UNIVERSIDAD POLITÉCNICA DE MADRID
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PLASMA SIMULATION IN SPACE PROPULSION:
THE HELICON PLASMA THRUSTER

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PLASMA SIMULATION IN SPACE PROPULSION:
THE HELICON PLASMA THRUSTER

PH.D. THESIS DISSERTATION

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*"All men by nature
desire knowledge."*

Aristotle

Als meus pares i gran germana, Pere, Paquita i Olga.

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Abstract

The Helicon Plasma Thruster (HPT) is an electrodynamic rocket proposed in the early 2000s. It matches an Helicon Plasma Source (HPS), which ionizes the neutral gas and heats up the plasma, with a Magnetic Nozzle (MN), where the plasma is supersonically accelerated resulting in thrust. Although the core of this thruster inherits the knowledge on Helicon Plasma sources, dated from the seventies, the HPT technology is still not developed and remains below TRL 4. A deep review of the HPT State-of-art has been performed, concluding that the propulsive figures reached for some prototypes abruptly differ from each other. This Thesis analyses some of the main physical phenomena taking place in a HPT from a theoretical point of view, in order to comprehend the feasibility of this technology as a competitive electric thruster.

A two dimensional three fluid model of the internal dynamics of electrons, ions and neutrals has been developed. As a result, this model identifies main loss mechanisms, and theoretically predicts the HPS production and utilization efficiencies and their dependences on the operational and design parameters. Asymptotic limits of this model have been derived yielding to reduced laws for highly efficient HPS, concluding that a good magnetic confinement and high electron temperatures are mandatory to reach optimum performances. An overall balance of the energy and plasma momentum have been performed after coupling the HPS model results with the results of a third party fluid model of the the MN. This provides an entire illustration of the HPT energy loss and thrust mechanisms, as well as the power and thrust efficiency.

After the analysis of the HPT internal dynamics and the use of third party results for the external fluid, a hybrid model of the MN has been developed based on a previous one for Hall Effect Thrusters (HALLMA). This model relies on a *Particle-in-Cell* method for ions and neutrals, and an anisotropic and isothermal fluid approach for electrons. This model describes the response of the plasma along the MN for different electron magnetization regimes, a capability not covered by the aforementioned MN fluid model. The obtained results shows the electrons demagnetization impact on the plume divergence and on the axial thrust generation.

Nonetheless, both the MN hybrid model and the MN fluid model consider electrons as an isothermal fully confined specie. This results in an inconsistent unbounded ambipolar potential drop, which yields infinite plasma acceleration and thrust. To face this problem, a one dimensional kinetic model of a fully magnetized expanding plasma throughout a convergent-divergent magnetic channel has been formulated and solved numerically. This model provides a preliminary insight on important phenomena like the collisionless electron cooling and the formation of a self-consistent potential drop along the magnetic channel. The total potential drop at the far field is finite as well as the ultimate ion velocity. This model concludes that most of the aforementioned phenomena, strongly depend on the ion to electron mass ratio, and weakly depend on other conditions such as the ion energy within the source, at least for the hypotheses considered here.

An experimental test campaign has been carried out at the Electric Propulsion and Plasma Dynamics Lab (Princeton University). These tests have pursued the experimental validation of some of the results obtained from the fluid model of the HPS, such as the characterization of plasma fluxes to the rear wall, and the axial density structure within the HPS. Source efficiencies have been also evaluated and compared against the model results. Theoretical to experimental fittings are poor in most of the studied cases, although some of the results can be compared qualitatively, such as the increasing trend of HPS efficiencies for stronger magnetic fields.

Lastly in this Thesis a preliminary design of a HPT prototype has been performed, based on the knowledge acquired on the discussed models. The implementation of a one dimensional (radial) model of the plasma wave interaction has been necessary in order to estimate the plasma impedance. Reduced laws of the internal fluid dynamics have been used to design the HPS, and select the best operational point. Later, results have been checked with the complete 2D fluid model of the HPS. Because of the simplification of some of the models developed in the current Thesis, thrust efficiency has been estimated about 50 %, which is at this moment, quite large in comparison to the available experimental results. This also indicates that the development of this technology requires to keep a strong and excellence research on both, experiments and theoretical modelling, but in a closed loop of continuous validation, with the goal of building advanced tools for the HPT design and assessment of its performances.

Resumen

El motor de plasma helicón es un cohete electrodinámico propuesto a principios de los años 2000. Se divide en dos etapas, la primera consiste en una fuente de plasma helicón, donde se ioniza el gas neutro y se calienta el plasma. La segunda etapa o tobera magnética, acelera el plasma supersónicamente, incrementándose así el empuje total del dispositivo. Aunque las fuentes de plasma helicón han estado presentes desde los años 70 en entornos industriales, su desarrollo como motor de plasma, así como las tecnologías adyacentes al mismo, siguen en un estado embrionario. Tras un análisis profundo del estado del arte del motor helicón, se concluye que las capacidades propulsivas sugeridas por los distintos grupos de investigación expertos en dicha tecnología, difieren las unas de las otras. En esta Tesis se analizan los principales fenómenos físicos que tienen lugar en el motor helicón desde un punto de vista teórico y numérico, con el objetivo de analizar la viabilidad de esta tecnología como motor de plasma eficiente y competitivo.

Primero, se ha desarrollado un modelo fluídico bidimensional para analizar la dinámica de los electrones, iones y neutros en la parte interna del motor. Como principal resultado, este modelo permite estimar la eficiencia en la producción del plasma y en la utilización del propulsante. También se identifican cada uno de los mecanismos, causantes del empeoramiento de dichas eficiencias. Se establecen relaciones sencillas y claras entre estos mecanismos y los parámetros de operación y de diseño del motor. Los límites asintóticos de este modelo conducen a leyes simplificadas para el diseño de fuentes de plasma helicón de alta eficiencia. Los resultados de este modelo se han interpretado conjuntamente con los resultados de otro modelo que simula la dinámica exterior en la tobera magnética. Realizando un balance global de las ecuaciones de energía y de momento, se caracterizan las pérdidas energéticas, los mecanismos de empuje, y las eficiencias internas y de empuje del motor helicón. Se concluye también que un buen confinamiento magnético y una alta temperatura electrónica son los requisitos mínimos para alcanzar unas actuaciones óptimas.

En una segunda fase se ha desarrollado un modelo híbrido de la tobera magnética. Este modelo se basa en HALLMA, una herramienta para la simulación de motores de efecto Hall (HALLMA). Las especies pesadas, iones y neutros, son tratadas con un modelo de partículas, mientras que los electrones se describen mediante un modelo de fluido anisotrópico e isoterma. Este modelo híbrido permite analizar los distintos regímenes de magnetización de los electrones y su efecto directo en la expansión del plasma a lo largo de la tobera magnética. Por ejemplo, permite estudiar las variaciones en la divergencia del chorro de plasma y en el empuje generado. El estudio de la desmagnetización electrónica es la principal ventaja de este modelo respecto al modelo fluido que se ha utilizado previamente en el balance global de energía y momento.

Sin embargo, los dos modelos de tobera magnética consideran los electrones como una población isoterma y confinada en todo el dominio de simulación, en otras palabras, se impone una distribución Maxwelliana para los mismos. Como resultado de dicha hipótesis la caída del potencial ambipolar no tiene límite inferior, por lo que el plasma se aceleraría infinitamente. Para atacar este problema se ha derivado un modelo cinético de un plasma totalmente magnetizado expandiéndose en una geometría unidimensional, tubo magnético convergente-divergente. Este modelo, resuelto numéricamente, permite estudiar fenómenos como el enfriamiento electrónico no colisional y permite describir consistentemente la evolución del potencial ambipolar, cuyo valor último es finito así como la velocidad última de los iones, que también es finita.

En el marco de esta Tesis se ha llevado a cabo una breve campaña experimental en el Laboratorio de Propulsión Eléctrica y Dinámica de Plasmas (Universidad de Princeton). Los experimentos propuestos han perseguido la validación de algunos de los resultados obtenidos con el modelo fluido de la fuente Helicón, como por ejemplo, la caracterización de los flujos de plasma que impactan en la sección trasera de la fuente, así como la estructura axial de la densidad del plasma a lo largo de la fuente helicón. La eficiencia de la fuente en la producción del plasma también ha sido evaluada y comparada con los resultados numéricos. Muchos de los resultados experimentales pueden ser comparados desde un punto de vista cualitativo con los resultados teóricos, sea un ejemplo, el aumento de las eficiencias de la fuente con la intensidad del campo magnético aplicado.

En la parte final de esta Tesis se ha diseñado de forma preliminar un prototipo de motor Helicón, basándose en el conocimiento adquirido con los modelos desarrollados. Para la estimación de la impedancia del plasma ha sido necesario implementar y extender un modelo radial de la interacción del plasma con la onda de radiofrecuencia. Las leyes simplificadas del modelo fluido-dinámico han sido utilizadas para llevar a cabo este diseño. La eficiencia del motor ha sido estimada entorno al 50 %, que es bastante alta en comparación con los

resultados experimentales, y probablemente se debe a la sencillez de los modelos utilizados. Se concluye entonces, que para un mejor desarrollo de la tecnología helicón con finalidades propulsivas, es necesario seguir investigando en las distintas áreas, teoría, simulación y experimentación, con el fin de obtener herramientas válidas para el diseño y cálculo de las actuaciones del motor Helicón.

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1 Introduction

The research on Electric Space Propulsion [1, 2, 3] began with the pionnering works of Goddard [4] in 1906 and Tsiolkovsky [5] in Russia in 1911. The interest on this field has been rising during the late XX century and early XXI, although ‘in-flight’ devices, such as the Ion or Hall effect thrusters, considered today as mature technologies, fit most of the propulsive mission requirements with an acceptable efficiency.

However, not all of the physics involved on their performances are well understood yet. More recently, the investment and research on these technologies has been speeded up, even more, due to private or pubic initiatives such as the ‘*All-electric propulsion satellite*’ promoted by Boeing, the new Eurostar E3000EOR satellite platform of Airbus Defence and Space, and the implementation of the Strategic Research Cluster on ‘*In-Space electrical propulsion and station keeping*’ promoted by the European Commission within the frame of the ‘*Horizon 2020 Programme*’ (2014-2020). Both initiatives aim to replace the chemical propulsion by electric devices capable of fulfilling all spacecraft manoeuvres after its release from the main launcher, e.g. GTO-GEO transfers, north-south station keeping, etc., and simplifying this way the platform architecture and the overall mission cost.

Nowadays, electric thrusters offer relatively low levels of thrust, 1 μN - 1 N, and very high specific impulses, $10^3 - 10^4$ s, in comparison to the chemical thrusters performances, with a maximum thrust about 1 MN and specific impulses always below 500 s. According to the ‘*Rocket Equation*’,

$$\Delta V = I_{sp} g_0 \log \left(\frac{m_0}{m_f} \right), \quad (1.1)$$

the total ΔV of a given mission [2], is proportional to the specific impulse and to the natural logarithm of the initial mass m_0 to the final mass m_f ratio. m_0 is the spacecraft dry mass plus the initial propellant mass, while m_f is the spacecraft dry mass plus the residual propellant.

Figure 1.1 – Exhaust velocities as a function of typical vehicle accelerations. Regions indicate approximate performance values for different types of propulsion systems. Source: reference [6].

Consequently, electric propulsion allows reducing the amount of propellant required by the mission because of its larger I_{sp} in comparison to chemical rockets, but the manoeuvring time is extended due to its lower thrust. The reduction of mass may reduce the launcher mass as well, so smaller launchers and its lower cost become more attractive. Figure 1.1, from Ref. [6], shows the performances, I_{sp} and thrust to spacecraft mass ratio, for different types of thrusters. The power available on the spacecraft, basically coming from the solar panels, constitutes another drawback for the electric propulsion. The development of more powerful thrusters or arrays of thrusters is limited by this factor. For a typical geostationary communications satellite, available power is about 20 kW, but most of this must be spent feeding the payload instruments.

EP devices are typically classified, based on the acceleration mechanism of thrust, into three categories: electrothermal, electrostatic, and electromagnetic. Within the first group, the resistojet and the arcjet are the most representative. The resistojet increases the exhaust velocity of the propellant on the solid divergent nozzle, by heating it on a pre-chamber using ‘common’ resistors. It provides I_{sp} below 500 s. The arcjet replaces the resistor by a high current arc aligned to the nozzle. This arc, apart from partially ionize the gas, adds more thermal energy to the propellant, than the resistojet does, reaching this way higher I_{sp} , up to 700 s.

Electrostatic devices are probably those offering the best performances. The ion gridded thruster provides I_{sp} on the 2000 - 10000 s range, and it is the most efficient, $\eta_t \sim 60 - 80$ %. The thrust efficiency is defined as $\eta_t = T^2/2\dot{m}P$ in which T is the thrust exerted by the thruster, $\dot{m}$ is the propellant mass flow rate and P is the power required to feed the thruster. Ion

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thrusters extracts and accelerates ions from the generated plasma by the use of an arrangement of biased grids at different potentials. The Hall effect thruster uses the cross-field concept ($\mathbf{E} \times \mathbf{B}$) to generate the plasma. An annular chamber with a transversal (radial) magnetic field is commonly used. At the back part, neutral gas is injected and a metallic plate positively biased constitutes the anode. Outside the annular channel, a hollow cathode provides enough free electrons to generate the plasma (by collisions) and self-neutralize the ion beam. The ion beam is formed because an electrostatic field develops between the anode and the cathode, practically perpendicular to the magnetic field, so generated ions are accelerated downstream. The magnetic field main role is to confine electrons and prevent them from short circuiting the discharge. Electrons are drifting azimuthally due to the aforementioned cross-field drift before reaching the anode due to collisions. Hall effect thruster efficiencies are lower than the ion gridded thruster, below 60 %, as well as the specific impulse, typically in the 1500-2000 s range. However, for a given power, thrust is larger than in gridded devices, mainly because of the lower I_{sp} ; the device is also more compact, and the power unit simpler.

Electrospray and the Field Emission electric propulsion (FEEP) thrusters, extract ions or charged droplets from conductive liquids or from metal liquids respectively. The liquid typically feeds a tiny and very small needle. Close to its tip, a biased aperture deforms the liquid surface because the high electric field developed there that compensates the liquid surface stress. This equilibrium generates the structure known as Taylor cone. Thrust is below the mN range, so thruster arrays have been already proposed [7].

The third group contains all electromagnetic thrusters, which use mainly the interaction among the electric currents within the plasma and applied or induced fields in order to accelerate the plasma downstream. The simplest one is the Pulsed plasma thruster (PPT). A fraction of a solid propellant bar is ionized by an electrical discharge in front of it, this generates a plasma arc. The electromagnetic response of the pulsed plasma accelerates ions downstream.

The Magnetoplasmadynamic Thruster (MPD) ionizes the neutral gas by using a very high current arc. Both currents and induced magnetic fields are generated by the plasma itself, and these accelerate the plasma downstream. Consequently, the MPD requires a very high power to offer competitive performances, tens of kW.

More recently, other thrusters with novel concepts have been proposed. Among others, the Electron Cyclotron Resonance Thruster (ECRT), the VArIable Specific Impulse Magnetoplasma-dynamic Thruster (VASIMR) and the Helicon Plasma Thruster (HPT). These devices share

Figure 1.2 – Princeton University HPT modified prototype firing at 700 W and 1.5 mg/s of Ar.

some similarities, the main one, the use of a Magnetic Nozzle (MN) to accelerate the plasma and enhance thrust. On the other hand, the differences appear on the plasma production and energizing stage: the ECRT uses, as its name indicates, the electron-cyclotron resonance ($\omega_{ec} \sim \omega$) to produce the plasma and energize electrons. This is achieved by establishing a good relation between the applied magnetic field B_0 , which defines the electron cyclotron frequency $\omega_{ec} \propto B_0$, and the microwave frequency ω that feeds the system and that it is in the order of few GHz, typically 2.4 GHz. Consequently, the resonance is satisfied on the surface in which

$$\omega_{ec} \sim \omega$$

. Ionization and electron heating only occur close to this area. The HPT implements a Helicon Plasma source (HPS) as the plasma generator and heating stage. These sources offer the advantage of producing a very dense plasma, if the RF wave is coupled efficiently to the magnetized plasma, meaning a correct propagation and absorption of the Helicon mode. Instead, the source behaves poorly as an inductive source. The VASIMR also uses a HPS to produce the plasma, but an intermediate ion cyclotron resonance stage energizes ions before expanding the plasma through the MN. This Thesis will be focused on the HPT, specifically on some of its physics, in order to assess theoretically its propulsive performances.

1.1 The Helicon Plasma Thruster

The HPT is an innovative technology for space propulsion, which, at present, is being researched extensively [8, 9, 10, 11, 12, 13, 14]. The device (see Figure 1.2) is constituted of a helicon source, where the plasma is generated and heated, and an external divergent magnetic nozzle, where the plasma is accelerated. The physical elements of a HPT are: a cylindrical dielectric chamber; a gas injection system, usually at the back of the chamber; an external an-

tenna wrapped around the chamber emitting RF waves, typically in the range 1-26 MHz, which propagate within the plasma; and a set of magnetic coils (or permanent magnets) that creates a longitudinal magnetic field, typically in the range 10^2 to 10^3 Gauss. In the ‘conventional’ design, the magnetic field is predominantly axial inside the chamber and divergent outside it, and has several roles. First, it makes the plasma column transparent to the propagation of the rf emission as helicon waves. Second, the magnetic field screens the chamber walls, thus reducing greatly plasma losses at them [15]. Third, outside the chamber, the divergent magnetic topology creates a magnetic nozzle that channels the supersonic plasma flow, transforming the plasma internal energy into axially-directed one, in a process very similar to the expansion of a hot gas in a conventional solid nozzle [16, 17].

The typical operation range of helicon sources is $\omega_{lh} \ll \omega \ll \omega_{ec} \ll \omega_{ep}$ [18], with ω_{lh} the lower-hybrid frequency, ω the wave frequency, ω_{ec} the electron cyclotron frequency, and ω_{ep} the plasma frequency. The Helicon mode pertains to the branch of whistler waves; in a cold, unbounded plasma, no other waves can propagate in that frequency range [19]. Although a unique theory for the absorption of the energy of helicon waves is not fully established yet, the collisional theory, for dense enough plasmas, states that absorption is achieved through the mediation of Trivelpiece-Gould surface waves, which are highly dissipative [18, 20]. The advantage of helicon sources over other RF sources, such as inductively-coupled ones, is that, adjusting conveniently the magnetic intensity, $\omega_{ec} \propto B$, there is not a severe cut-off of plasma density for wave propagation, and values of $10^{18} - 10^{20} \text{ m}^{-3}$ are achievable [21].

Other potential advantages of the HPT for space propulsion would be: the lack of electrodes, thus avoiding erosion limitations and promising a long thruster lifetime; the capability of operating with a wide range of propellants [22, 23]; and high throttleability, based on the capability of actuating, at constant power, on both the gas flow and the magnetic nozzle [24].

1.2 Scope of the Thesis

The current Thesis will inquire into the chances of the HPT facing other EP devices and achieving similar or competitive figures of merit. To reach this goal several works are carried out.

First, a detailed analysis of the HPT State-of-the-art is performed. This allows pointing out the HPT advantages and drawbacks of the existing prototypes, and identifying the lack of knowledge on several phenomena in order to determine both potential improvements and also theoretical models needs.

Second, an axisymmetric fluid model of the magnetized plasma flow inside the helicon thruster chamber is derived. Main mechanisms, such as neutral gas ionization, plasma magnetic confinement, subsonic flows, are discussed and the relevant HPS efficiencies are assessed in terms of design and operation parameters. For ideal plasma conditions, analytical solutions and simple scaling laws are obtained. The chamber model is then matched with the solution of a model of the external magnetic nozzle in order to characterize the whole plasma flow and estimate the thruster performances and main sources of inefficiencies are identified.

Third, a hybrid model of the fluid-dynamics throughout the diverging MN is developed based on an existing model, *HallMa* [25], devised to simulate plasma processes in a SPT-100 Hall effect Thrusters. The MN hybrid model results are compared against the results of the full fluid model that has been used to assess the HPT overall performances. This model implements Particle-in-cell (PIC) methods for ions and neutrals, and an anisotropic fluid model for electrons. In comparison to full fluid models, this model allows exploring the magnetization regimes, from the high magnetized and nearly collisionless regime to the low magnetized regime. The demagnetization effects on the plasma 2D supersonic expansion such as beam collimation and divergence, ion detachment and thrust gain are analysed in depth.

Fourth, due to the current uncertainties on the evolution of the electron thermodynamics of a magnetized plasma expansion, and its impact on the MN performances (thrust gain, maximum I_{sp} , etc, ...), a steady-state magnetized plasma expansion is analysed through a simplified paraxial kinetic model. This provides a self-consistent solution for the ambipolar electric field and its total finite drop, a magnitude that cannot be provided by neither the full fluid nor the hybrid model mentioned above. This result allows estimating the limits on the ion acceleration.

Fifth, a series of experiments on the modified EPPDyL-HPT prototype (Electric Propulsion and Plasma Dynamics Laboratory, Princeton University) [26], aim to proof some of the results derived from the fluid model of the plasma source (Chapter 3), including the characterization of the backstreaming flows, the axial structure of the plasma, and source performances.

Finally, a simplified model of the preliminary design of a HPT is presented. This takes asymptotic limits of the fluid models and implement different RF wave propagation models in order to characterize the plasma resistance. The effect of the rarefied plasma and induced field effects on the plasma impedance are treated carefully. HPT figures of merit and theoretical performances are predicted.

2 HPT: *State of the art*

Existing HPT prototypes are still far from achieving propulsive figures capable of competing with other mature plasma thrusters. For instance, thrust efficiency is below 10 % in the few cases in which it has been directly measured [27, 28, 29]. In this context, the understanding of the multiple physical processes taking place in the HPT, the interplay among them, and the assessment of HPT performances are much needed. Few prototypes of helicon thrusters have been built [10, 30, 22, 31] and they differ widely on the nominal power, ranging from few Watts up to tens of kW, and their design, some of them use permanent magnets that create non-axial, cusped magnetic topologies, making more difficult to build a comprehensive understanding of the device capabilities.

The bibliography about Helicon Plasma Sources and Helicon Plasma Thrusters is extremely vast, and with some controversial results between similar experiments. This makes difficult the literature review exercise, consequently this wide revision of the State of the Art only mentions references and relevant results for their application to the Helicon Plasma Thruster. First, general HPS experiments will be presented in subsection 2.1. Then, the main properties and results obtained by means of the continuous research on the most relevant existing HPT prototypes will be presented in subsections 2.2 and 2.3. Finally, main conclusions extracted from this literature review are summarized in Section 2.4.

2.1 General HPS experiments

Rod Boswell from the Plasma Research Laboratory of the Australia National University (ANU) and Francis Chen from the Electrical Engineering Department, University of California, Los Angeles (UCLA), published in the late 90' a series of two papers [32, 33], in which they reviewed the beginnings of the Helicon Plasma Sources. Most of the research on the HPS has been led by

the mentioned authors. Other laboratories with accredited work on HPS are the Department of Engineering Physics of the University of Wisconsin, led by N. Hershkowitz; the Department of Physics of the West Virginia University, led by E. Scime and his co-worker S. Cohen from the Princeton Plasma Physics Laboratory.

The ANU group and other institutions cooperating with them have disseminated the largest amount of HPS publications. Most of their work has inquired in the presence or not of a double layer somewhere in the expansion of a magnetized plasma, a topic that today becomes marginal, but in the early 2000 it was presented as the mechanism that generated thrust in a HPT [34] (via ion acceleration). A double layer (DL) is a thin structure which consist of two parallel layers of opposite electrical charge. This produces a steepened drop of the electric potential, that generates a strong and localized electric field. Charged particles might be accelerated, decelerated or bounced in a DL, depending on their motion properties. The formation of a DL is typically justified by the presence of large differences on the properties of the plasma that this barrier separates. Deviations from Maxwellian distributions, such as the presence of supra-thermal electrons [35] could explain the formation of the Double layer. However, Boswell and co-workers wrongly reported the detection of double layers in [36], in which the axial profile of the electric potential did not seem steep in a thin layer. Furthermore, in other experiments with similar prototypes carried out at ESTEC did not determine the trace of any double layer [9].

Other issues related to the electron trapping by the electrostatic waves, Landau damping, resonant coupling etc., were discussed in [19, 37]. In [38] they measured the depletion of the electron energy distribution function. This could be explained by the loss of plasma to the walls. In the same reference they mentioned also that the electron temperature was larger close to the walls, where the antenna was wrapped, indicating that RF absorption was probably localized at that thin region. Regarding the formation of an ion beam, this is reported in [39], although in that case the source was operated as an inductive plasma source (without magnetic field). The transition from the capacitive mode into the inductive mode was analysed in [40].

Lafleur *et al.* in [41] showed a very good correlation of measurements with the Helicon Theory of Chen [18, 42]. However, in that experiment the propellant was not injected into the plasma source but into the downstream diffusion chamber. Consequently, results of that work cannot be used to assess the HPT capabilities.

Engineering problems related to the RF operation were reported in [43], pointing out that

grounded antennae could carry currents from the plasma if these were in contact with them. The use of Faraday cages to shield the RF power was proposed, also in combination with ceramic spacers like in [44], eliminating parasitic discharges and reducing the deterioration of the antenna.

The UCLA group has focused its research on characterizing helicon wave propagation, excited modes, absorption mechanisms, optimization of parameters, and the experimental verification of the theory. Chen and co-workers set the standard bounded helicon theory of the wave propagation in a magnetized plasma column [18, 42, 45]. They concluded that for radially uniform density profiles the solution is analytical in terms of Bessel functions and corresponds to the cylindrical analogue of the helicon and Trievpiece-Gould (TG) modes of the more basic planar helicon theory. In [46] Landau damping versus collisional absorption mechanisms were discussed as well, underlining the issue that the experimental damping rate of the helicon wave was consistent with theoretical predictions based on collisions alone, without the assumption of non-classical processes such as the presence of fast electron tails. Furthermore, three years later, experiments summarized in [47] pointed out that Landau accelerated electrons was a too weak mechanism to explain the high ionization efficiency. Thus, the collisional theory was more consistent to explain RF absorption and ionization. In spite of this conclusion, in these series of experiments they proved the presence of this hot electron tail (15-25 eV).

Important results of several different experiments are summarized in Table A.2. Tests at 27 MHz, 2 kW, 800 G and around 12 mTorr of Ar [48], produced the highest density plasma of all Chen group experiments, $n = 2 \cdot 10^{19} \text{ m}^{-3}$ and $T_e = 4 - 5 \text{ eV}$. They also checked the use of different kind of antennas, the right-hand helical antenna and the Nagoya type III type. Similar results of the plasma density were reproduced later in Ref. [49], in which they also pointed out the better performances of the half-wavelength helical antenna against the full-wavelength helical antenna, however they did not report any physical explanation for this response. This work compared experimental results with the standard theory of helicon waves by the use of the HELIC code, which indeed, predicts just the opposite, a better performances of the full-wavelength antenna, a conclusion also underlined in [50].

The West Virginia University obtained interesting results with the Hot hELIcon eXperiment (HELIX). A parametric investigation was performed and summarized in [51]. The variation of plasma parameters was explored in [52] as a function of the wave frequency, 6-13.5 MHz,

at constant power, 750 W, field strength, 700 G within the source and 50 G at the expansion chamber, and 3.6 sccm of Ar. Laser Induced Fluorescence techniques were applied to the determination of the ion velocity distribution function (IVDF), upstream and downstream of the DL formation, which was the central topic of that study. Density upstream of the DL increased abruptly at frequencies above 12 MHz, while ion beam velocities about 9 km/s were measured in the expansion area. The authors justified the formation of the DL as a mechanism to compensate the weak ionization by thermal electrons in the upstream region in comparison to the large wall losses that occur there, thus the DL accelerates backwardly electrons to enhance ionization. Electron temperatures of 10 eV were measured in [53] by implementing spectroscopy based on the helium line ratio method. Ion beam velocities about 7 km/s for Argon were shown in [54] together with the presence of a cold ion population. Ion acceleration through a double layer was claimed again in [55]. Ion heating was investigated in [56] showing a remarkable anisotropy with a higher perpendicular temperature than the parallel one. However the ion temperature was still low, < 1 eV.

The Princeton Plasma Physics Laboratory designed the Magnetic Nozzle experiment (MNX). The magnetic field within the source was purely axial, thanks to the use of a Helmholtz-coil. The transition from a purple inductive mode to the helicon one (or “blue mode”) was reported in [57]. A fractional density about 0.3 % of supra-thermal electrons with temperatures ten times the temperature of the bulk electrons was found. Ion energies over 70 eV were measured in the plasma beam. Subsonic to supersonic ion flows transition was encountered close to the source exit (MN throat) as reported in [58], measuring ion energies about 30 eV (or equivalently $I_{sp} = 1100$ s), plasma density $2 \cdot 10^{19} \text{ m}^{-3}$ and electron temperatures about 4-9 eV, working at 27 MHz (other settings were not reported, RF power about 200-2000 W, and argon pressures about 0.4-30 mT). The role of a small mechanical aperture located at the source exit was investigated persistently in the PPPL and the WVU experiment, and also numerically in [59] by using a 3D full PIC code of the plasma. Table A.3 includes the main characteristics of experiments developed at the PPPL and at the WVU.

The Wisconsin University devoted its research to different issues, some of the most relevant are presented next. Neutral depletion and plasma acceleration in helicon plasmas was studied in depth in [60]. Its dependences on the RF power were illustrated, showing a stronger ionization, higher plasma density (10^{19} m^{-3}) and temperature (1-5 eV) as RF power increases up to 3 kW, reaching a high propellant utilization 92 %. The DL formation was attributed to the divergence of the magnetic field, hot electrons, and the strong neutral depletion that reduces

collisions. In [61] they identified the helicon mode as a dark blue core (10^{19} m^{-3}), indicating that in that region ion emission was large due to the strong excitation of these particles. Different magnetic topologies were investigated and antenna impedance was measured at different levels of the input power. Gradients caused by the unusual magnetic topologies modified source efficiency, wave propagation and power absorption. They reported the blue core between 500 W and 1.1 kW, and the measured antenna resistance increases from 0.78Ω (200 W) to 0.92Ω (1.1 kW), indicating that antenna coupling efficiency is better when the blue mode is observed. Similar to other groups, they also investigated the contributions of bulk and supra-thermal electrons to plasma ionization, see Ref. [62], and the role of Landau damping and collisions on the plasma - RF power coupling phenomena for different kind of antennas, Ref. [63]. Tysk *et al* [64] identified three regimes as a function of the power and magnetic field: first, an inductive mode below 650 W and 200 G, presenting evanescent wave structures; second, a transition regime between 650-900 W and 200-400 G, characterized by the observation of travelling waves; and third, the Helicon regime above 700 W and magnetic fields higher than 400 G, showing the blue core and a complex standing wave structure. Density levels were 10^{17} , 10^{18} , 10^{19} m^{-3} respectively for each mode. They combined multiple diagnostics: microwave interferometry, optical spectroscopy, LP and B-dot measurements. These provide density, temperature, and the fluctuation of the magnetic component of the wave. Table A.4 includes the main properties of the experiments performed at the Wisconsin University.

2.2 HPT Prototypes with conventional magnetic circuit

Once general HPS experiments have been presented, it is the turn to summarize the SoA of HPT existing prototypes. Up to eight HPT prototypes will be analysed, these are listed below:

1. The Helicon Double Layer Thruster (HDLT), by ANU (Australia).
2. The Mini Helicon Thruster Experiment (mHTX), by MIT-SPL (USA).
3. The Cylindrical HPT (CHPT), by Georgia Tech (USA).
4. The High Power Helicon Thruster (HPHT), by University of Washington (USA).
5. The Variable Specific Impulse Magnetoplasma Rocket with HPS only (VASIMR-HPSO), with the ICRH stage off, designed by Ad Astra Rocket (USA).
6. The Helicon Plasma Hydrazine COMbined Micro by HPHCOM consortium of 15 partners (Europe).

Figure 2.1 – (a) Helicon Double Layer Thruster at the ANU [29]. (b) mini-Helicon Thruster Experiment at MIT [65].

7. The Compact HPT with Permanent Magnets (CoHPT), by INR-NAS (Ukraine).
8. The Permanent Magnet Expanding Plasma (PMEP), by Iwate University (Japan).

In the previous list, two groups could be distinguished. Those prototypes that implement a conventional magnetic circuit, constituted by a set of coils that produces a nearly axial magnetic field inside the plasma chamber, i.e. HDLT, mHTX, CHPT and HPHT. Instead, HPHCOM, CoHPT and PMEP prototypes use different arrangements of permanent magnets and the magnetic topology is more complex, usually presenting cusps, separatrices and turning points. The special case of VASIMR operating with the ion cyclotron resonance heating stage off, belong to the first group. Regarding the power ranges, prototypes above cover the full range, from the low power level of the HPHCOM, only few tenths of Watts, up to the high power level of the HPHT and VASIMR (for instance its HPS), both consuming tenths of kW. Conventional prototypes are described next, and prototypes with PMs will be analysed in the following subsection.

The Helicon Double Layer Thruster (HDLT) designed by Boswell and Charles, from the Australian National University (ANU), was first named in 2006 [66] (see Figure 2.1), after three years of they had observed the formation of a current-free DL on the expanding plasma of their HPS [34]. In that experiment, Charles and Boswell claimed that the double layer, which accelerated ions from subsonic (within the source) into supersonic (in the discharge vacuum chamber), enhanced the thrust generated by the HPT [34, 67]. The ANU group established a strong cooperation with Surrey University and Iwate University to further develop the HDLT.

Most of the HDLT research was invested in detecting that DL. However, a common mistake

was to associate the presence of the supersonic ion beam to the existence of a current-free DL upstream. Furthermore, other authors proved that a DL does not generate additional thrust [68, 69], as the ANU group finally recognized in 2011 [27], concluding that a DL only transforms electron momentum into ion momentum, producing zero thrust. The HDLT research has helped to understand some relevant aspects of the complex physics of the plasma discharge, but has not provided any remarkable improvement on the HPT capabilities as a pure thruster: the maximum efficiency measured in the HDLT was 3% [27]. This low efficiency could be explained because of the poor vacuum conditions in most of the HDLT experiments, far from space conditions, e.g. $6 \cdot 10^{-4}$ mbar in [29]. This leads to produce plasma in both regions, within the source and also in the expansion chamber, a larger vacuum vessel to which the HPT is attached. This undesirable phenomena of producing plasma in the expansion area is evidenced by the detection of two ion populations on the measured ion velocity distribution function [70, 29]. Plasma production and plasma heating needs to take place inside the source to achieve good HPT propulsive performances.

In 2009, thrust was indirectly estimated through a momentum flux measuring instrument (MFMI) [71]. Later, in 2011, direct measurements of thrust were obtained using the Surrey prototype [29]. The prototype delivered 1 mN of thrust for 250 W and it increased linearly with power up to 2.8 mN at 650 W, injecting a constant mass flow rate of 1 mg/s of Krypton. Other propulsive figures were estimated: thrust efficiency about 1 % and propellant utilization about 20-35 % at 250 W and 500 W respectively. A second prototype at ANU with a conical chamber reached a thrust efficiency of 2.5 % with 650 W and 5 mN [72].

Other experiments of the HDLT group investigated the use of alternative propellants, magnetic control, magnetic steering, and plasma detachment. Regarding the use of other propellants, such as nitrogen, methane, ammonia, this was investigated in [73]: ion beam velocity was in the 14–16 km/s range for N_2O , 17–19 km/s for N_2 and 21–27 km/s range for CH_4 and NH_3 . Concerning the magnetic control, [74] shows that the beam energy (i.e. the I_{sp}) is tuneable with the strength of the exhaust coil current. The energy increases from 30 to 45 eV when the magnetic field strength increases from 100 to 240 G forming a magnetic throat at the exit section. The beneficial effect of a strong magnetic throat was also suggested by the aforementioned work of Takahashi [27], in which Mode B offered a better performance than Mode A. In references [75, 76] they demonstrated that the ion beam could be magnetically steered up to 20 deg off axis by using a solenoid placed normal to the two axial solenoids of the helicon plasma source without changing the beam exhaust velocity. The detachment of the ion beam from the magnetic nozzle was observed in [77], they pointed out that the

ion stream-tubes separated clearly from the magnetic streamlines. Furthermore, in [76] they reported a very low plume divergence of the ion beam of the HDLT, 6 degrees for xenon and 9 degrees for argon. Table A.1 summarizes HDLT data.

The mini Helicon Thruster Experiment (mHTX) was designed and tested at the Space Propulsion Laboratory of MIT from 2005 to 2009, under the supervision of Oleg Bathishchev [10, 65], see Figure 2.1 - (b). The experiment was envisaged specifically to demonstrate the propulsive capabilities of a HPT, in particular, the high propellant utilization, high I_{sp} , acceptable efficiency, and operation with different propellants: Argon, nitrogen and gas mixtures, N_2 -Ar, even air. The power range was about 700-1100 W for Ar mass flows rates of 0.3-1.5 mg/s. The coil system provided a magnetic field strength of 1500 G, much larger than in the HDLT.

The significant results of this group are summarized hereafter: (i) The mHTX prototype reached a propellant utilization larger than 90%, with a single peak of the ion energy distribution function, which is a proof of no ionization downstream; (ii) The measured ion energy versus the mass flow rate showed clearly that the available power was split between the energy invested on heating the plasma and the energy spent on gas ionization, concluding that for constant power the I_{sp} decreases as the mass flow increases; (iii) thrust was measured directly with a pendulum stand, but the magnetic thrust contribution was not measured since only the quartz tube was supported on the thrust balance. For 700W and 0.6 mg/s of Ar, 10 mN of pressure thrust were obtained which means thrust efficiency between 12 % and 48 %, if the magnetic thrust were included. The reported efficiency was 20 % for Ar and 18 % for N_2 . (iv) The plume divergence was narrower than in other prototypes, only 10 degrees [78], and indicates both the plasma beam collimated near the axis and plasma detachment. (v) Electron temperature measurements were about 6 eV with a possible hot population around 30 eV. (vi) A 50 % power loss in the antenna feeding line was reported in [10], and it was classified as an avoidable loss, that would increase thruster efficiency up to a promising 40 %. (vii) Concerning alternative propellants such as N_2 , the specific impulse was in the 2000-4000 s range, with a lower efficiency, and different types of ions (singly vs double charged ions) seemed to be present. Other mHTX properties are detailed briefly in Table A.5.

The cylindrical helicon plasma thruster (CHPT) was designed at the High-Power Electric Propulsion Laboratory (HPEPL), Georgia Tech, led by Prof. Walker (Figure 2.2). According to reference [14], the CHPT was operated with Ar in the forward power range 0-1.5 kW, and

2.2. HPT Prototypes with conventional magnetic circuit

Figure 2.2 – (a) Cylindrical Helicon Plasma Thruster at Georgia Tech [14]. (b) Sketch of the VASIMR, designed by AD Astra Rocket.

magnetic strengths about 0-1100 G. RF frequencies explored were from 2 up to 15 MHz, and the thruster was fed with argon mass flow rates between 0.74 and 4.45 mg/s. Thrust measurements were not performed in that experiment, while detailed parametric analyses of n and T_e and their dependences on the operational parameters f_{RF} , P_{RF} , $\dot{m}$ were carried out. Curiously, measurements under the antenna centreline, for $\dot{m} = 3$ mg/s and $f_{rf} = 7$ mg/s at different levels of B_0 and increasing P_{rf} , showed a null dependence of n against P_{rf} for magnetic fields weaker than 500 G, and the author concluded that the CHPT could be operating at the inefficient capacitive mode. For magnetic strengths larger than 750 G, density increased linearly with power, say inductive mode, $n = 0.5 - 4 \cdot 10^{18} \text{ m}^{-3}$ for 100–800 W, while $T_e \sim 3 - 4$ eV was kept constant. At 900 W the density jumped up to $6 - 7 \cdot 10^{18} \text{ m}^{-3}$ and T_e drop below 2 eV, this could correspond to the helicon mode, and both magnitudes remained constant with RF power still increasing. In another experiment performed by Williams *et al* [28], although they used a different set-up and parametric range, they reported some interesting propulsive figures. The explored range was 215 – 840 W, 12 – 13.56 MHz, 150 – 450 G and 1.5 – 4.5 mg/s of Ar. Within the mentioned range, they reached a maximum thrust of 6.3 mN, and $I_{sp} = 140$ s, and an optimum specific impulse $I_{sp} = 380$ s, providing less thrust 5.55 mN. The maximum thrust efficiency was about 1.4 %, while propellant utilization inside the source was probably poor. External ionization took place as proven by the two-peak ion energy distribution and the difference between the low I_{sp} and the measured ion speed according to the IEDF results, $v_{ex} \sim 7 - 10$ km/s. CHPT properties are summarized in Table A.6.

Other experiments at the HPEPL [79, 80, 81, 82], introduced the concept of an innovative annular helicon source, which was conceived as the ionization stage of a 5 kW HET. The annular chamber was delimited by two coaxial dielectric tubes. Density and temperature measurements, 200 mm downstream from the antenna location, reported promising results:

Figure 2.3 – (a) High Power Helicon Thruster at the Washington University [8]. (b) HPHT: RF power coupled to the plasma vs argon mass flow rate (Torr-L/s) [8].

$n \sim 1.2 \cdot 10^{18} - 7.2 \cdot 10^{19} \text{ m}^{-3}$ and $T_e \sim 16.9 - 19.2 \text{ eV}$, when the operational parameters were set $P_{RF} = 1 \text{ kW}$, $\dot{m} = 2.97 \text{ mg/s}$ of argon, $f_{rf} = 13.56 \text{ MHz}$ and $B = 450 \text{ G}$. High propellant utilization was estimated about 80-90 %. Curiously, two antennas were used in that source, one wrapped externally at the outer wall of the annular chamber, and the other placed within the inner Pyrex tube. Finally, Kieckhafer *et al.* reported in [83] a clear overview on the RF subsystems used at the HPEPL and its limitations and engineering solutions in order to increase the power transmission efficiency. More properties about the annular helicon source experiments are detailed in Table A.7.

The High Power Helicon Thruster (HPHT) was developed at the Department of Earth and Space Sciences of the University of Washington (Fig. 2.3), led by Prof. Winglee. The initial purpose of that experiment consisted on the use of the high-density plasma source as a solar wind simulator. The researchers claimed that the HPHT, which was operated at the higher power range, 20-50 kW, could perform as a thruster delivering up to 2 N of thrust, based on the direct measurements of the downstream ion energy [22]. Regarding the use of different propellants, the HPHT was fed with Argon, Xenon, Hydrogen, Nitrogen, and mixtures of $H_2 - N_2$, with apparently good results in terms of ionization and ion beam energy. The use of different propellants impacted on the measured I_{sp} and estimated thrust. Concerning the specific impulse, the following levels were suggested, 1500, 3000 and 5000 s for Argon, Nitrogen and Hydrogen respectively [22]. Based on these numbers, a thrust efficiency of 55% was claimed.

The HPHT is the prototype that operates at the lowest frequency, 0.5-1 MHz. This low frequency would have several advantages: RF power generators and amplifiers are typically more efficient and compact at low frequencies. Moreover, the lower the frequency is, the weaker

2.2. HPT Prototypes with conventional magnetic circuit

magnetic field strength should be applied, thus reducing the weight and electrical power consumed by the magnetic circuit generator, and also, enhancing the plasma detachment from the magnetic nozzle.

The RF subsystem carried large antenna currents, above 2000 Amp, and it was based on an arrangement of Insulated-Gate Bipolar Transistors, each one managing up to 75 A and 600 V [84]. Note also that at this low frequency and low magnetic field, the wave frequency is in the same order of the lower-hybrid frequency $\omega_{RF} \sim \omega_{lh}$, which is not common in conventional helicon discharges. According to the authors in Ref. [85], this could lead to some ion particle-wave interaction and different heating mechanisms than in conventional HPT. They also point out that wave propagates into the nozzle region and its properties are more similar to a free whistler wave than to a bounded helicon wave. They also reported plasma density peaks up to $2 \cdot 10^{20} \text{ m}^{-3}$, and $T_e \sim 25 \text{ eV}$, with no presence of any DL. A single-peak ion energy distribution function was measured in the plume region [8]. That prototype was apparently the top of its class, and almost among all existing prototypes. However, a deeper analysis of the HPHT results has unveiled worrying inconsistencies on some of the presented figures, which have given rise to strong doubts on the whole body of data published until today.

The first inconsistency lies in the injected mass flow rate $\dot{m}$, which is too low for the prototype nominal power. In reference [8], the authors claimed that the RF power coupled to the plasma was effectively 55 kW for $\dot{m}$ below 2 Torr L/s, said 160 sccm, which for argon is only 5 mg/s (see Reference [84]). In other studies [22], they reported a mass flow rate of 5 Torr L/s, said 13.2 mg/s approximately.

Doing a simple analysis at the highest mass flow rate $\dot{m} = 13.2 \text{ mg/s}$, the equivalent maximum current I_{eq} is computed as follows,

$$I_{eq} = e\dot{m}/m_i,$$

which assumes that all ions are singly-charged. For this case, the equivalent current is $I_{eq} = 32 \text{ A}$. On the other hand, the measured electron temperature was $T_e = 5 \pm 0.5 \text{ eV}$ [22]. Considering an electric potential drop $\Delta\phi$ of 10 times the plasma temperature, $\Delta\phi \sim 10T_e$, ions would sense an equivalent acceleration of $\Delta\phi = 50 \text{ eV}$. The kinetic power of the beam can be approached as follows,

$$P_{beam} \simeq I_{eq}\Delta\phi.$$

Thus, for the current case it yields $P_{beam} \simeq 1.6 \text{ kW}$. Next, the internal efficiency is computed as,

$$\eta_{int} = P_{beam}/P_{RF}.$$

This results an efficiency about 3-8 % for the RF power in the range 20-50 kW. Even if anoma-

lous thermodynamic effects were assumed to boost the ion energy, say larger $\Delta\phi$, the above efficiencies would remain poorer than the 55 % claimed by the authors. Continuing with the power balance, the cost of argon ionization mainly depends on the electron temperature. At $T_e = 5$ eV the cost per ion-electron pair would be around $E_{ion} \sim 80$ eV/ion, which already includes excitation and radiation losses. The ionization cost of the ejected plasma would be,

$$P_{ion} \simeq E_{ion} I_{eq}.$$

At the given $\dot{m}$, the ionization power results $P_{ion} \sim 2.5$ kW. Since not all the generated plasma is effectively ejected from the thruster, the previous power could be increased by a factor of two. Furthermore, some of the plasma is also lost to the walls, P_{wall} , as it will be demonstrated in Chapter 3. However, the total power coupled to the plasma $P_{ion} + P_{beam} + P_{wall}$ remains lower than the nominal power reported by the authors, see Figure 2.3.

A second issue is related to the thrust and specific impulse estimation [84]. They presented two different experimental ways to obtain these parameters: The first one computes the exhaust velocity measuring the time of flight of ejected ions. This time of flight, is based on measuring the elapsed time between the plasma density peaks that are captured in two different positions along the plasma expansion: one at the source exit section, and the other 70 cm downstream. Then, they link the measured exhaust velocity to the specific impulse, $I_{sp} = c_{ex} / G_0$, and to the thrust $T = \dot{m}_i c_{ex}$ ($G_0 = 9.81$ m/s² is the gravity constant). Nevertheless, note that the exhaust velocity is a confusing parameter in electric propulsion, since the external plasma jet (far plume) is still being accelerated, thus generating more thrust. Moreover, the correct definition of the specific impulse is $I_{sp} = T / \dot{m}$, the authors' definition of T and I_{sp} being just a crude description of these parameters. The second method they used was based on the ion beam energy measurement with a retarding field analyser (RFA). They claimed the RFA measurements were consistent with the other methodology. However, the beam power presented in Ref. [84] pointed out just the opposite. The ion beam energy was wrongly estimated because both, overestimation of the jet area downstream ($A = 0.5$ m², 70cm downstream from the HPS exit section) and because they assumed radial uniformity of the ion axial kinetic energy and plasma density $n_0 = 5 \cdot 10^{17}$ m⁻³. With these methods described above and considering that each ion carries an energy equivalent to 70 eV, which accounts, according to them, for ionization losses and beam kinetic energy. The equivalent current taking the previous values of A , n_0 , c_{ex} should be $I_{eq} = 200$ A, while the theoretical limit derived above is just 32 A. Consequently, they obtained an equivalent power around 14 kW, which it is clearly out of the theoretical limits. To conclude, the promised thrust efficiency $\eta = 55$ %, cannot be accepted as valid. Actually, the low $\dot{m}$ used could be an operational limit of the 4000 L vacuum chamber, in which they tested the thruster. This chamber size is not appropriate, too small

for this high RF power, which in fact, requires higher $\dot{m}$ and consequently larger pumping capacity, even in the pulse shooting mode. The mass flow rate managed by this group is far from the one injected in the VASIMR-HPS as presented next. Finally, there is not too much updated data on this thruster to cross-validate with other prototypes and HPS theory. The main HPHT figures are listed in Table A.8.

The Variable Specific Impulse Magnetoplasma Rocket (VASIMR), patented and built by Ad Astra Rocket Co. [86, 87, 88, 89] is not a simple HPT (see Figure 2.2). The thruster includes a HPS as a plasma production stage, an ion cyclotron resonance heating (ICRH) stage, and a magnetic nozzle as the acceleration stage. In normal operation with the ICRH on, most of the internal plasma energy is coupled to the ions and the nozzle expansion transfers the ion internal energy into the ion axial kinetic energy, through the inverse magnetic mirror effect, which is not the same process than in a HPT. The ICRH stage also needs well magnetized ions, and this requires stronger magnetic fields (above 1 Tesla) or the use of lighter propellants, and this is not the most convenient choice for an efficient and light plasma thruster. The main drawback concerns on the complexity and the weight of the magnetic field generator, typically based on superconductors.

Nonetheless, the available research about the VASIMR provides useful information on HPSs and magnetic nozzles thanks to all tests conducted with the ICRH stage off [90]. An experiment with 25 mg/s of Argon, $P_{RF} \sim 30$ kW and a magnetic field strength of 1700 G [91], reported a 95 % of propellant utilization, $I_{sp} \sim 1000$ s, which means an efficiency about 4 %. Plasma density peaks about 10^{20} m^{-3} and $T_e \sim 9$ eV were measured, with significant cooling on the nozzle and with no presence of any DL. Another experiment [92] operated at 28 kW and 107 mg/s of argon, provided 0.7 N of thrust, before igniting the booster, or ICRH stage. The thrust was indirectly measured with a plasma momentum flux sensor [93]. This results in an efficiency of 8 % and $I_{sp} \sim 700$ s. For the conditions of the last experiment, but with the ICRH on, all performances were improved: the VASIMR rated at 108 kW (ICRH + HPS stage), provided 3.6 N of thrust, and 3500 s of specific impulse, resulting a thrust efficiency about 54%. Finally, in the aforementioned reference [91] helicon source performances with krypton and argon were also compared. Krypton showed a higher propellant utilization than argon at the same power. For krypton, the lowest ionization cost they measured was 70 eV/ion, while it was about 80 eV/ion for the argon. Further details on the VASIMR are compiled in Table A.9.

Figure 2.4 – (a) Picture of the HPHCOM laboratory prototype [30]. (b) Sketch of the PMEP, designed at the Iwate University [27].

2.3 HPT Prototypes with non-conventional magnetic circuit

In this subsection, the important characteristics of HPT prototypes that implemented a magnetic circuit based mainly on permanent magnets are briefly presented.

The Helicon Plasma Hydrazine COMbined Micro (HPHCOM) has been the main European project in the HPT field. Funded by the European Commission within the 7th Framework Programme conducted by a consortium of 15 institutions from 7 different European countries, and led by Prof. Pavarin (Padua University). The project aimed to design, build, and test a HPT in the 50 W class, with 1.5 mN of nominal thrust and $I_{sp} = 1200$ s, using argon. These ambitious figures would mean a thrust efficiency about 20 %, somehow competitive in that low power range. The second objective was to study the feasibility of combining chemical/electric thruster concepts, by using the plasma as a catalyst of the hydrazine decomposition.

Several laboratory, engineering, and qualification models were built, by both University of Padua, and KhAI (Ukraine). Different kinds of RF antennas, magnetic field topologies and chamber designs were tested [30, 94]. Plasma density was found to increase by reducing the RF frequency from 10 to 3 Mhz, apart from reducing parasitic losses at the RF power subsystem. Different ceramic diaphragms were placed at the exit section of the plasma ionization chamber in order to constrict the flow and enhance ionization. Solid expansion bells (or solid nozzle) were added to the discharge chamber in order to improve performances in both qualification models, the one developed by CISAS and the one by KhAI. Different sets of electromagnets and permanent magnets were tested, the last one providing a non-conventional magnetic topology with some cusps and separatrices.

