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ESCUELA TÉCNICA SUPERIOR DE INGENIERÍA Y
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*CONTRIBUTIONS AND REFLECTIONS TO
THE STUDY OF STABILITY AND
OSCILLATION OF ACTIVE ANTENNAS*

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THE STUDY OF STABILITY AND
OSCILLATION OF ACTIVE ANTENNAS***

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Dissertation for the Degree of Doctor of Philosophy

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ABSTRACT

This thesis was carried out in the *DIEMAG* Group and *TSC* department of *ETSI y Sistemas de Telecomunicación* from *Universidad Politécnica de Madrid*. Its title is “*Contributions and Reflections to the Study of Stability and Oscillation of Active Antennas*” and it has been developed by Ángel Parra Cerrada, Electrical Engineer MSc, under the supervision of Prof. Vicente González Posadas, PhD and Prof. José Luis Jiménez Martín, PhD.

The current interest on active antennas is very important, mainly on small antennas and integrated antennas. The patch antennas are maybe the type of integrated antennas more used at present designs. The design of active patch antennas must always include a study of stability, because they must be stable for amplifier function or because they must oscillate in a proper way for TX-oscillator, mixing-oscillator, etc.

The fundamental aspects of analysis of patches and their modes are covered in a general review. The modes and internal fields of the patches are fundamental for the proper understanding of the radiation diagram, impedance at the feed point, interaction with active elements and compatibility with the feeding uses for active functions.

The main problem to study the stability of any circuit is as several authors have pointed out that “*there is not a compact formulation of an oscillation criterion that is both necessary and sufficient*”. The linear methods for the study of the stability of the RF circuits are reviewed and the conditions for their proper use, limitations and problems are exposed and summarized. The presentation of a new method based on the NDF for the linear stability analysis is a significant contribution of this work. This proposed method is used for several designs of active antennas and compared with classic linear methods, non-linear simulations and measurements. These comparatives state the NDF Method as correct predictor of stability, but not the classic linear methods. The new proposed NDF Method is a compact procedure that is necessary and sufficient criterion on the study stability.

There are multiple elements that can be used to *tune* or control the impedance of the patches. Two of these elements are the metamaterials (passive) and the Negative Impedance Converters, NIC (active). The inclusion of these elements on active patches and their implications on the stability of active patches are studied using the NDF Method and the classic methods. The results outcome that the use of NICs built with discrete transistors is a high risk for the stability.

The proposed NDF Method opens a new way for the research of new oscillators, active antennas and low phase noise oscillator topologies. The lack of additional conditions (*proviso*) for the use of the NDF Method provides it the advantage of being a universal method for studying the linear stability of any topology of oscillator or active antenna.

The thesis covers in detail the field of the stability of integrated active patch antennas. This study of the stability is disseminated with multiple publications as it is detailed in the contributions section.

RESUMEN

Esta tesis ha sido realizada en el *Grupo DIEMAG* y en el *Departamento TSC* de la *ETSI y Sistemas de Telecomunicación* de la *Universidad Politécnica de Madrid*. Su título es “*Contributions and Reflections to the Study of Stability and Oscillation of Active Antennas*” (Contribuciones y Reflexiones al Estudio de la Estabilidad y Oscilación de Antenas Activas) y ha sido realizada por Ángel Parra Cerrada, Ingeniero de Telecomunicación (MSc), bajo la dirección de el Prof. Dr. Vicente González Posadas y el Prof. Dr. José Luis Jiménez Martín.

Hoy en día existe un gran interés en el campo de las antenas activas, especialmente en las antenas integradas y de pequeño tamaño. Posiblemente, las antenas de parche sean, al día de hoy, las antenas más utilizadas en los nuevos diseños. El diseño de antenas activas siempre debe incluir un análisis de su estabilidad, ya que esta debe ser estable para aplicaciones como la amplificación e inestable de forma controlada para otras como oscilador, mezclador-oscilador, etc.

Los principales aspectos del análisis de los parches y de sus modos se cubren en una presentación general de estos. Conocer los campos internos de los parches es imprescindible para poder tener un correcto entendimiento de los diagramas de radiación, impedancia de entrada, interacción con los elementos activos y compatibilidad con puntos de acoplo al parche usados por los elementos activos.

El mayor problema para el estudio de la estabilidad de cualquier circuito es, como declaran diversos autores, “*no existe una formulación compacta del criterio de oscilación que sea condición necesaria y suficiente*”. Se realiza una revisión de los métodos clásicos de análisis lineal de la estabilidad de circuitos de RF y se enumeran y resumen sus condiciones para un uso apropiado, limitaciones y problemas. Una importante aportación de esta tesis es la presentación de nuevo método lineal de análisis de estabilidad basado en la NDF. El método propuesto es utilizado en el diseño de varias antenas activas; y los resultados de este análisis, de los análisis con los métodos clásicos, de los análisis no lineales y de las medidas son comparados. Estas comparaciones demuestran que el método basado en la NDF es una herramienta correcta para el análisis de estabilidad, mientras que los métodos clásicos no lo son. Este nuevo método basado en la NDF es un método compacto que es un predictor necesario y suficiente para el estudio de la estabilidad.

Existen múltiples elementos que pueden ser utilizados para ajustar y controlar la impedancia de los parches. Dos de estos elementos son los metamateriales (pasivos) y los Conversores de Impedancia Negativa, NIC (activo). Las implicaciones y efectos de incluir estos elementos en parches activos se han estudiado utilizando el método basado en la NDF y los métodos clásicos. Los resultados demuestran que los NICs realizados con transistores discretos presentan un elevado riesgo de inestabilidad.

El método basado en la NDF que se ha propuesto posibilita nuevos caminos para la búsqueda de nuevas topologías de osciladores, antenas activas y osciladores de bajo ruido. El no requerir ninguna condición previa (*proviso*) para utilizar el método basado en la NDF, le confiere a este la ventaja de ser un método universal para el estudio de la estabilidad válido para cualquier topología de osciladores y antenas activas.

En esta tesis se desarrolla en detalle el estudio de la estabilidad y antenas activas. Este estudio de la estabilidad se ha difundido ampliamente con múltiples publicaciones, tal como se detalla en el capítulo de Contribuciones.

*A Sonia,
por su insistencia y
paciencia.*

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PREFACE

The scope of this thesis is active antennas, mainly the oscillating patches. The discussion principally focuses on stability, but not only on this topic. The study of the stability of oscillators and the application of the conclusions on the methods for linear stability study is the thread of the dissertation. On this dissertation, the focused type of active antennas is patch antennas.

The first element to study is the patch antenna (Chap. 1). The patches consist of a patch that is capable of radiating on one side of a dielectric substrate with a ground plane on the other side. The substrate normally has a low dielectric constant and in some cases air is used as dielectric. A low ϵ_r increases the fringe fields that are the radiation source of the patch. The microstrip patch antennas present a large list of interesting advantages such as: lightweight, low profile, small in size, suitable for linear and circular polarizations, suitable for conformal structures, several radiation diagrams on a single patch, multi-band, etc. The fields and current distribution inside the patches can be very difficult to calculate, but for a first approximation and easy understanding approximations can be used. The two most used approximations are the transmission line model and the cavity model. Then, the sides of the patch are considered as narrow slots that can radiate energy to calculate an approximation of the radiation diagram of the patch antenna. The cavity model is very useful to study and understand the modes of the patches. The cavity model considers a magnetic wall around the patch, so only TM modes are possible. Then, the fields near to the edges of the patch are considered for studying the radiation of the patch.

Once the antenna element to be used is studied, the next aspect to cover is the stability of active circuits (Chap 2). The linear study of the stability is a fundamental step in the design of any RF circuit. The circuit must be stable, or unstable in a controlled way. Examples of stable circuits are amplifiers, mixers, etc.; and the controlled unstable circuits are the oscillators. It is important to point out that the internal and external stability are different concepts and that the internal stability is the required condition for stability. A circuit can have an internal instability that is not observed from the external ports, so it seems to be stable, but it does not work properly. A typical example of this type of instability is the ring-oscillator. An important conclusion of internal stability is that it implies external stability of system, but the converse is not true. The internal stability is guaranteed if all the roots of the characteristic function have a negative real part, so there are not any roots of the characteristic function on the Right Half Plane (RHP). There are multiple methods to analyze the stability during the lineal design step, but the linear solution must be checked on a non-linear step that requires much more computing power and time. The current problem is that there is a lack of confidence in the linear analysis; because, very often, problems with stability are detected on non-linear analysis, and they were not previously detected on the

linear design step. These *discrepancies* are the motivation for the review of the linear stability issues that are covered in this work. After the study of the stability, the classic methods for linear study of stability are classified and their provisos are summarized.

Another interesting passive elements to include in a design with patches are the metamaterials (Chap. 3). The basic approximation of a 1D metamaterial is a Transmission Line (TL) with a Left Handed (LH) behaviour. The Metamaterials are those materials that have a negative ε and a negative μ at the same time. These 1D metamaterials make possible the impedance matching and tuning of the patches for including several bands or functions. Some of their most relevant applications are functions such as hybrids, diplexers, etc. The combination of Left Handed and Right Handed behaviours on a single TL are the defined as Dual Composite Right/Left Hand, Tri Composite Right/Left Hand, etc. These models of metamaterial transmission lines make possible the impedance matching of a circuit or patch for several frequency bands.

The oscillators (Chap. 4) are fundamental elements for all RF and microwave systems, so the oscillators are the main active circuits that are studied on this work for its integration into active antennas, but other interesting active circuits are also studied. The linear design as a first approximation is widely used for RF and microwave oscillator design. This linear approximation is very useful because it needs less computational resources than the non-linear analysis and it is appropriate for the understanding of the oscillator. Simple linear models speed up designs and they give the chance of looking for new oscillator topologies. The linear design methods for analysis of oscillators can be divided into two main groups: Loop-Gain and Reference Plane. The classic linear design methods predict the oscillation frequency and gain margin, but only the Loop-Gain can predict the quality factor (Q_L). The use of each design method is conditioned by the circuit topology and the possibility of using the Virtual Ground. This is a limitation when the circuit includes distributed components as patches or transmission lines. An important handicap of the classic linear methods is that they require some conditions to be satisfied, *provisos*, before they can be used. If the provisos are not satisfied, the classic methods can provide wrong solutions. Another important problem is that the classic methods have different solutions for the same circuit.

A significant contribution of this thesis is the presentation of a novel method for linear analysis of stability: *NDF Method* (Sec. 4.4). This method is based on the use of the NDF (Normalized Determinant Function) as single tool for the linear study of stability of any circuit. The proposed NDF Method also makes possible the optimization of the phase noise for any oscillator topology, so it is a useful tool for the research of new low noise oscillator topologies. Then the study and

methodology, including provisos, of the classic linear methods and the proposed NDF Method are applied to the design of oscillators and active antennas.

Another interesting active function covered in this thesis is the Negative Impedance Converter (Chap. 5). The metamaterials are based on resonance, so their response is narrowband. NICs have a broad bandwidth range of use, they are applicable for group delay or tune frequency response. So, NICs present a new behaviour that is impossible to obtain with passive structures. They are suitable to be used on distributed amplification to overcome loss, reduction of equivalent resistance on patches and resonator, etc. The NICs are non-Foster devices so they do not comply with Foster's reactance theorem. They are used to overcome one of the fundamental limitations of Electrical Small Antennas (ESAs): the impedance bandwidth.

The previous studies and conclusions are applied to the study of Active Patch Antennas (Chap. 6). The main advantage of the active microstrip patch antennas is that they are easily integrated with active RF circuits. The integration is so deep in a lot of cases, that there is not any reference plane (typically defined with an impedance of $50\ \Omega$) between the radiating element and the active circuit. The output, or input, port of an active patch antenna is not a conventional $50\ \Omega$ interface, the port is the free space. The study of the fields inside the patch has a great importance because the integration of active devices "inside" the patch and the feed must be considered to ensure the compatibility with the fields of the patch. The active functions that may be integrated on patches are: oscillator, power amplifier, LNA, switcher, NIC, etc. It is possible to integrate in an active patch from a single function up to a complete RF transceiver. The problem of the study of the stability of active antennas is presented and the conclusions from previous studies are applied. Then, some examples are used to illustrate and demonstrate the classic analysis methods and the use of the NDF Method as a tool to study the stability of active patch antennas.

Chapter 1

PATCHES

1.1 Introduction to Patches

A simple and general definition of patch antenna is: *it consists on a patch that is capable of radiating on one side of a dielectric substrate with a ground plane on the other side*, Fig. 1.1. This substrate use to be $\epsilon_r < 10$ and in some cases it is air. The ϵ_r use to be low to increase the fringe fields that are the radiation source of the patch. There are antennas that do not have a ground plane, they are not patches but printed antennas. Some of these printed antennas are printed dipoles, slot antennas, traveling-wave antennas, etc. The patch material is a conductor, usually gold or copper and the shape can be any one, but regular shapes are generally used. The substrate use to have $\epsilon_r \leq 2.5$, but it can be materials with different properties as metamaterials or a stack of different dielectrics. The concept of patch antenna was given by Deschamps [Des53] but the first practical models took over 20 years to be available [How72]. The Spanish group *CSIC-Distcom* whos head investigator is Carlos Martín, PhD contributed with relevant works on the patch antennas field. These contributions could be summarized in 2001 AIC Congress, Rhodes [MGR01]. This chapter expects to be a overview of some of the contributions of *CSIC-Distcom Group*.

Figure 1.1: Patch Antenna

The main advantages of microstrip antennas include:

- They are lightweight.
- They are small in size and low profile.
- Both linear and circular polarizations can be obtained.
- Conformal structures are possible (it is easy to form curved surfaces and to vaporize metal over a dielectric).

- They can work in multi-band of frequencies (the use of metamaterials provides more flexibility to control the frequency).
- Several radiation diagrams can be achieved by a single antenna (using the “Modes” and shapes).
- Can be made compact (integrating the electronic).
- Low cost to manufacture (ease for mass production using the printed circuits).

Some of the problems referenced in the literature, but that are considered to be wrong on nowadays works are:

- Limited bandwidth (usually 1 to 5%). But it is possible to achieve up to 100% with thicker substrate, increasing complexity, using active devices or metamaterials [MGR01].
- Low power handling. But the design of arrays, new substrate materials and the active patches minimize losses.
- Somewhat lower gain ($\sim 6\text{ dB}$), but several shapes as rings increase the gain, up to 11 dB , and active patched reduce the losses.
- High purity on polarization is difficult to achieve, but different dielectric materials multilayer PCB and high precision manufacturing allow good purity, up to 40 dB [MGR01].

The modeling of the patches for the first linear design step of active patches is performed by using the cavity model. The models and works used for the study and modeling of patches during this work are mainly based on the works and contributions of *Consejo Superior de Investigaciones Científicas (CSIC) - Instituto de Teledetección y Telecomunicaciones*.

1.1.1 Radiation Mechanism

To calculate the fields or currents distribution on a patch can be a very complicated work, but simple approximations can be used to understand the radiation of a patch. The basis ideas of two of the most common approximations are outlined in this section.

The radiation of a patch can be illustrated on a simple way with the radiation of a *microstrip* antenna. The fields in a microstrip rectangular patch are similar to the ones on a microstrip line, but the patch that is usually built on a substrate with lower ϵ_r . The example of a rectangular patch is used to illustrate the basic ideas of these two approximations, Fig. 1.2.

Figure 1.2: Rectangular Patch Antenna

The first one considers the four sides of the patch as narrow slots that can radiate energy. The patch can be represented by an equivalent current density at the top surface using the Huygen field equivalence principle. These four slots can be represented by their equivalent currents $\vec{M}_s$ and $\vec{J}_s$ that have their equivalent magnetic and electric fields in the slots, Equ. 1.1.

$$\begin{aligned}\vec{J}_s &= \hat{n} \times \vec{H}_a \\ \vec{M}_s &= -\hat{n} \times \vec{E}_a\end{aligned}\tag{1.1}$$

For thin substrates, the the current at the bottom of the patch can be considered zero; and the tangential magnetic fields on the edges of the patch can also be considered as zero. Then, the unique current that is not considered zero is the magnetic current along the periphery of the patch, $\vec{M}_s$. These magnetic currents are equivalent to one magnetic dipole at each edge of the patch. The far field radiation produced by the two slots directed on the x axis is 0, because they are the same magnitude but opposite direction. Then the far field is the result of the radiation of the two magnetic dipoles along the y axis.

The second approximation considers that the microwave excitation produces movements of charges on the bottom of the patch and a small amount of current on the upper face of the patch. The $\vec{E}$ fields and the zooms of the edges of a

transversal section of a circular ring with a central short-circuit are presented for the two first modes in Fig. 1.3, where the fields that extend out of the patch edge (fringe fields) can be observed. These movements of charges produce current densities $\vec{J}_b$ on the bottom of the patch and $\vec{J}_t$ on the top of the patch. When the substrate is thin most of the charges and current are concentrated at the bottom of the patch, so it is possible to make the approximation of a magnetic wall around the patch. Then the patch is modelled as electric walls (up and down) and magnetic walls around, so only TM modes are possible in this cavity model. The fringe fields that are present near to the edges of the patch must be considered to study the radiation of the patch.

Figure 1.3: E Field on a Circular Ring Patch Antenna with Central Short-Circuit

1.2 Microstrip Antennas Configurations and Feeding

The configuration and characteristics of microstrip antennas are controlled by a large number of physical parameters. They are grouped in three sets: shape geometry, substrate (characteristics, stacked, etc.) and feeding.

Some authors [JE89] classify the microstrip antennas in four categories :

- microstrip patches
- microstrip dipoles
- printed slots
- microstrip traveling-wave

On this chapter only microstrip patches are covered, but the rest of microstrip antennas can be considered in a similar way as the type used as example. The microstrip patches can have any shape, but the basic shapes used on practice are in Fig. 1.4. The basic shapes are widely used and there are empirical and analytical equations for an approximated design. The rectangular, circular and circular ring shapes are the models under the scope of this work. These models are widely used on practical designs but most of the others are not, they are used mainly on academic publications.

Figure 1.4: Patch Antenna Shapes

Typical basic microstrip patch antennas (MPA) have a gain around 6 dB and a dipole type radiation diagram (which is the most common for basic typical patches) with beam-width on the range from 90° to 70° . But more complex stacked substrates or patches can provide greater gains or wider beam-widths.

The patches are on one side of a dielectric substrate with a ground plane on the other side. The feeding can be directed or coupled. Some of the most relevant coupling techniques are: coaxial, microstrip, coplanar, aperture coupled, proximity coupled and coplanar waveguide. The feeding technique has high importance and must be compatible with the desired mode of the patch. The coupling mode is the most important element to control the impedance and the efficiency of the patch antenna. The transfer impedance relation, the return losses and the transmission coefficient are vital parameters when the patch is used as a two port element on a traveling-wave antenna or on an oscillator.

The typical model used as equivalent circuit of a patch at the resonant frequency is the parallel R-L-C circuit. This equivalent circuit models the bandwidth (Q of the resonant circuit) and the radiation power (R, which includes the ohmic losses). As an example, a circular ring with central short-circuit ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$), Fig. 1.9, that includes the coaxial feed, Fig. 1.7, has for the Mode 1 the R-L-C equivalent circuit in Fig. 1.5. This equivalent R-L-C circuit has the same S_{11} that the patch for frequencies near the Model resonance, Fig. 1.6. The low height of the substrate makes these examples to have low radiation efficiency, but it has been chosen for simplicity in the examples, because they are focused on the impedance of the feeding.

Figure 1.5: Equivalent R-L-C Circuit of a Circular Ring with Central Short-Circuit and Coaxial Feed ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

Figure 1.6: S_{11} of (Patch and Equivalent R-L-C Circuit) of a Circular Ring with Central Short-Circuit and Coaxial Feed ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

1.2.1 Coaxial Feed

The coaxial feed coupling method is one of the basic methods that is widely used. The most typical structure is a coaxial line or a connector which inner conductor is used as a line to feed the patch, while the outer conductor of the coaxial line is connected to the ground plane, Fig. 1.7. The current on a circular ring patch with central short-circuit and coaxial feed is shown in Fig. 1.8.

Figure 1.7: Coaxial Feed

Figure 1.8: Current on Coaxial Feed of a Circular Ring with Central Short-circuit

The position of the feed point is a key parameter for the impedance of the patch, but also the wide of the inner conductor and the height (length of the feed) of the dielectric of the patch. The coupling (C) is function of the distance to the patch edge (x_0) and of the resonant length (L), according to the mode of the patch. When the dominant E_z field is considered, the coupling can be approximated as Equ. 1.2. The feeding also introduces a parasitic effect, which can be modelled as a parallel L-C circuit, where the most present effect at working frequencies is the inductive. This inductive effect may be compensated with a series capacitor. This capacitor may be a lumped or a distributed one. A possible distributed capacitor is a ring, in the patch, around the feeding point, a coupling disk spaced from the patch with a dielectric.

$$C(x_0) = \int \int \int E_z J_z dv \simeq \cos(x_0\pi/L) \quad (1.2)$$

To illustrate the impedance variation with the feeding point two circular patch rings are simulated, Fig. 1.9. One circular ring has a central open-circuit and the other has a short-circuit. They are built on a $\epsilon_r = 2.5$ substrate with a thickness of 0.8 mm . The external radius of the ring is 20 mm and the internal one is 3 mm . The substrate and ground plane are considered finites with a radius of 40 mm .

Figure 1.9: Model of Circular Ring with Central Short-Circuit

The real part of the impedance is presented for a sweep of positions of the feeding probe. Fig. 1.10 presents the real part of the impedances versus frequency for the ring patch with a central open circuit and Fig. 1.13 for the ring patch with a central short-circuit. The real part of the impedance is presented, for each mode of the patches, using the distance, in mm, of the coaxial feeding to the internal edge as parameter, Fig. 1.11 and Fig. 1.12. It can be observed that the relation of the impedance with the feeding position is not so easy as the simple model of Equ. 1.2 predicts.

Figure 1.10: Magnitude of Impedance of Circular Ring with Central Open-Circuit ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

Figure 1.11: Real Part of Impedance of Circular Ring with Central Open-Circuit vs. Feeding Point in mm. ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

Figure 1.12: Real Part of Impedance of Circular Ring with Central Short-Circuit vs. Feeding Point in mm. ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

Figure 1.13: Magnitude of Impedance of Circular Ring with Central Short-Circuit ($R_{in} = 3\text{ mm}$, $R_{out} = 20\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

The impedance is not easy to get when upper modes are considered or when complex feeding are used, but for single input coax feeding and first mode there are formulas widely used [Lie82].

1.2.2 Microstrip/Coplanar Feed

The microstrip feed is a direct connection of the feeding line to the patch on the same layer where the patch is, Fig. 1.14, so they are the most natural feeds for patches. They do not need any via, hole or soldering, but they have some problems. This feeding method may also include a capacitor as a gap between the line and the patch, but this capacitor introduces a reduction of the managed power, that is limited by the dielectric withstanding of the gap in the same way as these gaps limit power for any other feeding that includes them.

The patches present high impedance compared with the typical $50\ \Omega$ impedance of the line. This mismatch may be solved by an impedance matching network, but this network requires a significant space and it produces spurious undesired radiation, but it has a good back radiation. The The required space for the matching network is not available when the antenna is an array of patches, so other more convenient way is used. When this feeding is desired, it can be combined with vias to implement the feeding network on a new back layer behind the ground plane [MGR01].

Figure 1.14: Structure of Microstrip/Coplanar Feed of a Rectangular Patch ($L = 47\text{ mm}$, $W = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

The most used coplanar feeding is an *insertion* of the feeding line into the patch, but other methods are used to control the impedance of a patch without using additional matching elements [Chr04]. Then the microstrip line is changed to a coplanar line “inside” the patch. This coplanar line must be modelled as a grounded coplanar line with finite surrounding ground. The impedance of a patch with the *insertion* of the feeding line is presented for the “Mode 1” in Fig. 1.15 and Fig. 1.16 with the deep of the insertion of the feed in mm as parameter.

Figure 1.15: Magnitude of Impedance of Rectangular Patch ($L = 47\text{ mm}$, $W = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

There are several models to determine the impedance of the rectangular patch [CC05], but complex feeding and effects of feeding network must be considered [WAF92]. So the easy access to full wave 3D simulation is the best option to ensure the radiation diagram and the impedance of the patch.

Figure 1.16: Real Part of Impedance of Rectangular Patch vs. Deep of Feeding Insertion in mm ($L = 47\text{ mm}$, $W = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$)

When the patch is excited on a mode that only has two radiating edges the feeding can be done at one of the non radiating edges. It is the most common feeding when the patch is used as two port element on arrays [BG87]. This feeding on a chain of patches has the problem of generate a high spurious radiation, because the feeding lines became high radiating elements.

1.2.3 Coupled Feed

The coupled feeding methods are a set of patch feed techniques that do not require direct connection of the feeding network to the patch, so the feeding network is on a different layer. This feeding network can be directly coupled to the patch (Sec. 1.2.3.1) or can be coupled by a “hole” or slot on the ground plane that is between the patch and the feeding network (Sec. 1.2.3.2). The structure that uses a slot minimizes the effect on the radiation of the feeding network in the main direction of the antenna, and if no-resonant slot is used the back radiation can be controlled.

1.2.3.1 Proximity Coupled Microstrip Feed

The structure of the patch and feeding is a pair of dielectric layers with a ground plane on the bottom, microstrip lines between the two dielectrics and a patch over the upper dielectric layer, Fig. 1.17.

This structure usually has a thin dielectric on the microstrip layer and a wider one on the upper layer. This dielectric configuration allows a low radiation effect of the feeding network, but a wider operation band [PK87].

Figure 1.17: Structure of Proximity Coupled Microstrip Feed of a Circular Patch

The main problem of this feeding method is to align the patch with the feeding. It is possible to build it on a multilayer PCB, but the microstrip tracks are buried and soldering of active components is not possible. Nowadays, there are on the market PCB that combine several dielectric materials at competitive prices, so it can be a good solution for passive arrays, but not for active ones.

The parasitic equivalent model of this coupling is mainly capacitive, so it can be modelled as a series capacitor with the patch model.

The example patch, Fig. 1.17, has a “Mode 1” response. The example patch is built on two dielectric layers with $\epsilon_r = 2.5$, and each layer has $h = 0.8\text{ mm}$. The patch has $R = 28\text{ mm}$ with a feeding track with $W = 2.6\text{ mm}$ and a “coupling track” with $W_{\text{coupl}} = 0.3\text{ mm}$. The fields inside the patch and the radiation diagram are according to the expected. Even using a relative small PCB the diagram has a good cross-polarization and a low back radiation, Fig. 1.18.

1.2.3.2 Aperture Coupled Feed

The aperture coupled feed has the interesting feature of wide bandwidth and no effect of the feeding network on the main radiation diagram. The structure of the patch and feeding is a pair of dielectric layers with a ground plane between them, microstrip lines are on the bottom layer, a patch over the upper dielectric layer and a slot of different shapes on the ground plane under the patch, Fig. 1.19. This patch is built with on a two dielectric layers with $\epsilon_r = 2.5$ and each layer has $h = 0.8\text{ mm}$ and a ground plane in the middle. The patch has $R = 28\text{ mm}$ with a feeding track with $W = 2.6\text{ mm}$ and a “coupling slot”, which is a circular ring with $R_{\text{in-coupl}} = 5\text{ mm}$ and $R_{\text{out-coupl}} = 7\text{ mm}$.

This feeding method has a great number of design parameters as feeding line impedance, feeding stub length, coupling aperture shape and size, upper dielectric ϵ_r and width, etc. This great number of parameter make easy to optimize the bandwidth of the patch up to values greater than 20% without stacked patches [LL92, TWP98]. It is also easy to control the input impedance of the patch without requiring any impedance matching network [Chr04], so the losses of the

Figure 1.18: Circular Patch with Proximity Coupled Microstrip Feed ($R = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 2.6\text{ mm}$, $W_{coupl} = 0.3\text{ mm}$)

Figure 1.19: Structure of Aperture Coupled Microstrip Feed of a Circular Patch ($R = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 2.6\text{ mm}$, $R_{in-coupl} = 5\text{ mm}$, $R_{out-coupl} = 7\text{ mm}$) with Hidden Dielectric Layers

feeding or beam-forming network can be minimized. Another advantage of this feeding method is that the microstrip lines are on an external layer, so the active components integration is easy. The radiation diagram of this model of patches, Fig. 1.20, has good cross-polarization, but the feeding network contributes to the back radiation. This model of patches can be manufactured using PCBs that combine several dielectric materials, with competitive prices, so it can be a good solution for active and passive arrays.

Figure 1.20: Aperture Coupled Microstrip Feed of a Circular Patch Radiation Diagram ($R = 28\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 2.6\text{ mm}$, $R_{in-coupl} = 5\text{ mm}$, $R_{out-coupl} = 7\text{ mm}$)

The coupling is complex model that includes the mutual impedance, coupling factor, the resonant RLC of the slot and the line impedance. The coupling slot is not usually used resonant, this way the back radiation is minimized. The parasitic equivalent model of this coupling is mainly inductive, so it can be modelled as a series inductor with the patch model. The coupling slot allows also to change the polarization of the patch.

1.2.4 Coplanar Waveguide Feed

The coplanar waveguide (CPW) is the most used transmission line on the design of Monolithic Integrated Circuits (MMICs). So, when an antenna is designed to be included with active devices in MMIC technology, the use of coplanar waveguide may be an interesting option.

In this case the coplanar line is built on the metallic plane that is the ground plane of the patch antenna. The coplanar line is divided into a coupling “shape” under the patch, and this slot couples the energy to the patch. These coupling elements can have multiple shapes as for example “T” (or dipole), Fig. 1.21, “II” (or slot), Fig. 1.22 and “ring”, Fig. 1.23.

(a) View with Transparent Layers

(b) Radiation Diagram

Figure 1.21: Coplanar Waveguide Feed in “T” of a Rectangular Patch ($L = 50\text{ mm}$, $W = 30\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 1.28\text{ mm}$, $G_{feed} = 0.2\text{ mm}$, $L_{coupl} = 4.7\text{ mm}$, $W_{coupl} = 2.2\text{ mm}$)

(a) View with Transparent Layers

(b) Radiation Diagram

Figure 1.22: Coplanar Waveguide Feed in “II” of a Rectangular Patch ($L = 50\text{ mm}$, $W = 30\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 1.28\text{ mm}$, $G_{feed} = 0.2\text{ mm}$, $L_{coupl-slot} = 6.28\text{ mm}$, $W_{coupl-slot} = 2.2\text{ mm}$)

(a) View with Transparent Layers

(b) Radiation Diagram

Figure 1.23: Coplanar Waveguide Feed in “ring” of a Rectangular Patch ($L = 50\text{ mm}$, $W = 30\text{ mm}$, $\epsilon_r = 2.5$, $h = 0.8\text{ mm}$, $W_{feed} = 1.28\text{ mm}$, $G_{feed} = 0.2\text{ mm}$, $R_{in-coupl} = 4\text{ mm}$, $R_{out-coupl} = 5\text{ mm}$)

The circular slot in Fig. 1.23 is suitable to reduce the back radiation of the patch antenna, but the ring slot must not be resonant if it is desired to reduce the back radiation. In general, these feedings are similar to the aperture coupled, Sec. 1.2.3.2, but with different parasitic effect: inductive for Fig. 1.21 and capacitive for Fig. 1.22. But a more convenient way to model the circular slot is as an open-circuited patch with a parasitic rectangular patch [MGR01].

1.3 Simple Analytical Models

The Analytical Models have a great importance because they are necessary for the first steps of the design. These must have several properties to be considered “good” models:

- Their result must be good enough for the purpose.
- They must have an interpretation in terms of known physical phenomena.
- They provide understanding of antenna principles.
- They must be as simple as possible.
- They must provide information about the impedance and the radiation characteristics.

There are multiple techniques for the analysis of patches. Some of the most important analytical models are the transmission-line model [CM81] and the cavity model [RL79].

1.3.1 Transmission-line Model

This model is based on a characterization of the patch antenna as a transmission line with fringe fields that go over the ends of the transmission line. These fields are considered as slots and the radiating field of these slots is the approximation used to determine the radiated field of the patch antenna.

It is important to determine the ϵ_{eff} of the transmission-line model to determine the characteristics of the patch antenna. As the patch is on a dielectric with $\epsilon_r > 1$ and with air on the top the effective dielectric constant is on the range $1 < \epsilon_{eff} < \epsilon_r$. The ϵ_{eff} is a function of the ratio of the width of the line (W) with the height of the substrate (h) and the frequency. There are several models to determine the value of the ϵ_{eff} , but it goes from a lower value at low frequencies (ϵ_{eff_L}), Equ. 1.3, increasing up to ϵ_r for mid-high frequencies [Bal12].

$$\epsilon_{effL} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12\frac{h}{W}}} \quad (1.3)$$

for $W/h \gg 1$.

1.3.1.1 Transmission-line Model of Rectangular Patch

The rectangular patch is easy to model as a transmission-line. For the first dominant mode, TM_{010} , there are only two radiating slots at the ends of the longest dimension and the two non-radiating slots at edges of the longest dimension of the patch, Fig. 1.24. Then the effective length of the patch on a first approximation is considered in terms of the ϵ_{eff} , Equ. 1.4, and for an effective radiation the width for the TM_{010} can be approximated as in Equ. 1.5. Farther adjustments and more precise models must be introduced for a good starting point of the design [MGR01].

Figure 1.24: Transmission-Line Model of a Rectangular Patch

$$L = \frac{c}{2f_r \sqrt{\epsilon_{eff}}} \quad (1.4)$$

$$W = \frac{c}{2f_r} \left(\frac{2}{\epsilon_{eff} + 1} \right)^{1/2} \quad (1.5)$$

where c is the speed of light and f_r is the design frequency.

The fringe fields on the longest dimension of the patch introduce an extra length (ΔL) [Ham75], Equ. 1.6, to the physical length to consider for the effective length of the patch (L_{eff}), Equ. 1.7.

$$\Delta L = 0.412 \cdot h \cdot \frac{(\epsilon_{eff} + 0.3) \left(\frac{W}{h} + 0.264 \right)}{(\epsilon_{eff} - 0.258) \left(\frac{W}{h} + 0.8 \right)} \quad (1.6)$$

$$L_{eff} = L + 2 \cdot \Delta L \quad (1.7)$$

Then the resonant frequency for the dominant mode TM_{010} can be defined as Equ. 1.8.

$$f_{010} = \frac{c}{2L_{eff}\sqrt{\epsilon_{eff}}} \quad (1.8)$$

Some authors define a “fringe factor”, q , to quantize the relation of the physical length of the patch with the height of the substrate, Equ. 1.9. The “fringe factor” is increased with the increase of the height of the substrate.

$$q = \frac{L\sqrt{\epsilon_r}}{(L + 2 \cdot \Delta L) \sqrt{\epsilon_{eff}}} \quad (1.9)$$

So the resonant frequency of the TM_{010} , Equ. 1.8, can be re-written as Equ. 1.10.

$$f_{010} = q \cdot \frac{c}{2L\sqrt{\epsilon_r}} \quad (1.10)$$

The next step in the patch analysis is to determine the impedance of the slots at the radiating edges of the patch. These slots are characterized by the admittance, $Y_i = G_i + j \cdot B_i$, which is identical for both slots, $Y_1 = Y_2$, as it is defined by Equ. 1.11. This equation is valid for $\lambda_0/h \gg 10$ and it is a reasonable approximation for greater height of substrates. It is important to remember that substrates with low height cannot radiate properly.

$$\begin{aligned} G_1 = G_2 &= \frac{W}{120\lambda_0} \left(1 - \frac{1}{24} (k_0 h)^2 \right) \\ B_1 = B_2 &= \frac{W}{120\lambda_0} (1 - 0.636 \cdot \ln(k_0 h)) \end{aligned} \quad (1.11)$$

where $k_0 = 2\pi/\lambda_0$.

Using the transmission line equation, it is possible to translate the impedance of one radiating slot to the other, Fig. 1.25. When it is calculated for the resonance of the patch antenna and the length is a reasonable value ($0.48\lambda < L < 0.49\lambda$) the total admittance of the patch at one radiating edge is $Y_{in} = 2G_1 = 2G_2$. When the mutual impedance of both slots is considered, the input impedance of the patch is Equ. 1.12.

$$Z_{in} = \begin{cases} \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left(\frac{8h}{W_0} + \frac{W_0}{4h} \right) & \frac{W_0}{h} < 1 \\ \frac{120\pi}{\sqrt{\epsilon_{eff}} \left[\frac{W_0}{h} + 1.393 + 0.667 \ln \left(\frac{W_0}{h} + 1.444 \right) \right]} & \frac{W_0}{h} > 1 \end{cases} \quad (1.12)$$

where W_0 is the width of the feeding microstrip line and h is the height of the substrate.

Figure 1.25: Z_{in} on Radiating Edge of a Rectangular Patch

If an “insertion” of the feeding is made as in Fig. 1.14, the input impedance for an insertion depth, d , is given by Equ. 1.13.

$$R_{in}(d) = Z_c \left(\cos^2 \left(\frac{\pi}{L} d \right) + \frac{G_1^2 + B_1^2}{Y_c^2} \sin \left(\frac{\pi}{L} d \right) - \frac{B_1}{Y_c} \sin \left(\frac{2\pi}{L} d \right) \right) \quad (1.13)$$

where Y_c is the admittance of the line-model of the patch.

1.3.2 Cavity Model

The cavity model considers the fields inside the patch, but it does not consider the fields out of it. So, it is a suitable method to determine the fields inside the patches and the frequencies of the different modes. On a first approximation, it is not suitable for the study of the radiation, but it can be approached the fields on the boundaries and then it is suitable for the estimation of impedance, resonant frequencies (Modes) and radiation patterns [Har01]. The patch is considered as a resonant cavity with electric walls on bottom (ground) and top (patch), external magnetic wall and, if required, internal magnetic or electric wall for rings.

To manage the radiation, losses must be included on the magnetic walls (“open” boundaries of the patch). Two resistance values must be considered: loss resistance (R_L) and radiation resistance (R_R). When these losses must be considered on the close cavity model, loss tangent is introduced.

The height of the patch is small compared with the wave-length, $h/\lambda \ll 1$, so the fields in the patch are considered constant with this dimension, so only TM^z modes are possible. The terms on z variation on the vector potential equation and on the solutions could be omitted, but they are kept for formulation consistency.

A general solution of the cavity model, Equ. 1.14, when it is considered that the fields do not extend beyond the edges of the patch, that means that the vector potential must satisfy Equ. 1.15.

$$\nabla^2 A_z + k^2 A_z = 0 \quad (1.14)$$

$$A_z = [A_1 \cos(k_x x) + B_1 \sin(k_x x)] \cdot [A_2 \cos(k_y y) + B_2 \sin(k_y y)] \cdot [A_3 \cos(k_z z) + B_3 \sin(k_z z)] \quad (1.15)$$

1.3.2.1 Cavity Model of Rectangular Patch

With a rectangular patch as defined in Fig. 1.2, the vector potential to solve is A_z , but it is analogous in any position of the patch within a Cartesian reference system.

