performance of the lens in for the field of principal interest.

4.7 Conclusions

Accurate measurements of freeform surfaces are still a standing challenge to the surface metrology community, and have proven to be a genuine bottleneck in their usage in real world applications, especially in imaging systems. Metrology is essential, not only to validate the manufactured parts, but also in providing a real-time feedback to the fabrication process.

This Chapter was dedicated to the experimental characterization of a double channel freeform lens, for a near eye optical system of a Head Mounted Display.

Two different sets of lenses were manufactured: the first set was a full lens, where both channels of the lens were manufactured together, by injection molding. The second set consisted in half lens, where the channels were manufactured separately, by diamond turning. Later on, they are opto-mechanically attached to each other to use them in the near-eye system.

The lenses were measured in collaboration with two different companies. The half lens was measured with OptiPro, using UltraSurf, a 5-axes CMM non-contact system, with a low-coherence interferometry probe, and both half lens and full lens were measured with Zeiss, using UPMC ACC 850 Carat, a 3 axes CMM contact system.

The measurements displayed some limitations of freeform metrology, in particular in multichannel systems, with discontinuities between surfaces. Regions of the lens with large slope and near the intersection between the channels, where the kink between them was convex, were left unmeasured.

The measurements agreed: in the periphery of the lens, where the slopes were higher, both diamond turned and injection molded lens presented large surface deviations, up to 55 μm

and 50 μm , especially in the tangential direction. In the peripheral sagittal region, the injection molded lens presented a larger region with high surface deviation than the diamond turning, in the order of 15 μm . Both lenses present a surface deviation of $\pm 2\mu\text{m}$ in the center regions, although the diamond turned lens had a small region within the center region with a deviation of 5 μm . The measurements revealed that the surfaces injection molded lens had a displacement of 7 μm . Overall, the surface deviation of the diamond turned lens is better than the injection molded one, but this does not reflect in the performance of both lenses.

A performance evaluation of both lenses was made. The point cloud data obtained from the measurements was interpolated using XY polynomials, and the performances of the lenses were analyzed. Using raytracing, the results with the design lens were compared, using a RMS spot size map, which differentiates the performance in different regions of the lens. The results showed that, although the overall surface deviation of the diamond turned lens is better than the injection molded ones, the performance of the last ones is more similar to the design lens. As the image formation requirements are higher in the center regions of the lens, the injection molded lens show a higher performance than the diamond turned ones. The largest surface deviation in the peripheral regions of the lens does not seem to affect the performance in a considerable matter, since the resolution required in this regions is considerably lower than the require for the center regions.

The roughness of both lenses was also measured. The injection molded lens reveled different kinds of roughness in both surfaces, one surface exhibit randomly distributed micro irregularities, probably caused by polishing, while the other surface had grating grooves, probably caused by machinery. Both surfaces had the average roughness values below 10 μm , value appropriate for the application. The diamond turned lenses, however, displayed different roughness values for different samples. Three samples were chosen, according to some visible diffraction effects. All of them reveled highly periodical grating grooves, caused by the diamond tool, but with different average roughness values: 18 μm , 26 μm and 84 μm .

The RMS roughness values were used to calculate the Total Integrated Scattered (TIS) light for both types of lenses, through a first order approximation. The diamond turned reveled, for just one surface, different values for the tree samples. The sample with the highest roughness value had a TIS value too high to be evaluated through the first order approximation, and the estimation of the value is above 30%. The two other samples had a TIS value of 1.9 ± 0.8 (%) and 3.8 ± 1.7 (%), when weighted to spectrum of the display. The injection molded lens presented better and more similar values, with TIS values weighted to the spectrum of less than 0.7% for each surface, and around 1% when considering the contribution of both lens to the overall scattered light.

The lens were characterized and its performance evaluated through simulation. Both simulations and TIS calculation indicate that the injection molded lens have a higher performance than the diamond turned ones. The current limitations of freeform metrology prevent the complete measurement of the lenses topography. Some regions of high slope were not measured by the optical probe, and the region near the kink between the channels of the lens was not measured. This can be a crucial point since, depending on the multichannel configuration, the regions between the discontinuities may be optically relevant, and its measurement is crucial for the correct evaluation and performance of the lenses.