2.3. HPT Prototypes with non-conventional magnetic circuit

The Padua prototype of Figure 2.4, reached its best performances when an unusual magnetic topology was implemented. The field was generated by a double ring of axially-oriented SmCo magnets and a ring of radially-oriented ones. This layout produced a non symmetric field with a strong central peak of about 1400 G with two inversion zones and two secondary peaks of different intensity, 800 G and 200 G, the external MN being almost cancelled. Most plasma data was inferred from spectroscopy measurements. Plasma density increased with power up to $2 \cdot 10^{18} \text{ m}^{-3}$, which probably means incomplete ionization. An engineering model of the HPHCOM [30], was built and tested at KhaI and constituted a whole thruster assembly. Its final mass was 420 g, the main dimensions were 14 cm x 8 cm. During the design process, permanent Magnet Electro-Analogous (PMEA) coils were implemented. The optimum magnetic field configuration was characterized by one inversion and a sharp divergence in the antenna zone (near the tube exit). With this topology, plasma density was an order of magnitude higher than in other experiments that implemented inefficient magnetic topologies. The final design of the engineering model was based on permanent magnets. There was no data on its measured performances in the final publishable report of the HPHCOM project.

The group of Padua realized that standard helicon systems introduce several issues related to the RF power plasma coupling phenomenon, which could limit their application as an efficient space thruster. The simplest example is the potential damage of the RF subsystem due to the power reflection in case of having an impedance mismatch between the RF generator output impedance and the antenna impedance, even if the mismatch is small. For this reason, a new technology called S-Helicon resonant antenna was investigated. Nevertheless, the details of that technology were confidential and under patenting process.

A qualifying model of the HPHCOM prototype was built and manufactured at KhaI, based on the S-Helicon technology and was intended as a performance demonstrator. The thruster assembly weighted 2 kg but it was claimed that the use of lighter magnets would reduce the weight to 1kg. Measurements made at KhaI, using argon, yielded thrust about 0.5 mN and $I_{sp} = 422 \text{ s}$, for 0.12 mg/s and only 8 W, which implies an efficiency of 13 %. At higher power, errors in thrust measurements were unacceptable. The thruster was later tested at ONERA facilities and the above operational mode could not be reproduced again: performances, in terms of current and thrust, were much lower than those obtained at KhaI. They just concluded that differences could be attributed to the changes on the chamber or set-up configuration. Table A.10 summarizes the most important HPHCOM properties.

The Compact Helicon Plasma Thruster (CoHPT) was developed by the Institute of Nuclear Research of the Ukrainian National Academy of Sciences [13, 95, 96]. It implemented an arrangement of permanent magnets, to generate the field, some of them axially and others radially magnetized. Electromagnets were included in some of the experiments as well. Different arrangements of the permanent magnets were investigated. The emphasis of the research was in the sensitivity analysis of the magnetic topology on the plasma production. It was claimed that configurations with highly non-uniform magnetic fields, including cusps, could confine more effectively the plasma and improve ionization. The nominal power was between 600-1000 W, and the source was fed with argon at 3 mTorr of pressure. Measurements had shown bulk electron temperatures up to 18 eV, ion beam energies near 100 eV, and plasma density up to $5 \cdot 10^{18} \text{ m}^{-3}$. There was some evidence of the presence of a hot electron tail downstream of the outlet section. The group did not publish any thrust measurement. CoHPT characteristics are listed in Table A.11.

The Permanent Magnet Expanding Plasma (PMEP) is the permanent magnet version of the HDLT and has been developed and tested recently at Iwate University, Japan. Design and power range are similar to the original HDLT but the 2 coils are substituted by a double concentric arrays of neodymium iron boron (Nd-Fe-B) magnets, that create a convergent-divergent field inside the chamber [97]. The double loop antenna was commonly implemented in PMEP tests. Thrust measurements in [98] reported that thrust increased linearly with the applied power, from 0.5 mN at 140 W up to 3 mN at 725 W, which means a poor thrust efficiency of 1 % and $I_{sp} = 500 \text{ s}$. Plasma density and electron temperature were 10^{18} m^{-3} and 6 eV respectively for the optimum conditions. Later, according to [27], the prototype improved its best performance: 6 mN for 800 W and 0.75 mg/s of Ar, which means a thrust efficiency of 3 % and a specific impulse of 800 s. In the same experiment, Takahashi *et al* measured independently the magnetic thrust contribution on the magnetic circuit, almost 3 mN at 800 W, and the pressure thrust contribution on the plasma chamber walls, also 3 mN. In that experiment the magnetic topology was convergent within the source and divergent outside. This confirmed that the magnetic thrust was about 40-50 % of the total thrust, which agrees with theoretical predictions [17, 12]. The power was split differently into the mentioned thrust contributions when the magnetic topology was modified. For an approximate Helmholtz coil configuration (axial field within the source and divergent outside), at 750 W, magnetic thrust was approximately 0.5 mN and the total thrust was almost 2 mN. The increase of thrust with power was also demonstrated, 2.5-6 mN (200-800 W) in the convergent-divergent configuration and 1-2 mN (200-750 W) in the axial-divergent topology. The same author

claimed in October 2013, thrust efficiencies closer to 10 % during an informal conversation in the International Electric Propulsion Conference, at Waswhington DC. But in Refs. [99, 100] they concluded that the PMEP delivered 15 mN of thrust at 2 kW of power and 25 sccm of Ar (0.78 mg/s), which means a thrust efficiency of only 7.2 %. General properties of the PMEP are listed in Table A.12.

2.4 Conclusions

From the beginnings of the research on Helicon waves [19], a large theoretical and experimental investigation has been carried out. The works of Chen, Cohen and Hershkowitz among other authors have reported high plasma densities about 10^{19} m^{-3} , at 2 kW or above [48, 58, 60]. This was the main factor that attracted the interest of the space propulsion community to use this technology as a propulsive device, or as a high performance plasma source in a multi-stage thruster, e.g. VASIMR. However, most of the basic research on the HPS has not been focused on the propulsive capabilities, and has been centred on other issues: the double layer structures [36, 61, 53, 101], Landau damping [46, 63], conditions for the inductive to Helicon modes transition [26, 64, 57], etc.,.

The research on the HPT, more focused on its propulsive performances, has shown controversial results. It is very promising the high propellant utilization, above the 90 %, of the VASIMR [90], the mHTX [10], the HPHT [22], the HPHCOM [94] and the CoHPT [13]. A high plasma density, about 10^{19} m^{-3} or above, has been also measured in Refs. [14, 10, 22, 90], but the electron temperature sweeps a large range, from very low levels 1 eV, up to reasonably high values, 20 eV [10]. Both parameters must be high because HPT thrust scales with the density-temperature product. Thrust has been only measured in few experiments: direct measurements by Takahashi et al. [99] accredited 15 mN of thrust and 2000 s of specific impulse at 2 kW of power, which results a thrust efficiency about 10 %. Other authors reported surprising results for the thrust efficiency: 55 % for the HPHT [22], 20 % for the mHTX [10] and 13 % for the HPHCOM [30], although the veracity of these results is in question because of physical inconsistencies, the premature cancellation of the programme and the incapacity of repeating the experimental validation, respectively. Indirect measurements of the VASIMR-HPS thrust [90] report 500 mN of thrust at 28 kW of RF power, with an efficiency about 16 %. In order to understand these ‘poor’ experimental results it is crucial to increase the modelling capabilities. This could offer a better insight of the loss mechanisms that damage the overall thruster efficiency, and set the guides to improve this technology.

The main conclusion after reviewing the capabilities of HPT existing prototypes is that, some authors are confident on achieving good values of the thrust efficiency [10] –a crucial propulsive figure–, but others obtain disappointing low efficiencies, below 10 % [99]. Therefore, there is a high need of models capable of describing consistently the main processes, interpreting correctly the experimental data, identifying fundamental limits in the thruster operation, and helping on the design of a competitive device.

3 Two dimensional fluid model of a HPS

An axisymmetric macroscopic model of the magnetized plasma flow inside the helicon thruster chamber is derived. Ionization, confinement, subsonic flows, and production efficiency are discussed in terms of design and operation parameters. Analytical solutions and simple scaling laws for ideal plasma conditions are obtained. The chamber model is then matched with the solution of a model of the external magnetic nozzle in order to characterize the whole plasma flow and assess thruster performances. Thermal, electric, and magnetic contributions to thrust are evaluated. The energy balance provides the power conversion between ions and electrons within the chamber and along the nozzle. The power distribution among beam power, ionization losses, and wall losses is determined as well. Thruster efficiency is assessed and main causes of inefficiency are identified.

3.1 Introduction

This chapter deals with the fluid-dynamics problem and assumes that the plasma column has absorbed a known amount of RF energy in the form of electron energy, taking constant de electron temperature. To accomplish this goal, an axisymmetric model of plasma dynamics inside the chamber is developed. HPT performances in terms of thrust, power balance and thrust efficiency are assessed as well. Some of these figures require to match the HPS plasma conditions at the exit section with the results of a plasma simulation of the MN stage (results from DIMAGNO [17]).

The first part of the chapter derives the axisymmetric model of the chamber and analyzes plasma generation, heating, wall interaction, and internal flows. The model is based on decoupling the radial and axial dynamics through a variable-separation technique, already applied successfully to the plasma discharge in a Hall thruster [102]. The wall-recombination

or its equivalent frequency of particle loss to the wall acts as the coupling parameter between the axial and radial dynamics. This technique was already proposed by Fruchtman et al. [103]. However the current model introduces some improvements. Fruchtman took a constant neutral density along the tube, and later he studied neutral depletion [104] but then, he omitted wall losses, essentially, he took a 1D axial model. The current model deals with neutrals as a third fluid, but limits the analysis to its axial dynamics. Neutral depletion is controlled by the balance between plasma volumetric-production and wall-recombination, and the last one depends on the plasma radial dynamics and how walls are magnetically screened. Fruchtman also assumed an adiabatic condition for electrons at the chamber exit, while here, the solution is matched to the structure of the plasma expansion along the nozzle in order to assess the global momentum and energy balance. Concerning ion dynamics, Fruchtman considered a diffusive transport for them, however ions are typically weakly collisional and weakly magnetized, and their motion is controlled by the ambipolar electric field. The effect of the induced magnetic field on the plasma radial dynamics was not investigated neither by Fruchtman nor in the work in Ref. [105], in this chapter this effect is further explored.

Asymptotic regimes of the radial and axial dynamics are also investigated in this chapter. These asymptotic laws are quite useful in order to simplify the HPS modelling, and will be the essence of the model implemented in Chapter 7. A parametric investigation of main operational parameters, magnetic field strength and mass flow rate, is also carried out. This underlines the operational envelope to maximize plasma production efficiency and propellant utilization.

This chapter is structured in the following sections. The 2D chamber model is derived in Section 3.2. The plasma response inside the chamber is discussed in Section 3.3. The matching of the chamber model results and results of the MN model is carried out in Section 3.4, discussing also the different contributions to thrust. Section 3.5 presents the energy balance that allows determining the theoretical thrust efficiency. Conclusions reached in this chapter are summarized in Section 3.6.

3.2 Formulation of the chamber model

Figure 3.1 sketches the HPT, with the chamber and nozzle regions. The magnetic topology is generated by a Maxwell 3-coil arrangement. This configuration produces a magnetic field that is nearly axial and uniform inside the chamber ($\mathbf{B} = B\mathbf{1}_z$), and divergent at the nozzle. The

Figure 3.1 – Sketch of the HPT model.

rectangle symbolizes the cylindrical chamber of radius R and length L . A , W , and E are the labels for the chamber back-wall, lateral, and front-exit, respectively.

A mass flow $\dot{m}$ of neutral gas is injected from the back-wall of the cylinder, $z_A = -L$, and is ionized by the impact of electrons along the source. In steady-state operation, it is assumed that electrons have been energized by the RF wave, acquiring a uniform temperature T_e . The resulting plasma is composed of singly-charged ion, electrons, and neutrals (subscripts i , e , and n , respectively). Plasma density is high enough for assuming the zero-Debye-length limit, thus plasma is quasineutral with $n \equiv n_e = n_i$ except in Debye sheaths around the chamber walls, which constitute surface discontinuities in the quasineutral scale. The sonic Bohm criterion applies to the perpendicular flow at the entrance of the back (B) and lateral (Q) sheaths (Fig. 3.1). The perpendicular flow is also assumed sonic at the chamber exit section E , $z_E = 0$.

Continuity and momentum equations for each species, $j = i, e, n$ ions, electrons and neutrals, are

$$\nabla \cdot (n_e \mathbf{u}_e) = \nabla \cdot (n_i \mathbf{u}_i) = -\nabla \cdot (n_n \mathbf{u}_n) = n_e n_n R_{ion}, \quad (3.1)$$

$$\nabla \cdot (m_j n_j \mathbf{u}_j \mathbf{u}_j) = -\nabla p_j + q_j n_j (-\nabla \phi + \mathbf{u}_j \times \mathbf{B}) - \mathbf{S}_j. \quad (3.2)$$

m_j is the particle mass, q_j is the electric charge, with $q_e = -e$ for electrons. $\mathbf{u}_j$ is the macroscopic velocity, n_j is the density and $p_j = T_j n_j$ is the isotropic pressure. ϕ is the ambipolar electric potential, and $\mathbf{S}_j$ groups different collisional processes on each species. These include ionization and elastic electron-neutral, electron-ion, and ion-neutral collisions, with subindexes ion , en , ei , and in , respectively. Collisional rates for these processes, R_k ($k = ion, en, ei, in$), are defined in the Appendix B and plotted in Figure B.1 to show their dependence on the temperature T_e .

Chapter 3. Two dimensional fluid model of a HPS

According to the analyses in Refs. [102, 103, 15], and for an elongated chamber, $L \gg R$, the following assumptions are considered:

1. Axial symmetry: $\partial/\partial\theta = 0$.
2. Neutrals are assumed cold, with $\mathbf{u}_n = u_n \mathbf{1}_z$, and their density and velocity depend only on z .
3. Ion pressure is much smaller than electron pressure, because $T_i \ll T_e$ is assumed by default.
4. The plasma current $\mathbf{j}$ is taken purely azimuthal, thus $\mathbf{j} - \mathbf{1}_\theta j_\theta = \mathbf{0}$. In other words, there is no longitudinal currents, $u_{ir} = u_{er} \equiv u_r$ and $u_{iz} = u_{ez} \equiv u_z$. This is rather plausible for an elongated dielectric chamber and the nozzle model considered later.

5. Plasma density is expressed as

$$n(z, r) = n_z(z) n_r(z, r),$$

with $(2/R^2) \int_0^R r n_r(z, r) dr = 1, \forall z$.

6. The electric potential is split as

$$\phi(z, r) = \phi_z(z) + \phi_r(z, r),$$

with $\phi_r(z, 0) = 0$ for all z .

7. $u_{\theta i} \ll u_{\theta e} \equiv u_\theta$, so that magnetic effects of ions are negligible, and $j_\theta = j_{i\theta} + j_{e\theta} \simeq j_{e\theta}$.
8. Longitudinal electron-inertia is negligible, but azimuthal electron-inertia (due to u_θ) is retained.
9. Spatial gradients satisfy the following orderings,

$$\begin{aligned} \partial n_r / \partial z &\ll \partial n_r / \partial r, & \partial \phi_r / \partial z &\ll \partial \phi_r / \partial r, \\ \partial(u_r, u_\theta) / \partial z &\ll \partial(u_r, u_\theta) / \partial r, & \partial u_z / \partial r &\ll \partial u_z / \partial z. \end{aligned}$$

These assumptions reduce the 2D model into an axial and a radial sub-models coupled mainly through the wall recombination function,

$$S_w(z) = n_z v_w(z).$$

The frequency $v_w(z)$ is an eigenfunction to be determined. Then, Eqs. 3.1, 3.2 are projected onto the axial direction, obtaining the following set of equations,

$$n_z u_z + n_n u_n = g_0, \tag{3.3}$$

$$\frac{\partial}{\partial z}(n_z u_z) = n_z(n_n R_{ion} - v_w), \quad (3.4)$$

$$u_z \frac{\partial u_z}{\partial z} = -c_s^2 \frac{\partial \ln n_z}{\partial z} - n_n(R_{in} + R_{ion})(u_z - u_n), \quad (3.5)$$

$$u_n \frac{\partial u_n}{\partial z} = -n_z \left[R_{in}(u_n - u_z) + \frac{v_w}{n_n} u_n (1 - \alpha_w) \right], \quad (3.6)$$

$$e \frac{\partial \phi_z}{\partial z} = T_e \frac{\partial \ln n_z}{\partial z}. \quad (3.7)$$

$c_s = \sqrt{T_e/m_i}$ is the sound velocity, $g_0 = \dot{m}/(m_i \pi R^2)$ is the axial flux of heavy species, i.e. ions and neutrals, and $\alpha_w u_n$ is an effective axial velocity of neutrals created from plasma recombination at the lateral wall.

Analogously, projecting Eqs. 3.1,3.2 onto the radial direction, the set of equations for the radial sub-model is,

$$\frac{1}{r} \frac{\partial}{\partial r}(r n_r u_r) = n_r v_w, \quad (3.8)$$

$$u_r \frac{\partial u_r}{\partial r} = -c_s^2 \frac{\partial \ln n_r}{\partial r} - \frac{eB}{m_i} u_\theta + \frac{m_e}{m_i} \frac{u_\theta^2}{r} - n_n(R_{in} + R_{ion}) u_r, \quad (3.9)$$

$$u_r \frac{\partial u_\theta}{\partial r} = \frac{eB}{m_e} u_r - [n_n(R_{en} + R_{ion}) + n R_{ei}] u_\theta - \frac{u_\theta u_r}{r}, \quad (3.10)$$

$$e \frac{\partial \phi_r}{\partial r} = T_e \frac{\partial \ln n_r}{\partial r} + eB u_\theta - m_e \frac{u_\theta^2}{r}. \quad (3.11)$$

As a result of this variable separation technique, the axial model determines the set $(n_z, n_n, u_z, u_n, \phi_z)$, which depends only on z , while the radial model provides, at each z , the set $(n_r, u_r, u_\theta, \phi_r)$. Notice that equations for ϕ_z and ϕ_r are decoupled from the rest, Eqs. 3.7 and 3.11 respectively.

3.2.1 The radial model

Dimensionless variables are r/R , $n_r/n_r(z, 0)$, $e\phi_r/T_e$, u_r/c_s , and u_θ/c_e . $c_e = \sqrt{T_e/m_e}$ is used for normalizing u_θ instead of c_s . Boundary conditions at the axis, $r = 0$, are

$$u_r = u_\theta = \ln[n_r/n_r(z, 0)] = \phi_r = 0. \quad (3.12)$$

The boundary condition at the entrance of the Debye sheath attached to the lateral wall is the classical Bohm criterion, $u_r = c_s$ at $r = R$. This condition determines the eigenvalue $v_w(z)$ in the functional form,

$$\frac{v_w}{\omega_r} = \hat{v}_w \left(\frac{\omega_{lh}}{\omega_r}, \frac{v_{ion}}{\omega_r}, \frac{v_{en}}{\omega_r}, \frac{v_{ei0}}{\omega_r} \right). \quad (3.13)$$

$\omega_r = c_s/R$ is the radial-transit frequency. $\omega_{lh} = eB/\sqrt{m_e m_i}$ is the lower hybrid frequency. $v_{ei0} = (R_{ei} n)_{r=0}$, $v_{en} = R_{en} n_n$, and $v_{ion} = R_{ion} n_n$ are the frequencies of electron-ion, electron-neutral and ion-neutral collisions. Reference [15] concluded that the main magnetized regime

corresponds to the following ordering,

$$\omega_{lh} \gg \nu_{en} + \nu_{ei} + \nu_{ion} + \nu_{in} \geq O(\omega_r). \quad (3.14)$$

Notice that the magnetized plasma condition $\omega_r/\omega_{lh} \ll 1$ is equivalent to $\ell_e/R \ll 1$, with ℓ_e the electron Larmor radius. In the magnetized regime, the radial structure of the plasma column consists of a bulk diffusive region, a thin inertial layer, which is quasineutral and collisionless, and the thinner Debye sheath. At constant electron collision rates $\nu_e \equiv \nu_{en} + \nu_{ei} + \nu_{ion} = \text{const}$, the asymptotic solution for the bulk region according to Tonks [106] is:

$$\frac{n_r(z, r)}{n_r(z, 0)} = J_0\left(a_0 \frac{r}{R}\right), \quad \frac{u_r}{c_s} = a_0 \frac{\nu_e \omega_r}{\omega_{lh}^2} \frac{J_1(a_0 r/R)}{J_0(a_0 r/R)}, \quad \frac{u_\theta}{c_e} = \frac{u_r}{c_s} \frac{\omega_{lh}}{\nu_e}. \quad (3.15)$$

The inertial layer covers the range $u_r/c_s \sim \nu_e/\omega_{lh}$ to $u_r/c_s = 1$. The plasma balance condition, Eq. (3.13), reduces asymptotically to

$$\nu_w = a_0^2 \frac{\omega_r^2}{\omega_{lh}^2} \nu_e, \quad (3.16)$$

with $a_0 \simeq 2.405$, the first-zero of the Bessel function of the first kind J_0 .

3.2.2 Extension of the radial model: more effects and detailed parametric analyses

Even though the radial model has been already written down, this subsection explores and introduces into the previous model more physical effects, among them the non-neutral effects and the role of the induced magnetic field.

In Refs. [15, 107] the radial structure of a high density magnetized plasma column was explored, highlighting the effect of demagnetization due to the rise of collisions and/or the induced magnetic field. The balance between magnetic confinement vs electrostatic confinement was also discussed. Those analyses were performed at the zero-Debye limit $\lambda_{eD} = 0$, and at the zero-inertia limit for electrons, $m_e/m_i \rightarrow 0$. The derivation of this extended radial model includes Poisson's and Ampère's equations, which are necessary to account for effects such as non-neutrality and the induced magnetic field. These equations are:

$$\nabla \cdot \mathbf{E} = \frac{e}{\epsilon_0} (n_i - n_e), \quad (3.17)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \quad (3.18)$$

in which ϵ_0 and μ_0 denote the free space permittivity and permeability. The full set of equations is listed below and includes all the non-simplified equations of the previous subsection (Eqs. 3.8-3.11) and the radial projection of Eqs. 3.17 and 3.18,

- Continuity equation for both ions and electrons in the radial direction,

$$\frac{1}{r} \frac{\partial(r n_i u_{ir})}{\partial r} = \frac{1}{r} \frac{\partial(r n_e u_{er})}{\partial r} = n_e v_w. \quad (3.19)$$

$n_i = n_{ir}$ and $n_e = n_{er}$ in the previous equation and hereafter. This equation also states that zero radial current is assumed as well $j_r = e(n_i u_{ir} - n_e u_{er}) = 0$, $\forall r \in [0, R]$. $v_w = v_{ion} - v_z$ will be again one of the eigenvalues of this extended model. $v_{iz} = v_{ez} = v_z$ is also taken, thus assuming that there is not neither a local axial current nor global axial current.

- Ions radial momentum equation,

$$\begin{aligned} m_i n_i u_{ir} \frac{\partial u_{ir}}{\partial r} &= e n_i E_r - m_i n_i v_{in} u_{ir} - m_e n_e n_i R_{ei} (u_{ir} - u_{er}) \\ &\quad - m_i n_e v_{ion} u_{ir}. \end{aligned} \quad (3.20)$$

- Electrons radial momentum equation,

$$\begin{aligned} m_e n_e u_{er} \frac{\partial u_{er}}{\partial r} &= -k T_e \frac{\partial n_e}{\partial r} - e n_e (E_r + u_{e\theta} B_z) - m_e n_e n_i R_{ei} (u_{er} - u_{ir}) \\ &\quad - m_e n_e v_{en} u_{er} - m_e n_e v_{ion} u_{er} + m_e \frac{n_e u_{e\theta}^2}{r}. \end{aligned} \quad (3.21)$$

- Electrons azimuthal momentum equation,

$$\begin{aligned} m_e n_e u_{er} \frac{\partial u_{e\theta}}{\partial r} &= e n_e B_z u_{er} - m_e n_e v_{en} u_{e\theta} - m_e n_e n_i R_{ei} u_{e\theta} \\ &\quad - m_e n_e v_{ion} u_{e\theta} - \frac{m_e n_e u_{er} u_{e\theta}}{r}. \end{aligned} \quad (3.22)$$

- Poisson's equation,

$$\frac{\partial E_r}{\partial r} = \frac{e}{\epsilon_0} (n_i - n_e) - \frac{E_r}{r}, \quad (3.23)$$

- Ampère's equation,

$$\frac{\partial B_z}{\partial r} = \mu_0 e n_e u_{e\theta}. \quad (3.24)$$

The above equations constitute a set of six differential equations and an algebraic equation for determining: $n_e(r)$, $n_i(r)$, $u_{ir}(r)$, $u_{er}(r)$, $u_{e\theta}(r)$, $E_r(r)$, $B_z(r)$. The problem depends on multiple dimensional parameters as aforementioned m_e , m_i , v_{en} , v_{ion} , R_{ei} , kT_e , etc.,

In this model it is also assumed that ions are not magnetized, $l_i = m_i v_{th} / eB \gg R$ that it is the ion gyroradius, being much more larger than the plasma column radius R , so $u_{i\theta} \ll u_{e\theta}$ is expected. This justifies why the ion momentum equation in the azimuthal direction has not been included in the previous set of equations, and also the azimuthal current $j_\theta \simeq j_{e\theta} = -en_e u_{e\theta}$ has been simplified in the Ampère's law.

Magnitude	Reference value
Radial position, r	R
Ion density, n_i	$n_{i0} = n_i(r = 0)$
Electron density, n_e	$n_{e0} = n_e(r = 0)$
Ion velocity, u_{ir}	$c_s = \sqrt{kT_e/m_i}$
Electron velocity, $u_{er}, u_{e\theta}$	$c_e = \sqrt{kT_e/2\pi m_e}$
Electric force, eE	kT_e/R
Frequencies, ν, ω	c_s/R
Magnetic field, B	B_0

Table 3.1 – Reference values to normalize the main magnitudes in the radial model of Section 3.2.2.

Homogeneous boundary conditions are still imposed on the axis $r = 0$, $n_e(0) = n_i(0) = u_{er}(0) = u_{ir}(0) = u_{e\theta}(0) = E_r(0) = 0$. Note that $B_z(0)$ is unknown and together with ν_w are the eigenvalues to be determined. Eigenvalues can be found after setting boundary conditions at the wall $r = R$. Here is where the differences from the original model arise. For the magnetic field $B_z(R) = B_0$, being B_0 the externally applied magnetic field. The Bohm's criterion, $u_{ir}(R) = c_s$ does not apply here since equations will be integrated from $r = 0$ to the lateral wall $r = R$, passing through the non-neutral region $n_e \approx n_i$. $u_{er}(R) = c_e = \sqrt{kT_e/2\pi m_e}$ replaces the Bohm condition. This model allows on one side, tackling with the study of very tenuous plasmas, in which the Debye length $\lambda_D = \sqrt{\epsilon_0 kT_e / e^2 n_0} \neq 0$ could be large enough and makes it trickier the fulfilling of the Bohm criterion. Aside from this, for denser plasmas, the model deals with the arising of the induced magnetic field.

By determining the main dimensionless parameters it is possible to study properly the different regimes. First, let us consider reference magnitudes according the list in Table 3.1. With these magnitudes, Eqs. 3.19 - 3.22 and Eqs. 3.23, 3.24 are normalized as it follows, in which the tilde indicates normalized variables,

$$\hat{n}_i \hat{u}_{ir} = \gamma_0 \sqrt{\frac{m_i}{2\pi m_e}} \hat{n}_e \hat{u}_{er}, \quad (3.25)$$

$$\hat{u}_{ir} \frac{\partial \hat{n}_i}{\partial \hat{r}} = \gamma_0 \hat{n}_e \hat{\nu}_w - \frac{\hat{n}_i \hat{u}_{ir}}{\hat{r}} - \hat{n}_i \frac{\partial \hat{u}_{ir}}{\partial \hat{r}}, \quad (3.26)$$

$$\hat{u}_{ir} \frac{\partial \hat{u}_{ir}}{\partial \hat{r}} = \hat{E}_r - \hat{\nu}_{in} \hat{u}_{ir} - \gamma_0 \hat{\nu}_{ion} \frac{\hat{n}_e}{\hat{n}_i} \hat{u}_{ir} - \gamma_0 \frac{m_e}{m_i} \hat{\nu}_{ei0} \hat{n}_e \left(\hat{u}_{ir} - \sqrt{\frac{m_i}{2\pi m_e}} \hat{u}_{er} \right), \quad (3.27)$$

$$\begin{aligned} \hat{u}_{er} \frac{\partial \hat{u}_{er}}{\partial \hat{r}} &= -2\pi \hat{E}_r - \frac{2\pi}{\hat{n}_e} \frac{\partial \hat{n}_e}{\partial \hat{r}} - \sqrt{2\pi} \hat{\omega}_{lh0} \hat{u}_{e\theta} \hat{B}_z \\ &\quad - \frac{2\pi m_e}{m_i} \hat{\nu}_{ei0} \hat{n}_i \left(\sqrt{\frac{m_i}{2\pi m_e}} \hat{u}_{er} - \hat{u}_{ir} \right) \end{aligned}$$

$$-\sqrt{\frac{2\pi m_e}{m_i}}(\hat{v}_{en} + \hat{v}_{ion})\hat{u}_{er} + \frac{\hat{u}_{e\theta}^2}{\hat{r}}, \quad (3.28)$$

$$\begin{aligned} \hat{u}_{er} \frac{\partial \hat{u}_{e\theta}}{\partial \hat{r}} &= -\sqrt{\frac{2\pi m_e}{m_i}} \left[(\hat{v}_{en} + \hat{n}_i \hat{v}_{eio} + \hat{v}_{ion}) \hat{u}_{e\theta} - \sqrt{\frac{m_i}{m_e}} \hat{\omega}_{lh0} \hat{B}_z \hat{u}_{er} \right] \\ &\quad - \frac{\hat{u}_{er} \hat{u}_{e\theta}}{\hat{r}}, \end{aligned} \quad (3.29)$$

$$\frac{\partial \hat{E}_r}{\partial \hat{r}} = \frac{1}{\hat{\lambda}_{eD0}^2} \left(\frac{\hat{n}_i}{\gamma_0} - \hat{n}_e \right) - \frac{\hat{E}_r}{\hat{r}}, \quad (3.30)$$

$$\frac{\partial \hat{B}_z}{\partial \hat{r}} = \frac{1}{\sqrt{2\pi}} \beta_0 \hat{\omega}_{lh0} \hat{n}_e \hat{u}_{e\theta}. \quad (3.31)$$

In the previous equations, $\gamma_0 = n_{e0}/n_{i0}$ is the ratio of densities at the axis; $\lambda_{eD0} = \sqrt{\epsilon_0 k T_e / e^2 n_{e0}}$ is the reference Debye length in the bulk plasma; $\omega_{lh0} = \sqrt{e B_0 / m_e m_i}$ is the lower hybrid frequency. $\hat{v}_{en}, \hat{v}_{in}, \hat{v}_{eio}, \hat{v}_{ion}$ are the normalized frequencies of electron-neutral, ion-neutral, electron-ion and ionization collisions. For the definition of v_{ei} , the density dependence of $R_{ei}(n_{ref}, T_e)$ has been neglected, $R_{ei}(n, T_e) = R_{ei}(T_e)$, and consequently $v_{ei}(r, T_e) = n_i(r) R_{ei}(T_e) = n_{i0} R_{ei}(T_e) \hat{n}_i(r) = v_{ei0}(T_e) \hat{n}_i(r)$. In Eq. 3.31, $\beta_0 = \mu_0 n_{e0} k T_e / B_0^2$ is the plasma beta parameter. This compares plasma thermal pressure against magnetic pressures, identifying the regime in which the induced field becomes important.

The set of equations 3.25 - 3.31 are singular at $\hat{r} = 0$, so these require to be expanded at $\hat{r} \rightarrow 0$ in order to determine the initial conditions to integrate them. Considering the following approximations $\hat{u}_{ir} = a\hat{r}$, $\hat{E}_r = 2d\hat{r}$, which is equivalent to $\hat{\phi}_r = 0 - d\hat{r}^2$, $\hat{u}_{er} = f\hat{r}$, $\hat{u}_{e\theta} = g\hat{r}$, $\hat{n}_i = 1 - O(\hat{r}^2)$, $\hat{n}_e = 1 - O(\hat{r}^2)$ and the aforementioned equations, the constants (a, d, f, g and γ_0) are determined from the set of equations listed below,

$$\begin{aligned} 0 &= \hat{\lambda}_{eD0}^2 \hat{v}_w \left[\hat{v}_w + 2 \left(\hat{v}_{ion} + \frac{m_e}{m_i} \hat{v}_{eio} \right) \right] \gamma_0^2 \\ &\quad + 2 \left[\hat{\lambda}_{eD0}^2 \hat{v}_w \left(\hat{v}_{in} - \frac{m_e}{m_i} \hat{v}_{eio} \right) + 1 \right] \gamma_0 - 2, \end{aligned} \quad (3.32)$$

$$a = \frac{\gamma_0 \hat{v}_w}{2}, \quad (3.33)$$

$$d = \frac{1 - \gamma_0}{4 \hat{\lambda}_{eD0}^2}, \quad (3.34)$$

$$f = \frac{1}{\gamma_0} \sqrt{\frac{2\pi m_e}{m_i}} a, \quad (3.35)$$

$$g = \frac{\hat{\omega}_{ce0}}{\hat{v}_{en} + \hat{v}_{ion} + \hat{v}_{eio}} f. \quad (3.36)$$

Regarding β_0 , it can be written in terms of others parameters previously defined, so it is not considered an independent parameter. To relate it with others, consider the plasma skin depth

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$d_e = c/\omega_{ep}$ in which ω_{ep} is the electron plasma frequency and c the light speed,

$$d_e = \sqrt{m_e/\mu_0 n_{e0} e^2}, \quad (3.37)$$

and normalizing it with R , it yields,

$$\hat{d}_e = \frac{d_e}{R} = \hat{\lambda}_{eD0} \frac{c}{c_e^*} \quad (3.38)$$

$$\text{with } c_e^* = \sqrt{kT_e/m_e},$$

In addition, the electron Larmor radius is

$$l_e = m_e c_e^* / e B_0, \quad (3.39)$$

And normalizing it, this becomes $\hat{l}_e = l_e/R = \hat{\omega}_{lh}^{-1}$. So, β_0 is redefined as,

$$\beta_0 = \frac{\hat{l}_e^2}{\hat{d}_e^2} = \frac{\hat{l}_e^2}{\hat{\lambda}_{eD0}^2} (c_e^*/c)^2. \quad (3.40)$$

As a result, the product $\beta_0 \hat{\omega}_{lh0}$ in Eq. 3.31, can be rewritten as $\hat{l}_e / \hat{d}_e^2 \equiv \hat{l}_e \hat{\lambda}_{eD0}^{-2} (c_e^*/c)^2$.

Finally, imposing the aforementioned boundary conditions, the radial problem becomes an eigenvalue problem of the form: $\{\hat{B}(0), \hat{v}_w\} = F(m_i/m_e, \hat{v}_{in}, \hat{v}_{ion}, \hat{v}_{en}, \hat{v}_{ei}, \hat{\omega}_{lh}, \hat{\lambda}_{eD0}, c_e^*/c)$. At this point it is important to connect all these parameters with dimensional quantities, $m_i, \dot{m}, R, kT_e, B_0, u_{n0}$ in order to inspect which is the parametric space of interest. Considering, $n_{n0} u_{n0} = \dot{m}/m_i \pi R^2$ and $\eta_u n_{n0} u_{n0} = n_0 c_s$ with $\eta_u = \dot{m}_i/\dot{m}$ the local propellant utilization factor. Here, η_u is defined as the ions to neutrals mass flow rate at any HPS section. Later in this chapter, η_u is uniquely defined at the exhaust section of the HPS. The reference Debye length and plasma frequency are written as,

$$\hat{\lambda}_{eD0}^2 = \frac{\epsilon_0 \pi \sqrt{m_i} (kT_e)^{3/2}}{e^2 \eta_u \dot{m}}, \quad \text{or equivalently} \quad \hat{\omega}_{ep}^2 = \frac{m_i}{m_e \epsilon_0 \pi \sqrt{m_i} (kT_e)^{3/2}} e^2 \eta_u \dot{m}, \quad (3.41)$$

Also, the frequency of ionization ν_{ion} can be expressed in terms of the physical parameters,

$$\hat{\nu}_{ion} = \frac{n_{n0} R_{ion}(T_e)}{c_s/R} \equiv \frac{\dot{m} R_{ion}(T_e)}{m_i \pi R u_{n0} \sqrt{kT_e}}. \quad (3.42)$$

All other frequencies can be expressed in terms of collisional rates that mainly depend on different functions of the electron temperature:

$$\frac{\hat{\nu}_{in}}{\hat{\nu}_{ion}} = \frac{R_{in}(T_e)}{R_{ion}(T_e)}, \quad \frac{\hat{\nu}_{en}}{\hat{\nu}_{ion}} = \frac{R_{en}(T_e)}{R_{ion}(T_e)}, \quad \frac{\hat{\nu}_{ei}}{\hat{\nu}_{ion}} = \frac{\eta_u u_{n0}}{c_s} \frac{R_{ei}(T_e)}{R_{ion}(T_e)}, \quad (3.43)$$

Parametric analyses of the complete radial model

Figure 3.2 illustrates the variation of principal dimensionless parameters around the nominal values, $\dot{m} = 1$ mg/s, $T_e = 10$ eV, $R = 0.01$ m, $u_{n0} = 400$ m/s and $B_0 \in [50 - 1000]$ G, and Argon as the propellant, which satisfies the following orderings, $\hat{\omega}_{ep} \gg \hat{\omega}_{lh} > \hat{\nu}_{ion} \gtrsim 1$. According to this, solutions of the radial model are explored within the following parametric ranges:

Figure 3.2 – Parametric space of interest for the complete radial model, around the nominal values $\dot{m} = 1$ mg/s, $T_e = 10$ eV, $R = 0.01$ m, $u_{n0} = 400$ m/s and $B_0 \in [50 \text{ G} - 1000 \text{ G}]$, and for Argon: (a) Dimensionless plasma frequency $\hat{\omega}_{ep}$. (b) Dimensionless lower hybrid frequency, $\hat{\omega}_{lh}$. (c) β_0 parameter (assuming $\eta_u \rightarrow 1$). (d) β_0 parameter (assuming $\eta_u = 0.25$). (e) Dimensionless ionization frequency, $\hat{\nu}_{ion}$. (f) Ratios of frequencies of collisions for electron-neutral $\hat{\nu}_{en}$, ion-neutral $\hat{\nu}_{in}$ and electron-ion collisions $\hat{\nu}_{ei}$.

$\hat{\omega}_{ep}^2 \in [10^5, 10^6]$, that is equivalent to $\hat{\lambda}_{eD0}^2 \in [10^{-6}, 10^{-4}]$, $\hat{\omega}_{lh} \in [1, 10^2]$ or $\hat{l}_e \in [10^{-2}, 1]$, $\hat{v}_{ei} \in [10^{-2}, 10]$, $\hat{v}_{ion} = 1$, $\hat{v}_{en}/\hat{v}_{ion} = 20$, $\hat{v}_{in}/\hat{v}_{ion} = 0.1$, $c_e^*/c \sim 5 \cdot 10^{-3}$, and $u_{n0}/c_s \sim 0.08$.

The parametric results for this model are summarized in Fig. 3.3, for different values of $\hat{\lambda}_{eD}^2$. The variation of $\hat{v}_{ei}$, within the range of interest, has been determined to be small. However, averaged results are plotted for this variable. The dependence of $B(0)/B_0$ on the parameter $\hat{\lambda}_{eD}$ is depicted in Fig. 3.3 (a), as expected, reducing $\hat{\lambda}_{eD}$, say increasing electron density, leads to a larger demagnetization of the plasma due to the increasing effect of the induced field. The induced field rises with the azimuthal currents, which in turn are proportional to the mentioned density. However, this variation does not impact on the plasma flux to the wall, $n_{iw}u_{iw}$, as stressed in subplot 3.3 (b). Subplots 3.3 (c) - (d) depict u_{iw} and ϕ_w respectively. At the low magnetized regime $\omega_{lh}/v_e \ll 1$ the structure of a classical Debye sheath develops close to the wall, and Bohm condition for ions occur somewhere at $r \sim R - \lambda_D$. However, when the hybrid Hall parameter increases, $\omega_{lh}/v_e \gtrsim 1$, and for larger values of $\hat{\lambda}_{eD}^2 \simeq 10^{-5}, 10^{-4}$ then, ions cannot reach the sonic velocity and the classical Debye sheath does not form anymore. This is the case of a magnetized and extended sheath, and its thickness is not small. Larger λ_{eD} values also means that space charge effects are accentuated, so the difference between n_e and n_i becomes more noticeable as shown in subplot 3.3 (e), n_{ew}/n_{iw} . Small variations of $u_{e\theta w}(\hat{\lambda}_{eD})$ are detected in subplot 3.3 (f). The increasing trend as ω_{lh}/v_e rises, reproduces the same behaviour of the model with $\lambda_{eD} \rightarrow 0$. Note that the defined hybrid Hall parameter ω_{lh}/v_e is proportional to the classical Hall parameter χ_e , and for Argon it is roughly, $\chi_e = \omega_e/v_e \simeq 270\omega_{lh}/nu_e$.

Comparison against the simplified radial model

The main results of the complete model are compared against those obtained in the simplified model described in section 3.2.1 (model B hereafter).

Parametric analyses are carried out within an equivalent parametric space in terms of all dimensionless parameters involved. Even though model B assumes the zero-Debye limit, different values of $\hat{\lambda}_{eD0} = 10^{-4}, 10^{-5}, 10^{-6}$ are considered in order to compute β_0 in agreement to Eq. 3.40.

Results are shown in Figure 3.4. There is no apparent differences on the $B(0)/B_0$ results (3.4 a) when comparing both models. $B(0)/B_0$ drops as ω_{lh}/v_e decreases or $\hat{\lambda}_{eD0}$ increases, which is equivalent to the increase of d_{ep} in Ref. [107].

Fluxes to the wall are illustrated in Figure 3.4 (b). Both models get almost the same result.

Figure 3.3 – Radial model parametric results when including both the non-neutral effects, $\lambda_{eD0} \neq 0$, and the induced effects $\beta_0 \neq 0$. Other parameters are: $\hat{v}_{ion} = 1$, $\hat{v}_{en}/\hat{v}_{ion} = 20$ *cnt*, and variable ω_{lh} . $\nu_e = \nu_{ei} + \nu_{en} + \nu_{ion}$. The effect of varying ν_{ei} has been determined small in the range of physical interests, thus the value showed in the figure is the averaged result in the range $\hat{v}_{ei} \in [10^{-2}, 10]$. (a) Magnetic field at the axis, $B(0)/B_0$. (b) Ion flux to the wall $n_{iw} u_{iw}$. (c) Dimensionless ion speed at the wall, $\hat{u}_{iw}$. (d) Dimensionless electric potential at the wall, $e\phi/kT_e$. (e) Ratio of electron to ion density at the wall, n_{ew}/n_{iw} . (f) Dimensionless electron azimuthal velocity, $\hat{u}_{e\theta}$.

Figure 3.4 – Comparison of complete radial model against the simplified model B (lines with markers). Results are for $\hat{\lambda}_{eD0}^2 = 10^{-4}, 10^{-5}, 10^{-6}$ (thick, thin and dashed line respectively), $\hat{v}_{ion} = 1$, $\hat{v}_{en}/\hat{v}_{ion} = 20$ and variable ω_{lh} . The effect of varying ν_{ei} has been determined small in the range of physical interest, plotted results are averaged in the analysed range of ν_{ei} . (a) Magnetic field at the axis, $B(0)/B_0$. (b) Ion flux to the wall $n_{iw}u_{iw}/n_0c_s$. (c) Electric potential $e\phi/kT_e$ and (d) electron azimuthal velocity $\hat{u}_{e\theta}$, both variables evaluated at the position of $\hat{u}_{ir} = 1$.

Magnetic screening of the lateral wall is reached when ω_{lh}/ν_e increases. Even for scenarios in which plasma demagnetizes at its core, the measured flux to the wall does not present appreciable differences.

The electric potential depicted in Fig. 3.4 (c) corresponds to the potential at the radial position of $u_i/c_s = 1$. Note that some curves of the the complete model do not cover all the ω_{lh}/ν_e range because there the Bohm criterion is not fulfilled, and larger differences on the potential arises there.

The azimuthal electron velocity response obtained from each model presents similar behaviours when plasma is not magnetized as shown in Fig. 3.4 (d). But differences appear at the strong magnetization regime, in which the result obtained by the complete model differs

from the simplified model.

In Figure 3.5 radial profiles of ion density and ion velocity are depicted for the three models at different levels of magnetization $\hat{\omega}_{lh} = 100, 10, 1$. Model B does not add any information of the sheath expansion effect (sheath thickness), which is only recovered by the model that includes Poisson's equation. However, the simplified model allows assessing properly wall losses (plasma flux to the wall), which is one of the most important issue to estimate the HPS efficiencies. For this reason, it is considered that the simplified model is a good approach and justifies the choice of implementing it in the radial model of the HPS two-dimensional model. However, the final version of the radial model also includes the induced field. This decision is taken according to the conclusions of Chapter 7, in which it is concluded that the depletion of the magnetic field within the core of a dense plasma dramatically impacts on the plasma-wave coupling phenomena.

3.2.3 The axial model

After some manipulation, the set of Eqs. (3.3)-(3.7) yields,

$$(c_s^2 - u_z^2) \frac{\partial u_z}{\partial z} = (u_z - u_n) u_z n_n (R_{in} + R_{ion}) + c_s^2 (n_n R_{ion} - v_w), \quad (3.44)$$

$$(c_s^2 - u_z^2) \frac{\partial n_z}{\partial z} = -n_z [u_z (n_n R_{ion} - v_w) + (u_z - u_n) n_n (R_{in} + R_{ion})], \quad (3.45)$$

$$n_n u_n \frac{\partial u_n}{\partial z} = n_z [u_n v_w (\alpha_w - 1) + (u_z - u_n) n_n R_{in}]. \quad (3.46)$$

Boundary conditions for these equations are imposed at the back-wall sheath edge B, $z_B = -L$ and at the front exit E, $z_e = 0$,

$$u_{nB} = u_{n0}, \quad u_{zB} = -c_s, \quad u_{zE} = c_s. \quad (3.47)$$

Normalization of the previous equations with c_s , g_0 , and L yields that the axial solution depends on the following dimensionless parameters,

$$L/L_\star, \quad R_{in,s}/R_{ion}, \quad u_{n0}/c_s, \quad \alpha_w, \quad \text{and } v_w/v_{ion}. \quad (3.48)$$

$v_w(z)$ is the aforementioned eigenfunction of the simplified radial model. $L_\star$ is an effective ionization mean-free-path, computed as the ratio of the scaled ionization cross section R_{ion}/c_s , which depends only on T_e , and the neutral density $n_{n0} = g_0/u_{n0}$,

$$L_\star = c_s/(R_{ion} n_{n0})$$

If T_e and B are high enough, then,

$$v_w/(n_n R_{ion}) \ll 1, \quad R_{in,s}/R_{ion} \ll u_{n0}/c_s \ll 1,$$

Figure 3.5 – Ion density (a-c) and velocity (d-f) radial profiles given by the complete model (solid thick line) and model B (thin line). Simulations are carried out at $\hat{\lambda}_{eD0}^2 = 10^{-6}$, and other parameters as in Fig. 3.4. $\hat{\omega}_{lh} = 100$ in (a,d), $\hat{\omega}_{lh} = 10$ in (b,e) and $\hat{\omega}_{lh} = 1$ in (c,f).

In addition to this ordering, if $\alpha_w = 1$, the axial plasma flow admits the ideal solution, that corresponds to a perfect magnetically confined plasma.

$$u_n = u_{n0}, \quad \frac{u_z}{c_s} = \tan \xi, \quad \frac{n}{n_0} = 2\eta_u \cos^2 \xi, \quad \frac{n_n}{n_{n0}} = 1 - \eta_u \sin 2\xi, \quad (3.49)$$

$$\frac{z+L}{L_\star} = \int_{-\pi/4}^{\xi} \frac{1 - \tan^2 \xi'}{1 - \eta_u \sin 2\xi'} d\xi'. \quad (3.50)$$

$n_0 = g_0/c_s$ is the plasma reference density, ξ is an auxiliary variable, and $\eta_u = n_E/n_0$ coincides with the propellant utilization. By setting $z = 0$ at $\xi = \pi/4$ in Eq. 3.50, an implicit relation for $\eta_u(L/L_\star)$ is found,

$$\frac{L}{L_\star} = \int_{-\pi/4}^{\pi/4} \frac{1 - \tan^2 \xi}{1 - \eta_u \sin 2\xi} d\xi. \quad (3.51)$$

Although the functions in Eq. (3.49) are symmetric with respect to ξ , the function $z(\xi)$ is not symmetric and the point $\xi = 0$, where $u_{iz} = 0$ and n is maximum, shifts towards the chamber rear wall.

3.3 Plasma response inside the chamber

This section discusses the two-dimensional structure and the resulting performances of the plasma flowing throughout the chamber, in terms of the three main operation parameters: the magnetic field strength B , the propellant mass flow rate $\dot{m}$, and the plasma temperature T_e , which will be later related to the absorbed power P_a . Although the discussion can be done in terms of dimensionless parameters, dimensional results are presented here because of their clearness and simplicity. Thus, let us consider a cylindrical chamber of radius $R = 1$ cm and length $L = 10$ cm, operating nominally at $B = 600$ G, $\dot{m} = 0.1$ mg/s of Argon, and $T_e = 10$ eV. We also take $\alpha_w = 1$ and $u_{n0}/c_s = 0.07$. For these conditions the typical values of dimensionless parameters are $\omega_{lh}/\omega_r = 80$, $\nu_{en}/\omega_r \sim 10$, $\nu_{ei}/\omega_r \sim 3$, $\nu_w/n_n R_{ion} \sim 0.2$, $R_{in,s}/R_{ion} = 0.04$, and $L/L_\star = 3.7$.

3.3.1 2D plasma profiles

Figure 3.6 plots profiles of main axial magnitudes for two magnetic intensities, 200 G and 600 G, and compares them with the ideal axial solution of Eqs. (3.49)-(3.50). Fig. 3.7 plots profiles of two radial magnitudes at the chamber rear wall ($z = -L$) and front exit ($z = 0$), for the same magnetic intensities, and compares them with the ideal radial solution of Eq. (3.15).

Figure 3.6 – Dimensionless axial profiles inside the HPS. Solid lines are at $B_0 = 600$ G (thick) at 200 G (thin), and dashed lines are for the ideal axial solution. Normalization constants are defined in the main text. (a) neutral density, (b) plasma density, (c) plasma velocity, (d) radial averaged electron collision frequency, (e) plasma wall recombination-to-ionization ratio.

Figure 3.6(a) shows how the injected neutral flow is effectively depleted by ionization. In Fig. 3.6(b) we observe that the plasma density presents a positive gradient at the back of the chamber, caused by ionization, and then, a negative gradient, caused by ion acceleration. Fig. 3.6(c) shows the region of backward and forward plasma flow, with $u_{iz} = 0$ marking also the location of the maximum plasma density. Observe that the ion back-streaming region occupies only a small part of the chamber; in contrast, the constant- n_n model of Ref. [103] yields symmetric profiles of axial variables around the chamber mid-section, $z = -L/2$. Figure 3.6(d) plots the effective electron-collision frequency, which decreases by a factor of 8 between the chamber back and front sections, because of the decrease of v_{en} ($v_{en} \propto n_n$). As a consequence, the plasma is more magnetized near the chamber exit, which affects the radial profiles of Figure 3.7 and the local wall recombination. In fact, electron collisionality is dominated by collisions with neutrals, near the back wall, and with ions, near the front exit. Figure 3.6(e) depicts the ratio between wall-recombination and ionization along the chamber, which defines the net plasma production along the chamber. Wall-recombination is moderate for 200 G and small for 600 G, which explains why the ideal axial solution, see dashed line in Figure 3.6(a)-(c), is almost indistinguishable from the exact 600 G solution.

The radial profiles plotted in Fig. 3.7 do not cover the near-axis region $r/R < 0.4$, where gradients of u_r are very small at the high magnetized regime. The agreement of the exact solution with the ideal radial solution is excellent for $B = 600\text{G}$ at the back-wall section. Otherwise, at the exit section, the dominance in v_e of electron-ion collisions, which are proportional to the local n_e , makes the radial profile more steepened than in the ideal solution. The profiles of u_r illustrate how a large magnetic confinement prevents the development of large perpendicular velocities until the pre-sheath. The same is true for the radial electric field, $-e\phi_r \simeq m_i u_r^2/2$, which is negligible outside the thin inertial layer, of thickness ℓ_e , preceding the Debye sheath.