Then the boundary conditions, Equ. 1.16, are applied to Equ. 1.15 and the wavenumbers of the cavity can be determined as Equ. 1.17.

$$\frac{\partial A_z}{\partial z} = 0 \text{ for } x' = 0, x' = L, y' = 0, y' = W, z' = 0 \text{ and } z' = h \quad (1.16)$$

$$\begin{aligned} k_x &= \frac{m\pi}{L} \quad , \quad m = 0, 1, 2, \dots \\ k_y &= \frac{n\pi}{W} \quad , \quad n = 0, 1, 2, \dots \\ k_z &= \frac{p\pi}{h} \quad , \quad p = 0, 1, 2, \dots \end{aligned} \quad (1.17)$$

These wavenumber must comply with the constrain equation

$$k_x^2 + k_y^2 + k_z^2 = k_r^2 = \left(\frac{\omega_r}{c}\right)^2 \quad (1.18)$$

These mnp indexes represent the Mode of the patch, so the frequency of each mode is defined by Equ. 1.19.

$$f_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2 + \left(\frac{p\pi}{h}\right)^2} \quad (1.19)$$

Then the dominant mode of this rectangular patch is the TM_{100}^z , so the frequency of the first mode is Equ. 1.20.

$$f_{100} = \frac{c}{2L\sqrt{\epsilon_r}} \quad (1.20)$$

The electric and magnetic fields in the patch cavity for the dominant mode TM_{100}^Z according Equ. 1.15 are the ones defined in Equ. 1.21, using x' as a relative coordinate system aligned with the patch.

$$\begin{aligned} E_z &= E_0 \cos\left(\frac{\pi}{L}x'\right) \\ H_y &= H_0 \sin\left(\frac{\pi}{L}x'\right) \end{aligned} \quad (1.21)$$

$$E_x = E_y = H_z = H_x = 0$$

where $E_0 = -j\omega A_{100}$ and $H_0 = (\pi/\mu L) A_{100}$

Most of the practical designs of rectangular patch antennas comply with

$$\begin{aligned} h &\ll W \\ h &\ll L \end{aligned} \quad (1.22)$$

so, the frequencies of the modes, Equ. 1.19, can be simplified as Equ. 1.23.

$$f_{mn} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{L}\right)^2 + \left(\frac{n\pi}{W}\right)^2} \quad (1.23)$$

With the fields in the patch, using the Huygen field equivalence principle, it is possible to calculate the current density M of the equivalent magnetic dipoles and this way the radiation of the patch. First the radiation of a single slot is calculated, Equ. 1.24.

$$\begin{aligned} E_{\phi(slot)} &= j \frac{k_0 h W E_0 e^{-jk_0 r}}{2\pi r} (\sin(\theta) \cdot \text{sinc}(X) \cdot \text{sinc}(Z)) \\ X &= \frac{k_0 h}{2} \sin(\theta) \cos(\phi) \\ Z &= \frac{k_0 W}{2} \cos(\theta) \end{aligned} \quad (1.24)$$

To calculate the radiation of the patch, the radiation of the two equivalent slots are considered as the elements of an array spaced L_e . Then using the array factor the radiation of the patch is obtained as Equ. 1.25.

$$E_{\phi} = E_{\phi(slot)} \cdot 2 \cos\left(\frac{k_0 L_e}{2} \sin(\theta) \sin(\phi)\right) \quad (1.25)$$

The directivity, Equ. 1.26, of this patch for TM_{100}^Z mode is obtained, Equ. 1.27. This directivity can be approximated as Equ. 1.28. The cases with $W \gg \lambda_0$ are not practical because undesired higher modes can be excited on the patch.

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} \quad (1.26)$$

$$D_0 = \left(\frac{2\pi W}{\lambda_0} \right)^2 \frac{\pi}{I_2} \quad (1.27)$$

$$I_2 = \int_0^\pi \int_0^\pi \left(\frac{\sin\left(\frac{k_0 W}{2} \cos(\theta)\right)}{\cos(\theta)} \right)^2 \sin^3(\theta) \cos^2\left(\frac{k_0 L_e}{2} \sin(\theta) \sin(\phi)\right) d\theta d\phi$$

$$D_0 = \begin{cases} 6.6 \equiv 8.2 \text{ dB} & W \ll \lambda_0 \\ 8 \left(\frac{W}{\lambda_0} \right) & W \gg \lambda_0 \end{cases} \quad (1.28)$$

1.3.2.2 Cavity Model of Circular Patch

The second most used patch is the circular, so the model of this patch is a useful one. In this case the cylindrical coordinate system (ρ, ϕ, z) is used for simplicity.

In this case the vector potential that must be satisfied is Equ. 1.29.

$$\nabla^2 A_z(\rho, \phi, z) + k^2 A_z(\rho, \phi, z) = 0 \quad (1.29)$$

When it is solved, applying the contour conditions, the obtained vector potential is

$$A_z = B_{mnp} J_m(k_x \rho') [A_2 \cos(k_\phi \phi') + B_2 \sin(k_\phi \phi')] \cos(k_z z') \quad (1.30)$$

with the constrain equation

$$k_\rho^2 + k_\phi^2 + k_z^2 = \left(\frac{\omega_r}{c} \right)^2 \quad (1.31)$$

where the coordinates ρ' , ϕ' and z' are referenced to the patch.

If a suitable reference coordinates system is used and the field variation in z axis is not considered ($h \ll \lambda$), Equ. 1.30 can be simplified as Equ. 1.32, so the wavenumbers do not include z (p index) variation.

$$A_z = A_{mn} J_m(k_\rho \rho') \cos(n\phi') \quad (1.32)$$

The solution of this wave provides the next solution

$$\begin{aligned} k_\rho &= \frac{\chi'_{m,n}}{a} & m &= 0, 1, 2, \dots \\ & & n &= 1, 2, 3, \dots \\ k_z &= \frac{p\pi}{h} & p &= 0, 1, 2, \dots \end{aligned} \quad (1.33)$$

where χ'_{mn} are the zeros of the derivate of the first kind Bessel function $J_m(x)$. The first six values listed from the lowest are

$$\begin{aligned}\chi'_{11} &= 1.84118378134065 \\ \chi'_{21} &= 3.05423692822714 \\ \chi'_{01} &= 3.83170597020751 \\ \chi'_{31} &= 4.20118894121052 \\ \chi'_{41} &= 5.31755312608399 \\ \chi'_{12} &= 5.33144277352503\end{aligned}\tag{1.34}$$

So the resonant frequency of any mode is defined as Equ. 1.35. To obtain the correct resonant frequency, the effect of the fringe fields on the radius is considered, Equ. 1.36. Then, it can be determined the dominant mode of the circular patch. This Mode is the TM_{110}^z , this mode is usually named “Mode 1” and the resonant frequency of this mode is Equ. 1.37.

$$f_{mn0} = \frac{\chi'_{mn} \cdot c}{2\pi a_e \sqrt{\epsilon_r}}\tag{1.35}$$

$$a_e = a \left(1 + \frac{2h}{\pi a \epsilon_r} \left(\ln \left(\frac{\pi a}{2h} \right) + 1.7726 \right) \right)^{1/2}\tag{1.36}$$

$$f_{110} = \frac{1.84118378134065 \cdot c}{2\pi a_e \sqrt{\epsilon_r}}\tag{1.37}$$

It is possible to calculate the required physical radius for any desired TM_{mn0}^z patch with Equ. 1.35 and Equ. 1.36. The physical radius is defined by Equ. 1.38.

$$a = \frac{F}{\left(1 + \frac{2h}{\pi \epsilon_r F} \left(\ln \left(\frac{\pi F}{2h} \right) + 1.7726 \right) \right)^{\frac{1}{2}}}\tag{1.38}$$

$$F = \frac{\chi'_{mn} c}{2\pi f_r \sqrt{\epsilon_r}} \cdot 10^3 \text{ for } h \text{ in mm}$$

The study of the radiation of this patch antenna for the “Mode 1” takes to obtain the radiation conductance of the fringe fields around the patch, Equ. 1.39.

$$G_{rad} = \frac{(k_0 a_e)^2}{480} \int_0^{\pi/2} \left((J'_{02})^2 + \cos^2(\theta) (J_{02})^2 \right) \sin(\theta) d\theta\tag{1.39}$$

$$\begin{aligned}J'_{02} &= J_0(k_0 a_e \sin(\theta)) - J_2(k_0 a_e \sin(\theta)) \\ J_{02} &= J_0(k_0 a_e \sin(\theta)) + J_2(k_0 a_e \sin(\theta))\end{aligned}$$

The variation of the input resistance of a circular patch for the “Mode 1” is defined as Equ. 1.40, where R_{in} is the input resistance at the edge of the patch and d the depth of the feeding insert.

$$R_{in}(d) = R_{in} \frac{J_1^2(k \cdot d)}{J_1^2(k \cdot a_e)} \quad (1.40)$$

1.3.2.3 Cavity Model of Circular Ring Patch with Central Short-Circuit

The circular ring patches with central short-circuit have some interesting advantages over the classical circular rings (central open-circuit). Some of these properties are an easier feeding, due to the close-to-open boundaries conditions and a reduction of surface-waves [JWB⁺93] (Sec. 1.4). The short-circuit model of the circular ring also has a larger electrical size, so it has greater gain than the open-circuit model.

In this case, the vector potential that must be satisfied is the same as for the circular patch, Equ. 1.29.

When it is solved, applying the contour conditions and considering no variation with z ($h \ll \lambda$) [GPSVRI⁺06], the obtained vector potential is Equ. 1.41, so the wavenumbers do not include z (p index) variation.

$$A_z = A_{mn} [J_m(k_{nm}\rho') Y_n'(k_{mn}a) - J_m'(k_{nm}a) Y_n(k_{mn}\rho')] \cos(n\phi') \quad (1.41)$$

So, the characteristic equation of the circular ring with central short-circuit is

$$J_m(k_{nm}) Y_n'(k_{mn}r) - J_m'(k_{nm}r) Y_n(k_{mn}) = 0 \quad (1.42)$$

where the coordinate a is the inner radius, b is the outer radius and $r = a/b$.

Then Equ. 1.42 can be solved using r as parameter. The solution for the first TM_{n1} modes (“Mode n”) are in Fig. 1.26.

So, the frequencies of the modes are:

$$f_{mn} = \frac{k_{mn} \cdot c}{2\pi a \sqrt{\epsilon_r}} \quad (1.43)$$

To consider the fringe fields, the outer radius (a) must be corrected to effective radius (a_e). A simple approximation is Equ. 1.44, but a more exact one can be calculated using the effective area as described on Sec. 1.3.2.4.

$$a_e = a + \frac{3}{4}h \quad (1.44)$$

Figure 1.26: Parametric Circular Ring with Central Short-circuit k_{in} solution

1.3.2.4 Cavity Model for other Patch Shapes

One general approximation of the cavity model for any shape is to use the eigenfunction, Ψ_{mn} , of the shape without considering any field variation on z axis. The eigenfunction must comply with Equ. 1.45 and with the required contour conditions. For example the most common contour conditions are the magnetic wall, then Equ. 1.46 must be complied at the magnetic wall.

$$\nabla^2 \Psi_{mn} + k_{mn}^2 \Psi_{mn} = 0 \quad (1.45)$$

$$\frac{\partial \Psi_{mn}}{\partial n} = 0 \quad (1.46)$$

This method does not consider the fringe fields. To include the effect of the fringe fields on the dimensions of any shape a general method is to calculate the area of the desired shape, A_s , and then calculate equivalent area of a circular patch with the same area, Equ. 1.47.

$$A_{eq} = a_{eq}^2 \pi \quad (1.47)$$

where a_{eq} is the result of Equ. 1.36 when a is

$$a = \sqrt{\frac{A_s}{\pi}} \quad (1.48)$$

With the equivalent area of the shape, A_{eq} , the equivalent dimensions of the patch can be obtained and the eigenfunction, Ψ_{mn} , are adjusted to include the fringe fields effect.

Several eigenfunctions, Ψ_{mn} , for different shapes and a more detailed explanation can be found in [GBBI01].

1.4 Surface Waves and Coupling

The surface waves are a mayor problem in the patches. They are caused by the gap in ϵ on the dielectric-air interface beyond the edge of the patch. These surface waves are caused by the reflexion of the waves on the dielectric-air interface making them to travel on a “guide” much far than the fringe area of the “radiating slot”.

The effect can be approximated by the ray reflexion theory. The wave is reflected by the dielectric-air interface when it arrives with an angle smaller than Equ. 1.49. So the critical angle increases with greater dielectric ϵ_r .

$$\theta_c = \arcsin \left(\frac{1}{\sqrt{\epsilon_r}} \right) \quad (1.49)$$

These surface waves have a low loss, their loss is as $1/\sqrt{r}$. Then they can cause several undesired effects on patches and circuits that include patches:

- Loss of radiating efficiency. The more energy is captured on the surface wave, the less energy is radiated by the “slots”.
- End-fire radiation.
- Coupling with adjacent patches on arrays.
- Coupling and undesired effects with active components and tracks.

The amount of power captured on surface waves depends on the dielectric constant of the material and on the height of the dielectric. To consider that the losses by surface waves can be ignored the parameters of the substrate must comply Equ. 1.50 [JH79]. Values of h that avoid the surface waves makes the substrate so thin that the patch efficiency is negligible, so other techniques to reduce the surface waves must be used [MGR01].

$$h \leq \lambda_0 \frac{0.3}{2\pi\sqrt{\epsilon_r}} \quad (1.50)$$

To reduce the surface waves is vital for the proper operation of an array and for most of active antennas models. The coupling of energy to the adjacent elements on an array causes important effects on the radiation diagram as on the array

efficiency. On active antennas, the surface waves may introduce additional feed-back path that cause problem on amplifiers and mixers.

To reduce the presence of surface waves several methods are used. These methods can reduce or eliminate the surface waves at the working frequency by the inclusion of some modification or design characteristics on the ground plane, on the dielectric or on the patch. The most used ground plane method to reduce the surface waves is the inclusion of Electromagnetic Band Gap Structures (EGB) [JDG15]. Other interesting methods are the use of stacked-patches [XJW⁺04] and patches geometries that includes a short-circuit [ABDM05, BWJC08].

In any case, the study and control of the surface waves is a complex work and when active elements or antenna feeding circuits are integrated on the same PCB, it is reasonable to use a full-wave simulator as Finite element method (FEM), Finite-difference time-domain (FDTD), Finite Integration Technique (FIT) or any other equivalent method. [SSCM02].

1.5 A Practical Example

As practical example, a modification of a short circuited ring patch antenna is presented. This design is based on the designs and works of *CSIC-Distcom Group* [MGR01] that introduced commercial SCRCP in 90's. The antenna is designed to operate in two bands and with different polarization on each band: the lower band is designed for linear polarization and the upper one for circular polarization.

The short-circuited ring-patch (SCRCP) [JWB⁺93, LS88, LS90] is an evolution of the classic open-circuit ring patch that replaces the central open-circuit condition by a central short-circuit. The effect of this central short-circuit is that the fundamental mode of this SCRCP is the TM_{01} and it is not the TM_{11} [GPSVRI⁺06] as it is on the classic ring patch, Fig. 1.27. The TM_{01} produces a monopole radiation diagram and the TM_{11} , that is the second mode of the SCRCP, produces a directional diagram.

By using the two first modes of a SCRCP antenna, that have linear polarization, the antenna can be used as a dual band and dual mode antenna. The proposed patch antenna uses a pair of diagonal metallic annular sectors with a via (with radius $r_v = 0.4\text{ mm}$) that short-circuited them to the ground plane of the patch antenna, Fig. 1.28. These parasitic annular sectors are placed at a distance (r) of the centre of the patch and at 45° of the feed, this way a circular polarization can be generated for the upper band, TM_{11} mode.

The antenna is designed to operate at the frequencies of 1300 MHz , TM_{01} with linear polarization and 1950 MHz , TM_{11} with circular polarization. The frequencies of the modes can be obtained using Equ. 1.51.

Figure 1.27: Electric Fields for TM_{01} and TM_{11} Modes

Figure 1.28: Geometry of the Modified SCRP Antenna

$$f_{mn} = \frac{k_{mn} \cdot s}{2\pi b \sqrt{\epsilon_r}} \quad (1.51)$$

where b is the external radius, s is the speed of light in the vacuum and the mn indexes the order of the TM mode defined by the roots of Equ. 1.52.

$$J'_m(k_{mn}) N_m(k_{mn}g) - J_m(k_{mn}g) N'_m(k_{mn}) = 0 \quad (1.52)$$

where $g = a/b$, a is the internal radius, $J_m(x)$ the m^{th} first kind Bessel function, $N_m(x)$ the m^{th} second kind Bessel function, and ' denotes the first derivative.

The design results for this antenna are:

- $\epsilon_r = 2.5$
- $c = 35 \text{ mm}$
- $a = 5 \text{ mm}$
- $b = 25.15 \text{ mm}$

The upper operation frequency of the antenna is designed to have RHCP (Right Hand Circular Polarization). Some designs with dual-band and dual polarized operations have been reported in the literature [MN12, DZS04, CW03], most of them for application in mobile communication systems, but this is a novel structure for such purpose. The combination of a monopole diagram and a directional one makes this antenna model suitable to be used for terrestrial and satellite communication systems, that usually have circular polarization. The CP antennas have spawned wide applications of military and mobile wireless communications because they are not only able to reduce the multi-path effect, but also allow a more flexible orientation of the transmitter and receiver antennas. The CP radiation can be achieved by the introduction of some symmetric or asymmetric perturbations into the patch antenna. These perturbations excite two orthogonal modes with equal magnitudes and 90° phase difference. In order to generate CP, the perturbations of the proposed antenna have been developed by simply embedding two metallic pads and a via hole connected to the ground plane for each pad. These metallic pads are asymmetrically placed on the ring to make the surface current paths of the original patch antenna TM_{11} mode to split into two resonant modes that are tuned to obtain a good CP radiation. The dimensions of the antenna are modified on the full-wave electromagnetic design step to assure that the resonance frequencies f_1 and f_2 of the two orthogonal modes are close enough one to each other. Then, the antenna is excited at a frequency f_0 between the resonance frequencies of these two modes excited by the perturbations in such a way that the magnitudes of the two excited modes are equal. This double resonance corresponds to the two orthogonal TM_{11} modes, Fig. 1.29, the resonances with lower and higher frequency are denoted by TM_{11A} (f_1) and TM_{11B} (f_2), respectively. The resonances have 90° on phase shift, therefore, a circular polarization is obtained.

The current distribution over the patch can be decomposed in two orthogonal components: a radial and an azimuthal one. The annular sectors have an influence in the azimuthal component. If the feeding-point is placed along the y-axis (Fig. 1.28), a RHCP (Right Hand Circular Polarization) is achieved. However, if the feeding-point is placed along the x-axis, a LHCP (Left Hand Circular Polarization) is achieved. The final placement of the feeding-point is based on the search of the impedance matching of the resonant frequencies for both modes using electromagnetic simulation.

The proposed model of SCRP antenna with circular polarization at the upper band has been manufactured, Fig. 1.30, and measured, Fig. 1.31. The vias are built by drilling holes into the substrate and inserting metallic wires, which are soldered to the metallic annular sectors and to the ground plane. The numerical simulations are performed using CST Microwave Studio©. The measured and

simulated S_{11} parameters have a good match for the frequencies of the TM_{11A} and TM_{11B} , a slight difference is found for the frequency of the TM_{01} mode. The values of the parameters of the manufactured antenna, also used for the simulation, are listed in the Table 1.1.

Figure 1.29: H-field Distribution of the Two Excited Orthogonal Modes for the TM_{11} Mode of the Patch

Parameter	Name	Value
a	Central shot-circuited radius	5 mm
b	Patch radius	25.15 mm
c	Substrate and ground plane radius	35 mm
d	Radius to inner side of annular sectors	6.7 mm
R_S	Length (radius) of annular sectors	5 mm
ϕ_S	Angle of annular sectors	28°
t_p	Thickness of patch metal	0.35 mm
t_d	Thickness of dielectric	13 mm
t_g	Thickness of ground metal	1.6 mm
f	Radius to coaxial feed point	14 mm
$r = d + R_S/2$	Radius to vias of annular sectors	9.2 mm

Table 1.1: Parameters of SCRP Antenna

Fig. 1.32 shows the radiation patterns for both TM_{01} mode, 1.3 GHz , and TM_{11} , 1.95 GHz . In general, good match is found between measurements and numerical simulations. It is clear that the TM_{01} mode produces a monopole-like radiation pattern and that the TM_{11} mode has a broadside radiation pattern. The asymmetry in the y-z plane is caused by the position of the feeding, that is placed in the y axis. Table 1.2 shows the main performances of the proposed antenna, and a

comparison between measurements and numerical simulations values is presented. The circular polarization bandwidth ($AR < 3dB$) is closed to $40 MHz$, with a great match between measurements and simulations.

Figure 1.30: Photograph of Manufactured Dual Band and Dual Polarization SCRP Antenna

Figure 1.31: Simulated and Measured S_{11} of Dual Band and Dual Polarization SCRP Antenna

Figure 1.32: Simulated Radiation Patterns of SCRP Antenna

Parameter	Measurement		Simulation	
	$f = 1.3 \text{ GHz}$	$f = 1.95 \text{ GHz}$	$f = 1.3 \text{ GHz}$	$f = 1.95 \text{ GHz}$
Axial Ratio	Linear Pol.	0.8 dB	Linear Pol.	1.1 dB
Beam-width	YZ: 98° XZ: 105°	YZ: 102° XZ: 97°	YZ: 92° XZ: 91°	YZ: 85° XZ: 87°
Gain	1.6 dBi	7.2 dBi	1.4 dBi	6.9 dBi
Bandwidth ($AR < 3 \text{ dB}$)	Linear Pol.	40 MHz	Linear Pol.	40 MHz
S_{11}	-8 dB	-9 dB	-20 dB	-7.5 dB

Table 1.2: Parameters of SCRP Antenna

A parametric study, using the numerical simulations, varying some parameters of the antenna such as the external radius of the patch (b), the radius of the circular ground plane (c) and the position (d) and size (R_S and ϕ_S) of the annular sectors is done. It is shown, in Fig. 1.33, the location of the resonances of the TM_{01} , TM_{11A} and TM_{11B} modes for the different values of the parameters. The variation of ϕ_S between 25° and 60° has not significant influence in the resonant frequencies. The size and the position of the annular sectors have a small influence on the TM_{01} , but their influence in the TM_{11A} and TM_{11B} the key to obtain two resonances close enough in frequency to get low values of axial ratio. For this reason, the annular sectors must be placed close to the central short-circuit. As the size of the annular sector is increased, the separation of the two orthogonal TM_{11} modes makes greater and therefore, worse circular polarization is achieved.

This antenna is capable to work in two bands with two types of polarizations: LP (TM_{01} mode) and CP (TM_{11} mode) with a single feed point. The CP polarization has a good beam-width, better than 95° , and a 2% bandwidth (40 MHz) for an $AR < 3dB$. This design presents an interesting topology that has important advantages in several applications. For example the CP is the preferred one for mobile communication due to the reduction of multi-path effect and a more flexible orientation of antennas. In addition, it has the advantage of an easy manufacturing process and integration with electronic.

a) Variation with d b) Variation with R_s c) Variation with c d) Variation with b e) Variation with ϕ_s Figure 1.33: Resonances of Modes TM_{01} , TM_{11A} and TM_{11B} with Variation of Some Parameters

1.6 Conclusion on Patches Study

The patches have a great advantage compared with other antennas, they can be integrated with electronics on a easy way to build active antennas. The manufacture process of the antenna, or array, and the active elements can be completely automatic on an industrialized process.

The different modes and topologies of the patches provide a wide group of radiation patterns, were the most important ones are the Mode 0, that exist on short-circuited ring patches and the Mode 1. The control of the fields and currents on the patches can produce combinations of modes with different polarizations, as it is demonstrated in the example exposed in this chapter.

The study of the mode of the patches and the fields inside them have vital importance to be able to understand and integrate the patch with active devices. The first steps of the design of a patch requires analytical models and approximations for a quick design. This analytical models are really useful, because they help to understand the physics of the patches and help on the research of new active patches research.

The study of the fields inside the patch, the impedance and the S parameters of the patches is the key to get a proper integration of the patch and the active components. This can be a hard task or nearly impossible using analytical models, so full wave analysis of the patch is required. The fields distribution, phases and other important characteristics of the patch must be compatible with coupling and feeding points, track used on the oscillator and active function that is integrated in the patch.

One important problem on the design of arrays and active patches is the surface waves. There are several techniques and methods to reduce the surface waves, but on the case of active patches not only these surface waves can cause coupling problems. The integration of active devices, tracks and biasing of components on the patches requires a deep study of the coupling and undesired resonances. So it is advisable to make full wave analysis of the patches with integrated active devices.

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Chapter 2

STABILITY

2.1 Introduction to Stability

First, it is necessary to define what is denoted as internal and external stability. The external, or bounded-input, bounded-output, stability of a linear time-invariant system is described by the impulse response. The impulse response must be such as Equ. 2.1 [JNP65]. It must be noted that external stability is not equivalent to internal stability.

$$\int_0^{\infty} |h(t)| dt < M < \infty \quad (2.1)$$

Internal stability of a linear time-invariant system is not equivalent to external stability. Internal stability is implicitly determined by implementation; it is defined by the requirement that all the roots of the characteristic polynomial have negative real parts. More detailed analysis and demonstration of the internal stability based in Lyapunov's Method [LS61] proves that stability is guaranteed if all the roots of the characteristic polynomial have negative real parts. Another important conclusion of the internal stability is that it implies the external stability of the system, so it complies with Equ. 2.1. However, the converse is not true.

The oscillators are non-linear circuits and autonomous, they do not have any input ports, so the external stability does not have sense for them, only the internal stability. The stability methods and theory of linear time-invariant systems can be applied to a linear approximation of the non-linear circuit to infer the stability of the non-linear system [Nik13].

This chapter defines the requirements and methods for the determination of the stability of a circuit. It is focused on linear stability analysis of time-invariant systems.

The stability is easy to define, but it is not so easy to check. A system is stable if its s domain description function has not any poles in the Right Half Plane (RHP). But it is not easy to obtain the analytical expression of the system description function on a practical case, and it is sometimes impossible to obtain.

If the position of the poles is known the stability of the self-response of the system can be confirmed, Fig. 2.1.

Most of the methods are based on the practical analysis or measurements that can be performed on real circuits and they are simplifications of another more general method. These methods cannot determine the exact position of the poles, these methods are just valid for the determination of the existence of poles, and the number of poles, in the RHP.

A general problem in the use of the classic methods is that they were developed to be used with real measurements (S parameter, open Loop-Gain, etc.) of the devices, but now most often the input parameter of these methods is obtained from a circuit simulator, so the absence of poles in the parameters is not verified.

Figure 2.1: Poles position and system response

The main problem when studying the stability of any circuit is as several authors declare: “*there is not a compact formulation of an oscillation criterion that is both necessary and sufficient*” [vW10]. On this thesis, a compact method based on the NDF is presented and several examples and discussions on the use and provisos of stability methods are presented.

2.2 Classic Stability Criteria

There are multiple stability criteria that are used nowadays. Some of them are more generic but most of them are focused on two port quadrupoles and on feedback systems. The methods used for feedback systems are based on the analysis of the open-loop response of the system.

This work focuses on the study of active devices in active antennas. Some of the functions and active devices used on patches are amplifiers, oscillators, negative impedance converters, etc. The analysis of the stability or non-stability of these active patches is a hard task. The study of oscillators must guarantee the instability of the circuit at a unique frequency, the oscillation frequency.

Some particularizations of classic linear methods and some specific methods are focused in oscillators. In the oscillator design, the purpose of the analysis is to guarantee the instability at a unique frequency of the system. The analysis of

oscillators is one of the main subjects of this thesis, in particular the oscillators and NICs that are integrated in antennas.

There are a large list of methods and criteria, but some of the most important stability criteria are:

- BIBO
- Routh-Hurwitz
- Bode
- Nyquist
- Place of the roots
- General oscillator characterization using linear Open-Loop S-parameters
- Rollett Proviso
- Criterium in terms of the single stability parameter μ_1
- Reference plane methods (Negative admittance/impedance and Γ) (See Chapter 4)
- Loop-Gain Method (See Chapter 4)

Brief descriptions of these methods are presented in the following sections, but other specific methods for some circuits, mainly oscillators, as the Reference Plane Methods and the Loop-Gain Method are described in Chapter 4.

2.2.1 BIBO

The Bounded Input Bounded Output (BIBO) stability assures that the system results in a bounded output any time over the time interval $[t_0, \infty)$ for a finite input condition. This fact must be assured for any initial time (t_0). As long as the input is not infinity, the output of the system must be finite, so if the input to the system is

$$\|u(t)\| \leq 1 \text{ for } t \geq t_0 \quad (2.2)$$

then the output of the system is

$$\|y(t)\| \leq k \quad (2.3)$$

where $u(t)$ is the input to the system and $y(t)$ the output of the system. So while $k < \infty$ for any t_0 the system is stable.

For Linear Time-Invariant (LTI) systems, it can be demonstrated that the BIBO stability condition is verified, Equ. 2.4, so the BIBO stability is guaranteed if Equ. 2.5 is complied.

$$|y(t)| = \left| \int_{-\infty}^{\infty} h(s) u(t-s) ds \right| \leq \int_{-\infty}^{\infty} |h(s)| |u(t-s)| ds \leq K \int_{-\infty}^{\infty} |h(s)| ds \quad (2.4)$$

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty \quad (2.5)$$

where $h(t)$ is the response to the impulse of the system.

The BIBO stability is a stability input-to-output, but there are systems that have a bounded output, but an internal instability that is not transferred to the output (see Sec. 2.5).

2.2.2 Routh-Hurwitz

This method [Cla92] is based on the independent works of Routh [Rou77] and Hurwitz [Hur64]. When this method is used for a system studied by the closed-loop definition of a system, the characteristic polynomial is the denominator, $D(s)$, of the closed-loop response, $G(s)$ (Equ. 2.6), of the system, Fig. 2.2.

Figure 2.2: Closed-Loop System

$$G(s) = \frac{G_{oc}(s)}{1 + G_{oc}(s)} = \frac{N(s)}{D(s)} = \frac{(s - r_{n1})(s - r_{n2}) \cdots (s - r_{nN_n})}{(s - r_{d1})(s - r_{d2}) \cdots (s - r_{dN_d})} \quad (2.6)$$

The characteristic function of the system is defined as $D(s) = 0$, and the roots of $D(s)$ are the poles of the system. If none of the roots of $D(s)$ are in the RHP ($\forall n \Re(r_{dn}) < 0$) then the system is stable. But the problem is how to determine if any roots have real positive part.

In 1874, Routh introduced a method that uses combinations of coefficients Based on Sturm's theorem. The Routh method is based on a table with combinations of the coefficients of $D(s)$ when it is written in polynomial form, Equ. 2.7.

$$D(s) = a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots a_1 s + a_0 \quad (2.7)$$

The table or matrix must be filled with the following method:

- Write the coefficients in 2 rows, first row starts with a_n and second row starts with a_{n-1} . Other coefficients alternate between rows. Both rows should be the same length, and they are filled with zeros if it is necessary.

$$\bullet \quad b_n = -\frac{\det \begin{bmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{bmatrix}}{a_{n-1}}, \quad b_{n-1} = -\frac{\det \begin{bmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{bmatrix}}{a_{n-1}} \dots$$

$$\bullet \quad c_n = -\frac{\det \begin{bmatrix} a_{n-1} & a_{n-3} \\ b_n & b_{n-1} \end{bmatrix}}{b_n}, \quad c_{n-1} = -\frac{\det \begin{bmatrix} a_{n-1} & a_{n-5} \\ b_n & b_{n-2} \end{bmatrix}}{b_n} \dots$$

Then, the obtained matrix is Equ. 2.8 for the case of odd number of coefficients in $D(s)$.

$$\left[\begin{array}{cccc} a_n & a_{n-2} & \cdots & a_0 \\ a_{n-1} & a_{n-3} & \cdots & 0 \\ b_n = -\frac{\det \begin{bmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{bmatrix}}{a_{n-1}} & b_{n-1} = -\frac{\det \begin{bmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{bmatrix}}{a_{n-1}} & \cdots & b_0 = -\frac{\det \begin{bmatrix} a_n & 0 \\ a_{n-1} & 0 \end{bmatrix}}{a_{n-1}} \\ c_n = -\frac{\det \begin{bmatrix} a_{n-1} & a_{n-3} \\ b_n & b_{n-1} \end{bmatrix}}{b_n} & c_{n-1} = -\frac{\det \begin{bmatrix} a_{n-1} & a_{n-5} \\ b_n & b_{n-2} \end{bmatrix}}{b_n} & \cdots & c_0 = -\frac{\det \begin{bmatrix} a_{n-1} & 0 \\ b_n & 0 \end{bmatrix}}{b_n} \\ \cdots & \cdots & \cdots & \cdots \end{array} \right] \quad (2.8)$$

Before using the Routh-Hurwitz method, if all the coefficients of the Characteristic Function, Equ. 2.7, do not have the same sign there are roots in the RHP. If all the coefficients have the same sign further study of the polynomial is required using the Routh-Hurwitz method.

The Routh-Hurwitz theorem states that the number of sign changes in the first column of the Routh matrix equals the number of roots with positive real part (in the RHP) of the characteristic polynomial. So, a system is stable if all the elements of the first column of the Routh Matrix, Equ. 2.8, are positive. When any element of the first column is zero further consideration must be taken [Lev95], but they are out of the scope of this work.

The problem of this method is that it is required to have the analytical expression of the Characteristic Function and, in most of cases, it is nearly impossible to obtain for a real high frequency circuit. So the Routh-Hurwitz method is not valid for the problems and objectives of this work.

2.2.3 Nyquist

The Nyquist Stability theorem is based in the Cauchy's Principle of Variation of the Argument for complex variables. The key result of the Nyquist method is the following theorem.

Theorem. (*Cauchy's Principle of Variation of the Argument*). Let D be a closed region in the complex plane and let Γ be the boundary of the region. Assume the function $f : \mathbb{C} \rightarrow \mathbb{C}$ is analytic in D and on Γ except at a finite number of poles and zeros, then

$$\omega_n = \frac{1}{2\pi} \Delta_{\Gamma} \arg f(z) = \frac{1}{2\pi i} \int_{\Gamma} \frac{f'(z)}{f(z)} dz = Z - P$$

where Z is the number of zeros, P the number of poles in D and Δ_{Γ} denotes the variation along the contour Γ . The number w_n is called the winding number. It equals the number of encirclements ($N = Z - P$) of the critical point when the curve Γ is traversed in the positive direction. Poles and zeros of multiplicity m are counted m times.

For example, if for a closed region in the s plane, a Γ is defined as it is shown in Fig. 2.3 and it contains 4 zeros and 7 poles of the function under inspection, the $f(z)$ plot will have ($N = -3$) negative (counterclock) encircles around the origin, Fig. 2.4 in the $F(s)$ -plane. The argument of $f(z)$ will be $-3 \cdot (2\pi)$, so the winding number will be $w_n = -3$.

The theorem of the variation of the argument is useful to determine the difference of the number of poles and zeros of a function of complex variables in a given region. This is used as principle for the Nyquist's stability theorem. To do that, Γ is chosen as the Nyquist contour that encloses the right half plane (RHP). Then the number of clockwise encircles around the origin indicates the number of zeros (considering that there are not any poles) in the RHP.

Figure 2.3: Cauchy's Principle of Argument s-plane

Figure 2.4: Cauchy's Principle of Argument $F(\omega)$ -plane

A closed loop system with the open loop transfer function $G_{oc}(s)$, Fig. 2.2, is considered. The closed loop poles are the zeros of the function CF, Equ. 2.9. To determine the number of zeros in the RHP, the number of clockwise encircles around the origin of the function Equ. 2.9 as s moves along the Nyquist contour Γ in the clockwise direction, from 0 to $+i \cdot \infty$ over positive imaginary axis. In this case the contour just moves along positive half of the imaginary axis and we must determine the existence of zeros, because the Nyquist analysis is made to the CF. Then, the system is stable if it does not have any clockwise encircle around the origin, considering that the CF does not have any poles in the RHP.

$$CF = 1 - G_{oc}(s) \quad (2.9)$$

The Nyquist stability analysis has great advantages as it does not require the analytical characteristic function of the system and it can be used with measurements or simulations of the response of the open loop system.

A lot of references define $G_{oc}(s) = G(s) \cdot H(s)$, where the $G(s)$ is the direct gain of the loop and $H(s)$ is the feedback response, that has been defined as 1 in the proposed model, Fig. 2.2. For any of the cases, when the open loop Nyquist plot does not encircle the origin, stable network, the Gain Margin (G_m) and the Phase Margin (P_m) are defined, Fig. 2.5. These two parameters are the figures that define the stability robustness of the network.

Figure 2.5: Nyquist Gain and Phase Margin

The main problems are that the theorem of the variation of the argument determines the difference of zeros and poles ($Z - P$) and that a close loop architecture is required to use this criteria. Further considerations about the problems and provisos of this method for the analysis of high frequency active circuits are presented in Sec. 4.3.

2.2.4 Bode

This method of studying the stability is based on the Bode plots [Kai80] and it is valid for a closed-loop definition of a system as in Fig. 2.2. The method is based in the representation of the gain and phase response of the open loop circuit. So, this method is easily applicable to a circuit whatever its complexity if it is possible to open the loop and make measurements of the phase and magnitude of the direct gain. This method can be considered as a simplification of the Nyquist criteria that uses the Bode plots that are the traditional representation of the response of an amplifier or any two port circuits.

There are two important points to define in the plots. In the magnitude plot, the Gain cross over frequency (f_{go}) is the frequency that makes the Gain equal to 0 dB . In the phase plot the relevant point is the phase crossover frequency (f_{po}), that is the frequency where phase shift is equal to -180 degrees.

The Bode stability criteria is: If at the phase crossover frequency (f_{po}), the corresponding log modulus of the Gain is less than 0 dB , then the feedback system is stable.

In the Bode plots, Fig. 2.6 two systems are represented, they have different magnitude response, but the same phase response. The system with blue magnitude is not stable and the system with red magnitude is stable. Then, it is possible to define the margins of the gain magnitude and of the phase, they are represented in the colour of each system in Fig. 2.6. The Gain Margin (G_m) is given by Equ. 2.10 and the Phase Margin (P_m) is given by Equ. 2.11.

$$G_m = \frac{1}{g(f_{po})} \quad (2.10)$$

$$P_m = 180 + \arg(g(f_{go})) \quad (2.11)$$

Figure 2.6: Open-Loop Bode plots

2.2.5 Barkhausen

The Barkhausen stability criterion is a mathematical condition to determine when a linear circuit will oscillate. It was presented by Heinrich Georg Barkhausen in 1921. It is widely used in the design of feedback oscillators and to study the stability of operational amplifiers.

The criterion determines that a circuit defined as in Fig. 2.2 will oscillate if the magnitude of the open Loop-Gain $|G_{oc}(f)|$ is greater than 1 ($> 0\text{ dB}$) for any f_o that complies that the phase is -180 , $\arg(G_{oc}(f_o)) = \pi + 2n\pi$, where n is any integer.

It can be considered as a simplification of the Nyquist criterion. Barkhausen's criterion applies to linear circuits with a feedback loop. Therefore it cannot be applied to negative resistance or admittance oscillators. Barkhausen's criterion is a necessary condition for oscillation but not a sufficient condition.

It is well-known that multiple circuits satisfy the criterion but they do not oscillate. In a Nyquist sense, knowing that a circuit satisfies Barkhausen criterion only means that there may be encirclements of the $s = -1$ point, but maybe there are not any encirclements, Fig. 2.7. However, this fact alone does not have enough information to assess stability or oscillation. Without more information, it cannot be known how many encirclements of the -1 there are or which is the direction of the encirclements. A more complex criterion can be defined considering the sloped of the phase crossings, it could be considered Bode plots of Nyquist analysis, but the base problem is not solved: the encirclements determine $Z - P$. So, it cannot be determined how many open-loop poles are in the RHP.

Figure 2.7: Barkhausen Problem

2.2.6 Rollett Proviso

Classic Rollett Proviso and the provisos required for traditional two-port circuits, typically amplifiers, are interesting because their similarity with the problematic of the methods of analysis for other circuits with one port as oscillators and NICs.

The Rollett proviso [Rol62] is suitable for determining the unconditional stability of a two port network. There have been defined multiple derived tests over the years. These tests are suitable to determine if any terminated two ports network have unstable characteristic frequencies (poles in the RHP). These tests

evaluate the stability in terms of the two-port parameters of the network, but these tests are only valid if another condition (proviso) is first satisfied. The provisos are often ignored because the omission of the verification is not typically a problem for circuits with one active device and because these provisos are not easily evaluated. Another reason for omitting the proviso verification is that during years the S-parameters were obtained by measurement of the two-port circuit, so it is known that it is stable for the load conditions used during the measurement.

When the stability criteria is based on parameters obtained by CAD, the omission of the verification of the proviso is a risk, this risk is increased with the number of active devices that the circuit contains.