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CONCLUSIONS AND FUTURE WORK

Conclusions

The objectives of this thesis were the design and development of multichannel optical systems, both for imaging and non imaging applications. The development was pursued following three different paths: (i) optical design of Concentrated Photovoltaics (CPV) concentrator, combining a 9 channel approach with Köhler integration (Chapter 2); (ii) development of a new method to design diffractive surfaces, a type of multichannel optics where the control of light coming from each channel is achieved through interference and phase control (Chapter 3); (iii) experimental characterization of a double-channel freeform lens for a head mounted display imaging application (Chapter 4).

Chapter 2 has been devoted to the design of a CPV system, the 9-fold Fresnel-Köhler. The design is thought to pave the way for high efficient CPV devices with 4, 5 and 6 junction cells, and it is prepared to fulfill the expected requirements of these cells. The Chapter shows that the concentrator fulfills all the desirable properties in CPV optics: high concentration factors, large acceptance angle, high optical efficiency and spatial and spectral uniform irradiance distributions on the cell.

This Chapter also introduces to a new concept to avoid light crosstalk between optical channels, a Fresnel Lens with Variable Focal Point (FL-VFP). Multichannel systems always have to struggle with optical crosstalk between channels. The fact that the incoming bundle of light is divided into multiples channels has the potential to increase the acceptance of the system. However, when using a larger number of channels, the optical crosstalk between

channels becomes the limiting factor for an even higher concentration-acceptance angle product (CAP). In FK9, although the concentrator has already a high CAP, the major limitation to further increase this quantity is the crosstalk between different channels, which occurs when the system is at a considerable off-axis angle. Crosstalk happens between sub-lenses of the SOE, especially for rays that are refracted in areas of the Fresnel lens which are farther away from the optical axis. The main limiting errors for this situation are the chromatic aberrations of Fresnel lens. FL-VFP has the ability to control the rays coming from extreme points, i.e., apart from the optical axis and, as a consequence, to increase the CAP of the system. Although the focal point is variable, it varies along a line, such that each teeth of the lens is still rotationally symmetric.

FK9 and FK9 with FL-VFP concentrators present a CAP of 0.62 and 0.66, respectively, and are among the highest reported for concentrators based in Fresnel lens. A comparison of CAP with other concentrators with excellent uniformity is shown in Chapter 2.

High concentration and ambient temperature variation in Fresnel lens create chromatic aberrations. Most concentrators are affected by them, and the spectral irradiance distribution in different junctions gets extremely non-uniform, affecting considerably the power production of the multijunction cells. Silicon on Glass (SoG) lenses are especially affected by ambient temperature variations. Chapter 2 shows a comparative study between FK9 and other concentrators known for having a uniform irradiance at specific concentrators. The systems were designed with SoG lens. The study shows that FK9 is the only concentrator prepared for extreme conditions, as it is the only one that maintains the uniformity in spectral distributions for both high concentration ($C_g=2000\times$) and high ambient temperature ($T=45^\circ\text{C}$). The capability to maintain a uniform spatial and spectral distribution, combined with the high CAP, proves that FK9 is prepared to meet the expected requirements of 4, 5 and 6 junction cells. In these cells, the spectral balance between junctions is increasingly important, and they are likely to require higher concentration. FK9 is an excellent candidate for CPV systems with 4, 5 and 6 junction cells.

Chapter 3 presents a new method to design diffractive surfaces. The Simultaneous Multiple Surface (SMS) method (a geometrical optics design with a long history in nonimaging optics and recent new applications in imaging optics) is extended for designing diffractive surfaces. The diffractive SMS method constructs the optical system by designing simultaneously diffractive and refractive/reflective surfaces. Using the phase-shift properties of diffractive elements as an extra degree of freedom, the method controls two rays for each point on each diffractive surface, in opposition to refractive/reflective surfaces, where only one ray for each point can be controlled.

The concept of generalized diffractive oval (GDO) is introduced, as a freeform diffractive

surface with the ability to couple two incident wavefronts. A complete analytical description of the surface is obtained. Different design examples are provided, such as freeform monochromatic diffractive oval coupling two wavefronts, and a freeform bichromatic diffractive oval, coupling two wavefronts with different wavelengths. For these examples, all the design wavefronts are perfectly coupled, creating perfect imaging (neglecting the Airy disk diffraction limit). An interesting result of the freeform bichromatic diffractive oval is the fact, not only the design wavelengths present a perfect imaging, the spectrum band between them also presents very low RMS spot sizes.