Figure 3.8 plots, for 200 G, the $r - z$ contour maps of the plasma density and the ion velocity u_i . The constant-velocity lines are also good approximations for isopotentials. Plasma magnetization, even if moderate as here, tends to concentrate the gradients of the plasma flow around the lateral and rear walls of the chamber. Notice that the radial gradients of n in the bulk region are sustained not by the tiny radial electric field, but by the radial magnetic force generated by the azimuthal electron current. At the chamber exit, the plasma beam is radially non uniform and sonic.

If magnetic confinement is not large, plasma losses to the lateral wall increases the amount of neutrals created by wall recombination. These new neutrals are injected back into the plasma

Figure 3.7 – Dimensionless radial profiles inside the chamber of (a,b) plasma density and (c,d) radial velocity u_r/c_s , at (a,c) the rear wall, $z = -L$, and (b,d) the exit section, $z = 0$. Solid lines are at $B_0 = 600$ G (thick) and 200 G (thin), and dashed lines are for the ideal radial solution. Normalization constants are defined in the main text.

Figure 3.8 – Two-dimensional contour maps of plasma (a) density $\log_{10}(n/n_0)$ and (b) velocity $|u|/c_s$, inside the HPS, at $B_0 = 200$ G.

Figure 3.9 – Axial profiles inside the chamber of neutral (a) velocity and (b) density, for different values of the re-emission velocity parameter: $\alpha_w = 1$ thick black line, 0 thin black line, and 2 red line.

with a lower energy than the recombined ions, due to particle energy accommodation, and not specularly, thus increasing the neutral thermal energy. In the current model, this neutral heating cannot be reproduced accurately but the sensitivity of the solution to the properties of recombined neutrals can be estimated by varying the parameter α_w in Eq. (3.46). Figure 3.9 compares the solution for three cases: $\alpha_w = 1$, which keeps u_n almost constant; $\alpha_w = 0$, which assumes that neutrals from recombination are injected back with zero energy; and $\alpha_w = 2$, which assumes that new neutrals keep some of the ion axial directed energy before recombination. Although the macroscopic neutral velocity is affected by recombination conditions, the profiles of plasma density as well as other magnitudes remain unaltered.

3.3.2 Chamber performance parameters

The two main parameters characterizing plasma production in the chamber are the *propellant utilization* and the *production efficiency*, defined, respectively, as

$$\eta_u = \frac{\dot{m}_{iE}}{\dot{m}}, \quad \eta_p = \frac{\dot{m}_{iE}}{\dot{m}_{iT}}, \quad \text{with} \quad (3.52)$$

$$\dot{m}_{iT} = \dot{m}_{iE} + \dot{m}_{iA} + \dot{m}_{iW}. \quad (3.53)$$

$\dot{m}_{iT}$ is the total ion production rate in the chamber. This production is the sum of the ion mass flows at the chamber exit E , the back wall A , and the lateral wall W ,

$$\dot{m}_{iE} = m_i \pi R^2 c_s n_E, \quad \dot{m}_{iA} = \dot{m}_{iB} = m_i \pi R^2 c_s n_B, \quad \dot{m}_{iW} = m_i 2\pi R \int_0^L dz (n u_r)_{r=R}, \quad (3.54)$$

At the perfect confinement limit, the ideal law of $\eta_u(L/L_\star)$, (Eq. 3.51), is plotted in Fig. 3.10(a). This is the scaling law for the propellant utilization in terms of L , n_{n0} and T_e . The high propellant utilization regime requires a large ratio $L/L_\star$. Taking $L/L_\star \geq 2.5$ it yields $\eta_u \geq 95$ %. In comparison to this ideal response, Figure 3.10(b) plots the influence of the lack of confinement on the resulting η_u and for different plasma temperatures. For each T_e -curve, its knee separates a low-confinement, low-ionization regime from the high-ionization regime. As $\dot{m}$ increases, the curves of Fig. 3.10(b) shift towards higher η_u see Fig. 3.11. Achieving a high propellant utilization at the source exhaust section η_u , is the first requirement to be accomplished in order to design a competitive thruster. External ionization should be avoided in any case.

The production efficiency η_p measures the fraction of the produced plasma that is effectively ejected from the chamber and thus contributing efficiently to thrust. In the idealized magnetic confinement scenario, $\dot{m}_W \rightarrow 0$, $\dot{m}_B = \dot{m}_E$, and the production efficiency would reach a theoretical maximum of only 50 %. Figure 3.10(c) plots the influence of T_e and B on η_p . The qualitative behaviour is similar to $\eta_u(B)$, and the curve knee separates the two regimes. η_p tends to the limit ≈ 50 % at the high confinement limit. The production efficiency increases weakly with T_e due to a decrease of electron-ion collisionality.

Figure 3.11 plots the parametric curves of constant η_u and η_p , $\eta_u(B, \dot{m}) = \text{const}$ and $\eta_p(B, \dot{m}) = \text{const}$ respectively, at two levels of $T_e = 10, 20$ eV. These figures determine the optimum operational range, in terms of B and $\dot{m}$. To achieve a high η_u it is necessary to operate above a certain threshold of both B and $\dot{m}$. For T_e increasing, the optimal values of B and $\dot{m}$ decrease. Note also that at constant B , if $\dot{m}$ increases, η_u increases but η_p decreases. This is due to the loss of magnetic confinement because of the increasing of collisions (higher neutral-ion density) at larger $\dot{m}$.

Figure 3.10 – Parametric investigation of chamber performances. (a) Propellant utilization, $\eta_u(L/L_*)$, at the high magnetized plasma limit, i.e. ideal scaling law. (b) Propellant utilization $\eta_u(B, T_e)$ and (c) production efficiency $\eta_p(B, T_e)$, at $\dot{m} = 0.1$ mg/s, by using the two-dimensional model of the HPS.

The correct screening of the lateral wall make losses there negligible. However, the lack of magnetic screening at the rear wall penalizes strongly η_p and thus thruster performances. Within this ideal magnetic topology, the plasma flow to the rear-wall is similar to the plasma ejected at the front side, and thus requires to be re-ionized. This large loss would be avoided by screening the back wall too. Magnetic screening of both the rear and lateral chamber walls is feasible by appropriate design of the magnetic circuit, by implementing either coils or permanent magnets. Nevertheless the plasma structure within this magnetic topology could not be solved by the current model. As a first approach, a quantitative assessment can be made at the limit of large local screening of the rear wall, by just assuming that the plasma backflow to that wall is negligible. This implies imposing the boundary condition $u_z(-L) = 0$ instead of $u_z(-L) = -c_s$. Figure 3.12(a) shows that the maximum density is at the back wall, indicating that the forward-flow region $u_z > 0$ occupies the whole chamber. The screening of the rear-wall allows to double the production efficiency, and theoretically this would approach 100 %, as shown in Figure 3.12(b).

Figure 3.11 – Constant-level contours of propellant utilization and production efficiency in the parametric plane $\dot{m} - B$ at $T_e = 10$ eV (a) and 20 eV (b).

Figure 3.12 – Effects of magnetic screening of the back-wall: (a) Axial profile of plasma density and (b) production efficiency versus axial magnetic field. Solid and dashed lines are for $u_{zB}/c_s = -1$ and 0, respectively, modeling zero and total magnetic screening.

3.4 Evaluation of Thrust

In order to estimate the HPT thrust, it is necessary to deal also with the plasma expansion at the MN stage. Firstly, this section deals with the matching of the previous solution with the solution of the DIMAGNO code. Then, thrust contributions are identified and analysed in depth.

3.4.1 Matching chamber and nozzle models

The two-dimensional chamber model can be matched now to the solution of the model for the plasma expansion in the divergent nozzle of Ahedo and Merino [108]. This MN model assumes a collisionless, and slightly supersonic plasma at the throat (E section of the HPS). This can be only assumed if a high $\eta_u \approx 1$ and a hot plasma, $T_e > 10$ eV, is produced at the HPS. Both, the collisionless and the supersonic condition, $M_i = u_{iZE}/c_s \approx 1.01$, cause a mismatch

with the conditions of the plasma at the section E according to the previous HPS model. Another mismatch appears due to the limit $\ell_e/R \rightarrow 0$ assumed in the nozzle model. This states that the thin inertial layer next to the chamber lateral wall is neglected or does not exist at the nozzle plasma/vacuum edge. In the results presented here this means a 2-3 % loss in mass flow. The overall mismatching in terms of other properties, momentum, energy, etc., could be around 5 %. The shape of the wall-less magnetic nozzle, $r = R_V(z)$ with $R_V(0) = R$, sketched in Fig. 3.1, corresponds to the plasma/vacuum edge V. Two-dimensional profiles of the supersonic plasma expansion are discussed in Ref. [17].

The nozzle model cannot be extended into the far-downstream region because two important issues, the plasma/nozzle detachment [109] and the uncertainties in the electron thermodynamics are not solved yet. In order to close the problem in a reasonable way, an isolated plate, labelled as P in Fig. 3.1, will mark here the end of the nozzle region. The plate is located at a distance L_n from the chamber exit and collects the plasma beam, without recombining it. Surface D in Fig. 3.1 is the edge of the Debye sheath developing in front the plate. Observe that the plate is not merely an artifact, it could model a material surface for processing [110], a plasma momentum flux sensor for indirect thrust measurement [111, 112], or the downstream wall of the vacuum chamber.

B_0 and $\dot{m}$ have an important role on the chamber performances, but they have a minor role on the plasma expansion in the nozzle. Consequently, B_0 and $\dot{m}$ are fixed to their nominal values of 600 G and 0.1 mg/s, and the discussion of thruster performances is focused on the the influence of T_e and the nozzle length L_n .

3.4.2 Thrust contributions

Adding for the three species, the momentum flux equation of the whole plasma is,

$$\nabla \cdot \bar{\bar{M}} = e(n_e - n_i)\nabla\phi - en_e\mathbf{u}_e \times \mathbf{B}, \quad (3.55)$$

in which $\bar{\bar{M}}$ is the plasma momentum flux tensor,

$$\bar{\bar{M}} = \sum_{j=i,e,n} \left(m_j n_j \mathbf{u}_j \mathbf{u}_j + p_j \bar{\bar{I}} \right). \quad (3.56)$$

Then, the axial momentum flow across section z is,

$$F_z(z) = 2\pi \int_0^{R_V(z)} dr r M_{zz}(z, r). \quad (3.57)$$

Physically, the thrust F is the net backwards force exerted by the whole plasma on the thruster. This axial force is the sum of three different contributions:

$$F = F_{pres} - F_{elec} + F_{mag}. \quad (3.58)$$

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F_{pres} is the axial dynamic pressure of the plasma at the chamber walls, F_{elec} is the axial electric force between the positive electric charge in the sheath AB and the negative electric charge at the back wall. F_{mag} is the axial magnetic force of the azimuthal plasma current on the thruster magnetic circuit. Concerning F_{pres} , this is computed as follows,

$$F_{pres} = F_{zA} - D_W, \text{ with} \quad (3.59)$$

$$F_{zA} = F_z(-L), \quad (3.60)$$

$$D_W = \pi R^2 m_i \int_{-L}^0 dz n_z v_w (u_{iz} - \alpha_w u_n). \quad (3.61)$$

F_{zA} is the axial momentum flow to the wall A, and D_W is the loss of axial momentum at the lateral wall. The electric force obeys the following equation,

$$F_{elec} = \pi \epsilon_0 \int_0^R dr r \left(\frac{d\phi}{dz} \right)_A^2. \quad (3.62)$$

The last contribution, the magnetic force, is computed as the reaction force of the applied magnetic field on the plasma azimuthal currents j_θ integrated over all the volume of the plasma plume,

$$F_{mag} = 2\pi \int_0^{L_n} dz \int_0^{R_V(z)} dr r (-j_\theta) B_r. \quad (3.63)$$

Considering the HPS isolated from the MN, the chamber or internal thrust [104] could be defined as,

$$F_{cham} = F_{pres} - F_{elec}. \quad (3.64)$$

The magnetic thrust F_{mag} , for the current topology without a magnetic choke, is exclusively external thrust. Particularizing $F_z(z)$ at sections A, B, E, and D yields the relations between the plasma momentum flow and the different contributions to thrust:

$$\begin{aligned} F_{zB} &= F_{zA} - F_{elec}, \\ F_{zE} &= F_{zB} - D_W = F_{cham}, \\ F_{zD} &= F_{zE} + F_{mag} = F. \end{aligned} \quad (3.65)$$

The chamber thrust depends on the plasma temperature, $F_{cham}(T_e)$. For $B = 600$ G, when lateral wall screening is large, the plasma ‘drag’ on the lateral wall is negligible: $D_w/F_{cham} \simeq 0.01$. On the contrary, the negative contribution of the electric force is significant: taking $F_{pres} \simeq F_{zA}$ and the well-known Debye sheath solution for a floating wall, the ratio F_{elec}/F_{pres} becomes,

$$\frac{F_{elec}}{F_{pres}} \simeq 1 - \frac{F_{zB}}{F_{zA}} \simeq 1 - \frac{2}{\sqrt{1 + \ln(m_i / (2\pi m_e))}}. \quad (3.66)$$

$F_{elec}/F_{pres} = 0.38$ for argon. If wall secondary-electron emission is important, F_{elec} decreases but F_{cham} does not change. The magnetic thrust depends on both T_e and L_n , κ_F measures the increase of external thrust against the chamber thrust,

$$\kappa_F = F_{mag}/F_{cham}. \quad (3.67)$$

Figure 3.13 – (a) Thrust gain κ_F and (b) ion-power gain κ_P , both against the dimensionless nozzle length L_n/R .

This is depicted in Figure 3.13(a) (κ_{noz} in Ref. [109]). κ_F is almost independent of the plasma temperature T_e and it is monotonic with L_n due to the MN isothermal model considered here. Consequently, the total thrust can be written as,

$$F(T_e, L_n) \simeq F_{cham}(T_e) [1 + \kappa_F(L_n)]. \quad (3.68)$$

Figure. 3.14(a) plots the thrust of the simulated thruster versus the nozzle length and the plasma temperature. Recent experimental measurements on a HPT [27] yield values of κ_F about 0.4-0.7 which agrees well with the results of Fig. 3.13(a).

It is worth to observe that the net force exerted by the plasma beam on the downstream plate P, F_{plate} , is the dynamic pressure on the plate *minus* the electric force due to positive electric charge in the adjacent Debye sheath,

$$F_{elec,P} = \pi \epsilon_0 \int_0^{R_V(L_n)} dr \, r \left(d\phi/dz \right)_P^2. \quad (3.69)$$

Then, the force on the plate is written as follows,

$$F_{plate} = F_{zP} + F_{elec,P} = F_{zD} = F. \quad (3.70)$$

This equivalence between the thrust on the thruster and the plasma force on a downstream plate has been validated experimentally, the average discrepancy being a 2 % [112]. Notice that the equivalence is valid as long as (i) the plate presence does not modify substantially the upstream plasma beam, and (ii) there is no thrust contribution of the beam downstream of the plate location. This last condition requires to know well the plasma behavior far-downstream, which is still and open problem.

The monotonic behavior of $\kappa_F(L_n)$ means that, for given T_e , the total thrust and the plasma momentum flow increase with the length of the nozzle region, Fig. 3.14(a).

3.5 Thrust and internal efficiency

Following the same order as in the previous section, this section presents first the energy balance, identifying each contribution, and then analyses the global and partial HPT efficiencies.

3.5.1 Energy balance

The energy equation determines the plasma temperature T_e in terms of the plasma absorbed power P_a , which is the dominant contribution to the energy balance of the discharge.

The assumption of electron isothermality has the advantage that a *global* energy balance relates easily P_a to the rest of discharge parameters. The discussion of the energy balance will be restricted here to the relation among P_a , T_e and L_n , for given values of B , $\dot{m}$, R , and L .

The energy equation for the plasma, grouping contributions from all species can be expressed as,

$$\nabla \cdot \dot{\mathbf{P}} = \mathbf{j} \cdot \mathbf{E} + \dot{P}_a - \dot{P}_{ion}. \quad (3.71)$$

$\dot{\mathbf{P}}$ is the plasma power density flux, $\dot{P}_a$ is the absorbed power density from the RF wave, and $\dot{P}_{ion}$ groups energy losses due to ionization and excitation. $\dot{\mathbf{P}}$ and $\dot{P}_{ion}$ are computed as follows,

$$\dot{\mathbf{P}}(z, r) = \frac{n_n}{2} m_i u_n^2 \mathbf{u}_n + \frac{n}{2} [m_i u_i^2 \mathbf{u}_i + (m_e u_{e\theta}^2 + 5T_e) \mathbf{u}_e] + \mathbf{q}_e, \quad (3.72)$$

$$\dot{P}_{ion} = E'_{ion} n n_n R_{ion} \equiv \nabla \cdot (E'_{ion} n \mathbf{u}_i), \quad (3.73)$$

$\mathbf{q}_e$ the electron heat flux. In the MN, the contribution of neutrals to $\dot{\mathbf{P}}$, is negligible and the contribution of the electron azimuthal energy $m_e u_{e\theta}^2/2$ must be kept small for the nozzle model being consistent [17, 113]; in the results presented here it remains below 10 %. $E'_{ion}(T_e)$ is the effective ionization energy defined in the Appendix B.

Considering that there are not space charge effects in the MN stage $\nabla \cdot \mathbf{j} = 0$, the work of the electric field satisfies:

$$\mathbf{j} \cdot \mathbf{E} = -\nabla \cdot (\phi \mathbf{j}). \quad (3.74)$$

Then, the energy equation Eq. 3.71 takes the conservation form,

$$\nabla \cdot [\dot{\mathbf{P}} + E'_{ion} n \mathbf{u}_i + \phi \mathbf{j}] = \dot{P}_a. \quad (3.75)$$

Integrating this equation over the whole plasma domain, limited by chamber walls A and W , the nozzle/vacuum edge V , and the downstream plate P , the energy conservation balance

can be expressed as

$$P_{ion} + P_{wall} + P_{beam} = P_a. \quad (3.76)$$

On the left-hand side, there are the contributions of ionization and radiation, wall losses, and the power of the ejected beam respectively,

$$\begin{aligned} P_{ion} &= E'_{ion} \dot{m}_{iT} / m_i, \\ P_{wall} &= P_W + P_A, \\ P_{beam} &= P_P. \end{aligned} \quad (3.77)$$

P_W, P_A and P_P are computed as follows,

$$\begin{aligned} P_W &= P_Q = 2\pi R \int_{-L}^0 dz \mathbf{1}_z \cdot \dot{\mathbf{P}}(z, R), \\ P_A &= P_B = -2\pi \int_0^R dr r \mathbf{1}_z \cdot \dot{\mathbf{P}}(z_B, r), \\ P_P &= P_D = P_E = 2\pi \int_0^{R_V(z_D)} dr r \mathbf{1}_z \cdot \dot{\mathbf{P}}(z_D, r). \end{aligned} \quad (3.78)$$

The equalities $P_A = P_B$, $P_Q = P_W$, and $P_D = P_P$ express that there is no energy spent by the current-free plasma in sheaths AB , QW , and DP , just an energy transfer from electrons to ions. Then, the equality $P_E = P_D = P_{beam}$ also means that there are no energy sources in the nozzle.

The chamber model determines $P_{ion}(T_e)$ and $P_{wall}(T_e)$. Then, the nozzle model states that the power of the plasma beam is uniform, and can be split into ion power and electron power,

$$P_{beam} = P_i(z) + P_e(z) = \text{const}, \quad (3.79)$$

$$P_i(z) = \pi \int_0^{R_V(z)} dr r n m_i u_i^2 u_{iz}, \quad (3.80)$$

$$P_e(z) \simeq \pi \int_0^{R_V(z)} dr r (5T_e + 2q_{ez}). \quad (3.81)$$

Equation 3.79 means that there is a transfer of electron ‘internal’ energy into ion ‘kinetic’ energy along the nozzle. In Eq. 3.81 the assumption $m_e u_\theta^2 \ll 2T_e$ has already been taken on its derivation.

The nozzle model determines $P_i(z)$, with $P_i(0)$ corresponding to the sonic ion flow at section E. The ratio κ_P can be defined as,

$$\kappa_P = P_i(z) / P_i(0) - 1. \quad (3.82)$$

This is almost independent of T_e , and the parametric dependence of the ion power can be expressed as,

$$P_{iD}(T_e, L_n) \simeq P_{iE}(T_e) [1 + \kappa_P(L_n)]. \quad (3.83)$$

Remind that E refers to properties evaluated at the exit section (throat) and D is the cross

section at L_n , just at the sheath entrance of the downstream plate. Figure 3.13 (b) plots $\kappa_P(L_n)$, which is approximately proportional to the square of the local r -averaged Mach number in the nozzle; thus $\kappa_P = 10$ corresponds to a Mach number about $M_i \approx 3$. The electron energy flow is the sum of enthalpy and heat flows. Since it is an isothermal model, the enthalpy flow, $(5/2)T_e \dot{m}_{iE}/m_i$, is constant along the nozzle, and the non-zero electron heat flux, q_{ze} , can be determined directly only at the plate sheath edge D. The fluid-to-kinetic correspondence for energy fluxes of a near-Maxwellian population at the edge of a collisionless sheath according to Ref. [114],

$$\frac{5}{2}T_e n u_{ez} + q_{ez} = \left(2 + \frac{1}{2} \ln \frac{m_i}{2\pi m_e}\right) T_e n u_{ez}. \quad (3.84)$$

Thus P_{eD} can be evaluated as,

$$P_{eD}(T_e) = \left(2 + \frac{1}{2} \ln \frac{m_i}{2\pi m_e}\right) T_e \frac{\dot{m}_{iE}}{m_i}. \quad (3.85)$$

By replacing Eqs. 3.83, 3.85 in Eq. 3.79, the parametric dependence of the beam power becomes

$$P_{beam}(T_e, L_n) = P_{eD}(T_e) + P_{iE}(T_e)[1 + \kappa_P(L_n)]. \quad (3.86)$$

Let us consider that $P_{iE} \simeq T_e \dot{m}_{iE}/(2m_i)$, so the ratio P_{iE}/P_{eD} is written as,

$$P_{iE}(T_e)/P_{eD}(T_e) \simeq \left(4 + \ln \frac{m_i}{2\pi m_e}\right)^{-1}, \quad (3.87)$$

This means that at the chamber exit, only a small fraction of the plasma energy is deposited on the sonic ion flow, about one-fifteenth for argon.

The absorbed power P_a required to generate and expand a plasma of temperature T_e along a MN of length L_n , at given $\dot{m}$ and B , according to Eq. 3.76 is,

$$P_a = P_{ion}(T_e) + P_{wall}(T_e) + P_{beam}(T_e, L_n). \quad (3.88)$$

Figure 3.14 (b) plots $P_a(L_n)$ at different plasma temperatures. At any given T_e , the following equalities relate $P_a(L_n)$, $P_{iD}(L_n)$ to zero-length MN values $P_a(0)$, $P_{iD}(0)$, and $P_{eD}(0)$ to $P_{eD}(L_n)$

$$P_a(L_n) - P_a(0) = P_{iD}(L_n) - P_{iE} = P_{eD}(0) - P_{eD}(L_n). \quad (3.89)$$

This allows concluding that the increase of absorbed power with L_n is supplied to the beam at the chamber exit as an increase of the electron heat flow. Considering the inverse problem, if the absorbed power is kept constant, and the plate location is moved, it is evident from Fig. 3.14(b) that, as L_n increases, the plasma temperature must decrease. The consequences of having been taken this isothermal electron model are discussed below, after introducing partial efficiencies and specifically the expansion efficiency.

Figure 3.14 – Influence of nozzle length and plasma temperature on the HPT performances. Curves are obtained at $T_e = 10$ eV, 20 eV, and 30 eV, $B = 600$ G and $\dot{m} = 0.1$ mg/s. (a) thrust F , (b) required absorbed power P_a , (c, solid) thrust efficiency η , and (c, dashed) internal efficiency η_{int} .

3.5.2 Partial efficiencies

The thrust efficiency is defined as,

$$\eta = F^2 / 2 \dot{m} P_a. \quad (3.90)$$

F is computed according to $\kappa_F(L_n)$ (MN thrust) and the plasma momentum provided by the HPS model at the chamber exit, Eq. 3.68. P_a is evaluated according to the previous energy balance, Eq. 3.88.

Figure 3.14(c) shows that thrust efficiency is enhanced when either T_e or L_n are increased. Both cases imply an increase of the absorbed power. For the example considered here, thrust efficiency remains below 30 %. In order to understand this relatively modest performance figure, the different phenomena taking place in the plasma and how these affect the thrust efficiency are investigated. An alternative to the thrust efficiency, based on plasma beam properties, is the internal efficiency, defined by Sutton [6],

$$\eta_{int} = P_{izD} / P_a. \quad (3.91)$$

P_{izD} is the flow of ion axial kinetic energy at the final section D, and is computed as,

$$P_{izD} = \pi \int_0^{R_V(L_n)} dr r n m_i u_{iz}^3(L_n, r). \quad (3.92)$$

The two efficiencies, η and η_{int} coincide only if the beam expansion is complete. The internal

Figure 3.15 – Dependence of (solid lines) thrust and internal efficiencies and (dashed lines) ionization and wall losses with (a) T_e and (b) P_a , for $L_n = 30$ cm, $B = 600$ G, and $\dot{m} = 0.1$ mg/s.

efficiency can be factorized as,

$$\eta_{int} = \eta_{cham} \eta_{exp} \eta_{div}, \quad (3.93)$$

$$\eta_{cham} = P_{beam}/P_a \equiv 1 - \epsilon_{ion} - \epsilon_{wall}, \quad (3.94)$$

$$\eta_{exp} = P_{iD}/P_{beam}, \quad (3.95)$$

$$\eta_{div} = P_{ziD}/P_{iD}. \quad (3.96)$$

η_{cham} is the partial efficiency related to chamber processes, η_{exp} is related to the internal to kinetic energy conversion on the nozzle, and η_{div} is the beam or plume divergence. Let us consider that the nozzle efficiency, can be written as $\eta_{noz} = \eta_{exp} \eta_{div}$. $\epsilon_{ion} = P_{ion}/P_a$ and $\epsilon_{wall} = P_{wall}/P_a$, are the relative losses due to ionization/radiation and losses to the wall.

Figure 3.15 (a) shows for a long nozzle, $L_n/R = 30$, the dependence on T_e of ϵ_{ion} , ϵ_{wall} , η_{int} , and η . The two other efficiencies are independent of T_e , being $\eta_{exp} \simeq 0.44$ and $\eta_{div} \simeq 0.95$ according to the DIMAGNO results. The back wall contributes the most to ϵ_{wall} and its increase with T_e is due to $P_{wall} \propto T_e^{3/2}$. The decrease of ϵ_{ion} when T_e increases is due to the transition to the high-ionization region and the decrease of excitation/radiation losses. The difference between η and η_{int} for large T_e is due to the electron contribution to thrust not having a correspondence on η_{int} ; the negative difference at low T_e is due to the poor propellant utilization. Figure 3.15 (b) plots the same partial efficiencies versus the absorbed power instead of T_e . The difference between both groups of curves is summarized in the fact that increments of P_a are spent in ionizing more gas, at low power, and in heating the plasma, at high power.

The maximum thrust efficiency is below 30 %. The main causes reducing efficiency are summarized below. First, the thruster must operate in the high ionization regime (high η_u

is mandatory) and inelastic losses should be kept as low as possible. An idealized scenario would be about 10 %, according to Fig. 3.15 (b). Second, in this simplified model, a 30 % of the efficiency loss is due to energy losses at the chamber rear wall. Therefore, screening adequately that wall would yield $\eta_{cham} \sim 80$ % instead of 60 %. Third, other losses take place in the nozzle. For this short magnetic nozzle length $L_n/R = 30$, the beam divergence efficiency is quite high, $\eta_{div} \simeq 95$ %, but this may drop dramatically depending on the magnetic nozzle configuration, and existing uncertainties on the plasma detachment and electron thermodynamics response. The last efficiency loss is caused by the conversion of electron-to-ion energy, results from DIMAGNO states this is very poor $\eta_{exp} \simeq 44$ % (assuming isothermal and fully magnetized electrons). This result is very dependent on the hypotheses of the electron equation of state. An isothermal electron population has been postulated here on both sides, chamber and MN. This has been proposed in previous simplified analyses of MNs [16]. This allows the application of the Boltzmann relation on the plume expansion [115].

The choice of isothermal versus adiabatic has important consequences on the downstream plasma expansion, the energy balance, and the thruster internal efficiency. For an isothermal, collisionless magnetic nozzle, the ambipolar electric potential decreases with no bounds as L_n increases, the electron enthalpy flow is constant along the nozzle, while the electron heat flow at the chamber exit should increase as L_n becomes larger, in order to balance the total gain of ion kinetic energy in the nozzle. Consequently, a complete expansion with this model requires an infinite amount of energy coupled to the plasma upstream, and thus, it is not consistent. The electron to ion energy conversion efficiency is poor as well for the isothermal model.

In an adiabatic expansion of a collisional plasma, the ambipolar electric field tends to zero downstream. The electron heat flow at the chamber exit and within the nozzle is zero as well. The conversion of electron to ion energy is highly efficient, and ion energy gain is balanced by the loss of electron enthalpy. The beam power does not depend on the plasma expansion along the nozzle, $P_{beam}(T_e) \simeq 3T_{eE}\dot{m}_{iE}/m_i$ [103, 116].

Due to these uncertainties, Chapter 5 is devoted to investigate the electrons thermodynamics of a fully magnetized plasma, with a one-dimensional kinetic method.

3.6 Conclusions

In this chapter, a two dimensional fluid model of a collisional and magnetized plasma within a cylindrical dielectric chamber has been developed. Main processes, such as ionization, ion acceleration, plasma-wall losses have been analysed based on the design and operational

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parameters: chamber dimensions, mass flow rate of argon, magnetic field strength, and plasma temperature. Under the assumption of high magnetization, analytical solutions have been derived, and provide simple scaling laws for the plasma parametric response, which will be used in the preliminary design of a high power HPT in Chapter 7. Optimal values of design and operational parameters that maximize propellant utilization and production efficiency have been investigated.

The chamber model has been matched with the results of an existing nozzle model (DI-MAGNO), providing a complete picture of the processes of the plasma discharge in a helicon thruster: plasma generation, confinement, and acceleration. Thruster performances, such as thrust, power balance, and thruster efficiencies, assuming isothermal electrons, have been estimated. The analysis of the momentum and energy equations of the whole plasma has determined the thermal, electric, and magnetic contributions to thrust, the power conversion between ions and electrons within the chamber, and the power distribution among beam power, ionization losses, and wall losses.

The theoretical thrust efficiency has been estimated at about 30 %. Main losses comes from the poor screening of the chamber walls, mainly the rear wall for a nearly axial magnetic field, and the incomplete plasma expansion in the nozzle. Furthermore, the increase of the expansion efficiency depends too much on the electrons thermodynamics along the nozzle divergent geometry.

As a future work, the current model should be improved in order to overcome some limitations, the 1D magnetic topology in the chamber, the absence of double-charged ions or metastable atoms (which for Argon could be important in the expected range of temperatures). The use of hybrid schemes, instead of fluid ones, as the strategy followed in Chapter 4 could be a better solution.

4 Hybrid model of a plasma expansion in a diverging Magnetic Nozzle

The goal of this chapter is to explore the plasma expansion throughout a MN for different magnetization regimes, an important issue that cannot be analysed by simply fluid models. This study describes the 2D plasma structure, in terms of beam collimation, divergence, ion detachment, thrust generation, etc., for a broad range of magnetization regimes, from the highly magnetized and nearly collisionless regime to the weakly magnetized regime. This hybrid model is based on Particle-in-cell (PIC) methods for ions and neutrals, and an anisotropic fluid model for electrons.

4.1 Introduction

The hybrid model used here postulates electrons as an anisotropic fluid, while heavy particles, ions and neutrals, are given by a kinetic description, using a conventional particle-in-cell (PIC) method. This kind of methods have been optimized for several applications; among them stand out the pioneering work of Fife [117, 118] on the simulation of Hall Effect Thrusters (HET), followed by some improvements by Parra *et al.* [119]. Here, this kind of model is applied to an expanding plasma that flows through a divergent magnetic nozzle. The current model also includes some features described in Refs. [120],[121] for dealing with plasma-wall interaction. The core of the current model/code is based on the heritage of *HallMa* in the simulation of HETs. *HallMa* is the last version of the *HPHall* code, which is the simulation tool initially developed by Fife, and later, from the early 2000 until 2010 updated by the group of Prof. Ahedo, according to the works mentioned above. However, large modifications are required on this code to adapt it to the current MN problem, such as the magnetic topology, the fluid model boundary conditions and the injection algorithm at the MN throat. Some of these modifications were carried out in the Master Thesis of Navarro-Cavallé, Ref. [122]

Electrons are considered isothermal over the whole domain of the simulation. The recent work in Ref. [123] inquires into this issue and concludes that under the expected collisionless regime of a divergent magnetized plasma jet, the isothermal hypothesis clearly fails. But it can be still applied in the vicinity of the nozzle throat, i.e. the near plume. A non-isothermal electron formulation based on the results of Ref. [123] cannot be applied easily to the current model, mainly because of the disparity on some hypotheses, geometric constraints, etc.

The current work aims to simulate how plasma beam collimation, magnetic thrust, and nozzle efficiency behave due to variations of the electron magnetization regime. To achieve this goal, this model includes the cross field diffusion of electrons due to collisions. This model also reproduces the inward ion detachment as in Ref. [109]. In the mentioned reference, the plasma-vacuum edge, delimited by the magnetic field, set the domain boundary and did not allow to explore the transition of the plasma to the vacuum region. This smooth transition is tackled in this chapter.

This chapter is structured as follows. In the first place, the hybrid model is summarized stressing the key points for its application to the divergent magnetic nozzle. Secondly, a series of simulations through a divergent magnetic nozzle will be compared for different magnetization regimes. Section fourth is for the discussion of some specific phenomena. Finally, conclusions and future improvements are presented.

4.2 The magnetic nozzle hybrid model

An axisymmetric diverging magnetic field is considered. At the MN throat, a plasma source, Helicon plasma source for instance, feeds the MN with a hot plasma, which indeed presents sonic conditions there, as in a “*de Laval*” nozzle throat. The injected plasma is considered to be in radial equilibrium, an issue further discussed later, and related to the magnetization regime upstream. Thermal electron energy is converted into ion kinetic energy through the ambipolar electric field. The quasi-neutral condition is invoked everywhere, $n_i \sim n_e = n$.

The hybrid model is divided into two different parts, the particle-in-cell model for ions and neutrals, and the electron fluid model. The first one provides the plasma density n and the ion particle flux $\mathbf{g}_i = n\mathbf{u}_i$ or the ion current density $\mathbf{j}_i = en\mathbf{u}_i$. On the other side, the electron sub-model returns the perpendicular electric current $I_\perp$ through each magnetic surface and the thermalized electric potential ϕ_0 , which will be defined later and it is necessary to compute the electric potential ϕ . These two variables, ϕ_0 and $I_\perp$, are one-dimensional and constant along each magnetic surface and inputs for the PIC sub-model. Furthermore, plasma is

Figure 4.1 – Left: structured mesh on the r, z meridional plane (physical domain). Centre: sketch of the computational mesh, plane (ξ, η) . Right: detail of particle coordinates (ξ_p, η_p) in a cell of the computational mesh.

considered isothermal, so T_e is uniform everywhere and constant in time. Each sub-model will be discussed separately.

Following the PIC model [124] approach, ion and neutral particles are moved according to electromagnetic and inertial forces on a radial-axial structured mesh, which is indeed the lengthwise section of a cylindrical prism, and it is presented in Figure 4.1. An auxiliary rectangular mesh is used for weighting PIC algorithms to evaluate the different moments of the ion distribution function: density, particle fluxes, ion temperatures, etc.,. Weighting algorithms use a first-order method (bilinear interpolation method) for the calculation of the mentioned macroscopic properties. A *leap-frog* scheme is implemented to integrate particle trajectories. It results in a first-order model for particle velocities, and second-order for particle positions. Spatially, the model is 2D because it only retains the information on the radial-axial $r - z$ plane. But it is 3D in the velocity space, and retains radial, azimuthal and axial velocities.

4.3 The PIC model

The PIC model allows recovering the particle velocity distribution function necessary to obtain macroscopic fluid properties. This method requires tracking the evolution of a large number of particles and this is very challenging in terms of the computational cost. Let us consider for example an average particle density of $5 \cdot 10^{18} \text{ m}^{-3}$ in a cylindrical domain of radius 1 cm and length 10 cm (HPS size), this would require to deal approximately with more than 10^{14} particles. To reduce the computational cost this model considers clusters of particles, called macro-particles hereafter. Its mass is several orders of magnitude larger than an individual particle.

Figure 4.2 – Weighting algorithm sketches. Top left: nearest node or 0th-order weighting algorithm in a 1D geometry. Top right: linear weighting algorithm in a 1D geometry. Bottom: bilinear weighting algorithm in a 2D geometry, particle located at $(\xi, \eta) = (2.25, 2.75)$.

4.3.1 PIC meshes and weighting algorithms

The PIC sub-model requires to define a non-uniform structured mesh on the meridional plane (r, z) , as the one presented in Figure 4.1. This is defined during the pre-process time using the mesh generator tool of *TECPLOT*^{SV}, which implements elliptic algorithms to generate the mesh. Ions and neutrals are moved on this mesh according to their momentum equation.

However, the weighting algorithms, which are necessary to obtain macroscopic properties, such as density, particle fluxes, etc., are defined in a uniform and structured rectangular mesh (ξ, η) . There is a direct transformation between the nodes of this computational mesh, and the mesh on the physical plane, $(\xi, \eta) \rightarrow (r, z)$. Apart from that, the nodes of the computational mesh are always defined as integers (j, k) . This simplifies the weighting algorithms because the integer part of the particle coordinates $(Int(\xi_p), Int(\eta_p))$ identifies the corner (left side - bottom) of the occupied cell, $(j_p, k_p) \equiv (Int(\xi_p), Int(\eta_p))$. The rest of the corners in counter-clockwise direction are $(j_p + 1, k_p)$, $(j_p + 1, k_p + 1)$ and $(j_p, k_p + 1)$, as depicted in the sketch of Figure 4.1.

The current model uses a first order method for the weighting functions $S_{jk}(\xi, \eta)$. Figure 4.2 shows an sketch of several weighting algorithms: the 0th order weighting algorithm in a 1D geometry, which links the particle kinetic properties to the nearest node; the 1st order (or

linear) algorithm in a 1D geometry, which splits linearly particle properties into the two nodes closest to the particle; and finally, the bilinear weighting algorithms in a 2D geometry, which is the one used in the current work and distributes particle properties between the four nodes that surround the particle (cell corners). The bilinear weighting function $S_{jk}(\xi, \eta)$ is defined as follow,

$$\begin{aligned} S_{j_p, k_p} &= (1 - \xi_{p, rel})(1 - \eta_{p, rel}), \\ S_{j_p+1, k_p} &= \xi_{p, rel}(1 - \eta_{p, rel}), \\ S_{j_p+1, k_p+1} &= \xi_{p, rel}\eta_{p, rel}, \\ S_{j_p, k_p+1} &= (1 - \xi_{p, rel})\eta_{p, rel}, \end{aligned} \quad (4.1)$$

in which $\xi_{p, rel} = \xi_p - j_p$ and $\eta_{p, rel} = \eta_p - k_p$. According to this, the weighting of any property χ at the node (j, k) is obtained adding the contribution of all particles p of the kind α (ions or neutrals)

$$\chi_{jk} = \frac{1}{\Delta V_{jk}} \sum_p \chi_p S_{j,k}(\xi_p, \eta_p). \quad (4.2)$$

V_{jk} is the differential volume attributed to this node, and it is obtained by the revolution of the cross section Ω_{jk} around the axis. Ω_{jk} is the sub-domain on the plane (r, z) in which the weighting function $S_{jk}(\mathbf{x})$ is not null.

$$\Delta V_{jk} = \int_{\Omega_{jk}} 2\pi r S_{jk}(\mathbf{x}) d\Omega. \quad (4.3)$$

Accounting for all the expressions listed in the Appendix C, all macroscopic quantities can be defined according to them and implemented properly in Eq. 4.2.

The ion/neutral density at the node (j, k) is $n_{\alpha, jk}$,

$$n_{\alpha, jk} = \frac{1}{\Delta V_{jk}} \sum_p \frac{m_p}{m_\alpha} S_{jk}(\mathbf{x}_p). \quad (4.4)$$

in which m_p is the macro-particle mass, while m_α is the singly ion/neutral mass. The sum include all macro-particles p of kind α .

The ion/neutral particle flux $\mathbf{g}_{\alpha, jk}$,

$$\mathbf{g}_{\alpha, jk} = \frac{1}{\Delta V_{jk}} \sum_p \frac{m_p}{m_\alpha} \mathbf{v}_p S_{jk}(\mathbf{x}_p). \quad (4.5)$$

The second order tensor of ion/neutral momentum $\bar{\bar{P}}_{\alpha, jk}$,

$$\bar{\bar{P}}_{\alpha, jk} = \frac{1}{\Delta V_{jk}} \sum_p m_p \mathbf{v}_p \mathbf{v}_p S_{jk}(\mathbf{x}_p). \quad (4.6)$$

The average flux of total energy $\mathbf{e}_{\alpha, jk}$,

$$\mathbf{e}_{\alpha, jk} = \frac{1}{\Delta V_{jk}} \sum_p (1/2) m_p v_p^2 \mathbf{v}_p S_{jk}(\mathbf{x}_p). \quad (4.7)$$

Figure 4.3 – Leap-frog numerical integration scheme.

4.3.2 Particle trajectory integration

The motion of ion macro-particles is obtained by integration of the Newton equation presented below.

$$m_p \frac{d^2 \mathbf{x}_p}{dt^2} = \mathbf{F}(\mathbf{x}, \mathbf{v}, t) = q_p (\mathbf{E}(\mathbf{x}_p, t) + \mathbf{v}_p \times \mathbf{B}(\mathbf{x}_p, t)), \quad (4.8)$$

$q_p = e$ is the particle charge. The force term on the right hand side only includes the electromagnetic force or Lorentz force, since the exchange of momentum due to collisions is not considered here. In the current PIC model macro-particles collisions are treated macroscopically, this means that their motion is perturbed accounting for the averaged properties of all particles contained in the same simulation *cell*. This topic is out of the scope of this study. Integrating the last equation, the particle velocity $\mathbf{v}$ and its position $\mathbf{x}$ can be determined at any time.

Regarding neutrals, since these are not influenced by any electromagnetic force, they will describe ballistic trajectories. As aforementioned, collisions between heavy species could be analysed macroscopically. However, in all the simulations presented in this study, these effects are disregarded. Consequently, some mechanisms such as CEX or ion-neutral elastic collisions are not considered at all. Assuming an electron temperature of 10 eV using Argon, the rate of electron-neutral collisions is $R_{en} \sim 4.23 \cdot 10^{-13} \text{ m}^3/\text{s}$ [125], while the rate of CEX collisions is $R_{CEX} \sim 2.77 \cdot 10^{-15} \text{ m}^3/\text{s} \ll R_{en}$, considering that the relative ion-neutral velocity is on the order of the ion acoustic speed c_s . This allows excluding CEX collisions.

The integration scheme implemented in this model is the *leap-frog* (LF), which is commonly used because, apart from its simplicity, it ensures the preservation of the total energy of the system of particles. The particularity of this method is that particle velocity and position are known with an offset in time. Position is known at the instants $t, t + \Delta t, \dots$, while velocity is given at $t + \Delta t/2, t + 3\Delta t/2, \dots$. For simplicity, a sketch of the LF numerical scheme is depicted in Figure 4.3. Truncation errors are $O(\Delta t)^2$ and $O(\Delta t)^4$, for velocity and position respectively, as described in Ref. [126]. However, this approach is not implemented exactly in the current

version of the code and in any previous version of *HallMa* [127, 119], and the error incurred is larger than in the original LF scheme.

According to the LF scheme and Eq. 4.8, particle velocity and position should be computed as follows,

$$\mathbf{v}_{p,+1/2} = \mathbf{v}_{p,-1/2} + \Delta t \cdot \mathbf{f}(\mathbf{x}_{p,0}, \mathbf{v}_{p,0}), \quad (4.9)$$

$$\mathbf{x}_{p,+1} = \mathbf{x}_{p,0} + \Delta t \cdot \mathbf{v}_{p,+1/2}. \quad (4.10)$$

The term $\mathbf{f}(\mathbf{x}_{p,0}, \mathbf{v}_{p,0})$ is the electromagnetic force per unit mass m_p , acting on the particle p at the position $\mathbf{x}_{p,0}$ (instant t_0 or 0). The time offset between position and velocity must be accounted for in the weighting algorithms requiring the estimation of particle position at $t_0 + \Delta t/2$, and particle velocity at t_0 , to reach consistent results. Subscripts 0, 1/2 and 1 denote the time t_0 , $t_0 + \Delta t/2$ and $t_1 = t_0 + \Delta t$ respectively.

Regarding the cylindrical geometry of the problem, the error incurred in the position increases when approaching the axis. To solve this problem, Ref. [124] proposes the use of a reference frame intrinsic to the particle. However, the evolution of this frame depends on both, position and velocity, which are defined at different times. Operating accurately, the particle displacement $\Delta \mathbf{x}_0$ projected on the t_0 reference frame $\{\mathbf{1}_r, \mathbf{1}_\theta, \mathbf{1}_z\}^{(0)}$ is,

$$\Delta \mathbf{x}^{(0)} = (\Delta x_r^{(0)}, \Delta x_\theta^{(0)}, \Delta x_z^{(0)}) = \Delta t \cdot \left(v_{r,+1/2}^{(0)} \mathbf{1}_r^{(0)} + v_{\theta,+1/2}^{(0)} \mathbf{1}_\theta^{(0)} + v_{z,+1/2}^{(0)} \mathbf{1}_z^{(0)} \right), \quad (4.11)$$

Note that velocity at $t_0 + \Delta t/2$ (+1/2) should be also projected on the reference frame at t_0 . Reference frame at t_0 and t_1 are denoted hereafter by superscripts (0) and (1). Up to here the model is the one described by Cartwright *et al* [126]. Below, the numerical scheme implemented in *HallMa* and in the current model is further detailed. The main difference in comparison to the commented LF scheme is that there is not time offset between particle position and velocity, yielding an increase of the truncation error. Also, the azimuthal velocity is written in terms of the angular momentum $v_\theta = h_\theta / x_r$. The new position $\mathbf{x}_1^{(1)}$ and velocity $\mathbf{v}_1^{(1)}$ are given by the following set of equations. First, the acceleration term $\mathbf{a}_0^{(0)}$ is computed according to the force $\mathbf{f}(\mathbf{x}_0^{(0)}, \mathbf{v}_0^{(0)})$ which is considered constant in a time step Δt . For a singly charged ion it yields,

$$a_{z0}^{(0)} = \frac{f_{z0}^{(0)}}{m_i} = \frac{e}{m_i} \left(E_{z0}^{(0)} - \left(h_{\theta 0}^{(0)} / x_{r0}^{(0)} \right) B_{r0}^{(0)} \right), \quad (4.12)$$

$$a_{r0}^{(0)} = \frac{f_{r0}^{(0)}}{m_i} = \frac{e}{m_i} \left(E_{r0}^{(0)} + \left(h_{\theta 0}^{(0)} / x_{r0}^{(0)} \right) B_{z0}^{(0)} \right), \quad (4.13)$$

$$a_{\theta 0}^{(0)} = \frac{f_{\theta 0}^{(0)}}{m_i} = \frac{e}{m_i} \left(v_{z0}^{(0)} B_{r0}^{(0)} - v_{r0}^{(0)} B_{z0}^{(0)} \right). \quad (4.14)$$

The displacement vector $\Delta \mathbf{x}^{(1)}$ is obtained with the following expressions:

$$\Delta x_z^{(1)} = \Delta x_z^{(0)} = \left(v_{z0}^{(0)} + 0.5 a_{z0}^{(0)} \Delta t \right) \Delta t, \quad (4.15)$$

$$\Delta x_r^{(0)} = \left(v_{r0}^{(0)} + 0.5 a_{r0}^{(0)} \Delta t \right) \Delta t, \quad (4.16)$$

$$\Delta x_\theta^{(0)} = \left(h_{\theta 0}^{(0)} / x_{r0}^{(0)} + 0.5 a_{\theta 0}^{(0)} \Delta t \right) \Delta t. \quad (4.17)$$

New positions at $t_1 = t_0 + \Delta t$ are given by next equations,

$$x_{z1}^{(1)} = x_{z0}^{(0)} + \Delta x_z^{(0)}, \quad (4.18)$$

$$x_{r1}^{(1)} = \sqrt{\left(x_{r0}^{(0)} + \Delta x_{r0}^{(0)} \right)^2 + \left(\Delta x_\theta^{(0)} \right)^2}, \quad (4.19)$$

$$x_{\theta 1}^{(1)} = x_{\theta 0}^{(0)} + \Delta x_\theta^{(0)}, \text{ and } \theta_1 = \theta_0 + \arcsin \left(\frac{\Delta x_\theta^{(0)}}{x_{r1}^{(1)}} \right). \quad (4.20)$$

Finally, new velocities are computed and projected on the new frame (1),

$$v_{z1}^{(1)} = v_{z1}^{(0)} = v_{z0}^{(0)} + a_{z0}^{(0)} \Delta t, \quad (4.21)$$

$$v_{r1}^{(1)} = \left(v_{r0}^{(0)} + a_{r0}^{(0)} \Delta t \right) \frac{x_{r0}^{(0)} + \Delta x_r^{(0)}}{x_{r1}^{(1)}} + \left(h_{\theta 0}^{(0)} / x_{r0}^{(0)} + a_{\theta 0}^{(0)} \Delta t \right) \frac{\Delta x_\theta^{(0)}}{x_{r1}^{(1)}}, \quad (4.22)$$

$$h_{\theta 1}^{(1)} / x_{r1}^{(1)} = \left(h_{\theta 0}^{(0)} / x_{r0}^{(0)} + a_{\theta 0}^{(0)} \Delta t \right) \frac{x_{r0}^{(0)} + \Delta x_r^{(0)}}{x_{r1}^{(1)}} - \left(v_{r0}^{(0)} + a_{r0}^{(0)} \Delta t \right) \frac{\Delta x_\theta^{(0)}}{x_{r1}^{(1)}}. \quad (4.23)$$

Once the particle position is determined in the physical grid ($r - z$), it is necessary to determine its new position on the numerical mesh ($\xi - \eta$), where weighting functions are defined as aforementioned. However, the well-known transformation that is shown next, is just the inverse of the one required,

$$\begin{aligned} z_p &= z_p(\xi_p, \eta_p) \\ &= z(j_p, k_p) S_{j_p, k_p}(\xi_p, \eta_p) + z(j_p + 1, k_p) S_{j_p+1, k_p}(\xi_p, \eta_p) + \\ &\quad z(j_p + 1, k_p + 1) S_{j_p+1, k_p+1}(\xi_p, \eta_p) + z(j_p, k_p + 1) S_{j_p, k_p+1}(\xi_p, \eta_p), \end{aligned} \quad (4.24)$$

$$\begin{aligned} r_p &= r_p(\xi_p, \eta_p) \\ &= r(j_p, k_p) S_{j_p, k_p}(\xi_p, \eta_p) + r(j_p + 1, k_p) S_{j_p+1, k_p}(\xi_p, \eta_p) + \\ &\quad r(j_p + 1, k_p + 1) S_{j_p+1, k_p+1}(\xi_p, \eta_p) + r(j_p, k_p + 1) S_{j_p, k_p+1}(\xi_p, \eta_p), \end{aligned} \quad (4.25)$$

These expressions, determine the position $\mathbf{x}_p$ by using the weighting function S , and the coordinates of the four nodes that define the cell: $z(j_p, k_p)$, $r(j_p, k_p)$, $z(j_p + 1, k_p)$, so $\mathbf{x}_p = \mathbf{x}_p(S(\xi_p, \eta_p))$. To solve the inverse problem, $(\xi_p, \eta_p) = G(r_p, z_p)$, this is linearised around (ξ_{p0}, η_{p0}) , and (ξ_{p1}, η_{p1}) is determined iteratively through the Newton's algorithm.