The general two-port stability condition defined by Rollett determines that:

Theorem. *A circuit is unconditionally stable if the characteristic frequencies remain in the left half of the complex frequency plane for any arbitrary passive terminations.*

The invariant stability factor defined by Rollett is Equ. 2.12.

$$k \triangleq \frac{2\rho_{11}\rho_{22} - \Re(\gamma_{12}\gamma_{21})}{|\gamma_{12}\gamma_{21}|} \quad (2.12)$$

where γ is any of the z , y , h , g two-port parameters; $\rho_{11} = \Re(\gamma_{11})$ and $\rho_{22} = \Re(\gamma_{22})$.

The k parameter can get values from $+\infty$ and -1 when $\rho_{11}, \rho_{22} \geq 0$. The k factor can be written as a function of different parameters, for example as a function of S parameters, Equ. 2.13.

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |S_{11}S_{22} - S_{21}S_{12}|^2}{2|S_{12}S_{21}|} \quad (2.13)$$

The k factor [Jac06, PS94] must be greater or equal to unit ($k \geq 1$) for unconditionally stable condition. It is a necessary condition, but not sufficient or unconditional stability condition. To achieve unconditional stability, it must satisfy Equ. 2.13 and either Equ. 2.14, Equ. 2.15 or Equ. 2.16.

$$\Delta = |S_{11}S_{22} - S_{21}S_{12}| < 1 \quad (2.14)$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 > 0 \quad (2.15)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 > 0 \quad (2.16)$$

The k factor is presented in multiple textbooks [Rhe10, VPR05, Eve00, Gon84, Ha81] as a stability analysis, but it is not a stability condition, it could be defined as a parameter to study the conservation of the stability.

The proper definition of k factor must be:

Proposition. *If a network is stable on a particular load condition, it is that it must not have any poles in the RHP for a defined load condition. It will be stable for any passive load condition if and only if $k > 1$ and $|\Delta| < 1$ for all ω .*

Where $|\Delta|$ is the determinant of the S matrix of the network.

The condition $|\Delta| < 1$ can be replaced by Equ. 2.15 or Equ. 2.16.

2.2.7 Criterium in Terms of the Single Stability Parameter μ

The μ stability factor [KM10] can be derived from the k factor and the additional condition. This μ factor formulation is an easier alternative. It can be defined at source (μ_s) and load (μ_l) of the two port network [BGP01]: Equ. 2.17 and Equ. 2.18.

$$\mu_s = \frac{1 - |S_{11}|^2}{|S_{22} - \Delta(S_{11}^*)| + |S_{21}S_{12}|} \geq 1 \quad (2.17)$$

$$\mu_l = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta(S_{22}^*)| + |S_{21}S_{12}|} \geq 1 \quad (2.18)$$

The analysis of any of the two formulations, load or source, is enough to guarantee the stability of the circuit with any loads on the ports if the stability with a set of loads has been previously confirmed. The confirmation of the stability of the network with a set of loads is out of the scope of this method, the μ parameter confirms the stability of the network with any load, but just after checking the stability with a set of loads using any other stability analysis method. The method to determine the stability with a particular set of loads can be any of the linear stability methods.

2.3 Unconditional Stability of a Quadrupole Defined by S Parameters

There are multiple tests to evaluate the unconditional stability of a two port system. Some of these test are the $K - \Delta_S$ pair and the μ . Any of these tests can be defined using any of the parameters that define a two ports quadrupole as

Z , Y , S , etc. It is important to remember that any of these tests are only valid if some previous provisos are satisfied, but in some cases the previous provisos are difficult to check and in a lot of cases they are simply forgotten.

The stability conditions for a two ports quadrupole that is defined by the S -parameters is discussed in the next section.

2.3.1 Stability Conditions of Two Ports System

This section shows the requirements set that guarantee the unconditional stability of a two ports system. The case of a system defined with its S parameters is used, this is the case defined as Rollett proviso [Rol62]

Figure 2.8: Quadrupole defined by S parameters

Fig. 2.8 shows a generic quadrupole that is defined by its S parameters and it is loaded with arbitrary impedances: Z_S and Z_L . This quadrupole is unconditionally stable if it does not have any poles with positive real part. In other words it must not have any poles in the complex Right Half Plane (RHP). Then, the quadrupole is unconditionally stable if it is not possible to find a pair of loads that make the quadrupole have poles in the RHP.

The system is defined using the scattering parameters (S -parameters). The S -parameters are defined using the impedances Z_{0S} y Z_{0L} as reference impedances. When this system is loaded with other impedances than the ones used to define the S parameters, the system response is defined by $[A]$. The $a_1(s)$ y $a_2(s)$ are the Laplace transformation of the incident waves, in this way the response is defined by Equ. 2.19.

$$\begin{bmatrix} b_1(s) \\ b_2(s) \end{bmatrix} = [A] \begin{bmatrix} a_1(s) \\ a_2(s) \end{bmatrix} \quad (2.19)$$

$$[A] = \frac{1}{D(s)} \begin{bmatrix} S_{11}(1 - S_{22}\Gamma_L) + S_{12}S_{21}\Gamma_L & S_{12} \\ S_{21} & S_{22}(1 - S_{11}\Gamma_S) + S_{12}S_{21}\Gamma_S \end{bmatrix} \quad (2.20)$$

$$D(s) = (1 - S_{11}\Gamma_S)(1 - S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_L\Gamma_S \quad (2.21)$$

This is when the excitations are considered impulses, $\delta(t)$, it is considering the excitation as a unit matrix. Then the response has the same poles than the matrix $[A]$ when the quadrupole has any poles. This way, the quadrupoles defined by their scattering parameters will be unconditionally stable when none of its poles for any passive Z_s and Z_L loads are in the RHP.

The problem of this methodology to determine the unconditionally stability of a system is that the poles of the $[A]$ matrix are a sub-group of the poles of the system. It can exist poles that are not visible from the ports of the system, as it was described by Platzker [PS94]. Using the control theory, it can be assumed that may exist poles or modes that cannot be observed from the ports, and they cannot have any influence in the impedances of the ports. To use the Rollett proviso for a two ports quadrupole it is necessary to define the S -parameters using a pair of loads that do not present any poles in the RHP (proviso for Rollett test).

The simplest case to analyze $[A]$ is when the load impedances are the ones that were used to define the scattering parameters, in this case the $[A]$ matrix is simplified to $[S]$, because Γ_L and Γ_S are zero.

When passive loads but different from the ones used to define the S parameters are used, Γ_L and Γ_S are not zero. Assuming that the S parameters have not any poles in the RHP, if the system has any RHP poles they must be determined by the poles of $D(s)$.

As a first step, when the case of $\Gamma_S = 0$ is considered, $D(s)$ can be simplified to Equ. 2.23.

$$D(s) |_{\Gamma_S=0} = (1 - S_{22}\Gamma_L) \quad (2.22)$$

When the Nyquist criteria is applied to this case, Equ. 2.23, it can be determined that the system is unconditionally stable; it has not any poles in the RHP, for $\Gamma_S = 0$ and any passive Γ_L if $|S_{22}| < 1$ for any ω .

Now the case when $\Gamma_S \neq 0$ is analyzed. The equation of $D(s)$ can be re-written as Equ. 2.23.

$$D(s) = (1 - S_{22}\Gamma_L)(1 - \Gamma_{in}\Gamma_S) \quad (2.23)$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (2.24)$$

It has been concluded that if $|S_{22}| < 1$ for any ω , the first term of Equ. 2.23 does not have any poles in the RHP, so now the second term of the equation must be analyzed. Then the $\Gamma_{in}\Gamma_S$ clockwise encircles around the +1 determine the number of poles in the RHP. As $|\Gamma_S| < 1$, there are not any poles if $|\Gamma_{in}| < 1$ for any ω .

Then the provisos that guarantee the unconditional stability of a S -parameters defined quadrupole are:

A conditions:

1. The network must not have any RHP poles when it is loaded with Z_{01} and Z_{02} .
2. $|S_{22}| < 1$ for any ω .
3. $|\Gamma_{in}| < 1$ for any ω and any passive Γ_L .

If the system does not have any hidden poles, the first condition of A is equivalent to say that the S -parameters do not have any poles in the RHP.

If these rationale is performed taking as a first step $D(s)$ and $\Gamma_L = 0$, the condition A.1 is “ $|S_{11}| < 1$ for any ω ” and the condition A.3 is “ $|\Gamma_{out}| < 1$ for any ω and for any passive Γ_S ”.

The problem of these provisos is that it is necessary to examine infinite number of passive Γ_L to conclude the stability of the circuit. Multiple authors use the well-known K-factor to study the stability. Using the K-factor, the stability provisos are:

B conditions:

1. The network must not have any RHP poles when it is loaded with Z_{01} and Z_{02} .
2. $1 - |S_{22}|^2 > |S_{21}S_{12}|$ for any ω .
3. $K > 1$ for any ω .

where

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta_S|^2}{2 |S_{12}S_{21}|} \quad (2.25)$$

and Δ_S is the determinant of the S -parameter matrix.

An alternative set of provisos for B is the following set.

C conditions:

1. The network must not have any RHP poles when it is loaded with Z_{01} and Z_{02} .
2. $|\Delta_S| < 1$ for any ω .
3. $K > 1$ for any ω .

A most convenient way to condense these three conditions of any of the previous sets in two conditions is the use of μ parameter [ES92]. The two conditions are:

D conditions:

1. The network must not have any RHP poles when it is loaded with Z_{01} and Z_{02} .
2. $\mu > 1$ for any ω .

where

$$\mu = \frac{1 - |S_{11}|^2}{|S_{22} - S_{11}^* \Delta_S| + |S_{12} S_{21}|} \quad (2.26)$$

In all the previous conditions sets, the first point guarantee the “Rollett proviso”. In systems where there are not hidden poles (all the poles are observable from the ports), it is equivalent as not having any poles in the RHP in the S -parameters.

The conditions set can also be defined for other parameters, as example the A and C conditions sets for Z -parameters are shown.

A_Z Conditions:

1. The must not have any RHP poles when it is loaded with open-circuits.
2. $\operatorname{Re}(Z_{22}) > 0$ for any ω .
3. $\operatorname{Re}(Z_{in}) > 0$ for any ω and any passive Z_L .

C_Z Conditions:

1. The must not have any RHP poles when it is loaded with open-circuits.
2. $Re(Z_{22}) > 0$ for any ω .
3. $K_Z > 1$ for any ω .

where

$$K_Z = \frac{2Re(Z_{11})Re(Z_{22}) - Re(Z_{21}Z_{12})}{|Z_{21}Z_{12}|} \quad (2.27)$$

There are well-known cases (Sec. 2.5) where the omission of the check of one of the steps of the condition set take to an erroneous conclusion. But the main problem of these stability check lists is that the Rollett proviso cannot be guaranteed when the poles are not visible from the ports.

2.4 NDF

Platzker and Struble [PS94] presented that it is not possible to determine the stability of a N-node network when a two-ports simplification is used. This limitation can be extended to any of the previous classic methods, they are not rigorous and can provide wrong conclusion including the K and μ factors because they depend on another method to ensure the stability with a load condition.

The reason of the problem to determine the stability of a network suffered by the classic methods is not caused by symmetries (odd-even modes) or by the lack of additional conditions [PSH93]. The problem is that the stability of a network can only be studied on the network itself, but it cannot be studied on a reduced version of the network.

To determine the number of poles of a network on an easy way and without any restriction imposed by simplifications of the network, Platzker and Struble defined a new function, Normalized Determinant Function (NDF) Equ. 2.28, and then they use the Principle of the Argument Theorem on it (Sec. 2.2.3).

$$NDF = \frac{\Delta(S)}{\Delta_0(S)} \quad (2.28)$$

where $\Delta(S)$ is the determinant of the network under study and $\Delta_0(S)$ is the determinant of an identical network but with all the active generators disabled.

The stability test can be defined as:

A N -node network is stable if the Nyquist analysis for positive frequencies (from 0 to ∞) of the NDF of the complete network does not have any clockwise encircles around the origin.

This new function has some interesting properties:

- The NDF does not have any poles in the RHP, because $\Delta_0(S)$ is the determinant of a passive network and none of the zeros of $\Delta_0(S)$ are in the RHP.
- The denominators of $\Delta(S)$ and $\Delta_0(S)$ are the same, so they are cancelled in the NDF.
- $NDF \rightarrow 1$ as $\omega \rightarrow \infty$ and $\omega \rightarrow -\infty$, because $\Delta(S)$ and $\Delta_0(S)$ are polynomials with the same order of S .
- The NDF can be reduced to the $j\omega$ formulation, for values of S over the imaginary axis.
- As the response of the network must be a real function on time: $NDF(-j\omega) = NDF^*(j\omega)$.

The NDF and its relation with the Return Ratios (RR) [Bod45, Mac82] was demonstrated by Platzker and Struble and proposed as a general procedure to obtain the $NDF(j\omega)$ [SP93, PS94]. The RR was used by Bode, Maclean and others to study the stability of feedback amplifiers. The RR is defined by the ratio of an external voltage or current source that controls a controlled current or voltage source and the voltage or current on the original control point, Fig. 2.9, Equ. 2.29.

$$RR = -\frac{V_{12}}{V_C} \quad (2.29)$$

It can be proved that the RR can be used to calculate the NDF, for example, taking the Y matrix of the network under study, Equ. 2.30., and calculating the determinant of this matrix, it can be concluded that the relation between the RR and the NDF is Equ. 2.31, [SP93].

$$\begin{bmatrix} [y_{11}] & [y_{12}] & [y_{13}] & [y_{14}] & \cdots \\ [y_{21}] & [y_{22}] & [y_{23}] & [y_{24}] & \cdots \\ [y_{31} + g_m] & [y_{32} - g_m] & [y_{33}] & [y_{34}] & [Y_{15}] \\ [y_{41} - g_m] & [y_{42} + g_m] & [y_{43}] & [y_{44}] & \cdots \\ \cdots & \cdots & [Y_{51}] & \cdots & [Y_{55}] \end{bmatrix} \quad (2.30)$$

Figure 2.9: RR of N-Node Network with Single Dependent Current Source

$$RR = \frac{|Y|}{|Y_0|} - 1 = NDF - 1 \quad (2.31)$$

This discussion is applicable to networks with one dependent source, but it was extended by Struble and Platzker for network with any number of dependent sources, [SP93]. The general NDF equation as function of the RR of a network with several dependent sources is Equ. 2.32:

$$NDF = \frac{\Delta}{\Delta_0} = \frac{\Delta}{\Delta_0^{(1)}} \cdot \frac{\Delta_0^{(1)}}{\Delta_0^{(1,2)}} \cdot \frac{\Delta_0^{(1,2)}}{\Delta_0^{(1,2,3)}} \cdots \frac{\Delta_0^{(1,2,\dots,n-1)}}{\Delta_0^{(1,2,3,\dots,n)}} = (1 + RR_1) \cdot (1 + RR_2) \cdot (1 + RR_3) \cdots (1 + RR_n) \quad (2.32)$$

where the sup-indexes are the representation of the dependent sources. Then determinant of the network is Δ and the determinant where only some dependent sources, from 1 to i , have been disabled are $\Delta_0^{(1,\dots,i)}$.

The Transpose Return Relation (RR_T) can be defined as Equ. 2.33. The RR_T is the *natural* open Loop-Gain of the active device. Then the NDF can be expressed as Equ. 2.34.

$$RR_T = \frac{V_{12}}{V_C} = -RR \quad (2.33)$$

$$NDF = \prod_{i=1}^n (1 - RR_{T_i}) \quad (2.34)$$

The definition of the NDF, Equ. 2.34, is suitable for the calculation of the NDF and for the stability study of any network using any general linear simulator. This function has been programmed by the author and integrated in the NI-AWR© software as an alternative to the build-in NDF function. The use of the NDF was integrated by the author into AWR V9, September 2011 (Appendix A), before the actual built-in function was included on AWR V10 on February 2012.

2.5 Examples of Problematic Circuits

Some circuits with topologies as the Ring Oscillator, presented by Platzker, and the extension of the concepts of this ring oscillator to other circuits, as amplifiers built with parallel amplifier stages, do not match with classic stability methods. These circuits have solutions that indicate that they are stable, but they are not.

2.5.1 Ring Oscillator

One example of the problems that may happen when the assumption that there are not hidden poles in the RHP, just considering that the absence of poles in the RHP in the quadropole parameters is enough to satisfy the Rollett proviso is the Ring Oscillator.

A Simplified version of Platzker's oscillator [PSH93] is shown in Fig. 2.10. This circuit has two FETs (Field-Effect Transistor) connected in a ring and the ports are connected through resistors.

When the conditions (Sec. 2.3) $C_z.2$ and $C_z.3$ are checked, omitting the condition $C_z.1$, the result determines that the real part of Z_{11} is R and $K > 1$ when $R > g_m^{-1}$. So it is concluded that the circuit is unconditionally stable, but it has a pole when $s = g_m/C_g$. Then this circuit is unstable for open circuit loads and the condition $C_z.1$ is not satisfied.

Figure 2.10: Platzker's Ring Oscillator

The problem of the Ring oscillator can be presented on any circuit that has several active devices. When the number of active devices is increased, the complexity of the problem is increased in a greater factor.

An example of one simple circuit based on the ring oscillator is presented in Fig. 2.11. This circuit can be analyzed using any of the classic methods and the results indicate that the circuit is stable. In this case, the k factor and the μ factor are calculated, and both of them predict unconditional stability, Fig. 2.12.

These inconsistent solutions of the classic methods where the start point of Platzker's work to introduce the use of the NDF as an alternate method for amplifier stability analysis without the deficiencies of the classic methods. The NDF can be obtained using the RR_T , Equ. 2.34, with any linear analysis tool.

Figure 2.11: Ring Oscillator Example

Figure 2.12: Ring Oscillator Example stability Factors

The circuits to solve for a case with two active devices is shown in Fig. 2.13. The automatic NDF tool for NI-AWR© based on the use of RR_T , built by the author is used for this example.

The use of the AWR native NDF or the NDF obtained applying Equ. 2.34 to the circuits in Fig. 2.13 provide the same solution, Fig. 2.14. Then the NDF predicts an unstable condition of the circuit in Fig. 2.11 when it is loaded with Z_0 , as a contradiction of the classic stability methods, Fig. 2.12. So the NDF is able to solve these set of cases that classic methods cannot.

(a) Circuit to Obtain RR_{T_1} (b) Circuit to Obtain RR_{T_2} Figure 2.13: Circuit to Calculate the NDF Using the RR_T of the Ring Oscillator Example

Figure 2.14: NDF of Ring Oscillator Example

2.5.2 Amplifier of Two Parallel Stages

Designing amplifiers presents the problem of parametric oscillations. This problem is more common in amplifiers built with parallel stages and it is impossible to detect during linear analysis with classic methods when it is a Odd-Mode oscillation, because the odd-mode oscillations are not observable from the ports

of the simplified network. There are several approximations for these problems as the Odd-Mode oscillation analysis [ACP⁺03, Fre92]. But the chosen solution is to include resistors between stages, due to the root problem cannot be analyzed on linear steps with classic methods. Then, to eliminate the oscillation that is detected on non-linear analysis, additional resistances are included, so some available gain is sacrificed.

The NDF is a suitable tool to analyze this problem on the linear steps of the design. To illustrate this, an amplifier based on two parallel stages, Fig. 2.15, is analyzed. The gain of each basic amplifier is shown in Fig. 2.16.

Figure 2.15: Amplifier Stages

Figure 2.16: Gain of Amplifier Stages

The amplifier, Fig. 2.17, includes two basic amplifiers, DC-block capacitors and two extra 3 pF capacitors to tune the frequency response. The gain of the amplifier is tuned for not providing any gain above 3.3 GHz , Fig. 2.18. The shunt resistor used to eliminate the odd-mode oscillations is not included, but even when it is included, if is too high, it cannot eliminate the odd-mode oscillation. The shunt

resistor causes loss of gain, and it is required because the root problem of the odd-mode oscillation is hard or impossible to analyze on linear steps, so it is included to solve the problem, in case it exists, during non-linear analysis.

Figure 2.17: Amplifier of Two Parallel Stages

Figure 2.18: Gain of Amplifier of Two Parallel Stages

The classic linear analysis of an amplifier usually starts with the S parameters analysis of the amplifier, Fig. 2.19. The response of this amplifier shows a response without any poles on any of the S parameters, so this amplifier is stable when it is loaded with Z_0 according to classic methods theory.

Furthermore, the unified μ stability factor can be obtained for this amplifier, Fig. 2.20. The stability factor confirms, according to classic theory, that the amplifier is unconditionally stable.

But when this amplifier is analyzed using the proposed NDF Method, Fig. 2.21, the existence of a pair of poles in the RHP is detected. These poles were not detected by classic methods due to they use a simplified network. The poles are at a frequency of 5.722 GHz , so this amplifier must be unstable at this frequency.

Figure 2.19: S Parameters of Amplifier of Two Parallel Stages

Figure 2.20: μ_1 Factor of Amplifier of Two Parallel Stages

An HB analysis confirms that the amplifier is unstable. It presents an oscillation at 5.8366 GHz , that is an odd-mode oscillation. The oscillation has an amplitude of 28.75 dBm at the point VP1 in Fig. 2.17 and 16.31 dBm at VP4 while the amplitude is -82.63 dBm and -86.47 dBm at VP2 and VP3, respectively. So this oscillation is not visible from the external ports of the amplifier, then the poles can be defined as hidden poles of the simplified network of the quadrupole.

The traditional solution for this undesired oscillation is to include a shunt resistor that produces gain lost. If the NDF is used, other solutions can be found as for example, in this circuit it can be confirmed, using the NDF and the HB, that to change the length of transmission lines at the input of the basic amplifier from 12 mm to 8 mm is a valid solution to solve the odd-mode oscillation that does not require any shunt resistor, Fig. 2.23.

Figure 2.21: NDF of Amplifier of Two Parallel Stages

Figure 2.22: Spectrum of Amplifier of Two Parallel Stages

Figure 2.23: NDF of Amplifier of Two Parallel Stages with Input Lines of 8 mm

2.6 Conclusion on Stability Study

The most important conclusion that can be taken from the study of the classic stability methods is that all of them require a set of provisos before they can be used. The provisos can be studied or presented in different forms and formulations that have changed on time, but all of them are required to guarantee that the network is stable with a particular condition (unloaded, short-circuit or Z_0 loaded). A common practice to check the stability of a network with a particular condition is to consider that a network loaded with Z_0 is stable when its S parameters do not have any poles, but this method and any other methods that use a simplified network are not valid because hidden poles can exist. Other important problem is that the classic methods provide different solutions for autonomous circuits, even when provisos are checked, when different virtual grounds or network drawing are used. Table 2.1 summarizes the problems of the most popular classic methods.

Method	Problems
BIBO	It uses a simplified version of the network, so hidden poles can exist.
Routh-Hurwitz	It requires the analytical expression of the Characteristic Function.
Nyquist	It provides the figure $Z - P$ so the number of one of them must be determined by an alternative method.
Bode	It is a simplification of Nyquist. It has problems to detect when there are multiple crosses over real negative axis.
Barkhausen	It is a simplification of Nyquist. It has similar problems to Bode.
Rollett Proviso (k)	It uses a simplified version of the network, so hidden poles can exist. It requires a previous check of stability with a load condition.
Single Parameter (μ)	It is an improvement of Rollett Proviso, but with the same problems.
Loop-Gain	It requires previous provisos, it has dependence with virtual ground and it uses a simplified version of the network, so hidden poles can exist.

Table 2.1: Stability Methods Comparison

So, to solve the first step, in order to ensure that the network does not have any poles in the RHP (that it is stable) for a load condition, the NDF is the best tool as it does not use any simplification of the network and it can detect any kind of instability, wherever it is observable or not from the ports of the network.

The NDF Method solves the problem of several circuits that have been a handicap for the classic methods as the ring-oscillator [PSH93] and circuits with the called “*odd-mode*” instabilities [Fre92].

To summarize, the NDF is a tool for linear stability study of any network. It is applicable to autonomous and non-autonomous circuits and it is independent of the topology or geometry of the circuit.

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Chapter 3

METAMATERIALS

3.1 Introduction

This chapter covers the basic Transmission Lines (TL) concepts and the approximation of the Metamaterials as a Left Handed (LH) behaviour of a transmission line. The Metamaterials are those materials that have a negative ε and a negative μ at the same time. These materials were formulated by Viktor Veselago [Ves68] and his theoretical idea was the start point for the study of Left Handed Materials. These LH materials are artificial materials and they can be 1-Dimension, 2-Dimensions or 3-Dimensions. The 1-D metamaterials are the scope of this study and for convenience aspects they are approximated as a TL.

Metamaterials are useful elements on the design of active antennas because they can be used for adjusting the band-width of the antenna and the matching response of active devices. Metamaterials are also interesting to include band diplexing on TX/RX active antennas. The use of metamaterials allows the combination of transmitter and receiver function on active antennas without requiring a switch device. They are studied in this work because their interesting properties make them candidates for being used on future active patches and arrays. They make possible the frequency tuning of active and passive devices, so they can be used in patches, hybrids, arrays and active circuits such as amplifiers or oscillators; and of course on combinations of these circuits in active antennas.

3.2 Pure Right Hand (PRH) Transmission Lines

The transmission lines theory considers that the circuit size is comparable or greater than the λ , while the general circuit theory considers that λ is bigger than the circuit on study. The transmission line theory considers that it has distributed parameters, that are function of the network length. This distributed parameters make the voltage and current change along the transmission line. The transmission line theory is a particular simplification of the Maxwell equations. The general transmission line theory is covered in multiple bibliography as for example in the classic book of Pozar [Poz98].

As a simplification of the study of the transmission lines and of the metamaterials models as transmission lines, the models are considered lossless. The model of a conventional (Right Hand) transmission lines is in Fig. 3.1.

The general equations in the time domain for the PRH transmission line are Equ. 3.1, but for the case of sinusoidal signals they can be simplified to Equ. 3.2.

Figure 3.1: Pure Right Hand (PRH) Transmission Line Model

$$\frac{\partial v(z,t)}{\partial t} = -R' i(z,t) - L' \frac{\partial i(z,t)}{\partial t} \quad (3.1)$$

$$\frac{\partial i(z,t)}{\partial t} = -RG' v(z,t) - C' \frac{\partial v(z,t)}{\partial t}$$

$$\frac{dV(z)}{dz} = -(R' + j\omega L') \cdot I(z) \quad (3.2)$$

$$\frac{dI(z)}{dz} = -(G' + j\omega C') \cdot V(z)$$

The solution of the wave equations that define the transmission line is Equ. 3.3.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (3.3)$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

where

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R' + j\omega L') \cdot (G' + j\omega C')} \quad (3.4)$$

The propagation constant, Equ. 3.4, is an important factor of the transmission line and when it is considered loss-free, it has not real part. The conventional transmission lines have positive imaginary part, while the metamaterials have negative imaginary part.

Another important parameter of a transmission line is its characteristic impedance (Z_0), which is Equ. 3.5 in the case of a conventional transmission line.

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \sqrt{\frac{L'}{C'}} \quad (3.5)$$

In more general expressions the characteristic impedance and the propagation constant of any transmission line are Equ. 3.6 and Equ. 3.7.

$$Z_0(\omega) = \sqrt{\frac{Z'(\omega)}{Y'(\omega)}} \quad (3.6)$$

$$\gamma(\omega) = \sqrt{Z'(\omega) \cdot Y'(\omega)} \quad (3.7)$$

where Z' is the series impedance and Y' is the parallel admittance, Fig. 3.1.

From Equ. 3.3 it is possible to obtain other important parameters of the transmission lines. These parameters are the wave length (λ), Equ. 3.8, the phase speed (V_f), Equ. 3.9, and the group speed (V_g), Equ. 3.10.

$$\lambda = \frac{2\pi}{\beta} \quad (3.8)$$

$$V_f = \frac{\omega}{\beta} \quad (3.9)$$

$$V_g = \left(\frac{\partial \beta}{\partial \omega} \right)^{-1} \quad (3.10)$$

3.3 Composite and Dual Right-Left Hand (CRLH/D-CRLH) Transmission Lines

Metamaterials are artificial structures that must be homogeneous, in practical cases a material can be considered as homogeneous if the cell size is less than $\lambda/4$. Then this artificial material has a *normal* behaviour as a real homogeneous material for wave propagation. If the possible combinations of signs of ϵ and μ are considered, there are four cases, but only two of them have wave propagation. These cases are $\epsilon > 0, \mu > 0$ and $\epsilon < 0, \mu < 0$. The positive case is the one of the conventional materials and the negative one is the case of the metamaterials (LH).

In 1967, the first theoretical study [Ves68] of the behaviour of a hypothetical ($\epsilon < 0, \mu < 0$) LH material was presented, but a LH material was not developed until 1996 [PHSY96, PHRS98, PHRS99]. The first study of a LH transmission line was presented by Brillouin, but the transmission line that was proposed cannot be considered a metamaterial because it cannot be considered homogeneous, so it cannot be characterized using β and μ ; these lines are usually refereed as a backward-wave circuit. The first metamaterial is dated in 2000 as a result of the work of Smith [SPV+00]. After Smith's paper, a lot of works were published, and

some of the most relevant are by Cristophe Caloz [Cal06, CN07, CI13]. Actual technology have always a parasitic series inductance and a parallel capacitance, so the practical lines have always a RH response for upper frequencies.

Some of the models of Composite Right-Left Hand transmission lines (CRLH) models are shown in Table 3.1. The properties and a deep study of multiple models are shown in [CI13, CPnMGEP09].

	Line	Dispersion Diagram
1
2
3
4
5
6

Table 3.1: First Eight Lossless Transmission Line Models

	Line	Dispersion Diagram
7
8

Table 3.1: First Eight Lossless Transmission Line Models

These CRLH transmission lines are really interesting elements to tune and control the response of microwave circuits. They can be applied to improve the bandwidth of antennas [GWH11, TBML11, NER09], antennas efficiency [LLW⁺11] or microwave circuits tuning [HTY11, pCGL10].

Two models of CRLH lines are detailed in this work, D-CRLH and Tri-CRLH. These models of CRLH are used to develop some microwave devices that are mainly used in front-end of communication systems as diplexers and hybrids.

3.3.1 D-CRLH Line

The Dual Composite Right-Left Hand transmission lines (D-CRLH) (Table 3.1, Dual 2) were proposed by Caloz [Cal06]. Caloz posed that the structure in Fig. 3.2 behaves as the complementary structure of the conventional CRLH cell. This structure has a series parallel and a shunt series LC tanks. The D-CRLH indeed exhibits its left-handed band at high frequencies, its right-handed band at low frequencies and it has a nature band-stop between left-handed and right-handed bands.

The equations that define the performances of the balanced D-CRLH line are given in Equ. 3.11 and Equ. 3.13. If the resonance frequencies of the shunt LC tank and the series LC tank are ω_{sh} and ω_{se} , a balanced condition can be achieved by satisfying Equ. 3.11. Most of the designs are built for this balanced condition.

$$\omega_{sh} = \sqrt{\frac{1}{L_L C_R}} = \omega_{se} = \sqrt{\frac{1}{L_R C_L}} \quad (3.11)$$

Once the D-CRLH line is balanced, a right-handed frequency (ω_R) and a left-handed frequency (ω_L), Equ. 3.12, can be defined.

$$\omega_R = \sqrt{\frac{1}{L_R C_R}}, \quad \omega_L = \sqrt{\frac{1}{L_L C_L}} \quad (3.12)$$

Figure 3.2: Schematic of the D-CRLH Line

For the balanced case, Equ. 3.11 can be rewritten as Equ. 3.13.

$$\omega_0 = \sqrt{\omega_{sh} \cdot \omega_{se}} = \sqrt{\omega_L \cdot \omega_R} = \omega_{sh} = \omega_{se} \quad (3.13)$$

The characteristic impedance of any transmission line is defined by Equ. 3.14 and the propagation constant is defined by Equ. 3.15.

$$Z_0(\omega) = \sqrt{\frac{Z'(\omega)}{Y'(\omega)}} \quad (3.14)$$

$$\gamma(\omega) = \sqrt{Z'(\omega) \cdot Y'(\omega)} \quad (3.15)$$

The characteristic impedance of the D-CRLH line can be defined as Equ. 3.16 and for the balanced case, $\omega_{se} = \omega_{sh}$, as Equ. 3.17.

$$Z_0(\omega) = Z_R \sqrt{\frac{1 - (\omega/\omega_{sh})^2}{1 - (\omega/\omega_{se})^2}} \quad (3.16)$$

$$Z_0(\omega) = Z_0 = Z_R = Z_L = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{L_L}{C_L}} \quad (3.17)$$

Finally, the propagation constant per unit cell (γ) is given as Equ. 3.18 and for the balanced case, $\omega_{se} = \omega_{sh}$, as Equ. 3.19.

$$\gamma(\omega) = \sqrt{j \cdot \frac{\omega L_R}{1 - (\omega/\omega_{se})^2} \cdot j \cdot \frac{\omega C_R}{1 - (\omega/\omega_{sh})^2}} = j \cdot \frac{\omega}{\omega_R} \sqrt{\frac{\omega_{se}^2 \omega_{sh}^2}{\omega_{se}^2 \omega_{sh}^2 - \omega_{se}^2 \omega^2 - \omega_{sh}^2 \omega^2 + \omega^4}} \quad (3.18)$$

$$\gamma(\omega) = j \cdot \frac{\omega \omega_L}{\sqrt{\omega_{se}^2 \omega_{sh}^2 - \omega^2 (\omega_{se}^2 - \omega_{sh}^2) + \omega^4}} = j \cdot \frac{\omega \cdot \omega_L}{\omega^2 - \omega_0^2} = j \cdot \beta(\omega) \quad (3.19)$$

If the unit cell of Fig. 3.2 is repeated to form a periodic infinite transmission line, the input impedance is recursively repeated at the successive nodes. Then, the overall input impedance is given as Equ. 3.20.

$$Z_{in} = Z' + \left[\frac{1}{Y'} \parallel Z_{in} \right] = Z' + \frac{Z_{in}/Y'}{(1/Y') + Z_{in}} \Rightarrow Z_{in} = \frac{Z'}{2} \left[1 \pm \sqrt{1 + \frac{4}{Z' \cdot Y'}} \right] \quad (3.20)$$

Applying the procedure presented in [CI13] for CRLH lines, the cut-off frequencies of the infinite transmission line approach of the D-CRLH can be calculated. This approach provides accurate results for the -10 dB cut-off frequency when the number of cells is larger than 3. Then, by substituting Z' and Y' with the expressions in terms of ω_{sh} and ω_{se} ; and calculating the roots of the radical of Z_{in} in Equ. 3.20 results in the cut-off frequencies for the left-handed and the right-handed regions defined in Equ. 3.21.

$$\omega_{CL,R} = \omega_0 \sqrt{1 + \frac{\omega_L}{8\omega_R} \pm \sqrt{\frac{\omega_L}{4\omega_R} \sqrt{1 + \frac{\omega_L}{16\omega_R}}}} \quad (3.21)$$

where the sign “+” is for the left-handed high-pass cut-off and the sign “−” is for the right-handed low-pass cut-off. It must be noted that $\omega_{CL} > \omega_{CR}$.

3.3.2 Tri-CRLH Line

This line is a Composite Left-Right-Left Hand Line that is suitable for three band tuning. The equivalent circuit of the unit cell of this composite line is similar to the models proposed for composite lines by Caloz [Cal06] and Itoh [SCI04].

The model of the Tri-CRLH lossless transmission line is shown in Fig. 3.3.

It is possible to analyze this line using the same steps as in the D-CRLH case. The first step is to calculate the series impedance (Z') and the parallel admittance (Y'), they are defined respectively by Equ. 3.22 and Equ. 3.23.

$$Z'(\omega) = j \cdot \left[\frac{\omega \cdot L_R}{1 - (\omega/\omega_s)^2} - \frac{1}{\omega \cdot C_P} \right] \quad (3.22)$$

Figure 3.3: Tri-CRLH Line Model

$$Y'(\omega) = j \cdot \left[\frac{\omega \cdot C_R}{1 - (\omega/\omega_P)^2} - \frac{1}{\omega \cdot L_P} \right] \quad (3.23)$$

where

$$\omega_S^2 = \frac{1}{L_R \cdot C_L} \quad \text{and} \quad \omega_P^2 = \frac{1}{L_R \cdot C_P} \quad (3.24)$$

The characteristic impedance of any transmission line is defined by Equ. 3.14. When this equation is formulated as a function of the equivalent capacitors and inductors of the Tri-CRLH the characteristic impedance is obtained in Equ. 3.25.

$$Z_0(\omega) = \sqrt{\frac{\omega \cdot L_R - \frac{1 - (\omega/\omega_S)^2}{\omega \cdot C_P}}{\omega \cdot C_R - \frac{1 - (\omega/\omega_P)^2}{\omega \cdot L_P}}} \quad (3.25)$$

In the same way, taking Equ. 3.15, the propagation constant can be obtained, Equ. 3.27, as a function of $L_{R,P}$ and $C_{P,R}$.

$$\gamma(\omega) = \sqrt{Z'(\omega) \cdot Y'(\omega)} \quad (3.26)$$

$$\beta(\omega) = \sqrt{\left[\omega \cdot L_R - \frac{1 - (\omega/\omega_S)^2}{\omega \cdot C_P} \right] \cdot \left[\omega \cdot C_R - \frac{1 - (\omega/\omega_P)^2}{\omega \cdot L_P} \right]} \quad (3.27)$$

The study of Equ. 3.26 shows that the propagation constant has a left behaviour ($\beta < 0$) for low and high frequencies and a right behaviour ($\beta > 0$) for frequencies in the middle of those bands. This line can be balanced ($\omega_S = \omega_P$) or unbalanced ($\omega_S \neq \omega_P$), but as in most of the designs [CN07, CPnMGEP09, HMGPIVSV07], the balanced case is the most common.

3.4 Numerical Solution of Composite Right-Left Hand (CRLH) Transmission Lines

The CRLH transmission lines are suitable for tuning the frequency response of multiple microwave circuits. To use the CRLH lines to tune these circuits, it is necessary to design the CRLH lines with the proper phase response at each frequency of interest. The problem is that the phase of the propagation constant of the CRLH lines must be solved by numerical approximation.

This numerical problem can be easily solved nowadays with numerical software tools as Matlab. The first step is to define the equations in the proper form to be used by the numerical solver. To illustrate the procedure the Tri-CRLH line example is used.

The objective is to get a Tri-CRLH with the required phases at three frequencies, but as the transmission line is formed with an arbitrary number of series cells (N), the phase contribution of each cell is $1/N$.

The characteristic impedance and the constant of phase of this line are defined by Equ. 3.28 and Equ. 3.29.

$$Z_0(\omega) = \sqrt{\frac{\omega L_R - \frac{1 - (\omega/\omega_S)^2}{\omega C_P}}{\omega C_R - \frac{1 - (\omega/\omega_P)^2}{\omega L_P}}} \quad (3.28)$$

$$\beta(\omega) = \sqrt{\left[\frac{1}{\omega C_P} - \frac{\omega L_R}{1 - (\omega/\omega_S)^2} \right] \left[\frac{1}{\omega L_P} - \frac{\omega C_R}{1 - (\omega/\omega_P)^2} \right]} \quad (3.29)$$

where

$$\omega_S^2 = \frac{1}{L_R C_L} \quad \text{and} \quad \omega_P^2 = \frac{1}{L_R C_P}$$

The goal is to design a Tri-CRLH line that presents specific phases at three defined frequencies, $\beta(f_1)$, $\beta(f_2)$ and $\beta(f_3)$. For this purpose the transmission line equations must be rewritten in a suitable form to be solved by numerical approximation with Matlab [Mol04].

Component	Value
C_P	32.5 pF
C_R	0.36 pF
C_L	15 pF
L_P	82.1 nH
L_R	0.93 nH
L_L	37.8 nH

Table 3.2: Components of Each Cell of the Tri-CRLH

First a function with one vector as parameters input is defined. This function must contain all the equations, conditions and range restrictions. The restrictions include some important facts as that the capacitors and inductors must have a positive value.

When the function that defines all the equations and conditions of the problem is prepared, it must be evaluated. To evaluate the function *fsolve* is used. Function *fsolve* finds a root (zero) of a system of nonlinear equations. The problem is that the system defined by these equations has multiple solutions and in order to find a suitable solution, initial conditions and the weight of each equation must be adjusted by hand.

Using the proposed code it is possible to calculate a Tri-CRLH of 3 cells to obtain the required phases for an application in the bands of 425 MHz , 910 MHz and 1850 MHz .