The diffractive SMS method calculates different portions of the diffractive surfaces by a sequential application of the GDO procedure. To design simultaneously diffractive and deflective surfaces, GDO (for diffractive surfaces) and Generalized Cartesian oval (GCO) (for deflective surfaces) are calculated simultaneously and sequentially. A 2D example of a hybrid (1 diffractive and 1 refractive surfaces) lens, designed with the diffractive SMS, is provided, coupling three wavefronts of two different wavelengths. The designs points were perfectly imaged, and the RMS curve of the design points is typical of aplanatic systems. The most interesting result is a set of combinations of field angle/wavelength that, even though they were not controlled, present a very low RMS spot size value, forming *ring-shaped* RMS spot size map (with one axis representing the field and the other the wavelength).

The diffraction efficiency of the designs examples were analyzed, based on the extended scalar theory. The study showed that, even though SMS-based systems are designed for multiple wavefronts, there are no relevant differences on the diffraction efficiencies for the different wavefronts. The SMS-based diffractive systems do not need require any more caution than other diffractive designs to account for diffractive efficiency effects.

The diffractive SMS proved to be a reliable method, which can be use as a direct method for the final design, or as good starting point for further optimization. Its potential to design freeform diffractive surfaces may have an impact in imaging and spectroscopy applications, among others, by decreasing size, weight, number of elements of the optical system, and correcting for chromatic aberrations.

Chapter 4 is dedicated to the experimental characterization of a double channel freeform lens, for a near eye optical system of a Head Mounted Display. The lenses were manufactured both using diamond-turned and injection molding technique, and their topography and roughness are measured and compared, and the Total Integrated Scatter light was calculated. The lenses were measured in collaboration with two different companies. The half lens was measured with OptiPro, using UltraSurf, a 5-axes CMM non-contact system, with a low-coherence interferometry probe, and both half lens and full lens were measured with Zeiss, using UPMC ACC 850 Carat, a 3 axes CMM contact system.

Using the point cloud data obtained from the measurements, the performance of the lenses performance was analyzed using raytracing, and compared to the design lens.

The injection molded lens, although having a worse overall surface deviation and a slight displacement between the surfaces, has a better performance and a similar one to the design lens than the diamond turned one. The largest surface deviation of the injection molded lens was in its periphery, and the fact that in the center the deviation was in the range $\pm 2\mu\text{m}$, the region where the highest resolution is demanded, ensured that the injection molded lens had a higher performance, appropriate for the optical system for which it was designed. This work highlighted the importance of defining correct tolerances for different regions of the optical components, to find a balance between precision across the entire surfaces of the lens. As some freeform components may have "wild shapes", the correct tolerance definition across the entire surface can be decisive in the trade-off between performance/cost of manufacture.

As a result of this doctoral thesis, the multichannel configurations were further explored: a high performance CPV system was designed, with the potential to pave the way to 4, 5 and 6 junction cells-based systems; a new direct (no optimization) and analytical method for designing simultaneously deflective and diffractive surfaces was developed; a experimental characterization of a double-channel freeform lens was performed, using cutting-edge freeform metrology.

Future work

Futures lines of research of this thesis are directly related with the results of each Chapter.

FK9 was thought for four, five and six junction cells. The first four-junction cells have recently been developed, and already achieved record efficiencies of 46%, as shown in Section 2.1.1 . These cells are not commercially available, and information about the spectral balance between junctions is not available as well. Plus, as a technology under development, the manufacturers still have to optimize the appropriate material thickness, among other parameters. Nevertheless, the first CPV minimodule using four junction cells was recently demonstrated by Fraunhofer ISE (Section 2.1.1), setting a new world record efficiency of 43.3%. The natural line of development would be to manufacture a FK9 prototype with four-junction cells for different concentration levels, and measure its performance on efficiency and power production for different solar spectrums and different temperatures.

Another line of development in the FK concentrators family would be to further increase the number of channels, which could further increase the CAP level. The optical crosstalk between channels would also increase, and the CAP level increase is expected to be

mitigated by the crosstalk. A further expansion of FL-VFP to address the crosstalk problem might enhance the CAP level. A suggestion is to redesign the Fresnel teeth such that the focal point varies on a region instead of just a line, by removing the rotationally symmetric imposition for each teeth. This design, although optically compelling, might be too expensive to be cost-effective.