Defining the objective coordinates (ξ_p^*, η_p^*) , and knowing the real coordinates at t_0 , (z_{p0}, r_{p0}) and at t_1 , (z_{p1}, r_{p1}) , it is possible to determine $(\Delta \xi, \Delta \eta) = (\xi_{p1} - \xi_{p0}, \eta_{p1} - \eta_{p0})$ inverting the

following system:

$$\begin{bmatrix} \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} \\ \frac{\partial r}{\partial \xi} & \frac{\partial r}{\partial \eta} \end{bmatrix} \begin{bmatrix} \Delta \xi \\ \Delta \eta \end{bmatrix} = \begin{bmatrix} z_{p1} - z_{p0} \\ r_{p1} - r_{p0} \end{bmatrix}, \quad (4.26)$$

Then, the new approach for (ξ, η) is (ξ^*, η^*)

$$\begin{bmatrix} \xi_p^* \\ \eta_p^* \end{bmatrix} = \begin{bmatrix} \xi_{p0} + \Delta \xi \\ \eta_{p0} + \Delta \eta \end{bmatrix}. \quad (4.27)$$

Iteratively $(\xi_{p0}, \eta_{p0}) = (\xi_p^*, \eta_p^*)$ is taken, and $(z_{p0}(\xi_{p0}, \eta_{p0}), r_{p0}(\xi_{p0}, \eta_{p0}))$ is recalculated according Eqs. 4.24 , 4.25. The iterative procedure (Eqs. 4.26 , 4.27) is repeated until the error ϵ is much more smaller than a certain tolerance. The error in this case is computed as following,

$$\epsilon = \sqrt{(z_{p1} - z(\xi^*, \eta^*))^2 + (r_{p1} - r(\xi^*, \eta^*))^2}. \quad (4.28)$$

4.3.3 PIC Boundary conditions

In this subsection the conditions at the boundaries of the PIC domain are described. It includes the injection of ions at the MN throat, the plasma wall recombination at the lateral dielectric wall, and the discussion of particle integration difficulties at the axis of symmetry and at the plasma-vacuum edge. Ions or neutrals that cross other boundaries, such as the one at the top and the one downstream, which are “free” or “vacuum” boundaries, are lost and excluded in later time.

MN throat - Particle injection

At the MN throat, the plasma density profile $n(r)$ is defined according the radial equilibrium of the plasma at the source upstream. More details on this equilibrium will be addressed in Section 4.4. In the current section the description is more focused on the algorithm implemented to inject macro-particles at the throat. Assuming sonic conditions there, the injected particle flux is given by $g(r) = n(r)c_s$. The ion mass flow rate at the throat is obtained integrating radially the particle flux,

$$\dot{m} = m_i c_s \int_0^R 2\pi r n(r) dr. \quad (4.29)$$

However, this continuous radial profile $g(r)$ has to be discretized and the amount and initial position of macro-ions evaluated. A fraction of the total (ion) mass injected in each PIC time step $\dot{m}\Delta t$ is assigned to each panel that forms the MN throat, $\Delta m_i(\xi = 1, \eta)$. This can be approximated by,

$$\Delta m_i(\xi = 1, \eta) = \bar{g}_i(\eta) A_\eta m_i \Delta t, \quad (4.30)$$

$\bar{g}_i$ is the averaged particle flux at the panel labelled by η (lower node), $A_\eta(\eta) = \pi(r^2(\eta+1) - r^2(\eta))$ is the panel surface (circular ring) on the physical mesh, and Δt the PIC time-step, $\Delta t \sim 5 \cdot 10^{-8}$ s usually.

In order to ensure a certain uniformity of the macro-ion mass, the size of new particles is determined by the panel with the smallest fraction of injected mass, $\min(\Delta m_i(\eta))$, $\forall \eta \in [1, \eta_t - 1]$. η_t is the node that defines the upper end of the throat. The number of macro-particles injected through the panel of minimum mass, is set as an input parameter, M . Then, all injected macro-ions have the same size: $m_I = \min(\Delta m_i(\eta)) / M$.

In all other throat panels, the number of injected macro-ions per panel M_η is set accordingly to the following criteria:

$$M_\eta = \text{Int}\left(\frac{\Delta m_i(\eta)}{m_I}\right) + \delta M$$

$$\delta M = \begin{cases} 1 & \text{if } P < \frac{\Delta m_i(\eta)}{m_I} - \text{Int}\left(\frac{\Delta m_i(\eta)}{m_I}\right) \\ 0 & \text{if } P \geq \frac{\Delta m_i(\eta)}{m_I} - \text{Int}\left(\frac{\Delta m_i(\eta)}{m_I}\right) \end{cases} \quad (4.31)$$

P is a random number taken from a uniform distribution. This law ensures that on average the mass assigned to each panel is the one given by Eq. 4.30.

Once m_I and M_η have been set, it is necessary to define the initial position and the initial velocity of each macro-particle. Both components of the initial position (ξ_{p0}, η_{p0}) are defined in the computational mesh. $\xi_{p0} = 1 + \delta\xi$ and $0 < \delta\xi < 1$ is a small offset that is set randomly. The η_{p0} coordinate is determined arbitrarily along each single panel. In comparison to the method implemented in *HallMa* to inject neutral particles, in that work the radial position was also determined randomly but along all the injection section, without distinguishing between panels. In the current work, this would not be a good approach because both, it does not allow to reproduce radially varying profiles $g_i(r)$, and it introduces large errors due to not considering radial effects. After defining (ξ_{p0}, η_{p0}) , (r_0, z_0) is determined by using the weighting/interpolation functions $S_{jk}(\xi_p, \eta_p)$ (Eqs. 4.24 and 4.25).

Finally, the algorithm to set the initial velocity $\mathbf{u}_{i0}$ of injected macro-ions is the one proposed in Refs. [126, 124]. The particle velocity $\mathbf{u}_{i0}$ is split into the axial component u_{z0} (perpendicular to the panel) and the component on the transversal plane $r - \theta$, $\mathbf{u}_{t0}$, thus $\mathbf{u}_{i0} = u_{z0}\mathbf{1}_z + \mathbf{u}_{t0}$. For the axial component u_{z0} , a drifting Maxwellian flux is considered, while $\mathbf{u}_{t0}$ is set from a full Maxwellian. Note that the axial drift velocity at the throat, $u_d = c_s$, is much more larger than the ion thermal speed. So the distribution shapes as a quasi mono-energetic beam like distribution as shown in Figure 4.4. The parameter T_e/T_i plays an important role defining the

Figure 4.4 – Left: Sketch of the MN throat panels. Right: normalized distribution function of the axial velocity of injected particles $f_{iz}(u_{z0}/c_s)$.

width of the ion velocity distribution function $f_i(u_{z0}, \mathbf{u}_{t0})$.

$$f_i(u_{z0}, \mathbf{u}_{t0}) = \frac{\frac{m_i n_0}{k T_i} \exp\left(-\frac{m_i (u_{z0} - u_d)^2}{2k T_i}\right)}{\exp\left(-\frac{m_i u_d^2}{2k T_i}\right) + \frac{u_d}{\sqrt{2k T_i / m_i}} \sqrt{\pi} \operatorname{erfc}\left(\frac{-u_d}{\sqrt{2k T_i / m_i}}\right)} \times \\ \times \frac{m_i}{2\pi k T_i} \exp\left(-\frac{m_i \|\mathbf{u}_{t0}\|^2}{2k T_i}\right), \\ u_{z0} \in [0, \infty[\text{ and } \mathbf{u}_{t0} \in \mathbb{R}^2, \quad (4.32)$$

n_0 is the reference particle density. Then, normalizing the axial part with the ion sound speed c_s , $\hat{u}_{z0} = u_{z0}/c_s$, and $\hat{u}_d = u_d/c_s = 1$,

$$f_i(\hat{u}_{z0}, \mathbf{u}_{t0}) = f_{iz}(\hat{u}_{z0}) \frac{m_i}{2\pi k T_i} \exp\left(-\frac{m_i \|\mathbf{u}_{t0}\|^2}{2k T_i}\right), \text{ with} \\ f_{iz}(\hat{u}_{z0}) = \frac{n_0 \frac{T_e}{T_i} \hat{u}_{z0} \exp\left(-\frac{T_e}{2T_i} (\hat{u}_{z0} - 1)^2\right)}{\exp\left(-\frac{T_e}{2T_i}\right) + \sqrt{\frac{\pi T_e}{2T_i}} \operatorname{erfc}\left(-\sqrt{\frac{T_e}{2T_i}}\right)}, \text{ and} \\ \hat{u}_{z0} \in [0, \infty[\text{ and } \mathbf{u}_{t0} \in \mathbb{R}^2. \quad (4.33)$$

Figure 4.4 shows the sketch of throat panels and the distribution function of the macro-ion initial axial velocity $f_{iz}(\hat{u}_{z0})$. Note that the definition of the distribution function is consistent with the kinetic description of particles passing through a panel with a given velocity $(u_{z0}, \mathbf{u}_{t0})$ in a time step,

$$dM(u_{z0}, \mathbf{u}_{t0}) = u_{z0} f_i(u_{z0}, \mathbf{u}_{t0}) A_\eta \Delta t du_{z0} d\mathbf{u}_{t0}, \quad (4.34)$$

and the total number of ions through the panel surface would be $M = \int dM$, which indeed is

equivalent to $\sum_{\eta=1}^{\eta_t-1} M_\eta$ from Eq. 4.31.

Lateral dielectric wall, plasma-wall recombination

Ions that impact to the lateral wall, which is assumed dielectric, must be recombined and reinjected into the PIC domain as neutral particles. The distribution function of new neutrals is characterized by the emission temperature T_{emi} . This temperature depends on both the wall temperature T_{wall} and the energy of particles that impact the wall, which indeed is written in terms of an equivalent temperature, T_{imp} :

$$kT_{imp} = \frac{q_w}{2g_w}. \quad (4.35)$$

q_w is the energy flux of particles after travelling through the electrostatic sheath, so it includes the gain of energy due to the acceleration caused by the electric potential drop at the Debye sheath. g_w is the particle flux toward the wall. Defining α_{wall} as the wall accommodation factor that depends on the material type. The emission temperature can be written as follows,

$$T_{emi} = (1 - \alpha_{wall})T_{imp} + \alpha_{wall}T_{wall}. \quad (4.36)$$

The distribution function of injected neutrals due to recombination is factorized in a similar fashion as the distribution of ions at the MN throat (Eq. 4.33), thus distinguishing the perpendicular direction to the wall (z) from the parallel direction ($r - \theta$ plane). A Maxwellian distribution is taken in the parallel direction. In the perpendicular direction, a non-drifting Maxwellian flux is considered. This differs from the drifting Maxwellian flux imposed at the injection algorithm. Both components are controlled by the emission temperature T_{emi} . The distribution function accounting for all these hypotheses is,

$$f_{i,wall}(u_{z0}, \mathbf{u}_{t0}) = \frac{m_i n_w}{kT_{emi}} \exp\left(-\frac{m_i u_{z0}^2}{2kT_{emi}}\right) \frac{m_i}{2\pi kT_{emi}} \exp\left(-\frac{m_i \|\mathbf{u}_{t0}\|^2}{2kT_{emi}}\right),$$

$$u_{z0} \in [0, \infty[\text{ and } \mathbf{u}_{t0} \in \mathbb{R}^2. \quad (4.37)$$

n_w is the wall particle density of injected particles. In order to compute fluxes to the lateral wall and other moments of the IDE, the Extended Surface Weighting algorithms described in Ref. [127] are implemented. This ensures the fulfilment of the Bohm kinetic condition and avoids oscillatory problems reported in *HallMa*.

The initial position of new macro-neutrals is set randomly along each panel that defines the lateral wall. In the perpendicular direction to the wall, the macro-particle is set a small distance apart from the wall, following the same criterion that has already been explained for the injection algorithm (small parameter $\delta\xi$).

In the current step of development, neither CEX nor *ion-neutral* elastic collisions are included in the code. Consequently, all neutrals introduced from the wall, which is the only possible source of neutrals, describe a ballistic trajectories until reaching the end of the PIC domain.

The neutral path is integrated with the same scheme than for ions (Sec. 4.3.2), except that there are no forces applied to them. The unique role of these neutrals now is its contribution to the *electron-neutral* collisional frequency, ν_{en} , that will be further discussed in the electrons fluid model section.

Particle reflection at the axis

The particle integration scheme implemented in *HallMa* fails when particles approach the axis of symmetry. This issue was not investigated during the development of that model because the area modelled was only the cylindrical channel and the near plume area of HETs, a region that did not include the axis.

Considering the cylindrical frame presented before, it is easy to identify that Eqs. 4.13 - 4.23, those involving h_θ/x_r , may introduce large errors or integration difficulties when $x_r \rightarrow 0$. In particular it has been found that the term $h_{\theta 0}^{(0)} \Delta t / x_{r0}^{(0)}$ in Eq. 4.17 could be huge for some particles approaching the axis.

To solve this problem Δt should be strongly reduced, but the code would become much more costly in terms of computational resources. Another option, that is the one implemented here, consists in forcing a specular reflection of the particle if its kinetic parameters (position, velocity, etc.,) are within some tolerance ranges or conditions defined next,

- $x_{r0}^{(0)} < 5 \cdot 10^{-4}$ m. Particle must be close to the axis. $5 \cdot 10^{-4}$ is approximately a third of the radial length of cells attached to the axis.
- $v_{r0}^{(0)} < 0$, particle must approach the axis.
- The radial displacement due to the term $h_\theta \Delta t / x_r$ of Eq. 4.19, must be shorter than half of the radial length of the cell attached to the axis. This condition can be rewritten as $\Delta t^{**} = 7.5 \cdot 10^{-4} x_{r0}^{(0)} / h_{\theta 0}^{(0)} < \Delta t$. If $\Delta t^{**} < \Delta t$ reflection is forced.

The new radial position and velocity, $x_{r1}^{(1)}, v_{r1}^{(1)}$, and the angular velocity $v_\theta = h_\theta / x_r$ for all macro-particles that fulfil the previous conditions is given by the following expressions:

$$x_{r1}^{(1)} = x_{r0}^{(0)} - v_{r0}^{(0)} \Delta t, \quad (4.38)$$

$$v_{r1}^{(1)} = -v_{r0}^{(0)}, \quad (4.39)$$

$$\Delta x_r = -v_{r0}^{(0)} \Delta t, \quad (4.40)$$

$$h_{\theta 1}^{(1)} = h_{\theta 0}^{(0)}. \quad (4.41)$$

Particle integration at the plasma-vacuum edge

The last issue concerns the failure of the particle integrator scheme caused by the crude estimation of the electric field at the plasma-vacuum edge/transition. This border is determined by the shape of the magnetic field as it will be described in section (4.4). However, the transition between these areas in terms of density gradients (or ambipolar field gradients) depend on different parameters, such as the magnetization regime and the boundary conditions of the electron fluid submodel.

It has been identified that close to this area the gradients of the ambipolar field can be very large causing huge electric forces acting on the charged particles. Consequently, the computed displacement vector according to Eqs. 4.12 - 4.23 is larger than a typical cell length, and trajectory integration fails unacceptably. For this reason, all particles, whose displacement on the computational mesh is larger than the unity $\Delta\xi\eta = \sqrt{(\Delta\xi)^2 + (\Delta\eta)^2} > 1$, are removed from the simulation in order to avoid numerical noise or incorrect results.

Since it is not possible to compute the integration error of each particle because it is computationally expensive, the global conservation of mass is computed instead. This balance requires to define several quantities: $\dot{m}_{i0}$ is the mass flow rate of injected ions at the throat. Due to the injection algorithm already described, this quantity presents a small error in comparison to the theoretical mass flow rate. $\dot{m}_{i,right}$ is the average mass flow rate lost at the downstream boundary of the PIC domain, $\dot{m}_{i,top}$ at the top boundary, $\dot{m}_{i,left}$ at the lateral dielectric wall. Accounting for these definitions, the relative error of the macro-particle mass balance $\epsilon_{\dot{m}_i}$ is computed as following,

$$\epsilon_{\dot{m}_i} = \frac{\dot{m}_{i,right} + \dot{m}_{i,top} + \dot{m}_{i,left} - \dot{m}_{i0}}{\dot{m}_{i0}}. \quad (4.42)$$

Small imbalances are usually due to poor statistics of the PIC model, and can be disregarded. On the contrary, in some cases large imbalances appear due to a massive removal of particles at the plasma-vacuum edge for the aforementioned reasons. In sections 4.5-4.6 this issue is further investigated through the evaluation of mass imbalance.

4.4 The electron fluid model

In this section the electron fluid model is briefly described. To make easier the comprehension of the model, first, the magnetic frame is clearly defined. After presenting the magnetic mesh, fluid equations are projected on it. This allows defining the one dimensional variables on the magnetic coordinate λ : the thermalized potential $\phi_0(\lambda)$, and the perpendicular current $I_\perp(\lambda)$.

These are either imposed or solved, depending on the region. This will be discussed in the last part of this section.

4.4.1 The magnetic mesh

The magnetic field $\mathbf{B}$ that defines the MN, is generated by a single loop coil, and the magnetic topology is solved during the pre-process using a commercial FEM code (*MAXWELL SV*). This ring is located at the throat section and its radius is $R_c = 4.5R$, R being the radius of the plasma source tube that feeds the MN. The magnetic field is determined by,

$$\nabla \cdot \mathbf{B} = 0, \quad (4.43)$$

$$\nabla \times \mathbf{B} = \frac{1}{\epsilon_0 c^2} \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}. \quad (4.44)$$

However, the zero-vorticity $\nabla \times \mathbf{B} = 0$ is assumed. This is justified by considering only the steady state problem and also because the plasma currents are expected to be small in comparison to the applied field $\mathbf{B}$. Thus, the influence of the induced magnetic field on the MN performances, investigated numerically in Ref. [109] is also disregarded as a first approximation. This strongly simplifies the model, because otherwise, the magnetic mesh should be updated accordingly the induced field caused by plasma currents, adding to the current model a high complexity. The zero-divergence and the zero-vorticity of $\mathbf{B}$ (Eqs. 4.43, 4.44) ensure that a magnetic potential function σ and a magnetic stream function λ exist and satisfy,

$$\frac{\partial \sigma}{\partial z} = B_z, \quad \frac{\partial \sigma}{\partial r} = B_r. \quad (4.45)$$

$$\frac{\partial \lambda}{\partial z} = -r B_r, \quad \frac{\partial \lambda}{\partial r} = r B_z. \quad (4.46)$$

where B_z and B_r are the magnetic field components expressed on a cylindrical reference frame $\{\mathbf{1}_z, \mathbf{1}_r, \mathbf{1}_\theta\}$. Hereafter, the magnetic reference frame will be $\{\mathbf{1}_\perp, \mathbf{1}_\parallel, \mathbf{1}_\theta\}$ in which $\mathbf{1}_\parallel = \mathbf{B}/B = \cos \alpha \mathbf{1}_z + \sin \alpha \mathbf{1}_r$ is the parallel direction, $\mathbf{1}_\perp = \sin \alpha \mathbf{1}_z - \cos \alpha \mathbf{1}_r$ is the perpendicular direction (pointing inward to the axis), $\alpha(r, z)$ is the local magnetic angle that forms between $\mathbf{B}$ and the axial reference $\mathbf{1}_z$, and the azimuthal direction is determined by $\mathbf{1}_\theta = \mathbf{1}_\perp \times \mathbf{1}_\parallel$. Each different λ value, belongs to a different magnetic streamline. The streamline rotation around the axis of symmetry, defines a magnetic surface. A differential volume can be defined between two magnetic surfaces and their intersections with the PIC boundaries. The magnetic mesh is depicted in Figure 4.5 together with a detailed sketch of a magnetic differential volume.

Figure 4.5 – Left: Magnetic mesh for the electron fluid model. The magnetic streamline that defines the nozzle is labelled as “edge” λ_{ed} coloured in blue (plasma-vacuum edge in full fluid models), λ_0 coincides with the axis (dashed green line), λ_w is the longest line with both ends intersecting the lateral wall. Right: differential volume determined by two magnetic streamlines. It draws the geometry main lengths, the magnetic frame, and the variables involved in the charge conservation law (Eq. 4.51).

4.4.2 Fluid equations and projection on the magnetic mesh

Before presenting the electron fluid equations it is of interest to state the divergence theorem on the magnetic mesh. Considering the geometry of a magnetic differential volume Ω , the divergence of any vectorial field (e.g. velocity $\nabla \cdot \mathbf{u}$) is written as,

$$\begin{aligned} \iiint_{\Omega} (\nabla \cdot \mathbf{u}) d\Omega &= \iint_{S(\lambda+d\lambda)} (\mathbf{u} \cdot \mathbf{1}_{\perp}) dS \\ &\quad - \iint_{S(\lambda)} (\mathbf{u} \cdot \mathbf{1}_{\perp}) dS + 2\pi r (\mathbf{u} \cdot \mathbf{1}_n) dS|_0 + 2\pi r (\mathbf{u} \cdot \mathbf{1}_n) dS|_1 \\ &= \left\{ \frac{\partial}{\partial \lambda} \left(\int_{l(\lambda)} 2\pi r (\mathbf{u} \cdot \mathbf{1}_{\perp}) dl \right) + \frac{2\pi (\mathbf{u} \cdot \mathbf{1}_n)}{B \cos \gamma} \Big|_0 + \frac{2\pi (\mathbf{u} \cdot \mathbf{1}_n)}{B \cos \gamma} \Big|_1 \right\} d\lambda \end{aligned} \quad (4.47)$$

$S(\lambda)$ refers to any surface that bounds the differential volume Ω , $l(\lambda)$ is the streamline length and dl is its differential length. Sub-indexes indicate properties evaluated at the end of the magnetic lines, where these cross PIC domain boundaries: 0 identifies the end located closer to the MN throat and 1 the opposite end. $\mathbf{1}_n$ is the perpendicular outward vector to the PIC boundary and γ is the angle between $\mathbf{1}_{\perp}$ and $\mathbf{1}_n$ at the position in which the magnetic streamline intercepts the PIC boundary.

Electrons are considered as an isothermal fluid, this restricts the area of analysis to the near plume region. Beyond that zone, this hypothesis is not necessarily true. Although it is used extensively in the literature [109], other studies claim that electrons must cool down, as in Refs. [128] and [123], the last one is indeed part of the work presented in Chapter 5. As a consequence of the isothermal condition, the electron fluid equations considered here are

only the particle conservation and momentum equation, which are listed below.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = \dot{n}_e, \quad (4.48)$$

$$n_e m_e \left(\frac{\partial \mathbf{u}_e}{\partial t} + \mathbf{u}_e \cdot \nabla \mathbf{u}_e \right) = -\nabla p_e - e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + \mathbf{M}. \quad (4.49)$$

$\mathbf{u}_e$ is the electron flow velocity, m_e the electron mass, $p_e = n T_e$ the electron pressure which is considered isotropic, $\dot{n}_e$ is the particle production/loss term due to ionization/recombination, and $\mathbf{M}$ includes all collisional effects that modify electron momentum. $\mathbf{M}$ can be written as $\mathbf{M} = -n_e m_e \nu_e \mathbf{u}_e$, where $\nu_e = \nu_{ei} + \nu_{en}$ is the effective electron collision frequency. This takes into account both *electron-ion* ν_{ei} , and *electron-neutral* ν_{en} inelastic collisions, $\nu_e = \nu_{ei} + \nu_{en}$, which are computed as indicated in the Appendix B. Concerning $\dot{n}_e$, neither ionization nor recombination effects are considered in the current work, because it is assumed that at the MN stage, the mean-free path for these collisions is much larger than other collisional process $\lambda_{ion} \gg \lambda_{col}$. Taking $T_e = 10$ eV, $n_n \sim 10^{17} \text{ m}^{-3}$, $T_n = 900$ K, and $n \sim 5 \cdot 10^{17} \text{ m}^{-3}$, the ionization mean free path at the MN stage is $\lambda_{ion} \sim 0.29$ m, while $\lambda_{en} \sim 0.013$ m and $\lambda_{ei} \sim 0.008$ m. ν_e could also include other physical processes related to the plasma wall interaction, virtual cathode considerations and anomalous diffusion as done in the investigation of HETs [119]. But in the current work, these contributions are not included.

Charge conservation law

Equation 4.48 can be combined with the ion continuity equation, which indeed has the same form. Thanks to the quasineutral plasma hypothesis and the lack of ionization/recombination processes, adding together electron and ion continuity equations yields the charge conservation law,

$$\nabla \cdot \mathbf{j} = 0, \quad (4.50)$$

where $\mathbf{j} = en(\mathbf{u}_i - \mathbf{u}_e)$ is the electric current density. Applying the result of Eq. 4.47 to Eq. 4.50, it can be rewritten as follows,

$$\frac{\partial I_{\perp}(\lambda)}{\partial \lambda} = - \frac{2\pi j}{(B \cos \gamma)} \Big|_0 - \frac{2\pi j}{(B \cos \gamma)} \Big|_1. \quad (4.51)$$

Current densities j are positive according to the orthogonal outward vector to the PIC domain boundary, $\mathbf{1}_n$ (see Fig. 4.5). $I_{\perp}(\lambda)$ depends on both electrons and ions, and it is calculated as the integral over the magnetic surface of the local current density,

$$I_{\perp}(\lambda) = \iint_{S(\lambda)} (\mathbf{j} \cdot \mathbf{1}_{\perp}) dS(\lambda) = \int_{I(\lambda)} 2\pi r e n_e (\mathbf{u}_i - \mathbf{u}_e) \cdot \mathbf{1}_{\perp} dl = I_{i\perp}(\lambda) + I_{e\perp}(\lambda) \quad (4.52)$$

The ion current $I_{i\perp}$ is provided by the PIC sub-model, while the electron current $I_{e\perp}$ requires to solve first the electron dynamics, specifically, the cross-field velocity $\mathbf{u}_{e\perp}$, which is discussed in the next subsection. Equation 4.51 indicates that the variation of the perpendicular current

through each magnetic surface $I_\perp(\lambda)$ is equivalent to the loss or gain of net charge at the streamline boundary.

Electron momentum equation

Regarding the momentum equation (Eq. 4.49), the inertial term on the left side drops because this convective term is usually much more smaller than the electromagnetic force, on the right side of the equation, $\|n_e m_e \mathbf{u}_e \cdot \nabla \mathbf{u}_e\| / \|e n_e \mathbf{u}_e \times \mathbf{B}\| \sim r_L u_e / R v_{et} \ll 1$. Here, r_L is the electron Larmor radius, R the magnetic nozzle throat radius and v_{et} the electron thermal velocity. Moreover, if the plasma momentum equation were considered (including ions), this term would be disregarded because the significant difference between the electron mass and the ion mass $m_e/m_i \ll 1$.

The electric field $\mathbf{E}$ derives from a scalar potential as well as the magnetic field, $\mathbf{E} = -\nabla\phi$. This is because both, the magnetic field fluctuations in time are negligible $\nabla \times \mathbf{E} = 0$ and plasma is quasineutral $\nabla \cdot \mathbf{E} \simeq 0$. Accounting for all these hypotheses, the electron momentum equation is reduced to,

$$\mathbf{0} = -T_e \nabla n_e + e n_e \nabla \phi - e n_e \mathbf{u}_e \times \mathbf{B} - n_e m_e \nu_e \mathbf{u}_e, \quad (4.53)$$

which can be projected on the magnetic frame. Let us consider first the parallel direction $\mathbf{1}_\parallel$,

$$0 = -T_e \frac{\partial n_e}{\partial \mathbf{1}_\parallel} + e n_e \frac{\partial \phi}{\partial \mathbf{1}_\parallel}. \quad (4.54)$$

The collisional term in Eq. 4.54 has already been dropped because it is expected to be small in comparison to pressure gradients,

$$\frac{\|n_e m_e \nu_e \mathbf{u}_e\| / B}{\|T_e \partial n_e / \partial \mathbf{1}_\parallel\|} \sim \frac{u_e R}{v_{et} \lambda_{col}} \ll 1,$$

in which $\lambda_{col} = \min\{\lambda_{en}, \lambda_{ei}\}$ is the electron collision mean-free-path with ions and neutrals, λ_{ei} , λ_{en} respectively. The ordering of the different lengths for the helicon plasma at the MN stage has been assumed the following: $\lambda_D \ll r_L \ll R \lesssim \lambda_{col} \ll \lambda_{ion}$; $\lambda_D \sim 10^{-5} \text{ m}$ is the Debye length taking $T_e = 10 \text{ eV}$ and $n \sim 5 \cdot 10^{17} \text{ m}^{-3}$; $r_L \sim 3.2 \cdot 10^{-4} \text{ m}$; others, such as $\lambda_{ion} \sim 0.29 \text{ m}$, $\lambda_{en} \sim 0.013 \text{ m}$ and $\lambda_{ei} \sim 0.008 \text{ m}$ have been already computed above.

Integrating Eq. 4.54 along each magnetic line it yields the well-known Boltzmann relation,

$$\phi(r, z) = \phi_0(\lambda) + \frac{T_e(\lambda)}{e} \ln \left(\frac{n_e(r, z)}{n_{ref}} \right); \quad (4.55)$$

$\phi_0(\lambda)$ is the thermalized potential and n_{ref} is a reference density, here $n_{ref} = n(r=0, z=0)$ is taken. The Boltzmann relation determines the electric potential in terms of one-dimensional variables ($T_e(\lambda)$ and $\phi_0(\lambda)$) and the plasma density distribution $n_e(r, z)$ determined by the PIC sub-model.

Analogously, the momentum equation is projected on the other two directions, $\mathbf{1}_\perp$ and $\mathbf{1}_\theta$ respectively,

$$0 = -T_e \frac{\partial n_e}{\partial \tilde{\mathbf{1}}_\perp} + en_e \frac{\partial \phi}{\partial \tilde{\mathbf{1}}_\perp} + en_e u_{e\theta} B - v_e m_e n_e u_{e\perp}, \quad (4.56)$$

$$0 = -en_e u_{e\perp} B - v_e m_e n_e u_{e\theta}. \quad (4.57)$$

$u_{e\theta}$ is written as a function of $u_{e\perp}$ using Eq. 4.57.

$$u_{e\theta}(r, z) = -\chi_e u_{e\perp}. \quad (4.58)$$

After some straightforward algebra, and combining Eqs. 4.55 and 4.58 into Eq. 4.56, $u_{e\perp}$ is easily determined,

$$u_{e\perp}(r, z) = \frac{r\chi_e}{1 + \chi_e^2} \left(\frac{\partial \phi_0(\lambda)}{\partial \lambda} - (1 - \ln(n_e/n_0)) \frac{\partial T_e/e}{\partial \lambda} \right), \quad (4.59)$$

and considering the isothermal hypothesis and χ_e ,

$$u_{e\perp}(r, z) = \frac{r\chi_e}{1 + \chi_e^2} \frac{\partial \phi_0(\lambda)}{\partial \lambda}, \quad (4.60)$$

$\chi_e = \omega_{ec}/\nu_e$ is the Hall parameter, and $\omega_{ec} = eB/m_e$, is the electron gyrofrequency that has already been defined. The Hall parameter χ_e compares electron magnetization against electron collisions. Cross-field diffusion increases with χ_e decreasing, so in this work the parametric analysis could be done in terms of χ_e . Note that in Chapter 3, analyses have been done according to χ_{lh} that is written in terms of B_0 , n_0 and the electron-ion rate of collisions, R_{ei} . They are related through the mass ratio, $\chi_e = \sqrt{m_i/m_e} \chi_{lh}$,

$$\chi_{lh} = \frac{eB_0}{n_0 R_{ei} \sqrt{m_e m_i}}. \quad (4.61)$$

Note that Eq. 4.60 is the Ohm's law in the direction perpendicular to the magnetic field. This expression for $u_{e\perp} = \mathbf{u}_e \cdot \mathbf{1}_\perp$ allows computing the $I_{e\perp}(\lambda)$ term of Eq. 4.52.

$$I_{e\perp}(\lambda) = - \int_{l(\lambda)} 2\pi r e n_e \mathbf{u}_e \cdot \mathbf{1}_\perp dl = - \int_{l(\lambda)} \frac{2\pi e n_e r^2 \chi_e}{1 + \chi_e^2} dl \left(\frac{\partial \phi_0}{\partial \lambda} \right). \quad (4.62)$$

4.4.3 Boundary conditions and solver

$I_\perp(\lambda)$ and $\phi_0(\lambda)$ are the one-dimensional variables determined by the current model, and it is required to set the boundary conditions for them. When this code was applied to the characterization of plasma dynamics in HETs, natural boundary conditions were both the net current j at the intersection of each streamline with the boundary of the PIC domain, and the electric potential $\phi(r, z)$ at a certain position of the first and the last streamlines considered (say the total potential drop between λ_1 , typically the cathode, and λ_n , typically the anode). Then, Eqs. 4.51, 4.60, were integrated in λ , to determine $I_\perp(\lambda)$ and $\phi_0(\lambda)$.

However, the scenario here is quite different. The first difference is that magnetic lines do not

cross “physical” walls as in the *HallMa* code, so the condition for j at the end of each line is in some cases unknown. Second, plasma is injected at the MN throat according to a given density profile, and this indirectly sets a condition for ϕ_0 , which will be discussed next.

The plasma equilibrium at the MN throat

For all lines that pass through the MN throat, some variables, e.g. $\phi_0(\lambda)$ depend on the plasma conditions upstream. In Ref. [129], plasma at the source was considered fully magnetized, and the radial equilibrium consisted of a balance between the plasma pressure and the magnetic pressure. Consequently, the radial electric field was zero or almost null (i.e. $\phi(r) = 0, r \in [0, R]$), excepting at the vicinity of the Debye sheath attached to the cylindrical wall of the plasma source, as shown in Chapter 3. Accounting for a given radial density profile $n_e(r)$ and the Boltzmann relation, this yields a condition for the thermalized potential for all lines crossing the MN throat, $\lambda \in (\lambda_0, \lambda_{ed})$,

$$\phi_0(\lambda) \equiv \phi_0(r) \Big|_{z=0} = -(T_e(\lambda)/e) \ln (n_e(r)/n_0) \Big|_{z=0},$$

n_0 is the plasma density at the centre of the MN throat, $n_0 = n(r = 0, z = 0)$. The previous expression assumes $\phi_0(r = 0) = 0$. The density profile under that conditions is expressed in terms of the 0th Bessel function, $n_e(r)/n_0 \propto J_0(r/R)$ [107]. For simplicity, in that work the density radial profile was taken parabolic because of its resemblance to the Bessel function solution of the internal plasma density.

However, the present work moves a step forward, and the radial equilibrium at the MN throat is solved for each regime during the pre-process stage. The model implemented here requires solving the 2D fluid-dynamics of a magnetized plasma column as described in Chapter 3.

Then, with the obtained density and potential radial profiles, $n(r)$ and $\phi(r)$ respectively, the condition for $\phi_0(\lambda)$ can be set,

$$\phi_0(\lambda) \equiv \phi_0(r) \Big|_{z=0} = \phi(r) - (T_e(\lambda)/e) \ln (n_e(r)/n_0) \Big|_{z=0},$$

Recall that the model in Ref. [105] assumes zero axial current everywhere within the source, which is actually a plausible assumption. However, due to this condition, the radial *plasma* confinement is influenced by the ion inertia too, and the parameter that controlled the HPS response was the *hybrid* Hall parameter $\chi_H = \omega_{lh}/\nu_e$, as was in the 2D fluid model. But in the current model, χ_e is the parameter that controls whether electrons are magnetized or not on the MN diverging side. These parameters differ by a factor of $\sqrt{m_i/m_e}$, for instance $\sqrt{m_i/m_e} = 490$ considering Xenon as a propellant. χ_e controls the *electron* magnetization regime in the nozzle, in a similar fashion as it controls electron cross-field diffusion in a

Hall effect thruster, in which cases current ambipolarity is not forced [130]. This study is indeed more focused on the analysis of the expansion stage, and the transition between magnetized/demagnetized regimes in the MN, without considering the loss of confinement at the source walls.

Solving $I_{\perp}(\lambda)$ and $\phi_0(\lambda)$

$\phi_0(\lambda)$ is constant in time and uniform along each one of the magnetic lines that intercept the MN throat, $\lambda \in (\lambda_0, \lambda_{ed})$ as sketched in Fig. 4.5. λ_{ed} is the magnetic stream line that coincides with $r = R$ at the MN throat, and defines the MN “edge”; and λ_0 corresponds to the axis of symmetry. The area between λ_0 and λ_{ed} corresponds to the dense plasma jet region. For all these lines, the only one-dimensional variable that has to be evaluated is the perpendicular current through magnetic surfaces $I_{\perp}(\lambda)$. This is obtained by solving Eq. 4.52, 4.62, in which the ion current $\mathbf{j}_i(z, r) = en\mathbf{u}_i$ is provided by the PIC sub-model according to Eq. 4.5.

Concerning the boundary condition of the net charge flux at the intersection of each magnetic streamtube with the PIC domain, this model assumes zero axial current at the throat, $j_{|0}(\lambda) = 0$, for consistency with the source conditions. This hypothesis together with Eq. 4.51 and the computation of $I_{\perp}(\lambda)$ according to Eq. 4.52, determines the net current at the downstream boundary $j_{|1}(\lambda)$,

$$j_{|1}(\lambda) = -\frac{\partial I_{\perp}(\lambda)}{\partial \lambda} \frac{B \cos \gamma}{2\pi}. \quad (4.63)$$

Considering the lack of any external electrode/cathode, the plasma must be globally current-free. In comparison to fully fluid models, such as the one in Ref. [131], which considered a perfect edge between the plasma and the external vacuum, here, the imbalance on the current-free condition within the dense plasma domain is computed as following,

$$\epsilon_c = -I_{\perp}(\lambda_{ed}) + \int_{\lambda_0}^{\lambda_{ed}} (\mathbf{j}(\lambda) \cdot d\mathbf{S}(\lambda))_{|1}. \quad (4.64)$$

Outside the region in which magnetic lines do not cross the MN throat, $\lambda \in (\lambda_{ed}, \lambda_w)$, a closure of the electron fluid model has to be postulated. λ_w is the magnetic line, contained in the PIC volume, that intersects the dielectric wall at its both ends. Its top end is the one located closest to the left-upper corner of the PIC mesh. There are still some uncertainties as to which are the most appropriate conditions. This depends on several aspects: the dynamics of electrons trapped by the magnetic field, wall properties, such as whether it is dielectric or conducting, the geometry of the magnetic field, and whether lines intersect the wall or the vacuum boundary.

In the current scenario it is proposed that the lateral wall is made of a dielectric material. This

means that local and global zero net current is imposed there. The problem now remains of the transition to this zero net current from the usually non-zero perpendicular current determined at the edge of the dense plasma $I_{\perp}(\lambda_{ed})$ according to Eq. 4.52 and Eq. 4.60. The drop of this magnitude in the λ coordinate, with δ a parameter to control it, is suggested to be the following,

$$I_{\perp}(\lambda) = I_{\perp}(\lambda_{ed}) \left(\frac{\lambda - \lambda_w}{\lambda_{ed} - \lambda_w} \right)^{\delta} \quad (4.65)$$

The main idea is that the net current through λ_w , and the rest of lines that also intersect the wall in the same manner must be 0. Since in the current domain, there are magnetic streamlines that intersect the wall at one end but not in the other, because these intersect the “free” boundary, a zero net current along these lines fraction is not a mandatory condition. According to this situation it is reasonable to impose the mentioned law for $I_{\perp}$. Also note that in this region, the plasma density is expected to be very low, so electron collisionality becomes smaller, and consequently electron cross field diffusion is smaller as well. As a result, the perpendicular current should also decrease. There is no strong physical basis for Eq. 4.65 and it is just imposed to force the fulfilment of plasma-wall boundary conditions. At this low density/outer region, $\phi_0(\lambda)$ is determined recombining Eqs. 4.51, 4.62, with $j_0 = 0$. The Adams-Bashforth scheme is used to integrate backwards $\phi_0(\lambda)$ as it was done in *HallMa*.

$$\phi_{0,i} = \phi_{0,i+1} + \Delta\lambda_i \left(\beta_{1i} \frac{I_{e\perp,i+1}}{\gamma_{i+1}} + \beta_{2i} \frac{I_{e\perp,i+2}}{\gamma_{i+2}} \right), \quad (4.66)$$

with

$$\beta_{1,i} = \frac{\Delta\lambda_i + 2\Delta\lambda_{i+1}}{2\Delta\lambda_{i+1}}, \quad (4.67)$$

$$\beta_{2,i} = -\frac{\Delta\lambda_i}{2\Delta\lambda_{i+1}}, \quad (4.68)$$

$$\gamma_i = \int_{l(\lambda_i)} \frac{2\pi e n_e r^2}{\chi_e} dl_i \text{ (from Eq. 4.62),} \quad (4.69)$$

Here i is the index that identifies each magnetic streamline λ_i . The influence of δ (Eq. 4.65) on the model results is analysed parametrically in the results section for two different magnetization regimes. The previous scheme allows for a non-uniform spacing of λ nodes, $\Delta\lambda_i$.

4.5 Simulation results

The Hall parameter χ_{lh} and the plasma electron temperature T_e are considered inputs and identify all the cases studied. χ_{lh} is defined at the center axis line just at the MN section ($r = 0, z = 0$), which is the interface between the helicon tube and the MN plume. At the mentioned location, plasma density is set to $n_0 = 10^{19} \text{ m}^{-3}$ and it is kept constant for all

Figure 4.6 – Left: Required B_0 (G) to keep $\chi_{lh} = 2$ constant when varying the electron temperature. Right: variation of $R_{ei}(T_e) \propto \ln \Lambda \propto T_e^{-3/2}$. Asterisks corresponds to $T_e = 5, 10, 15$ eV.

analysed cases.

Once these parameters T_e and χ_{lh} are set, the plasma radial equilibrium is solved according to the model in Section 4.4.3. Then, the necessary ion mass flow rate $\dot{m}$ at the throat is determined by integrating the density radial profile according to Eq. 4.29. After computing this flow, all conditions for the hybrid model are known, and the numerical simulations can be performed. All the results that will be shown in this section, have been obtained from time-averaged simulations that cover a total elapsed real time $\Delta t_T = 4\Delta z/c_s$, Δz being the axial length of the simulation domain, for instance $\Delta z = 0.3$ m ($\Delta z = 30R$).

Two different studies have been carried out. The first one is a validation of the highly magnetized regime, for the three mentioned temperatures. The second one sets up the temperature to 10 eV and explores the transition from the magnetized to the demagnetized regimes by decreasing χ_{lh} progressively.

4.5.1 The highly magnetized regime

In this first analysis, the high magnetized regime $\chi_e \gg 1$ is investigated for different electron temperatures $T_e = 5, 10, 15$ eV. To keep $\chi_{lh} = 2$ ($\chi_e \sim 1000$ for Xenon) constant, the magnetic field B_0 has been tuned according to Figure 4.6. The variation of B_0 must be proportional to $R_{ei}(T_e) \propto T_e^{-3/2}$, see the mentioned figure. The dimensionless electron-ion frequency of collisions varies depending the temperature case: $\hat{\nu}_{ei} = \nu_{ei}R/c_s = 195.72, 50.67, 23.84$ at $T_e = 5, 10, 15$ eV respectively.

Figure 4.7 depicts the radial profiles of the plasma density n , the electric potential ϕ and the thermalized potential ϕ_0 at the throat. The flat potential of ϕ , except in the vicinity of the Debye sheath, and the fact that the density drops at least two orders of magnitude when

Chapter 4. Hybrid model of a plasma expansion in a diverging Magnetic Nozzle

T_e (eV)	B_0 (G)	$\nu_{ei}R/c_s$	$\log_{10}(n_s/n_0)$	$I_\perp(\lambda_{ed})$ (Amp)	Δt_T (ms)	$\dot{m}_{i0}$ (mg/s)	$\epsilon_{\dot{m}_i}$ (%)
5	1544	195.72	-2.82	-0.11	0.75	0.966	-9.92
10	610	50.67	-2.23	-0.19	0.6	1.36	-2.32
15	334	23.84	-1.91	-0.12	0.45	1.67	-1.16

Table 4.1 – Summary of main input data of the different simulations performed in the highly magnetized regime $\chi_{lh} = 2$ and $\delta = 32$ for several temperatures. In the last column, $\epsilon_{\dot{m}_i}$ is the error incurred in the macro-particle mass balance.

Figure 4.7 – Radial equilibrium at the MN throat according to Sec. 4.4.3, at different electron temperatures, $T_e = 5, 10, 15$, thick, thin, and dashed line respectively. All results are evaluated at constant magnetization $\chi_{lh} = 2$ ($\chi_e = 1000$). Left: density profile; centre: electric potential profile; right: thermalized potential profile for setting the MN electron model conditions.

approaching the lateral wall, indicate that although $\chi_{lh} \sim 1$, plasma within the helicon source can be still considered *well* confined.

Table 4.1 summarizes some of the important parameters of each simulated case: required magnetic field B_0 , reference frequency of electron collisions $\nu_{ei}R/c_s$ (computed at $(r, z) = (0, 0)$), the density at the sheath transition of the HPS dielectric wall according to the plasma radial equilibrium $\log_{10}(n_s/n_0)$, the total real time of the simulation Δt_T , the injected mass flow rate $\dot{m}_{i0}$, and the relative error of the macro-particle mass balance, $\epsilon_{\dot{m}_i}$.

contour maps of the plasma density are drawn in Figure 4.8, from the throat section ($z = 0$) and until 30 cm downstream. Concerning the effect of the temperature, the results can be considered qualitatively the same. The denser plasma is located close to the MN throat and along the axis. Magnetic streamlines, which define the nozzle geometry, are depicted in black, overlying the density contour map (note that nozzle divergence is rather small). For each magnetic streamtube, an ion streamtube with the same origin at the MN throat is depicted in red. The figure shows that ions effectively detach from magnetic streamlines, reproducing qualitatively the results in Refs. [17, 108].

contour maps of the ion Mach number, i.e. the module of the ion velocity over the ion sound speed $\|\mathbf{u}_i\|/c_s$, are also depicted in Fig. 4.8. All of them present a very similar response

Figure 4.8 – Left side: plasma density contour maps (m^{-3}), ion streamlines in red, and magnetic streamlines in black. Each graph is obtained at different electron temperature: 5 eV, 10 eV and 15 eV. Right side: contour maps of the ion Mach number $\|u_i\|/c_s$. Black arrows indicate the ion flow direction. Arrow lengths scale with the velocity. All results are at constant magnetization $\chi_{lh} = 2$.

and differences can be attributed to the numerical noise intrinsic to the PIC model. These maps stress the fact that ions are also radially accelerated, and this causes beam divergence inefficiencies. Larger axial ion acceleration is located close to the axis (red coloured area).

The dimensionless electric potential $e\phi/kT_e$ is depicted in Figure 4.9. Due to the crude closure of the fluid equation in the vacuum area, the electric potential oscillates in time and depend strongly on the sign of $I_\perp(\lambda_{ed})$. White coloured areas identify the zones in which the potential, after averaging in time, reaches too large negative values (below the dark blue scale). On the contrary, areas where the potential reach large positive values are coloured in dark red. Although the solution in the dense plasma region is reasonable, these results indicate that the current model requires further development when it approaches the near vacuum region or beyond.

The axial component of the magnetic force density, $F_m = -j_\theta B_r$, is drawn in Figure 4.9, only in the region delimited by $\lambda_0 - \lambda_{ed}$. At higher temperatures the magnetic force density is more intense, basically because F_m scales with the electron temperature, $F_m \propto j_\theta \equiv j_{e\theta} \propto u_{e\theta} \propto \phi_0 \propto T_e$, according to Eqs. 4.55 , 4.58 and 4.60. However, this force should be normalized with the injected plasma momentum at the throat, which indeed it is also proportional to T_e , yielding a dimensionless thrust almost invariant with the temperature.

Finally, the thrust gain F_z along the nozzle should be equivalent to the volume integral of the magnetic force density. Fig. 4.10 depicts the thrust provided by the increase of ion momentum F_{iz} (see Eq. 3.57 in Chapter 3), normalized by the ion momentum at the throat F_{iz0} . Ion momentum gain throughout the nozzle increases monotonically. However, in the current simulations there is an evident limitation on the computational domain. The plasma beam intercepts the upper boundary, approximately at $z = 0.15$ m causing a loss of particles and consequently an artificial drop of the total ion momentum gain. Results beyond that location do not reproduce the expected thrust trend and are not shown.

Apart from this issue, there is another matter in question regarding the non negligible difference between these F_{iz} curves obtained at each temperature, while these were expected to follow approximately the same trend. The problem here is due to how the current model behaves at the plasma-vacuum edge or its surrounding area. Note that in this highly magnetized regime, plasma density drops abruptly at the “edge” and consequently the electric potential and the electric field could reach high values or be very steep. Macro-ions at this region feel this electric force, and the current particle trajectory integration scheme is not able to provide a feasible integration of either its trajectory, or its velocity. The error incurred

Figure 4.9 – Left side: contour maps of the dimensionless ambipolar electric potential $e\phi/kT_e$, and countourmaps of the magnetic force density (N/m^3), for the same cases in Fig. 4.8. The origin of the potential $\phi = 0$ is set at $(z, r) = (0, 0)$, and also is the upper cut-off level, so in the outer region the potential increases strongly and it is not shown (dark red area). White area corresponds to an extremely negative potential, due to the crude estimation of the potential at the vacuum region. The black line is the λ_{ed} streamline. Right side: axial magnetic force $F_m = j_\theta B_r \mathbf{1}_z$ in the region in which magnetic streamlines are connected to the MN throat, $\lambda \in (\lambda_0, \lambda_{ed})$.

Figure 4.10 – Ion momentum gain along the diverging MN, normalized with the ion momentum at the MN throat, F_{iz}/F_{iz0} , for the same cases in Fig. 4.8.

Figure 4.11 – Tracked ions at the plasma edge, which are suppressed because of the failure of the particle trajectory integration scheme. Left: position (r_0, z_0) at t_0 . Right: the new radial position (r_1, z_0) which would be erroneously computed. This is for the case at $T_e = 5$ eV, $\chi_{lh} = 2$, $\delta = 32$, in which the global mass balance error in the PIC model was $\epsilon_{\dot{m}_i} = 10\%$ as reported in Tab. 4.1.

cannot be accepted and these particles are also suppressed as they were being lost at the upper boundary. The ion mass flow rate lost at the edge due to the failure of particle trajectory integration is 10%, 2.5% and 1.2% for $T_e = 5, 10$ and 15 eV respectively. This also explains the noisy behaviour of the local ion velocity vectors at the edge (see again Fig. 4.8). If these particles were kept in the simulation, they would introduce larger errors in the computation of plasma properties in other areas of the computational domain. Figure 4.11 shows, for the simulation at $T_e = 5$ eV, the position $(r, z)_0$ of all suppressed particles, and compares it with the new radial position (r_1, z_0) that would be obtained with the current trajectory integration scheme. This clearly exemplifies that the trajectory integration fails.

χ_{lh}	B_0 (G)	$\log_{10}(n_s/n_0)$	$I_{\perp}(\lambda_{ed})(Amp)$	$\dot{m}_{i0}$ (mg/s)	$\epsilon \dot{m}_i$ (%)
2	610	-2.82	-2.19	1.36	-2.32
1	305	-1.63	-0.03	1.43	-0.40
0.2	61	-0.56	0.88	2.11	-0.12
0.1	30.6	-0.43	3.35	2.31	-0.08
0.02	6.1	-0.42	66.51	2.39	+0.08

Table 4.2 – Summary of main data of the simulations performed at the different magnetized regimes at constant temperature $T_e = 10$ eV, $\delta = 32$, $n_0 = 10^{19} \text{ m}^{-3}$, $v_{ei}R/c_s = 50.67$ and $\Delta t_T = 0.6$ ms.

Figure 4.12 – Radial equilibrium at the MN throat according to Sec. 4.4.3, at different levels of magnetization, $\chi_{lh} = 1, 0.2, 0.1$, thick, thin, and dashed lines respectively. All results evaluated are at constant electron temperature $T_e = 10$ eV. Left: density profile; centre: electric potential profile; right: thermalized potential profile for setting the MN electron model conditions.

4.5.2 The transition to the demagnetized regime

The transition to the demagnetized regime at the MN is explored by reducing the Hall parameter at constant electron temperature $T_e = 10$ eV. This is achieved by weakening the magnetic field strength. The radial equilibrium within the tube is solved in order to set the boundary conditions at the MN throat. The same hypotheses as in the previous analysis are assumed: $v_e \sim v_{ei}$, $n_0 = 10^{19} \text{ m}^{-3}$ constant at the axis line, which implies that the mass flow rate has to be readjusted. Four new cases, $\chi_{lh} = 1, 0.2, 0.1, 0.02$, are considered here in addition to the previous one $\chi_{lh} = 2$ ($T_e = 10$ eV). The required magnetic field is $B_0 = 305, 61, 30.6, 6$ Gauss respectively. Considering the use of Xenon, the mass flow rate is $\dot{m} = 1.42, 2.10, 2.28, 2.37$ mg/s for each case. The discretization of the injected flow in macro-ions presents a small error. Averaging in time, the injected mass flow rate is $\dot{m}_i = 1.43, 2.11, 2.31, 2.39$ mg/s instead of the theoretical values. The maximum error is 1.3% and occurs in the $\chi_{lh} = 0.1$ scenario. Table 4.2 summarizes some of the important parameters of each case, as done in Table 4.1 for highly magnetized scenarios.