The design parameters are:

- $f_1 = 425\text{ MHz}$, $\beta(f_1) = -30^\circ$
- $f_2 = 910\text{ MHz}$, $\beta(f_2) = 15^\circ$
- $f_3 = 1850\text{ MHz}$, $\beta(f_3) = -50^\circ$
- $47\Omega < Z_0 < 53\Omega$
- $N = 3$

The solution of this Tri-CRLH provides a response of each cell and a set of values of the components, Table 3.2. The response of the base cell can be represented as the $\beta(\omega)$ response, Fig. 3.4, the dispersion diagram, Fig. 3.5, or the refraction index, Fig. 3.6.

The response of the complete Tri-CRLH line, that is built with three cells, can be presented with the characteristic impedance, Fig. 3.7, and the phase response, Fig. 3.8. The phase response of the designed Tri-CRLH line is in a good agreement with the values defined on the design requirements.

Figure 3.4: Tri-CRLH Cell β Response

Figure 3.5: Tri-CRLH Cell Dispersion Diagram

Figure 3.6: Tri-CRLH Cell Refractive Index

Figure 3.7: Tri-CRLH characteristic impedance

Figure 3.8: Tri-CRLH 3 cells line phase response

3.5 Works on CRLH Circuits

In this study, the use of Dual-Composite Right/Left-Handed (D-CRLH) transmission lines and Tri-CRLH lines is exemplarized with several designs. The D-CRLH lines are used to design two diplexers and the Tri-CRLH line is used to design a 90° hybrid. The design procedure is shown and the measurements of the prototypes are compared with the simulated results.

3.5.1 Dual-composite Right-Left-Handed Transmission Lines for the Design of Compact Diplexers

In the early 1960s [MJY64] the diplexers were proposed to separate two different frequencies from a single input signal, and they have been widely used since then. The increase of applications for wireless services makes a challenge to get compact and costly efficient diplexers. A generalized procedure to design diplexers loaded with any complex load was recently proposed [WM07], but the most important demand nowadays is to get compact designs. The traditional lumped-element technology presents obvious difficulty to achieve compact circuits. New possibilities, as the use of ring resonators [XHH07] have been proposed, but they do not have a direct use to design compact diplexers. Other process technologies as the Low-Temperature Cofired Ceramics (LTCC) [FW06, TY06] have been tested to reduce the size of the diplexers and they present good expectations.

The CRLH technology offers new potentials for designs at multi-frequencies and miniaturization [SCI04, Cal06, CI13]. Recent years, different topologies of diplexers that use CRLH lines, as the split-ring-resonators [BGGGM05], have been published. Other designs, as the three-dimensional multilayer CRLH diplexer, have been proposed [HCI05]. This multilayer diplexer was able to achieve a good compactness, but with large losses and an important frequency shift. Planar and original design of diplexers was proposed by Wong [WBE06], but this design was not suitable for compact designs. Other designs as the dual-mode rat-race [dCGGMSVGP09] was proposed, but it is not suitable for realist compact designs at low frequencies and this topology cannot be extended to diplexers with more than two output ports. The use of conventional CRLH lines seems to present problems to be applied to the design of diplexers, but the recently proposed Dual-CRLH (D-CRLH) [TNHC07, Cal06] has interesting properties to be used in diplexers. The D-CRLH lines have a right-handed behaviour at low frequencies, a left-handed behaviour at high frequencies and a natural band-stop between the right and left bands. These D-CRLH have been seldom used in microwave circuits [XP06, XPSL06].

The D-CRLH diplexers present advantages over conventional diplexers such as smaller size (especially in the ultra high frequency band). The design strategy and the design equations are presented in this work. The D-CRLH lines have interesting properties as their non-linear phase response that makes them an interesting element for designing diplexers at any arbitrary frequencies. The design parameters are, of course, the frequency bands and the number of sections, but the number of sections is mainly defined by the separation of the bands. Two diplexers are shown as examples and the good match of simulation with measurements is demonstrated. In order to check the ruggedness of the designs a sensitivity analysis is also provided.

3.5.1.1 Diplexer Design Principles

It has been observed that the designs of diplexers using CRLH lines have two important problems: the losses at the band-pass and poor rejection to the undesired band. The use of D-CRLH introduces some band-stop that can be used to increase the rejection to the diplexed band, so the natural band-stop of the D-CRLH lines is the key to get good rejection for near bands.

For a proper use the band-stop of the D-CRLH lines on a diplexer that separates the lower frequency (f_{lower}) from the higher one (f_{higher}), one branch must be designed with a band-stop at f_{higher} and a band-pass at f_{lower} and the other branch must be designed with a band-stop at f_{lower} and a band-pass at f_{higher} . So, for the proposed diplexer, Fig. 3.9, the design frequency of the upper branch must be f_{higher} (to allow f_{lower} pass to port 2) and f_{lower} for the bottom branch (to allow f_{higher} pass to port 3). Then, according to the phase response of the D-CRLH lines, the D-CRLH line of the upper branch (which allows f_{lower} to pass) will be right-handed at f_{lower} and the D-CRLH line of the lower branch (which allows f_{higher} to pass) will be left-handed at f_{higher} . This right/left behaviour is the opposite of the one of the diplexers designed with conventional CRLH lines.

Figure 3.9: D-CRLH Proposed Diplexer Structure

The proposed design needs to ensure that the propagation at the pass frequencies are right-handed for the upper branch (pass for f_{lower}) and left-handed for the lower branch (pass for f_{higher}). A factor $k(N)$ that compares the ratio of the band-stop of a branch with the passing frequency for each branch is defined. This factor is the ratio of the band-stop with the right-handed frequency, f_r , for the upper branch and the ratio of its band-stop with the left-handed frequency, f_l , for the lower branch. With this factor, the correct performance of the D-CRLH line to separate the right or left handed band-pass from the band-stop can be assured. The $k(N)$ factor depends on the number of cells on the D-CRLH line and on the frequencies separation.

If a lossless and one cell line is considered the design procedures would be as follows:

- The design will use balanced D-CRLH lines for both diplexer branches. The band-stop frequencies of both D-CRLH lines must be chosen. The upper branch is defined with the sub-index 2, and the lower one with the sub-index 3. The stop bands of the D-CRLH lines have the following values:

$$\omega_{02} = \omega_{higher}, \quad \omega_{03} = \omega_{lower}, \quad \omega_{02} > \omega_{03} \quad (3.30)$$

- Define the right-handed frequency of the D-CRLH section of the upper branch, ω_{2R} , and the left-handed frequency of the D-CRLH section of the bottom branch, ω_{3L} , as follows:

$$\omega_{3L} = \omega_{lower} \cdot k(N), \quad \omega_{2R} = \frac{\omega_{higher}}{k(N)} \quad (3.31)$$

- For symmetry, the left-handed frequency of the D-CRLH section of the upper branch, ω_{2L} , and the right-handed frequency of the D-CRLH section of the bottom branch, ω_{3R} , can be defined as follows:

$$\omega_{3R} = \frac{\omega_{lower}}{k(N)}, \quad \omega_{2L} = \omega_{higher} \cdot k(N) \quad (3.32)$$

- Introduce the values of the right-handed and left-handed frequencies for the branch 2 and the branch 3 in Equ. 3.21 to obtain the cut-off frequencies (in order to generalize the expression ω_{lower} or ω_{higher} have been changed by ω_0):

$$\begin{aligned} \omega_{CL,R} &= \omega_0 \sqrt{1 + \frac{\omega_0 k(N)}{8 (\omega_0/k(N))} \pm \sqrt{\frac{\omega_0 k(N)}{4 (\omega_0/k(N))} \sqrt{1 + \frac{\omega_0 k(N)}{16 (\omega_0/k(N))}}} \\ &= \omega_0 \sqrt{1 + \frac{(k(N))^2}{8} \pm \sqrt{\frac{(k(N))^2}{4} \sqrt{1 + \frac{(k(N))^2}{16}}}} \end{aligned} \quad (3.33)$$

Figure 3.10: K Factor

- Finally, to assure that the diplexer can properly separate the two frequencies, it has been chosen that the right-handed cut-off frequency of the upper branch is at the right side of the left-handed cut-off frequency of the bottom branch. This means that $\omega_{C2R} > \omega_{C3L}$. If this condition is introduced in Equ. 3.33, it leads to the transcendent Equ. 3.34.

$$\omega_{lower} < \omega_{higher} \frac{\sqrt{1 + \frac{(k(N))^2}{8}} - \sqrt{\frac{(k(N))^2}{4}} \sqrt{1 + \frac{(k(N))^2}{16}}}{\sqrt{1 + \frac{(k(N))^2}{8}} \pm \sqrt{\frac{(k(N))^2}{4}} \sqrt{1 + \frac{(k(N))^2}{16}}} \quad (3.34)$$

- If the ratio between the higher and lower frequency is called R ($R = \omega_{higher}/\omega_{lower}$) then, the limit for the previous inequality will be given by Equ. 3.35.

$$(R^2 - 1) + (R^2 - 1) \frac{(k(N))^2}{8} - (R^2 - 1) \frac{k(N)}{2} \sqrt{1 + \frac{(k(N))^2}{16}} = 0 \quad (3.35)$$

Equ. 3.35 is a transcendent equation that is represented in Fig. 3.10. The k factor can be approached by a second-, third- or fourth-order polynomial without making an important error. For the third-order case, the expression of the third-order polynomial is given as Equ. 3.36. The third order polynomial approximation of Equ. 3.35 is also shown in Fig. 3.10.

$$k(N) = 0.1309R^3 - 1.0143R^2 + 3.5343R - 2.6442 \quad (3.36)$$

- Once the value of the k factor and of the frequency ratio R has been determined, the values for each branch left-handed and right-handed capacitors and inductances can be solved. For the the branch rejecting the higher frequency and allowing to pass the lower one (port 2 in Fig. 3.9), the values for the components are given as (sub-index of the component is taken from the number of the output port):

$$\begin{aligned} L_{2R} &= \frac{Z_0 k(N)}{\omega_{higher}}, & L_{2L} &= \frac{Z_0}{\omega_{higher} k(N)} \\ C_{2L} &= \frac{L_{2L}}{Z_0^2}, & C_{2R} &= \frac{L_{2R}}{Z_0^2} \end{aligned} \quad (3.37)$$

- For the branch which rejects the lower frequency and allows to pass the higher frequencies (port 3 in Fig. 3.9), the values for the components are given as (sub-index of the component is taken from the number of the output port):

$$\begin{aligned} L_{3R} &= \frac{Z_0 k(N)}{\omega_{lower}}, & L_{3L} &= \frac{Z_0}{\omega_{lower} k(N)} \\ C_{3L} &= \frac{L_{3L}}{Z_0^2}, & C_{3R} &= \frac{L_{3R}}{Z_0^2} \end{aligned} \quad (3.38)$$

Equ. 3.37 and Equ. 3.38 provide the overall inductances and capacitances for the D-CRLH lines when they have a single section ($N = 1$). When the number of sections is increased, as all the sections of the D-CRLH transmission line are identical, the values for the corresponding inductances and capacitances of any D-CRLH section are obtained as follows:

$$\begin{aligned} L_{2,3R,N} &= \frac{L_{2,3R}}{N}, & L_{2,3L,N} &= L_{2,3L} \cdot N \\ C_{2,3R,N} &= \frac{C_{2,3R}}{N}, & C_{2,3L,N} &= C_{2,3L} \cdot N \end{aligned} \quad (3.39)$$

Now it can be analyzed the effect of the number of cells in the D-CRLH lines. The cells or sections of a D-CRLH line are identical, but in classic filter theory each section is different. A D-CRLH line has right/left-handed propagation without any dependence with the number of cells. It has been proved [Cal06] that a D-CRLH transmission line maintains its performances up to 10 cells and it also has been proved with low number of cells [HMGPIVSV07]. The main effect of the number of cells is the slope of the response between up and low frequencies, it is more abrupt for larger number of cells. The diplexer design must find the compromise between rejection and losses to choose the suitable number of cells. Most of simple designs use low order D-CRLH lines (1 or 2) to achieve the rejection with low losses.

3.5.1.2 Experimental Results

To illustrate the described process, two diplexers have been designed and manufactured. The two diplexers have been designed for low microwave frequencies to demonstrate the important miniaturization possibilities. One model is for separating the TETRA band and the GSM band. The other model is for separating more near bands, GSM and GALILEO bands.

The simulated parameters are compared with the measurements for both models. And as a final step the sensitivity analysis of the TETRA-GSM model is performed to evaluate the robustness of this diplexer.

3.5.1.2.1 Diplexer for Separate Frequencies (TETRA-GSM)

This diplexer must be suitable for separating 380 MHz and 960 MHz bands. Using the procedure described in this chapter, Equ. 3.30 and Equ.3.36, the components of this diplexer are calculated. The diplexer for separating these bands has a value of 2.01 for the $k(N)$ factor. The components values are adjusted to the most suitable commercial available values. The used components and information of their tolerance and self resonance are shown in Table 3.3. The numerical sub-index on the component is the reference to the diplexer branch, Fig. 3.9. The upper branch is designed to reject the upper band (960 MHz) and to allow the lower band (380 MHz), while the lower branch is designed with the complementary purpose.

The D-CRLH lines have been designed as balanced lines with a characteristic impedance of 50 Ω . These two D-CRLH lines have been interconnected using 50 Ω conventional lines and a “T”. This diplexer is manufactured using Arlon600 substrate, with a $\epsilon_r = 6$ and a height of 0.6 mm, and using SMA connectors to allow easy interconnection and measurement. The complete schematic of this

	$k(N) = 2.01$		
	Designed	Commercial	Tolerance, %
L_{2L}	4.12 nH	08HQ3N9	± 5
C_{2L}	1.65 pF	GRM0222C1C1R7BD0	± 5
L_{2R}	16.66 nH	08HQ16N	± 5
C_{2R}	6.66 pF	GRM0222C1C6R6BD0	± 5
L_{3L}	10.42 nH	08HQ10N	± 5
C_{3L}	4.17 pF	GRM0222C1C4R3BD0	± 5
L_{3R}	42.13 nH	08HQ42N	± 5
C_{3R}	16.66 pF	GRM0222C1C16R0BD0	± 5

Table 3.3: Information on Chip Components of TETRA-GSM Diplexer

diplexer is shown in Fig. 3.11 and the photograph of the final manufactured model in Fig. 3.12.

Figure 3.11: Schematic of the Proposed Design with one D-CRLH Cell for Separating 380 MHz and 960 MHz

The simulated parameters have a good match with the measurements. The insertion losses of the diplexers are lower than 0.4 dB for upper and lower bands, Fig. 3.14 with good return losses, Fig. 3.13, better than -22 dB . The isolation of both diplexed ports is about 40 dB , Fig. 3.15. These good performances have been achieved with an interesting compactness for a circuit at low microwave frequencies (UHF).

Figure 3.12: Photo of the Proposed Design with one D-CRLH for Separating 380 MHz and 960 MHz

Figure 3.13: Simulated and Measured Output Return Losses for the D-CRLH TETRA-GSM Diplexer at the Two Output Ports (2 and 3)

Figure 3.14: Detailed of the Simulated and Measured Insertion Losses for the D-CRLH TETRA-GSM Diplexer

Figure 3.15: Simulated and Measured Isolation between Output Ports for the D-CRLH TETRA-GSM Diplexer

3.5.1.2.2 Diplexer for Near Frequencies (GSM-GALILEO)

The second model of diplexer has the handicap of the near frequencies to be separated. The factor $k(N)$ of this diplexer for the 1.0 GHz and 1.2 GHz bands is 0.36. The D-CRLH diplexer technology allow to get good performances for a compact diplexers even with near bands. These performances have not been achieved in previous works on diplexing circuits [BGGGM05, HCI05, WBE06, dCGGMSVGP09]. Using the same procedure as it was described for the previous model of diplexer the values of the commercial components for this diplexer are obtained, Table 3.4. The design of the D-CRLH lines has also been for a 50Ω characteristic impedance and for balanced lines. Conventional micro-strip lines have been used for proper interconnection of the D-CRLH lines and SMA connectors. The circuit has been manufactured using Arlon600 with $\epsilon_r = 6$ and a height of 0.6 mm . The numerical sub-index of the components indicates the upper or lower branch of the diplexer to which they belong to. The photograph of the manufactured circuit is shown in Fig. 3.16.

The results of this model of diplexer have good match with the simulations, but in this case, near bands diplexer, the tolerance of the components do not allow such a good performances. The insertion losses of this diplexer are lower than 0.8 dB for the low band and lower than 1.6 dB for the upper band, Fig. 3.18. The return losses are better than -12 dB , Fig. 3.17 and the isolation better than 32 dB , Fig. 3.19.

	$k(N) = 0.36$		
	Designed	Commercial	Tolerance, %
L_{2L}	25.2 nH	08HQ25N	± 5
C_{2L}	10 pF	GRM0222C1C10RBD0	± 5
L_{2R}	1.75 nH	08HQ1N8	± 5
C_{2R}	0.7 pF	GRM0222C1CR7BD0	± 5
L_{3L}	30.03 nH	08HQ30N	± 5
C_{3L}	12.01 pF	GRM0222C1C12RBD0	± 5
L_{3R}	2.11 nH	08HQ2N2	± 5
C_{3R}	0.84 pF	GRM0222C1CR8BD0	± 5

Table 3.4: Information on Chip Components of GSM-GALILEO Diplexer

Figure 3.16: Photo of the Proposed Diplexer for Separating Frequencies in the GSM-GALILEO Bands

Figure 3.17: Simulated and Measured Output Return Losses for the D-CRLH GSM-GALILEO Diplexer at the Two Output Ports (2 and 3)

Figure 3.18: Detailed of the Simulated and Measured Insertion Losses for the D-CRLH GSM-GALILEO Diplexer

Figure 3.19: Simulated and Measured Isolation between Output Ports for the D-CRLH GSM-GALILEO Diplexer

3.5.1.3 Sensitivity Analysis

Parasitic effects are always present, and these parasitic effects can be classified into two groups: effects due to the lumped-element tolerance that makes that the actual value differs from the nominal one and effects due to the presence of a parasitic inductance in the capacitors or a parasitic capacitance in the inductors. The sensitivity analysis of the tolerance of these two models of diplexers is performed. The parasitic inductance or capacitance of the elements must be considered in the selection of devices with a self resonance in concordance with the design bands of the diplexer.

For the sensitivity analysis, a tolerance of $\pm 5\%$ has been considered for both capacitors and inductances. The statistical distribution for the values of the components has been considered Gaussian and the number of repetitions is 3000.

It has been shown that the insertion losses of the TETRA-GSM diplexer are very robust to the tolerance of the components, Fig. 3.20. But in the case of the GSM-GALILEO diplexer, the insertion losses, and other parameters, are not so robust to the tolerance of the components, Fig. 3.21.

Figure 3.20: Sensitivity Analysis for the Insertion Losses for the D-CRLH TETRA-GSM Diplexer for a Component Variation of 5%

Figure 3.21: Sensitivity Analysis for the Insertion Losses for the D-CRLH GSM-GALILEO Diplexer for a Component Variation of 5%

3.5.2 Tri-CRLH Lines for the Design of Hybrids at Arbitrary Bands

The conventional lines hybrids as many other microwave circuits have periodical response at odd multiplies of the design frequency. The phase response of the CRLH lines is really interesting to adjust these responses. The response of distributed microwave devices can be modified using any CRHL for multiple purposes as for example to increase the usable band [MT06, OCI04]. Other application of the CRLH lines is to adjust the response frequency of the odd responses, so the periodical microwave circuits can be designed for arbitrary frequencies. There are some works that use CRLH lines to adjust the response of hybrids to be used in two bands [YLS105], but there has not been any previous works on the use of other metamaterial lines for adjusting hybrids to be used in three bands [PCGPJM08]. This usage of the metamaterial lines is illustrated using a Tri-CRLH to adjust the response of a 90 degrees hybrid for being used in three bands that do not have odd relation.

3.5.2.1 Hybrids Design Principles

The use of the Tri-CRLH or any other metamaterial lines in hybrids or other microwave circuits allows to adjust the response for a requisite with multiple bands, then they are valid for designing lines and circuits with specific relative phases at desired frequencies.

The first step on the hybrid design is to analyze a conventional hybrid and determine the relative phases that are required to tune the harmonic responses to be matched to the required design frequencies. Then a phase offset must be introduced to this phases to be able to get a series of proper phase signs (positive or negative phase) according the right/left behaviour of the Tri-CRLH.

When the desired phases of the Tri-CRLH have been determined, it is necessary to design a pair of Tri-CRLH lines, one for the $50\ \Omega$ branch and other for the $33\ \Omega$ branch.

The problem to design Tri-CRLH lines with specific phases is that there are multiple solutions and the equations are transcendental, so they must be solved using numeric techniques, Sec. 3.4. So the solutions must be guided using proper conditions and limitations on the response and on components values until a valid and feasible solution is obtained.

3.5.2.2 Experimental Results

The design of a hybrid for the TETRA-GSM900-GSM2100 bands is performed to illustrate the design methodology and the good performances that hybrids for multiple bands have when Tri-CRLH lines are used in their design. Then a sensitivity analysis is performed to demonstrate the robustness and industrialization possibilities of this technique for hybrids manufacturing.

3.5.2.2.1 TETRA-GSM900-GSM2100 Tri-CRLH Hybrid

The design of this hybrid working at Tetra (425 MHz), GSM900(910 MHz) and GSM2100 (2100 MHz) starts with the design and simulation of a conventional 90 degrees hybrid, Fig. 3.22. The response of this hybrid has the main coupling band at 425 MHz (f_h), Fig. 3.23, and the periodic responses at $3f_h$, $5f_h$, etc.

The Tri-CRLH line is suitable for moving up the frequencies of the main (f_0) and $5f_0$ responses and down the response of $3f_0$. Other CRLH lines can be used in a similar way to adjust the frequencies as it is required.

The required phases for the design bands are measured from the response of the conventional hybrid, they are:

$$\begin{aligned} \text{TETRA} : \quad f_1 &= 425\text{ MHz} & \beta_1 &= -15^\circ \\ \text{GSM900} : \quad f_2 &= 910\text{ MHz} & \beta_2 &= 50^\circ \\ \text{GSM1800} : \quad f_3 &= 2100\text{ MHz} & \beta_3 &= -50^\circ \end{aligned} \quad (3.40)$$

The two required Tri-CRLH lines, 33Ω and 50Ω , can be designed for the required phase response, Equ. 3.40, according the phase response of the Tri-CRLH,

Figure 3.22: Conventional 90 Degrees Hybrid

Figure 3.23: Magnitude Response of the Conventional 90 Degrees Hybrid

Figure 3.24: β of the $50\ \Omega$ Tri-CRLH Line

Equ. 3.27, using the method described in Sec. 3.4. The theoretical response of the $50\ \Omega$ Tri-CRLH line is shown in Fig. 3.24.

The Tri-CRLH are designed as one cell transmission lines. The theoretical components and the commercial available ones for both impedances are shown in Table 3.5 and Table 3.6.

The schematic of the $50\ \Omega$ Tri-CRLH cell is in Fig. 3.25 and its responses in phase and magnitude are in Fig. 3.26.

The two models of Tri-CRLH lines are inserted in the middle of the $33\ \Omega$ and $50\ \Omega$ arms. The circuit has been designed for ARLON600 ($\epsilon_r = 6.0$) with a height of $0.6\ \text{mm}$. The complete schematic of this circuit is in Fig. 3.27.

	$Z_0 = 50 \Omega$		
	Designed	Commercial	Tolerance
C_P	13 pF	ATC400S120F	$\pm 1 \%$
C_R	1.84 pF	ATC400S1R8P	$\pm 0.02 \text{ pF}$
C_L	3.03 pF	ATC400S3R0Q	$\pm 0.03 \text{ pF}$
L_P	32.4 nH	0603WL330G	$\pm 2 \%$
L_R	4.59 nH	L06034R7BHL	$\pm 0.01 \text{ nH}$
L_L	7.56 nH	0603WL7R5G	$\pm 2 \%$

Table 3.5: Information on Chip Components of 50Ω Tri-CRLH Line of TETRA-GSM900-GSM2100 Hybrid

	$Z_0 = 33 \Omega$		
	Designed	Commercial	Tolerance, %
C_P	19.6 pF	ATC400S200F	$\pm 1 \%$
C_R	2.78 pF	ATC400S2R7Q	$\pm 0.03 \text{ pF}$
C_L	4.59 pF	ATC400S4R6A	$\pm 0.05 \text{ pF}$
L_P	21.4 nH	0603WL330G	$\pm 2 \%$
L_R	3.03 nH	L06033R3BHL	$\pm 0.01 \text{ nH}$
L_L	5 nH	L06034R7BHL	$\pm 0.01 \text{ nH}$

Table 3.6: Information on chip components of 33Ω Tri-CRLH line of TETRA-GSM900-GSM2100 Hybrid

Figure 3.25: Schematic of the 50Ω Tri-CRLH Cell

Figure 3.26: Phase and Magnitude of the 50 Ω Tri-CRLH Cell

Figure 3.27: Schematic of the Tri-CRLH Hybrid

The response of the TETRA, GSM900, GSM2100 Hybrid including the commercial components in the Tri-CRLH lines is in Fig. 3.28. The response of this circuit is good enough to be used in amplifiers or any combination circuit for real application systems.

Figure 3.28: Magnitude Response of the Tri-CRLH Hybrid

3.5.2.3 Sensitivity Analysis

As the tolerance of the components is critical for evaluating if a design can be industrialized, a sensitivity analysis has been performed considering the tolerance of the commercial inductors and capacitors. For the sensitivity analysis, the tolerance of components, Table 3.5 and Table 3.6, has been considered for both capacitors and inductances. The statistical distribution for the components values has been considered Uniform and the number of repetitions is 2000. The design is robust enough for being constructed with conventional components, Fig. 3.29.

Figure 3.29: Sensitivity Analysis of Magnitude Response of the Tri-CRLH Hybrid

3.6 Conclusions on Metamaterials Study

D-CRLH transmission lines have shown to be a good solution for the design of circuits that are alternatively used for rejecting or allowing different frequency bands. Then, two diplexer-circuits based on D-CRLH transmission lines have been designed. A general design procedure for implementing arbitrary frequency compact diplexers has been proposed. As the D-CRLH transmission lines do not present a frequency periodic performance, the proposed diplexer can be designed for any given frequency ratio $R = f_{02}/f_{03}$. Two first-order prototypes, one for TETRA-GSM and other for GSM-GALILEO, have been designed and manufactured.

A model of Hybrid for TETRA, GSP900 and GSM2100 bands has been proposed to illustrate how can the CRLH lines (Tri-CRLH in this case) be used to tune the harmonic repetitions of microwave circuits.

As the propose of the presented designs is to illustrate the technique, low frequencies have been used to allow the use of conventional lumped components, but the techniques and circuit models can be extended to other integrated techniques as MMICs.

In order to quantify the effect of the tolerances, a sensitivity analysis has been done to show if the effect of the tolerances in the performances of the circuits is acceptable or not. However, this sensitivity analysis concerns to the tolerance of the elements but not the effect of parasitic elements. The effect of these parasitics is controlled by the fact that the chosen lumped elements must present a very high self-resonance frequency, which indicates that the performance of the chosen lumped element is the one indicated by its nominal value.

The sensitivity analysis has been done for all examples. It has been shown that the first prototype of the diplexer and the hybrid are very robust to the tolerance of the components. But, the specifications for the second prototype of diplexer are much harder and, so the sensitivity is higher. The use of CRLH lines with lumped elements require a study of the suitable tolerances and in some cases selection or tuning of the components may be required.

The interesting properties of the metamaterials are applicable to the patch antennas and arrays. The metamaterias can be integrated on active patches to adjust the phase for each mode of the patch, to include diplexing, etc. The possibility of designing compact diplexers to be integrated in arrays with active patches is an interesting solution than the metamaterial can offer to the study of active patches and arrays.

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Chapter 4

OSCILLATORS

4.1 Introduction

The oscillators are fundamental elements for all RF and microwave systems, as Radar systems [JQS⁺09]. They are one of the most problematic circuits on design process. Nowadays, the linear design as a first approximation is widely used for RF and microwave oscillator design [Ale97, Ale99, Ale00, EH81, Rhe10]. Nonlinear simulation needs more computational resources and the non-linear models for active devices must be available. These non-linear models are not always available and in some cases they do not have enough accuracy [CS10]. However, it is necessary, before starting a nonlinear simulation, to have a good approximation of the frequency and start-up conditions. To conclude, in nonlinear simulation, it is desirable to have a good background in nonlinear simulation and nonlinear approximation of these circuits [SQC⁺07]. In some cases, knowledge in nonlinear solution stability is necessary [VA08b, VA08a], specially if harmonic balance is used [SQ03].

The simulation using linear methods for the design of oscillators has an important advantage, it only needs the S parameters of the lineal model of the active device, it is quicker and simpler than the nonlinear and it is suitable for tuning the circuit [VPR05, KO81]. When a simple linear model is used, the speed of the linear simulations gives the chance of looking for new oscillator topologies. On the other hand, with the linear simulation only the oscillation frequency, gain margin and oscillator quality factor (Q) can be predicted, but not the phase noise, harmonic levels or output power. Nowadays, the methodology used for oscillator design consists on a linear simulation as a first step to determine the validity of the topology and then an harmonic balance simulation and a transient simulation to validate the correct behaviour of the design.

The linear design methods for oscillator analysis can be divided into two main groups: Loop-Gain [Ale97, RH01] and Reference Plane [EH81, Jac92b, BS79, Kur69]. The negative resistance, the negative conductance and the reflection coefficient (for microwave circuits) methods are members of the second group.

This chapter will make a deep analysis of oscillator plane reference and Loop-Gain design methods. It defines the applicable conditions and the expected accuracy that can be achieved with these methods. Some examples will be shown to illustrate wrong solutions that the use of linear reference plane and Loop-Gain methods can produce. The wrong solutions will be justified by necessary conditions for a proper use of these methods. These required conditions that the classic methods must satisfy, *provisos*, are defined on this chapter for all the classic methods. The strengths and weaknesses of the, widely used, plane reference and Loop-Gain methods are also described. Several classic topologies of microwave

oscillators, as Grounded Collector Tuned Bases (GCTB) and Grounded Bases Tuned Oscillator (GBTO), are used to illustrate these results and the additional required conditions.

4.2 Classic Linear Methods

4.2.1 Reference Plane Methods

The principles of the linear Reference Plane Methods will be described and their main strengths, weaknesses and limitations of use. The Reference Plane Methods include the methods that use the Z , Y or Γ network functions to analyze oscillators. It is important to remark that all the system poles are included on any network function, however they are not included on general transfer functions.

The required condition for a circuit to be a proper oscillator is that it must have a pair of complex conjugate poles in the Right Half Plane (RHP) of the s complex plane. Any pole factorized network function will have Equ. 4.1 time response, for the p order pole with k multiplicity.

$$L^{-1} \left[\frac{a_k}{(s - s_p)^k} \right] = \frac{a_k \cdot t^{k-1} \cdot e^{s_p t}}{(k-1)!} \quad (4.1)$$

It can be demonstrated that if a system has more than a pair of poles in the RHP, the time will not be a periodic solution, it will be a quasi-periodic solution and it is not desirable for an oscillator [SQ03]. It can also be demonstrated that if the real part of the poles is negative, the time solution will be a dumped sine, Fig. 2.1.

The drawing used for these methods is conditioned in order to find two dipoles, one with a resonant structure and other with a negative $Z/Y/\Gamma$ generator that can be separated by a reference plane 4.1. The reference plane can be any plane, without a real division between resonator and generator, because the denominator of any network function has all the information about system poles. But, to use one of the traditional divisions, $Z/Y/\Gamma$, simplifies the necessary conditions to assure a correct linear analysis. The division plane can be any point of the circuit and it must provide the same solution, but only if the used linear method can be applied (Sec. 4.3).

In the next sections, it will be shown that these traditional methods are really the application of the method of Nyquist for the detection of poles in the RHP of each particular network function. The classic interpretation of oscillators, when they are analyzed by reference plane methods, is that the resonator defines the frequency and the active circuit provides the gain to compensate the losses of the resonator. This interpretation is the one used by Kurokawa for the first harmonic

approximation or descriptive function of an oscillator [Kur69]. The classic linear methods provide the frequency of the pair of complex poles in the RHP at the linear analysis, but this frequency is not the linear oscillation frequency. The gain of the active device must be compressed to the required level for moving the poles to the imaginary axis. This compression shifts the frequency of the poles to the frequency of linear oscillation, the Kurokawa first harmonic approximation. The solution must also comply the Kurokawa conditions for being an stable solution.

Figure 4.1: Oscillator As Two Sub-systems

4.2.1.1 Admittance Method (Impedance Network Function)

The negative conductance method needs the reference plane in a position that divides the oscillator into two sub-circuits: negative conductance generator and resonator [Kur69, Rhe04].

The first harmonic Kurokawa approximation defines the resonator as the sub-circuit that fixed the oscillation frequency and the negative generator as the circuit that offsets the losses of the resonator. This interpretation is widely used in the literature, because it is intuitive and easy to understand.

The network function is a more powerful and formal method, but less intuitive. The network function can be obtained by injecting with an ideal current generator a current in the network as it is shown in Fig. 4.2. The network function is then

defined by Equ. 4.2. The poles of this network function are the addition of the two sub-circuits. If the circuit is an oscillator, the network function must only have a pair of complex poles in the RHP. In Equ. 4.2, I_g is the inducted current, V is the circuit response and Z is the inverse of the sum of the admittances of Fig. 4.2.

Figure 4.2: Negative Admittance Method Conceptual Diagram

$$V = Z \cdot I_g \quad (4.2)$$

$$Z = \frac{1}{Y_{res} + Y_{osc}}$$

The poles of the network function, Equ. 4.2, are determined by the zeros of the Characteristic Function (CF) of the circuit, Equ. 4.3.

$$Y_T = Y_{res} + Y_{osc} = 0 \quad (4.3)$$

The Kurokawa first harmonic approximation of the descriptive function concludes on the classical oscillator start-up condition: $\Im(Y_T) = \Im(Y_{res} + Y_{osc}) = 0$ and $\Re(Y_T) = \Re(Y_{res} + Y_{osc}) < 0$. Using the first harmonic approximation, it is also possible to determine the oscillation stability and the minimum phase noise, Table 4.1. The variable V is the voltage of the fundamental tone at the reference plane and ω the frequency, then V_0 and ω_0 are the voltage of the fundamental tone and its frequency at the oscillation condition. As this is an approximation for the first harmonic it will be more accurate when the oscillator has less harmonics and the signal is a more pure tone. The position of the reference plane is not arbitrary, the solution differs from one position to another, so the experience on these analysis methods is required to place the reference plane in the best place and obtain the most accurate estimation of the oscillator signal. There is another important approximation used in the equations in Table 4.1, the conductance of the negative generator does not vary with the frequency. This condition is easily satisfied if the resonator has a high Q , but it is not for low Q oscillators.

Parameter	Definition
Characteristic Equation	$Y_T(V, \omega) = Y_{osc}(V) + Y_{res}(\omega) = 0$
Oscillation Condition	$Y_T(V_0, \omega_0) = Y_{osc}(V_0) + Y_{res}(\omega_0) = 0$
Oscillation Stability	$-Y_{osc}(V)$ with $Y_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum Noise	$-Y_{osc}(V)$ with $Y_{res}(\omega)$ cross into a $\pi/2$ clockwise angle

Table 4.1: Admittance Oscillation Conditions

When the resonator has a parallel resonator equivalent circuit at the oscillation frequency, the active circuit must be a negative conductance generator. It is also required that the magnitude of conductance of the negative generator is reduced with the compression of the transistor.

4.2.1.2 Impedance Method (Admittance Network Function)

As in the case of the Admittance Method, the circuit must also be divided into two sub-circuits to be able to use the Impedance Method. One sub-circuit must be the resonator and the other the active circuit. The network function can be obtained by inserting an ideal voltage generator in the network as it is shown in Fig. 4.3. The network function is then defined by Equ. 4.4. If the circuit is an oscillator, the network function must only have a pair of complex poles in the RHP.

Figure 4.3: Negative Impedance Method Conceptual Diagram

In Equ. 4.4, V_g is the inducted voltage, I is the circuit response and Y is the inverse of the sum of the impedances of Fig. 4.3.

$$I = Y \cdot V_g$$

$$Y = \frac{1}{Z_{res} + Z_{osc}} \quad (4.4)$$

The poles of the network function, Equ. 4.4, are determined by the zeros of the Characteristic Function (CF) of the circuit, Equ. 4.5.

$$Z_T = Z_{res} + Z_{osc} = 0 \quad (4.5)$$

The first harmonic approximation of the descriptive function concludes on the classical oscillator start-up condition: $\Im(Z_T) = \Im(Z_{res} + Z_{osc}) = 0$ and $\Re(Z_T) = \Re(Z_{res} + Z_{osc}) < 0$. Using the first harmonic approximation, it is also possible to determine the oscillation stability and the minimum phase noise, Table 4.2. The variable I is the current between both sub-circuits at the fundamental tone and ω the frequency, then I_0 and ω_0 are the current of the fundamental tone and its frequency at the oscillation condition. This approximation will be more accurate when the oscillator has less harmonics and the signal is a more pure tone and when the resonator has a higher Q .

Parameter	Definition
Characteristic Equation	$Z_T(I, \omega) = Z_{osc}(I) + Z_{res}(\omega) = 0$
Oscillation Condition	$Z_T(I_0, \omega_0) = Z_{osc}(I_0) + Z_{res}(\omega_0) = 0$
Oscillation Stability	$-Z_{osc}(I)$ with $Z_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum Noise	$-Z_{osc}(I)$ with $Z_{res}(\omega)$ cross into a $\pi/2$ clockwise angle

Table 4.2: Impedance Oscillation Conditions

When the resonator has a series resonator equivalent circuit at the oscillation frequency, the active circuit must be a negative resistance generator. It is also required that the magnitude of resistance of the negative generator is reduced with the compression of the transistor.

4.2.1.3 Reflection Coefficient Method (Reflection Coefficient Network Function)

The last method based on reference plane is the Reflection Coefficient. The plane separates the resonant circuit and the active device with its associated passive components. The active device works as a reflection generator with a Γ that

varies with a_g . This method was used traditionally due to the availability of the reflection coefficient of antennas and resonators when the computational models and simulation were not available. This method is maybe the less suitable of the group of the reflection plane methods because is the method that presents the higher difficulty to determine the zeros of the CF.

The Kurokawa first harmonic approximation can be used to define the network function and the conditions for oscillation stability and minimum phase noise. The network function for reflection coefficients is defined by Equ. 4.6.

$$b_{osc} = \frac{\Gamma_{osc}}{1 - \Gamma_{osc} \cdot \Gamma_{res}} \cdot a_g = \Gamma \cdot a_g \quad (4.6)$$

In Equ. 4.6, Γ is a function of the reflection coefficients of the resonator and of the active circuit [Jac92b], a_g is the incident wave of the active circuit; and b_{osc} is the reflected wave from the active sub-circuit. In this case, the oscillation condition is satisfied if Γ has a pair of conjugate complex poles in the RHP. These poles are determined by the zeros of the Characteristic Function, Equ. 4.7. But, it is not only necessary the existence of a pair of zeros in the Characteristic Function, Γ_{osc} must not have any poles in the RHP.

$$\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res} = 0 \quad (4.7)$$

Figure 4.4: Reflection Method Conceptual Diagram

For the parameters defined in Table 4.3, a is the incident wave, considering only the first harmonic, at the reference plane and ω is the frequency. So, at the stated oscillation condition, a_0 and ω_0 are the incident wave and the oscillation frequency.

Parameter	Definition
Characteristic Equation	$\Gamma_T(a, \omega) = -\frac{1}{\Gamma_{osc}(a)} + \Gamma_{res}(\omega) = 0$
Oscillation Condition	$\Gamma_T(a_0, \omega_0) = -\frac{1}{\Gamma_{osc}(a_0)} + \Gamma_{res}(\omega_0) = 0$
Oscillation Stability	$\frac{1}{\Gamma_{osc}(a)}$ with $\Gamma_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum Noise	$\frac{1}{\Gamma_{osc}(a)}$ with $\Gamma_{res}(\omega)$ cross into a $\pi/2$ clockwise angle

Table 4.3: Reflection Coefficient Oscillation Conditions

It is important to point that it is more appropriate to use the Z or Y network methods than the reflection coefficient network function, even though using the reflection coefficient network function it is possible to determine if the poles are in the RHP. The use of the reflection coefficient network function is less intuitive and the compression behaviour is more complex than for other cases. This method has important practical issues; because the reflection coefficient was easily measured years ago when good simulation tools were not available.