An interesting research option would be to explore FL-VFP or similar strategies in other multichannel systems, to address to the cross talk issue.

Regarding the diffractive SMS described in Chapter 3, the next logical steps would be to design a freeform hybrid system, as the one presented in this thesis, by extending the method from two to three dimensions. Also, the method can be extended for the simultaneous design of two or more diffractive surfaces. This extension can be especially interesting, due to the fact that a diffractive doublet enables an extension of the diffraction efficiency for a wider spectrum range. Another interesting line of research would be to explore the possibility of designing simultaneously the surface relieves of different diffractive surfaces, to optimize a system for two or more wavelengths. Diffractive SMS systems revealed interesting RMS spot sizes maps, when considering the distribution in both the wavelength and the field angle. The hybrid SMS system forms a thought-provoking ring-shaped RMS spot size map. A future line of research would explore and understand the mathematical reasons behind this RMS spot size distribution.

PUBLICATIONS

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1. **J. Mendes-Lopes**, P. Benítez, and J. C. Miñano, "Simultaneous Multiple Surface Method for diffractive surfaces," *Opt. Express* **24**, 5584-5590 (2016)
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Patents

1. "Concentrator for polychromatic light", Inv: P. Benítez, J.C. Miñano, P. Zamora, **J. Mendes-Lopes**, J. Chaves, R. Mohedano, WO2013158634 A1, 16 April 2012

Oral presentations in international conferences

1. J.C. Miñano, **J. Mendes-Lopes**, B. Narashimhan, P. Benítez, "Recent advances in the SMS method: Diffraction and 3D aplanatism", Nonimaging Optics: Efficient Design for Illumination and Solar Concentration XIII: Commemorating the 50th Anniversary of Nonimaging Optics, August 2016, **(invited talk)**
2. M. Nikolic, P. Benítez, J. Liu, **J. Mendes-Lopes**, B. Narasimhan, J. C. Miñano, "Design of compact optical systems using multichannel configurations", Novel Optical Systems Design and Optimization XIX, SPIE Optics + Photonics, San Diego, California 29-30 August 2016 (to be presented)
3. **J. Mendes-Lopes**, P. Benítez and J. C. Miñano, "Design of diffractive optical surfaces within the nonimaging SMS design method", SPIE Optical System Design, Jena, Germany, 7-10 September 2015
4. **J. Mendes-Lopes**, P. Benítez and J. C. Miñano, "Design of diffractive optical surfaces within the SMS design methods, Nonimaging Optics: Efficient Design for Illumination and Solar Concentration XII, SPIE Optics + Photonics, San Diego, California 9-10 August 2015 **(Invited talk)**
5. **J. Mendes-Lopes**, P. Benítez and J. C. Miñano, "Design of diffractive optical surfaces

within the SMS design method”, Freeform Optics 2015 Arlington, Virginia United States 7–11 June 2015

6. **J. Mendes-Lopes**, P. Benítez, P. Zamora and J.C. Miñano, "9-Fold Fresnel-Köhler concentrator for increased irradiance uniformity on high concentrations", *at*: High and Low Concentrator Systems for Solar Electric Applications VIII, ,SPIE Optics + Photonics, San Diego, USA, August 2013
7. **J. Mendes-Lopes**, P. Benítez, P. Zamora and J.C. Miñano, "New Fresnel-Köhler optical design: 9-Fold photovoltaics concentrator", *at* Nonimaging Optics: Efficient Design for Illumination and Solar Concentration XII, SPIE Optics + Photonics, San Diego, USA, August 2013
8. **J. Mendes-Lopes**, P. Benítez, P. Zamora and J.C. Miñano, "9-Fold Fresnel-Köhler concentrator for increased irradiance uniformity on high concentrations", *at* : 9. International Conference on Concentrator Photovoltaics Systems (CPV-9), Miyazaki, Japan, April 2013

POSTER PRESENTATIONS

9. **J. Mendes-Lopes** ; P. Benítez ; P. Zamora ; J. C. Miñano; "9-Fold Fresnel-Köhler Concentrator for Increased Irradiance Uniformity on High Concentrations", Proceed. CIOMP-OSA Summer Session on Optical Engineering, Design and Manufacturing, Changchun, China 2013