Abstract: In the design process of array antennas, coupling is one of the most important elements to be counted. The real feeding radiated coefficients can be quite different from the theoretical ones because of this effect. In this paper, a compensation method is presented allowing matching each element from the array. The parameters of the array coupling model are obtained through the measurement of radiation pattern of the elements without the feeding network. An application to linear patch array is presented as an example.

INTRODUCTION

Actual input impedance and radiated field from each element in the array can differ very much from the single element model. Input impedance can be computed through an N-port network model described by impedance \( Z \) or scattering \( S \) matrix. Radiated field can be computed if we know the active radiated field from each element that takes into account the complete array influence in each element radiated field, as presented by Millioux [1]. In receiving antennas the model would be the same, giving an equivalent active diagram and active output impedance that depend on the loading circuit. In large arrays, most elements present similar conditions: their class, position and load. That situation provides almost equal active impedance and diagram for many elements in the array except the extreme ones. In [2], Wasylikowski shows the relation between input impedance and radiated field for Minimum Scattering Antennas (MSA). Many times, real antennas have been assumed as MSA with good results [3]. In other cases, that assumption is not as fortunate and relation between input coupling matrix \( Z \) and active radiated field is not so clear. One of the most important questions in the design of array antenna is if the coupling can be modelled by an \( N \times N \) coupling matrix \( C \) as described in most studies of adaptive arrays [4].

In this paper, a matrix model is presented for array antennas fed through linear networks. The model is based in the several active modes of the array elements and active diagram computation as a function of these modes. If \( k \) independent modes can describe the electromagnetic behaviour of each element, \( N \times k \) modes will be needed to define all the array radiation characteristics. An \( N \times (k+1) \) square matrix will describe the array behaviour. This description is independent of the feeding network, feeding distribution or transmission-reception application [5]. For many printed patches only one resonant radiation mode can be used to describe the radiated field of each element. When this happens a \( 2N \times N \) matrix describes the complete array model. Based on this model, applying the feeding network parameters, a coupling \( C \) matrix can be obtained to compensate the design model from the influence of element coupling. The parameters of this model can be obtained through the measurements of the radiation pattern for each individual element in the final array disposition. As an example, a printed antenna of rectangular patches as that described by Pozar in [6], allows us to demonstrate how accurate the matrix model introduced in this paper is.

RADIATED/RECEIVE FIELD MODEL.

Transmission model for an individual antenna

The antenna electrical behaviour can be defined by its input impedance and its radiated field (1):
\[ E_{\text{sw}} = v_e \hat{E}(0,0) F(0,0) \text{exp}(-jkr) \]

(1)

where \( v_e \) is a voltage proportional to the input current, \( F(0,0) \) is the radiation pattern, \( \hat{E}(0,0) \) is the polarisation vector. The antenna can also be seen as a function of its scattering matrix. Then the radiated field and the radiated power can be expressed as (2) and (3):

\[ E_{\text{sw}}(0,0) = b_s \sqrt{S_r} F(0,0) \text{exp}(-jkr) \hat{E}(0,0) \]

(2)

\[ P_{\text{rad}} = \eta_s |b_s|^2 \left( 1 - |S_r|^2 \right) = |b_s|^2 \left[ |F(0,0)|^2 \right] \text{d}2 = |b_s|^2 \]

(3)

\( S_r \) represents the reflection coefficient at the input defined respect \( Z_0 \). \( b_s \) is a power wave proportional to the amplitude and phase of the input wave \( v_e \). The input reference impedance is \( Z_0 \) while the output reference impedance is \( Z_\text{ref} \). The antenna gain can be expressed using the normalised radiation pattern and the antenna efficiency as (4):

\[ G(0,0) = n \times |F(0,0)|^2 \frac{|b_s|^2}{1 - |S_r|^2} \]

(4)

Reception model for an individual antenna

When the antenna works in receiving way the equivalent surface \( A_e(\theta,\phi) \) represents the amount of power taken by the antenna. If \( S_r \) represents reflection coefficient of the circuit, then a power wave \( b_0 \) can be extracted at the input port:

\[ A_e(\theta,\phi) = \frac{n \times |F(0,0)|^2}{2\pi} \]

(5)

\[ P_{\text{rad}} = |b_0|^2 \left[ 1 - |S_r|^2 \right] = |b_0|^2 \left[ 1 - |S_r|^2 \right] \text{d}2 = |b_0|^2 \]

(6)

The reception power wave proportional to the impinging field is (7):

\[ z_e = \frac{b_0 \cdot \hat{E}(0,0)}{\sqrt{2} \eta_s} \]

(7)

Finally, if the reciprocity principle is applied then \( S_r = S_e \).