4.2.2 Loop-Gain Method

The Loop-Gain Method [RH01] and the reference plane methods are two different ways of studying an oscillator, Fig. 4.5. The Loop-Gain is suitable to analyze some additional parameters than the reference plane methods cannot do. The Loop-Gain Method can provide parameters such as the Gain Margin and the Loaded Quality Factor (Q_L). These parameters have relation with other parameters: the Q_L is related with the Phase Noise; and the Q_L and the Gain Margin are related with the Start-up time [RH01, Rhe10, RC04]. When an oscillator is analyzed with any of the reference plane methods, the Q_L and the Gain Margin cannot be easily estimated. If the reference plane methods are compared with the Loop-Gain, it is more suitable for the investigation of new oscillator topologies [Rhe04, RC04].

The Loop-Gain method would be the preferred one as it can provide a bigger set of parameters, but it has the problem that some circuits cannot be re-drawn to identify a feedback path. These set of circuits include the active device in the feedback loop or they use distributed elements that cannot be re-drawn using the virtual ground [Ale99].

Figure 4.5: Oscillator Analysis: Reference Plane (Left) and Feedback (Right)

The general equation of the feedback system is Equ. 4.8, but to use this equation in microwave systems is really difficult. The problem of this equation is that the analytical expression in Laplace domain for RF and MW systems is difficult or impossible to obtain. Most of RF designers use the Nyquist Criteria or simplifications of this criteria, as the Barkhausen Criteria, to determine the start-up condition of the oscillator [NM92]. It is important to point that some of the simplifications of the Nyquist criteria have important limitations, Sec. 2.2.

$$O(s) = \frac{G(s)}{1 + G(s) \cdot H(s)} \cdot I(s) \quad (4.8)$$

When no simplification is applied, the Nyquist Criteria is used to determine the zeros of the Characteristic Function, Equ. 4.9, because the zeros of CF are poles of the feedback equation, Equ. 4.8. The Nyquist Criteria determines the difference of poles and zeros that a function has in the RHP. So, if CF has a pair of conjugate complex zeros in the RHP, the system will have a periodic solution, it will be a proper oscillator.

$$1 + G(s) \cdot H(s) \quad (4.9)$$

Randall and Hock [RH01] perform an analysis of Loop-Gain Method with the identification of a feedback structure as the one in Fig. 4.6. With this feedback structure, the network function as a function of Z parameters is obtained, Equ. 4.10. Randall and Hook's feedback equation is also transformed in a function of S parameters, Equ. 4.11.

$$\frac{I_o}{I} = \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}} \quad (4.10)$$

Figure 4.6: Randall Feedback Diagram

$$\frac{I_o}{I} = \frac{S_{22} - S_{11} + 2S_{21} - S_{12}S_{21} + S_{11}S_{22} - 1}{1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22})} \quad (4.11)$$

Then, the system represented as a feedback structure similar to Fig. 4.6 will be an oscillator if the characteristic function I_o/I has a pair of poles in the RHP. In order to determine the number of poles of the network function, Equ. 4.11, the Characteristic Function, Equ. 4.12, is analyzed by means of the Nyquist Criteria.

$$CF = 1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22}) \quad (4.12)$$

The CF, Equ. 4.12, is suitable for predicting the oscillation, but it has the disadvantage of having a dependence with the open loop point. This problem was solved by Randall and Hock defining a new function, Equ. 4.13. This new equation is defined using an infinite chain of quadrupoles and extracting the eigenvalues. With this equation the problem with the open point and the proposed partial solution proposed by Alencho [Ale97, Ale00] are solved.

$$G_L = \frac{Z_{21} - Z_{12}}{Z_{11} + Z_{22} - 2Z_{12}} \quad (4.13)$$

$$G_L = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}$$

G_L is divided into two terms: the active part G_{osc} , and the resonator term G_{res} . The A variable is the amplitude, usually the incident wave for S parameters; ω is the frequency; A_0 and ω_0 are the amplitude and frequency at the oscillation condition. This is named first harmonic approximation, and it is more accurate when the A_0 is a more pure tone, it is when the resonator has a higher Q and the compression is lower. The conditions for oscillation start-up, oscillation stability and minimum noise are defined in Table 4.4.

Parameter	Definition
Characteristic Equation	$G_T(A, \omega) = -\frac{1}{G_{osc}(A)} + G_{res}(\omega) = 0$
Oscillation Condition	$G_T(A_0, \omega_0) = -\frac{1}{G_{osc}(A_0)} + G_{res}(\omega_0) = 0$
Oscillation Stability	$\frac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$ cross into a clockwise angle from 0 to π
Minimum Noise	$\frac{1}{G_{osc}(A)}$ with $G_{res}(\omega)$ cross into a $\pi/2$ clockwise angle

Table 4.4: Loop-Gain Oscillation Conditions

The conditions in Table 4.4 are easy to fulfill if active and passive elements are properly isolated, but it is not easy to get in a real circuit.

4.3 Conditions for the Proper Use of Classic Linear Methods

The classic linear methods are widely used as a tools to determine the oscillation condition of circuits [Rhe10, Roh97]. In most cases the validity of these methods is assumed without taking any precaution to assure the proper use of them.

A circuit complies with the required conditions for a proper start-up of oscillation when it has a pair of conjugate complex poles in the RHP. The classic methods use the Nyquist Criteria or a simplification of it, so they determine the number of poles in the RHP only in certain conditions, Chap. 2. The applicability of these methods to determine the presence of the poles in the RHP is not assured if some provisos are not satisfied before their use. The required provisos for each classic method are described in detail in the following paragraphs.

In order to demonstrate that any classic method require additional provisos to be used, the work of Jackson [Jac92b] is very interesting. It is focused on the reflection coefficient method, but it can be expanded to all classic methods. Jackson concludes that the classic oscillator start-up conditions are not sufficient neither necessary for the oscillation condition.

It is possible to expand the zeros of any characteristic function (reflection coefficient, impedance or admittance) because all have the same zeros. In this demonstration the reflection coefficient is used, but it can be demonstrated for any other characteristic functions.

The characteristic function for the reflection coefficient method is Equ. 4.14.

$$\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res} \quad (4.14)$$

If the impedances and admittances are defined as function of their numerator and denominators as Equ. 4.15 and Equ. 4.16.

$$Z_{res}(s) = \frac{N_L(s)}{D_L(s)} \text{ and } Z_{osc}(s) = \frac{N_d(s)}{D_d(s)} \quad (4.15)$$

$$Y_{res}(s) = \frac{D_L(s)}{N_L(s)} \text{ and } Y_{osc}(s) = \frac{D_d(s)}{N_d(s)} \quad (4.16)$$

Now the characteristic function, Equ. 4.14, can be expressed as Equ. 4.17.

$$F(s) = 1 - \frac{Z_{res} - Z_0}{Z_{res} + Z_0} \cdot \frac{Z_{osc} - Z_0}{Z_{osc} + Z_0} = \frac{(N_L + Z_0 \cdot D_L) \cdot (N_d + Z_0 \cdot D_d) - (N_L - Z_0 \cdot D_L) \cdot (N_d - Z_0 \cdot D_d)}{(N_L + Z_0 \cdot D_L) \cdot (N_d - Z_0 \cdot D_d)} \quad (4.17)$$

The previous expressions can be simplified for a high value of Z_0 and also for a low value of Z_0 .

When Z_0 is a high impedance, Equ. 4.17 can be simplified as Equ. 4.18.

$$F(s) \approx \frac{2}{Z_0} \cdot (Z_{res} + Z_{osc}) \quad (4.18)$$

As $2/Z_0$ is a constant, it does not affect the position (semi-plane) of the poles and zeros of the function $F(s)$. As this function is studied by means of Nyquist, there is not any difference between $F(s) = \frac{2}{Z_0} (Z_{res} + Z_{osc})$ and $F(s) = Z_{res} + Z_{osc}$. The $F(s)$ function is simplified as Equ. 4.19.

$$F(s) \approx \frac{N_d \cdot D_L + N_L \cdot D_d}{D_L \cdot D_s} \quad (4.19)$$

When Z_0 is a small impedance, then Equ. 4.17 can be simplified as Equ. 4.20.

$$F(s) \approx 2 \cdot Z_0 \cdot \frac{Z_{res} + Z_{osc}}{Z_{res} \cdot Z_{osc}} \quad (4.20)$$

By means of a similar process, excluding the constant $2 \cdot Z_0$, Equ. 4.20 is expanded as Equ. 4.21.

$$F(s) \approx Y_{res} + Y_{osc} \quad (4.21)$$

$$F(s) \approx \frac{N_d \cdot D_L + N_L \cdot D_d}{N_L \cdot N_s}$$

The equivalence of the presented reference plane methods, as on Jackson papers [Jac92b], proves that the use of any of these methods is valid for all oscillator circuits, but the use of any of these methods is not enough to guarantee the start-up or the stability. The necessary and sufficient condition is the presence of a pair of conjugate complex poles in the RHP. As the analytical network functions in s domain are not practical to get, the Nyquist analysis of the network functions is used. But the Nyquist analysis only provides information of the difference between poles and zeros, so it is necessary to define the conditions to assure the proper use of Nyquist analysis on the network functions.

It is necessary to define the condition or conditions which assure the proper use of these methods. Based on the Jackson [Jac06], Ohtomo [Oht95] and Platzer [PS94] papers, it is possible to conclude the additional conditions for each sub-circuit (divided by the reference plane) to assure that the Nyquist analysis of the characteristic equations (Z_T , Y_T , Γ_T) provide, without possibility of error, information about the existence or not of one pair of conjugate complex poles in the RHP.

4.3.1 Conditions For the Proper Use of Reference Plane Methods

The reference plane methods have been unconditionally accepted and used in a lot of cases as a tool to analyze oscillators without any other condition or precaution. The applicability of these methods to determine the presence of poles in the RHP must be defined, so they cannot be unconditionally used.

The admittance, impedance and reflection coefficient methods have the start-up conditions summarized in Table 4.5, as they are described on Sec. 4.2.

	Impedance	Admittance
Start-up condition	$R_{osc}(\omega) + R_{res}(\omega) < 0$	$G_{osc}(\omega) + G_{res}(\omega) < 0$
	$X_{osc}(\omega) + X_{res}(\omega) = 0$	$B_{osc}(\omega) + B_{res}(\omega) = 0$
	Reflection Coefficient	
Start-up condition	$ \Gamma_{osc}(\omega) \cdot \Gamma_{res}(\omega) > 1$ $\Phi_{osc}(\omega) + \Phi_{res}(\omega) = 0$	

Table 4.5: Start-up Conditions for Reference Plane Oscillator Analysis Methods

It is important to point that the conditions in Table 4.5 are neither necessary nor sufficient to guarantee a proper start-up and oscillation condition of an oscillator. They are simplifications of the Nyquist analysis, so they must comply with some requirements to be used with proper validity. The necessary and sufficient

condition is the existence of only a pair of complex conjugate poles in the RHP. If the circuit has more than a pair of poles it can have multiple oscillation conditions, jumping from one to another, or a quasi-periodic response.

For the RF and MW circuits exist great difficulties to obtain an analytical expression in the Laplace's domain (S domain) and in most circuits, it is impossible to obtain. The Nyquist analysis is the tool to obtain the number of poles in the RHP of a function when it is not possible to have access to its analytical expression and it is only possible to have access to its response to $s = j\omega$.

The expressions in Table 4.5 are fully valid and it is not necessary to perform any additional verification for simple circuits with a parallel resonator with a negative admittance generator or a with a serial resonator with a negative resistance generator. When circuits are nearly ideal, they have only a zero cross of the imaginary part (Z , Y or Γ) and the imaginary part changes from negative to positive, the start-up conditions in Table 4.5 are valid. But real circuits with parasitic inductances and capacitors that introduce multiple feedback paths require a more complex set of provisos.

In multiple literature references, the reference plane methods are presented and illustrated with simple examples to assure the validity of Table 4.5. This way, many design engineers consider these conditions as valid without any doubt and they are not warned of the risk of using the classic methods without checking additional provisos.

4.3.1.1 Negative Conductance Additional Conditions (Impedance Network Function)

In the case of circuits with a negative conductance generator, the most common option is to simulate the frequency response of Y_T and apply the Nyquist criteria to it. The Nyquist criteria provides the figure $N_Z - N_P$, where N_Z is the number of zeros in the RHP and N_P is the number of poles in the RHP. The magnitude of $N_Z - N_P$ is the number of turning circles around the origin and it is clockwise for positive values. Then, as the zeros of Y_T are the poles of the Z network function, the presence of a pair of complex poles of Y_T , one clockwise encircle around the origin, is the correct oscillation condition for the analyzed network.

Then, to be sure that the number of clockwise turning circles around the origin is the number of zeros of the Y_T characteristic function, it must not have any poles in the RHP. The total admittance is the sum of the admittance of the passive resonator and of the admittance of the negative conductance generator (Y_{osc}). So, the additional condition is that Y_{osc} must not have any poles in the RHP. Now it is necessary to define the proper procedure to verify this fact.

The admittance of the active sub-circuit (Y_{osc}) must not have any poles in the RHP, visible or *hidden poles*. It is important to remember that Y_{osc} is a reduced network function from a bigger and more complex network function, so it is possible that some poles of the network are not visible (*hidden poles*) [PS94]. If any *hidden poles* exist they cannot be detected in Y_T but they will make the result of the Nyquist analysis of Y_T to provide a wrong result.

The only way to assure that Y_{osc} of a circuit does not have any visible or hidden poles in the RHP of the S plane is to use the NDF to analyze a circuit build with the active sub-circuit of the oscillator (negative conductance generator) loaded with a short circuit. This procedure to determine if a sub-circuit has any poles in the RHP was described by Jackson, and it is based on Platzzer and Ohtomo papers [Jac06, Oht95, PS94, PSH93, Oht93].

This way, the proviso that must be satisfied before performing the Nyquist analysis of the Y_T of a negative conductance oscillator is that the NDF for positive frequencies of the active sub-circuit (Y_{osc}) loaded with a short circuit must not clockwise encircle the origin.

4.3.1.2 Negative Resistance Additional Conditions (Admittance Network Function)

The admittance network function is used to analyze oscillators that have a negative resistance generator.

Using the same procedure as in Sec. 4.3.1.1, when the Z_T function of the circuit is used an additional proviso must be satisfied. To determine the existence of a pair of complex poles in the RHP in the Y network function, the Z_T characteristic function must have a pair of complex zeros in the RHP. The Nyquist criteria is used to determine the zeros of the Z_T CF. As the Nyquist criteria determines the $N_Z - N_P$, it is necessary to assure that Z_{osc} does not have any poles in the RHP.

The way to determine the number of poles of Z_{osc} in the RHP or the absence of them is through the use of the NDF, it must be calculated the NDF of a network formed by the active negative resistance generator loaded with an open circuit. The NDF will determine the number of visible and hidden poles of Z_{osc} , the number of clockwise encircles around the origin for positive frequencies indicates the number of pairs of poles in the RHP.

When the previous proviso for the Z_{osc} has been verified, it is possible to use the Nyquist criteria for the Z_T to determine the number of poles of the Y network function.

It is important to remark that the cross over the real axis of the Nyquist analysis of Z_T characteristic function is not the oscillation frequency (or the poles frequency). It is near to the poles frequency and its value would be closer when the Q of the resonator is higher. If the compression of the transistor (g_m) is

introduced in this analysis, it is visible that the compression modifies the cross frequency of the Nyquist analysis. When g_m is reduced enough to have a cross over the origin, the crossing frequency would be the oscillation frequency at first harmonic approximation, as it was defined by Kurokawa [Kur69].

The same effects on the crossing frequency and compression behaviours are presented on the analysis of the Z network function in Sec. 4.3.1.1.

4.3.1.3 Reflection Coefficient Additional Conditions (Reflection Coefficient Network Function)

The last reference plane that is studied to define the required conditions to be analyzed with the Nyquist criteria is the Reflection Coefficient Method.

This method is based in the simulation of the frequency response of the reflection coefficients of the resonator and active sub-circuit. Then, $\Gamma_T = 1 - \Gamma_{osc} \cdot \Gamma_{res}$ must be analyzed using the Nyquist criteria, but it is more common to analyze $\Gamma_{osc} \cdot \Gamma_{res}$ and check the clockwise encirclement around +1.

The Nyquist criteria determines the difference of zeros and poles of a function, so it must be assured that the parts of the function do not have any poles in the RHP. The passive sub-network cannot have any poles, so the check must be focused on the active sub-circuit.

The only possibility to determine that the active sub-circuit does not have any poles in the RHP is to use the NDF to analyze a network formed with the active sub-circuit loaded with Z_0 . If the NDF of this network does not have any clockwise encircle around the origin, the active sub-circuit does not have any poles in the RHP and it is possible to use the Nyquist criteria to Γ_T (or $\Gamma_{osc} \cdot \Gamma_{res}$) to determine the zeros of the Reflection Coefficient Characteristic Function (Γ_T), that are the poles of the Reflection Coefficient Network Function.

The previous discussion on the compression and on the effect of the Q of the resonator with the frequency of crossing the real axis are applicable to this method.

4.3.1.4 Illustrative Examples of Additional Conditions for Reference Plane Methods

Some examples are presented to illustrate the use of the NDF to verify the provisos of the Classic Reference Plane Methods. Two topologies are used: common base and common collector oscillators. They will be analyzed using the admittance, the impedance and the reflection coefficient methods.

The circuits are designed for the L band using a BFR380F BJT transistor. The transistor model contains all parasites, including the package model. The transistor is biased with a current of 40 mA and a collector to emitter voltage of 5 V.

4.3.1.4.1 Common Collector Oscillator

When a reference plane is used to analyze a common collector oscillator, the preferred option is the negative resistance method. This is the most common option because the first harmonic approximation of the active sub-circuit (right sub-circuit of Fig. 4.7) behaves as a negative impedance generator.

In order to compare the results of different classic methods, this common collector oscillator is analyzed using the three reference plane methods: impedance, admittance and reflection coefficient.

Figure 4.7: Common Collector Oscillator

For the reflection coefficient method, the schematic in Fig. 4.10 is used. The results of the classic methods are in Fig. 4.8, Fig. 4.9 and Fig. 4.11. The Z_T analysis predicts an oscillation frequency of $1244.2 MHz$, but the other two classic methods (Y_T and Γ_T) predict that the circuit is not an oscillator. This discrepancy can be explained if Y_{osc} and Γ_{osc} have hidden poles in the RHP.

The verification of the provisos for the classic reference plane methods must confirm if it is possible to use all the methods for this circuit or some methods cannot be used. The NDF of the circuit in Fig. 4.12 is used to verify the provisos of the three methods: open-circuit for Z_T , short-circuit for Y_T and $R_L = 50 \Omega$ for $\Gamma_{osc} \cdot \Gamma_{res}$ (Γ_T).

Figure 4.8: Nyquist Impedance of Common Collector Oscillator

Figure 4.9: Nyquist Admittance of Common Collector Oscillator

The NDF for the open-circuit, Fig. 4.13, does not encircle the origin, so the proviso for the admittance network method is complied. Then the Nyquist analysis of the Z_T can be used to predict the existence of a pair of poles in the Y network function in the RHP, the oscillation condition.

But the NDF for the short-circuit, Fig. 4.14, and $R_L = 50 \Omega$, Fig. 4.15 encircle the origin. The existence of poles in Y_{osc} masks the zeros of Y_T , so it cannot be used to determine the poles of the Z network function. Then, the Nyquist analysis of Y_T does not detect any zeros and the circuit seems to be stable, but it is unstable (it is an oscillator). It is important to remember that the Nyquist analysis indicates the number of zeros minus the number of poles ($N_Z - N_P$).

Figure 4.10: Reflection Coefficient Measurement Model of Common Collector Oscillator

Figure 4.11: Nyquist Reflection Coefficient of Common Collector Oscillator

For the case of the reflection coefficient method, the NDF of the active sub-circuit loaded with $R_L = 50 \Omega$ also encircles the origin, so the required proviso for this method is not satisfied. In an analogous way as the Y_T method, the reflection coefficient cannot be used for this circuit.

Figure 4.12: Provisos Verification for Common Collector Oscillator

Figure 4.13: Proviso Verification for Z_T of Common Collector Oscillator

At this point, it can be said that the active sub-circuit in Fig. 4.12 is not suitable for the analysis using the Y network function (Z_T) or the reflection coefficient method (Γ_T). So this sub-circuit is modified to get a new one that complies with the provisos for the three reflection plane methods, Fig. 4.16. The modification consists on adding a 10Ω resistor in the emitter.

Figure 4.14: Proviso Verification for Y_T of Common Collector Oscillator Figure 4.15: Proviso Verification for Γ_T of Common Collector Oscillator

The new active sub-circuit can now be checked for the provisos of the three reference plane methods. The NDFs for the three load conditions (open-circuit, short-circuit and $R_L = 50\ \Omega$) are in Fig. 4.17, Fig. 4.18 and Fig. 4.19. None of them encircle the origin, so the new active sub-circuit is suitable for using the three reference plane methods to analyze the oscillator.

Figure 4.16: Stabilized Active Sub-Circuit of Common Base Oscillator

Figure 4.17: Proviso Verification for Z_T of Stabilized Common Collector Oscillator

The most common analysis for this type of oscillator is the Z_T because it is a negative resistance topology. In this kind of oscillator is necessary to verify that the compression of the transistor makes the magnitude of the negative real part of the Z_{osc} decrease, Fig. 4.20.

Figure 4.18: Proviso Verification for Y_T of Stabilized Common Collector Oscillator Figure 4.19: Proviso Verification for Γ_T of Stabilized Common Collector Oscillator

Now that the provisos are satisfied, the analysis of Z_T , Y_T and Γ_T can be done with guarantee. They predict a correct condition of oscillation, but three different frequencies, Fig. 4.21, Fig. 4.22 and Fig. 4.23. This aspect about the existence of different solutions, Table 4.6, is covered in Sec. 4.5.

To end with the verification of this oscillator, an Harmonic Balance (HB) simulation, Fig. 4.24 is performed. The proper oscillation of the circuit is confirmed, Fig. 4.25.

Figure 4.20: Real part of Z_{osc} Variation with g_m Reduction

Method	Frequency [MHz]
Admittance (Y_T)	1653.5
Impedance (Z_T)	1473.1
Reflection Coefficient (Γ_T)	1557.2

Table 4.6: Frequency Solutions of Common Collector Oscillator for each Reference Plane Method

Figure 4.21: Nyquist Impedance of Stabilized Common Collector Oscillator

Figure 4.22: Nyquist Admittance of Stabilized Common Collector Oscillator

Figure 4.23: Nyquist Reflection Coefficient of Stabilized Common Collector Oscillator

Figure 4.24: Stabilized Common Collector Oscillator Schematic for HB Simulation

Figure 4.25: HB Spectrum of Stabilized Common Collector Oscillator

4.3.1.4.2 Common Base Oscillator

A common base oscillator (Grounded Base Tuned Oscillator), Fig. 4.26, is usually analyzed using the negative admittance method, but it will be analyzed using the three reference methods.

The provisos for these methods are tested. The NDF of the active sub-circuit for the three load conditions: open-circuit, Fig. 4.27, for negative resistance; short-circuit, Fig. 4.28, for negative admittance; and $R_L = 50 \Omega$, Fig. 4.29, for reflection coefficient. They are satisfied for all the reference plane methods, so the three methods must provide correct estimations of the oscillation condition of the circuit.

Figure 4.26: Common Base Oscillator

The Nyquist analysis of Z_T , Y_T and Γ_T are performed. These analysis are shown in Fig. 4.30, Fig. 4.31 and Fig. 4.32. In agreement with the correct verification of the provisos, the three methods indicate a correct oscillation condition. There are small differences in the linear predicted frequencies, Table 4.7, but this aspect is covered in Sec. 4.5.

Method	Frequency [MHz]
Admittance (Y_T)	1607.2
Impedance (Z_T)	1713.3
Reflection Coefficient (Γ_T)	1708.6

Table 4.7: Frequency Solutions of Common Base Oscillator for each Reference Plane Method

Figure 4.27: Common Base Oscillator NDF for Short-circuit

Figure 4.28: Common Base Oscillator NDF for Open-circuit

Another important aspect to verify is that the active sub-circuit must have a reduction of the magnitude of the real part of the admittance with the compression of the active device. The behaviour presented by this active sub-circuit is shown in Fig. 4.33. The circuits that present this response are usually analyzed by the Y_T method for its first harmonic approximation.

To finalize this example a verification of the oscillation condition of the circuit is performed using, as in the previous example, the HB analysis. This analysis is performed to the complete circuit, Fig. 4.34. The solution of the HB analysis predicts an oscillation at 1687 MHz , Fig. 4.35, in concordance with the linear predictions.

Figure 4.29: Common Base Oscillator NDF for $Z_L = 50\Omega$ Figure 4.30: Z_T of Common Base Oscillator

Figure 4.31: Y_T of Common Base Oscillator Figure 4.32: Γ_T of Common Base Oscillator Figure 4.33: Real part of Y_{osc} Variation with g_m Reduction

Figure 4.34: Common Base Oscillator Schematic for HB Simulation

Figure 4.35: HB Spectrum of Common Base Oscillator

4.3.2 Conditions For the Proper Use of Loop-Gain Method

The general function of a loop-back system, Equ. 4.8, is the start point for the Loop-Gain analysis, but the rigorous determination of the analytical feedback equation in Laplace domain of a real circuit is difficult or even impossible. The Nyquist criteria analysis is used to determine the start-up condition of oscillators [NM92] and sometimes other simplifications of this criteria that have more restrictions than the Nyquist criteria are used, as for example the Barkhausen criteria [BBM⁺13].

The Nyquist criteria uses the argument principle to determine the zeros of $(1 - G(s) \cdot H(s))$, so the Nyquist criteria determines the difference of the zeros and poles of this characteristic function. Then, when the zeros of this characteristic function are determined, the poles of the general feedback equation are found.

The Loop-Gain method is based mainly in Randall and Hock works [RH01]. They defined a feedback structure, Fig. 4.36. In this work they presented a feedback expression for the Z parameters. This expression can be rewritten as a S parameters expression, Equ. 4.22.

Figure 4.36: Randall-Hock Proposed FeedBack Structure

$$\frac{I_o}{I} = \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}} = \frac{S_{22} - S_{11} + 2S_{21} - S_{12}S_{21} + S_{11}S_{22} - 1}{1 + (S_{12} + S_{21} - S_{12}S_{21} + S_{11}S_{22})} \quad (4.22)$$

To define the required provisos to be satisfied before using the Loop-Gain Method, Equ. 4.22 is rewritten in a more convenient form as Equ. 4.23.

$$\frac{I_o}{I} = \frac{\frac{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12} - S_{22} + S_{11}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}}{1 - \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}}} \quad (4.23)$$

The improved definition of Randall and Hock Loop-Gain, Equ. 4.24, can be identified in Equ. 4.23.

$$G_L = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}} \quad (4.24)$$

So, it is possible to rewrite the general feedback system function using the Randall and Hock G_L expression as Equ. 4.25.

$$\frac{I_o}{I} = \frac{Z_{21} - Z_{12}}{Z_{11} - Z_{12} - Z_{21} + Z_{22}} = \frac{-\frac{G_L \cdot (S_{22} - S_{11})}{S_{21} - S_{12}} + 1}{1 - G_L} \quad (4.25)$$

The zeros of Equ. 4.25 can be determined with the Nyquist analysis of G_L , it must be checked the Nyquist encircling of +1. It is necessary to be sure that the zeros of $1 - G_L$ are all the poles of the system, then the Nyquist analysis of the $1 - G_L$ function or G_L function can be used as a suitable oscillator linear design equation. The provisos that must be verified before starting a linear design using the Loop-Gain Method are:

- G_L must not have any poles in the RHP. This condition is fulfilled if the TF (“Test Function”), $1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}$, does not have any zeros in the RHP; and S_{21} and S_{12} do not have any poles in the RHP.
- The numerator of Equ. 4.23 must not have any poles. This condition is verified if none of the S parameters have any poles in the RHP and the TF does not have any zeros in the RHP.

The verification of the zeros of the TF is easy to be performed with the Nyquist analysis of the TF once the S parameters are available. But, to determine that none of the S parameters do not have any poles in the RHP, it is necessary to apply the NDF to the quadrupole loaded with Z_0 . The NDF of this quadrupole must not encircle the origin. So, after checking that none of the S parameters do not have any poles, the Nyquist analysis of the TF must determine that it does not have any zeros in the RHP.

When these two conditions:

- NDF of the quadrupole loaded with Z_0 does not clockwise encircle the origin.
- Nyquist analysis of TF does not clockwise encircle the origin.

are satisfied, the Nyquist analysis of G_L (encircling the +1) can be performed to determine the poles of the system.

The Randal and Hock G_L does not have different frequency solutions with different open loop points, but it has different frequency solutions for different virtual grounds. This aspect about the existence of different solutions is covered in Sec. 4.5.

4.3.3 Summary of Provisos and Oscillation Conditions for Classic Methods

A contribution of this work is to search, summarize and arrange the required provisos before starting a linear design that uses any classic method to study the oscillation condition of a circuit. The provisos of any of the classic methods require the NDF as a tool to determine the non existence of poles of the active sub-circuit in the RHP. These provisos, the start-up condition and the stability of the oscillation condition are summarized in the following tables.

	Negative Admittance Method
Proviso	NDF of active sub-circuit loaded with a shot-circuit must not clockwise encircle the origin
Start-up Condition	$G_{osc}(\omega) + G_{res}(\omega) < 0$ $B_{osc}(\omega) + B_{res}(\omega) = 0$
Oscillation Stability	$-Y_{osc}(\omega)$ with $Y_{res}(\omega)$ cross into a clockwise angle from 0 to π

Table 4.8: Summary of Provisos and Oscillation Condition of Negative Admittance Method

	Negative Impedance Method
Proviso	NDF of active sub-circuit loaded with an open-circuit must not clockwise encircle the origin
Start-up Condition	$R_{osc}(\omega) + R_{res}(\omega) < 0$ $X_{osc}(\omega) + X_{res}(\omega) = 0$
Oscillation Stability	$-Z_{osc}(\omega)$ with $Z_{res}(\omega)$ cross into a clockwise angle from 0 to π

Table 4.9: Summary of Provisos and Oscillation Condition of Negative Impedance Method

	Reflection Coefficient Method
Proviso	NDF of active sub-circuit loaded with Z_0 must not clockwise encircle the origin
Start-up Condition	$ \Gamma_{osc}(\omega) \cdot \Gamma_{res}(\omega) > 1$ $\Phi_{osc}(\omega) + \Phi_{res}(\omega) = 0$
Oscillation Stability	$\frac{1}{\Gamma_{osc}(\omega)}$ with $\Gamma_{res}(\omega)$ cross into a clockwise angle from 0 to π

Table 4.10: Summary of Provisos and Oscillation Condition of Reflection Coefficient Method

	Loop-Gain Method
Provisos	- NDF of open-loop circuit loaded with Z_0 must not clockwise encircle the origin - Nyquist analysis of TF $(1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12})$ must not clockwise encircle the origin
Start-up Condition	$-\frac{1}{G_{osc}(\omega)} + G_{res}(\omega) > 0$
Oscillation Stability	$-\frac{1}{G_{osc}(\omega)}$ with $G_{res}(\omega)$ cross into a clockwise angle from 0 to π

Table 4.11: Summary of Provisos and Oscillation Condition of Loop-Gain Method

4.4 NDF Proposed Method

The classic methods need additional provisos that must be verified before starting the analysis of the circuit using any of the classic methods. These provisos (Sec. 4.3.3) require the use of the NDF. But after checking the provisos, there is another problem with the use of the classic methods: they provide different solutions for the same circuit depending of other aspects as the position of the virtual ground, the open point of the loop, the reference plane position, etc. These discrepancies are solved when the proposed NDF Method is used.

The *NDF/RR_T* method is proposed by the author, It is based on the stability studies published by Platzer and his deep analysis of the properties of the NDF [PS94, PSH93]. Platzer proposed the use of the NDF to verify the Rollett proviso [Rol62].

The provisos for a correct use of the classic methods have been studied and presented by several authors [Jac92b, Jac92a, Jac06, Oht95]. NDF must be used as a tool to study the proviso of any classic methods [JMGPPC⁺12b, GPJMPC⁺11], so the question that is the start point of this *NDF/RR_T* method is:

“Why is not the NDF directly used for oscillator linear design?”

The NDF is the quotient of the network determinant and the normalized network determinant, Equ. 4.26. The normalized determinant is obtained by disabling all the active devices.

$$NDF(s) = \frac{\Delta(s)}{\Delta_0(s)} \quad (4.26)$$

The problem is that to obtain directly the network determinant is not an easy task. But Platzzer describes how to calculate the NDF [PSH93] using the Return Relations (RR) defined by Bode [Bod45], Equ. 4.27.

$$NDF = \prod_{i=0}^n (RR_i + 1) \quad (4.27)$$

The term RR_i is the return relation of the i active device when the $i - 1$ and all the previous ones are disabled. The RR can be obtained using the linear model of the active device, as for example Fig. 4.37 for a BJT. If the linear model is not available, it can be extracted by simulation from the non-linear or S parameters model [Jac06, Gia69].

Figure 4.37: Linear BJT Model

When the linear models of the devices are available, the circuit must be redrawn to have access to the controlled current source of the linear model of the active device, Fig. 4.38. All parasitic elements of the active device are included in the Feedback circuit and only the ideal linear amplifier model is extracted from the complete circuit.

The process to obtain the RR of an active device is to change the dependence of the controlled current source to an independent voltage source and calculate the return voltage that would control the current source, Fig. 4.39. The negative relation of the applied control voltage with the return voltage is the RR of the active device, Equ. 4.28. When a circuit has more than one active device the RR of each device must be calculated with the previous ones disabled and the

following ones untouched. So this process of isolating the controlled current source must be done for each active device of the circuit, then the RR of the circuit can be calculated using Equ. 4.27.

Figure 4.38: Circuit with Access to the Controlled Current Source

Figure 4.39: RR of a Controlled Current Source

$$RR = \frac{-V_{13}}{V_{ext}} = \frac{-H(\omega) \cdot g_m \cdot V_{ext}}{V_{ext}} = -g_m \cdot H(\omega) \quad (4.28)$$

The Transposed Return Relation (RR_T) is defined as $RR_T = -RR$. This RR_T can be interpreted as the “*true Loop-Fain*” of an active device. The RR_T can be written as function of several terms, Equ. 4.29.

$$\text{True Loop-Gain} = RR_T = -RR = RR_{T_{osc}} \cdot RR_{T_{res}} = g_m \cdot H(\omega) \quad (4.29)$$

With the definition of the RR_T , it is possible to write the NDF as a function that is more suitable for the understanding of the stability of the circuit and it is directly related with the “*Loop-Gain*” of the active devices, Equ. 4.30.

$$NDF = \prod_{i=0}^n (1 - RR_{T_i}) \quad (4.30)$$

The NDF does not require any additional calculus or supposition to determine the oscillation frequency of the first harmonic approximation (Kurokawa approximation). It provides direct information about the number of poles that a system has in the RHP. A system must have only a pair of conjugate complex poles in the RHP to achieve a proper start-up and a stable oscillating condition.

The number of poles in the RHP is determined by the clockwise encircles of the origin of the Nyquist plot of the NDF when the frequency goes from $-\infty$ to ∞ , but as the NDF is symmetrical, it is only necessary to consider the positive frequencies. If the Nyquist plot is used with the RR_T , then the clockwise encircles of the +1 must be counted.

Considering the RR_T as an “*open Loop-Gain*”, it is possible to define the oscillation condition and the minimum noise condition as they were defined for the classic methods, Table 4.12. The RR_T is decomposed into the active contribution of the loop, $RR_{T_{osc}}$, and the passive contribution of the loop, $RR_{T_{res}}$. The active contribution only considers the controlled generator, $RR_{T_{osc}} = g_m$, and the passive contribution includes all the passive elements including the resonator, $RR_{T_{res}} = H(\omega)$. The ω variable is the frequency and the V is the control variable of the controlled generator. At the stable oscillation condition the frequency and the control variable are ω_0 and V_0 . This condition is defined for the Kurokawa first harmonic approximation, so it will be more accurate when the signal is a purer tone, the required compression is lower and the resonator bandwidth is smaller.

Parameter	Definition
Characteristic Equation	$1 - RR_T(V, \omega) = -\frac{1}{RR_{osc}(V)} + RR_{res}(\omega) =$ $-\frac{1}{g_m(V)} + H(\omega) = 0$
Oscillation Condition	$1 - RR_T(V_0, \omega_0) = -\frac{1}{RR_{osc}(V_0)} + RR_{res}(\omega_0) =$ $-\frac{1}{g_m(V_0)} + H(\omega_0) = 0$
Oscillation Stability	$\frac{1}{RR_{osc}(V)} \text{ with } RR_{res}(\omega)$ <p>cross into a clockwise angle from 0 to π</p>
Minimum Noise ⁽¹⁾	$\frac{1}{RR_{osc}(V)} \text{ with } RR_{res}(\omega)$ <p>cross into a $\pi/2$ clockwise angle</p>

(1): See Sec. 4.6.

Table 4.12: NDF/RR_T Oscillation Conditions

The NDF Method has multiple advantages, the main advantages are:

- It provides the oscillation frequency at the first harmonic approximation without requiring any compression of the transistor gain. The cross over the negative real axis is always at the same frequency. The frequency does not have any dependence with g_m because it is only a multiplicative figure of the feedback response, $H(\omega)$.
- It can be used for any oscillator topology, because it is not necessary to identify the resonator to define a reference plane or point for opening the loop.
- It is not necessary to redraw the circuit for any more suitable distribution. For some circuits, the use of the Alechno's virtual ground can help to prepare them for a particular classic method, but circuits with transmission lines cannot be redrawn using the virtual ground.
- The solution is unique. It does not have any dependence with the virtual ground.
- The solution is always correct. It is not necessary to check any proviso. The omission of the verification of the provisos sometimes provides wrong solutions when the classic methods are used.

- The upper frequency of the analysis can be assured. The NDF has an asymptotic behaviour towards +1, so it is easy to determine if the range of the frequency used in the analysis is high enough.
- It is suitable for estimating the gain margin and Q_L . These parameters can be estimated for any oscillator topology. The reference plane methods cannot determine the gain margin or the Q_L and the Loop-Gain method provides different solutions for each virtual ground position.

4.5 Multiple Solutions for the Same Circuit Problem

The problem of multiple solutions for the a same circuit when the classic linear methods are used is illustrated with two examples. The classic methods provide different solutions when the open point or reference plane are changed, and for different ground reference points. Randal and Hock G_L does not have different solutions with the open point, but it has the problem with different ground reference points.

These examples are also analyzed using the NDF Method and the problems of multiple solutions and, in some cases, wrong solutions are solved with this proposed method.

4.5.1 First Example

A general oscillator, Fig. 4.40, without any ground reference or distributed elements (just using lumped elements) is proposed to illustrate the problem of the existence of multiple solutions for the same circuit. The active device used for this example is a BFR360F transistor biased with $I_C = 10\text{ mA}$ and $V_{CE} = 3\text{ V}$. Different virtual ground points are defined using the Virtual Ground [Ale99]. The three most interesting positions of the virtual ground are the well-known classic topologies: common collector (Colpitts), common emitter (Pierce) and common base, but any other can be analyzed. These three oscillators, Fig. 4.41, Fig. 4.42 and Fig. 4.43, are different “topologies” but they are the same circuit, so they must have the same oscillation frequency, gain margin and Quality Factor (Q_L). The problems to assure a proper oscillator are significant and there are multiple works that present topologies and techniques to assure a robust oscillator [CC14].

They can be analyzed using any of the classic methods, as it has been previously demonstrated, but only the Randal and Hock G_L is used in this set of examples. Any other classic method can be used with similar conclusions.