Figure 4.12 depicts the radial profiles of the plasma density, the electric potential and the thermalized potential at the throat. Here, in comparison to the highly magnetized regime, the

Figure 4.13 – Plasma density contour maps (m^{-3}), ion streamlines in red, and magnetic streamlines in black. All results are at constant temperature $T_e = 10$ eV and each graph is for a different value of the Hall parameter, $\chi_{lh} = 1, 0.2, 0.1$ and 0.02 .

radial profile of the ambipolar potential ϕ drops gently, except at $\chi_{lh} = 1$ that is similar to the one obtained at $\chi_{lh} = 2$. This indicates that within the source, the role of the radial electric field becomes important, and compensates the lack of magnetic pressure. So, at $\chi_{lh} = 0.2, 0.1$ the plasma confinement within the source is already very poor. This issue is also stressed by their corresponding density profiles. At the sheath transition, plasma density values are much more larger than in other cases, pointing out that plasma losses to the wall will reduce source efficiency, but this is not the object of this chapter (see Ref. [105] or Chapter 3 for further details on this topic). The shape of these profiles allows us to conclude that plasma within the source is already demagnetized.

Plasma density countormaps are drawn in Figure 4.13, from the throat and until 30 cm downstream. The denser plasma is still located close to the MN throat and at the axis line. However, the effect of reducing the Hall parameter has a huge impact on the ion dynamics. Below $\chi_{lh} = 1$, their streamlines, coloured in red, are diverging more than magnetic field streamlines. This highlights that plasma demagnetizes and opens outward when χ_{lh} is reduced, causing a deterioration of MN performances. This cannot be fully predicted by current models [109].

Figure 4.14 – contour maps of the ion Mach number ($\|\mathbf{u}_i\|/c_s$). All results are at constant temperature $T_e = 10$ eV and each graph is for a different value of the Hall parameter, $\chi_{lh} = 1, 0.2, 0.1$ and 0.02 . Black arrows indicate the ion flow direction. Arrow lengths scale with the velocity.

Next, several plasma properties are presented to support these results.

The ion Mach number is depicted for the same scenarios than in Fig. 4.14. Because of the weaker magnetic field, a larger area is occupied by ions with higher radial velocities, meaning that the undesired radial acceleration emerges due to the loss of confinement.

A direct consequence of demagnetization is the increase of plasma-wall interaction. This can be shown in Figure 4.15, in which neutral density contour maps are plotted. The rise of this quantity in the current scenario in which a fully ionized gas is injected, indicates that more ions are colliding with the lateral dielectric wall, so new neutrals born due to recombination are introduced to the PIC domain.

Another effect is that neutrals induce “electron-neutral” collisions, $\nu_{en} = n_n R_{en}$. This is accounted for in ν_e , decreasing the local Hall number, and thus modifying the electron cross-field diffusion. Then, the solution for ϕ_0 outside of λ_{ed} is strongly altered, interfering on the ambipolar potential ϕ (see case $\chi_{lh} = 0.02$ in Fig. 4.16). At this weakly magnetized regime, it seems that the increase of electron perpendicular mobility (or inversely, the drop of its

Figure 4.15 – Neutrals density contour maps (m^{-3}), showing that ions impact the lateral wall and produce new neutrals due to wall-recombination. All results are at constant temperature $T_e = 10$ eV and each graph is for a different value of the Hall parameter, $\chi_{lh} = 1, 0.2, 0.1$ and 0.02 .

Figure 4.16 – Normalized electric potential $e\phi/kT_e$. The origin of the potential $\phi = 0$, is set at $(z, r) = (0, 0)$, and also is the upper cut-off, so in the outer region the potential increases largely and it is not shown (dark red area). All results are at constant temperature $T_e = 10$ eV and each graph is for a different value of the Hall parameter, $\chi_{lh} = 1, 0.2, 0.1$ and 0.02 . The black solid line is the λ_{ed} streamline.

Figure 4.17 – contour maps of the axial component of the magnetic force density F_m (N/m³), for the same magnetization regimes of Fig. 4.13. It is only shown in the region in which magnetic streamlines are connected to the MN throat.

resistivity), produces a smoother evolution of the potential in the outward direction, although the perpendicular current is larger. On the other hand, in the highly magnetized regime (see $\chi_{lh} = 1$ in Fig. 4.16), the resistivity is much higher and it seems to produce a sharp rising of the potential at the vacuum side of the plasma edge. This behaviour is reasonably justified by the Ohm's law.

The axial component of the magnetic force density F_m is drawn in Figure 4.17. These contour maps show that the magnetic force strength is larger in the cases with higher electron magnetization. However, there are several effects mixed together. First, the force is proportional to the applied field, $\mathbf{F}_m = \mathbf{j}_\theta \times \mathbf{B}_r$, so larger magnetic field yields larger forces. Second, the force is also proportional to the azimuthal plasma current, which is approximately $\mathbf{j}_\theta \simeq -en\mathbf{u}_{e\theta}$. In all cases simulated here, and due to the radial profiles imposed at the throat according the magnetization regime, the lower the χ_{lh} is, the weaker the azimuthal currents become. The strength of the interaction of these currents with the applied field [131], or the resulting magnetic thrust, drops when χ_{lh} decreases.

Figure 4.18 – Ion momentum gain along the diverging MN, normalized with the ion momentum at the MN throat for the same cases as well. Thick solid red line is for $\chi_{lh} = 2$, thin red line $\chi_{lh} = 1$, solid black line $\chi_{lh} = 0.2$, thin black line $\chi_{lh} = 0.1$ and solid blue line $\chi_{lh} = 0.02$.

Figure 4.19 – Left: axial evolution of the ion mass flow rate $\dot{m}_i/\dot{m}_{i0}$ in the PIC domain. It highlights the loss of particles through the upper vacuum boundary. Right: averaged axial Mach number $M_{iz} = (F_{iz}/F_{iz0})/(\dot{m}_i/\dot{m}_{i0})$. Thick solid red line is for $\chi_{lh} = 2$, thin red line $\chi_{lh} = 1$, solid black line $\chi_{lh} = 0.2$, thin black line $\chi_e = 0.1$ and solid blue line $\chi_{lh} = 0.02$.

Figure 4.20 – Perpendicular current $I_{\perp}(\lambda)$ in Amperes (blue line), the solid black is the perpendicular ion current $I_{i\perp}$ (Amp), and the red line is the electron perpendicular current $I_{e\perp}$ (Amp). The strong gradient of $\phi_0(\lambda)$ close to the plasma edge λ_{ed} (see Figure 4.7) induces the increasing trend of $I_{\perp}$ and $I_{e\perp}$. Cases $\chi_{lh} = 1.0, 0.2, 0.1, 0.02$ at $T_e = 10$ eV.

Once again the ion momentum gain along the MN, normalized by the ion momentum at the throat is depicted in Fig. 4.18. Ion momentum gain must be a monotonically increasing function in isothermal models. However, these are peaked because the particle loss through the upper “vacuum” boundary. Note that the ion mass flow rate $\dot{m}_i / \dot{m}_{i0}$ drops along the axial direction as highlighted in Figure 4.19. The peak of F_{iz} / F_{iz0} is larger in the highly magnetized regime concluding that it produces thrust more efficiently.

The averaged axial Mach number $M_{iz} = (F_{iz} / F_{iz0}) / (\dot{m}_i / \dot{m}_{i0})$ is defined in Ref. [6]. According to this, Figure 4.19 proves that the axial velocity of ion particles increases monotonically.

Regarding the particle integration issue at the surroundings of the plasma edge, which has been discussed in the highly magnetized regime, the performances of the current model are better in the medium-low magnetized regimes. This is mainly because the electric potential presents a smoother transition to the vacuum area when plasma opens gently (see again Fig. 4.16).

Concerning the one dimensional variables of the electron fluid model, Figures 4.20 and 4.21 depict the shape of $I_{\perp}(\lambda)$ and $\phi_0(\lambda)$ respectively. These evolve differently depending on the

Figure 4.21 – $\phi_0(\lambda)$ for cases in Fig. 4.20: solid black line ($\chi_{lh} = 1$), dashed black line ($\chi_{lh} = 0.2$), dot-dashed black line ($\chi_{lh} = 0.1$) and blue line ($\chi_{lh} = 0.02$).

Figure 4.22 – Electron perpendicular velocity $u_{e\perp}$ (m/s) according to Eq. 4.56 at $\chi_{lh} = 1$ (left) and $\chi_{lh} = 0.2$ (right). Both at $T_e = 10$ eV. The negative value indicates that electrons flow outward.

magnetization regime. Considering first the results at $\chi_{lh} = 1$ and $T_e = 10$ eV, $I_{\perp}(\lambda)$ receives contributions from ions that detach inward and the electron perpendicular diffusion, both contributing positively to the current. The match at λ_{ed} , even if it is not completely smooth, allows accepting the proposed law in Eq. 4.65 as a good approach.

On the other hand, profiles at $\chi_{lh} = 0.2, 0.1, 0.02$ present some peculiarities: the perpendicular current is higher than in the previous case, and this behaviour is driven by the stronger cross field diffusion of electrons (lower χ_{lh} , larger $I_{e\perp}$). Note also that $I_{e\perp}$ and consequently $I_{\perp}$, increases abruptly close to the edge. This response is caused by the $\phi_0(\lambda)$ profile imposed in the range $\lambda \in (\lambda_0, \lambda_{ed})$ according to the MN throat conditions, and by the fact that the local Hall parameter decreases downstream along the magnetic streamlines (electron demagnetization). Both induce an increase of the electron cross field diffusion. The overshoot of $I_{e\perp}$ is also supported by Figure 4.22, in which the electron perpendicular velocity $u_{e\perp}$ is depicted.

Concerning the ion detachment, at $\chi_{lh} = 1.0$, the ion perpendicular current $I_{i\perp}$ is positive

in the $\lambda_0 - \lambda_{ed}$ range, meaning that ions detach inwardly in the dense plume. The negative values for $I_{i\perp}$ at the near vacuum area means outwardly detachment. For all other cases at a lower magnetized regime $\chi_{lh} = 0.2, 0.1, 0.02$, the perpendicular ion current becomes negative already in the dense plume, $\lambda_0 - \lambda_{ed}$ range, meaning that a larger amount of ions are detaching outwardly. This response is consistent with the trend of the axial momentum gain F_{iz} presented in Fig. 4.18.

The response of the thermalized potential ϕ_0 depicted in Fig. 4.21 partially justifies and explains the transition of $\phi(r, z)$ to the vacuum area drawn in Fig. 4.16.

4.5.3 Sensitivity analysis of the electron model boundary conditions

The last analysis concerns the study of how the proposed law for the closure of the perpendicular current, Eq. 4.65, influences the obtained results. The scenario with $T_e = 10$ eV and $\chi_{lh} = 0.1$ is taken as the reference case. Table 4.3 summarizes the resulting PIC mass balance for each analyzed δ .

Figure 4.23 shows the plasma density contour maps for all cases summarized in the aforementioned table. When δ decreases, keeping constant other input parameters, the divergence of the plasma jet is reduced in a similar way of the one observed when plasma magnetization increases, i.e. larger χ_{lh} . However the PIC mass balances in these scenarios are much better than those obtained when exploring the $\chi_{lh} = 2$ cases, and the difficulties in the integration of ion trajectories close to λ_{ed} have not been repeated here. This is probably because of the smother ϕ transition at the plasma edge.

The ion Mach number contour maps, which are depicted in Figure 4.24, indicate that axial acceleration increases when δ decreases. Which is again similar to the stronger beam collimation observed when plasma magnetization increases.

Figure 4.25 shows the electric potential contour maps. These illustrate that the variation of δ modifies the outward separation of the plasma-vacuum “effective” edge from the MN edge, λ_{ed} streamline.

Finally, the one dimensional variables of the electron fluid model could explain some of the results presented above. $I_{\perp}(\lambda)$ and $\phi_0(\lambda)$ are depicted in Figures 4.26 and 4.27 respectively. A smaller δ forces $I_{\perp}(\lambda)$ to decrease from $I_{\perp}(\lambda_{ed})$ (tabulated in Tab. 4.3) to $I_{\perp}(\lambda_w) = 0$ more slowly. Taking also into account that in this outer area the resistivity is larger, due to the lower density and usually stronger field, the derivative $\partial_{\lambda}\phi_0$ according to Eq. 4.59 is higher as well.

Figure 4.23 – Plasma density contour maps (m^{-3}), ion streamlines in red, and magnetic streamlines in black, for several δ values at $T_e = 10$ eV, and $\chi_{lh} = 0.2$.

Figure 4.24 – contour maps of the ion Mach number for the same cases in Fig. 4.23. Black arrows indicate the ion flow direction. Arrows length scale with the velocity.

Figure 4.25 – Normalized electric potential $e\phi/kT_e$, for the same cases in Fig. 4.23. The origin of the potential $\phi = 0$, is set at $(z, r) = (0, 0)$, and also is the upper cut-off, so in the outer region the potential increases largely and it is not shown (dark red area). The black line is the MN edge, say λ_{ed} streamline.

Figure 4.26 – Perpendicular current $I_{\perp}(\lambda)$ in Amperes (blue line), the solid black is the perpendicular ion current $I_{i\perp}$ (Amp), and the red line is the electron perpendicular current $I_{e\perp}$ (Amp). All cases are from Fig. 4.23.

Figure 4.27 – $\phi_0(\lambda)$ for all cases in Fig. 4.23. $\delta = 256$ (solid black), 32 (dashed black), 16 (dash-dotted black), 8 (solid blue), 4 (dashed blue) and 2 (dash-dotted blue).

δ	$\dot{m}_{right}$ (kg/s)	$\dot{m}_{top}$ (kg/s)	$\dot{m}_{left}$ (kg/s)	$I_{\perp}(\lambda_{ed})$ (Amp)	$\epsilon \dot{m}_i$ (%)
256	5.94E-07	1.63E-06	8.76E-08	2.82	0.23
32	6.22E-07	1.61E-06	7.14E-08	3.35	-0.08
16	6.45E-07	1.63E-06	3.13E-08	4.57	-0.08
8	8.32E-07	1.46E-06	7.95E-09	6.35	-0.19
4	7.81E-07	1.53E-06	3.52E-12	5.47	0.29
2	7.50E-07	1.56E-06	5.53E-12	5.60	0.13

Table 4.3 – Summary of the PIC mass balance error for different δ scenarios and constant $T_e = 10$ eV, $\chi_{lh} = 0.1$ and $\dot{m}_{i0} = 2.31 \cdot 10^{-6}$ kg/s. $\dot{m}_{right,top,left}$ refers to the mass loss at the right, top or left boundaries respectively. $\Delta t_T = 0.45$ (ms) for all simulated cases.

Consequently this yields higher values of ϕ_0 as depicted in Fig. 4.27, thus overconfining the plasma or reducing its divergence (see again Fig. 4.24).

4.6 Discussion

According to the results presented in the previous section, the hybrid model reproduces the results of other models as compared in Ref. [132] in the highly magnetized regime. Furthermore, this adds some realism to the physics, for instance, it includes electron collisions, which are relatively important in the near plume close to the MN throat.

Regarding the electron thermodynamics, this model is still suffering of the same uncertainties of other fluid models. It takes the electrons isothermal everywhere, an assumption that is not fully consistent. Considering thermalized electrons along magnetic streamlines yields some unrealistic limits of the potential drop. For this reason and referring to the conclusions in Chapter 5, the region where isothermality is guaranteed is reduced to a small area close to the throat. Beyond that point, Boltzmann condition is broken, as well as the formulation of the thermalized potential ϕ_0 . For example, the electron temperature drops 20% of its initial value at the source when the magnetic field is reduced 5 times. This would mean that in the current magnetic topology, isothermality could not be assumed beyond $z = 0.065$ m.

About the law of the closure of $I_{\perp}$, it has been proven that the value chosen for δ impacts the shape of the potential in the vicinity of the magnetic nozzle edge λ_{ed} , modifying the results of this magnetic nozzle model. This issue requires further work.

Concerning the aforementioned failure of the particle trajectory integration, which occurs mainly on the plasma-vacuum edge, this is also influenced by the resolution of the electric

potential at this edge, and consequently by the $I_{\perp}(\lambda)$ law. Related to this, it is important to note that the assumption of quasi-neutrality is another limitation of the hybrid model. Note that in the highly magnetized regime, the drop of the density at the edge is of several orders, thus the premise of quasi-neutrality is not necessarily preserved there. So, the edge could be understood as a sheath that separates the high density area from the near vacuum area, in which non-neutral effect become important. This issue is not addressable by the current model.

4.7 Conclusions

A hybrid model, fluid for electrons and PIC for ions has been developed to reproduce the main phenomena of a magnetized/demagnetized plasma flow through a diverging magnetic nozzle. This work has been based on the heritage of *HallMa* in the simulation of HETs performances.

In the high magnetized regime, or nearly collisionless regime, the model matches the results of other models, for instance DIMAGNO [17]. This is because of the similarity of hypotheses concerning electron thermodynamics. Consequently, the same uncertainties of these models also apply to the results presented here.

This model illustrates the transition between the highly magnetized and demagnetized regimes. Ions detach inward in the first regime, while in the second regime, their streamlines become much more divergent than the magnetic streamlines. Special attention has been paid to the study of boundary conditions at the throat, which are determined by the radial equilibrium of the plasma within the cylindrical source. A closure of the charge conservation equation has been also proposed in the area of low density plasma. However, further investigation is required to improve the model performances in this near-vacuum area.

5 Kinetic effects in a magnetized plasma expansion

A steady-state plasma expansion is analysed through a 1D - kinetic model. Plasma is considered fully ionized, strongly magnetized and collisionless. Ion and electron distribution functions are set far upstream, say at the plasma reservoir, and the paraxial evolutions of them are solved consistently with the formation of the ambipolar electric field, which keeps the plasma beam quasi-neutral. Electrons sense the effective potential barriers along the expansion, which is a phenomenon related to both magnetic mirroring and the adverse electric potential. This keeps a large amount of them confined, while a small fraction will escape downstream together with the ion beam to ensure the null current condition. The evolution of the potential field along the expansion, its finite total drop, as well as several macroscopic properties (density, temperature, etc ...) are determined based on the solution provided by this model. Collisionless electron cooling and heat fluxes associated to the energy depletion of the distribution function are discussed as well.

5.1 Introduction

This study is motivated by the current uncertainties on collisionless magnetized plasma expansions, which is the case of the HPT as already reported in Chapters 3 and 4. Although this study can be also useful for some other space plasma thrusters that use a MN to guide and accelerate the plasma jet. For instance, the VASIMR [31, 90], the ECRT (Electron Cyclotron Resonance Thruster) [133] and even the AF-MPD [134, 135].

The precedents on MN theory has motivated modellers to take isothermal, polytropic or adiabatic approaches for the electron temperature. Concerning the isothermal approach, this yields the Boltzmann density-potential relation. However, this is only valid if electrons are close to Maxwellian, an hypothesis that cannot be guaranteed in a collisionless plasma plume.

Furthermore, this presents several inconsistencies: an isothermal plasma expansion requires an infinite source of energy to be coupled to the plasma. The potential drops without bounds, thus an isothermal model, such the one in Ref. [17] and the one in Chapter 4, are not valid to predict essential HPT performances, such as the thrust or the I_{sp} , and these approaches should be disregarded. Only the plasma dynamics in the near field, very close to the MN throat can be accepted as a valid approach. On the other hand, polytropic models are hardly justifiable in a collisionless plasma plume, from a kinetic point of view.

Arefiev and Breizman [136, 128] also studied this problem with a similar approach but they looked for a time-dependent solution. They also proposed the existence of a trapped electron population. However, the justification of this trapped population was conditioned by the presence of a travelling expansion wave far downstream. This wave somehow added momentum to the plasma, even at the far field, apart from connecting the quasi steady flow at the MN throat vicinity to the unsteady flow downstream. Other authors proposed a PIC based model to explore the collisionless and steady state plasma expansion [137], showing that part of the electron population is trapped during the transient start-up. However, that model was for non-magnetized electrons, and it is also difficult to extract a simple and general physical picture from a particular PIC implementation.

The finite potential drop that develops throughout the plasma expansion, including both convergent and divergent sides, is one of the most important results of the current model. It depends on the initial electron temperature, and defines the total ion acceleration, thus it allows an estimation of the specific impulse. So, this model overcomes the limitations of the isothermal and polytropic models on the prediction of these propulsive figures. Since the electron and ion distribution functions are computed at each location, all moments of these distributions can be calculated as well. This includes the perpendicular and parallel temperatures of both species, and the electron heat flux.

This chapter continues with a detailed description of the model, focusing on some non-trivial aspects of the related variables, and presenting all hypotheses as well. Then, it follows with a brief summary of the numerical model implemented to solve self-consistently the relation *magnetic field - ambipolar potential*, forcing both the quasi-neutrality and the zero net current conditions. A parametric analysis of the important plasma quantities (density, fluxes, etc.,) is carried out once the solution is identified. Finally, an in-depth discussion analyses with some peculiarities of this model, asymptotic trends and model limits.

Figure 5.1 – Sketch of the problem geometry and main phenomena.

5.2 Model formulation

The model presented here aims to kinetically describe the steady-state plasma flow through a slender magnetic channel, or magnetic field tube $\mathbf{B}$, assimilable to a convergent-divergent magnetic nozzle. Plasma is considered fully ionized and collisionless, and both species, ions and electrons, are strongly magnetized, i.e. their gyroradii are smaller than any other dimension, $r_{il}, r_{el} \ll R, L$. Although this model is quasi-one-dimensional in the spatial space (it does allow area change), it retains the three dimensions of the velocity space in order to preserve the difference between the particle dynamics along the field ($\mathbf{1}_{\parallel}$) and the dynamics on the plane perpendicular to it ($\mathbf{1}_{\perp}$). Figure 5.1 sketches the problem geometry, and also points out some of the phenomena and hypotheses.

Far upstream, within the plasma reservoir, Maxwellian electrons, of temperature $T_{e\infty}$, and mono-energetic ions, of energy $E_{i\infty}$ are considered. In terms of the velocity distribution function, both species are postulated isotropic within the source. Along the expansion, the model establishes the plasma as quasi-neutral, and consequently an ambipolar potential ϕ , must emerge to keep this condition. The spatial evolution of this potential, or its correspondence to the magnetic field, $\phi(B)$, is the result provided by this model. Regarding the net current carried by the plasma, the model allows dealing with a prescribed current, which would correspond to the case of placing electrodes upstream and downstream. Hereafter, the zero current condition is taken because it is more appropriate to the case of electrodeless EP devices, such as the HPT.

Charged particles sense two different mechanisms or a combination of them while travelling through the magnetic channel. First, the magnetic mirror effect reflects particles, backward at the convergent side, and forward (inverse mirror) at the divergent side, independently of the sign of the charge. In other words, on the convergent side, the perpendicular kinetic energy

increases at the expense of the parallel one, and vice versa on the divergent side. Second, the ambipolar electric field accelerates ions downstream, but it is as a barrier for electrons, in general, it reflects them back to the source. Later, it will be shown that both mechanisms, magnetic mirror and ambipolar field, have to be considered together to determine the electron trapped population.

For simplicity and because ions are typically cold in comparison to electrons, ions are taken to be mono-energetic.

$$f_i(E_i) = \frac{m_i n_{i\infty}}{4\pi} \left(\frac{m_i}{2E_{i\infty}} \right)^{1/2} \delta(E_i - E_{i\infty}), \quad (5.1)$$

which satisfies the normalization $\int f_i d^3 w = n_i$. k is the Boltzmann constant. The ion temperature far upstream $T_{i\infty}$ is proportional to the ion energy as usual $kT_{i\infty} = (2/3)E_{i\infty}$. Concerning electrons, they are considered Maxwellian far upstream,

$$f_e(E_e) = n_{e\infty} \left(\frac{m_e}{2\pi kT_{e\infty}} \right)^{3/2} \exp\left(-\frac{E_e}{kT_{e\infty}}\right), \quad (5.2)$$

which also allows the normalization $\int f_e d^3 w = n_e$. Let us consider now the Vlasov's equation, which is the collisionless limit of the Boltzmann equation,

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{x}} + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = 0. \quad (5.3)$$

Vlasov's equation states that along particle trajectories, the distribution function f_α is conserved, and it becomes a function of the constants of motion. In this problem, these two constants are the particle energy E_α and its magnetic moment μ_α . Since distribution functions are known at the source, and only depend on the energy (Eq. 5.1, 5.2), this same dependence applies at every point connected to the source.

Particle energy can be written as follows,

$$\begin{aligned} E_\alpha &= m_\alpha \frac{w_{\alpha\parallel}^2}{2} + m_\alpha \frac{w_{\alpha\perp}^2}{2} + q_\alpha \phi, \text{ with} \\ \mu_\alpha &= \frac{m_\alpha w_{\alpha\perp}^2}{2B} = \text{const}, \\ \text{yielding} \\ E_\alpha &= m_\alpha \frac{w_{\alpha\parallel}^2}{2} + \mu_\alpha B + q_\alpha \phi. \end{aligned} \quad (5.4)$$

$\alpha = i, e$ for ions and electrons respectively. $q_i = e$ for ions and $q_e = -e$ for electrons. The electric potential, ϕ , is assumed a monotonic decreasing function, from $\phi = 0$ at the source far upstream to $\phi = \phi_{out}$, which is unknown, at the end of the expansion far downstream.

For a given particle energy, E_α , and a specific position defined by $B(\phi)$, it is possible to define the particle turning point, or in other words, the location in which the parallel velocity becomes

Figure 5.2 – Left: $\hat{\mu}_{iM}(\varepsilon_i = E_{i\infty}/kT_{e\infty} = 0.1)$ (dashed line) and $\hat{\mu}_{eM}(\varepsilon_e)$ for $\varepsilon_e = E_e/|e\phi_{out}| = \{0.49, 0.8, 0.9, 1.26\}$. $\varepsilon_e = 0.49$ corresponds to $E_e = E_e^*$. No local minima exist for electrons with $\varepsilon_e < \varepsilon_e^*$. Right: Further details of the curve $\mu_{eM}(\varepsilon_e = 0.8, \Phi)$. This is valid for all electrons, whose energy belongs to the range $[E_e^*, -e\phi_{out}]$. Three different areas, A, B, C, are identified. The shaded area, B_2 , corresponds to the region filled by trapped electrons. All magnitudes are dimensionless. Both figures are for a known solution of $\Phi(\hat{B})$ and are presented beforehand in order to illustrate the problem topology.

zero, $w_{\alpha\parallel} = 0$, this is,

$$\mu_{\alpha M}(E, B) = \frac{E_\alpha - q\phi}{B}. \quad (5.5)$$

Since B is peaked at the throat and ϕ decreases monotonically, $\mu_{\alpha M}$ could present a local minimum at a point that will be henceforth identified by the subscript T,

$$\mu_{\alpha T}(E_\alpha) = \min\{\mu_{\alpha M}(E_\alpha, B)\} = (E_\alpha - q\phi_T)/B_T \quad (5.6)$$

5.2.1 Details on $\mu_{\alpha M}$ curves

Once the solution $B(\phi)$ is known, the T position (B_T, ϕ_T) is a function that only depends on the particle energy E_α . For ions it is straightforward to conclude that this minimum must appear on the convergent side. The physical interpretation of this minimum is that only ions with $\mu_i < \mu_{iT}(E_i)$ can escape downstream, and the others, $\mu_i \geq \mu_{iT}(E_i)$, are reflected back to the source.

For electrons, the situation is indeed more complex. The difference appears due to the opposite sign of $q = -e$ in Eq. 5.5. This makes the numerator to decrease continuously, and then $\mu_{eM} = 0$ at $e\phi = -E_e, \forall E_e \leq -e\phi_{out}$. This clearly sorts out electrons into two groups. First, high energy electrons, with $E_e > -e\phi_{out}$, have energy enough to escape downstream, after overcoming the magnetic bottle. For all these electrons, μ_{eM} presents an absolute minimum on the divergent side, $\mu_{eT}(E_e) = \min(\mu_{eM})$ at (B_T, ϕ_T) . Beyond this minimum μ_{eM} diverges to infinity because the numerator is positive and bounded ($E_e - e\phi_{out}$), while the denominator, B , approaches 0. Second, confined electrons, with $E_e \leq -e\phi_{out}$, do not have energy enough to

Figure 5.3 – First derivative of $B(\Phi)$, $\partial B/\partial\Phi$ (red line). Other lines depict $-\mu_{eM}^{-1} = -B/(\varepsilon_e - \Phi)$ for different energy levels: confined electrons (blue), critical energy ε_e^* (dashed blue), $\varepsilon_e = 1$ (black), $\varepsilon_e = 1.5$ (green). All magnitudes are dimensionless.

overcome the adverse potential. For each energy this occurs at the point in which the electron energy matches the local electric potential, $-e\phi = E_e$. Figure 5.2 advances how μ_{im} and μ_{em} behave for a given solution $\phi(B)$.

Local minima of curves $\mu_{eM}(E_e, \phi)$ for a given solution of the model $\phi(B)$, are always placed on the divergent side, because the numerator of Eq. 5.5 is a decreasing function, so it requires the denominator to decrease (divergent side) to generate a minimum. Mathematically, this can be demonstrated by taking the derivative of μ_{eM} against $-e\phi$, and equating it to 0. This yields the next condition for μ_{eT} ,

$$\frac{\partial \mu_{eM}}{\partial(-e\phi)} = 0 \rightarrow \mu_{eT}(E_e) = - \left(\frac{\partial B}{\partial(-e\phi)} \Big|_{e\phi_T(E_e)} \right)^{-1}. \quad (5.7)$$

Positive solutions of μ_{eT} are only possible if B decreases, meaning that these minima are always at the divergent side. Consequently, the shape of the unknown solution $B(\phi)$, is a key point to determine accurately these minima. Note also that $\partial B/\partial(e\phi)$ is zero at the throat, and negative everywhere on the divergent side. Far downstream, if the potential reaches an asymptotic limit, $\phi \rightarrow \phi_{out}$, the derivative approaches 0 again, $\partial B/\partial(-e\phi) \rightarrow 0^-$. This means, that $\partial B/\partial(-e\phi)$ presents at least a minimum on the divergent side, or in other words, B has an inflection point, $\partial^2 B/\partial(-e\phi)^2 = 0$.

Considering that $\mu_{eT}(E_e) = (E_e + e\phi_T)/B_T$ and accounting for all the details about $\partial B/\partial(-e\phi)$, it can be conclude that Eq. 5.7 could present a double solution for all electron energies contained within the range $E_e \in [E_e^*, -e\phi_{out}]$: the first solution corresponds to a local minimum of μ_{eM} and the other one to a local maximum. This is illustrated in Figure 5.3, which plots together $\partial B/\partial(-e\phi)$ and $-\mu_{eM}^{-1}$. Note that $\partial B/\partial(-e\phi)$ intersects these curves at locations where

Figure 5.4 – Ion velocity distribution upstream (left) and downstream (right) of B_{iT} . These figures are also valid for electrons with $E_e > -e\phi_{out}$. The radius of these spheres depends on the chosen kinetic energy level. As one moves upstream from point B_{iT} , the distribution approaches a full sphere. At point B_{iT} , it is the forward half of the sphere. As one moves further downstream, the distribution is a narrowing spherical segment, as on the right figure.

the minima or maxima of μ_{eM} are placed, pointing out that Eq. 5.7 presents two solutions within the mentioned interval, and only one solution for high energy electrons ($E > -e\phi_{out}$). E_e^* is defined as the energy whose $\mu_{eM}(E_e^*, \phi)$ curve presents an inflection point at (B_T^*, ϕ_T^*) , thus:

$$\frac{\partial \mu_{eM}}{\partial (-e\phi)} = \frac{\partial^2 \mu_{eM}}{\partial (-e\phi)^2} = 0 \text{ at } \phi_T^*(E_e^*),$$

$$\frac{\partial^2 B}{\partial (-e\phi)^2} = 0 \text{ at } \phi_T^*(E_e^*).$$

For the rest of electrons with $E_e < E_e^*$, their curves $\mu_{eM}(E_e, \phi)$ monotonically decrease to 0 without presenting any local minima.

The strong hypothesis of this model hides behind the shape of these curves μ_{eM} within the range of energies $E_e \in [E_e^*, -e\phi_{out}]$. *This model implicitly assumes that a trapped population of electrons exists in a wide area of the divergent side of the expansion, which is totally decoupled from the source. The model also assumes that they keep the same distribution function as if they were connected to the source (Eq. 5.2).* As a consequence of this hypothesis, confined electrons with $E_e < -e\phi_{out}$ are isotropic everywhere along the full expansion.

Figure 5.5 – Left: Isotropic distribution function in regions A and C of Fig. 5.2. Right: Isotropic distribution function in region B . Two spherical cones, one moving forward ($w_{e\parallel} > 0$) and the other moving backward ($w_{e\parallel} < 0$), corresponds to the region B_1 of Fig. 5.2. The velocity distribution function of trapped electrons is indicated by the shaded area (region B_2). This population fills out the sphere, and keeps the distribution isotropic as well.

5.2.2 Angular distribution functions

The behaviour of the ion distribution function upstream and downstream of B_{iT} can be further understood considering this function as a spherical shell of radius w_i , in the velocity space,

$$w_i = \sqrt{\frac{2(E_i - e\phi)}{m_i}}. \quad (5.8)$$

The angle that defines the spherical shell is $\theta_i(E_i, B)$,

$$\theta_i(E_i, B) = \sin^{-1} \sqrt{\frac{\mu_{iT}(E_i)}{\mu_{iM}(E_i, B)}}. \quad (5.9)$$

At $B = B_{iT}$, the distribution becomes a semi-sphere, $\theta_i(E_i, B_{iT}) = \theta_{iT} = \pi/2$, meaning that only ions marching forward exist at this location. The sketch of the ion distribution function is depicted in Figure 5.4.

Concerning electrons, it is necessary to distinguish between the response of high energy electrons and the behaviour of confined electrons. For the first group, the shape of the distribution function is similar to the one shown for ions except that for electrons the radius of the spherical shell always decreases due to the adverse potential,

$$w_e = \sqrt{\frac{2(E_e + e\phi)}{m_e}}. \quad (5.10)$$

The angle $\theta_e(E_e, B)$ is,

$$\theta_e(E_e, B) = \sin^{-1} \sqrt{\frac{\mu_{eT}(E_e)}{\mu_{eM}(E_e, B)}}. \quad (5.11)$$

Regarding the second group, the confined electrons, and specifically for electrons with energies below E_e^* , the distribution function is isotropic everywhere (full sphere). This is because all these electrons are reflected back to the source, the distribution is isotropic there, and this keeps the same form (Eq. 5.2) along these trajectories. For the special subgroup, $E_e \in [E_e^*, -e\phi_{out}]$, the distribution is isotropic as well. Nevertheless, μ_{eT} is only a local minimum, while the absolute minimum is $\min(\mu_{eM}) = 0$ at $\phi = -E_e/e$. So, the definition of Eq. 5.11 is not consistent everywhere. If this formula is applied at $\phi < \phi_{eT}$ (upstream of T, or area A in Figure 5.2), the distribution function would be a spherical shell with a loss cone. This is understood as the depletion of electrons with a pitch angle within the interval $\theta_e \in [\pi, \pi - \sin^{-1} \sqrt{\mu_{eT}/\mu_{eM}}]$. However, there is no loss of particles, because these electrons with $\mu_e < \mu_{eT}$, passing through the bottle defined by this local minima, are reflected back further downstream, because μ_{eM} drops finally to 0 at some point, see for instance region C in Figure 5.2. Once these electrons are reflected, the distribution is filled completing the sphere. Electrons flowing through region B_1 of Figure 5.2, define a double cone in the velocity space, one moving forward ($w_{e\parallel} > 0$) and the other moving backward $w_{e\parallel} < 0$ (see white areas in Figure 5.5). The semi-angle of these cones, can be computed with Eq. 5.11. Note also that these cones become hemispheres at two locations, ϕ_T and at the border between regions B_1 and C, where $\mu_{eM} = \mu_{eT}$. In region C, electrons behave as if they belong to the group of low energies $E_e < E_e^*$. The high pitch angles in the space velocity is refilled by trapped electrons in the parametric space labelled by B_2 , shaded zone in Figures 5.2, 5.5.

5.2.3 Quasi-neutrality equation: electron and ion density

After postulating the shape of the distribution functions, the electron/ion density is computed taking the 0^{th} moment of the corresponding distribution. Concerning ions, the average of any quantity χ_i is written as,

$$\langle \chi_i \rangle = \frac{n_{i\infty}}{n_i} \int_{\Omega_{w_i}} \chi_i f_i d^3 w_i = \frac{1}{n_i} \frac{2\pi B n_{i\infty}}{m_i^2} \int_{\Omega_{E_i}} \int_{\Omega_{\mu_i}} \frac{\chi_i(E_i, \mu_i)}{|w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i, \quad (5.12)$$

$$\text{and } w_{i\parallel} = \pm \sqrt{\frac{2}{m_i} \{E_i - e\phi - \mu_i B\}} = \pm \sqrt{\frac{2B}{m_i} \{\mu_{iM} - \mu_i\}}. \quad (5.13)$$

In this equation the coordinates transformation from the 3D velocity space to the $E - \mu$ space has already been applied, see Appendix C for further details on this transformation. Assuming that the potential profile is known $\phi(B)$, the ion density is found integrating Eq. 5.12 with $\chi_i = 1$. However, due to the dependence of f_i on μ_i , special attention must be focused on the integral limits over the μ space. So the integrand counts once in the range $\mu_i \in [0, \mu_{iT})$, because there are only forward ions. Otherwise, $\mu_i \in [\mu_{iT}, \mu_{iM}]$, the integrand counts twice

because there are ions moving forward and backward due to reflection. Then, the expression for the ion density is,

$$n_i(\phi) = \frac{2\pi B n_{i\infty}}{m_i^2} \int_{\Omega_{E_i}} \left[\int_0^{\mu_{iT}} \frac{f_i(E_i, \mu_i)}{|w_{i\parallel}|} d\mu_i + 2H(e\phi - e\phi_T) \int_{\mu_{iT}}^{\mu_{iM}} \frac{f_i(E_i, \mu_i)}{|w_{i\parallel}|} d\mu_i \right] dE_i. \quad (5.14)$$

H is the Heaviside or step function. Finally, integrating over the energy and magnetic moment space, the ion density results as

$$\frac{n_i(\phi)}{n_{i\infty}} = \frac{1}{2} \left(\sqrt{1 - \frac{e\phi}{E_i}} \mp \sqrt{1 - \frac{e\phi + \mu_{iT}B}{E_i}} \right). \quad (5.15)$$

The negative sign applies downstream of B_{iT} , where $\phi < \phi_{iT}$, otherwise the positive sign is taken. Analogously for electrons, the average of any quantity χ_e is written as,

$$\begin{aligned} \langle \chi_e \rangle &= \frac{n_{e\infty}}{n_e} \int_{\Omega_{w_e}} \chi_e f_e d^3 w_e = \\ &= \frac{1}{n_e} \frac{2\pi B n_{e\infty}}{m_e^2} \int_{\Omega_{E_e}} \int_{\Omega_{\mu_e}} \frac{\chi_e(E_e, \mu_e)}{|w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e. \end{aligned} \quad (5.16)$$

$$\text{and } w_{e\parallel} = \pm \sqrt{\frac{2}{m_e} \{E_e + e\phi - \mu_{eT}B\}} = \pm \sqrt{\frac{2B}{m_e} \{\mu_{eM} - \mu_e\}}, \quad (5.17)$$

The electron density is found by integrating Eq. 5.16 with $\chi_e = 1$. The definition of the integral limits also requires discussing some details. At a given position $-e\phi(B)$, only electrons with $E_e > -e\phi(B)$ add some contribution to the density or other higher order moments. So the integral limits over the energy space go from $-e\phi(B)$ to ∞ . The upper limit is the consequence of postulating electrons as Maxwellian at the source. Over the μ_e space, all confined electrons, either reflected or trapped, count twice for the density (forward and backward marching electrons), and the range is $\mu_e \in [0, \mu_{eM}]$. The high energy electrons behave as ions, so the integrand counts once in the range $\mu_e \in [0, \mu_{eT}]$, because there are only forward electrons, otherwise, in the range $\mu_e \in [\mu_{eT}, \mu_{eM}]$, the integrand counts twice. The integral over the energy space is divided according to the two groups of electrons: confined, $E_e \in [-e\phi, -e\phi_{out}]$, and high energy, $E_e > -e\phi_{out}$. The expression of the electron density is therefore found to be,

$$\begin{aligned} \frac{n_e(\phi)}{n_{e\infty}} &= e^{e\phi/kT_{e\infty}} e r f \sqrt{\frac{e(\phi - \phi_{\infty})}{kT_{e\infty}}} - \frac{2}{\sqrt{\pi}} e^{e\phi_{\infty}/kT_{e\infty}} \sqrt{\frac{e(\phi - \phi_{\infty})}{kT_{e\infty}}} + \\ &= \frac{1}{\pi (kT_{e\infty})^{3/2}} \int_{-e\phi_{\infty}}^{\infty} \left[\sqrt{E_e + e\phi} \mp \sqrt{E_e + e\phi - \mu_{eT}B} \right] \exp(-E_e/kT_{e\infty}) dE_e. \end{aligned} \quad (5.18)$$

The two first terms are analytical and come from the contribution of confined electrons. The integral term belongs to the contribution of high energy electrons. For them, the negative sign applies downstream of $B_{eT}(E_e)$, otherwise, the positive sign is taken.

Hereafter in this Chapter, the following dimensionless variables will be considered,

$\hat{B} = \frac{B}{B_{Max}}$	Magnetic field
$\Phi = \frac{\phi}{\phi_{out}}$	Electric potential
$\varepsilon_i = \frac{E_{i\infty}}{kT_{e\infty}}$	Ion energy
$\varepsilon_e = \frac{E_e}{ e\phi_{out} }$	Electron energy
$\hat{\mu}_i = \frac{\mu_i B_{Max}}{ e\phi_{\infty} }$	Ion magnetic moment
$\hat{\mu}_e = \frac{\mu_e B_{Max}}{ e\phi_{out} }$	Electron magnetic moment
$\Psi = \frac{ e\phi_{out} }{kT_{e\infty}}$	Total drop of the electric potential
$\hat{n}_\alpha = \frac{n_\alpha}{n_{\alpha\infty}}$	Densities
$\hat{u}_\alpha = u_\alpha / \sqrt{kT_{e\infty}/m_i}$	Velocities

Equations 5.15 and 5.18 in terms of the dimensionless variables are written below,

$$\hat{n}_i = \frac{1}{2} \left[\sqrt{1 + \frac{\Psi}{\varepsilon_i} \Phi} \mp \sqrt{1 + \frac{\Psi}{\varepsilon_i} \Phi - \frac{\Psi}{\varepsilon_i} \hat{\mu}_{iT} \hat{B}} \right]. \quad (5.19)$$

$$\begin{aligned} \hat{n}_e &= \exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{2}{\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} \\ &+ \frac{\Psi^{3/2}}{\sqrt{\pi}} \int_1^\infty \left[\sqrt{\varepsilon_e - \Phi} \mp \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \right] \exp(-\Psi\varepsilon_e) d\varepsilon_e. \end{aligned} \quad (5.20)$$

The quasi-neutrality equation $\hat{n}_i(\Phi) \sim \hat{n}_e(\Phi)$ determines an implicit equation for $\Phi(\hat{B})$,

$$\begin{aligned} \sqrt{1 + \frac{\Psi}{\varepsilon_i} \Phi} \mp \sqrt{1 + \frac{\Psi}{\varepsilon_i} \Phi - \frac{\Psi}{\varepsilon_i} \hat{\mu}_{iT} \hat{B}} &= \\ 2e^{-\Psi\Phi} \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{4}{\sqrt{\pi}} e^{-\Psi} \sqrt{\Psi(1-\Phi)} & \\ + \frac{2\Psi^{3/2}}{\sqrt{\pi}} \int_1^\infty \left[\sqrt{\varepsilon_e - \Phi} \mp \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \right] \exp(-\Psi\varepsilon_e) d\varepsilon_e. & \end{aligned} \quad (5.21)$$

In the equation above, ε_i is an input. Ψ , the total potential drop, is part of the solution, as well as the potential profile $\Phi(\hat{B})$.

5.2.4 Current-free condition: particle and charge fluxes

Particle fluxes are computed as the first moment of the distribution function, taking $\chi_i = w_{i\parallel}$ and $\chi_e = w_{e\parallel}$, for ions and electrons respectively. This is an odd moment, so only particles

that escape contribute to the net flux. This means that only the range $\mu_\alpha \in [0, \mu_{\alpha T}]$ must be accounted for when computing the integral over the magnetic moment space. Regarding electrons, only those overcoming the total potential drop are included, this reduces the integral over energies to the group of high energy electrons.

The ion flux, $\Gamma_i = n_i \langle w_{i\parallel} \rangle = n_i u_i$, is computed as follows,

$$\Gamma_i(\phi) = \frac{2\pi B}{m_i^2} \int_{\Omega_{E_i}} \int_0^{\mu_{iT}} \frac{w_{i\parallel}}{|w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i = \frac{n_\infty B \mu_{iT}}{\sqrt{8E_{i\infty}} m_i}. \quad (5.22)$$

Fluxes are dimensionless with the reference flux $n_\infty c_s$. $c_s = \sqrt{kT_{e\infty}/m_i}$ is the ion sound speed. According to this, the dimensionless ion flux becomes,

$$\Gamma_i(\phi) = \frac{\Psi \hat{\mu}_{iT} \hat{B}}{\sqrt{8\varepsilon_i}}, \quad (5.23)$$

For electrons, their flux is $\Gamma_e = n_e \langle w_{e\parallel} \rangle = n_e u_e$,

$$\Gamma_e(\phi) = \frac{2\pi B}{m_e^2} \int_{-e\phi_\infty}^\infty \int_0^{\mu_{eT}} \frac{w_{e\parallel}}{|w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e, \quad (5.24)$$

and using Eq. 5.2 for the distribution function f_e , it yields,

$$\Gamma_e(\phi) = \frac{B n_\infty}{\sqrt{2\pi m_e}} \left(\frac{1}{kT_{e\infty}} \right)^{3/2} \int_{-e\phi_\infty}^\infty \mu_{eT}(E_e) \exp\left(\frac{-E_e}{kT_{e\infty}}\right) dE_e, \quad (5.25)$$

that is rewritten in dimensionless variables using the same reference flux, $n_\infty c_s$,

$$\Gamma_e(\phi) = \sqrt{\frac{m_i}{2\pi m_e}} \Psi^2 \hat{B} \int_1^\infty \hat{\mu}_{eT}(\varepsilon_e) \exp(-\Psi \varepsilon_e) d\varepsilon_e. \quad (5.26)$$

Concerning charge fluxes, these are proportional to particle fluxes, under the hypothesis that ions are singly charged. Considering the reference charge flux, $e n_\infty c_s$, the dimensionless form of the ion charge flux is equivalent to the particle flux,

$$j_i(\Phi) = \Gamma_i(\Phi), \quad (5.27)$$

while for electrons is,

$$j_e(\Phi) = -\Gamma_e(\Phi). \quad (5.28)$$

Then, the net flux of charge $j(\Phi)$ is the sum of ion and electron charge fluxes,

$$j = j_i + j_e. \quad (5.29)$$

For any selected value of j^* , and for instance the current-free scenario that corresponds to $j^* = 0$, this yields a second equation, that together with Eq. 5.21 allows determining the solution $\Phi(\hat{B})$ and the total potential drop Ψ ,

$$j^* = \frac{\hat{B} \Psi \hat{\mu}_{iT}}{\sqrt{8\varepsilon_i}} - \sqrt{\frac{m_i}{2\pi m_e}} \Psi^2 \hat{B} \int_1^\infty \hat{\mu}_{eT}(\varepsilon_e) \exp(-\Psi \varepsilon_e) d\varepsilon_e. \quad (5.30)$$

This equation introduces the ion to electron mass ratio m_i/m_e , which is another input parameter of this model. The quasi-neutrality condition, Eq. 5.21, does not depend on this parameter. But its influence appears through the total potential drop Ψ , which is indeed determined by Eq. 5.30. Note that the current condition is a “global” condition because it is constant along

Figure 5.6 – Sketch of the mesh $(\Phi_j, \hat{B}_j)$ and the initial approach to the solution $\Phi_j(\hat{B}_j)$. The horizontal axis is a notional representation of axial distance.

the expansion. On the contrary, the quasi-neutrality condition is a local state that must be imposed everywhere along the expansion.

To compute the net current carried by the plasma, it is necessary to account for the continuity equation in steady state and without production/loss of particles.

$$\nabla \cdot (n_\alpha \vec{u}_\alpha) = 0, \quad (5.31)$$

and for the 1D-paraxial geometry of this model, this is

$$\frac{\Gamma_\alpha}{B} = \text{constant}. \quad (5.32)$$

So, the net current carried by the plasma according Eq. 5.30 and 5.32 is, in dimensionless form,

$$I = \frac{\Psi \hat{\mu}_{iT}}{\sqrt{8\varepsilon_i}} - \sqrt{\frac{m_i}{2\pi m_e}} \Psi^2 \int_1^\infty \hat{\mu}_{eT}(\varepsilon_e) \exp(-\Psi \varepsilon_e) d\varepsilon_e. \quad (5.33)$$

$I = 0$ for the plasma current-free condition.

5.3 Numerical method

The method implemented in this work consists of solving iteratively Eqs. 5.21 and 5.33 given the dimensionless inputs m_i/m_e and ε_i . The steps to follow are listed below:

1. Guess the function $\Phi(\hat{B})$ and the initial approximation for Ψ . Φ is assumed a monotonically increasing function of distance, between $\Phi = 0$ at the source, and $\Phi = 1$ infinitely downstream. $\Phi(\hat{B})$ is evaluated on a mesh of m nodes, so we consider pairs of values $(\Phi_j, \hat{B}_j)$. This mesh is sketched in Figure 5.6.
2. Compute the function in Eq. 5.5 for ions $\hat{\mu}_{iM}(\varepsilon_i, \hat{B}) = \frac{\varepsilon_i/\Psi + \Phi}{\hat{B}}$, and find its minimum $\hat{\mu}_{iT}(\varepsilon_i)$.

3. For each of a selected electron energy within the high energy range $1 < \varepsilon_e < \infty$, compute $\hat{\mu}_{eM}(\varepsilon_e, \hat{B}) = \frac{\varepsilon_e - \Phi}{\hat{B}}$, and find its minimum $\hat{\mu}_{eT}(\varepsilon_e)$.
4. Evaluate the integral term in Eq. 5.21 using the Laguerre-Gauss(LG) quadrature.
5. Evaluate ion and electron densities, Eq. 5.19 and 5.20 and compute the relative neutrality error in all the m nodes of the mesh, $\epsilon(\Phi_j) = |1 - n_e(\Phi_j)/n_i(\Phi_j)|$.
6. Compute also the imbalance on the current-free condition, Eq. 5.33. In this step, it is possible to impose another net current I^* , and its imbalance is then $I - I^*$.
7. Compute the modified sum of squares that quantifies the overall error, $Err = \sum_{j=1}^m \epsilon^2(\hat{B}_j) + m(I - I^*)^2$.
8. Iterate from step 1 until function Err is lower than a desired numerical tolerance. The numerical algorithm to minimize the error is a trust region method based on interior point techniques, as the one described in Refs. [138, 139]. This method is very robust for solving the current problem, although this robustness penalizes the convergence rate.

In the third step above, the selected energies for electrons correspond to the nodes of the LG quadrature. This quadrature is useful to efficiently compute the contribution of energetic electrons to any moment of the electron distribution function. In this work, the results are computed with only 5 nodes in the LG quadrature, which allows to integrate exactly polynomials of order 9 [140], and ensures a good convergence of the solution.

5.4 Results and Parametric analysis

Once the electric potential function $\Phi(\hat{B})$ and the total potential drop Ψ are determined, it is possible to compute all moments of the ion and electron distribution functions in order to show the evolution of different macroscopic variables: densities, fluxes, temperatures, heat fluxes, etc.

Before computing these macroscopic variables it is interesting to show the parametric response of $\hat{\mu}_{eT}$, $\hat{B}_{eT}$, Φ_{eT} , due to variations of input parameters, m_i/m_e and ε_i . All these variables have been necessary to determine each moment, including those for computing the solution itself, density and particle flux. These variables are plotted in Figure 5.7, illustrating some important trends.

Figure 5.7 – Parametric analysis of: (a) $\hat{\mu}_{eT}(\varepsilon_e)$, (b) $\hat{B}_{eT}(\Phi_{eT})$, and (c) $\Phi_{eT}(\varepsilon_e)$. These results are for different solutions $\Phi(\hat{B})$, keeping constant the ion energy $\varepsilon_i = 0.1$, and each line correspond to $m_i/m_e = 10^3, 10^4, 10^5$, the arrow indicates the mass ratio increases.

First, $\hat{\mu}_{eT}$ increases almost linearly with ε_e over all the energy range in which $\hat{\mu}_{eM}$ presents a minimum, $\varepsilon_e > \varepsilon_e^*$. Second, the higher the energy is, the lower the corresponding $\Phi_T(\varepsilon_e)$ becomes. Consequently, the minima of curves $\hat{\mu}_{eM}$ are closer and closer to the MN throat for higher energies. At the limit $\varepsilon_e \rightarrow \infty$ the minimum of this curve coincides exactly with the magnetic throat. This is a conclusion that can be deduced also from Eq. 5.7. Third, for heavier ions with larger m_i/m_e , the locations of $\hat{\mu}_{eM}$ minima, occupy a narrower range in terms of Φ and similar range in terms of $\hat{B}$.