Radiated/received field model for an array antenna

A first approach of a model to take into account the previous effects has been proposed in [5,7]. This new network takes into account the decomposition of the current distribution in multiple characteristic modes. Then, any \( N \)-array antenna can be represented as a \((k-1) \times N\)-network. The new network has one input and \( k \) output ports corresponding to any of the radiating modes of each antenna (by input ports we mean any of the actual probes of the array while by output ports we mean fictitious ports representing any radiating function). Figure 1 shows the new \( J \times N \)-port network (\( J \) corresponding to the antenna input and \( N \) to the radiation modes). When the radiating elements are resonant microstrip antennas, only one radiating mode may be considered resulting in a \( 2 \times N \)-port network.
network. This network is represented in Figure 2. The terminals at the left side of the 2N-
port network represent physical probes of the antenna that can be directly measured while
the ones at the right side allow us to define the radiation functions. They will never be
charged since they represent ideal radiating (b) or receiving (a) antennas. Then the matrix
equation relating previous variables is given in (8)
\[
\begin{bmatrix}
 b \\
 b_r \end{bmatrix}_{2N+1} = \begin{bmatrix}
 S_{a} \\
 S_{r} \end{bmatrix}_{2N+1} \begin{bmatrix}
 a \\
 a_r \end{bmatrix}_{2N+1}
\]
where \( S_a \) represents the reflection coefficient, vector \( S_t \) represents the transmission
parameter for each mode, \( S_r \) represents reception coupling for any of the defined modes and
matrix \( S_{r} \) indicates the scattered field by each mode.

When operating in a transmitting way, the array is fed through a set of generators
with an equivalent incident wave \( a_s = \sum_{j=1}^{N} a \cdot g_{j} \cdot \exp(-j k r) \) and source reflection coefficient
\( r_{g_{j}} \). In this case equation (8) can be simplified, because \( a_r = 0 \). The total radiated field can be expressed
as the sum of the contributions from any of the radiating elements. This can be written in
vector form as the dot product of the steering vector and the transmitting power wave (9). A
coupling matrix \( C_{c} \) can be defined (10);
\[
C_{c} = S_{r} \cdot (1 - \Gamma_{g} \cdot S_{a})^{-1}
\]
de the steering vector in \( (\theta, \phi) \) direction. Each term of matrix \( C_{c} \) represents the
amount of signal coupled from antenna \( j \) to \( i \). The radiation intensity can be expressed as
the following dot product
\[
I(\theta, \phi) = \frac{1}{2} |E_{max}|^2 = a_i^{*} C_{c}^{*} \cdot d^{*} C_{c} a_j = a_i^{*} M_{a} \cdot a_j
\]

MODEL COEFFICIENTS CALCULATION

Once the model is defined, \( S_{a} \) can be obtained through measurements with a vector
network analyzer. \( \Gamma_{g} \) represents the generator reflection coefficient and \( S_{t} \) matrix can be
calculated through measurements of the radiation pattern for each individual antenna of the
array, as follows:

If a linear array of \( N \) independent patches (without feeding network) is measured
exciting one patch at a time (the elements of vector \( a \) are equal to zero for all of them but
one), the columns of the matrix $S_e$ (considering $a_n=0$), $b_i$ is obtained after processing the radiation pattern. The radiated field in a point of the space situated in far field respect every patch will be:

$$E(\mathbf{r}_m) = \sum_{n} b_n \mathbf{e}_n(\mathbf{r}_m) \mathcal{F}_e(\mathbf{r}_{n,p}) \frac{\exp(-j k_0 |\mathbf{r}_m - \mathbf{r}_n|)}{|\mathbf{r}_m - \mathbf{r}_n|}$$  \hspace{1cm} (12)

where the positions are shown in figure 3.

The reception power is obtained if measurement probe pattern ($F_{sm}$) is known (13). This equation represents the relation between the $b_i$ and the measured pattern, as wanted.

$$a_m = \sum_n b_n \left( \mathbf{e}_n(\mathbf{r}_{n,p}) \mathcal{F}_e(\mathbf{r}_{n,p}) \mathcal{F}_{sm}(\mathbf{r}_{m,p}) \frac{\exp(-j k_0 |\mathbf{r}_m - \mathbf{r}_n|)}{|\mathbf{r}_m - \mathbf{r}_n|} \right)$$  \hspace{1cm} (13)

Normally, more points than elements are measured, so an optimization process can be developed to obtain the coefficients $b_i$, and the coupling matrix $C_t$. In [8] this procedure is followed to design the structure of figure 4. Measurements of this structure will be presented at the conference.

REFERENCES


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