Figure 4.40: General Oscillator Model without Ground Reference

Figure 4.41: Virtual Ground Common Collector Oscillator

Figure 4.42: Virtual Ground Common Emitter Oscillator

Figure 4.43: Virtual Ground Common Base Oscillator

The open-loop analysis, Fig. 4.44, predict that the common emitter topology will oscillate, but the other two will not.

The common collector circuit does not cross the imaginary axis with a positive real part greater than the unit, so it indicates that the circuit does not oscillate. But, it is interesting to point the solutions if the Barkhausen criteria [BCP11] is used. The common base circuit crosses the imaginary axis at two frequencies. The first one 1467 MHz is from positive to negative, so it is not an oscillation condition, but the second one 3927 MHz is a “Barkhausen” oscillation condition. This example can be considered as a complementary to the Nguyen examples [NM92].

If the method of representation is the Barkhausen, the provisos presented in Sec. 4.3.2 must be checked before using the G_L method.

Figure 4.44: Nyquist Plot of G_L for Common Collector, Emitter and Base

The verification of the proviso for the G_L is a set of two conditions: the Test Function (TF), Fig. 4.46, and the NDF of the open loop with both ports loaded with Z_0 , Fig. 4.45. The TF of the common collector and common base encircle the origin, so the proviso for the G_L is not satisfied, then the unexpected solutions of these two circuits for the Loop-Gain analysis, Fig. 4.44, are expected. The other proviso, NDF of the open loop loaded with Z_0 , Fig. 4.45, is satisfied, but both of them are required before use the G_L method.

In these cases, the NDF of the open loop circuits loaded with Z_0 do not encircle the origin, but the Nyquist analysis of the TFs do. So, as the proviso of the Characteristic Function (CF) is only the NDF of the open loop circuits loaded with Z_0 , these three circuits must be suitable to be analyzed by means of the CF. The Nyquist plots of the CF of these circuits are shown in Fig. 4.47. The CF predicts that the three circuits oscillate. The frequencies are nearly the same for the common collector and common base, but the common emitter has a completely different solution. The gain margins of the three solutions are completely different.

Figure 4.45: Nyquist Plot of NDF for Open-Loop Common Collector, Emitter and Base Loaded with Z_0

Figure 4.46: Nyquist Plot of TF for Common Collector, Emitter and Base

This example is a partial demonstration, further examples can be analyzed, of the multiple solutions for the same circuit when it is analyzed using the CF and with different ground reference or loop open points. These conclusions can be used for different transistors technology, as the CMOS, and the same provisos must be verified [KMB10, KMB11].

The proposed NDF Method can be used with the three topologies and the solution for them is shown in Fig. 4.48. When this method is used the solutions are identical and an unique pair of complex poles in the RHP is predicted. The analysis can be performed with any linear simulator using the build-in NDF function or the Platzer definition if it is not build-in.

Figure 4.47: Nyquist Plot of CF for Common Collector, Emitter and Base

Figure 4.48: NDF for Common Collector, Emitter and Base

4.5.2 Second Example

As a second example, an oscillator similar to the previous one but with biasing circuits, Fig. 4.49, is proposed. This example is a generalization of the first example, that now considers the bias and it is suitable for adding to the comparison of the classic methods the HB analysis of the example circuit. This oscillator is suitable to be redrawn using the virtual ground into the three well-known classic topologies. These three interesting cases are shown in Fig. 4.50, Fig. 4.51 and Fig. 4.52: Common Collector, Emitter and Base.

Figure 4.49: General Biased Oscillator Model without Ground Reference

Figure 4.50: Virtual Ground Common Collector Oscillator with Bias

As these circuits include the bias, it is possible to use the non-linear model of the transistor to perform an Harmonic Balance Analysis (HB) and a Time Domain (TD) analysis. These two non-linear analysis are performed to the three “topologies” of the circuit in Fig. 4.49 in order to demonstrate that the three circuits are the same oscillator. The time domain signal and the spectrum of the three circuits are in Fig. 4.53 and it can be checked that the three circuits have the same time signal and spectrum, so it is proved that they are the same circuit.

Figure 4.51: Virtual Ground Common Emitter Oscillator with Bias

Figure 4.52: Virtual Ground Common Base Oscillator with Bias

a) Time

b) Spectrum

Figure 4.53: Time (HB) and Spectrum of the CC, CE and CB Circuits

These models that include the non-linear model of the transistors allow to perform a transient simulation. A transition from 0 V to 5 V in 5 ns in the power supply is used as transient to excite the start-up of the oscillator. The start-up of the three circuits and their signals in steady state are shown in Fig. 4.54. The start-up and signals in time are identical for the three circuits, so in addition to the concordance in the HB simulation these results assure that the three circuits are really the same oscillator.

Figure 4.54: Transient Time Start-up and Steady State Signal of the CC, CE and CB Circuits

The non-linear simulations have confirmed that the three circuits under study are the same oscillator. Now they are analyzed using the CF, the G_L and the NDF linear methods. The schematics of the three circuits and the open-loop points are shown in Fig. 4.55, Fig. 4.56, and Fig. 4.57. These schematics include the required high value inductances and capacitors for biasing the circuit and DC-blocks. These biased circuits with a signal open-loop point are suitable for the CF and G_L with the use of the transistors non-linear models.

Figure 4.57: Schematic for CF and G_L Analysis of Common Base Oscillator

As it has been previously described, the first step before a CF and G_L analysis is to perform a verification of the required provisos. The first proviso is that the NDFs of the quadrupoles loaded with $Z_0 = 50\Omega$ do not encircle the origin. The NDF of the open-loop circuits with the 50Ω loads are in Fig. 4.58. These NDFs show that the three circuits comply the first proviso, so they can be analyzed using the CF. But the G_L needs an additional proviso, the “test function”. The “test function” of the three circuits are shown in Fig. 4.59. The TFs of the common collector and common emitter do not encircle the origin, so they can be analyzed using the G_L method. But the TF of the common base circuit encircles the origin, so this schematic cannot be analyzed using the G_L method because it will provide a wrong result.

Figure 4.58: NDF of Z_0 Loaded Quadrupole

Figure 4.59: "Test Function" of CC, CE and CB Quadrupole

The “Characteristic Functions” of the three circuits are simulated and the results are shown in Fig. 4.60. These analysis predict proper start-up conditions of the three circuits, but each circuit presents a different gain margin and oscillation frequency.

The results of multiple examples, similar to the previous ones, confirm that the “Characteristic Function” has dependence with the virtual ground point and with the open-loop point. So the CF analysis will provide different solutions for different open-loop points and different virtual grounds. Of course, the prediction of the oscillation condition will only be provided if the proviso is satisfied, but the gain margin and frequency depend on the virtual ground and open-loop point.

The G_L of the three circuits are simulated and shown in Fig. 4.61. It is important to remember that the common base circuit does not comply with the proviso. The Nyquist plots of the common emitter and common collector clockwise encircle the +1, so an oscillation is predicted. But the common base circuit does not clockwise encircle the +1, so there is not an expected oscillation in this case, but it is normal because it does not comply with the Randall and Hock oscillation condition.

Figure 4.60: CF of CC, CE and CB Circuits

a) Full View

b) Zoom

Figure 4.61: G_L of CC, CE and CB Circuits

Now that the CF and G_L methods have been used with the three circuits, it is the moment to apply the proposed NDF/RR_T method to them. The NDF/RR_T Nyquist plots of the three circuits are shown in Fig. 4.62 and Fig. 4.63. The three plots are identical, as it must have been in any analysis, because the three

circuits are the same one. The NDF clockwise encircles the origin with a real axis crossing frequency of 1496 MHz and the RR_T also enclose clockwise the $+1$ at the same frequency. The predicted frequency and gain margin for all the circuits are the same if the NDF/RR_T method is used to analyze them.

Figure 4.62: NDF of CC, CE and CB Circuits

Figure 4.63: RR_T of CC, CE and CB Circuits

As these examples are using the non-linear model of the transistor it is possible to extract a linear model of the transistor and apply a compression, g_m reduction, in the three circuits. When the g_m is reduced to the proper value for the $+1$ crossing point for the CF, G_L and NDF , all of them cross at the same frequency, Fig. 4.64. Then it is confirmed that the NDF/RR_T provides the solution for the first harmonic approximation without requiring compression.

a) Characteristic Function

b) G_L Figure 4.64: Compressed CF and G_L of CC, CE and CB Circuits

4.6 Phase Noise Optimization with NDF

Characterization of the noise is essential for communication, RF and Radar systems. It is a parameter that system engineers need because it defines the final performances of the system [Rob83, Ric11, Bar04]. There are important works on oscillator design as Randall and Hock [RH01] and Jackson [Jac92b], but the design of oscillators presents high difficulties [FGVHVA⁺11, Kur69, Rhe10]. The use of the NDF as a linear design tool for low noise oscillators is a contribution presented throughout this work. The NDF also does not require any proviso, it has the advantage of allowing optimization of phase noise for any oscillator topology.

4.6.1 Leeson's Model

The linear optimization of the phase noise of oscillators has been a great problem, because it is not possible to optimize some topologies when the classic methods are used in their linear analysis. A first attempt to get a linear interpretation of the noise of oscillators and to provide linear optimization keys was published by Leeson in 1966 [Lee66]. The Leeson noise model and the improvements of several authors [HL98b, HL98a, DMR00, EB97] developed a general linear noise model for feedback scheme oscillators, Fig. 4.65, similar as it was defined by Leeson.

Leeson's feedback diagram, Fig. 4.65, consists on an amplifier and a feedback network. The amplifier is characterized by its gain function, $G(A, \omega)$, and the feedback network is characterized by its frequency response, $H(\omega)$. The gain has dependence with the amplitude (A) and with the frequency (ω), and the feedback network only with the frequency. The feedback network is characterized as a filter or resonator that presents losses (B). The frequency response of the resonator is defined as Equ. 4.31. This resonator circuit model provides a frequency response

as function of the carrier offset frequency (Δf_m) where f_0 is the oscillator carrier frequency, B the resonator losses and Q_L the loaded Q of the resonator.

Figure 4.65: Leeson Oscillator Model for Phase Noise

$$H(\Delta f_m) = \frac{B}{1 + j \cdot \frac{2 \cdot Q_L \cdot \Delta f_m}{f_0}} \quad (4.31)$$

The phase noise defined by Leeson can be defined as Equ. 4.32 when Equ. 4.31 is used as the response of the resonator. The term $L_{out}(\Delta f_m)$ is the spectral density of the phase noise in $rad^2 \cdot Hz^{-1}$, it is defined as the ratio of the noise power density of one sideband and the power at the oscillation frequency. When it is expressed in logarithmic units dBc/Hz is used. P_{in} is the power at the oscillation frequency in W at the input of the amplifier block. F is the noise figure of the amplifier loop including the flicker noise up-conversion at frequencies near the carrier. This noise figure must also consider the effect of the compression of the amplifier. K is the Boltzmann's constant, T is the absolute temperature in Kelvin degrees and f_c is the flicker cut-off frequency.

$$L_{out}(\Delta f_m) = \left(\frac{F \cdot K \cdot T}{2 \cdot P_{in}} \cdot \left(1 + \frac{f_c}{\Delta f_m} \right) \right) \cdot \left(1 + \left(\frac{f_0}{2 \cdot Q_L \cdot \Delta f_m} \right)^2 \right) \quad (4.32)$$

It can be observed that the main noise contribution is different for each frequency offset range. These areas have a dependence with different powers of the frequency offset, Fig. 4.66. The phase noise can be modelled in a very simple way as Equ. 4.33 where K_α is a constant for each power of Δf_m^α .

Figure 4.66: Dependence of Phase Noise with Frequency Offset

$$L_{out}(\Delta f_m) = \sum_{\alpha=0}^4 (K_{\alpha} \cdot \Delta f_m^{\alpha}) \quad (4.33)$$

Unfortunately the Leeson's model is only valid for circuits that have a feedback that can be recognized and opened. So, the phase noise of circuits that can only be analyzed using the reference plane methods [Jac92b] cannot be analyzed using the Leeson's model. But when it is possible to use the Leeson's model, it provides results that have a great match with measurements [Rhe10, Eve86, EB97, Eve00]. The author will demonstrate that the NDF Method is suitable to optimize the phase noise of any oscillator topology [CMP⁺14, JMGPPC⁺12a].

4.6.2 Considerations about Leeson's Model

Several authors [DMR00, HL98b, Eve97] have modelled the phase noise as a transfer function of the noise at the input and at the output of the amplifier. They have use the Leeson's model as a start point and use the first term of Leeson's model as input noise, as for example Equ. 4.34. The first term is the noise of the amplifier and the second one is the noise due to flicker noise up-conversion.

$$L_{in}(\Delta f_m) = \left(\frac{F \cdot K \cdot T}{2 \cdot P_{in}} \cdot \left(1 + \frac{f_c}{\Delta f_m} \right) \right) \quad (4.34)$$

When Equ. 4.34 is considered as the input noise in a feedback, Fig. 4.67, the output phase noise is the input noise multiplied by the closed Loop-Gain, G_{cl} , Equ. 4.35. The closed Loop-Gain is shown in Equ. 4.36, where DG is the direct gain

from the input to the output points and OLG is the open Loop-Gain. The G_{cl} can also be expressed in terms of the amplifier gain (G_0), the loss of the resonator (B), the centre frequency (f_0) and the loaded Q of the resonator (Q_L) as Equ. 4.6.2.

Figure 4.67: Closed-Loop Model for Phase Noise

$$L_{out}(\Delta f_m) = L_{in}(\Delta f_m) \cdot |G_{cl}|^2 \quad (4.35)$$

$$G_{cl} = \frac{DG}{1 - OLG} \quad (4.36)$$

$$G_{cl} = \frac{G_o}{1 - \frac{G_o \cdot B}{1 + j \cdot \frac{2 \cdot Q_L \cdot \Delta f_m}{f_0}}} \quad (4.37)$$

When Equ. 4.35 and Equ. 4.6.2 are combined, the output phase noise can be expressed as Equ. 4.38. This equation differs from the Leeson's expression, Equ. 4.32, that is intensive tested and corroborated. But, it can be checked that the Leeson's expression is a particular case of Equ. 4.38. It must be considered that the oscillator is at the oscillation condition ($G_0 \cdot B = 1$) and that the $G_0 = 1$, but in a general case the insertion loss of the resonator determines the G_0 .

$$L_{out}(\Delta f_m) = \left(\frac{F \cdot K \cdot T}{2 \cdot P_{in}} \cdot \left(1 + \frac{f_c}{\Delta f_m} \right) \right) \cdot \left| \frac{G_0 \cdot \left(1 + j \cdot \frac{2 \cdot Q_L \cdot f_m}{f_0} \right)}{1 + j \cdot \frac{2 \cdot Q_L \cdot f_m}{f_0} - G_0 \cdot B} \right|^2 \quad (4.38)$$

As G_0 is determined by the loss of the resonator it must be considered, $(1 - Q_L/Q_0) = B = G_0^{-1}$. The output power of the oscillator is defined as $P_{out} = P_{in} \cdot G_0^2$. Now Equ. 4.38 can be rewritten as Equ. 4.39, but this expression differs from the Everard expression [Eve97].

$$L_{out}(\Delta f_m) = \frac{1}{\left(1 - \frac{Q_L}{Q_0}\right)^4} \cdot \left(\frac{F \cdot K \cdot T}{2 \cdot P_{out}} \cdot \left(1 + \frac{f_c}{\Delta f_m}\right)\right) \cdot \left(1 + \left(\frac{f_0}{2 \cdot Q_L \cdot \Delta f_m}\right)^2\right) \quad (4.39)$$

The Everard expression has been tested and fully corroborated, so Equ. 4.38 and model in Fig. 4.67 must be considered wrong for cases when $G_0 \neq 1$. They are only valid for $Q_L \ll Q_0$, that makes $G_0 = 1$. But Equ. 4.38 has another big problem, it is that it predicts a different phase noise for each measurement point inside the loop. The direct gain, G_0 , is measured as the relation of the signal in the output and input points while the Open Loop-Gain is fixed for a circuit wherever the loop is opened, $OLG = DG \cdot H(j\omega)$. This statement can be checked with a circuit as the one in Fig. 4.68 to measure the phase noise in several points inside the loop. The results of the simulation of the circuit in Fig. 4.68 are shown in Fig. 4.69, where the phase noise is proved to be unique wherever the measurement point inside the loop is placed.

Figure 4.68: Circuit for Checking Phase Noise using Leeson's/Everard's Model

A new model for phase noise is proposed by the author, Fig. 4.70. This model tries to solve the problem of the change of the estimated phase noise with the injection point presented by Equ. 4.38 and Equ. 4.39. The input and output point for the phase noise is the same one, but it can be placed at any point inside

the loop. As the input and output are the same point the DG is always 1 and the OLG is the same with independence of the chosen point, then the OLG is defined as Equ. 4.40.

Figure 4.69: Phase Noise in Several Point of Circuit in Fig. 4.68

Figure 4.70: Proposed Model for Phase Noise

$$OLG = \frac{1}{1 + j \cdot \frac{2 \cdot Q_L \cdot \Delta f_m}{f_0}} \quad (4.40)$$

The new defined OLG, Equ. 4.40, is used in the general feedback, then the closed Loop-Gain is defined as Equ. 4.41. And now the output phase noise for this model is defined as Equ. 4.42. This output phase noise is the same as the one defined by Leeson. This new equation, that is not really new because it is Leeson's expression, indicates that the phase noise has not dependence with the G_0 , but it is not true because the noise figure (F) of the amplifier has a strong dependence with the gain; and the compression of the amplifier also affects the noise figure. In addition to this, the flicker noise up-conversion has also influence in the noise figure.

$$G_{cl} = \frac{1}{1 - \frac{1}{1 + j \cdot \frac{2 \cdot Q_L \cdot \Delta f_m}{f_0}}} \quad (4.41)$$

$$L_{out}(\Delta f_m) = \left(\frac{K \cdot T \cdot F}{2 \cdot P_{in}} \cdot \left(1 + \frac{f_c}{\Delta f_m} \right) \right) \cdot |G_{cl}|^2 = \left(\frac{K \cdot T \cdot F}{2 \cdot P_{in}} \cdot \left(1 + \frac{f_c}{\Delta f_m} \right) \right) \cdot \left| \frac{1}{1 - \frac{1}{1 + j \cdot \frac{2 \cdot Q_L \cdot \Delta f_m}{f_0}}} \right|^2 \quad (4.42)$$

4.6.3 Important Factors for Phase Noise Optimization

Everard's method [Eve00, EB97, Eve97] is the linear method most used for phase noise optimization. Their main problem is that, as a method based on Leeson's model [Lee66], it is only applicable for oscillators that can be modelled as a feedback system.

When the input power of the oscillator is replaced for the output power and the gain of the amplifier as the relation of the loaded and unloaded Q , Equ. 4.43 is obtained. The noise figure has been expressed as function of the nominal gain (G_0), the compressed gain (G_c) and the flicker noise (F_n) to reflect the importance of these factors in the noise figure and to emphasize that the F term is not the amplifier noise figure.

$$L_{out}(\Delta f_m) = \left(\frac{F(G_0, G_c, Fn) \cdot K \cdot T}{2 \cdot P_{out} \cdot \left(1 - \frac{Q_L}{Q_0}\right)^2} \cdot \left(1 + \frac{f_c}{\Delta f_m}\right) \right) \cdot \left(1 + \left(\frac{f_0}{2 \cdot Q_L \cdot \Delta f_m}\right)^2\right) \quad (4.43)$$

Some approximations and simplifications can be applied to Equ. 4.43 and important conclusions for the phase noise optimization can be obtained. The equation can be approximated for a white frequency noise region as Equ. 4.44. It can be concluded that this equation has a minimum for $Q_L = Q_0/2$, then the minimum phase noise can be defined by Equ. 4.45.

$$L_{out}(\Delta f_m) \approx \frac{F(G_0, G_c, Fn) \cdot K \cdot T}{8 \cdot Q_0^2 \cdot P_{out} \cdot \left(\frac{Q_L}{Q_0}\right)^2 \cdot \left(1 - \frac{Q_L}{Q_0}\right)} \cdot \left(\frac{f_0}{\Delta f_m}\right)^2 \quad (4.44)$$

$$L_{out}(\Delta f_m)_{min} \approx \frac{2 \cdot F(G_0, G_c, Fn) \cdot K \cdot T}{Q_0^2 \cdot P_{out}} \cdot \left(\frac{f_0}{\Delta f_m}\right)^2 \quad (4.45)$$

The conclusions that can be obtained from Equ. 4.43 are:

- The noise figure (F) must be minimized. Nowadays it is very difficult to minimize the noise figure of the active devices, because it is near to the minimum, thanks to the advanced manufacturing techniques.
- The relation between Q_L and Q_0 for an optimum condition must be $Q_L/Q_0 = 1/2$. This condition is equivalent to have a S_{21} of -6 dB .
- The Q_0 must be as high as possible, then the Q_L will be the biggest.
- The output power of the amplifier, P_{out} , must be as high as possible.
- The flicker corner frequency must be the lowest.[GW11]

To design for the previous conditions, it is very important to have a linear technique for calculating the Q_L and for optimizing this value [WXW⁺11], because the phase noise is proportional to the inverse of the square of the Q_L . Most of these techniques are limited to the oscillators that can be modelled as a feedback system, but most of the times it is difficult because the oscillator is not designed with this topology.

Other important aspect is the selection of the active device, it is the key to get a low F . But, the active device must be also chosen by taking into account the up-conversion of the flicker noise [GW11].

This word provides several examples that demonstrate the use of the NDF on the phase noise optimization on the following examples, Sec. 4.7.

4.6.4 NDF for Phase Noise Optimization

Most of techniques for the linear optimization of the phase noise of oscillators are only suitable for feedback topologies. Most high frequency oscillators do not present this structure, because the space limitation and in most of the circuits the internal parasitic elements of the active device are included in the feedback. There are also other methods [HL98b, DMR00, EB97, Eve00, Eve97, RCG94] while none of them are suitable for being used to optimize the phase noise as a general method for any topology.

When the oscillator is a reference plane topology and it is not possible to define a feedback path, none of the classic methods are suitable for the phase noise optimization. The author propose [JMGPPC⁺12a] a generalization of the Everard's equation that uses the NDF which is suitable to be used for optimizing the phase noise of any oscillator topology.

The RR_T can be interpreted as the “*true Loop-Gain*”, Sec. 4.4. Using the NDF or the RR_T to analyze an oscillator the “*true Loop-Gain*” is available without requiring a feedback topology as it is required by other methods.

The RR_T is defined as Equ. 4.46. It is important to remember that the RR_T or NDF are suitable of being analyzed using the Nyquist method to determine the number of poles. The NDF determines the number of poles in the RHP and the frequency of Kurokawa's first harmonic approximation, it does not require any proviso or additional verification nor g_m compression.

$$RR_T = -RR = 1 - NDF = 1 - \prod_{i=1}^N (1 - RR_{T_i}) \quad (4.46)$$

The RR_T can be divided into the active part, RR_{osc} , and into resonator part, RR_{res} , Equ. 4.47. The active part is only the g_m of the active device, so it is a scale factor. The passive part does not only include the resonator, it also includes all other elements and all parasitic elements of the transistor, Fig. 4.71.

$$RR_T = RR_{osc} \cdot RR_{res} = g_m \cdot H(\omega) \quad (4.47)$$

The proposed method to optimize the phase noise of any oscillator [JMGPPC⁺12a] is based into optimize the “*true*” loaded quality factor, Q_L , of the oscillator. The Q_L of the oscillator can be determined by the variation of the RR_T , Equ. 4.48.

Figure 4.71: Model for RR_T Calculus

$$Q_L = -\frac{\omega}{2} \cdot \frac{d}{d\omega} [\arg(RR(\omega))] = -\frac{f}{2} \cdot \frac{d}{df} [\arg(NDF(f) - 1)] \quad (4.48)$$

When the first harmonic approximation, Kurokawa's approximation, is applied to the RR_T , the minimum phase noise condition can be defined. The condition for the minimum phase noise is that the AM-PM conversion must be the lowest. This condition is satisfied when the cross of the $-1/RR_{osc}(V)$ and the $RR_T(\omega)$ has a counter-clockwise angle of $\pi/2$ for increasing values of V and ω . In general it is possible to increase the Q_L making the RR_T to cross as quick as possible over the real axis.

A mayor contribution of this work is to propose the NDF/RR_T Method as a new way to analyze oscillators. It has several advantages over classic methods and one of these interesting advantages is that it can be used to optimize the phase noise of any topology, including reference plane oscillators. The use of the NDF/RR_T allows to use the Loop-Gain concept and the Leeson's equation for the optimization of the phase noise of any circuit topology.

4.7 Practical Examples

Two examples are presented and the measurements of the manufactured circuits are compared with the simulation results. These examples are a common collector oscillator and a common base oscillator. These examples are circuits that are usually analyzed using a reference plane method, but the proposed NDF Method can be used to analyze them. The use of the method presented in this work makes possible the optimization of the phase noise on reference plane oscillator models.

4.7.1 Common Collector Oscillator Example

The schematic of the oscillator to design and manufacture is in Fig. 4.72. If the circuit is going to be analyzed by means of any reference plane method the required provisos must be checked. The NDF of the active sub-circuit, negative resistance generator, must be calculated with the proper load for each case:

- open-circuit: if the admittance network function is used
- short-circuit: if the impedance network function is used
- Z_0 : if the reflection coefficient network function is used

The NDF for the three loads are shown in Fig. 4.73. The origin is not encircled in any case, so this circuit can be analyzed using any of the reference plane methods.

This circuit has a negative resistance generator, so it is usually analyzed by the admittance network function, the impedances of the circuit are calculated and the Nyquist plot is used to determine the oscillation condition.

The Nyquist plots for the three network functions are calculated and they are shown in Fig. 4.74. The three plane reference network functions predict an oscillation near 1.14 GHz , but each network function predict a different frequency.

As the reference plane methods do not provide information about the Q_L , gain margin and start-up time, the circuit is analyzed using the open-loop method. The chosen point for opening the loop is between the emitter and the centre point of the feedback capacitors, Fig. 4.75.

Before using the G_L method, it is necessary to check the provisos:

- NDF of the Z_0 loaded quadrupole
- Test Function (TF)

Figure 4.72: Schematic of Common Collector Oscillator Example

The NDF of the Z_0 loaded quadrupole, Fig. 4.76, clockwise encircles the origin, so the G_L method will provide a wrong result and it is not necessary to continue with the verification of the second proviso, TF. In any case the G_L is performed for illustrative purpose, Fig. 4.77. The Nyquist plot of the G_L crosses the real axis two times with a real part greater than +1, one time clockwise and the other counterclockwise. As the +1 is not encircled the G_L method does not predict an oscillation condition.

Now the NDF/RR_T method is used to analyze the common collector oscillator. This method does not require any provisos, so the Nyquist plot of the RR_T must clockwise encircle the +1 for a proper oscillator. The Nyquist plot, Fig. 4.78, predicts an oscillation frequency of 1.12 GHz.

The circuit is manufactured, Fig. 4.79, and the measurement of the spectrum is compared with the HB simulated one. These spectra are shown in Fig. 4.80 and they are in a great concordance for the frequency predicted by simulation.

Figure 4.73: NDF of the Negative Resistance Generator

a) Impedance

b) Admittance

c) Reflection Coefficient

Figure 4.74: Nyquist Plots of Common collector Oscillator

Figure 4.75: Common Collector Oscillator for G_L Analysis Figure 4.76: Common Collector Oscillator Proviso for G_L Method

Figure 4.77: Nyquist Plot of G_L of Common Collector Oscillator Figure 4.78: Nyquist Plot of RR_T of Common Collector Oscillator

Figure 4.79: Common Collector Oscillator

Figure 4.80: Common Collector Oscillator HB and Measured Spectra

4.7.2 Common Base Oscillator Example

The second example is an oscillator with a negative conductance generator, Fig. 4.81. It is not possible to define a feedback path in this circuit because the internal base-emitter capacitors of the transistor are in this path. Then it is not possible to use the G_L to analyze this circuit. The most common method used to analyze this topology is the impedance network function.

Figure 4.81: Schematic of Common Base Oscillator Example

The advantages of the NDF/RR_T are significant, so this is the method that is used to analyze the oscillator. The RR_T of this circuit, Fig. 4.82, shows the existence of only a pair of complex poles in the RHP, so it is an oscillator with the correct conditioning.

The NDF can also provide information about the Q of the oscillator that is related with the phase noise, this aspect is covered in Sec. 4.6.

The oscillator is built to compare the experimental results with the simulated ones, Fig. 4.84. The oscillator is analyzed by the HB and the spectra compared with the measured one, Fig. 4.83. The measured spectrum mostly agrees with the one obtained with the Harmonic Balance simulation.

Figure 4.82: Nyquist plot of RR_T of Common base oscillator

Figure 4.83: Common Base Oscillator HB and Measured Spectra

Figure 4.84: Common Base Oscillator

4.7.3 Phase Noise Optimization Example

An oscillator to illustrate the use of the NDF proposed method, Sec. 4.6.4, for optimizing the phase noise is presented and manufactured. The oscillator that is used is a common collector [GPJMPC⁺11], that is usually analyzed by a reference plane method. This oscillator has a capacitive feedback, Fig. 4.85, that is designed with micro-strip distributed elements that makes impossible to analyze it with a classic Loop-Gain method even if the virtual-ground is used. Fig. 4.85 only shows the components, the micro-strip lines and paths are not shown, they have been designed using the NI-AWR© iNet. This interesting feature allows a clean schematic but includes all the tracks in the layout.

Figure 4.85: Common Base Oscillator with Reference Plane for Phase Noise Optimization

The transistor that is used in this oscillator is a low cost device, BFR380F from Siemens Technologies. The feedback includes internal capacitors of the transistor and the layout technology used is micro-strip. These two factors make impossible to redraw the circuit to define a feedback path to use classic phase noise optimization methods as Everard's method.

First the traditional admittance network function is used to determine the oscillation condition and then the *NDF* method is used to determine the oscillation condition and to optimize the phase noise.

The total impedance, Z_T , must be calculated for the admittance network method. The zeros of the Z_T are the poles of the system. Of course, the proviso must be checked before using this method. In this circuit the proviso is satisfied, the NDF of the open-circuit loaded active sub-circuit does not encircle the origin.

The Z_T of the oscillator, before any optimization, is in Fig. 4.86. The oscillator is properly designed and it has an oscillator condition at 2100 MHz . But this analysis does not provide any useful information for the phase noise optimization.

Figure 4.86: Total Impedance of Common Base Oscillator (Before Optimization)

The NDF/RR_T method is used to analyze the circuit in Fig. 4.85. With NDF/RR_T plot, Fig. 4.87, a pair of conjugate complex poles are predicted in the RHP, this is the proper condition for an oscillator. These poles are at 1955 MHz , and this is the poles frequency for the Kurokawa's first harmonic approximation.

The NDF/RR_T method can be used to estimate the loaded Q of the oscillator, Sec. 4.6. This way, the conclusions of the phase noise studies of Leeson [Lee66] and Everard [EB97, Eve00, Eve97] are applicable to any oscillator circuit when it is analyzed using the NDF/RR_T method.

The loaded Q of the circuit in Fig. 4.85 is estimated using the RR_T and a Q figure of 16 is obtained for the first harmonic approximation oscillation frequency, Fig. 4.88.

The next step is to optimize the Q_L of the circuit to obtain a better phase noise, but to illustrate the optimization of the Q_L the Harmonic Balance (HB) simulation of the different circuits will be performed and the phase noise at 10 KHz used to measure the improvement of the phase noise. The circuit has been analyzed for several values of C_r , C_p , C_s and L ; the results of these Q_L estimations and of the HB phase noise are shown in Table 4.13.

Figure 4.87: NDF/RR_T Nyquist Plot of the Common Base Oscillator (Before Optimization)

Figure 4.88: Q_L Using the RR_T of the Common Base Oscillator (Before Optimization)

The phase noise is reduced with the increase of Q_L , Table 4.13, but there is a Q_L value above which the phase noise is not increased. This behaviour is explained by Everard [Eve97] and it is caused because the Q_L is increased above the optimum value. Then the optimum Q_L and phase noise is for the lumped values of the 4th column of the Table 4.13. The NDF/RR_T and the Q_L of the optimized oscillator are shown in Fig. 4.89 and Fig. 4.90. The frequency of the optimized oscillator at the first harmonic approximation is 1986 MHz and the Q_L 39.51.

Parameter	1	2	3	4	5	6
C_r (pF)	1.8	1.8	1.5	1.2	1.2	1.8
C_p (pF)	0.47	0.68	1.0	1.2	1.2	1.5
C_s (pF)	1.5	1.2	0.82	0.47	0.47	0.47
L (mm)	9.0	8.6	8.6	9.0	9.0	7.3
Q_{RR_T}	15.98	20.08	31.59	39.51	43.04	43.41
$f_{0_{RR_T}}$ (GHz)	1955	2000	1996	1986	1982	1986
GM_{RR_T}	9.3	7.9	7.2	3.84	3.55	2.44
$f_{0_{HB}}$ (GHz)	2163	2145	2058	2003	1999	1999
Ph. Noise HB (dBc/Hz @ 10KHz)	-90.5	-93.4	-99.0	-106.5	-103.1	-101.6

Table 4.13: Oscillator Q_L and Phase Noise vs. Lumped Values Figure 4.89: NDF/RR_T Nyquist Plot of the Common Base Oscillator (Optimized)

The two models of the oscillator, 1^{st} and 4^{th} , are manufactured and the results of the measurements are compared with the HB simulation result, Fig. 4.91 and Fig. 4.92. Both circuits have identical aspect, but different component values, Fig. 4.93. The phase noise of both have a good match between the measurement and simulation data. The phase noise at 10 KHz has been reduced from -90.53 dBc/Hz to -103.4 dBc/Hz , that is a good optimization as it was predicted by the linear NDF/RR_T optimization method. It is important to point that the noise floor in Fig. 4.92(b) is the noise floor of the spectrum analyzer ($\approx -153 \text{ dBc/Hz}$).

Figure 4.90: Q_L Using the RR_T of the Common Base Oscillator (Optimized)

Figure 4.91: Measurements of Common Base Oscillator (Before Optimization)

Figure 4.92: Measurements of Common Base Oscillator (Optimized)

Figure 4.93: Photograph of Common Base Oscillator

The NDF/RR_T has demonstrated a good matching between simulated and experimental results when it is used with a circuit that cannot be analyzed with a classic Loop-Gain method for phase noise optimization.

4.8 Conclusions on Oscillators Study

This chapter covers the linear design methods for oscillator analysis. After a brief presentation of the most used methods, that can be classified into two groups, reference plane and Loop-Gain, the problem and required provisos for their proper use are listed. A contribution of this work is to search, summarize and arrange the required provisos before starting a linear design that uses any classic method to study the oscillation (or stability) condition of a circuit. These provisos must be always verified before starting a linear analysis of an oscillator with any classic linear method. Some examples of the erroneous solutions that can be obtained if the provisos verification is omitted are shown.

It is also demonstrated with several examples that the classic linear methods, when they can be applied because the provisos are satisfied, provide different solutions. Another discrepancy is that all the classic methods provides different solutions for different open points or virtual ground of the same circuit. The Randall and Hock Loop-Gain Method solves the problem with the open point, but it does not solve the problem with different virtual grounds.

All these problems that cause different solutions are solved with the proposed NDF Method. The NDF Method does not require any proviso and it is always valid for any topology. The solution of the proposed method is always the same with independence of the virtual ground and it does not require any open point to analyze the circuit.

Another advantage of the NDF is that it can be used for the optimization of the Phase Noise (Q_L), then the phase noise optimization during linear steps is now available for any topology. In addition, as the proposed method is valid for any topology, this method opens the research of new low noise oscillator topologies.

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Chapter 5

NEGATIVE IMPEDANCE CONVERTERS (NICs)

5.1 Introduction

There are different types of material or structures that can present a negative effective permeability. The passive ones are the metamaterials, Chap. 3. They are based on resonance, so the response is narrowband and they are complex to build in 2D or 3D structures. There are other structures that have a negative effective permeability: the Negative Impedance Converters (NICs).

The NICs can achieve broad bandwidth ranges of use, this way they can tailor group delay or tune the material response. They are suitable to be used on distributed amplification to overcome loss. And they have new kinds of behaviour as superluminal propagation and nonlinear absorption, that are not possible with passive structures. These properties make the NICs suitable to new behaviour that is impossible with passive structures.

There have been many efforts to overcome the conflicting performance characteristics of Electrical Small Antennas (ESAs), including their efficiencies, bandwidths, and directivities, using various meta-structures. To overcome one of the fundamental limitations, the impedance bandwidth, the non-Foster elements have been introduced. In this application, as in many more, the NICs presents wider frequency matching than passive structures [ZZ13].

The NICs are non-Foster devices so they do not comply the Foster's reactance theorem. Foster's reactance theorem states that:

Theorem. *The reactance of a passive, lossless two-terminal (one-port) network always strictly monotonically increases with frequency.*

It is, that the reactance is a function of complex frequency variable that must comply for real frequency axis with Equ. 5.1 and Equ. 5.2 [Fos24].

$$F(j\omega) = jX(\omega) \quad (5.1)$$

$$\frac{\partial X(\omega)}{\partial \omega} \geq 0, \forall \omega \quad (5.2)$$

The reactance of capacitor and inductors on the Smith Chart always moves clockwise with the increase of frequency, but the the reactance of the NICs moves counterclockwise for a wideband, Fig. 5.1. The behaviour of the NICs as non-foster devices can be compared with the conventional passive components as it is shown on Table 5.1, where it is easy to see that the $\frac{\partial}{\partial \omega}$ of each non-foster NIC has an opposite sign than the conventional passive device.

A simple visual description of the *current path* and of the *voltage path* are shown in Fig. 5.2. The twist of the two paths is the reason because the NICs are “*inverters*” of impedances.

Figure 5.1: NICs Reactance Behaviour

Figure 5.2: NIC Current (Blue) and Voltage (Red) Paths

An early implementation of a NIC based on vacuum tube was realized by Merrill in 1951 [Mer51a, Mer51b]. Some years later, in 1953, the first NIC realization based on transistors was presented by Linvill [Lin53]. There are a wide variety of NIC topologies [KS69], but other widely used is the proposed by Yanagisawa [Yan57].

This chapter covers the basic concepts and fundamentals of the NICs and presents several examples of their use and analysis.

	Conventional	NIC (inverter)
Resistor	R	$-R$
Inductor	$j\omega L$	$-j\omega L = \frac{\omega L}{j}$
Capacitor	$\frac{1}{j\omega C}$	$\frac{1}{-j\omega C} = \frac{j}{\omega C}$

Table 5.1: Comparative of Conventional Passives with NIC

5.2 NICs Fundamentals

The NICs offer the possibility of circuits that are able to reduce the capacity, the inductance or the resistance of a network. These negative capacity and inductance behaviour can be used for the wide-band compensation of circuits and antennas. The negative resistors are suitable for increasing the Q of antennas and oscillators.

The NICs built with different devices and topologies as transistor and operational amplifiers. The NICs based on operational amplifiers are not suitable for high frequency, so their study is out of the scope of this work. There are multiple NICs for high frequency based on transistors, Fig. 5.3.

The two most used grounded topologies are the Linvill, Fig. 5.3(b), and Yanagisawa, Fig. 5.3(e). The ungrounded topology most used is the Linvill, Fig. 5.3(a), that is suitable for series NICs.

To illustrate the basic linear analysis of NICs, the grounded Linvill, Fig. 5.4, and Yanagisawa, Fig. 5.5, models are analyzed. It can be observed that they are really the same circuit, but considering different port and resistor to invert. The equivalent linear model circuit of both circuits is the same one, Fig. 5.6.