Figure 5.8 analyses parametrically the solution $\Phi(\hat{B})$ for different values of the mass ratio and the ion energy. The first one proves again that the dimensionless potential drop on the convergent side is smaller for heavier ions, and the magnetic field peak shifts to the left. On the other hand, higher ion energies make larger the drop of potential on the convergent side. So, magnetic field maxima move to the right. The total potential drop Ψ is analysed in Figure 5.8(c). This increases for larger m_i/m_e , while its variation with ε_i only seems significant for $\varepsilon_i > 1$. The high ion energy scenario does not have applicability in the case of the HPT, but may have some interest in the VASIMR case [Table A.9].

Figure 5.8 – Solution $\hat{B}(\Phi)$ for different values of (a) the mass ratio $m_i/m_e = 10^3, 10^4, 10^5$ (with $\epsilon_i = 0.1$) and (b) the ion energy $\epsilon_i = 0.01, 0.1, 1, 10$ (with $m_i/m_e = 10^4$). (c) Parametric analysis of the total potential drop $\Psi = \Psi(m_i/m_e, \epsilon_i)$.

5.4.1 Plasma density and velocity

The density $n_i(\Phi) = n_e(\Phi)$ and the ion velocity $u_i = \langle w_{i\parallel} \rangle$, or Mach number, are depicted in Figure 5.9. Density is also compared to the Boltzmann equilibrium, dashed lines in Figure 5.9 (a). The density predicted according to Boltzmann is,

$$n_e(\phi) = n_{e\infty} \exp\left(\frac{e\phi}{kT_{e\infty}}\right). \quad (5.34)$$

Because Boltzmann yields an unbounded drop of the potential, in the mentioned picture, the density variation is only compared within the range $\Phi = [0, 1]$, which is equivalent to say $\phi = [0, -\Psi kT_{e\infty}]$. Boltzmann's law is accurate in the subsonic part of the expansion (convergent side), and differs more and more while moving forward along the expansion.

Note that the electron velocity is also $u_e = \langle w_{e\parallel} \rangle = u_i$ because the forcing of the current free condition. The ion final velocity is larger as m_i/m_e increases because it is proportional to the total potential drop Ψ , and this increases with the ion to electron mass ratio (Fig. 5.8).

$$\hat{u}_i = \hat{u}_e = \frac{\langle w_{i\parallel} \rangle}{c_s} = \frac{\Psi \hat{\mu}_{iT} \hat{B}}{\sqrt{8\epsilon_i \hat{n}_i}}. \quad (5.35)$$

Figure 5.9 – (a) Dimensionless density $n_i(\Phi) = n_e(\Phi)$ at ion energy $\varepsilon_i = 0.1$ for mass ratio $m_i/m_e = 10^3, 10^4, 10^5$. Dashed lines show the Boltzmann equilibrium density for comparison. For them, the unbounded potential drop is cut at Ψ . (b) Dimensionless density at mass ratio $m_i/m_e = 10^4$ for ion energy $\varepsilon_i = 0.01, 0.1, 1, 10$. (c, d) Ion Mach number $u_i = u_e$ using the same parametric space as in Figure 5.8. Hereafter, asterisks indicate the MN throat location $\Phi(\hat{B} = 1)$.

5.4.2 Temperatures

Ion temperature is computed by taking the 2nd moment of the IDF. It is necessary to distinguish the parallel temperature $T_{i\parallel}(\phi)$ from the perpendicular temperature $T_{i\perp}(\phi)$ because here the isotropy is not necessarily being preserved. Consequently, this division is more convenient than dealing directly with the total temperature, $T_i = (1/3)T_{i\parallel} + (2/3)T_{i\perp}$. The first component, $T_{i\parallel}$, is a measure of the random ion axial kinetic energy, applying the definition in Eq. 5.12,

$$kT_{i\parallel}(\phi) = \langle m_i c_{i\parallel}^2 \rangle = \frac{1}{n_i} \frac{2\pi B}{m_i^2} \int_{\Omega_{E_i}} \int_{\Omega_{\mu_i}} \frac{m_i (w_{i\parallel} - u_i)^2}{|w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i, \quad (5.36)$$

$$\text{with } c_{i\parallel} = w_{i\parallel} - u_i. \quad (5.37)$$

Integrating the previous equation taking into account that it is an even moment, (so upstream of B_{iT} it is necessary to consider the double sign of $w_{i\parallel}$, depending whether the ion is moving forward or backward),

$$kT_{i\parallel}(\phi) = \frac{n_\infty}{n_i} \left\{ \frac{1}{3\sqrt{E_{i\infty}}} \left[(E_{i\infty} - e\phi)^{3/2} \pm (E_{i\infty} - e\phi - \mu_{iT}B)^{3/2} \right] - \left(\frac{m_i}{2E_{i\infty}} \right)^{1/2} \mu_{iT} B u_i + \frac{m_i}{2\sqrt{E_{i\infty}}} u_i^2 \left[(E_{i\infty} - e\phi)^{1/2} \pm (E_{i\infty} - e\phi - \mu_{iT}B)^{1/2} \right] \right\} \quad (5.38)$$

The double sign criterion is the same as in Eq. 5.15. Using $kT_{i\infty} = (2/3)E_{i\infty}$ as the reference temperature, the dimensionless ion parallel temperature results as follows,

$$T_{i\parallel}(\Phi) = \frac{kT_{i\parallel}(\phi)}{kT_{i\infty}} = \frac{1}{\hat{n}_i} \left[\frac{1}{2\epsilon_i^{3/2}} \left\{ (\epsilon_i + \Psi\Phi)^{3/2} \pm (\epsilon_i + \Psi\Phi - \Psi\hat{\mu}_{iT}\hat{B})^{3/2} \right\} - \frac{3\hat{\mu}_{iT}\Psi\hat{B}\hat{u}_i}{(2\epsilon)^{3/2}} + \frac{3\hat{u}_i^2}{4\epsilon_i^{3/2}} \left\{ (\epsilon_i + \Psi\Phi)^{1/2} \pm (\epsilon_i + \Psi\Phi - \Psi\hat{\mu}_{iT}\hat{B})^{1/2} \right\} \right] \quad (5.39)$$

Note that the previous equation is consistent at least at the boundaries of the spatial domain, because its evaluation at the source and far downstream ($\Phi = 0, 1$) results in $T_{i\parallel} = 1$ and $T_{i\parallel} = 0$ respectively as expected for a mono-energetic ion distribution.

The ion perpendicular temperature $T_{i\perp}$ measures the evolution of the ion perpendicular kinetic energy,

$$kT_{i\perp}(\phi) = \langle m_i \frac{w_{i\perp}^2}{2} \rangle = B \langle \mu_i \rangle = \frac{1}{n_i} \frac{2\pi B}{m_i^2} \int_{\Omega_{E_i}} \int_{\Omega_{\mu_i}} \frac{B\mu_i}{|w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i, \quad (5.40)$$

and following the same procedure as before, this is integrated resulting in,

$$kT_{i\perp}(\phi) = \frac{n_\infty}{n_i} \frac{1}{E_{i\infty}^{1/2}} \left\{ \frac{1}{3} (E_{i\infty} - e\phi)^{3/2} \pm \frac{1}{2} B \mu_{iT} \sqrt{E_{i\infty} - e\phi - \mu_{iT}B} \pm \frac{1}{3} (E_{i\infty} - e\phi - \mu_{iT}B)^{3/2} \right\}. \quad (5.41)$$

In dimensionless units this becomes,

$$T_{i\perp}(\Phi) = \frac{kT_{i\perp}}{kT_{i\infty}} = \frac{1}{\hat{n}_i \epsilon_i^{3/2}} \left\{ \frac{1}{2} (\epsilon_i + \Psi\Phi)^{3/2} \pm \frac{1}{2} \left(\epsilon_i + \Psi\Phi + \frac{\Psi\hat{\mu}_{iT}\hat{B}}{2} \right) \sqrt{\epsilon_i + \Psi\Phi - \Psi\hat{\mu}_{iT}\hat{B}} \right\}. \quad (5.42)$$

This expression is also consistent where $\hat{B} \rightarrow 0$, $T_{i\perp}(\Phi = 0) = 1$ at the source and $T_{i\perp, out} = 0$ far downstream.

$T_{i\perp}(\Phi)$ and $T_{i\parallel}(\Phi)$ are depicted in Figure 5.10. This highlights that ions become anisotropic through the expansion. The parallel temperature drops monotonically, while the perpendicular temperature follows approximately the magnetic field variation, reaching the maximum close to the magnetic throat. This peaked trend of $T_{i\perp}(\Phi)$ is due to the conservation of the magnetic moment and is more accentuated for higher ion energies. The ion beam that forms downstream has zero temperature, because the mono-energetic IDF postulated at the source does not allow any velocity dispersion when the expansion is completed. In other words, since there is no energy dispersion, and all ions sense the same potential drop, all escaping ions reach the same velocity downstream, which is purely axial ($w_{i\perp} \rightarrow 0$ when $B \rightarrow 0$) without any dispersion, this is $c_{i\parallel} = 0$. If another IDF at the source were postulated, the parallel temperature would not drop necessarily to 0. For instance, think on a Maxwellian distribution, its energy dispersion is kept throughout the expansion and it yields $c_{i\parallel, out} \neq 0$, and consequently

Figure 5.10 – (a, b) Dimensionless perpendicular and parallel ion temperature, for different mass ratios and energy levels (same cases as in Figure 5.9). Dashed lines are for $T_{i\perp}$ and solid lines are for $T_{i\parallel}$. (c, d) Total ion temperature $T_i = (1/3)T_{i\parallel} + (2/3)T_{i\perp}$. Arrows indicate the increasing direction of m_i/m_e or ϵ_i .

$$T_{i\parallel, out} \neq 0.$$

This expected behaviour is similar to the $T_{e\parallel}$ response that will be further discussed below. In Ref. [141]¹ it has been proved that $T_{i\parallel, out} \neq 0$ for a Maxwellian IDF at the source. However, this causes small differences on the general solution $\Phi(\hat{B})$ and Ψ . These can be totally disregarded except if ions are more energetic than electrons $T_{i\infty} \gg T_{e\infty}$.

Regarding electrons, an analogous procedure is followed to determine each component of the electron temperature, $T_{e\perp}(\phi)$ and $T_{e\parallel}(\phi)$. The electron parallel temperature according Eq. 5.16 is computed through the following expression,

$$kT_{e\parallel}(\phi) = \langle m_e c_{e\parallel}^2 \rangle = \frac{1}{n_e} \frac{2\pi B}{m_e^2} \int_{\Omega_{E_e}} \int_{\Omega_{\mu_e}} \frac{(w_{e\parallel} - u_e)^2}{|w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e, \quad (5.43)$$

$$\text{with } c_{e\parallel} = w_{e\parallel} - u_e. \quad (5.44)$$

The proper setting of integration limits is crucial to determine this variable. As an even moment of the EDF, the same limits used to compute $n_e(\phi)$ in Sec. 5.2.3 must be applied here.

- For confined electrons, $-e\phi < E_e < -e\phi_{out}$ the integral over the magnetic moment

¹Work mentored by the author of this Thesis.

ranges from $\mu_e \in [0, \mu_{eM}]$ and counts twice.

- For the rest of electrons, $E_e > -e\phi_{out}$:
  - Upstream of $B_{eT}(E_e)$ the integral over the magnetic moment is divided into two parts, $\mu_e \in [\mu_{eT}, \mu_{eM}]$ counting twice, and $\mu_e \in [0, \mu_{eT}]$ counting once.
  - Downstream of $B_{eT}(E_e)$ only the range $\mu_e \in [0, \mu_{eT}]$ contributes to $kT_{e\parallel}$ and counts once.

According to these ranges, it is also convenient to split the contribution to Eq. 5.43 depending on whether this comes from the confined electron population $T_{e\parallel,c}$ or from the high energy electrons $T_{e\parallel,h}$. The first contribution, after integrating over the magnetic moment is written as follows,

$$kT_{e\parallel,c}(\phi) = \frac{n_{e\infty}}{n_e} \frac{2\pi}{3} \left(\frac{m_e}{2\pi kT_{e\infty}} \right)^{3/2} \int_{-e\phi}^{-e\phi_{out}} \left[\left(\frac{2}{m_e} \right)^{3/2} (E_e + e\phi)^{3/2} + 3\sqrt{\frac{2}{m_e}} u_e^2 \sqrt{E_e + e\phi} \right] \exp(-E_e/kT_{e\infty}) dE_e, \quad (5.45)$$

and this can be analytically integrated,

$$kT_{e\parallel,c}(\phi) = \frac{n_{e\infty}}{n_e} \left\{ kT_{e\infty} \left[\exp\left(\frac{e\phi}{kT_{e\infty}}\right) \operatorname{erf} \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} - \frac{2}{3\sqrt{\pi}} \exp\left(\frac{e\phi_{out}}{kT_{e\infty}}\right) \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} \left(\frac{2(e\phi - e\phi_{out}) + 3kT_{e\infty}}{kT_{e\infty}} \right) \right] + m_e u_e^2 \left[\exp\left(\frac{e\phi}{kT_{e\infty}}\right) \operatorname{erf} \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} - \frac{2}{\sqrt{\pi}} \exp\left(\frac{e\phi_{out}}{kT_{e\infty}}\right) \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} \right] \right\}. \quad (5.46)$$

With the current normalization it is rewritten as follows,

$$T_{e\parallel,c}(\Phi) = \frac{1}{\hat{n}_e} \left\{ \exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{2}{3\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) + \frac{m_e}{m_i} \hat{u}_e^2 \left[\exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{2}{\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} \right] \right\}. \quad (5.47)$$

The second contribution (coming from the high energy electrons) does not allow an analytical integration over the energy space because of $\mu_{eT}(E_e)$ is found numerically. So, the expression for $T_{e\parallel,h}$ after integrating over the magnetic moment space is,

$$kT_{e\parallel,h}(\phi) = \frac{n_{e\infty}(kT_{e\infty})^{-3/2}}{n_e\sqrt{\pi}} \int_{-e\phi_{out}}^{\infty} \left\{ \frac{2}{3} \left[(E_e + e\phi)^{3/2} \pm (E_e + e\phi - \mu_{eT}B)^{3/2} \right] - \sqrt{2m_e\mu_{eT}Bu_e} \right\} \exp(-E_e/kT_{e\infty}) dE_e$$

$$+ m_e u_e^2 \left[(E_e + e\phi)^{1/2} \pm (E_e + e\phi - \mu_{eT} B)^{1/2} \right] \exp(-E/kT_{e\infty}) dE_e, \quad (5.48)$$

and in dimensionless form,

$$\begin{aligned} T_{e\parallel,h}(\Phi) = & \frac{1}{\hat{n}_e \sqrt{\pi}} \int_1^\infty \left\{ \frac{2}{3} \Psi^{5/2} \left[(\varepsilon_e - \Phi)^{3/2} \pm (\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B})^{3/2} \right] \right. \\ & - \sqrt{\frac{2m_e}{m_i}} \Psi^2 \hat{\mu}_{eT} \hat{B} \hat{u}_e \\ & \left. + \frac{m_e}{m_i} \Psi^{3/2} \hat{u}_e^2 \left[(\varepsilon_e - \Phi)^{1/2} \pm (\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B})^{1/2} \right] \right\} \exp(-\Psi \varepsilon_e) d\varepsilon_e. \end{aligned} \quad (5.49)$$

The double sign in some terms is interpreted as usual. $T_{e\parallel}(\Phi)$ is the result of summing Eqs. 5.47, 5.49,

$$T_{e\parallel}(\Phi) = T_{e\parallel,c}(\Phi) + T_{e\parallel,h}(\Phi). \quad (5.50)$$

The electron perpendicular temperature $T_{e\perp}(\phi)$ is computed as follows,

$$kT_{e\perp}(\phi) = \langle m_e \frac{w_{e\perp}^2}{2} \rangle = \frac{1}{n_e} \frac{2\pi B}{m_e^2} \int_{\Omega_{Ee}} \int_{\Omega_{\mu_e}} \frac{m_e w_{e\perp}^2}{2 |w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e, \quad (5.51)$$

Again, it is suitable to separate the contribution due to confined electrons $T_{e\perp,c}(\phi)$ from the contribution of high energy electrons $T_{e\perp,h}(\phi)$.

After integrating over the magnetic moment, using the same limits criteria, the confined electrons contribution is written as follows,

$$kT_{e\perp,c}(\phi) = \frac{n_\infty}{n_e} \frac{(kT_{e\infty})^{-3/2}}{\sqrt{\pi}} \frac{4}{3} \int_{-e\phi}^{e\phi_{out}} (E_e + e\phi)^{3/2} \exp(-E_e/kT_{e\infty}) dE_e. \quad (5.52)$$

Then, integrating over the energy space,

$$\begin{aligned} kT_{e\perp,c}(\phi) = & \frac{n_{e\infty}}{n_e} \left\{ kT_{e\infty} \exp\left(\frac{e\phi}{kT_{e\infty}}\right) \operatorname{erf} \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} \right. \\ & \left. - \frac{2}{3\sqrt{\pi}} \exp\left(\frac{e\phi_{out}}{kT_{e\infty}}\right) \sqrt{\frac{e\phi - e\phi_{out}}{kT_{e\infty}}} (2(e\phi - e\phi_{out}) + 3kT_{e\infty}) \right\}, \end{aligned} \quad (5.53)$$

and with dimensionless units,

$$\begin{aligned} T_{e\perp,c}(\Phi) = & \frac{1}{\hat{n}_e} \left\{ \exp(-\Psi \Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} \right. \\ & \left. - \frac{2}{3\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) \right\}. \end{aligned} \quad (5.54)$$

On the other hand, the contribution of high energy electrons after integrating over the magnetic moment space is

$$\begin{aligned} kT_{e\perp,h}(\phi) = & \frac{n_{e\infty} (kT_{e\infty})^{-3/2}}{n_e \sqrt{\pi}} \int_{-e\phi_{out}}^\infty \left\{ \frac{2}{3} \left[(E_e + e\phi)^{3/2} \pm (E_e + e\phi - \mu_{eT} B)^{3/2} \right] \right. \\ & \left. \pm \mu_{eT} B \sqrt{E_e + e\phi - \mu_{eT} B} \right\} \exp(-E/kT_{e\infty}) dE_e, \end{aligned} \quad (5.55)$$

which in dimensionless variables is rewritten as,

$$T_{e\perp,h}(\Phi) = \frac{\Psi^{5/2}}{\hat{n}_e \sqrt{\pi}} \int_1^\infty \left\{ \frac{2}{3} \left[(\varepsilon_e - \Phi)^{3/2} \pm (\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B})^{3/2} \right] \right.$$

Figure 5.11 – (a, b) Dimensionless perpendicular and parallel electron temperature, for different mass ratios and energy levels. Dashed lines are for $T_{e\parallel}$ and solid lines are for $T_{e\perp}$. Abscissa plots $1 - \Phi$ to point out the behaviour of the temperature far downstream, and the asymptotic trend of the parallel component. (c, d) Total electron temperature $T_e = (1/3)T_{e\parallel} + (2/3)T_{e\perp}$. Arrows indicate the increasing direction of m_i/m_e or ϵ_i .

$$\pm \hat{\mu}_{eT} \hat{B} \sqrt{\epsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \Big\} \exp(-\Psi \epsilon_e) d\epsilon_e, \quad (5.56)$$

Figure 5.11 displays the electron temperature profile and points out the existence of electron collisionless cooling. This cooling response, as well as the loss of isotropy, would not be present if electrons were well confined.

The perpendicular temperature $T_{e\perp}$ approaches zero as the magnetic field decreases and the potential approaches its limit, but the parallel component $T_{e\parallel}$ tends to a finite limit far downstream. In this region, high energetic or “hot” electrons control this and other variables, because lower energy electrons, or confined, have been bounced back electrostatically. These “hot” electrons are also the ones that ensure the current-free condition as well as the neutralization of ions far downstream. The electron cooling that occurs here is a consequence of the partial depletion of the EDF due to the electric potential barrier. This is also responsible for the loss of electron isotropy on the divergent side, with the perpendicular temperature becoming progressively smaller than the parallel temperature.

The finite limit of the parallel electron temperature is only due to electrons that escape to

Figure 5.12 – (a) Dimensionless ion parallel heat flux, which is analytically 0. This proves the consistency of numerical results. Dimensionless ion flux of perpendicular heat $\hat{q}_{i\perp}(\Phi)$ for different mass ratios (b) and energy levels (c). Arrows indicate the increasing direction of m_i/m_e or ϵ_i .

infinity, those with $\epsilon_e > 1$. They keep the energy dispersion they had in the plasma reservoir. The electron energy dispersion at the end of the expansion, as aforementioned, results in a parallel velocity dispersion, and the parallel temperature measures this deviation from the mean parallel velocity. This bound is quite dependent on the ion to electron mass ratio, and decreases for larger mass ratios. For the analysed range of ion energies the electron temperature does not vary too much, and only the case $\epsilon_i = 10$ shows appreciable differences.

5.4.3 Heat Fluxes

Ion heat fluxes are computed by taking the 3rd moment of the IDE. The parallel flux of “random” parallel kinetic energy, $q_{i\parallel}(\phi) = n_i \langle m_i c_{i\parallel}^3 / 2 \rangle$, becomes, using Eq. 5.12,

$$q_{i\parallel}(\phi) = n_i \langle m_i c_{i\parallel}^3 / 2 \rangle = \frac{2\pi B}{m_i^2} \int_{\Omega_{E_i}} \int_{\Omega_{\mu_i}} \frac{m_i (w_{i\parallel} - u_i)^3}{2 |w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i. \quad (5.57)$$

Integrating the previous equation considering that f_i is mono-energetic, this is found analytically to be zero, $q_{i\parallel}(\phi) = 0$, and this result is plotted in Figure. 5.12 (a). On the other hand, $q_{i\perp}(\phi) = n_i \langle m_i c_{i\parallel} w_{i\perp}^2 / 2 \rangle$ is the axial transport of perpendicular kinetic energy,

$$q_{i\perp}(\phi) = n_i \langle m_i c_{i\parallel} w_{i\perp}^2 / 2 \rangle = \frac{2\pi B}{m_i^2} \int_{\Omega_{E_i}} \int_{\Omega_{\mu_i}} \frac{m_i w_{i\perp}^2 (w_{i\parallel} - u_i)}{2 |w_{i\parallel}|} f_i(E_i, \mu_i) d\mu_i dE_i. \quad (5.58)$$

In comparison to the ion flux, Eq. 5.22, which was an “odd” moment in $w_{i\parallel}$, and consequently reflected ions did not contribute to the particle/charge flux. Now, because we are considering the moment on $c_{i\parallel}$ instead of the moment on $w_{i\parallel}$, reflected ions still contribute to $q_{i\perp}(\phi)$.

Figure 5.13 – (a,b) Dimensionless electron parallel heat flux $\hat{q}_{e\parallel}(\Phi)$ at different mass ratios and ion energy levels. (c,d) Dimensionless electron flux of perpendicular heat $\hat{q}_{e\perp}(\Phi)$ at different mass ratios and ion energy levels. Arrows indicate the increasing direction of m_i/m_e .

Integrating Eq. 5.58 properly, the result is,

$$q_{i\perp}(\phi) = \frac{n_{\infty}}{2\sqrt{2E_{i\infty}m_i}} \left\{ \frac{B^2\mu_{iT}^2}{2} - 2u_i\sqrt{\frac{m_i}{2}} \left[\frac{2}{3}(E_{i\infty} - e\phi)^{3/2} \pm \frac{2}{3}(E_{i\infty} - e\phi - \mu_{iT}B)^{3/2} \pm \mu_{iT}B\sqrt{E_{i\infty} - e\phi - \mu_{iT}B} \right] \right\} \quad (5.59)$$

We use here the same criterion for the double sign, positive upstream of B_{iT} and negative downstream. Using $n_{\infty}c_s k T_{i\infty}$ as the reference ion heat flux, the dimensionless form of Eq. 5.59 results as,

$$\hat{q}_{i\perp}(\Phi) = \frac{q_{i\perp}}{n_{\infty}c_s k T_{i\infty}} = \frac{3}{4\epsilon_i^{3/2}} \left\{ \frac{\hat{B}^2\hat{\mu}_{iT}\Psi^2}{2^{3/2}} - \Psi^{3/2}\hat{u}_i \left[\frac{2}{3}(\epsilon_{i\infty}/\Psi + \Phi)^{3/2} \pm \frac{2}{3}(\epsilon_i/\Psi + \Phi - \hat{\mu}_{iT}\hat{B})^{3/2} \pm \hat{\mu}_{iT}\hat{B}\sqrt{\epsilon_i/\Psi + \Phi - \hat{\mu}_{iT}\hat{B}} \right] \right\} \quad (5.60)$$

The flux of perpendicular heat $\hat{q}_{i\perp}(\Phi)$ is negative everywhere along the expansion, see for instance Figures 5.12 (b, c). Since it has been seen that $\hat{T}_{i\perp}(\Phi)$ decreases in the forward direction beyond the throat, the usual sign criterion of the Fourier heat transport law does not apply here, although it would apply in the convergent segment [142].

Concerning electrons, the same procedure is followed accounting for the same integral limits described in the computation of electron temperatures (Sec. 5.4.2). The axial transport of

random parallel kinetic energy $q_{e\parallel}(\phi) = n_e \langle m_e c_{e\parallel}^3 / 2 \rangle$ is computed according Eq. 5.16,

$$q_{e\parallel}(\phi) = n_e \langle m_e c_{e\parallel}^3 / 2 \rangle = \frac{2\pi B}{m_e^2} \int_{\Omega_{E_e}} \int_{\Omega_{\mu_e}} \frac{m_e c_{e\parallel}^3}{2 |w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e. \quad (5.61)$$

The contribution of confined electrons and high energy electrons is computed separately, $q_{e\parallel,c}$ and $q_{e\parallel,h}$ respectively. For the confined population note that $c_{e\parallel}$ alternates the sign $c_{e\parallel} = \pm w_{e\parallel} - u_e$ depending the electron is moving forward or backward. Integrating properly over both spaces, $\mu_e \in [0, \mu_{eM}(E_e)]$ and $E_e \in [-e\phi, -e\phi_{out}]$, and using $q_{e\infty} = n_{\infty} c_s k T_{e\infty}$ as the reference electron heat flux, the contribution of confined electrons admits an analytical expression in Φ ,

$$\begin{aligned} q_{e\parallel,c}(\Phi) &= \frac{q_{e\parallel,c}}{n_{\infty} c_s k T_{e\infty}} = \\ &= \left[\frac{3}{2} \exp(-\Psi \Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} \right. \\ &\quad \left. - \frac{1}{\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) \right] \hat{u}_e \\ &\quad - \left[\frac{1}{2} \exp(-\Psi \Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{1}{\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} \right] \frac{m_e}{m_i} \hat{u}_e^3. \end{aligned} \quad (5.62)$$

On the contrary, the contribution of high energy electrons has to be numerically determined because its dependence on $\mu_{eT}(E_e)$. Here, the limits of the energy integral are $E_e \in (-e\phi_{out}, \infty)$. The limits of the magnetic moment integral depend on whether it is evaluated downstream or upstream of $B_{eT}(E_e)$, $\mu_e \in [0, \mu_{eT}]$ and $\mu_e \in [0, \mu_{eM}]$ respectively. And for the second case, also accounting for the backward/forward possible displacement of electrons contained in the range $\mu_e \in [\mu_{eT}, \mu_{eM}]$. After integrating over the magnetic moment space and normalizing with $q_{e\infty}$ this contribution is expressed as follows,

$$\begin{aligned} \hat{q}_{e\parallel,h}(\Phi) &= \frac{q_{e\parallel,h}}{n_{\infty} c_s k T_{e\infty}} = \\ &= \frac{1}{\sqrt{\pi}} \int_1^{\infty} \left\{ \frac{1}{2^{3/2}} \left(\frac{m_i}{m_e} \right)^{1/2} \Psi^3 \hat{B}^2 \left[\hat{\mu}_{eM}^2 - (\hat{\mu}_{eM} - \hat{\mu}_{eT})^2 \right] \right. \\ &\quad \left. - \Psi^{5/2} \hat{u}_e \hat{B}^{3/2} \left[\hat{\mu}_{eM}^{3/2} - (\hat{\mu}_{eM} - \hat{\mu}_{eT})^{3/2} \right] \right. \\ &\quad \left. + \frac{3}{2^{3/2}} \sqrt{\frac{m_e}{m_i}} \Psi^2 \hat{u}_e^2 \hat{\mu}_{eT} \hat{B} \right. \\ &\quad \left. - \frac{1}{2} \frac{m_e}{m_i} \Psi^{3/2} \hat{u}_e^3 \hat{B}^{1/2} \left[\hat{\mu}_{eM}^{1/2} - (\hat{\mu}_{eM} - \hat{\mu}_{eT})^{1/2} \right] \right\} \exp(-\Psi \varepsilon_e) d\varepsilon_e. \end{aligned} \quad (5.63)$$

Regarding the electron flux of perpendicular heat or axial transport of perpendicular kinetic energy, $q_{e\perp}(\phi)$, its general expression is written as,

$$q_{e\perp}(\phi) = n_e \langle m_e c_{e\parallel} w_{e\perp}^2 / 2 \rangle = \frac{2\pi B}{m_e^2} \int_{\Omega_{E_e}} \int_{\Omega_{\mu_e}} \frac{m_e c_{e\parallel} w_{e\perp}^2}{2 |w_{e\parallel}|} f_e(E_e, \mu_e) d\mu_e dE_e. \quad (5.64)$$

It is also split into the two usual contributions, $q_{e\perp,c}(\phi)$ and $q_{e\perp,h}(\phi)$. Using $q_{e\infty}$ to normalize

Figure 5.14 – (Left) Contribution of confined electrons to the dimensionless electron parallel heat flux $\hat{q}_{e||,c}(\Phi)$. (Right) Contribution of *hot* electrons to the dimensionless electron parallel heat flux $\hat{q}_{e||,h}(\Phi)$. This contribution causes the asymmetry of $\hat{q}_{e||}(\Phi)$ (i.e. not peaked at the magnetic throat $\hat{B} = 1$), moving its peak into the convergent side. Arrows indicate the increasing direction of $m_i/m_e = 10^3, 10^4, 10^5$.

them, the contribution of confined electrons becomes the following analytical expression,

$$\begin{aligned} \hat{q}_{e\perp,c}(\Phi) &= \frac{q_{e\perp,c}}{n_\infty c_s k T_{e\infty}} = \\ &= -\hat{u}_e \left[\exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} \right. \\ &\quad \left. - \frac{2}{3\sqrt{\pi}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) \right]. \end{aligned} \quad (5.65)$$

Analogously, for the high energy electrons contribution, the dimensionless flux of perpendicular heat, after integrating over the magnetic moment space results in,

$$\begin{aligned} \hat{q}_{e\perp,h}(\Phi) &= \\ &= \frac{q_{e\perp,h}}{n_\infty c_s k T_{e\infty}} = \frac{1}{\sqrt{\pi}} \int_1^\infty \left\{ \frac{1}{2^{3/2}} \left(\frac{m_i}{m_e} \right)^{1/2} \Psi^3 \hat{B}^2 \hat{\mu}_{eT}^2 \right. \\ &\quad \left. - \Psi^{5/2} \hat{u}_e \hat{B}^{3/2} \left[\frac{2}{3} \hat{\mu}_{eM}^{3/2} \pm \frac{2}{3} (\hat{\mu}_{eM} - \hat{\mu}_{eT})^{3/2} \pm \hat{\mu}_{eT} \sqrt{\hat{\mu}_{eM} - \hat{\mu}_{eT}} \right] \right\} \exp(-\Psi\epsilon_e) d\epsilon_e. \end{aligned} \quad (5.66)$$

The parallel $\hat{q}_{e||,h}(\Phi)$ and perpendicular $\hat{q}_{e\perp,h}(\Phi)$ contributions are presented in Figure 5.13. Zero heat fluxes are obtained at both limits, infinitely upstream and infinitely downstream, $\hat{q}_{e||,h}, \hat{q}_{e\perp,h}(\hat{B} \rightarrow 0) \rightarrow 0$. The sharp maximum of the parallel component in the convergent channel cannot be satisfactorily explained. However, Figure 5.14 shows that this asymmetry is due only to *hot electrons*, $\hat{q}_{e||,h}(\Phi)$.

5.5 Discussion

The contribution of free electrons to the obtained results becomes residual against the high impact of the trapped population on all even moments of the EDF. Free electrons only keep the control of some of the odd moments, being the unique contributor to the particle flux.

The mechanism by which electrons are trapped on the divergent magnetic field area arises

as the weak point of the current model. Whether this trapping mechanism is due to transient effects or collisions, this model would require a further extension to include all these phenomena.

Even modes of the EDF would be the most influenced by any hypothetical depletion of this trapped population. Concerning electron density, the deviation from the Boltzmann equilibrium could be more premature because of the absence of this trapped electrons that were the responsible of keeping the Maxwellian character of the electron population. It can be anticipated that the anisotropy of the electron temperature would be accentuated, as justified above, the current isotropic behaviour, except at the far field, is due to this trapped electrons. Probably the overall cooling would be modified as well.

In this section, several issues are further investigated. Firstly, the spatial evolution of the electron distribution function, which is obtained self-consistently through the solution, although allowing the presence of the mentioned isotropic and trapped electron population. Secondly, the density and electron temperature are analysed, marking out the role or contribution of each electron kind. Thirdly, asymptotic trends and analytical approaches to some variables are presented. Finally, the model limits are investigated, playing with the current condition and implementing non-realistic lighter ions. The consequence of these variations on the depopulation/depletion of the trapped electrons are illustrated.

5.5.1 Electron distribution function

Figure 5.15 plots the evolution of the EDF $f_e = f_e(w_{e\parallel}, w_{e\perp})$ at several positions on the convergent side, at the magnetic throat, and on the divergent side, for the case $m_i/m_e = 10^4, \varepsilon_i = 0.1$. These figures point out the fact that even far downstream, for instance at $\Phi = 0.9$, the confined electrons population is still isotropic due to the subfamily of trapped electrons.

The beam appears progressively along the magnetic channel, as well as the loss of particles moving backwards is more noticeable. Those particles with $\varepsilon_e > 1$ pass progressively their individual bottle neck and they can only move forward.

5.5.2 Electron density contributions

The electron density written as indicated in Eq. 5.20 separates the contribution of hot electrons (integral term) from the contribution of confined electrons. However, that mixes reflected electrons with trapped electrons. Here, we sort electrons into three types: (1) *hot* or high energy

Figure 5.15 – Electron distribution function $f_e = f_e(w_{e\parallel}, w_{e\perp})$, at several positions on the convergent side $\Phi = 0.004, 0.02, 0.06$, magnetic throat $\Phi = 0.1$, and divergent side $\Phi = 0.5, 0.9$, for the case $m_i/m_e = 10^4$, $\varepsilon_i = 0.1$. Electron velocities are dimensionless with the electron thermal velocity $c_e = \sqrt{kT_{e\infty}/m_e}$.

electrons, (2) low energy reflected electrons, (3) trapped electrons. According to previous definitions, the different kinds of electrons ranges on a specific sub-domain of the $E_e - \mu_e$ space, which has been briefly introduced in Sec. 5.2.1, but here these limits are considered for the determination of partial densities.

$$\begin{array}{ll}
 \text{High energy (h)} & \varepsilon_e > 1 \\
 \text{Confined (c)} & \varepsilon_e \leq 1
 \end{array}
 \left\{ \begin{array}{l}
 \text{Reflected (hr): } \hat{\mu}_e > \hat{\mu}_{eT}(\varepsilon_e) \\
 \text{Free (hf): } \hat{\mu}_e < \hat{\mu}_{eT}(\varepsilon_e) \\
 \text{Reflected (cr): } \left\{ \begin{array}{l} \forall \hat{\mu}_e < \hat{\mu}_{eM}(\varepsilon_e, \hat{B}) \text{ if } \Phi < \Phi_T(\varepsilon_e). \\ \hat{\mu}_e < \hat{\mu}_{eT}(\varepsilon_e) \text{ if } \Phi > \Phi_T(\varepsilon_e). \end{array} \right. \\
 \text{Trapped (ct): } \hat{\mu}_e > \hat{\mu}_{eT}(\varepsilon_e) \text{ at } \Phi > \Phi_T(\varepsilon_e).
 \end{array} \right. \quad (5.67)$$

The free electron contribution to density is only the part that corresponds to the loss cone, and it is written as follows,

$$\hat{n}_{e,hf} = \pi^{-1/2} \Psi^{3/2} \int_1^\infty \left[\sqrt{\varepsilon_e - \Phi} - \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \right] \exp(-\Psi \varepsilon_e) d\varepsilon_e. \quad (5.68)$$

High energy reflected electrons only exist upstream of each T point, $\Phi < \Phi_T(\varepsilon_e)$. This includes all the convergent side and a small region, depending on input parameters, on the divergent side close to the magnetic throat. T points have been determined through Eq. 5.7, and reordering its terms this sets the upper limit for the integral over the energy space ε_e^h ,

$$\varepsilon_e^h = \begin{cases} \Phi - \frac{\hat{B}}{\partial \hat{B} / \partial \Phi} > 1 \text{ (divergent side)} \\ \infty \text{ (nozzle throat)} \end{cases} \quad (5.69)$$

The minimum value of ε_e^h is 1, corresponding to the escape energy. So, the partial density of high energy reflected electrons is,

$$\hat{n}_{e,hr} = 2\pi^{-1/2} \Psi^{3/2} \int_1^{\varepsilon_e^h} \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \exp(-\Psi \varepsilon_e) d\varepsilon_e, \quad (5.70)$$

and the partial density of high energy electrons is obtained adding $\hat{n}_{e,hf}$ to $\hat{n}_{e,hr}$,

$$\hat{n}_{e,h} = \hat{n}_{e,hf} + \hat{n}_{e,hr} = \frac{\Psi^{3/2}}{\sqrt{\pi}} \int_1^\infty \left[\sqrt{\varepsilon_e - \Phi} \mp \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B}} \right] e^{-\Psi \varepsilon_e} d\varepsilon_e \quad (5.71)$$

Partial density contributions are depicted in Figure 5.16, which also depicts the integral limits ε_e^h .

Trapped electrons only exist on the divergent side. They appear at $\Phi_T(\varepsilon_e = 1)$. However, due to the potential barriers the lower limit of the integral varies depending on the location at which density is evaluated.

$$\varepsilon_e^c = \begin{cases} \Phi - \hat{B} / (\partial \hat{B} / \partial \Phi) & \text{if } \Phi_T(\varepsilon_e = 1) < \Phi < \Phi^* \\ \text{Solve for } \varepsilon_e : \hat{\mu}_e(\varepsilon_e, \hat{B}) = \hat{\mu}_{eT}(\varepsilon_e) & \text{if } \Phi^* < \Phi < 1 \end{cases} \quad (5.72)$$

The minimum of ε_e^c is the one corresponding to the critical energy, ε_e^* . ε_e^c are drawn in Fig. 5.16.

Figure 5.16 – (Lef) Partial electron density contributions $\hat{n}_{e,c}$ (dashed blue line), $\hat{n}_{e,h}$ (black line), $\hat{n}_{e,cr}$ (thin blue line), $\hat{n}_{e,ct}$ (thick blue line), $\hat{n}_{e,hr}$ (thin black line), $\hat{n}_{e,hf}$ (dashed black line) and the total electron density $\hat{n}_e$ (red line). (Right) Energy integral limits for hot electrons ε_e^h (blue line), which is valid until $\varepsilon_e = 1$. The prolongation of this defines ε_e^c , in this case valid until the minimum of the curve, corresponding to ε_e^* . From this point until ($\varepsilon_e = 1, \Phi = 1$), this line depicts the relation $\Phi_t(\varepsilon_e)$, position of the local maxima of curves μ_{eM} , complemented by Fig. 5.3 , 5.2. In the range, $\Phi \in [\Phi_T^* - 1]$, ε_e^c is defined by the red dots. Dashed vertical line indicates the throat position.

Accounting for these limits the density of trapped electrons is written as,

$$\hat{n}_{e,ct}(\Phi) = 2\pi^{-1/2}\Psi^{3/2} \int_{\varepsilon_e^c(\Phi)}^1 d\varepsilon_e \exp(-\Psi\varepsilon_e) \sqrt{\varepsilon_e - \Phi - \hat{\mu}_{et}\hat{B}}. \quad (5.73)$$

Finally, the contribution of reflected electrons within the confined group $n_{e,cr}$ is obtained subtracting from the whole confined population (Eq. 5.74) the result of Eq. 5.73 appropriately in each location,

$$\hat{n}_{e,c}(\Phi) = \Psi^{3/2} \left[\exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{2}{\sqrt{\pi}} \sqrt{\Psi(1-\Phi)} \right], \quad (5.74)$$

$$\hat{n}_{e,cr}(\Phi) = \hat{n}_{e,c}(\Phi) - \hat{n}_{e,ct}(\Phi). \quad (5.75)$$

Figure 5.16 plots all these contributions for the case $m_i/m_e = 10^4$ and $\varepsilon_i = 0.1$. This underlines several issues: hot electrons are only a small fraction of the whole electron density; within this group, reflected electrons only dominate in the first part of the convergent channel, before reaching the throat; then, free electrons control the contribution of hot electrons; regarding reflected confined electrons, these spread their influence along the full convergent side and approximately until $\Phi = 0.5$, which corresponds to a position, relatively far downstream from $\hat{B} = 0.1$. At that position the influence of trapped electrons has already grown up abruptly.

5.5.3 Electron Temperature Analysis

In the results subsection the electron temperature has been split into the contribution of hot and confined electrons. Here, each one of these contributions is linked to the temperature of each electron kind if they were considered an isolated population, i.e. using partial density to

Figure 5.17 – Contributions to electron parallel temperature $T_{e,\parallel}(1-\Phi)$ (black line). Red lines depict the contribution due to hot electrons $T_{e,\parallel,h}(1-\Phi)$. Contribution of confined electrons $T_{e,\parallel,c}(1-\Phi)$ is depicted in blue. Dashed blue lines are for $T_{e,\parallel,c}^*(1-\Phi)$. Arrow indicates $m_i/m_e = 10^3, 10^4, 10^5$ increasing at constant ion energy $\epsilon_i = 0.1$.

average quantities or take moments of the EDF.

Concerning the confined population, its parallel temperature $kT_{e,\parallel,c}^*(\phi)$, is computed through Eq. 5.16, switching n_e by $n_{e,c}$, f_e by f_{ec} , truncated Maxwellian distribution function, and obviously considering a null drift velocity $c_{\parallel,e} = w_{e,\parallel}$, means $u_{e,c} = \langle w_{e,\parallel} \rangle = 0$ or null particle flux. In dimensionless units $T_{e,\parallel,c}^*(\phi)$ is analytical in Ψ, Φ ,

$$T_{e,\parallel,c}^* = \frac{1}{\hat{n}_{e,c}} \left[\exp(-\Psi\Phi) \operatorname{erf} \sqrt{\Psi(1-\Phi)} - \frac{2}{3\pi^{1/2}} \exp(-\Psi) \sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) \right], \quad (5.76)$$

which has the same form of the first two terms of Eq. 5.47, except for the density. In the mentioned equation, the second term that is multiplied by the factor $(m_e/m_i)\hat{u}_e^2$ is clearly proportional to $\hat{n}_{e,c}$, so $T_{e,c}(\Phi)$ can be rewritten in terms of $T_{e,\parallel,c}^*$ and $\hat{n}_{e,c}$:

$$T_{e,\parallel,c} = \frac{\hat{n}_{e,c}}{\hat{n}_e} T_{e,\parallel,c}^* + \frac{\hat{n}_{e,c}}{\hat{n}_e} \frac{m_e}{m_i} \hat{u}_e^2. \quad (5.77)$$

Note that Eq. 5.77 is undetermined at $\Phi = 1$. Solving the limit $\lim_{\Phi \rightarrow 1} T_{e,\parallel,c}^*$ it is proven that this drops to zero, so $T_{e,\parallel,c}^*(\Phi = 1) \rightarrow 0$. Consequently, Eq. 5.77 at the end of the expansion becomes,

$$T_{e,\parallel,c}|_{\Phi \rightarrow 1} = \frac{\hat{n}_{e,c}}{\hat{n}_e} \frac{m_e}{m_i} \hat{u}_{e,out}^2. \quad (5.78)$$

The contribution of confined population to the parallel temperature will be only 0 if the rate $n_{e,c}/n_e$ had become negligible at the end of the expansion, which is not the case for the current model. Another issue to underline here is the isotropic shape of the distribution function in the velocity space. Note that $T_{e\perp,c}^*(\Phi)$ could be written with the same form of $T_{e,\parallel,c}^*(\Phi)$ (Eq. 5.76), or just note that Eq. 5.54 exactly fits the first two terms of Eq. 5.47, so $T_{e\perp,c}^*(\Phi) = T_{e,\parallel,c}^*(\Phi), \forall \Phi$.

$T_{e\perp,c}(\Phi)$ can be written in terms of $T_{e\perp,c}^*(\Phi)$ as well,

$$T_{e\perp,c}(\Phi) = \frac{\hat{n}_{e,c}}{\hat{n}_e} T_{e\perp,c}^*. \quad (5.79)$$

Even though the density ratio $\hat{n}_{e,c}/\hat{n}_e$ is not zero when $\Phi \rightarrow 1$, the perpendicular temperature drops to zero far downstream. As a result, the confined population keeps its postulated isotropic shape, but not its temperature, which drops because of the potential barriers that reduce continuously its kinetic energy dispersion.

Regarding hot electrons, the analysis only focuses on the analysis of the parallel component, $kT_{e\parallel,h}(\phi)$. Considering the electron flux expression (Eq. 5.26), and the partial density of high energy electrons (Eq. 5.71), Eq. 5.49 is rewritten as,

$$T_{e\parallel,h} = \frac{1}{\hat{n}_e \pi^{1/2}} \int_1^\infty \frac{2}{3} \Psi^{5/2} \left((\varepsilon_e - \Phi)^{3/2} \mp (\varepsilon_e - \Phi - \hat{\mu}_{eT} \hat{B})^{3/2} \right) \exp(-\Psi \varepsilon_e) d\varepsilon_e \\ - 2 \frac{m_e}{m_i} \hat{u}_e^2 + \frac{\hat{n}_{e,f}}{\hat{n}_e} \frac{m_e}{m_i} \hat{u}_e^2 \quad (5.80)$$

The contribution of the last two terms constitutes an upper bound $-(m_e/m_i)\hat{u}_e^2$ and a lower bound $-2(m_e/m_i)\hat{u}_e^2$, because $0 < \hat{n}_{e,f}/\hat{n}_e < 1$. Consequently the first term must be the largest one to avoid a “non-physical” negative value of $T_{e\parallel,f}$. The behaviour of $T_{e\parallel}(\Phi)$, $T_{e\parallel,c}(\Phi)$, $T_{e\parallel,c}^*(\Phi)$ and $T_{e\parallel,h}(\Phi)$ are shown in Figure 5.17. It is important to observe that $T_{e\parallel,c}^*$ effectively drops to zero when $\Phi \rightarrow 1$, while $T_{e\parallel,c}$ reaches an asymptotic trend due to the second term of Eq. 5.77. The asymptotic trend of $T_{e\parallel,h}(\Phi)$ also controls the temperature at the end of the expansion, concluding that the limit on the parallel temperature is due to the high energy electrons, those that overpass the finite potential drop.

5.5.4 Asymptotic trends

In preceding subsections some quantities show the apparent existence of asymptotic trends when $\Phi \rightarrow 1$. Here, several results are further analysed, among them, the ratio $n_{e,h}/n_{e,c}(\Phi \rightarrow 1)$ and the finite limit of the parallel temperature $T_{e\parallel}(\Phi \rightarrow 1)$. Apart from this, the dependence of Ψ on m_i/m_e and the fitting or an asymptotic expression for $\Phi(\hat{B})$ far downstream are derived, since other limits depend on these results.

Ψ dependence on m_i/m_e and ε_i

The influence of input parameters m_i/m_e and ε_i on Ψ can be analysed through the current equation, indeed the current-free condition in Eq. 5.33. Its terms are reorganized yielding the

Figure 5.18 – $\Psi(m_i/m_e)$ relation according the simulation results (thick solid line). The thin black is the first term in Eq. 5.81 and the dashed line is the second term.

next implicit relation for Ψ ,

$$\Psi = \log \left(\frac{m_i}{2\pi m_e} \right)^{1/2} + \log \left(\frac{\int_0^\infty \hat{\mu}_{eT} \exp(-\varepsilon'_e) d\varepsilon'_e}{\hat{\mu}_{iT}} \right) + \log \sqrt{8\varepsilon_i} \quad (5.81)$$

Note that the first term is the same that appears in a classical electrostatic sheath attached to a wall [143]. So Ψ is almost linear against $\log(m_i/2\pi m_e)$ with a 1/2 slope. The deviation from the linearity is small and is due to the second term, which depends on Ψ because of $\varepsilon'_e = \Psi(\varepsilon_e - 1)$. However, this deviation is small and is luckily almost linear. Thus, a good global approximation could be simply $\Psi \propto 0.6 \log(m_i/2\pi m_e)$. The numerical solution for $\Psi(m_i/m_e)$ and the first two terms of Eq. 5.81 are depicted in Fig. 5.18. This result points out the similarity of electrostatic sheaths with the analysed problem.

Asymptotic law $\Phi - \hat{B}$ far downstream

Let us to consider the expression for the ion density, Eq. 5.19, but exclusively on the divergent side, so only taking the negative sign of the second square root term,

$$\hat{n}_i = \frac{1}{2} \left(\frac{\Psi}{\varepsilon_i} \right)^{1/2} \sqrt{\frac{\varepsilon_i}{\Psi} + \Phi} \left(1 - \sqrt{1 - \frac{\hat{\mu}_{iT}}{\frac{\varepsilon_i}{\Psi} + \Phi}} \hat{B} \right). \quad (5.82)$$

At $\Phi \rightarrow 1$, it allows an expansion in terms of the small parameter,

$$\frac{\hat{\mu}_{iT} \hat{B}}{\frac{\varepsilon_i}{\Psi} + \Phi} \ll 1, \quad (5.83)$$

resulting the asymptotic expression,

$$\hat{n}_i \sim \frac{1}{4} \frac{\Psi}{\varepsilon_i} \frac{\hat{\mu}_{iT}}{\sqrt{1 + \frac{\Psi}{\varepsilon_i}}} \hat{B} + O(\hat{B}^2). \quad (5.84)$$

Figure 5.19 – Fitting constant κ_{ec} for the asymptotic law of $\hat{B} = \hat{B}(\Phi)$ at the far field (Eq. 5.88), as a function of the ion to electron mass ratio at constant energy $\varepsilon_i = 0.1$

For the electrons, it is known that the density contribution of free electrons can be disregarded along the full expansion. Taking Eq. 5.74, and expanding it at $\Phi \rightarrow 1$,

$$\hat{n}_e \sim \frac{4}{3\sqrt{\pi}} \exp(-\Psi) (\Psi(1-\Phi))^{3/2} + O(\Psi(1-\Phi))^{5/2}. \quad (5.85)$$

Matching the above expressions, a law for $\hat{B} = \hat{B}(\Phi, \Psi, \hat{\mu}_{iT}, \varepsilon_i)$, is obtained. This is valid when $\Phi \rightarrow 1$, the far downstream region.

$$\hat{B} \sim \frac{16}{3\sqrt{\pi}} \frac{\varepsilon_i \sqrt{1+\Psi/\varepsilon_i}}{\Psi \hat{\mu}_{iT}} \exp(-\Psi) (\Psi(1-\Phi))^{3/2} + O(\Psi(1-\Phi))^{5/2}. \quad (5.86)$$

This law depends on input parameters ε_i and outputs, $\Psi, \hat{\mu}_{iT}$. However, these outputs can be parameterized as functions of m_i/m_e and ε_i . Hereafter, for compactness,

$$\kappa_{ec} = \frac{16\varepsilon_i \sqrt{1+\Psi/\varepsilon_i}}{3\sqrt{\pi} \hat{\mu}_{iT}} \exp(-\Psi) \Psi^{1/2}, \quad (5.87)$$

$$\text{and } \hat{B} \sim \kappa_{ec} (1-\Phi)^{3/2}. \quad (5.88)$$

Figure 5.19 depicts the dependency of κ_{ec} on the ion to electron mass ratio, $\kappa_{ec}(m_i/m_e)$ at constant ion energy $\varepsilon_i = 0.1$. This monotonically decreases as the mass ratio increases. The asymptotic law for $\hat{B}(\Phi)$ in Eq. 5.88 is shown in Figure 5.20, for different mass ratios. This confirms the validity of this approach at the last 10% of the expansion, $\Phi \gtrsim 0.9$.