The circuit in Fig. 5.6 can be analyzed to solve the input impedance on ports. The input current can be expressed as Equ. 5.3.

$$I = -V_2 \cdot g_m \quad (5.3)$$

Then the internal voltages V_1 and V_2 are expressed as Equ. 5.4.

$$\begin{aligned} V_1 &= (-V_2 \cdot g_m \cdot R_1) - (V_1 \cdot g_m \cdot R_2) \\ V_2 &= (-V_1 \cdot g_m \cdot R_3) \end{aligned} \quad (5.4)$$

Now the voltage at the input port can be solved as function of internal voltages, Equ. 5.5.

$$V = \left[\frac{g_m^2 R_1 R_3}{g_m R_2 + 1} \right] V_2 \quad (5.5)$$

The input impedance of circuit Fig. 5.6 can be solved as $R = V/I$ using Equ. 5.5 and Equ. 5.3. So, the input impedance is defined on general cases and for high values of g_m as Equ. 5.6.

$$R = \frac{V}{I} = -\frac{g_m R_1 R_3}{g_m R_2 + 1} + \frac{1}{g_m} \quad (5.6)$$

$$R \approx -\frac{R_1}{R_2} R_3 \quad \text{when } g_m R_2 \gg 1 \text{ and } g_m \gg 1$$

It can be observed that circuits in Fig. 5.4 and in Fig. 5.5 are completely equivalent, because the load to invert in each circuit, R_1 and R_3 , are commutable terms, Equ. 5.6.

On a similar way any other NICs can be analyzed. As additional example, the ungrounded Linvill NIC, Fig. 5.7, is analyzed. This circuit has an equivalent circuit as it is shown in Fig. 5.8.

To analyze the circuit in Fig. 5.8, the current and the voltage at the input port is solved as Equ. 5.7 and Equ. 5.8.

$$I = -V_1 \cdot g_m \quad (5.7)$$

$$V = -V_1 + V_1 \cdot g_m \cdot Z + V_2 = (g_m \cdot R_1 - 2) \cdot V_1 \quad (5.8)$$

Now the input impedance can be solved as Equ. 5.9.

$$R = \frac{V}{I} = -\frac{g_m R_1 - 2}{g_m} \quad (5.9)$$

$$R \approx -R_1 \quad \text{when } g_m R_1 \gg 2$$

These examples are the most used NICs on practical circuits, but any other NIC in Fig. 5.3 can be analyzed with a similar procedure.

The basic analysis of any NICs indicates that it has a wide-band behaviour that presents the negative impedance of the desired load at the input ports. The band is, at a first approximation, as wide-band as the transistor gain, g_m , keeps a high value.

Figure 5.3: NICS Topologies Examples

Figure 5.4: NIC Example 1, Grounded Linvill

Figure 5.5: NIC Example 2, Yanagisawa

Figure 5.6: Model of NIC1 and NIC2

Figure 5.7: NIC Example 3, Ungrounded Linvill

Figure 5.8: Model of NIC3

5.3 NICs Stability

The NICs present stability problems, and their stability analysis is difficult to study. This is one of their major obstacles for their wide practical application.

The classic methods have been used and still are in use, to determine the stability of the NICs. But the topologies of the NICs circuits are easily susceptible of having hidden poles. So the classic methods present a high risk of wrong stability prediction.

5.3.1 Classic NICs Stability

The classic methods for studying the stability of NICs are mainly based on analyzing some network function and then use Nyquist theorem to see the network function root-loci. The first methods, called reference plane methods, use the impedance, Z ; admittance, Y ; or reflection coefficient, Γ ; seen from a plane towards the right side (the loaded NIC); and the Z , Y or Γ towards the left side (external load). A first approach [Abe08], studies how should be the performance in one port when the NIC port is loaded with a short circuit (s.c.) or with an open circuit (o.c.) and then the k or μ stability factor is used, but multiple examples of NICs comply this criteria and they are unstable, because hidden poles are not observable when this approach is used. It has been stated that the necessary conditions for a device to be stable are that the input impedance or admittance, seeing at either port, when the other port is open-circuited or short-circuited, should be positive. This implies that there are not any poles in the RHP. But, this approach has a problem, because only s.c. or o.c. impedances are defined and not other ones such as ideal inductances or capacitances. This is an important problem, because most loads to be inverted on practical cases are inductances or capacitances. In addition, it must be highlighted that the loaded NIC cannot be considered by itself without the presence of the external load (Y_L , Z_L or Γ_L). Then, for example, for an impedance analysis it can be seen that for a network current, I_g , the voltage, V , at the reference plane is given as Equ. 5.10.

$$V = Z_T \cdot I_g \quad (5.10)$$

where Z_T is given as

$$Z_T = \frac{1}{Y_T} = \frac{1}{Y_{NIC} + Y_L} \quad (5.11)$$

So, to have a stable circuit, the function Z_T cannot have any poles in the RHP. As the poles of Z_T match to the zeros of Y_T function. Then, the required condition is given for SCS port as follows:

$$\begin{aligned} |Z_{L1}| &\leq |Z_{in}| \\ \text{or} \\ |Y_{L1}| &\geq |Y_{in}| \end{aligned} \quad (5.12)$$

Equ. 5.12 states that to have a stable NICE the magnitude of the external impedance, Z_{L1} , has to be lower than the magnitude of input impedance of the NIC under SCS conditions. So, it implies that the analysis on the stability of a NIC has to take into account the external load. In this way, successive considerations can be obtained for different ports and loads. These conditions are summarized in Table 5.2.

Parameter	Expression
Characteristic Equation Z	$Z_T(I, \omega) = Z_{NIC}(I) + Z_L(\omega) = 0$
Characteristic Equation Y	$Y_T(V, \omega) = Y_{NIC}(V) + Y_L(\omega) = 0$
Characteristic Equation Γ	$\Gamma_T(A, \omega) = -\frac{1}{\Gamma_{NIC}(A)} + \Gamma_L(\omega) = 0$

Table 5.2: NICs Characteristic Equations of Reference Plane

Now, the stability conditions of a NIC, depending on the output port (1 or 2), can be summarized as follows:

$$\begin{aligned} |Z_{L1}| &\geq |Z_{in}| & \text{or} & & |Y_{L1}| &\leq |Y_{in}| & \text{for OCS port} \\ |Z_{L1}| &\leq |Z_{in}| & \text{or} & & |Y_{L1}| &\geq |Y_{in}| & \text{for SCS port} \\ |Z_{L2}| &\geq |Z_{out}| & \text{or} & & |Y_{L2}| &\leq |Y_{out}| & \text{for OCS port} \\ |Z_{L2}| &\leq |Z_{out}| & \text{or} & & |Y_{L2}| &\geq |Y_{out}| & \text{for SCS port} \end{aligned} \quad (5.13)$$

The stability of a NIC depends not only on its load conditions (load to be inverted) but also on the external load. The condition for a NIC to be stable is that the function, Z_T , Y_T or Γ_T , must not have any poles in the RHP.

These classic linear methods are widely used on antenna designs [NRH14] and problems to determine the stability are totally accepted [NHS⁺15].

5.3.2 Study of Classic Stability Methods on NICs

A set of three examples is used to illustrate the study of the stability of NICs circuits. The NIC model used for these examples is the ungrounded Linvill topology. This NIC is used to invert a capacitor, an inductance and a resistor.

When the admittance or impedance methods are used, Fig. 5.9, the NIC is considered as an inverter of the suitable load as impedance or admittance. For a

more general analysis the three proposed NICs (capacitor, inductance and resistor inverters) are analyzed using the admittance, impedance, Loop-Gain and NDF methods.

Figure 5.9: NICs as Admittance and Impedance Inverters

The transistor used for the study of the stability is a BFR360F. First the study of the NIC considering the transistor as the isolated “die”, Fig. 5.10(a), is performed, then the effect of the package of the transistor, Fig. 5.10(b), is considered. To simplify the schematics presented on the study of the stability of NICs, the circuits do not use the bias network and their effects are not considered.

Figure 5.10: BFR360F

5.3.2.1 NIC as Capacitor Inverter

The first example is a NIC that inverts a capacitor to compensate the capacitive effect of a R-L-C parallel resonant circuit. An ideal transformer is used to obtain the desired value, impedance or admittance, of the ungrounded circuit. This transformer is an interesting tool, because in multiple ungrounded circuits the lumped elements do not allow the use of the virtual ground.

The analysis of the stability is performed using the Y_T , Z_T , Loop-Gain, NDF and, finally, transient time.

5.3.2.1.1 Study of NIC as Capacitor Inverter Using Total Admittance, Y_T

The analysis of the ungrounded NIC with the R-L-C resonant circuit is performed using an ideal transformer, that is placed in parallel with the other elements, Fig. 5.11. The use of the transformer allows to reference the measurement of the circuit to ground. This analysis is done using an “*ideal*” version of the BFR360F transistor that does not consider the package. All the simulations consider the transistor as a “*die*” while it is not pointing something else.

Figure 5.11: Admittance Analysis of NIC with a Capacitor

Before performing the Y_T analysis, it is necessary to check the proviso. The required proviso is that the active part, NIC, does not have any poles in the RHP when it is loaded with a short-circuit. So, the NDF of the left sub-circuit (NIC) in Fig. 5.11 loaded with a short-circuit is performed, Fig. 5.12. The proviso is satisfied, so the admittance analysis can be properly used on this example circuit.

When the total admittance is obtained for this circuit, Fig. 5.13, this analysis indicates that the circuit is unstable. Then an ideal transistor (just an ideal controlled current source) is considered, without any parasitic effects, and in this case the NIC is stable, Fig. 5.13(c). This fact seems to confirm that minor parasitic effects are a high risk for the stability of NICs.

Figure 5.12: NDF of Short-Circuited NIC with a Capacitor

Figure 5.13: Y_{TOT} of NIC with a Capacitor

5.3.2.1.2 Study of NIC as Capacitor Inverter Using Total Impedance, Z_T

Now the circuit is analyzed using the Z_T , Fig. 5.14. This is not the preferred method that designers would choose, but it is an interesting work to illustrate the study of the stability of NICs.

Figure 5.14: Impedance Analysis of NIC with a Capacitor

The proviso that must be verified before performing the Z_T analysis, that is the active part, NIC, does not have any poles in the RHP when it is loaded with an open-circuit. So, the NDF of the left sub-circuit (NIC) in Fig. 5.14 in open-circuit is performed, Fig. 5.15. The proviso is not satisfied, so the impedance analysis can provide incorrect result, but it is analyzed to illustrate the risk of using a linear method without the verification of the suitable provisos.

Figure 5.15: NDF of Open-Circuited NIC with a Capacitor

The Z_T analysis of this NIC circuit shows that the total impedance does not encircle the origin, Fig. 5.16, so it indicates that the NIC is stable. This solution must be considered with utmost caution because the proviso was not satisfied. It is known that the NIC that uses the real BFR360F (even without package) it is not stable, so the conclusion of the Z_t is incorrect (because the proviso is not satisfied). This example as many others presented in this work demonstrate the risk of not checking the suitable provisos.

Figure 5.16: Z_{TOT} of NIC with a Capacitor

5.3.2.1.3 Study of NIC as Capacitor Inverter Using Loop-Gain

Before analyzing this circuit using the Loop-Gain method it must be redrawn in a more suitable form. Using Alechno Virtual Ground [Ale99] the circuit in Fig. 5.11 and Fig. 5.14 can be redrawn, to use the Loop-Gain analysis, in several forms as it is shown in Fig. 5.17.

The provisos that must be verified before using the Randall and Hock [RH01] Loop-Gain method are:

1. The Open-Loop network must be stable when both ports are loaded with Z_0 . This proviso can be verified using the NDF on the Z_0 loaded network and verifying that the Nyquist plot of the NDF does not have any clockwise encircles around the origin.
2. The Test Function, $TF = 1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}$, Nyquist analysis does not clockwise encircle the origin.
3. None of the S parameters have any poles. This proviso is verified with the first proviso of this list.

Figure 5.17: NIC with a Capacitor with Several Virtual Grounds

The NDF of the Z_0 loaded circuits, Fig. 5.17, and their TF are calculated. The NDFs do not present any clockwise encircles around the origin, Fig. 5.18, and the TFs, Fig. 5.19, do not have any clockwise encircles, so both conditions are verified. As the required provisos are satisfied for the three circuits in Fig. 5.17, the Loop-Gain Method can be used with all of them.

The Loop-Gain analysis, Fig. 5.20, of all the circuits in Fig. 5.17, confirm that all of them are not stable. This conclusion is obvious, because all of them are really the same circuit, but the analysis of different draws of the same circuit (using Virtual Ground) provide different solutions. When the proviso is not satisfied for any “topologies” the Loop-Gain will provide incorrect solutions, some of these examples are presented when the oscillators are analyzed using the Loop-Gain method, Sec. 4.5.

Figure 5.18: NDF of Z_0 loaded NIC with a Capacitor for Several Virtual Grounds

Figure 5.19: TF of NIC with a Capacitor for Several Virtual Grounds

Figure 5.20: GL of NIC with a Capacitor for Several Virtual Grounds

5.3.2.1.4 Study of NIC as Capacitor Inverter Using NDF

As the last method used to verify the stability of the circuit in Fig. 5.11, the NDF of the complete circuit is performed. Of course, the transformer is not used as the NDF is a function of the network which does not require any port for any measurement. The NDF of this circuit, Fig. 5.21, presents a clockwise encircle around the origin, so it is confirmed that the NIC has a pair of complex poles in the RHP, so this NIC is unstable.

Figure 5.21: NDF of NIC with a Capacitor

5.3.2.1.5 Study of NIC as Capacitor Inverter Using NDF, Including Transistor Package Effect

When the effect of the package of the transistor, Fig. ??, is considered, the NIC has more stability problems. The study of the stability of this circuit using any of the methods shows multiple instabilities. The NDF of the complete network, Fig. 5.22, two pairs of complex poles in the RHP.

Figure 5.22: NDF of NIC with a Capacitor Including Transistor Package Effect

The transient time analysis of the NIC confirms the stability problem of the circuit and shows the existence of at least two oscillation frequencies, Fig. 5.23. The lower frequency oscillation starts the first, and then the higher one appears in combination with the first one.

Figure 5.23: Transient of NIC with a Capacitor Including Transistor Package Effect

5.3.2.2 NIC as Inductor Inverter

This second example is a NIC that inverts an inductor to compensate the capacitive effect of a R-L-C serial resonant circuit. On a similar way as in the previous examples an ideal transformer is used for obtaining the impedance or admittance of the ungrounded circuit.

This analysis of the stability is also performed using the Z_T , Y_T , Loop-Gain, and NDF.

5.3.2.2.1 Study of NIC as Inductor Inverter Using Total Impedance, Z_T

The analysis of the ungrounded NIC (which is placed in a series with the other elements) with the R-L-C serial resonant circuit is performed using an ideal transformer, Fig. 5.24. The use of the transformer allows to reference the measurement of the circuit to ground. This analysis is done using an “*ideal*” version of the BFR360F transistor (die) which does not consider the package, while it is not pointing something else.

Figure 5.24: Impedance Analysis of NIC with an Inductor

The proviso that must be checked before using the total impedance method is to verify that the NDF of the active sub-circuit loaded with an open circuit does not encircle the origin. As it is shown in Fig. 5.25 the NDF does not clockwise encircle the origin, then the NIC does not have any poles in the RHP for an open condition, so the Z_T method can be applied.

Figure 5.25: NDF of Open-Circuited NIC with an Inductor

The origin is not clockwise encircled by the Nyquist analysis of the Z_T of this NIC, Fig. 5.26, so it indicates that the circuit is stable.

Figure 5.26: Z_{TOT} of NIC with an Inductor

5.3.2.2.2 Study of NIC as Inductor Inverter Using Total Admittance, Y_T

The Y_T analysis of the ungrounded NIC with the series R-L-C resonant circuit is performed using an ideal transformed, as it is shown in Fig. 5.27.

Figure 5.27: Admittance Analysis of NIC with an Inductor

A proviso must be verified before performing the Y_T analysis. As it has been pointed, the required proviso is that the active part of the circuit does not have any poles in the RHP when it is loaded with a short-circuit. This proviso can be verified using the NDF to analyze the left sub-circuit (NIC) in Fig. 5.27 loaded with a short-circuit, Fig. 5.28. The proviso is not satisfied, so the admittance analysis cannot be used to analyze this example circuit with guarantee of correct results. But, to illustrate the problems when the proviso is not checked the Y_T analysis is performed.

Figure 5.28: NDF of Short-Circuited NIC with an Inductor

When the total admittance is obtained for this circuit, Fig. 5.29, it indicates the existence of a pair of poles of the Y_T in the RHP. This analysis is not determinant on the stability of the NIC, but as the proviso was checked it can be verified that the Nyquist criteria of the Y_T shows $N = Z - P$ (positive is clockwise). The

NDF has a clockwise encirclement, so the NIC has a pole, $P = 1$. The Y_T has a counter-clock encirclement, so $N = -1$. As $Z = P + N$, the number of poles of the Y_T function is zero ($Z = 1 - 1$). Then it provides that the NIC is stable, but this analysis method is not suggested as the proviso was not satisfied.

Figure 5.29: Y_{TOT} of NIC with an Inductor

5.3.2.2.3 Study of NIC as Inductor Inverter Using Loop-Gain

Using the Virtual Ground, three different representations of the same circuit, Fig. 5.24, are obtained. The three options used for illustrating the Loop-Gain analysis are shown in Fig. 5.30.

As it has been shown in this work, before going on the Loop-Gain analysis the suitable provisos must be verified. The NDF of the Open-Loop network when it is loaded with Z_0 on both ports is shown in Fig. 5.31. The NDF proviso is verified for the three “circuits” in Fig. 5.30, because none of the NDFs have any clockwise encirclement around the origin.

Figure 5.30: NIC with an Inductor with Several Virtual Grounds

Figure 5.31: NDF of Z_0 loaded NIC with an Inductor for Several Virtual Grounds

The verification of the second proviso for the Loop-Gain method is the Nyquist analysis of the TF. This analysis shows that the configurations Fig. 5.30(b) and Fig. 5.30(c) do not satisfy the proviso, so the Loop-Gain method could be only used for the circuit on configuration Fig. 5.30(a).

Figure 5.32: TF of NIC with an Inductor for Several Virtual Grounds

The Loop-Gain analysis of all the circuits is shown in Fig. 5.33. The circuit a) is the only one that satisfies the provisos. The Nyquist analysis of the Loop-Gain of this circuit (Fig. 5.30(a)) confirms that it is stable. The Nyquist analysis of the other two circuits that do not satisfy the provisos have different results. The circuit (b) has two crosses over the positive real axis, it complies the Nyquist stability criteria, but it does not comply the Barkhausen criteria. But the circuit (c) shows an instability as a result of the Loop-Gain analysis. The results of the circuits (b) and (c) must be considered taking into account that these circuits do not satisfy the required provisos for the Loop-Gain analysis.

Figure 5.33: GL of NIC with an Inductor for Several Virtual Grounds

5.3.2.2.4 Study of NIC as Inductor Inverter Using NDF

The NDF analysis of the complete NIC circuit, Fig. 5.24, does not require any proviso and it is always applicable to any circuit topology, so its solution is always correct. In this case the NDF, Fig. 5.34, shows that the NIC is stable, as it was predicted by the classic methods that comply with provisos.

Figure 5.34: NDF of NIC with an Inductor

5.3.2.2.5 Study of NIC as Inductor Inverter, Including Transistor Package Effect

Now the effect of the package of the transistors are considered in the study of the stability of the NIC that inverts an inductor. The study of the stability follows that same steps as in the study without package effect.

Before performing the stability study using the Admittance Network Method, Z_T , the absence of poles in the RHP of the active sub-circuit loaded with an Open-Circuit must be verified using the NDF. The proviso for the Z_T is not satisfied because the NDF, Fig. 5.35, has three encirclements around the origin. Sometimes, it is interesting to use the unwrap phase of the NDF, Fig. 5.36, to check the number or encirclements, $360 \cdot (-3) \simeq -1081$. Then, the Z_T stability analysis is not done because the information that it will be provided is going to be incorrect.

Figure 5.35: NDF of Open-Circuited NIC with an Inductor, Including Transistor Package Effect

Figure 5.36: Phase of NDF of Open-Circuited NIC with an Inductor, Including Transistor Package Effect

The proviso for the Y_T , NDF of short-circuit loaded active sub-circuit, is not satisfied because it has two pairs of poles in the RHP. So the Impedance Network stability analysis is also omitted.

The analysis of the stability of the NIC that inverts an inductor using the Loop-Gain method can be studied for the three circuits in Fig. 5.30 when the parasitic effects of the transistor package are considered. The first step verifies the required provisos for the three “*topologies*” of the NIC. The NDF of the open-loop circuits loaded with Z_0 , Fig. 5.37, show that the circuits (a) and (c) comply this first proviso while the circuit (b) does not.

Now the second proviso required for the Loop-Gain method is verified for the circuits in Fig. 5.30. The TF of these circuits, Fig. 5.38, have encirclement around the origin for the circuit (a), but it does not have for the circuits (b) and (c). The TFs have several loops that make difficult to study the plot, so it may be useful to use the representation of the phase, Fig. 5.39. The phase plots show that the circuit (b) and (c) do not have any encirclements, while the circuit (a) has a complete encirclement around the origin.

Figure 5.37: NDF of Z_0 loaded NIC with an Inductor for Several Virtual Grounds, Including Transistor Package Effect

Then, as conclusion of the study of the provisos required for the Loop-Gain methods only the circuit in Fig. 5.30(c) complies both. The Loop-Gain of the circuits are shown in Fig. 5.40, but it is complex to obtain conclusion of circuits that do not comply the provisos. The detail of the Loop-Gain of the circuit (c) is shown in Fig. 5.40(c). The Loop-Gain method indicates that the NIC is not stable when the package of the transistor is considered, while it was stable when the package effect was omitted.

Figure 5.38: TF of NIC with an Inductor for Several Virtual Grounds, Including Transistor Package Effect

Figure 5.39: Phase of TF of NIC with an Inductor for Several Virtual Grounds, Including Transistor Package Effect

Figure 5.40: GL of NIC with an Inductor for Several Virtual Grounds, Including Transistor Package Effect

Finally, the proposed NDF Method is used to analyze the NIC that inverts an inductor when the package effect of the transistor is considered on the circuit model. The NDF of the complete circuit, Fig. 5.24, is analyzed, Fig. 5.41, and it confirms that the NIC goes unstable when the package of the transistors are considered.

Figure 5.41: NDF of NIC with an Inductor, Including Transistor Package Effect

The transient time analysis of the NIC confirms the unstable condition of the circuit. In Fig. 5.42, it can be observed the damped response of the resonant R-L-C circuit and the start of the undesired oscillation.

Figure 5.42: Transient of NIC with an Inductor, Including Transistor Package Effect

5.3.2.3 NIC as Resistor Inverter

The stability of the NIC is also studied for the NIC inverting a resistor. Two antenna equivalent circuits are used for this study, parallel and serial R-L-C resonant circuits. The NIC is used to reduce the equivalent resistor of the antenna increasing the Q . These two models are analyzed using the classic linear stability

methods and then using the NDF. On a first step the transistors of the NIC will not consider the package effect, which are going to be considered on the last step of the stability analysis.

5.3.2.3.1 NIC with Parallel R-L-C Circuit

The NIC that is used to invert a resistor is placed in parallel with the parallel R-L-C resonant circuit. This circuit is analyzed using the Admittance Network Function (Z_T), the Impedance Network Function (Y_T), the Loop-Gain and the NDF.

5.3.2.3.1.1 Study of NIC as Resistor Inverter Using Total Impedance, Z_T

The schematic used for performing the analysis of the total impedance, Z_T , is shown in Fig. 5.43. The total impedance can be obtained by using an auxiliary transformer, that is useful for circuits where the virtual ground is not easy to use.

Figure 5.43: Impedance Analysis of Resistor-NIC with a Parallel R-L-C Circuit

The required proviso must be verified before using Nyquist plot of the Z_T to study the stability of the circuit in Fig. 5.43. As it is well-known, the proviso consists on the study of the NDF of the active sub-circuit loaded with an open circuit, which NDF, Fig. 5.44, must not encircle the origin. The proviso is partially satisfied, the NDF has 0.5 clockwise encirclement around the origin. The Z_T analysis can be performed, but further considerations must be done.

Figure 5.44: NDF of Open-Circuited Resistor-NIC for a Parallel R-L-C Circuit

The study of the Z_T of the circuit in Fig. 5.43 shows 0.5 counter-clock encirclement around the origin (the proviso has 0.5 clockwise encirclement around the origin). So the result is that this NIC is stable, Fig. 5.45. This stability considers the real transistor die, but it does not include the package parasitic effect, that will be considered later. These cases with partial encirclements on proviso or in total admittance or impedance must be considered with special care.

Figure 5.45: Z_{TOT} of Resistor-NIC with a Parallel R-L-C Circuit

5.3.2.3.1.2 Study of NIC as Resistor Inverter Using Total Admittance, Y_T

Now, the schematic in Fig. 5.46 is used to perform the analysis of the total admittance, Y_T . In this case also an auxiliary transformer is used to obtain the total admittance without requiring any virtual or real ground on the circuit.

Figure 5.46: Admittance Analysis of Resistor-NIC with a Parallel R-L-C Circuit

The NDF of the active sub-circuit loaded with a short-circuit, Fig. 5.47, must not encircle the origin. This is the required proviso to verify before using Nyquist plot of the Y_T to study the stability of the circuit. The proviso is also satisfied, so it is possible to use the Y_T analysis to study the stability of the NIC in Fig. 5.46.

Figure 5.47: NDF of Short-Circuited Resistor-NIC for a Parallel R-L-C Circuit

The Y_T of this NIC with a parallel resonant R-L-C, Fig. 5.48, does not encircle the origin, so the circuit is stable as it was also predicted by the total impedance analysis in the previous steps.

Figure 5.48: Y_{TOT} of Resistor-NIC with a Parallel R-L-C Circuit

5.3.2.3.1.3 Study of NIC as Resistor Inverter Using Loop-Gain

To study the stability of the NIC inverting a resistor, being used on a parallel resonant R-L-C circuit, three topologies of the same circuit are proposed, Fig. 5.49.

Before any further analysis using the Loop-Gain method it is necessary to verify the two provisos that are required by this method. Both of them, NDF of Z_0 loaded open loop circuit, Fig. 5.50, and TF, Fig. 5.51, are satisfied for all the circuits in Fig. 5.49. So it is possible to use the Loop-Gain analysis for all the proposed topologies of the NIC.

Figure 5.49: Resistor-NIC with a Parallel R-L-C Circuit with Several Virtual Grounds

Figure 5.50: NDF of Z_0 loaded Resistor-NIC with a Parallel R-L-C Circuit for Several Virtual Grounds

Figure 5.51: TF of Resistor-NIC with a Parallel R-L-C Circuit for Several Virtual Grounds

The Nyquist analysis of Loop-Gain of the different redraws of the NIC, Fig. 5.52, show that all circuits are stable.

Figure 5.52: GL of Resistor-NIC with a Parallel R-L-C Circuit for Several Virtual Grounds

The analysis using the Z_T , Y_T and Loop-Gain methods of the NIC with a parallel R-L-C circuit that inverts a resistor are possible, because all the provisos are satisfied. All the analysis show the same result: the NIC is stable without considering transistor package effect. Later in this chapter the parasitic effects of the transistor package will be considered.

5.3.2.3.1.4 Study of NIC as Resistor Inverter Using NDF

The NDF of the complete circuit, Fig. 5.53, shows that the circuit is stable. The conclusion of the NDF is the same as all the other methods have shown. For this circuit, it has been possible to use all the classic methods shown in this work because the provisos were satisfied for all of them. But in more complex circuits that include multiple parasitic effects the provisos are not always satisfied, so the best tool to analyze them is the NDF Method as it is proposed in this work.

Figure 5.53: NDF of Resistor-NIC with a Parallel R-L-C Circuit

5.3.2.3.2 NIC with Serial R-L-C Circuit

The same ungrounded NIC that was used to invert a resistor in the previous section is placed in serial with a serial R-L-C resonant circuit. This circuit is also analyzed using the Admittance Network Function (Z_T), the Impedance Network Function (Y_T), the Loop-Gain and the NDF.

5.3.2.3.2.1 Study of NIC as Resistor Inverter Using Total Impedance, Z_T

The Z_T of the circuit is analyzed using an auxiliary transformer, Fig. 5.54, with similar procedure as in previous cases.

Figure 5.54: Impedance Analysis of Resistor-NIC with a Serial R-L-C Circuit

The required proviso for the Z_T stability method is shown in Fig. 5.55. The NDF of the open-circuit loaded NIC (active sub-circuit) does not encircle the origin, so the proviso is satisfied and the Z_T analysis can be performed.

Figure 5.55: NDF of Open-Circuited Resistor-NIC for a Serial R-L-C Circuit

The study of the Z_T of the NIC with the serial R-L-C circuit, Fig. 5.54, does not encircle the origin, so the NIC is stable, Fig. 5.56.

Figure 5.56: Z_{TOT} of Resistor-NIC with a Serial R-L-C Circuit

5.3.2.3.2.2 Study of NIC as Resistor Inverter Using Total Admittance, Y_T

Now, the schematic in Fig. 5.57 is used for the total admittance, Y_T , of the circuit.

Figure 5.57: Admittance Analysis of Resistor-NIC with a Serial R-L-C Circuit

The NDF of the active sub-circuit loaded with a short-circuit, Fig. 5.58, does not encircle the origin. The proviso is satisfied, so it is possible to use the Y_T analysis to study the stability of the NIC. It must be pointed that it has 0.5 clockwise encirclement around the origin.

The Y_T of this NIC with a serial resonant R-L-C, Fig. 5.59, has 0.5 counter-clock encirclement around the origin, so the circuit is stable as it was also predicted by the total impedance analysis in the previous steps.

Figure 5.58: NDF of Short-Circuited Resistor-NIC for a Serial R-L-C Circuit

Figure 5.59: Y_{TOT} of Resistor-NIC with a Serial R-L-C Circuit

5.3.2.3.2.3 Study of NIC as Resistor Inverter Using Loop-Gain

Three topologies of the same circuit are proposed, Fig. 5.60, for the study of the stability of the NIC inverting a resistor on a serial with a serial resonant R-L-C circuit.

The NDF of the Z_0 loaded circuits and the TF provisos are satisfied for the three configurations, Fig. 5.61 and Fig. 5.62, so it is possible to analyze the three configurations using the Loop-Gain method.

The Nyquist analysis of Loop-Gain of the three redraws of the NIC, Fig. 5.63, show that they are stable. Later in this chapter the parasitic effects of the transistor package will be considered for this resistor-NIC in serial with the serial R-L-C resonant circuit.

Figure 5.60: Resistor-NIC with a Serial R-L-C Circuit with Several Virtual Grounds

Figure 5.61: NDF of Z_0 loaded Resistor-NIC with a Serial R-L-C Circuit for Several Virtual Grounds

Figure 5.62: TF of Resistor-NIC with a Serial R-L-C Circuit for Several Virtual Grounds

Figure 5.63: GL of Resistor-NIC with a Serial R-L-C Circuit for Several Virtual Grounds

Figure 5.64: NDF of Resistor-NIC with a Serial R-L-C Circuit

5.3.2.3.2.4 Study of NIC as Resistor Inverter Using NDF

5.3.2.3.2.5 Study of NIC as Resistor Inverter, Including Transistor Package Effect

Now the parasitic effect of the transistors packages, Fig. ??, are considered for the two proposed models of NICs that invert a resistor.

The first model, a NIC in parallel with a parallel R-L-C resonant circuit, Fig. 5.43, is analyzed. The NDF of the complete circuit, Fig. 5.65, shows that the NIC is now unstable, while it was stable when the package effect of the transistors were not considered. The NDF shows two clockwise encirclements around the origin, one at 5378 MHz and other at 7405 MHz .

The transient time analysis of the NIC, Fig. 5.43, confirms the unstable condition of the circuit at least at one frequency, Fig. 5.66.

Figure 5.65: NDF of Resistor-NIC with a Parallel R-L-C Circuit, Including Transistor Package Effect

Figure 5.66: Transient of Resistor-NIC with a Parallel R-L-C Circuit, Including Transistor Package Effect

The second model is a NIC in serial with a serial R-L-C resonant circuit, Fig. 5.54. The NDF, when the package of the transistor is considered, of the complete circuit, Fig. 5.67, shows that the circuit becomes unstable, but it is stable for transistors without package effect. The transient time analysis of the circuit also confirms the oscillation condition, Fig. 5.68.

Figure 5.67: NDF of Resistor-NIC with a Serial R-L-C Circuit, Including Transistor Package Effect

Figure 5.68: Transient of Resistor-NIC with a Serial R-L-C Circuit, Including Transistor Package Effect

5.4 Conclusions on NICs Study

A detailed analysis of several NICs models has been presented in this chapter. Multiple examples of NICs and their stability study have been presented with different level of parasitic effects on consideration. The real model of a transistor has been used, but the stability study has been made considering the transistor package and without this parasitic effect.

The first conclusion that can be obtained from the study of the stability of the NICs is that the preferred method for the study of the stability is the NDF Method, but the classic methods can also be used if the provisos are verified. Lately, some authors consider the use of the Return Relation or NDF as the preferred methods for the study of the stability of NICs [TX14, TX15]. The use of the NDF is not only the most suitable method to analyze the stability of an isolated NIC with a passive load, but also the method to determine the stability and the possible hidden poles of a NIC combined with other active elements as active wide-band antennas, that use a NIC.

The inclusion of the package of the transistors has a disastrous effect on the stability of the NIC. It can be assumed that the effect of parasitics on the transistor interconnection has mayor influence on the stability of the circuit. So, the parasitic effects of transistors and elements used in the interconnection of the two transistors must be considered up to high frequencies to discard any problems on stability.

The problems on stability when discrete transistors are used for high frequency applications of NICs make the use of these NICs very difficult to be applied to practical applications. It seems interesting to open future investigation lines on the stability of NICs implemented on MMIC technology, due to the low parasitic effects on interconnections of this technology.

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Chapter 6

ACTIVE PATCHES

6.1 Introduction

The definition of an active patch is: “An **active patch antenna** is a patch antenna that contains active electronic components”. A patch antenna with any active components, from a simple diode for tuning to a complex function, inside the antenna is an active patch antenna, Fig. 6.1.

Figure 6.1: Active Patch Antenna

Active patches have a great list of advantages for nowadays RF communication applications. The main advantages are that, in addition to the important advantages that the microstrip antennas have, they are easily integrated with that active RF circuit. The integration is so deep in a lot of cases that there is not any reference plane (typically defined with an impedance of 50Ω) between the radiating element and the active circuit. The active devices can be “*inside*” the patch and the patch can be a fundamental “*internal*” element of the active circuit, as for example the resonator on an oscillator. The active antennas can be considered as an active microwave circuit where one output/input port is the free space instead of a conventional 50Ω interface.

In this chapter the great importance of the study of the fields inside the patch, when the patch is used as a two port device, is shown. Then the problem of the study of the stability of the active antennas is presented and some examples are used to illustrate and demonstrate the stability study and the use of the NDF as a tool to study the stability.

Some active functions that may be integrated in patches are: oscillator, power amplifier, LNA, switcher, NIC, etc. In fact, the complete RF transceiver may be integrated in the patch.

The integration of the oscillator function and the NIC use are studied in this chapter. These two functions are used to exemplify the problems and techniques to study the stability of any active patch.

6.2 Active Patches Analysis

The active patches present some issues that must be carefully considered on the design process. Some of these aspects are:

- The size of the pad of interconnection point (or points) of the active devices must be considered as a distributed feed because the patch has a quick variation of the impedance.
- The patch can be used as one port or two ports, but to use it as a two port element increases the complex of the analysis and design.
- The active element or elements may disturb the currents in the patch, exciting upper modes. So a more complex and detailed modeling using full-wave 3D analysis must be used.

The purpose of using active devices in patch antennas can cover multiple purposes as: increasing the effective length of short antenna, increasing antenna bandwidth, decreasing the mutual coupling between array elements, improving the noise factor, resonating, filtering, mixing, duplexing, power amplifying, phase shifter, etc.

In the active patches the freedom to optimize the oscillator and antenna as independent elements does not exist, because there is not an obvious distinction between the active circuit and the radiating structure. The patch works as a load or resonator and as radiator of the active patch. Multiple examples of active patches can be found in the literature, to provide some examples functions as a Rectangular patch with a gun diode as TX and FM modulator, [BBS96] and a self-oscillating down-converter patch, [ZWC99, CKI94], can be listed.

The classic analysis of the active patches uses the same techniques as on amplifiers or oscillators to determine the stability or oscillating behaviour of the antenna. So, the same problems with the provisos that are present on the oscillators analysis, Cap. 4, are applied to the active patches analysis. The use of NDF as a tool for the linear study the stability of active patches makes possible the research of new topologies and the study of the stability without provisos and problems that classic methods infer. So, the discussion on the analysis of oscillators is fully applicable to the active patches.

The active patches design requires in addition to the determination of the stability and active function performances of the circuit, the radiation performances of the antenna. And these studies must be performed considering the lack of independence between them.

The study of active patches is illustrated with two examples. First, an oscillating rectangular patch antenna that includes a NIC is presented, and then, a patch as feedback network of an oscillating antenna. The second example shows the importance of the study of the fields inside the patch when the interconnection points of the active devices are *inside* the patch.

6.3 Practical Examples

Two examples are presented and the measurements of the manufactured circuits are compared with the simulation results. These examples are an active patch antenna with an oscillator function with a NIC; and an active patch that uses the patch as feedback network.

6.3.1 Active Patch with NIC

This example consists on an oscillator that has a patch antenna as resonator. A NIC is used to increase the Q of the oscillating antenna. The Q of an antenna is inverse related to its size [Chu48, Whe47], so the optimum size antennas with high radiation efficiency have low Q . The optimization of the Q must improve the phase noise of the oscillator, but other aspects must be considered. The oscillating antenna is analyzed using the classic Admittance Method and the NDF Method [JMGPPC+12]. There are some discrepancies between the classical method and the NDF, but the HB analysis [Rhe10, VPR05, SQ03, KO81] and the experimental results show that the correct results during the linear analysis are provided by the NDF Method.

First, the different elements that are used on the active patch antenna are presented and analyzed. Then, a patch antenna with an ideal NIC is analyzed to get an upper limit of Q . And finally, a real NIC patch oscillator is analyzed.

6.3.1.1 Brief Description of Active Patch Elements

6.3.1.1.1 Patch Antenna

The patch antenna used for this example is a Rectangular Microstrip Patch (RMP). It is build on a FR4 substrate with a height of 0.5 mm and a $\epsilon_r = 4.5$. This PCB material is also used for the rest of the circuit of the active patch antenna. The dimensions of the RMP can be approximated using the Cavity Resonator

Mode, Sec. 1.3.2.1. The design frequency of the patch antenna is 1414 MHz, so for the Mode TM_{10} the length must be 5 cm; and 3 cm is chosen as width to have a Mode TM_{01} high enough. The adjust of the patch dimensions and the exact determination of the feed is obtained by using the Axiem© simulator in NI-AWR©.

The S_{11} of the patch antenna with a finite ground plane (real size of the manufactured PCB. 9.2x6.0 cm) is obtained. In Fig. 6.2 the S_{11} of the infinite patch antenna and real patch antenna are shown. This patch antenna has an equivalent circuit model of a parallel R-L-C resonant circuit [GA10] with a series parasitic L. This circuital model is most suitable for the initial linear simulations, for the understanding of the oscillator topology and for the optimization of the design. But the S parameters obtained from the electromagnetic model of the patch must be used on the stability analysis, because using a simplified model may cause undetectability of undesired instabilities.

Figure 6.2: S_{11} of RMP Antenna

6.3.1.1.2 Negative Conductance Generator

The equivalent circuit model of the patch antenna used as resonator, that it a parallel resonant circuit, determines that the required active circuit must be a negative conductance generator. A common base circuit (Colpitts) [NAS05] is used for this work, Fig. 6.3. The conductance of this Colpitts negative generator is shown in Fig. 6.4. It is also required to check that the magnitude of the conductance is properly reduced with the compression of the transistor.

Figure 6.3: Colpitts Negative Conductance Generator

Figure 6.4: Conductance of Negative Generator

6.3.1.1.3 Negative Impedance Converter (NIC)

The NIC used in this work, Fig. 6.5, synthesizes a negative impedance to reduce the equivalent conductance of the patch. The reduction of the equivalent conductance of the patch makes the Q_L of the active oscillator antenna to be increased. The equivalent conductance of the patch antenna is $11.9 \times 10^{-3} S$, so a NIC with a conductance of $-10.87 \times 10^{-3} S$ ($Z = -92 \Omega$) is used for this example.

The admittance of the real NIC is not constant with the frequency, so it is tuned for the frequency range in which the oscillator operates. The conductance of the real NIC, including PCB and real model of all the components, is shown in Fig 6.6. The NIC is used as a parallel lumped element to the patch antenna in order to increase the Q_L .

Figure 6.5: Negative Impedance Converter

Figure 6.6: Conductance of Negative Impedance Converter

6.3.1.2 Linear Analysis of Active Patches

Two different models of proposed active patches are analyzed. There is only one difference between the two models of active patch antennas, it is the inclusion of the NIC to increase the Q of the resonator. The analysis of each model is performed using both linear methods: classic Y_T method and NDF Method. The classic linear method is always used, but the provisos that most of times are omitted are checked in order to determine if the discrepancies of the methods are caused by the omission of the verification of the provisos.

It is important to point that to perform an analysis using the Admittance Method, by searching the poles of the Impedance Network Function, some provisos must be verified [JMGPPC⁺12, GPJMPC⁺11]. In this case, the proviso that must be satisfied before performing the Admittance analysis is that the NDF of the negative conductance generator loaded with a short-circuit must not clockwise

encircle the origin. When the origin is not encircled, the negative conductance generator (or complete active sub-circuit), Y_{NegGen} , does not have any poles in the Right Half Plane (RHP) [Jac92, PSH93, PS94, Jac06], and the oscillator is suitable to be analyzed with the Admittance Method.

6.3.1.2.1 Simple Oscillator

The objective of using a NDF on the oscillator is to increase the Q_L , so an oscillator without NIC must be analyzed to have a reference to compare with. The “*simple oscillator*” is composed by the patch and the negative conductance generator.

The first analysis is the classic linear one, it is analyzed using the Admittance Method. The proviso that must be checked is the NDF of the negative conductance generator, loaded with a short-circuit, must not encircle the origin. The verification of the proviso is shown in Fig. 6.7, it does not encircle the origin, so the circuit is suitable to be analysed using the Admittance Method.

Figure 6.7: NDF of Short-circuited Negative Conductance Generator

Once the proviso is satisfied, the linear analysis of the characteristic function of this circuit, Equ. 6.1, can be used. This analysis, Fig. 6.8, indicates that the oscillator will operate properly with an oscillation frequency of 1454 MHz.

$$Y_T = Y_{Patch} + Y_{NegGen} = 0 \quad (6.1)$$

The NDF method that has been proposed is also applied to the oscillator used as example. The NDF of the complete oscillator circuit, Fig. 6.9, predicts a proper conditioned oscillator with an oscillation frequency of 1439.4 MHz. When the NDF Method is used, as it was explained in Sec. 4.6, it is possible to estimate the Q_L of the oscillator, Fig. 6.10.

Figure 6.8: Y_T of Simple Oscillator

Figure 6.9: NDF of Simple Oscillator

Figure 6.10: Q_L of Simple Oscillator

Now that a reference circuit has been analyzed the oscillator that includes a NIC can be analyzed and the improvements on performances that are expected verified.

6.3.1.2.2 Oscillator with NIC

To get a best performance limit of the Q_L and of the phase noise, a design using an ideal NIC, -92Ω in this case, is performed. The circuit is formed with the negative conductance generator, Fig. 6.3, the patch and a parallel ideal negative resistor (-92Ω) as ideal NIC.

The analysis of the Y_T of the oscillator with the ideal NIC shows a figure similar to the one of the simple oscillator, and the same oscillation frequency, 1454 MHz .

The NDF analysis of the circuit, Fig. 6.11, shows a proper operation as oscillator and an oscillation frequency of 1435.9 MHz , similar to the one of the simple oscillator.

Figure 6.11: NDF of Ideal NIC Oscillator

To verify if the ideal NIC has the expected effect on the Q_L of the oscillator, it is estimated using the NDF and it is compared with the Q_L of the simple oscillator, Fig. 6.12. The Q_L presents an improvement around a factor of $\times 10$. Then the effect of an ideal NIC on the Q_L of the oscillator is confirmed at the linear design steps.

Now the analysis of an active patch oscillator that includes a NIC can be performed and the improve of the Q_L compared with the simple oscillator and the ideal NIC oscillator.

The first linear method used to analyze the oscillator is the Y_T . Then the oscillator is divided into two sub-circuits, one with the active part (negative conductance generator and NIC) and other with the passive patch. The CF of this oscillator is the same as in Equ. 6.1, but considering that Y_{NegGen} includes all the active sub-circuit. The first step must be to verify the proviso, but this verification is delayed to illustrate the risk of the classic methods when the provisos

are not checked. So, the Y_T is studied for the NIC oscillator, Fig. 6.13. It can be observed that the Admittance Method predicts a proper oscillation at 1406 MHz, and it does not indicate any problem on upper frequencies.

Figure 6.12: Q_L of Ideal NIC Oscillator vs. Simple Oscillator

Figure 6.13: Y_T of NIC Oscillator

Now the NDF Method is used, but the result shows multiple pairs of poles in the RHP, Fig. 6.14. So the NIC oscillator will not have a correct operation.

Figure 6.14: NDF of NIC Oscillator

The discordance of results of the Admittance Method and the NDF Method has an easy explanation: the proviso was not verified for the Admittance Method and it is not satisfied. To confirm this fact, the proviso is checked, Fig. 6.15. The proviso is not satisfied, so the Admittance Method provides incorrect results, the correct solution for the linear analysis of this oscillator is the one provided by the NDF Method.

Figure 6.15: NDF of Short-circuited Active Sub-circuit of NIC Oscillator

6.3.1.3 HB Analysis and Measurement of Active Patches

The next step on the design of the active patch is to perform a non-linear analysis. The Harmonic Balance (HB) is used for both models of active patches: the “*simple oscillator*” and the “*NIC oscillator*”.

The HB of the NIC oscillator presents some solutions that confirm that the circuit has several unstable frequencies as it was predicted by the NDF, Fig. 6.16. So, the prediction of the Admittance Method is wrong, of course, because the proviso is not satisfied.

Figure 6.16: Several HB Solutions of Real NIC Patch

Several HB simulations are performed, they are presented and compared with the measurements of the prototypes. The results of the phase noise, Fig. 6.17, must be discussed in a first stage. The phase noise of the simple oscillator is -122.58 dBc/Hz @ 100 KHz and the phase noise of the NIC oscillator has 2.59 dB degradation, but it does not have the expected improvement. The NIC oscillator presents a phase noise 4.71 dB worse than the ideal NIC oscillator. The reason of this degradation is not because the lack of Q_L improvement, but the extra active devices of the NIC also add noise to the circuit.

So, as the phase noise cannot confirm if the NIC has been able to increase the Q_L of the active patch, another analysis must be proposed to confirm the Q improvement. The study and measurements of the pushing of the patches, Fig. 6.18, simple and NIC models, confirm that the Q of the oscillator is increased with the inclusion of a NIC. This way, it can be confirmed that NICs are suitable for increasing the Q . But in this case, the noise contribution of the active devices of the NIC is greater than the contribution of the increased Q_L , so the final phase noise is worse, Fig. 6.17.

Figure 6.17: HB and Measurements of Phase Noise of Patches

The simple oscillator and the oscillator with a NIC were manufactured, the image of the active patch that includes the NIC is shown in Fig. 6.19. Their phase noise, Fig. 6.17, pushing, Fig. 6.18, and spectrum, Fig. 6.20 and Fig. 6.21, were measured and compared with HB simulations.

The oscillation frequencies of both oscillators are properly predicted by the HB simulations, Fig. 6.20 and Fig. 6.21. The frequencies of the simulations and measurements of the active patch with NIC are exactly the same, while the simple oscillator frequencies have a small mismatch. The exact match frequency of the NIC oscillator is a strange coincidence, in general such a good match is not obtained for this type of oscillators. In any case the simulations provide a good approximation of the behaviour of both active patches.

Figure 6.18: Pushing Simulations and Measurements of Patches

Figure 6.19: Photograph of Active Patch Antenna with NIC

The manufactured active patch that includes the NIC also presents the instability predicted by the NDF Method. It has been observed that with small variations on the voltage supply a high number of power-up's of the oscillator (about 10%) result on a chaotic oscillation, Fig. 6.22. This problem could exist with a high risk in broadband oscillators [Kur69, KBK69], then special care must be considered if the network includes NICs or active devices. The NDF Method is a good tool to detect those undesired conditions during linear analysis steps.

Figure 6.20: Simple Active Patch Spectrum

Figure 6.21: NIC Active Patch Spectrum

Figure 6.22: Measured of Unstable Condition

6.3.2 Active Feedback Patch

The proposed design is an active antenna that uses the patch antenna as a two-ports series resonator, and this patch is used as feedback network on a closed-loop oscillator. The patch antenna must be studied to confirm the required conditions; and the crucial compatibility of the Modes inside the patch with the feed network inserted in the patch.

The inclusion of the patch on the oscillator makes really difficult to use some of the classic oscillator design methods, and sometimes it is nearly impossible to use them, because the ground reference is fixed by the patch and the Virtual Ground [Ale99] cannot be applied. The NDF Method solves the limitations of the classic methods and the use of the Virtual Ground is not required.

6.3.2.1 Brief Description of Active Patch Elements

The elements used on the design of this active antenna are two: a short-circuited ring patch and an amplifier.

The proposed patch is a circular ring patch with a central short-circuit. The use of a circular ring provides an additional design parameter, internal radius, and set the Mode 0 as the fundamental mode of the patch antenna. The simplified design approximations for this antenna can be found on Sec. 1.3. Then a quasi-coplanar line is inserted on this patch to obtain a 180 degree series resonator.

The first step is to ensure the fields compatibility of the line and the coupling structure with the fields of the desired mode of the patch. The fields on a 180 degree line, that can be considered as a micro-strip for the phase and as a coplanar for coupling to the patch, are studied. The Z-axis (line to ground plane) electric field, E_z , of the grounded coplanar line with the desired length for the 180 degree is shown in Fig. 6.23.

Figure 6.23: E field of 180 Degree Grounded Coplanar Line

Now, the electric field of the coupling structure, quasi-coplanar line in this case, must be compared with the field inside the patch. This example uses a short-circuited CRP that has the Mode 0 as the fundamental mode. The E_z fields for the two first modes, Mode 0 and Mode 1, of this patch are shown in Fig. 6.24.

It can be observed that the desired coupling (and feedback) line has an E_Z field that is compatible with the Mode 1 of the patch, but it is not with the Mode 0. The line must be a ring sector suitable to have the desired insertion phase and coupling to the patch.

Figure 6.24: E_Z field of a Short-circuited CRP Antenna

The desired design frequencies are 3.8 GHz for the Mode 1 and 2.2 GHz for the Mode 0, Fig 6.25. Once, a first design of the patch is available the desired feedback and feeding network is included. The coplanar line does not have significant losses, but when it is included inside the patch with the suitable coupling and insertion phase, it has good impedance matching on a wide frequency, however it presents important losses at the frequency of the Mode 1 of the patch, Fig. 6.26.

Figure 6.25: S_{11} of Single Feed CRP

The internal and external radius of the patch must be calculated to have the desired mode and frequency. In this example, the required mode is “Mode 1” and the design frequency is 3.8 GHz . The patch is designed for a substrate with height of 0.8 mm and $\epsilon_r = 2.5$. The design dimensions are a radius of 1.5 mm for the internal short-circuit and a radius of 14.7 mm for the external radius, Fig. 6.27. The coupling line and the gaps of the quasi-coplanar line have been designed to get a phase of 180 degrees and an insertion loss of 5 dB , Fig. 6.26. The losses of the line are the energy coupled to the patch. As the coupling is performed at

the frequency of the “Mode 1”, 3.8 *GHz*, and with a field distribution compatible with this mode, the patch radiates this energy but the resistive losses.

Figure 6.26: S_{11} of CRP with a 180 Degree, Two Ports, Feed Network

Figure 6.27: 3D View of Electromagnetic Model for Simulation of CRP with a 180 Degree, Two Ports, Feed Network

For the amplification function, two models of amplifiers are used: one is based on an IC amplifier (MAAM26100-B1) and other on a discrete transistor. The amplifier based on a discrete transistor is designed using a BFR460 BJT as active device, Fig. 6.28. The amplifier must have enough gain, Fig. 6.29, to compensate the losses of the resonator, Fig. 6.26. The losses of the ring antenna can be adjusted with the gap of the grounded coplanar line. This line is not a pure coplanar line, it is an hybrid of micro-strip and coplanar. The amplifier must be designed to have an insertion phase of 180 degrees at the desired oscillation frequency, Fig. 6.29.

Figure 6.28: Amplifier Schematic

Figure 6.29: Amplifier Gain

For the next analysis, it is important to point that the results of the linear analysis are not adjusted to get exactly the design frequency (3.8 GHz), they are the results of the linear simulation once the oscillator has been adjusted to match the desired frequency on the HB non-linear analysis.

6.3.2.2 Loop-Gain Analysis

The linear analysis of this feedback oscillator can be performed using the Loop-Gain Method [RH01] or any other linear method. In this case the most common is the Loop-Gain Method, but it is important to point that the required provisos must be checked before using this classic method, Sec. 4.3. These provisos are:

- The NDF of the open loop quadrupole, loaded with Z_0 on both ports, must not clockwise encircle the origin.
- The Nyquist analysis of the TF, denominator of Equ. 6.2, must not clockwise encircle the origin.

$$G_L = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22} + S_{12}S_{21} - 2S_{12}} \quad (6.2)$$

When these provisos are satisfied, the Nyquist analysis of $1 - G_L$ can be performed. In this example the provisos are satisfied, Fig. 6.30. So the classic Bode or Nyquist Analysis of $1 - G_L$ can be performed, Fig. 6.31. The oscillation on Bode plot is determined for the point at which the real part is smaller than -1 and the phase crosses -180 degrees with a negative slope. This condition must only exist at one frequency.

Figure 6.30: G_L Provisos Figure 6.31: Classic G_L Nyquist Analysis

The classic G_L Method predicts a proper oscillation at 3508.3 MHz. In this example the provisos are satisfied and the Alechno Virtual Ground [Ale99] has not been required, but in multiple cases the Virtual Ground cannot be applied and the provisos of the classic method are not satisfied, so the best linear analysis to be used is the NDF Method.

6.3.2.3 NDF Analysis

The NDF Method may be the best option for the analysis of this oscillator topology or any other topology, because this method can be used without any proviso and it does not require neither any re-drawn, nor open the loop.

The NDF Method determines that the active CRP is a proper conditioned oscillator, Fig. 6.32. It is important to remember that, the estimated oscillation frequency is the oscillation frequency at the first harmonic approximation [Kur69, KBK69] and compression is not required to determine if the NDF Method is used, but compression is required for classic methods. The estimation of the Q_L of this oscillator is also estimated using the NDF Method, Fig. 6.33, but in this case it is also possible to estimate it using the Loop-Gain Method. When a circuit does not satisfy the Loop-Gain provisos and the virtual ground cannot be used, it cannot be analyzed using the Loop-Gain Method. Then, it would not be possible to get an estimation of the Q_L , but using the NDF Method. The Q_L is a fundamental parameter for the design of any oscillator because it is the key for a linear optimization of the phase noise of the oscillator.

Figure 6.32: NDF of Active Short-Circuited CRP

Figure 6.33: Q_L of Active Short-Circuited CRP

6.3.2.4 Measurements and Non-Linear Results

The prototype of this active patch antenna includes the amplifier over the central short-circuit of the patch. Models of this topology with an amplifier based on a transistor, 6.34(a), and with an amplifier IC (MAAM26100-B1), Fig. 6.34(b), have been manufactured and measured.

a) Model Based on a Transistor Z_0
Amplifier IC

b) Photo of Model with

Figure 6.34: Models of Active CRP

The analysis of the complete active CRP is performed using the HB method. The spectrum of this oscillator is shown in Fig. 6.35. The oscillation frequency of the active patch has been adjusted to match the frequency of the “Mode 1” of the patch. The matching with the linear prediction is not good, but it is a normal fact on a low Q oscillator.

The measured phase noise of the manufactured prototype has a good match with the estimated by the non-linear simulation, Fig. 6.36, the manufactured model is about 1.5 dB worse than the predicted by the non-linear simulation.

Figure 6.35: Spectrum of Active Short-Circuited CRP

Figure 6.36: Phase Noise of Active Short-Circuited CRP

Using the HB analysis to “force” the circuit to oscillate at different frequencies, the losses on the resonator are measured and they confirm that if the oscillator is not adjusted to the frequency of the “Mode 1” there is not any radiated power, Fig 6.37.

Using the results provided by the non-linear simulation, it is possible to obtain the radiation diagram of the CRP, Fig. 6.38. This radiation diagram is in concordance with the expected on a “Mode 1” patch antenna.

Figure 6.37: Radiated Power vs. Frequency of Active CRP

a) 3D Radiation Diagram

b) Radiation Diagram

Figure 6.38: Active CRP Radiation Diagrams

6.4 Conclusions on Active Patches Study

The design of active patches presents, in a lot of cases, the problem that the required “topology” is difficult or impossible to find because the virtual ground cannot be applied in some cases due to the patch physical implications that fix the ground reference point. Other important aspect is that the active patches do not have any conventional 50Ω interface, so the division and partial analysis of the circuit must consider this fact and take some precautions.

The inclusion of the active elements in the patches requires a study of the impedance and fields of the patch to ensure the compatibility of the required feed with the desired patch mode. And also, it must be taken into consideration the size of the active components and consider their pads as a distributed feeding point. The lack of an easy geometry in the patches when the active devices are included and the field distortion effect of the feeding and bias network of active devices make necessary, for most cases, a 3D full wave simulation of the patch.

The inclusion of active elements in patches has multiple advantages as the ones listed on the introduction of this chapter, but care must be taken of undesired effects. The greater the number of active devices are used on a patch the greater the risk of undesired behaviour is. The possible odd-mode oscillations and undesired response of the circuit can be studied during the linear design steps. The problems to use the virtual ground on circuits with distributed elements, such as a patch, and the requirement of the verification of provisos make difficult and insecure to use classic methods. But the NDF Method does not have these problems, then the NDF Method is an ideal tool for the analysis of this type of active patches.

NICs in an active patch are suitable to be used to compensate the inductive or capacitive frequency response of the patch, to improve the bandwidth, to tune small antennas and to increase the Q_L . It is important to consider the risk of undesired poles in the RHP that can make the circuit operate in an unpredicted way when additional active devices of the NIC are included. The use of the NDF as an analysis tool is a good option to assure the non existence of undesired hidden or visible poles.

A NIC is used to increase the Q_L of an active patch oscillator in the first example. The increase of the Q_L of the oscillating patch has been predicted by the NDF and proved with the pushing response of the oscillator. But, this increment of the Q is not transformed into a better phase noise, because the contribution to the phase noise of the new active devices is not considered in a linear analysis. The results of the HB simulations show the degradation in phase noise that has been observed in the measurements.

The analysis of the E field inside the antenna is the first important step to determine and understand if a coupling topology is compatible with the desired radiation diagram and frequency. An example with a two port grounded coplanar line, that is inserted in the patch structure and used as resonator for a closed loop oscillator topology, is used to illustrate the importance of this study. The fields in a circular ring patch and in a grounded coplanar line are compatible with the “Mode 1” of the patch, but they are not compatible for the “Mode 0”. The example can be analyzed using any of the classic linear methods, but it is important to remember that the suitable provisos must be checked before. The *natural* method for this second example is the Loop-Gain Method. But the preferred method is the proposed NDF Method, which does not require any proviso, open the loop or change the ground, as the classic linear methods do.

The NDF Method always provides the estimation of the Q_L , but only the classic Loop-Gain method does; the rest of classic methods do not provide this information, so the NDF is a generic tool suitable to study the phase noise and stability on linear design steps of any active patch topology.

It has been proved that the NDF has a good ability to predict unwanted operation of circuits and active patches. The use of the NDF as an analysis tool has demonstrated a good match and correct prediction of active patches. The correlation of the simulations and the measurements has been illustrated with the results of two examples.

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Chapter 7

CONCLUSIONS AND FUTURE RESEARCH LINES

7.1 Final Conclusions

It is difficult to make a summary of the achievements that have been reached during the progress of this thesis. As any complex research work, multiple achievements have been obtained, which are detailed in the chapter that covers each topic of this thesis. The main contribution of this thesis is to provide a step in the formal understanding and methods for the design of active antennas, mainly focused on patches that include an oscillation function. All the developments presented on this work are oriented to the objective of a better understanding of active antennas and their stability study, with special emphasis on antennas that include oscillator functions. The result of this work provides conclusions on multiple topics and topologies of active antennas.

The works of this thesis can be classified into two main topics: passive elements and active functions with their stability issues.

The first passive elements presented on this are the patches. The patches are the base elements for most of the discussions carried out in this work, because they are used as antenna elements during all the study. The patches are one of the most interesting radiating elements because they have a wide range of parameters for adjustment, topologies and they are easily integrated with active elements. Therefore they are the current main working topic on active antennas. The patches allow automatic manufacturing process of antennas and arrays with a high precision and low cost, and they are suitable for including active functions on a single manufacturing process. The wide list of topologies of patches allows the design of antennas with several simultaneous radiation patterns and polarizations, thus they are used to combine several bands and services on a single antenna.

It is possible to use simple analytical models for the preliminary design of patches, but when they are used on active antennas and some active devices are included *inside* the patch, a more complex study of the fields inside the patch and the wide-band frequency response are required. The inclusion of bias and feeding points for active elements inside the patches makes to consider these feeding points as distributed due to the quick change of impedance with the position of the feeding point on patches. All these aspects and the importance, for multiple designs, of the exact distribution of the fields inside the patch make the use of a full 3D electromagnetic simulation of the patch a must. Several examples of patches and integration of active devices have been shown in this work.

The other passive elements discussed in this work are the metamaterial transmission lines. These passive elements are useful for the adjustment of the frequency response of elements for several frequencies that do not have a periodic ratio. The use of the metamaterial lines has been illustrated with the design of diplexers and hybrids for arbitrary frequencies. The example of hybrid is designed to operate at 424 MHz, 910 MHz and 2100 MHz; frequencies without

any harmonic relation as it is natural on a 90 degree hybrid. This property of the metamaterial lines is also applicable to adjust the phase relation for several modes of a patch with a single feed, diplexing, etc. These artificial lines are interesting for the design of active antennas and they are studied as any other passive element. They can provide a frequency adjustment for narrow-band applications, but when a wide-band adjustment is required, active functions such as NICs must be used.

The main topic of this work are active antennas, but one of the first aspects to study in any RF active circuit is the stability. The classic methods to study the stability of oscillators are reviewed. The classic methods require provisos to be checked before using them. These provisos are presented, classified and summarized. But classic methods are linked to the topology of the circuits. The virtual ground can be used in some cases to redraw a circuit for being analyzed with the desired method, but other times it is impossible due to distributed elements, as for example patches. The classic methods have another problem in addition to the requirement of provisos and circuit redraw, they provide different solutions for the same circuit for different open point or virtual ground. These problems and limitations of the classic methods are solved with the method proposed in this work: NDF Method.

My original contribution to knowledge is the proposal of a method for linear design of any oscillator topology and for linear stability study of any circuit, this method is based on the use of the Normalized Determinant Function (NDF) as a direct tool to study these circuits. The use of the NDF has several advantages over the classic methods. One of these advantages is that it is suitable to be used for any topology, so it is suitable for the research of new topologies of active antennas and oscillators. The NDF does not require any proviso and it has an asymptotic to +1, thus it is a direct tool for stability study for which the upper frequency to study is easy to determine. It is possible to estimate the Q_L of an oscillator using the NDF, then it is a universal linear design tool to optimize the phase noise of any oscillator topology and an interesting tool for the research of new low noise oscillator topologies.

The proposed NDF Method has been used with success in several examples of oscillators and active antennas. The results of the linear classic methods and NDF Method have been compared with the non-linear simulations and measurements; and for all cases the experimental results are in accordance with the NDF Method.

During the study of active antennas, the oscillator active function and wide-band active compensation circuits, Negative Impedance Converters (NIC), are used. These circuits that include NICs are hard to study using classic methods, but in some cases they can be studied using these methods. Several examples demonstrate that NICs are highly unstable circuits and in some cases the classic linear methods do not detect this undesired behaviour. Ideal or nearly ideal NICs,

that use transistors without the parasitic effect of package are stable, but when real parasitic effects are considered they became unstable. NICs could be used as negative resistors to increase the Q_L of oscillators, but the noise contribution of new active devices and the risk of undesired poles must be considered. So, NICs may be useful on high integrated circuits as the MMIC, but circuits that use discrete transistors are not advisable for the implementation of NIC functions on RF circuits.

The study of several topologies of oscillators has demonstrated that the proposed NDF Method is a better design tool than the classic linear methods. These examples show that the NDF Method is suitable for the phase noise optimization of any topology during the linear design steps. The circuits present the behaviour predicted by the NDF in opposition to the predicted by the classic design methods when the provisos are omitted.

Several examples of active patches, that combine the experience and concepts presented in this work and illustrated on previous circuits and sections, are used to demonstrate several models of active patch antennas. When some examples of active antennas that use oscillation and NIC functions are studied, the main conclusion is that the proposed NDF Method is valid for the linear study of these circuits and the phase noise optimization. The experimental results and measurements are correctly predicted by the NDF and the practical usability of NICs is questioned due to their instability for RF applications.

As a personal conclusion, I can state that a work of this extension and complexity has provided me a wider understanding of the relation of different topics and a higher level of maturity which other projects, that took place in my previous experience, did not help me to achieve.

7.2 Future Research Lines

The future research lines proposed as continuation of this work are focused on three aspects: The active antennas and their study on arrays that use them as self-synchronized circuits; the study of the stability of NICs implemented on MMIC, or other high integrated technology; and the stability study using the proposed method based on the NDF for the research on topologies of oscillators for low noise applications.

The study of arrays that use active antennas with a high independence between them, but with self-synchronization, is an interesting aspect for several reasons. These arrays have fault tolerance because the fault of a single amplifier does not significantly affect the performances of the array antenna. They can manage a high power because they use spacial RF combination and the amplifier function is distributed. Then, the thermal dissipation can be managed on an easier way

than for single concentrated power amplifiers. These arrays provide the best performances in terms of noise figure, because the low noise amplifier is in the antenna and there are not any losses in transmission lines. This group of interesting properties of the arrays built as combination of active antennas make their study an interesting future research line.

It has been demonstrated that the NDF Method is a good tool for linear analysis of the stability of NICs. The stability of NICs that are built with discrete active devices is unsatisfactory for most of the designs, but if the parasitic effects of packaging is omitted, the circuits seem to have more possibilities of being useful for practical applications. Then, the implementation of NICs using MMICs for a high integration is the natural line to follow in the study of these circuits, and then, evaluate their possibility for being used in high frequency RF applications.

Considering the advantages of using the proposed NDF Method for the study of the phase noise during linear design steps; the use of this method for the research of new topologies of low phase noise oscillators is a research line to explore. Some of the great advantages of the NDF Method are that it is not linked to any topology and it makes possible the estimation of the Q_L for any oscillator topology. The union of these properties may be useful in the research of new oscillator topologies.

Chapter 8

CONTRIBUTIONS

8.1 Summary

The contributions and dissemination performed during this work can be classified in the following groups:

- Publication on journals of reviews and comment on classic design methodology, including practical examples of circuits.
- Dissemination of the work on National and International Congresses.
- Intensive use and exemplification of the use of the NDF Method as linear stability analysis and oscillator design tool.
- Dissemination of examples and practical circuits to illustrate the risk and errors of classic stability methods.
- Custom plug-in software for NI-AWR© to perform NDF analysis before it was included by manufacturer.
- Instrumentation and low cost measurement application for transmitters, focused on Radar transmitters.

The dissemination can be summarized as:

- 10 Publications on International JCR Journals.
- 3 International Congresses.
- 6 National Congresses.
- 5 Works on Development and Research Projects.
- 1 Success Story Contribution on NI-AWR Web-page.
- 4 Other Contributions.

The published works are useful as reference for future works on linear stability and design of active antennas and oscillators. They have special emphasis on the problems and considerations to consider for a correct analysis of any active antenna or oscillator.

These disseminations are an indicator of the quality and extension of the work of this thesis, but they are not the unique measurement figure of the quality.

8.2 Publications on Journals

- V. González-Posadas, J. L. Jiménez-Martín, Á. Parra-Cerrada, L. E. García-Muñoz, and D. Segovia-Vargas. Dual-composite right-left-handed transmission lines for the design of compact diplexers. *IET Microwaves, Antennas & Propagation*. volume 4, pages 982–990, August 2010. [doi:10.1049/iet-map.2009.0571](https://doi.org/10.1049/iet-map.2009.0571)
- V. Gonzalez-Posadas, J. L. Jimenez-Martin, Á. Parra-Cerrada, D. Segovia-Vargas, and L. E. Garcia-Muñoz. Oscillator accurate linear analysis and design. classic linear methods review and comments. *Progress In Electromagnetics Research.*, volume 118, pages 89–116, 2011. [doi:10.2528/PIER11041403](https://doi.org/10.2528/PIER11041403)
- J. L. Jimenez-Martin, V. Gonzalez-Posadas, Á. Parra-Cerrada, Á. Blanco del Campo, and D. Segovia-Vargas. Transpose return relation method for designing low noise oscillators. *Progress In Electromagnetics Research*. volume 127, pages 297–318, 2012. [doi:10.2528/PIER12022305](https://doi.org/10.2528/PIER12022305)
- J. L. Jimenez-Martin, V. Gonzalez-Posadas, Á. Parra-Cerrada, L. E. Garcia-Muñoz, and D. Segovia-Vargas. Comments and remarks over classic lineal Loop-Gain method for oscillator design and analysis. new proposed method based on NDF/RRT. *Radioengineering.*, volume 21(1), pages 478–491, April 2012. [doi:10.13164/re](https://doi.org/10.13164/re)
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- Á. Parra-Cerrada, J. L. Jiménez Martín, V. González Posadas, and D. Segovia-Vargas. Advantages of NDF as oscillator linear design tool: Design of a NIC active patch antenna. *Journal of Electromagnetic Waves and Applications.*, volume 30(1), pages 98–108. 2016. [doi:10.1080/09205071.2015.1093433](https://doi.org/10.1080/09205071.2015.1093433)
- J. L. Jiménez-Martín, Á. Parra-Cerrada, R. Fernandez-Regio, D. Segovia-Vargas, L. Garcia, and V. González-Posadas, Dual band and dual

polarization short circuited ring patch antenna, *Journal of Electromagnetic Waves and Applications.*, volume 30(1), pages 1198-1206, June 2016. [doi:10.1080/09205071.2016.1187093](https://doi.org/10.1080/09205071.2016.1187093)

- Á. Parra-Cerrada, J. L. Jiménez-Martín, D. Segovia-Vargas, and V. González-Posadas, Analysis of new active antenna with a patch as feed-back network, using the NDF Method, *Journal of Electromagnetic Waves and Applications.*, volume 30(13), pages 1763-1772, July 2016. [doi:10.1080/09205071.2016.1215265](https://doi.org/10.1080/09205071.2016.1215265)
- D. Segovia-Vargas, J. L. Jiménez-Martín, Á. Parra-Cerrada, F. Albarracin-Vargas, E. Ugarte-Muñoz and V. González-Posadas, Stability Analysis and Design of Negative Impedance Converters: application to circuit and small antennas, *Radioengineering.* 25(3):409–418, Sept. 2016. [doi:10.13164/re.2016.0409](https://doi.org/10.13164/re.2016.0409)

8.3 International Congresses

- V. González-Posadas, J. L. Jiménez-Martín, Á. Parra-Cerrada, L. E. García-Muñoz, and D. Segovia-Vargas. Full conditions for the stability analysis of negative impedance converters. pages 135–138. 6th European Conference on Antennas and Propagation (EuCAP). Prague, Czech Republic, March 2012. [doi:10.1109/EuCAP.2012.6206680](https://doi.org/10.1109/EuCAP.2012.6206680).
- Á. Parra Cerrada, J. L. Jiménez Martín, and V. González Posadas. Ndf Method for linear oscillator design, new method for linear analysis of any oscillator topology. IEEE International Symposium on Consumer Electronics (ISCE). pages 1–2, Madrid, Spain, June 2015. [doi:10.1109/ISCE.2015.7177842](https://doi.org/10.1109/ISCE.2015.7177842).
- Á. Parra-Cerrada, V. González-Posadas, J. L. Jiménez-Martín, and D. Segovia-Vargas. Design of a nic active oscillating patch antenna using NDF as linear design tool. In The 10th European Conference on Antennas and Propagation (EuCAP), Davos, Switzerland, Apr 2016. Poster.

8.4 National Congresses

- V. González-Posadas, J. M. Rodríguez-Martín, C. Martín-Pascual, and Á. Parra-Cerrada. Sistema de localización de excursionistas mediante el uso de antenas activas en transmisión. XVII Symposium Nacional URSL., 2002.
- V. González-Posadas, J. L. Jiménez-Martín, Á. Parra-Cerrada, L. E. García-Múñoz, and D. Segovia-Vargas. Diplexor TETRA-GSM basado en líneas d-crlh. XXIII Symposium Nacional URSL., 2008.
- Á. Parra-Cerrada, V. González-Posadas, and J. L. Jiménez-Martín. Línea tri-crlh para aplicaciones en tres bandas e híbrido tri-crlh (TETRA-GSM900-GSM1800). XXIII Symposium Nacional URSL., 2008.
- Á. Parra-Cerrada, J. L. Jiménez-Martín, V. González-Posadas, Á. Blanco-del-Campo, and D. Segovia-Vargas. Verificación de provisos de los métodos clásicos de plano de referencia para diseño lineal de osciladores por medio de la NDF. Uso de la NDF como herramienta de diseño. XXVII Symposium Nacional URSL., September 2012.
- Á. Parra-Cerrada, J. L. Jiménez-Martín, V. González-Posadas, Á. Blanco-del-Campo, and D. Segovia-Vargas. Verificación de provisos del método de ganancia del lazo para diseño lineal de osciladores por medio de la NDF y el uso de esta como herramienta directa de diseño. XXVII Symposium Nacional URSL., September 2012.
- Á. Parra Cerrada, J. L. Jiménez Martín, V. González Posadas, Á. Blanco del Campo, and D. Segovia Vargas. Diseño y optimización de ruido de fase en osciladores utilizando la NDF. XXIX Symposium Nacional URSL., September 2014.

8.5 Works on Development and Research

- 2000-2002 Design of hiker localization system by the use of active antennas ("Sistema de Localización de Excursionistas Mediante el Uso de Antenas Activas en Transmisión") (5.75 GHz). Government Research Department of Madrid (Dirección General de Investigación de CAM), exp. 09/0061/1999. Distcom Antenas, S.L.U. Collado Villalba - Spain.
- 2013 Differential Amplifier for the Wideband Amplifier of Project RAEGE (Amplificador Diferencial para el Receptor de Banda Ancha del Proyecto RAEGE). Ministerio de Fomento-OAN. Universidad Politécnica de Madrid (UPM). Madrid - Spain. [webpage](#)

- 2012-2014 Implementation of Remote and Guided Electromagnetic Simulation Works. School of Telecommunications Engineering (ETSIST-UPM). Universidad Politécnica de Madrid (UPM). Madrid - Spain. [webpage](#).
- 2014-2015 Preparation of documentation for autonomous learning in the area of Electromagnetics and Telecommunications. School of Telecommunications Engineering (ETSIST-UPM). Universidad Politécnica de Madrid (UPM). Madrid - Spain. [webpage](#).
- 2014-2017 Photonics and Radio-frequency Developments and Applications for Space Geodesy (Difrageos). Comunidad Autónoma de Madrid (I+D Technology). Universidad Politécnica de Madrid (UPM). Madrid - Spain. [webpage](#)

8.6 Success Story

- 2015 Vicente González Posadas and Ángel Parra Cerrada, Success story: University of Madrid empowers students to master RF circuit design concepts using NI AWR software, AWR Corporation - National Instrument, [webpage](#).

8.7 Other Contributions

8.7.1 Publications on Journals

- Á. Parra-Cerrada, V. González-Posadas, J. L. Jiménez-Martín, A. Blanco-del-Campo, W. Hernandez, and C. Calderón-Cordova. Low-cost measurement for a secondary mode S radar transmitter. Instrumentation and Measurement, IEEE Transactions on., volume 64(12), pages 3217–3225, Dec 2015. [doi:10.1109/TIM.2015.2450357](https://doi.org/10.1109/TIM.2015.2450357)

8.7.2 International Congresses

- Á. Parra-Cerrada, V. González-Posadas, J. L. Jiménez-Martín, Measurement of MSSR Transmitters Using IF Direct Conversion - Built with General Purpose Instrumentation. European Microwave Week (EUMI), London, UK, Oct 2016. Accepted.

8.7.3 National Congresses

- J. M. Rodríguez, A. Benito, V. González, A. Parra, J. E. González y J. L. Jiménez. Medidas de propiedades dieléctricas de materiales de construcción utilizando técnicas de reflexión I. XXVI Symposium Nacional de la URSI, September 2011.
- J. M. Rodríguez, A. Benito, V. González, A. Parra, J. E. González y J. L. Jiménez. Medidas de propiedades dieléctricas de materiales de construcción utilizando técnicas de reflexión II. XXVI Symposium Nacional de la URSI, September 2011.

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Appendix A

NDF Script History

The author developed a NDF tool to use in the NI-AWR© circuit simulator before the inclusion of the native NDF function on the software. The first version of the author was in September 2011 for AWR V9. The AWR software introduced the NDF function on the V10, in February 2012, but it presented some important discrepancies with the results of the version developed by the author. Then the custom version of NDF was updated and maintained for AWR V10 until May 2014 the new release of AWR V11 included a new version of NDF which does not present discrepancies. Up to nowadays, the last version of the custom NDF is valid for the latest version of AWR V12.03.

A.1 Custom NDF Version Changes

PROGRAM: NDF analysis using transistors linear models and Transpose Return Relations (RRt)

Version: 0.81

Date: 11 Feb 2012

Authors: A. Parra-Cerrada (angel@parranet.com), V. Gonzalez-Posadas,
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EUITT - UPM - Spain

Description:

This script is suitable to perform an analysis of the Normalized Determinant Function (NDF) of a network using the RR_T .

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Requirements:

- Equations script Version 0.8
- NI-AWR 10.x
- Root Schematic needs at least one port for proper frequency analysis. If it is portless, an isolated port must be added.
- Schematic names must NOT include spaces

Reserved names:

- For schematics: 000_NDF_*
- For datafiles: RR_[circuit_name]_t*
- For output equation document: NDF_[circuit_name]
- For V_Meter: NDF_VM1
- For VCCS: NDF_U1
- For ACCS: NDF_I1
- For graphics:
 - NDF_BJT_parameters
 - NDF_[circuit_name]
 - NDF_[circuit_name]_fosc
 - NDF_[circuit_name]_Q_RRt

Reserved internal equations:

- fosc_NDF
- mkdata2
- fmkdata3

Versions:

- V 0.1:
 - First version AWR 9.x
 - Only GBJT3 model supported
 - Support for one BJT
 - No hierarchical support
- V 0.2:
 - Simple hierarchical support
 - Up to 3 BJT support
- V 0.3:
 - Complex hierarchical support (up to 50 dept)
- V 0.4:
 - Up to 100 BJTs support (up to 50 dept)
- V 0.5:
 - New internal matrix structure for BJTs analysis
- V 0.6:
 - New Internal names for schematics
- V 0.7:
 - VCCS, ACCS, V_METER new names to avoid conflicts
- V 0.8:
 - Output Equations changes for compatibility with AWR 10.x (not compatible with AWR 9.x)
 - Support for analysis of multiple schematics on a project
 - Now it supports GBJT3 and GBJT models

- V 0.81:
 - Minor change to solve problems to activate sch (it has been disabled)

TO DO:

- It does not support spaces in sch names
- Add support for other transistors Models.
- Add support for parametric simulation
- Add support for ACE extractions

USE:

- The script is called from scripts->simulation->NDF_new and the root schematic must be chosen, in this example the root schematic is "test_oscillator_for_NDF"