Asymptotic law for $n_{e,h}$ far downstream

Once $\kappa_{ec} = \kappa_{ec}(m_i/m_e, \varepsilon_i)$ is known, it is easy to analyse the asymptotic behaviour of $n_{e,h}$ at $\Phi \rightarrow 1$. The density contribution of hot electrons evaluated far downstream, beyond their points $B_{eT}(E_e)$, requires considering exclusively the contribution of free electrons, Eq. 5.68. In that equation the numerical uncertainty relies on the unknown law for $\hat{\mu}_{eT}(E_e)$, which indeed

Figure 5.20 – Asymptotic law $\hat{B} = \hat{B}(\Phi)$ at the far field, Eq. 5.88, as a function of the ion to electron mass ratio $m_i/m_e = 10^3, 10^4$ (thin and thick lines respectively) at constant ion energy $\varepsilon_i = 0.1$. In both figures, solid lines are for the numerical solution and dashed lines for the approximation. The figure on the right side uses the logarithmic abscissa $1 - \Phi$ to stress the behaviour in the far field. Blue, red and black lines are for $m_i/m_e = 10^3, 10^4, 10^5$ respectively.

cannot be predicted before solving the complete problem. However, accounting for the results obtained in the parametric analysis, see Fig. 5.7 in Sec.5.4, the approximation $\hat{\mu}_{eT}(\varepsilon_e) \simeq \varepsilon_e/\hat{B}$ for the minimum of each $\hat{\mu}_{eM}$ curve can be taken. In addition, all these minima occur close to the throat, so $\hat{B}_{eT} \simeq 1$ and $\hat{\mu}_{eT}(\varepsilon_e) \simeq \varepsilon_e$, except for $m_i \simeq m_e$ (discussed later). Accounting for all these assumptions, and using the approximation of Eq. 5.88 into Eq. 5.68, the last one can be easily integrated and results in,

$$n_{e,h}(\Phi) \simeq \frac{\exp(-\Psi\Phi)}{2} \left(1 - \operatorname{erf} \sqrt{\Psi(1-\Phi)}\right) + \frac{\exp(-\Psi)}{\sqrt{\pi}} \sqrt{\Psi(1-\Phi)} - \sqrt{1 - \kappa_{ec}(1-\Phi)^{3/2}} \left[\frac{\exp(-\Psi\Phi')}{2} \left(1 - \operatorname{erf} \sqrt{\Psi(1-\Phi')}\right) + \frac{\exp(-\Psi)}{\sqrt{\pi}} \sqrt{\Psi(1-\Phi')} \right] \quad (5.89)$$

$$\text{with } \Phi' = \frac{\Phi}{1 - \kappa_{ec}(1-\Phi)^{3/2}} \quad (5.90)$$

Expanding it at $\Phi \rightarrow 1$ the asymptotic form of $\hat{n}_{e,h}$ is,

$$\hat{n}_{e,h}(\Phi) \sim \frac{3}{8} \exp(-\Psi) (1 + \Psi) \kappa_{ec} (1-\Phi)^{3/2} + O(1-\Phi)^{5/2} \quad (5.91)$$

The decreasing rate of $n_{e,h}(\Phi)$ is also proportional to $(1-\Phi)^{3/2}$. This is an important and probably surprising result, but it means that even close to the end of the expansion, say infinitely downstream, the decreasing rate of the confined and free electrons is the same, so the ratio between them will be constant,

$$\frac{\hat{n}_{e,h}}{\hat{n}_{e,c}} \sim \frac{3}{2} \left(1 + \frac{1}{\Psi}\right) \exp(-\Psi) \frac{\varepsilon_i}{\hat{\mu}_{iT}} \sqrt{1 + \frac{\Psi}{\varepsilon_i}} \quad (5.92)$$

This density ratio is depicted in Figure 5.21. The asymptotic approximation fits satisfactorily

Figure 5.21 – Density ratios of high energy electrons over confined electrons contribution as indicated in Eq. 5.92 (dashed line). For different mass ratios at constant ion energy $\varepsilon_i = 0.1$. The open circles are the results obtained at the end of the expansion, accordingly the numerical method.

the numerical results, except for very low mass ratios. In such a case, this simplified model might fail because of the non-negligible contribution of high energy electrons to the total density, an hypothesis that has been taken in the derivation of Eq. 5.88.

Asymptotic law for $T_{e\parallel} \sim T_{e\parallel,h}$ far downstream

Regarding $T_{e\parallel}$, the influence of the hot electrons contribution on its asymptotic behaviour far downstream has already been noted in the Results section. This constant value reached by the temperature is strongly dependent on the ion to electron mass ratio and also means that the first term in Eq. 5.80 must be constant far downstream. In other words the integral term in the numerator decreases at the same rate of the electron density. Performing a similar procedure as the one carried out to get the asymptotic trend of $\hat{n}_{e,h}$, but in this case the simplifications $\hat{\mu}_{eT} \simeq \varepsilon_e$ and $\hat{B} \propto (1 - \Phi)^{3/2}$ are introduced in the first term of Eq. 5.80, results in the following expression $T_{e\parallel,h1}$ ²,

$$T_{e\parallel,h1} = \frac{2\Psi^{5/2}}{3\pi^{1/2}\hat{n}_e} \int_1^\infty \left\{ (\varepsilon_e - \Phi)^{3/2} - (1 - \kappa_{ec}(1 - \Phi)^{3/2})^{3/2} (\varepsilon_e - \Phi')^{3/2} \right\} \exp(-\Psi\varepsilon_e) d\varepsilon_e, \quad (5.93)$$

with Φ' as in Eq. 5.90. After integrating over the energy space it yields,

$$T_{e\parallel,h1} = \frac{2\Psi^{5/2}}{3\sqrt{\pi}\hat{n}_e} \times \left\{ \frac{3\pi^{1/2}}{4\Psi^{5/2}} \left[\exp(-\Psi\Phi) \left(1 - \operatorname{erf} \sqrt{\Psi(1 - \Phi)} \right) - a_{ec}^{3/2} \exp(-\Psi\Phi') \left(1 - \operatorname{erf} \sqrt{\Psi(1 - \Phi')} \right) \right] \right\}$$

²Here, $T_{e\parallel,h1}$ is only the first term of Eq. 5.80 and it is evaluated far downstream from all $\hat{B}_{eT}$ locations

Figure 5.22 – Solid lines: evolution of the electron parallel temperature $T_{e||}(\Phi)$ for different mass ratios $m_i/m_e = 10^3, 10^4, 10^5$ (blue, red and black respectively) at constant ion energy $\varepsilon_i = 0.1$. Dashed lines: asymptotic value reached by $T_{e||,out}$ according to Eq. 5.99 at the far field, for the same analysed cases.

$$+ \frac{\exp(-\Psi)}{2\Psi^{5/2}} \left[\sqrt{\Psi(1-\Phi)} (2\Psi(1-\Phi) + 3) - a_{ec}^{3/2} \sqrt{\Psi(1-\Phi')} (2\Psi(1-\Phi') + 3) \right], \quad (5.94)$$

with $a_{ec} = 1 - \kappa_{ec}(1-\Phi)^{3/2}$. Expanding the previous equation at $\Phi \rightarrow 1$, the asymptotic form of $T_{e||,h1}$ results in a constant value that fits satisfactorily the asymptotic trend of $T_{e||}(\Phi)$, so $T_{e||}(\Phi) \sim T_{e||,h1}(\Phi)$ at $\Phi \rightarrow 1$

$$T_{e||,h1} = \frac{1}{\hat{n}_e} \frac{\exp(-\Psi)}{2} \left(\Psi + \frac{3}{2} \right) \kappa_{ec} (1-\Phi)^{3/2} = \frac{3\sqrt{\pi}}{8\Psi^{3/2}} \left(\Psi + \frac{3}{2} \right) \kappa_{ec}, \quad (5.95)$$

or equivalently,

$$T_{e||,h1} = \left. \frac{n_{e,h}}{n_e} \right|_{\Phi \rightarrow 1} T_{e||,h1}^*, \text{ with } T_{e||,h1}^* = \frac{4}{3} \left(1 + \frac{1}{2(\Psi+1)} \right). \quad (5.96)$$

The complete contribution of hot electrons to $T_{e||}$ approaches asymptotically to the following expression,

$$T_{e||,h}|_{out} = \left. \frac{\hat{n}_{e,h}}{\hat{n}_e} \right|_{out} T_{e||,h1}^* - 2 \frac{m_e}{m_i} \hat{u}_{e,out}^2 + \left. \frac{\hat{n}_{e,h}}{\hat{n}_e} \right|_{out} \frac{m_e}{m_i} \hat{u}_{e,out}^2, \quad (5.97)$$

and replacing $\hat{u}_{e,out}^2$ by $2(\varepsilon_i + \Psi)$, which is correct for the mono-energetic ions considered here,

$$T_{e||,h}|_{out} = \left. \frac{\hat{n}_{e,h}}{\hat{n}_e} \right|_{out} T_{e||,h1}^* - 2 \frac{m_e}{m_i} (\varepsilon_i + \Psi) \left(2 - \left. \frac{\hat{n}_{e,h}}{\hat{n}_e} \right|_{out} \right). \quad (5.98)$$

Adding to this equation the contribution from confined electrons, Eq. 5.78, the parallel electron temperature far downstream reaches the following constant value,

$$T_{e||,out} = \left. \frac{\hat{n}_{e,h}}{\hat{n}_e} \right|_{out} T_{e||,h1}^* - 2 \frac{m_e}{m_i} (\varepsilon_i + \Psi). \quad (5.99)$$

The second term can be usually disregarded since the mass ratio is typically very small $m_e/m_i \ll 1$. Figure 5.22 shows the evolution of $T_{e||}(\Phi)$ along the expansion and compares its

asymptotic limit against the one derived in Eq. 5.99 (dashed lines).

5.5.5 Model limits: depleting trapped electrons

In the following pages the model is tuned to explore how this trapped population behaves when some hypotheses or conditions are modified. Before going into the new conditions, the list below summarizes the main conclusions extracted from all previous results:

1. The plasma expansion depends on the upstream distribution function of each species, as well as the mass ratio.
2. The plasma is assumed quasi-neutral and current-free. It has been demonstrated that only free electrons contribute to the current.
3. The presence of trapped electrons on the divergent region is necessary to ensure quasi-neutrality without penalising the null current condition, highlighting that reflected electrons, those connected to the source, and free electrons, those that escape, are not enough to ensure simultaneously both hypotheses.
4. Trapped electrons compensate for the large difference between the ion/electron dynamic response caused by the disparity of their masses and energy distribution.
5. If a $\phi - B$ solution without trapped electrons were possible for the given conditions and hypotheses, then μ_{eM} would decrease monotonically to 0, $\forall \varepsilon < 1$, without any local minima.

Hereafter, some hypotheses will be relaxed to illuminate the possibility of gradually depleting these trapped electrons and transferring their density contribution to the high energy electrons. Two different ways are proposed: (1) reduce the ion mass to make its dynamics more similar to electrons; (2) imagine the presence of a virtual electrode downstream, that allows controlling the current carried by the plasma. For negative currents, this reduces the potential drop, allowing more electrons to scape, and increasing their contribution to density.

Lighter ions

Here, the ion to electron mass ratio is reduced to unity, $m_i/m_e = 1$, with the aim of making the particle dynamics more similar. However, distribution functions are kept the same as in previous analyses, mono-energetic and Maxwellian for ions and electrons respectively,

Figure 5.23 – (a) $\hat{B} - \Phi$ comparison for $m_i/m_e = 1, 10^4$, blue and black line respectively, at $\varepsilon_i = 0.1$. (b) Maximum magnetic moment curve for the critical energy $\hat{\mu}_{eM}^*$, $\varepsilon_e^* = 0.971$ corresponding to the case $m_i/m_e = 1, \varepsilon_i = 0.1$ is depicted in blue. Black line plots the negative inverse of the derivative $\partial_\Phi \hat{B}$, $(-\partial \hat{B}/\partial \Phi)^{-1}$, which is tangential to $\hat{\mu}_{eM}^*$ according to Eq. 5.7. The dashed red line shows the solution $\hat{B}(\Phi)$ to stress the fact that $\hat{\mu}_{eT}^*$ occurs further downstream than in usual/nominal cases. (c) Curves $\hat{\mu}_{eM}$ for several electron energies in blue. The red one corresponds to $\varepsilon_e = 1$ and the black to $\varepsilon_e^* = 0.971$. Asterisks indicate the location of $\hat{\mu}_{eT}^*$, and the vertical dashed line is the magnetic throat location. (d) Partial density contributions. The red line is the total electron density $\hat{n}_e$. Confined population $\hat{n}_{e,c}$ (dashed blue line), which split into $\hat{n}_{e,cr}$ (thin blue line) and $\hat{n}_{e,ct}$ (thick blue line). Here, asterisks correspond to the throat location Φ_0 . High energy electrons $\hat{n}_{e,h}$ (solid black line), and its partial contributions, $\hat{n}_{e,r}$ (thin black line) and $\hat{n}_{e,hf}$ (dashed black line).

with the nominal ion energy $\varepsilon_i = 0.1$. It is expected that this should yield a smaller total potential drop, and consequently, most of the electrons would belong to the “hot” population. Consequently, the inflection point of $\hat{B}(\Phi)$ should move downstream.

The results obtained confirm most of the guesses. First, the shape of the solution $\hat{B}(\Phi)$, which is illustrated in Figure 5.23 (a), is modified in the way predicted: (1) The total drop of the potential is much smaller than in the nominal case, $\Psi = 0.637$ vs $\Psi = 6.21$. Because the ion mass has been reduced, the required ambipolar field to sustain the current free plasma condition must be weaker. (2) The drop of the potential on the subsonic region is larger $\Phi_0 = 0.35$, against $\Phi_0 = 0.1$ of the nominal case ($m_i/m_e = 10^4$); (3) The inflection point of $\hat{B}(\Phi)$ on the divergent region has moved mildly downstream, as predicted, see for instance Fig. 5.23 (b). Consequently its tangential point with the curve $\hat{\mu}_{eM}^*$ at $\Phi_{eT}^* = 0.8$ ($\hat{\mu}_{eT}^* = 0.464$), occurs now far away from the magnetic throat, specifically at $\hat{B}_{eT}^* = 0.37$, but the critical energy

$\varepsilon_e^* = 0.971$ is closer to the escape energy $\varepsilon_e = 1$. This indicates that the interval of trapped electrons has been reduced strongly, and their effects moved downstream, to the last part of the expansion.

The fact of cutting the Maxwellian tail at a lower energy $E_e = \Psi k T_{e\infty}$, makes these electrons (with $\varepsilon_e > 1$) to have an important contribution to the density. So, these are playing part of the role of the confined population in the nominal case. This can be visualised by the fact that their last point of reflection, $\hat{\mu}_{eT}$, is located now further downstream for energies slightly higher than the escape energy, Fig. 5.23 (c). For comparison, Fig. 5.7 showed that turning points T of electrons with $\varepsilon_e > 1$ were located very close to the magnetic throat, and only minima corresponding to the trapped population were effectively pushed away from the throat.

Finally, Figure 5.23 (d) shows the electron density $\hat{n}_e$ together with all partial contributions as defined in 5.5.2. It proves that the energetic population controls the plasma discharge, just from the beginning, $\hat{n}_{e,h}$ (black line). Furthermore, from close to the throat and until the end of the expansion, free electrons $\hat{n}_{e,hf}$ (dashed black line) represents the most important contribution to density. Concerning the confined population $\hat{n}_{e,c}$ (dashed blue line), its larger contribution belongs to electrons that are reflected to the source $\hat{n}_{e,cr}$ (thin blue line). Beyond $\Phi = 0.6826$, which corresponds to $\Phi_{eT}(\varepsilon_e^*)$, the contribution of trapped electrons $\hat{n}_{e,ct}$ fairly rises, this is the hump-shaped thick blue line.

The main conclusion extracted from this analysis is that energetic electrons are now dominating the whole expansion and the contribution of the trapped population has been reduced to marginal levels.

Solution for a current-carrying plasma

The code developed allows considering a net current carried by the plasma, $j/en_\infty c_s \equiv \Gamma_i - \Gamma_e$. A maximum (negative) current corresponds to the limit case of a null potential drop, or flat potential $e\phi = 0$ (everywhere), only pressure and magnetic forces are active. At this limit, all electrons should belong to the “hot” population, because $E_e > -e\phi_{out} = 0, \forall E_e$, and trapped electrons cannot exist. Obviously, increasing potentials (with positive slopes) would provide a larger electron current, but the flat potential is considered the limit case of a “*monotonic decreasing potential*”, which is the shape of the solution postulated in Sec. 5.2.

The other limit corresponds to an infinite drop of the ambipolar field, and this should be equivalent to imposing the Boltzmann equilibrium. The current work shows that this condition requires the presence of “trapped” electrons everywhere on the divergent region. This case

results in an unbounded ion acceleration because of the infinite drop of the electric potential.

Considering first the case of the flat potential, the notation used previously has to be addressed again because $\Psi = 0$, and some expressions cannot be evaluated correctly. The peculiarity of this scenario is that all curves $\mu_{\alpha M}$ present their minima at the magnetic throat, where the magnetic field is maximum. All quantities are going to depend only on the field strength. Eq. 5.15 and 5.18, with $\mu_{iT} = E_{i\infty}/B_{Max}$ and $\mu_{eT} = E_e/B_{Max}$ respectively, and integrating them properly results on the following quasi-neutral condition,

$$\hat{n}_i(\hat{B}) = \hat{n}_e(\hat{B}) = \frac{1}{2} \left(1 \pm \sqrt{1 - \hat{B}} \right), \quad (5.100)$$

with the positive sign on the convergent side and the negative along the divergent side. Note that the density drops to $\hat{n} = 1/2$ at the throat. Deviations from this value are related to the loss of the Maxwellian character due to the presence or higher influence of free-electrons. The ion flux is,

$$\Gamma_i = \frac{1}{c_s} \sqrt{\frac{E_{i\infty}}{8m_i}} \hat{B} = \sqrt{\frac{\varepsilon_i}{8}} \hat{B}, \quad (5.101)$$

which is valid in all the domain, convergent and divergent sides indistinctly. Analogously, the electron flux results,

$$\Gamma_e = \frac{1}{c_s} \sqrt{\frac{kT_{e\infty}}{2\pi m_e}} \hat{B} = \sqrt{\frac{m_i}{2\pi m_e}} \hat{B}. \quad (5.102)$$

Combining Eqs. 5.101-5.102, the net charge flux is,

$$j = \left(\left(\frac{\varepsilon_i}{8} \right)^{1/2} - \sqrt{\frac{m_i}{2\pi m_e}} \right) \hat{B}. \quad (5.103)$$

Taking the nominal values for the dimensionless parameters, $\varepsilon_i = 0.1$ and $m_i/m_e = 10^4$, the negative bound for the charge density flux is $j = -39.89\hat{B}$.

On the other extreme, imposing an infinite drop of the potential in Eq. 5.18, this results the well-known Boltzmann relation,

$$n_e(\phi) = n_{e\infty} \exp(e\phi/kT_{e\infty}). \quad (5.104)$$

Hence, the electron flux must be 0 because electrons are fully confined. Equating the last equation and the one for the ion density (Eq. 5.15), and after some algebra this provides an implicit relation for $\phi(B)$,

$$\frac{\mu_{iT}B}{E_{i\infty}} = 4 \exp\left(\frac{e\phi}{kT_{e\infty}}\right) \left[\sqrt{1 - \frac{e\phi}{E_{i\infty}}} - \exp\left(\frac{e\phi}{kT_{e\infty}}\right) \right], \quad (5.105)$$

which is the same result obtained in the study of the magnetic cusp confinement [144]. At the magnetic throat the derivative of the magnetic field against the electric potential must be zero,

Figure 5.24 – (a) Solution $\Phi(\hat{B})$ when the current condition is switched to $j/\hat{B} = 0, -5, -10, -15, -35$, solid blue, dashed blue, solid red, dashed red and black line respectively. (b) Dimensional curves $\hat{\mu}_{eM} B_{Max} / kT_{e\infty}$ for different currents: *Maxwell-Boltzmann* equilibrium in green, solid line is for $\mu_{eM}(E^* = 3.13kT_{e\infty}, \phi)$, dashed line for $\mu_{eM}(1.5E^*, \phi)$ and dash-dotted line for $\mu_{eM}(2E^*, \phi)$; $j/B = 0$ in blue, solid line is for $\mu_{eM}(E^* = 3.07kT_{e\infty}, \phi)$, dashed line for $\mu_{eM}(0.5(E^* + E_{out}), \phi)$ and dash-dotted line for $\mu_{eM}(E_{out} = 6.21kT_{e\infty}, \phi)$; $j/B = -10$ in red, solid line is for $\mu_{eM}(E^* = 2.02kT_{e\infty}, \phi)$, dashed line for $\mu_{eM}(0.5(E^* + E_{out}), \phi)$ and dash-dotted line for $\mu_{eM}(E_{out} = 2.34kT_{e\infty}, \phi)$; $j/B = -20$ in black, solid line is for $\mu_{eM}(E^* = 1.22kT_{e\infty}, \phi)$, dashed line for $\mu_{eM}(0.5(E^* + E_{out}), \phi)$ and dash-dotted line for $\mu_{eM}(E_{out} = 1.28kT_{e\infty}, \phi)$; (c) Density ratio $n_{eh,out} / n_{e,out}$ for different values of the current j/B . (d) Critical energy $E^* / kT_{e\infty}$ (blue) and total dimensionless potential drop Ψ (red) as a function of the current carried by the plasma j/B .

$$\left. \frac{\partial B}{\partial e\phi} \right|_{\phi_0} = 0, \quad (5.106)$$

in which ϕ_0 is the potential at the throat. This condition allows to determine $\Phi_0 = e\phi_0 / kT_{e\infty}$ as a function of ion energy,

$$\frac{1}{2\varepsilon_i} = \sqrt{1 - \Phi_0 / \varepsilon_i} \left(\sqrt{1 - \Phi_0 / \varepsilon_i} - 2 \exp(\Phi_0) \right). \quad (5.107)$$

Once Φ_0 is determined, μ_{iT} is evaluated setting throat conditions in Eq. 5.105. For $\varepsilon_i = 0.1$, $\Phi_0 = 0.6827$ and $\hat{\mu}_{iT} = 0.4633$ (dimensionless with $kT_{e\infty} / B_{Max}$). And the ion flow according Eq. 5.22 $\Gamma_i = 0.518\hat{B}$.

All solutions $\phi(B)$ obtained by modifying the net current condition but ensuring the quasi-neutrality of the plasma are between these two limits. Figure 5.24 (a) shows the shape of the solution $\Phi(\hat{B})$ for different values of the net current, $j = \Gamma_i - \Gamma_e$. Note that the inflection

point of $\hat{B}(\Phi)$, at the divergent side, moves downstream (higher Φ) for larger negative currents. This has an impact on the contribution of the trapped population, whose contribution is also reduced, see for instance Figure 5.24 (b). This points out how the domes of μ_{eM} curves are reduced when the negative current is larger. In the case of the flat potential, the trapped population does not exist, and all electrons belong to the reflected or the free electrons classes. On the contrary, for the Boltzmann equilibrium, trapped electrons exist everywhere along the divergent side and they are responsible to keep the isotropic/isothermal behaviour of the electron population. Figure 5.24 (c), which draws the ratio $n_{e,f,out}/n_{e,out}$, highlights again that free electrons are more dominant for negative currents. Another variable that stresses the importance of trapped population is the critical energy ε_e^* , introduced already in Sec. 5.2.1. This is depicted in Figure 5.24 (d), dimensionless with $kT_{e\infty}$. This energy bound is zero for the flat potential case, although in such a case the minimum energy of escaping electrons is also 0, so $E^*/| -e\phi_{out}| \rightarrow 1$. On the other side, for the Maxwell-Boltzmann scenario, ε_e^* can be computed and results $E^*/kT_{e\infty} = 3.1343$ or $E^*/|e\phi_{out}| \rightarrow 0$. The conclusion is that there exists an infinite range of energies $E \in (E^*, \infty)$, in which electrons of high magnetic moment $\mu_e > \mu_{eT}$ are fully decoupled from the source. These appear from the MN throat until the end of the expansion, infinitely far downstream. For convenience, Ψ is also depicted in the same figure, highlighting that the electron range of trapped electrons becomes larger by increasing the current. Of course, for the last case Ψ diverges to infinity. These results are computed for $\varepsilon_i = 0.1$ and $m_i/m_e = 10^4$. Limit cases, say maximum and minimum current scenarios, $j = 0.518\hat{B}$ and $j_m = -39.89\hat{B}$ respectively, have been predicted properly.

High order moments, say temperature and heat fluxes, can be computed within all the explored range, $\phi_{out} \in (-\infty, 0)$. For the Boltzmann limit the temperature is imposed to be isothermal and isotropic everywhere. However, on the divergent side, electrons that sustain these conditions are mostly decoupled from the source as discussed before (see Fig. 5.24 (b)). So either collisions keep them thermalized or a non-steady process should trap/de-trap them in this area. Since all electrons are confined, heat fluxes are also 0, so this scenario cannot be justified by this collisionless model.

Concerning the flat potential scenario, the temperature is analytical in $\hat{B}$, and depends only on the magnetic field strength, as indicated next,

$$kT_{e\parallel}(\hat{B}) = \frac{n_\infty}{n_e} \frac{2\pi B}{m_e} \left(\frac{m_e}{2\pi kT_{e\infty}} \right)^{3/2} \int_0^\infty \int_{\mu_e} \frac{(w_{e\parallel} - u_e)^2}{\sqrt{\frac{2B}{m_e}(\mu_{em} - \mu_e)}} \exp(-E/kT_{e\infty}) dE d\mu_e$$

$$\text{Dimensionless: } T_{e\parallel} = \frac{1}{\hat{n}_e} \left\{ \frac{1 \pm (1 - \hat{B})^{3/2}}{2} - \frac{\hat{B}^2}{\pi(1 \pm \sqrt{1 - \hat{B}})} \right\}. \quad (5.108)$$

Figure 5.25 – Dimensionless electron temperatures and heat fluxes according Eqs. (5.108,5.109, 5.112,5.113)

Far upstream ($\hat{B} \rightarrow 0$), the limit of the expression above is unity and far downstream it reaches the constant value $T_{e||,out}/T_{e\infty} \rightarrow 3 - 8/\pi \approx 0.4535$. The dimensionless perpendicular temperature for this case is,

$$T_{e\perp}(\hat{B}) = \frac{1}{2\hat{n}_e} \left\{ 1 \pm \frac{3}{2} \hat{B} \sqrt{1 - \hat{B}} \pm (1 - \hat{B})^{3/2} \right\} \quad (5.109)$$

Regarding the heat flux (say random parallel flux axial kinetic energy), for the flat potential case, this results in the following analytical expression:

$$\begin{aligned} q_{e||}(\hat{B}) &= n_e \langle m_e \frac{c_{e||}^3}{2} \rangle = [...] \\ &= n_{\infty} k T_{e\infty} \sqrt{\frac{k T_{e\infty}}{2\pi m_e}} \left\{ 1 - (1 - \hat{B})^2 - \frac{3}{2} \hat{B} \frac{1 \pm (1 - \hat{B})^{3/2}}{1 \pm \sqrt{1 - \hat{B}}} + \frac{2}{\pi} \frac{\hat{B}^3}{(1 \pm \sqrt{1 - \hat{B}})^2} \right\} \end{aligned} \quad (5.110)$$

in which the limit $\hat{B} \rightarrow 0$ ensures $q_{e||} \rightarrow 0$. The flux of perpendicular heat becomes,

$$\begin{aligned} q_{e\perp}(\hat{B}) &= n_e \langle m_e c_{e||} \frac{w_{e\perp}^2}{2} \rangle = [...] \\ &= n_{\infty} k T_{e\infty} \sqrt{\frac{k T_{e\infty}}{2\pi m_e}} \left\{ \hat{B}^2 - \frac{\hat{B}}{1 \pm \sqrt{1 - \hat{B}}} \left[1 \pm \frac{3}{2} \hat{B} \sqrt{1 - \hat{B}} \pm (1 - \hat{B})^{3/2} \right] \right\} \end{aligned} \quad (5.111)$$

Note that heat fluxes expressions are proportional to $n_{\infty} k T_{e\infty} \sqrt{k T_{e\infty} / 2\pi m_e}$, which could be considered the reference heat flux to make them dimensionless:

$$\hat{q}_{e||}(\hat{B}) = 1 - (1 - \hat{B})^2 - \frac{3}{2} \hat{B} \frac{1 \pm (1 - \hat{B})^{3/2}}{1 \pm \sqrt{1 - \hat{B}}} + \frac{2}{\pi} \frac{\hat{B}^3}{(1 \pm \sqrt{1 - \hat{B}})^2} \quad (5.112)$$

$$\hat{q}_{e\perp}(\hat{B}) = \hat{B}^2 - \frac{\hat{B}}{1 \pm \sqrt{1 - \hat{B}}} \left[1 \pm \frac{3}{2} \hat{B} \sqrt{1 - \hat{B}} \pm (1 - \hat{B})^{3/2} \right] = \hat{B}^2 - \hat{B} \hat{T}_{e\perp} \quad (5.113)$$

Figure 5.25 plots together the temperatures and the heat fluxes for the flat potential scenario, according Eqs. (5.108,5.109, 5.112,5.113).

5.6 Conclusions

In this chapter, a steady solution of a fully magnetized and collisionless plasma expanding throughout a convergent-divergent magnetic channel has been determined, under the hypotheses of quasi-neutrality and zero current.

The total potential drop is solved self-consistently, and its parametric dependence on the mass ratio and the ion energy has been satisfactorily analysed. This result is more realistic than the unbounded potential drop of models that invoke the density-potential Boltzmann equilibrium, or polytropic models. The potential drop causes the partial depletion of the EDF, yielding to collisionless electron cooling, the loss of isotropy, and the formation of collisionless heat fluxes. The response of the plasma to this potential drop keeps some similarities with the structure and plasma behaviour in a classical electrostatic Debye sheath. However, the spatial distribution of this potential drop extends in all the domain for the scenario discussed in this Chapter, instead of the narrow width of the electrostatic sheath. Another difference is the non neutrality of the sheath, while in the studied case neutrality is invoked and kept thanks to the electrons trapped population.

The partial contribution of each electron population to the electron density and temperatures has been analysed in depth identifying the kind of population that dominates in each region. The trapped population has been identified as the responsible of sustaining the plasma quasi-neutrality on the divergent side for acceptable values of the dimensionless parameters. This is also essential to keep the whole confined population fully isotropic but not isothermal. These electrons considered as an isolated non-drifting population present a null temperature at the far field. The role of the high energy electrons consists of ensuring the zero net current condition of the plasma beam. These are also responsible for the asymptotic trend of the electron temperature. Asymptotic laws at the far field have been derived for the electric potential, density and temperatures.

The limits of the model have been explored as well. By decreasing the mass ratio $m_i/m_e \sim 1$, the particle dynamics of all species are more similar to each other, meaning a smaller presence of the trapped population, without penalizing the fulfilment of the quasi-neutrality and the current-free conditions.

By modifying the current condition while keeping the quasi-neutral hypothesis, two limits have been analytical determined: on one side, the flat potential scenario, and on the other side, the Maxwell-Boltzmann equilibrium. For the first one, all macroscopic moments of each distribution function are determined analytically because they do not depend on the

potential. At the Maxwellian limit, all of the divergent side is fully populated by trapped electrons, which indeed, maintain the isotropic and isothermal behaviour. This second limit could be justified by an eventual transient process that could fill forbidden orbits in velocity space, or by considering a fully confined electron population with eventual collisions that keep its distribution Maxwellian as well.

As future work several issues are proposed. First, to explore the effect of considering the superposition of two electron populations within the source, both Maxwellian but with different temperatures, say cold and suprathermal electrons. This should help to understand the formation of quasi-double layers, with more steepened drops of the electric potential, forming when there is a small fraction of hot electrons within the source. Second, it is of high interest the study of bi-Maxwellian electron/ion distributions at the plasma reservoir, because this is more consistent with the distribution shown by some devices as the ECR or the VASIMR. Third, transient effects or collisions are proposed here as a mechanism to explain the trapped population. To introduce time-dependent effects it is recommended to extend the current model to a Vlasov-Poisson model. For dealing with collisions it is necessary to switch to a more general model, Boltzmann equation solver, or to another kind of models, like full PIC codes. However, these models would lack the clarity of kinetic models.

6 HPT Experimental research

The HPS developed at the EPPDyL by Dr. Justin Little and Prof. Edgar Choueiri was initially conceived for studying the mechanisms of plasma detachment on the MN. Although thrust has never been measured directly, this source is regarded as one of the most efficient because of the relatively high propellant utilization [26]. The series of experiments presented in this Chapter aims to test some of the results derived in Chapter 3 [105], including the characterization of the backstreaming flows and the axial structure of the plasma within the source.

6.1 Introduction

The first test that was accomplished consisted of characterizing the plasma flux impinging on the rear wall. To do this, several molybdenum electrodes have been inserted in a redesigned back plate. This made necessary to change completely the antenna and feeding system in order to avoid RF signal interferences with the electrodes. The new set-up implements a single loop antenna, which is wrapped around the dielectric tube, and is spaced apart from the source by using small PTFE spacers. The antenna is supported by a renewed and larger coaxial feeder. Both antenna and feeder are made of copper and are water cooled as in the original design. Further details of the new set-up will be summarized in the following section.

The second test is related to the characterization of the plasma flux at the rear wall. It has also explored the possibility of magnetically screening this area as well. The proposed solution consisted in allocating an axially magnetized annular permanent magnet behind the injection section in order to generate a magnetic cusp, which is expected to reduce the plasma flux to the wall. The effect of this change on the magnetic topology has been explored in terms of propellant utilization and production efficiency.

Finally, the axial structure of the plasma density within the HPS was measured by a Langmuir probe at the ion saturation current, and assuming that plasma is almost isothermal within the source. These results should ideally behave as the profiles illustrated in Chapter 3.

The rest of the chapter is organized as follows. Section 6.2 describes the main characteristics of the HPS prototype used in this test campaign and the modifications implemented in some of its subsystems. Section 6.3 briefly summarizes the EPPDyL facilities and diagnostics. Section 6.4 presents all the noteworthy results obtained during the test, after post-processing the acquired data. Finally, Section 6.5 is for conclusions and discussion.

6.2 Prototype description

The HPS breadboard consists of the following elements: the discharge chamber and gas feeding system, the magnetic field generator and the RF subsystem. A sketch of the HPS implementing the planar antenna is presented in Figure 6.1 - (a).

The discharge chamber consists of a 7 cm inner diameter, 30.5 cm long quartz tube. This is concentrically aligned with the two electromagnetic coils, which constitute the magnetic generator. These coils are water cooled and connected to a DC power supply which provides the current that generates the magnetic field. Figure 6.2 - (a) shows the axial variation of the magnetic strength. This also indicates the conversion between the strength of the magnetic field at the nozzle throat B_0 (G), say at the middle point between both coils $(r, z) = (0, 0)$, and the applied current I_B (Amp).

A ceramic plate is placed in front of the planar antenna to isolate it from the generated plasma. This backplate (BP) can be moved along the quartz tube, like a piston in a cylinder, changing in this way the length of the discharge chamber. Figure 6.2 - (b) shows the shape of the convergent-divergent magnetic lines that originate at the BP edge for several positions of this. The gas injection tube, made of alumina to prevent thermal loads, is located at the center of the back plate. The following subsections inquire into the description of the RF subsystem and the new design of the back plate, both elements have been modified exclusively to achieve the experiment goals.

6.2.1 The RF subsystem and the single loop antenna

The RF architecture is schematically depicted in Figure 6.3. First, a wave generator produces the sinusoidal RF signal, typically at 13.56 MHz. This is connected to a power amplifier stage by

Figure 6.1 – Sketch of the HPS implementing the former planar antenna behind the back plate. Source: Ref [26].

Figure 6.2 – (a) Dimensionless magnetic field variation along the quartz tube, B/B_0 , and Magnetic field strength at the nozzle throat B_0 (G) vs applied DC current I_B (Amp). The axial distance is normalized with the equivalent coil radius, z/r_c , and it is measured from the MN throat, located at the centre point between both coils. (b) Magnetic streamlines shape. Source: Ref [26].

using a 50 Ohm coaxial cable. The load power is controlled by setting the gain (or amplitude) of the sinusoidal signal emitted by the wave generator. Just after the amplifier, a SWR (Standing Wave Ratio meter) will evaluate how much power is effectively delivered to the antenna. Since the impedance of the antenna is unknown, and highly depends on the signal frequency, a matching box is required to adapt the line impedance (50 Ohm) to the load. The matching box uses a L-type scheme, based on a series/shunt capacitors mechanically actuated. This box is fixed on the rear part of the test breadboard, already at the vacuum side in order to reduce RF losses. Finally, the single loop antenna, which replaces the planar one that was used in other experiments, is directly connected to the matching box through a copper coaxial feeder. The loop is approximately placed 14.5 – 15 cm away from the MN throat section.

An Agilent 8648B signal generator is used as the RF wave generator. Concerning the power amplifier, this is split into two elements: an ENI 2100L pre-amplifier, which increases the

Figure 6.3 – RF subsystem architecture flow diagram.

signal from below one W to 20 W; and the Alpha 9500 linear amplifier capable of providing up to 1.5 kW.

The capacitors implemented in the matching box are two Comet vacuum-variable capacitors with a range of 5 pF - 500 pF. Each capacitor is rated at 5 kV. Typically, the matching network is able to reduce the reflected power below the 20% of the total RF power, for any configuration of the set-up (antenna type and its position in the set-up).

In this experiment, a new single loop antenna is designed, built and implemented in order to not disturb the measurements of the electrodes inserted in the new back plate. This antenna wraps the cylindrical quartz tube whereas the planar one that was placed behind the ceramic back plate.

The antenna loop, made of copper, is fixed to the glass tube by using three Teflon spacers that reduce or almost cancel out all the mechanical oscillations of the whole antenna system, which would move as a cantilever beam without the spacers. Vibration of the structure is caused by the chamber pumps.

The antenna loop is connected to a coaxial feeder following a similar design, in contrast to the former planar antenna. The other end of the coaxial feeder is fixed to the matching box outlet connectors. This solution better confines the RF radiation. All the system is water cooled. A side view of the single loop antenna installed on the test breadboard is illustrated in Figure 6.4.

Figure 6.4 – Side view of the single loop antenna and partial view of the antenna feeder, at its final position before installing the quartz tube and the Faraday cage.

Figure 6.5 – New design of the back plate: (a) Front view of the back plate, with inserted molybdenum threaded rods, 2.18 mm diameter, and 2/56 thread size. (b) Back view with high temperature wires already connected to the molybdenum rods.

Electrode	R (mm)
E1	11.9
E2	14.7
E3	17.5
E4	20.0
E5	22.8
E6	25.7

Figure 6.6 – (a) Sketch of the backplate with the inserted molybdenum threaded rods (front view, from the HPS exit section). (b) Radial position of each electrode.

Figure 6.7 – (a) Orange Tank: large dielectric vessel, at the EPPDyL, University of Princeton. (b) RF compensated LP installed on the axial scanning/rotating arm system.

6.2.2 New back plate and planar electrodes

The back plate (BP) of the HPS has been modified in order to adapt it to the test requirements. This BP, made of Macor®, a machinable soft ceramic, is mainly a disc of 70 mm diameter and 16 mm of average thickness (Figure 6.5 (a)). entered on it, a 6.35 mm true hole supports the alumina tube that works as an injector.

Up to 6 molybdenum threaded rods have been inserted on it. These are electrically wired at the back part (Figure 6.5 (b)), and perform as an electrode when they are electrically biased. The Theory of planar Langmuir probes can be applied as noticed in the results (Sec 6.4).

Each rod is 2.18 mm diameter, having an exposed area or area of collection $A_{c,e} = 3.73 \text{ mm}^2$. The mentioned electrical wires are connected to an electrical box. Within the box, the wires are wrapped around three toroidal ferrites that filter out RF fluctuations and electrical noise on the collected current.

The radial and angular positions of each electrode are schematically drawn in Figure 6.6, and listed in the attached table.

6.3 Test facilities and main diagnostics

The HPS set-up is placed into the EPPDyL's Large Dielectric Pulsed Propulsion (LDPP) vacuum chamber, Figure 6.7 (a). This vessel roughly measures 2.5 m diameter and 7.6 m long, and it is made of fiberglass. Two mechanical Stokes roughing pumps, a root blower, and a 48 inch diameter CVC diffusion pump rated at 95,000 l/s of pumping capacity, evacuate the chamber. This vacuum system is able to reach ultimate pressures around $2 \cdot 10^{-6}$ Torr, however, due to leaks, the base pressure is around $5 \cdot 10^{-5}$ Torr in dry conditions. In all the tests performed

during this experiment the pressure never exceeded $\cdot 10^{-4}$ Torr.

Hot points at the magnetic generator, antenna feeder or matching box are monitored using a Flir i7 infrared camera, in order to prevent them from overheating, which could lead to a critical failure of the mentioned devices. Plasma diagnostics used in this experiment are mainly a RF compensated Langmuir probe and the aforementioned planar electrodes. Concerning the first one, its tip is made of tungsten wire 0.25 mm diameter, and 3 mm long. The acquisition scheme to obtain the current-voltage (I-V) curves, consists of a shifting oscillator that feeds a bi-polar operational amplifier (Kepco 1000M). These produce a sinusoidal signal at low frequency, 5 Hz, and around 100 V peak-to-peak waveform, which is the bias voltage for the LP. The current collected by the probe is measured across a low-pass filter, using 1 kOhm or 10 kOhm resistance, depending on the resolution requirements. Both, current and bias voltage are displayed in a Tektronix TDS 3034B digital oscilloscope. The LP is fixed on a rotating arm driven by a step motor, Figure 6.7 (b). The rotating axis of the arm is at the same axial position of the MN throat. A lead-screw, driven by a second step motor, allows moving the probe stand towards the HPS (radially or along the axis). Position accuracy is around 1 mm in the translation stage, and 1 deg in the rotating stage.

Radial or angular sweeps of the LP at the ion saturation current can be automatically performed by using DAQ and a dedicated LabView® tool.

6.4 Test results

In this test campaign several experiments have been performed for different configurations of the HPS set-up and with different settings of the operational parameters as well. These include the analysis of the HPS response at different lengths of the discharge chamber, mass flow rates, magnetic field strengths, and also the effect of including an axially magnetized permanent magnet behind the back plate in order to increase its shielding. Other studies attempted to unveil the HPS response at different RF power levels but they have not been included in this report.

Concerning what has been analysed and with which methodology, there are mainly 3 kind of measurements. First, the characterization of the plasma flux to the rear wall (back plate) is performed by biasing the electrodes at the ion current saturation regime (about -30 V). The collected current is measured through a low-pass filter, as in the LP. This allows to estimate the ion flux impinging the BP, and provides a first assessment of the non uniformities of this magnitude. Electron temperature and density at the wall are predicted by using them as

Figure 6.8 – Scenario A. Ion saturation at BP electrodes (markers) and at the LP located at the MN throat (black line), with increasing magnetic field strength I_B (Amp), and constant RF forward power $P_{RF} = 500$ W, for different mass flow rates: (a) $\dot{m} = 0.5$ mg/s, (b) $\dot{m} = 1.0$ mg/s, (c) $\dot{m} = 1.5$ mg/s and (d) $\dot{m} = 2.0$ mg/s.

planar Langmuir Probes. Second, axial sweeps along the HPS axis within it and along the MN near plume have been performed with the RF compensated LP at ion saturation. Assuming uniformity of the electron temperature, these sweeps illustrate the shape of the plasma density axial structure. Third, radial sweeps of the LP at the MN throat point out, among other figures, a first approximation of the propellant utilization efficiency.

6.4.1 Scenario A

This case considers a HPS length of $L = 18$ cm from the BP to the MN throat. Several mass flow rate levels are analyzed $\dot{m} = 0.5, 1.0, 1.5$ and 2.0 mg/s. RF forward power is maintained as constant as possible, $P_{RF} = 500$ W, during each measurement at different levels of the applied magnetic field I_B , typically from 0 to 20 Amp. The ion saturation current is measured in each electrode at the BP and compared with the ion saturation current collected by the LP at the MN throat. Based on these currents, the ion flux g_i is estimated and depicted in Figure 6.8 (a-d),

for the mentioned levels of $\dot{m}$ and increasing magnetic field I_B .

From these graphs several conclusions are extracted. First, there is a mode transition at a certain magnetic field I_B^* . This clearly depends on the injected mass flow rate, occurring at larger applied fields for higher $\dot{m}$. $I_B^* \approx 8.5, 10, 12.5, 14$ Amp for $\dot{m} = 0.5, 1.0, 1.5, 2.0$ mg/s respectively. Second, there is not a clear trend of the variation of g_i with I_B increasing within the range $I_B < I_B^*$, and this variation seems to depend again in $\dot{m}$. However, it is quite clear and evident the drop of the collected current at the BP within the range $I_B > I_B^*$. Third, the ion saturation current at the LP increases with I_B , if $I_B < I_B^*$; at $I_B \approx I_B^*$ it is maintained constant or barely drops; the current increases again once $I_B > I_B^*$.

It is noticeable that around I_B^* , large oscillations of the collected current appear in the measurements at the BP electrodes, as well as in the LP measurements. The frequency of these oscillations is about 3 kHz, and the amplitude of them is in the order of the averaged current. These oscillations are maintained along the full range $I_B > I_B^*$. This low frequency, is indeed lower than the ion (axial) transit frequency, $c_s/L \sim 20$ kHz, and could be attributed to a kind of breathing mode like in a HET [145]. This could mean that the fact of increasing the magnetic field enhances plasma ionization, and consequently, the propellant utilization, which will be investigated later.

Analysing in depth the results in Fig. 6.8, the ion flux at the BP electrode is within the range $10^{20} - 10^{21} \text{ m}^{-2} \text{ s}^{-1}$ (for $I_B \leq I_B^*$) and drops to $10^{20} \text{ m}^{-2} \text{ s}^{-1}$ at $I_B > I_B^*$. Fluxes measured at the LP are below $10^{21} \text{ m}^{-2} \text{ s}^{-1}$ for very low magnetic fields and increase up to $2.5 - 4 \cdot 10^{21} \text{ m}^{-2} \text{ s}^{-1}$ at larger fields. Note that the average ion flux, assuming sonic conditions and full ionization of the injected gas $\dot{m} = 0.5$ mg/s (Fig 6.8 a), is $1.83 \cdot 10^{21} \text{ m}^{-2} \text{ s}^{-1}$, which is in the same order of the measured flux at the MN throat, but an order of magnitude larger than the one measured at the electrodes. Also, the measured profile is not uniform in the MN throat cross-section. This non-uniformity is observed in the electrodes measurements as well. Typically, more current has been collected at the electrodes located closer to the centre of the plate (open circles, E1, E2, E3, in Fig. 6.8) in comparison to the lower current collected at the outer electrodes (open squares, E4, E5, E6). Radial sweeps of the LP at the MN throat also point out this non uniformity. Furthermore, in most of the cases a double peak profile has been detected, suggesting that the plasma discharge is inductively generated, coupling more power at the plasma column edge.

Plasma density and electron temperature have been determined at different radial positions in the MN throat section for this Scenario A, and $\dot{m} = 0.5$ mg/s (Figure 6.9). Assuming that

Figure 6.9 – Scenario A. (a) Plasma density and (b) electron temperature, at different radial positions (cm from the cylinder centreline) on the MN throat section, for increasing I_B , RF forward power $P_{RF} = 500$ W and $\dot{m} = 0.5$ mg/s. Subfigure b plot follows the same color code as (a).

Figure 6.10 – Scenario A, validation of LP ion saturation current $g_{i,sat}(r)$ (green curves) at the MN throat, against the ion sonic flux $g_s = nc_s$, computed according to the values of n and T_e presented in Figure 6.9. $I_B = 14$ Amp and 24 Amp in (a) and (b) respectively, constant RF forward power $P_{RF} = 500$ W and $\dot{m} = 0.5$ mg/s.

plasma is sonic at the throat, this allows validating the radial sweepings of the LP as shown as an example in Figure 6.10. These radial profiles are acquired for most of the analysed cases in order to estimate the propellant utilization. Furthermore, the ion flux measured by the LP at I_{sat} , is quite consistent with the ion sonic flux estimated by using n and T_e from the I-V curves. This allows making faster radial scans, without the need of obtaining I-V curves, whose DAQ is much slower.

The density at the throat increases quickly with I_B from $10^{16} m^{-3}$ up to low $10^{17} m^{-3}$, for $I_B < I_B^*$ as shown in Fig 6.9. Then, at $I_B > I_B^*$ the density is maintained pretty uniform. Regarding the electron temperature, it behaves in the opposite way, decreasing from a maximum of 8 eV at $I_B \sim 2$ Amp down to 2-5 eV, depending on the radial position. Radial profiles of $n(r)$ and $T_e(r)$ at constant I_B point out that plasma is denser and hotter within the range $r \in (-1, 1)$ cm.

Figure 6.11 – Scenario A, plasma density (a,c) and electron temperature (b,d), at each BP electrode with increasing I_B , and constant RF forward power $P_{RF} = 500$ W. $\dot{m} = 0.5$ mg/s in (a,b) and $\dot{m} = 1.0$ mg/s in (c,d). (b,d) plot follows the same color code as (a,c).

The electron density and temperature have been determined in the BP electrodes as well. Figure 6.11 provides the results at $\dot{m} = 0.5, 1.0$ mg/s. Density increases with I_B up to $1.5 - 2.5 \cdot 10^{17} \text{ m}^{-3}$, before reaching I_B^* , then drops abruptly and it is kept almost constant around 10^{17} m^{-3} . Concerning the electron temperature, $T_e = 5 \pm 1$ eV can be considered in the full range. The error in the estimation of T_e is around 15-20%, so the fluctuations are attributed to this error.

The total plasma flow is estimated at the BP and at the MN throat, based on all previous results: densities, temperatures, and fluxes. This provides a rough approximation to the source efficiency in terms of propellant utilization η_u and production efficiency η_p , as explained in Chapter 3, Section 3.3. Figure 6.12-(a) shows that η_u increases with I_B . However, this performance does not improve with $\dot{m}$, concluding that the larger mass flow rate does not enhance ionization, or in other words, the improved ionization gain is lower than the increment of injected gas amount. This response does not fit the results in Figure 3.11 of Chapter 3, that claimed that for a given I_B the larger $\dot{m}$ is, the larger η_u should be. The best $\eta_u \sim 0.4$ is reached

Figure 6.12 – Scenario A: (a) Estimation of the propellant utilization, η_u and (b) dimensionless plasma flow to the BP, $\dot{m}_i/\dot{m}$; with increasing I_B , constant RF forward power $P_{RF} = 500$ W, and different mass flow rates, $\dot{m} = 0.5, 1.0, 1.5, 2.0$ mg/s.

Figure 6.13 – Scenario A: Estimation of the highest production efficiency $\eta_{p,M}$ considering zero plasma losses to the lateral wall.

at $\dot{m} = 0.5$ mg/s and $I_B > I_B^*$. At all other $\dot{m}$ levels, the optimum η_u is between 0.15 and 0.25. Concerning the dimensionless ion flow rate to the BP, $\dot{m}_{i,BP}/\dot{m}$, Figure 6.12-(b) points out that it is minimized beyond the critical magnetic field strength $I_B > I_B^*$. As in the η_u response, this is also lower at larger mass flow rates $\dot{m}$, thus confirming the poorer ionization efficiency when $\dot{m}$ increases. Finally, assuming zero losses of plasma to the lateral wall $\dot{m}_w$, it is possible to estimate the maximum production efficiency, $\eta_{p,M}$,

$$\eta_{p,M} = \frac{1}{1 + \dot{m}_{i,BP}/\dot{m}_{i,E}}, \quad (6.1)$$

a result that it is shown in Figure 6.13 (a). The trend of $\eta_{p,M}(I_B)$ is almost the same for all different $\dot{m}$ levels, pointing out that independently of how much plasma is generated, its axial dynamics (forward vs backward plasma flow) preserves its proportionality, a ratio that it is more favourable at larger magnetic fields. In comparison to the theoretical results of Chapter 3, the maximum η_p was 0.5 for the unshielded BP, a worse result than that estimated in this experiment $\eta_{p,M} \sim 0.7 - 0.8$. However, let the reader note that the convergent geometry of the field within the source could probably guide a fraction of the produced plasma to the lateral

Figure 6.14 – Case B. Ion saturation current at BP electrodes (markers) and at the MN located at the MN throat (black line), with increasing magnetic field strength I_B (Amp), and constant RF forward power $P_{RF} = 500$ W, for different mass flow rates: (a) $\dot{m} = 0.5$ mg/s, (b) $\dot{m} = 1.0$ mg/s, (c) $\dot{m} = 1.5$ mg/s and (d) $\dot{m} = 2.0$ mg/s.

wall (close to the back part), so the amount of plasma that is actually moving backwards has been underestimated, and also to the fact that Eq. 6.1 omits lateral wall plasma losses due to cross field diffusion. The shape of the magnetic streamlines depicted in Figure 6.2 - (b) helps to understand the idea that plasma could be guided backwards to the lateral wall.

6.4.2 Scenario B

This case considers a HPS length of $L = 15.5$ cm. The same procedure as in Scenario A ($L = 18$ cm) has been followed, investigating several mass flow rates levels $\dot{m} = 0.5, 1.0, 1.5$ and 2.0 mg/s. RF Power is maintained constant at $P_{RF} = 500$ W, and the strength of the applied magnetic field I_B is typically rated at 0-20 Amp. The ion saturation current is measured in each electrode at the BP and compared with the ion saturation current collected by the LP at the the MN throat. Based on these currents, the ion flux g_i ($1/m^2s$) is estimated and depicted in Figure 6.14 (a-d), for the different $\dot{m}$ and increasing I_B .

Figure 6.15 – Case B: (a) Estimation of the propellant utilization, η_u and (b) dimensionless plasma flow to the BP, $\dot{m}_i/\dot{m}$.

Figure 6.16 – Case B: Estimation of the highest production efficiency $\eta_{p,M}$ considering zero plasma losses to the lateral wall.

In comparison to case A, the mode transition occurs at a lower I_B^* for the same $\dot{m}$. Furthermore, plasma discharge is lost at $I_B > I_B^*$ or it is very tenuous. So in this scenario of a short chamber, the study of the source performances at $I_B > I_B^*$ has no interest. This barrier occurs at $I_B^* = 6, 7.5, 9$ and 11 Amp for $\dot{m} = 0.5, 1.0, 1.5, 2.0$ mg/s respectively. g_i increases with I_B in all the measurements ($I_B < I_B^*$), including BP electrodes and LP at the MN throat.

Analogously to the previous case, the total plasma flow is estimated at the BP and at the MN throat, a result that is depicted in Figure 6.15. Regarding η_u , results are slightly worse than those obtained in the long chamber case, below 30 % in the best case at $\dot{m} = 0.5$ mg/s. The dimensionless mass flow rate of plasma into the BP is in the same order that the measured in Scenario A. Consequently, the maximum production efficiency $\eta_{p,M}$, omitting again lateral losses, becomes poorer than in case A and it is kept below 50 % for all $\dot{m}$. Apart from these drawbacks on the efficiency, the range of operability is also more limited due to the loss of the discharge (or jump to a capacitive mode) beyond I_B^* .

Figure 6.17 – Analysis of the influence of a PM placed behind the BP, $L = 18 \text{ cm}$. Ion saturation current at BP electrodes (markers) and at the LP located at the MN throat (black line), with increasing magnetic field strength I_B (Amp), and constant RF forward power $P_{RF} = 500 \text{ W}$, for different mass flow rates: (a) $\dot{m} = 0.5 \text{ mg/s}$, (b) $\dot{m} = 1.0 \text{ mg/s}$. (Markers color code is the same as in Fig. 6.14).

6.4.3 Influence of magnetically screening the BP

For the long chamber configuration, a permanent magnet PM, made of ferrite and axially magnetized, has been placed behind the Macor BP. This forms a magnetic cusp close to the rear BP. As a result, plasma should be screened, decreasing the total particle flux to the BP, and consequently, increasing the source efficiency (higher η_p).

At different mass flow rates, $\dot{m} = 0.5, 1.0 \text{ mg/s}$ fluxes to the BP electrodes and to the LP at the MN throat have been read against variations of I_B 6.17. In comparison to the results of Scenario A, the new configuration does not show any clear mode transition, and I_B^* cannot be defined. Fluxes to the electrodes also respond differently. Fluxes already decrease from low applied I_B , then, for intermediate applied fields these increase or are kept constant, and finally decrease again. So, the PM is affecting the whole range of applied fields. The current collected by the electrodes is in the same range, even larger, than in the case without PM (Scenario A). The $g_{i,sat}$ at the LP monotonically increases, a response, which has also been perturbed by the presence of the PM, however it is maintained in the same order of magnitude.

$\eta_u(I_B)$ and $\dot{m}_{i,BP}/\dot{m}$ are depicted in Figure 6.18 together with the previous results of Case A. Concerning the propellant utilization it is slightly larger when the PM is included in the set-up. However, the flux to the BP has been increased, although the PM was thought to reduce the plasma flow impinging the BP.

Figure 6.19 illustrates the maximum production efficiency $\eta_p(I_B)$, neglecting losses to the lateral wall. Although the response of η_u and $\dot{m}_{i,BP}/\dot{m}$ has been modified due to the presence

Figure 6.18 – Analysis of the influence of a PM placed behind the BP, $L = 18$ cm. (a) Estimation of the propellant utilization, η_u and (b) dimensionless plasma flow to the BP, $\dot{m}_i/\dot{m}$. Thin lines belong to the results in Scenario A without permanent magnets.

Figure 6.19 – Analysis of the influence of a PM placed behind the BP, $L = 18$ cm. Estimation of the highest production efficiency $\eta_{p,M}$ considering zero wall losses to the lateral wall. Thin lines belong to the results in Scenario A.

of the PM, $\eta_p(I_B)$ shows a very similar trend. This preliminary test with the PM allows concluding that the screening of the walls using magnetic cusps requires a better understanding of the whole discharge and the plasma transport in uncommon magnetic topologies.

6.4.4 Axial structure of the plasma discharge

The last study consists of exploring the structure of the plasma density along the source axis and in the near plume (~ 20 cm). Data is acquired by sweeping axially the LP at $I_{i,sat}$. Since this cannot reach the rear wall of the source, it is limited at $z = -16$ cm, values at the wall are interpolated with those measured by the BP electrodes (E1, E2 and E3).

Density is estimate assuming $T_e \sim 5$ eV, which is a good approximation according to the previous results. Figure 6.20 (a) depicts the density axial profile $n(z)/n_0$ normalized with n_0 , which corresponds to the density measured at the centre of the MN throat section. This

Figure 6.20 – (a) Axial structure of the plasma density n/n_0 , at $\dot{m} = 0.5$ mg/s, $P_{RF} = 500$ W, $L = 18$ cm; for several strengths of the magnetic field. n_0 is the density at the centre of the MN throat section, which is depicted in (b), $n_0(I_B)$.

Figure 6.21 – (a) Axial structure of the plasma density n/n_0 , at $\dot{m} = 1.0$ mg/s, $P_{RF} = 500$ W, $L = 18$ cm; for several strengths of the magnetic field. n_0 is the density at the centre of the MN throat section, which is depicted in (b), $n_0(I_B)$.

value varies with I_B as demonstrated in Fig.6.20 (b). These results are obtained using the configuration of Scenario A and $\dot{m} = 0.5$ mg/s, while Figure 6.21 is for $\dot{m} = 1.0$ mg/s.

In comparison to the results in Chapter 3 (Fig. 3.6), the axial structure is qualitatively similar for low magnetic fields, $I_B < I_B^*$, and it is peaked close to the rear wall. For stronger applied fields, beyond I_B^* the density peak within the source vanishes and the density profile is almost flat along the range $z/L \in (-0.75, 0)$. The density at the BP is lower than that predicted by the theory, in which $n(-L) \sim \eta_u \dot{m} / (m_i \pi R^2 c_s)$. This lower flux to the wall is the cause of reaching larger values of η_p than those estimated by the theory. Concerning the drop of the plasma density on the divergent plume side, all cases presents a similar trend, and this is almost independently of I_B , It is not straightforward to explain why the theory does not fit the results, but it is possible to point out main differences: (1) in the current experiment, the magnetic

Figure 6.22 – Axial structure of the plasma density, considering Scenario A, and $B_0 = 60, 100, 140$ G. Solid lines are for the simulation results and dashed for the experiments. (a) $\dot{m} = 0.5$ mg/s and $T_e = 7$ eV, resulting $P_{abs} = 100, 153, 156$ W for increasing B_0 (power balance). (b) $\dot{m} = 1$ mg/s and $T_e = 6$ eV, resulting $P_{abs} = 442, 450, 424$ W for increasing B_0 .

topology within the tube is not axial as in the model, here, there is a clear convergent channel until the MN throat; (2) this shape guides part of the plasma to the lateral wall, an amount of plasma which can be large, if this were measured, the estimated η_p would be reduced; (3) the theory does not include anomalous transport, a mechanism that could alter the plasma discharge as in other devices; (4) T_e is assumed uniform in the computation of the density axial profile $n(z)/n_0$, as well as in the theory, but this is not necessarily true; (5) experimental error can be large, e.g. around 15-20 % for T_e .

Figure 6.22 shows the axial density profile for several magnetic strengths $B_0 = 60, 100, 140$ G, considering the set-up of scenario A, two different mass flow rates, $\dot{m} = 0.5, 1$ mg/s and two different temperatures, $T_e = 7, 6$ eV. Experimental results, in dashed line, are depicted together with those obtained by using the fluid model, solid lines. The plasma peak, which corresponds to a region of high ionization, seems to be the only feature recovered by the theoretical model, although numerical values are quite different. The power balance applied to the model results indicate that for $\dot{m} = 0.5$ mg/s and $T_e = 7$ eV, P_{abs} should be about 150 W, a power level far from the experimental rated power, $P_{RF} \sim 500$ W. On the contrary for $\dot{m} = 1.0$ mg/s and $T_e = 6$ eV, the absorbed power is about $P_{abs} = 442, 450, 424$ W at $B_0 = 60, 100, 140$ G respectively, which approaches better the emitted RF power. The theoretical model does not recover any change of the axial profile trend when $B_0 > B_0^*$ (say $I_B > I_B^*$).

Concerning the theoretical predictions of the propellant utilization and production efficiency, Figure 6.23 compares these with the experimental estimation. Scenario A is considered again under the same conditions detailed above. $\eta_u(B_0)$, $\eta_p(B_0)$ follow an increasing trend with B_0 at weak fields and then a plateau for stronger fields. A similar trend is shown in both experi-

Figure 6.23 – Propellant utilization $\eta_u(B_0)$ in blue and production efficiency $\eta_p(B_0)$ in black, for the Scenario A. Solid lines are for the simulation results and dashed for the experiments. (a) $\dot{m} = 0.5$ mg/s and $T_e = 7$ eV. (b) $\dot{m} = 1$ mg/s and $T_e = 6$ eV.

mental and theoretical results. However, the theory overestimates these figures. η_p obtained from experimental data does not follow the same criteria of $\eta_{p,M}$ in Figures 6.13 and 6.16, in which a null loss of plasma to the lateral wall was postulated. The current interpretation of the data considers that part of the plasma generated impinges directly into the lateral wall due to the magnetic field divergence. This is accounted by enlarging the equivalent area of the rear wall, adding to this a fraction of 6 cm of the lateral wall. Consequently, η_p (black dashed line with squared markers) is reduced strongly in comparison to $\eta_{p,M}$.

6.5 Conclusions

The test campaign carried out with the modified HPS at the EPPDyL can be considered of a great success. Concerning the modification of the HPS, the design, fabrication and integration of the annular antenna with its own coaxial feeder has offered similar performances that the planar former antenna. The measured density and temperatures suggest that the HPS is operating in a high density inductive mode. The planar electrodes installed at the back plate have been useful to estimate the ion flux impinging the rear wall, as well as the temperature and plasma density when these are operated as planar Langmuir probes. This study has determined that the flux to the back wall is not comparable to the forward plasma flux as predicted by some theoretical models. These electrodes have been useful also for detecting plasma fluctuations in the kHz range, probably associated to ionization or ion transit processes. These measurements at the back plate together with the estimation of $\eta_u(B_0, \dot{m})$ have determined the maximum HPS production efficiency, $\eta_{p,M}(B_0, \dot{m})$, which seems to be independent of the injected mass flow rate, and monotonically increasing with

the applied magnetic field.

Regarding the possibility of magnetically screening the rear back wall in order to reduce plasma losses, this solution requires further assessment according to the obtained results. Furthermore, the permanent magnet modifies the whole discharge and HPS performances.

Finally, the axial structure of the plasma density measured within the source does not fit quantitatively the theoretical predictions. Discrepancies are attributed to the different magnetic topology of the experiment and the general simplicity of the theoretical model, which also overestimates the propellant utilization and production efficiency.

7 Pre-design of a HPT

This chapter sets the basis for the design of an experimental HPT. Firstly, simplified fluid and rf-wave propagation models are presented and implemented in order to define a suitable design point, and determine the main operational parameters: mass flow rate, magnetic strength, chamber dimensions, etc.. The second section deals with a 1D model of the RF-wave plasma coupling, and extends it to the scenario of non-uniform plasma density and magnetic field, with the aim of determining the plasma resistance and selecting the most efficient antenna. The last section analyses in depth the design point by applying the fluid model of Chapter 3 in order to assess HPS figures of merit. A preliminary insight of thermal loads due to conductive and radiative heat is also attempted in this section. Finally, the HPT theoretical performances are discussed.

7.1 Introduction to the HPT design methodology

A detailed design of a HPT should be carried out according to some of the models presented in the previous chapters of this Thesis. However, most of the computational tools, based on the mentioned models, are highly time demanding. In the current Chapter a simplified methodology is presented and applied to the specific case of a 15 kW HPT.¹

The simplified methodology proposed here consists of determining some of the main design parameters self-consistently through the assessment of the HPS energy balance, while other parameters are frozen assuming several hypotheses.

Table 7.1 lists all parameters that are kept constant in the current method. Some of them are taken from the literature review of Chapter 2, and others are justified by the results obtained in

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¹Work performed within the frame of the ESA project “HPT for Space Missions”, Ref. [146]

previous chapters or from the mentioned DIMAGNO model (in the case of MN performances). The total power or electric power P , in Watts, excludes in this analysis the power feeding the magnetic circuit. The utilization efficiency η_u assumes the quasi complete ionization of the neutral gas and disregards its dependence on the plasma temperature, the mass flow rate, and the HPS geometry. Once the HPT pre-design set-up is determined, this assumption is checked out by using the 2D fluid model. The efficiency of the RF subsystem and all mechanisms for which RF-power is coupled into the plasma are considered together in the same parameter, η_{RF} , which is taken constant, although these parameters depend on other plasma properties. The neutral gas temperature T_g , which is necessary to estimate its thermal velocity and guess the neutral gas injection velocity, is taken constant as well. The RF-wave frequency f_{RF} is set to the standard value of 13.56 MHz. The energy of ejected ions E_{beam} is assumed to be several times the plasma temperature T_e . This approximation assumes that the gain of kinetic energy per ion is due to the acceleration through the electric potential drop, which develops self-consistently along the plasma expansion. This simplifies the existing uncertainties on the thermodynamics of plasma species, electrons and ions, along the plume, independently whether it is magnetized or not. Energy losses due to the plume divergence are considered in η_{div} , which is assumed high for a well-designed MN. The chamber aspect ratio $k_{geom} = L/R$ will be taken constant in order to simplify the parametric analysis, although the chamber length is an implicit function of some other parameters, as indicated in the definition of η_u , Eq. 3.49. Argon is set as the propellant.

The parametric analysis is carried out in terms of the following dimensional properties in order to determine the optimum design:

1. Nominal plasma density n_0 .
2. The applied magnetic field B_0 , uniform and constant within the HPS.
3. The average electron temperature T_e .

The design process will define several central variables of the HPS, including the nominal mass flow rate $\dot{m}$ and all other parameters listed in Table 7.2, which shows how the HPS would behave within the parametric space $n_0 - B_0 - T_e$.

Given an electron temperature T_e , the mass flow rate of neutral gas $\dot{m}$ for each pair of $B_0 - n_0$ values is defined implicitly, say $\dot{m} = \dot{m}(n_0, B_0, T_e)$, by the HPS global energy balance:

$$P_a = P_{beam} + P_{inel} + P_{wall}, \quad (7.1)$$

where each term is defined as follows,

$$P_{beam} = \eta_u E_{beam} T_e \frac{\dot{m}}{m_i} \text{ and } P_{use} = \eta_{div} P_{beam}, \quad (7.2)$$

$$P_{wall} = \frac{\dot{m}_{wall}}{m_i} T_e \left(\frac{5}{2} + \frac{1}{2} \ln \left[\frac{m_i}{2\pi m_e} \right] \right), \quad (7.3)$$

$$P_{inel} = (\eta_u \dot{m} + \dot{m}_{wall}) \frac{E'_{inel}}{m_i}. \quad (7.4)$$

Here, E'_{inel} is the energy cost of ionization per electron-ion pair, which also includes losses due to excitation and radiation. P_{beam} is the power of the ejected plasma beam, P_{use} is the axial-directed kinetic energy of ions at the end of the expansion, P_{wall} is the power lost to the dielectric wall, and P_{inel} is the power cost of ionization, radiation and excitation. Eq. 7.1 is an implicit equation for $\dot{m}$. This would be linear and easily solvable if the mass flow to the wall $\dot{m}_{wall}$ were constant. However, this is not correct within all the analysed range, i.e., it is influenced by plasma confinement. Considering its dependence on the main parameters and other inputs and taking the results of Ref. [105] it is possible to use an asymptotic approximation to determine $\dot{m}_{wall}$ as a function of $\dot{m}$:

$$\dot{m}_{wall} \simeq \pi k_{geom} R^3 m_i n_0 \nu_w, \text{ with} \quad (7.5)$$

$$R^2 = \frac{\dot{m}}{m_i \pi n_0 c_s}, \text{ and} \quad (7.6)$$

$$\nu_w = 2.405^2 \left(\frac{c_s^2}{R^2} \right) \left(\frac{m_e m_i}{e^2 B_0^2} \right) (\nu_{ei} + \nu_{en}). \quad (7.7)$$

c_s is the ion sound speed, ν_{ei} and ν_{en} accounts for electron collisions with ions and electrons respectively, and ν_w is an equivalent frequency of plasma recombination at the wall. It is expected that neutrals should deplete along the dielectric tube. Thus, it is expected that the neutral density should drop more than one order of magnitude. For this reason in $\nu_{en} = n_n R_{en}$, n_n is taken as a small fraction of the neutral density at the feeding section $n_{n0} = n_0 c_s / u_{n0}$, with $u_{n0}^2 = T_g / m_i$.

After some algebra, Eq. 7.1 is solved as a 4th order polynomial on $\dot{m}$. After determining $\dot{m}$, the production efficiency, η_p is computed, assuming that any possible backward flow of plasma is magnetically-screened and thus negligible,

$$\eta_p = \frac{1}{1 + \frac{\dot{m}_{wall}}{\eta_u \dot{m}}}. \quad (7.8)$$

Finally, the chamber radius is determined by Eq. 7.6.

Input design parameter	Value
P	15 kW
η_u	0.95
η_{RF}	0.9
<i>Propellant</i>	Argon
T_g	800 K
f_{RF}	13.56 MHz
E_{beam} (times T_e)	5
η_{div}	0.8
k_{geom}	5.0

Table 7.1 – Input parameters for the pre-design of a kW class HPT and expected nominal figures.

Output design parameter	Value
Mass flow rate	$\dot{m}$
Plasma production efficiency	η_p
Chamber radius	R
Hall parameter	χ_H
Plasma Beta	β_H
Electrons Larmor dimensionless radius	l_e/R
Dimensionless Deby length	λ/R
Inelastic power ratio	$\epsilon_{inel} = P_{inel}/P_a$
Power to the wall ratio	$\epsilon_{wall} = P_{wall}/P_a$
Useful power ratio	$\epsilon_{use} = P_{use}/P_a$

Table 7.2 – Main output parameters of the parametric analysis, pre-design of a sW class HPT.

7.1.1 Fluid criteria

The fluid criteria are based on the evaluation of the main dimensionless parameters within the parametric space T_e, n_0, B . These identify the range in which the HPT would behave better, with larger efficiencies.

1. The **Hall parameter**, $\chi_H = \omega_{lh}/\nu_e = eB_0/\sqrt{m_i m_e} \nu_e$, is defined including collisions with neutrals, $\nu_e = \nu_{ei} + \nu_{en}$. It assesses magnetic confinement versus perpendicular collisional transport and it must be large, $\chi_H \gg 1$. At large values, plasma moves diffusively through the magnetic lines towards the lateral wall, plasma-wall losses being reduced.
2. The **Plasma beta**, $\beta_0 = \mu_0 T_e n_0 / B_0^2$. It compares thermal pressure over magnetic pressure. Small values of $\beta_0 \ll 1$ ensures that the induced magnetic field, which tends to demagnetize the plasma, is not strong, and can be fully disregarded.
3. The dimensionless **Larmor radius** of electrons, $l_e/R = \sqrt{T_e m_e} / e R B_0$. It measures the magnetic confinement in terms of the plasma size. Small values, $l_e/R \ll 1$, are preferable.
4. The dimensionless **Debye length**, $\lambda_D/R = \sqrt{\epsilon_0 T_e / e n_0} / R$, which is expected to be small, $\lambda_D/R \ll 1$, allows considering the plasma quasi-neutral everywhere except at the Debye sheaths attached to the walls.

7.1.2 Wave criteria: 0th order model of the RF wave propagation

The second criterion inquires into the analysis of the required conditions in terms of n_0, B_0 and T_e that ensure Helicon wave propagation within the plasma column. To deal with this phenomenon, the 0th wave-plasma model is considered as a first approximation, and it is briefly presented in this subsection. This states both magnetic field and plasma density to be uniform everywhere.

More generally, the study of the plasma-wave interaction phenomena aims to determine how efficient is the mechanism by which the RF power is damped or absorbed into the plasma. This involves the following processes, (1) the RF wave propagation through the magnetized plasma column, (2) the RF energy absorption by the plasma, and (3) the influence of plasma properties on the radio-frequency subsystem (RFSS) .

The propagation of the RF wave on the surrounding environment and plasma column is addressed with the Maxwell equations. Since the RF emission has a single frequency $\omega/2\pi$,

taking the temporal Fourier transform $\propto \exp(i\omega t)$, the Maxwell equations become

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}, \quad (7.9)$$

$$\nabla \times \mathbf{B} = i\omega \bar{\epsilon} \cdot \mathbf{E} + \mu_0 \mathbf{j}_a. \quad (7.10)$$

$\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field, $\bar{\epsilon}$ is the dielectric tensor, μ_0 is the vacuum permeability, i the imaginary unit, and the source term, $\mathbf{j}_a$, is the electric current density through the antenna.

The dielectric tensor carries all the information of the static magnetic field generated by the electromagnets, the material environment and plasma properties. Inside the plasma $\bar{\epsilon}$ is built upon three scalar functions related to the parallel, perpendicular, and Hall conductivities [21]. For usual conditions, these depend on the quasi-static electron and ion distribution functions.

In the literature, there are basically three different levels of analysis for the above Maxwell equations: (i) a zero-dimensional (0D) analysis of a uniform infinite plasma and uniform static magnetic field; (ii) a one-dimensional (1D) analysis of a cylindrical plasma column, axially uniform with an axial static magnetic field, surrounded by vacuum and the antenna; and (iii) a two-dimensional (2D) analysis of the plasma, magnetic field, chamber and antenna geometries.

Starting from the 0D analysis, this offers an easy and clear insight to comprehend the theoretical background of RF waves within unbounded magnetized plasmas. From Eqs. 7.9 and 7.10, and considering that waves are of the form $\propto \exp(i(\mathbf{k}\mathbf{x} - \omega t))$ the dispersion relation for the wavenumber vector $\mathbf{k}$ is easily derived. Based on a dimensional analysis, and in the most general case, although excluding ion effects, the dispersion relation depends on the following parameters, $\mathbf{k} = \mathbf{k}(\theta, \omega_{ce}/\omega, \omega_{pe}/\omega, \nu/\omega)$; with ω_{ce} the electron cyclotron frequency, ω_{pe} the plasma frequency, ν_e the effective collision frequency for electrons, ω the RF frequency, and θ the angle between $\mathbf{k}$ and the magnetic field $\mathbf{B}_0$, which is uniform. Defining also the vacuum wavenumber $k_0 = \omega/c$, in which c is the speed of light in vacuum, and taking $\omega_{pe} \gg \omega_{ce}, \omega, \nu$, the dispersion relation becomes [21],

$$\frac{N^2}{(\omega_{pe}/\omega)^2} = \frac{-1 \mp \frac{\omega_{ce}}{\omega} |\cos\theta| / \left(1 + i \frac{\nu}{\omega}\right)}{\left(1 + i \frac{\nu}{\omega}\right) \left(1 - \left(\frac{\omega_{ce}}{\omega} |\cos\theta| / \left(1 + i \frac{\nu}{\omega}\right)\right)^2\right)}, \quad (7.11)$$

where $N = k/k_0$ is the wave refraction index defined as usual. It is well known that the double sign is for the two allowed circular polarizations. However, under all hypotheses postulated here, only right-hand polarized (RHP) waves propagate, whereas left-hand polarized are all cut-off within this frequency hierarchy regime. Thus, considering only RHP waves Eq. 7.11

Figure 7.1 – (a) Example of HE & TG modes branches, $N_{\perp}$ vs $N_{\parallel}$ for $\omega_{ce}/\omega = 10^2$ and $\omega_{pe}/\omega = 10^4$. (b) HE and TG modes propagation limits: $|N_{\parallel}/(\omega_{pe}/\omega)|_{Max}$, $|N_{\parallel}/(\omega_{pe}/\omega)|_{min}$, thick & thin lines respectively. In region A neither HE nor TG modes propagate, and the RF wave is damped in a thin skin depth, $Im(k_{\perp}^{-1}) \sim d_e$, like in an inductive plasma. In B, both modes propagate. In C, the HE mode is evanescent and the TG mode propagates but is strongly damped as pointed out in figures (c) and (d). (c) Dimensionless transversal depth of penetration for the TG (δ_{TG} thick lines) and the HE (δ_{HE} thin lines) mode, as a function of $N_{\parallel}/(\omega_{pe}/\omega)$ and ω_{ce}/ω , and the collisional term $\nu/\omega = 0.1$. (d) The same in (c) with $\nu/\omega = 0.8$. Dash-dotted thin lines in (c) and (d) are the same limits plotted in figure (b).

reduces to

$$\frac{N^2}{(\omega_{pe}/\omega)^2} = \frac{1}{\frac{\omega_{ce}}{\omega} |\cos\theta| - \left(1 + i \frac{\nu}{\omega}\right)}. \quad (7.12)$$

In order to extract conclusions about wave propagation, the collisionless limit of Eq. 7.12 $\nu/\omega \ll 1$ is considered. Wave resonance emerges at $\omega^* = \frac{\omega_{ce}}{\omega} \cos\theta = 1$; for $\omega^* < 1$ waves are cut off and decay exponentially within a tiny layer in the transversal- B -field direction. For $\omega^* > 1$, RF waves propagate as conventional RHP waves (whistler-helicon branch)[21]. Within this regime and assuming also $\omega_{pe}/\omega \gg \omega_{ce}/\omega \gg 1$, two propagation modes may arise, known as the Helicon (HE) and the Trivelpiece-Goud (TG) modes; each mode corresponds to a different branch of the same wave, see Figure 7.1. As a consequence, wave conversion might occur in a rarefied plasma. HE is a long-wavelength mode, as opposed to TG, which

is a short-wavelength mode. TG mode is also highly electrostatic, and collisions affect on its behaviour, causing a strong damping of this mode; This is easily pointed out by solving Eq. 7.12 for the frequency, retaining the collisional term, and defining the collisionless plasma skin depth as $d_e = c/\omega_{pe}$,

$$\omega = \frac{k^2 d_e^2}{1 + k^2 d_e^2} (\omega_{ce} \cos\theta + i\nu). \quad (7.13)$$

The imaginary part indicates how strong is the wave damping. For the TG mode, $k^2 d_e^2 \gg 1$, the imaginary part reduces to $\sim \nu$, which is by far larger than in the HE mode, $\sim k^2 d_e^2 \nu$ (with $k^2 d_e^2 \ll 1$) [147].

Concerning propagation limits, HE and TG modes are limited in terms of the above dimensionless parameters. The point, at which TG and HE modes merge each with other, represents a lower limit for the quantity $N_{\parallel} / (\omega_{pe}/\omega)$,

$$|N_{\parallel} / (\omega_{pe}/\omega)|_{min} = (2\omega/\omega_{ce}), \quad (7.14)$$

and the upper limit corresponds to the condition $N = N_{\parallel}$ or $\cos\theta = 1$,

$$|N_{\parallel} / (\omega_{pe}/\omega)|_{Max} = 1 / \sqrt{\left(\frac{\omega_{ce}}{\omega} - 1\right)}. \quad (7.15)$$

$N_{\parallel}$ is the field aligned wave refraction index. It is common in the literature to consider $N_{\parallel}$ fixed; first, because it is related to the axial wavelength, λ_a , of waves excited by the antenna or proportional to the antenna axial length L_a , $k_{\parallel}^{-1} \sim \lambda_a \propto L_a$ [21]; second, because in the 1D-cylindrical model it is a natural result due to the “quantization rule” of axial modes, being an integer the number of wavelengths contained in the helicon tube. Both limits are perfectly depicted in figure 7.1, and this highlights that HE mode propagation is only possible between these limits. Below $|N_{\parallel} / (\omega_{pe}/\omega)|_{min}$ neither HE nor TG propagate. Over $|N_{\parallel} / (\omega_{pe}/\omega)|_{Max}$, HE mode is evanescent. TG mode is still propagating beyond this limit according to the dispersion relation. These limits are rewritten to set explicit relations for the plasma density n against the magnetic field, contained in ω_{ce} , and $N_{\parallel}$ [147]:

$$n > n_{min} = \frac{\epsilon_0 m_e}{e^2} \omega_{ce} \omega N_{\parallel}^2, \quad (7.16)$$

$$n < n_{Max} = \frac{1}{4} \frac{\epsilon_0 m_e}{e^2} \omega_{ce}^2 N_{\parallel}^2. \quad (7.17)$$

So, if $\omega > \omega_{ce}/4$, no propagation happens. The 0D model also provides useful information about the dimensionless transversal penetration depth of each mode, δ . This is $\delta = \text{Im}(k_{\perp}^{-1})/d_e$ [147], thus it depends on the dissipative part of the transversal wavenumber. This is numerically investigated, for given $N_{\parallel} / (\omega_{pe}/\omega)$, ω_{ce}/ω , and ν/ω . Figures 7.1(c,d) highlight again that the HE mode propagates weakly damped between $|N_{\parallel} / (\omega_{pe}/\omega)|_{min}$ and $|N_{\parallel} / (\omega_{pe}/\omega)|_{Max}$; Close to $|N_{\parallel} / (\omega_{pe}/\omega)|_{Max}$ and for $\omega_{ce}/\omega \gg 1$, the HE mode penetration depth is extremely large ($\delta_{HE} \gg 1$), indicating that this mode propagates easily through the

plasma. Otherwise, δ_{TG} is everywhere much shorter than that of the HE mode, $\delta_{TG} \ll \delta_{HE}$, pointing out that TG mode is strongly damped or more dissipative.

7.1.3 Preliminary results based on design criteria

This section presents the results obtained following all criteria described above. Results are for plasma temperature of 10 eV and 20 eV. Regarding the magnetic field, the analysed range is $B_0 = [100 - 1700]$ G, while $n_0 = [5 \cdot 10^{17} - 2 \cdot 20^{17}] \text{ m}^{-3}$ is taken for the plasma density. All other required parameters have already been defined in Table 7.1.

Figure 7.2 depicts the relation between the radius R of the HPS and the nominal plasma density n_0 and applied magnetic field B_0 . R decreases as n_0 increases, as it was expected. R is larger in the 10 eV case (when considering a constant plasma density along the source). The radius is almost constant when varying B_0 .

The Argon mass flow rate $\dot{m}$ required to fit the power balance and to reach the values in Table 7.1, is larger at the lowest temperature case, as shown in Figure 7.3. $\dot{m}$ decreases for decreasing R , while n_0 slightly increases. High B_0 fields also admit a larger density while maintaining a good confinement, with low power losses to the wall. The production efficiency η_p is over 90 % within a big region of the parametric space, as illustrated in Figure 7.4.

Power ratios in Figures 7.5 - 7.6 show how the main effect of temperature is in the power losses due to inelastic collisions: ionization, excitation and radiation. The higher the temperature is, the more efficient the plasma production is. Losses to the wall are kept very small in a wide range of the analysed parameters. The useful power or beam power increases with the temperature, as stated in previous sections. This is depicted in Figure 7.7. It drops abruptly at the high density/low magnetic field corner, because of the lack of magnetic confinement which increases the wall losses over 50 %, although the mass flow rate is smaller. It means that under these conditions most of the energy is lost to the wall.

Figures 7.8, 7.9, 7.10 and 7.11 depict the main dimensionless parameters presented in Section 7.1.1. Based on them the following conclusions are met. The break of plasma quasineutrality is not expected anywhere in the studied $B_0 - n_0$ range, as indicated by the dimensionless Debye length λ_D/R . The induced magnetic field will be only slightly significant at $\beta_0 > 0.1$ (right side of the -1 contour level). The plasma transport perpendicular to the magnetic field can be considered purely diffusive, since the Hall parameter is large $\chi_H > 10$ in the entire domain. Electrons are also well confined (i.e. small Larmor radius $l_e/R \ll 1$), except in the corner of high plasma density and low magnetic field strengths. In some regions of the para-

Figure 7.2 – Radius R (cm) of the HPS as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.3 – Mass flow rate $\dot{m}$ (mg/s) as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.4 – HPS production efficiency η_p as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.5 – HPS normalized plasma losses to the wall $\epsilon_{wall} = P_{wall}/P_a$ as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.6 – HPS normalized inelastic losses $\epsilon_{inel} = P_{inel}/P_a$ as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.7 – HPS normalized beam power $\epsilon_{beam} = P_{beam}/P_a$ as a function of the plasma nominal density n_0 ($1/m^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.8 – Dimensionless Debye length $\log_{10}(\lambda_D/R)$ as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.9 – Plasma Beta $\log_{10}\beta_0$ as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.10 – Dimensionless electron Larmor radius $\log_{10}(l_e/R)$ as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.12 – Minimum parallel wavelength $\lambda_{\parallel,min}/R$ for the HE mode propagation, as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.13 – Maximum parallel wavelength $\lambda_{\parallel,max}/R$ for the HE mode propagation, as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.14 – Perpendicular wavelength of the HE mode at the knee of the dispersion relation $\lambda_{\perp}(\theta_0)/R$ as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b).

Figure 7.15 – Trade-off and design point selection of the 15 kW class HPT, as a function of the plasma nominal density n_0 ($1/\text{m}^3$), and magnetic field strength B_0 (T) at 10 eV (a) and 20 eV (b). Thrust efficiency (thin black lines), $\log_{10} \chi_H$ (thick black lines), thrust density (red lines), and wave criteria $\lambda_{||}/R$ (blue lines).

7.1.4 Design point trade-off

The selection of the nominal design point is carried out considering together all phenomena introduced in previous subsections.

First, a good confinement of the generated plasma. This requires to ensure a Hall parameter $\chi_H \gg 1$, which corresponds to the black thick line in the graphs of Figure 7.15.

Second, the propagation of HE and TG wave modes must be ensured. The largest parallel wavelength, which is that obtained at the knee of the dispersion relation, see Figure 7.1 - (a), must be comparable to the length of the dielectric tube. Blue lines in Figure 7.15 depict $\lambda_{||,max}/R$ assuming $\omega = 13.56$ MHz. An analysis of the frequency (not presented here) shows up that the larger the frequency is, the shorter the tube could be. Regarding the magnetic field strength, it presents the opposite trend, the stronger the magnetic field is, the longer the tube should be.

Third, the thrust compactness of the thruster F/A , which measures the ratio of thrust over the thruster front area, leads to select a design point beyond $F/A = 10 \text{ N/m}^2$ (red lines in Fig. 7.15) in order to be attractively compact. Thrust is estimated as $F \approx \eta_u \dot{m} \sqrt{kT_e/m_i} (1 + \kappa_F)$, where κ_F is the ion axial momentum gain throughout the MN expansion, normalized against the ion axial momentum at the throat, which is derived from the parameters listed in Table 7.1.

Fourth, the estimated thrust efficiency η is depicted with thin black lines in Figure 7.15. η has been computed assuming all the aforementioned hypotheses. The region of high magnetic fields and low densities is preferable in order to obtain efficiencies over 25 %. Taking into

Output design parameter	$T_e = 10 \text{ eV}$	$T_e = 20 \text{ eV}$
Chamber aspect ratio, L/R	10.0	10.0
Magnetic field strength, B_0	1000 G	1000 G
Reference plasma density, n_0	$4.6 \cdot 10^{18} \text{ m}^{-3}$	$5.6 \cdot 10^{18} \text{ m}^{-3}$
Chamber radius, R	11.5 cm	7.35 cm
Mass flow, $\dot{m}$	64.2 mg/s	43.9 mg/s
Thruster compactness, F/A	14.29 N/m ²	34.26 N/m ²
Wave Frequency, f_{RF}	13.56 MHz	13.56 MHz

Table 7.3 – Summary of the design point main parameters of a 15 kW class HPT.

account the requirements described above, two design points are set at a different temperature, because of the existing uncertainties on this parameter. All other parameters of interest are listed in Table 7.3. Focusing on the 20 eV case, the nominal density is $n_0 = 5.6 \cdot 10^{18} \text{ m}^{-3}$, for a nominal mass flow rate of $\dot{m} = 43.9 \text{ mg/s}$ of Argon, and a chamber radius of $R = 7.35 \text{ cm}$. Thrust compactness is estimated around 34.26 N/m^2 . The 10 eV scenario suppose a larger chamber radius $R = 11.5 \text{ cm}$ and inject mass flow rate, $\dot{m} = 64.2 \text{ mg/s}$. Because thrust is proportional to T_e , the thrust compactness is reduced to 14.29 N/m^2 .

7.2 1D model of the RF wave propagation

In the previous section, the 0D wave-plasma model has been used to analyse the compatibility of the emitted RF wave with plasma properties, magnetic field strength and chamber geometry. This preliminary study shows how TG and HE waves could propagate within all the analysed parametric space $B_0 - n_0$, but under different conditions, i.e. differences in the wavelength or mode of propagation. This analysis concluded that some requirements on the chamber geometry must be satisfied to guarantee the propagation of the HE-TG modes. This also demonstrates that the propagation of the HE mode is only possible within a limited range of parallel wavenumbers, highlighting the existence of a maximum and a minimum value for the parallel wavenumber. The narrowness or wideness of the range between these two limits depends on the $B_0 - n_0$ conditions. Within that range, HE propagates with a low damping in comparison to the TG mode. Outside these limits both modes are strongly damped or become evanescent. Another peculiarity of the 0D model is that it deals with travelling, non-confined waves and the wavenumber space is continuous, so all values between the aforementioned limits are possible without considering any discretization in terms of modes.

However, in the case considered here, in which a resonator cage surrounds the HPT chamber, the geometry is cylindrical and waves behave as standing waves reflected at the conducting walls of the resonator. For the analysis envisaged here, suitable models are the one proposed by Cho [148, 149] and Shamrai [20, 150], which deal with the propagation of the RF-wave within a uniform cylindrical column of plasma of radius R_p and length L , surrounded by an antenna of radius R_a , and the full set-up contained into a resonator or conductive metallic shell, of radius R_w and length L , similarly to the sketch in Figure 7.16 - (a). Electromagnetic field patterns E, B are determined by solving the Maxwell equations within the plasma and through the vacuum gap between the plasma edge and the shell. These are numerically or analytically integrated along the radial direction, while Fourier-expanded in the azimuthal and longitudinal directions. Currents through the antenna j , which are the source terms in the equations set, are also expanded in the same way.

$$\{E, B, j\}(z, r, \theta) = \sum_{(m,l)} \{E, B, j\}_{m,l}(r) \exp(ik_l z + im\theta) + CC. \quad (7.18)$$

Longitudinal modes, $k_l = l\pi/L$ decouple easily into series of sines or cosines, which are consistent with boundary conditions. This requires that all tangential electric fields to the conductive wall are zero:

$$E_\theta, E_r(z=0, L) = 0, \quad (7.19)$$

$$E_\theta, E_z(r=R_w) = 0. \quad (7.20)$$

R_w is the radius of the resonator cage. In this analysis we consider that waves are confined within the chamber length L , which in fact can be considered a good approximation since the drop of the density along the nozzle could act as a mirror for the downstream propagation of electromagnetic waves.

Considering Maxwell equations 7.9-7.10 in cylindrical coordinates and the shape of electromagnetic field as in Eq. 7.18, a set of two algebraic equations and four differential equations is easily derived.

$$\frac{dE_z}{dr} = \frac{\epsilon_2 k_l}{\epsilon_1} E_\phi - \left(\omega - \frac{k_l^2}{\epsilon_1 \omega \epsilon_0 \mu_0} \right) B_\phi - \frac{k_l m}{\epsilon_1 r \omega \epsilon_0 \mu_0} B_z, \quad (7.21)$$

$$\frac{dE_\phi}{dr} = \left(\frac{m\epsilon_2}{r\epsilon_1} - \frac{1}{r} \right) E_\phi + \left(\omega - \frac{m^2}{r^2 \epsilon_1 \omega \epsilon_0 \mu_0} \right) B_z + \frac{k_l m}{\epsilon_1 r \omega \epsilon_0 \mu_0} B_\phi, \quad (7.22)$$

$$\frac{dB_z}{dr} = \left(\frac{k_l^2}{\omega^2 \epsilon_0 \mu_0} + \frac{\epsilon_2^2}{\epsilon_1} - \epsilon_1 \right) \omega \epsilon_0 \mu_0 E_\phi - \frac{mk_l}{r\omega} E_z - \frac{m\epsilon_2}{r\epsilon_1} B_z + k_l \frac{\epsilon_2}{\epsilon_1} B_\phi, \quad (7.23)$$

$$\frac{dB_\phi}{dr} = \frac{mk_l}{\omega r} E_\phi + \left(\epsilon_3 \omega \epsilon_0 \mu_0 - \frac{m^2}{r^2 \omega} \right) E_z - \frac{1}{r} B_\phi, \quad (7.24)$$

$$E_r = \frac{\epsilon_2}{\epsilon_1} E_\phi + \frac{k_l}{\epsilon_1 \omega \epsilon_0 \mu_0} B_\phi - \frac{m}{r \epsilon_1 \omega \epsilon_0 \mu_0} B_z \quad (7.25)$$

$$B_r = \frac{m}{r\omega} E_z - \frac{k_l}{\omega} E_\phi, \quad (7.26)$$

with $E_\phi = -iE_\theta$ and $B_\phi = iB_\theta$. In the above expressions, the following dielectric cold plasma tensor has been considered:

$$\bar{\epsilon} = \begin{bmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}, \text{ with} \quad (7.27)$$

$$\epsilon_1 = 1 - \sum_j \frac{\omega_{pj}^2 (\omega + i\nu_j)}{\omega [(\omega + i\nu_j)^2 - \omega_{cj}^2]}, \quad (7.28)$$

$$\epsilon_2 = - \sum_j \frac{s_j \omega_{cj} \omega_{pj}^2}{\omega [(\omega + i\nu_j)^2 - \omega_{cj}^2]}, \quad (7.29)$$

$$\epsilon_3 = 1 - \sum_j \frac{\omega_{pj}^2}{\omega (\omega + i\nu_j)^2}. \quad (7.30)$$

ω_{cj} and ω_{pj} are the cyclotron and plasma frequencies, ν_j the frequency of collisions and s_j the sign of the electric charge, distinguishing ions from electrons, $j = i, e$ respectively.

At the antenna location R_a , a current I_a flows through it and this is accounted in Eqs. 7.23, 7.24 as a finite discontinuity of $B_z(R_a)$ and $B_\theta(R_a)$ fields,

$$\Delta B_z = -\mu_0 j_\theta \delta r, \quad (7.31)$$

$$\Delta B_\theta = \mu_0 j_z \delta r. \quad (7.32)$$

These assume that the antenna is contained on a thin cylindrical layer at R_a , and its shape is defined on this $\theta - z$ plane. In the next subsection, the series for $j_\theta^{m,l}$ and $j_z^{m,l}$ are presented for the analysed antennae.

In this Thesis this model is applied to two different scenarios. The first one considers a uniform plasma-density n_0 and constant axial applied field B_0 . The second one includes cases in which the density drops radially, as in the known solution of Chapter 3, and with radially varying magnetic field as well, which is the case of a dense plasma, i.e. high - β_0 .

Uniform plasma

Concerning the uniform case, the set of equations can be simplified. This simplification also applies to the vacuum domain between the plasma edge and the resonator wall. Obviously, propagation conditions are different in both regions, and this information is contained in the dielectric tensor. Replacing Eqs. 7.22 and 7.24 into Eqs. 7.21, 7.23, after differentiating them,

reduces the set of equations to a pair of Bessel equations,

$$\frac{d^2 \hat{E}_z}{d\hat{r}^2} + \frac{1}{\hat{r}} \frac{d\hat{E}_z}{d\hat{r}} + \left(\epsilon_3 - \frac{\epsilon_3 \hat{k}_l^2}{\epsilon_1} - \frac{m^2}{\hat{r}^2} \right) \hat{E}_z = \frac{\epsilon_2 \hat{k}_l}{\epsilon_1} \hat{B}_z, \quad (7.33)$$

$$\frac{d^2 \hat{B}_z}{d\hat{r}^2} + \frac{1}{\hat{r}} \frac{d\hat{B}_z}{d\hat{r}} + \left(\epsilon_1 - \frac{\epsilon_2^2}{\epsilon_1} - \frac{\hat{k}_l^2}{\hat{r}^2} - \frac{m^2}{\hat{r}^2} \right) \hat{B}_z = \frac{\epsilon_2 \epsilon_3 \hat{k}_l}{\epsilon_1} \hat{E}_z. \quad (7.34)$$

Variables have been normalized according to

$$\hat{r} = \frac{r\omega}{c}, \quad \hat{k}_l = \frac{k_l c}{\omega}, \quad \hat{B} = \frac{B R_a}{\mu_0 I_a}, \quad \hat{E} = \frac{E R_a}{\mu_0 I_a c}. \quad (7.35)$$

Following Cho et al. [148], solutions for Eqs. 7.33-7.34, can be written down as

$$\hat{B}_z = a_1 J_m(p_1 \hat{r}) + a_2 J_m(p_2 \hat{r}), \quad (7.36)$$

$$\hat{E}_z = a_1 e_1 J_m(p_1 \hat{r}) + a_2 e_2 J_m(p_2 \hat{r}), \quad (7.37)$$

with J_m the Bessel function of the first kind and of order m ; p are the non-negative roots of the following biquadratic equation, which are equivalent to the radial wavenumber (short and long wavelength),

$$p^4 - (\alpha + \beta) p^2 + \alpha\beta - \gamma\delta = 0, \quad (7.38)$$

with

$$\alpha = \epsilon_1 - \frac{\epsilon_2^2}{\epsilon_1} - \hat{k}_l^2, \quad \beta = \epsilon_3 \left(1 - \frac{\hat{k}_l^2}{\epsilon_1} \right), \quad \gamma = \frac{\epsilon_2 \epsilon_3 \hat{k}_l}{\epsilon_1}, \quad \delta = \frac{\epsilon_2 \hat{k}_l}{\epsilon_1}. \quad (7.39)$$

In Eq. 7.37, e_j is computed as,

$$e_j = \frac{\alpha - p_j^2}{\gamma}, \quad \text{for } j = 1, 2. \quad (7.40)$$

a_1, a_2 are constants to be determined by the boundary conditions, or matching conditions at the plasma-vacuum edge. Equations 7.36 and 7.37 and the involved constants a_j, p_j, e_j must be determined for each pair of longitudinal-azimuthal modes $l - m$.

Analogously, at the vacuum side, where $\epsilon_1 = \epsilon_3 = 1$ and $\epsilon_2 = 0$, the solution of Eqs. 7.33, 7.34 is expressed in terms of the modified Bessel function of first and second kind,

$$\hat{B}_z = a_3 I_m(\kappa \hat{r}) + a_4 K_m(\kappa \hat{r}), \quad (7.41)$$

$$\hat{E}_z = a_5 I_m(\kappa \hat{r}) + a_6 K_m(\kappa \hat{r}), \quad (7.42)$$

with $\kappa = \hat{k}_l^2 - 1$. Let us consider that the plasma column of radius R_p is surrounded by a first vacuum region gap until the antenna at R_a , and a second one from the antenna until reaching the conducting wall R_w . According to this geometry, four more constants should be determined, $a_7 - a_{10}$. At the plasma-vacuum edge, the continuity of all $\mathbf{E}, \mathbf{B}$ fields is invoked (4 equations); at the antenna δB_z and δB_θ forcing terms are applied, as anticipated in Eqs. 7.31 - 7.32, and continuity for other field components (4 equations); and the two equations of the boundary conditions at the wall (2 equations) provide enough equations to determine all

the unknown constants, a_j , for $j = 1, 10$, and then determine $\mathbf{E}, \mathbf{B}$ patterns inside the plasma and throughout the vacuum gap. Expressions for all other components, $\hat{E}_\phi, \hat{B}_\phi, \hat{E}_r, \hat{B}_r$, are algebraic equations depending only on $\hat{E}_z, \hat{B}_z$, and coefficients a_j . All these expressions as well as the matching equations are annexed in Appendix D-D.2.

Non-uniform plasma

In the non-uniform case with variable density $n(r)$ and variable static magnetic field $B_0(r)$, the equations need to be solved numerically. First, the solution at the axis, $r \rightarrow 0$, needs to be regularized taking the following expansion shape:

$$\hat{E}_\phi = \alpha_1 \hat{r}^{p-1}, \quad \hat{E}_z = \alpha_2 \hat{r}^p, \quad \hat{B}_\phi = \alpha_3 \hat{r}^{p-1}, \quad \hat{B}_z = \alpha_4 \hat{r}^p. \quad (7.43)$$

These are introduced into Eqs. 7.21 - 7.24, and looking for the non-trivial solution, this yields to the condition $p = \pm m$, with double multiplicity. So, two of the α 's coefficients are undetermined. Depending on m , the following expressions determine other α 's:

$$m > 0, p = m \begin{cases} m\alpha_2 = \hat{k}_l \alpha_1 - \alpha_3 \\ m\alpha_4 = (\epsilon_2 - \epsilon_1) \alpha_1 + \hat{k}_l \alpha_3 \end{cases} \quad (7.44)$$

$$m < 0, p = -m \begin{cases} m\alpha_2 = \hat{k}_l \alpha_1 + \alpha_3 \\ m\alpha_4 = (\epsilon_2 + \epsilon_1) \alpha_1 + \hat{k}_l \alpha_3 \end{cases} \quad (7.45)$$

$$m = 0 \begin{cases} \alpha_2 = \frac{2}{\epsilon_3} \alpha_3 \\ \alpha_4 = 2\alpha_1 \end{cases} \quad (7.46)$$

On the vacuum side, the analytical solution is still valid. Fields are numerically integrated from $r = r_0$, with r_0 small enough, until $r = R_p$, using the *Matlab* ODE toolbox. For each pair of longitudinal-azimuthal modes $l - m$, a pair of solutions are integrated within the plasma column, the first one $(\hat{E}_{z1}, \hat{E}_{\phi1}, \hat{B}_{z1}, \hat{B}_{\phi1})$ takes $\alpha_1 = 1, \alpha_3 = 0$, and the second one $(\hat{E}_{z2}, \hat{E}_{\phi2}, \hat{B}_{z2}, \hat{B}_{\phi2})$ takes $\alpha_1 = 0, \alpha_3 = 1$. Then, the superposition principle applies here, and the solution within the plasma is,

$$\chi = \sum_{j=1,2} \alpha_j \chi_j, \quad (7.47)$$

with $\chi = \{E_z, E_\phi, B_z, B_\phi\}$.

$\mathbf{E}, \mathbf{B}$ solution on the vacuum side and its constants a_j , for $j = 3, 10$ are determined following the same procedure of the uniform case: continuity of the solution at the plasma edge R_p , antenna currents at R_a and wall boundary conditions at R_w . So, this yields an eigenvalue problem that is numerically solved [150, 148]. Note that the α 's coefficients might be scaled in order to match boundary conditions. Further details on this model are given in Appendix D.

Figure 7.16 – Sketch of the selected antennae. Half-loop (left), Nagoya III (centre), and half-turn helical antenna (right) from Ref. [63].

7.2.1 Introduction to RF-antennae: Shapes and Fourier modes

Three different kinds of RF-antennae have been analysed at the design point. These are the half-loop antenna, the Nagoya III antenna and the half-turn helical antenna, which are shown in Figure 7.16. For all of them, it is necessary to compute their Fourier $m - l$ (azimuthal-longitudinal) modes. For the half-loop antenna its modes are

$$\begin{aligned} j_{\theta,ml}(r) &= -\frac{iI_a}{mL\pi} \sin\left(\frac{l\pi z_a}{L}\right) \delta(r - R_a), \\ j_{z,ml}(r) &= \frac{I_a}{\pi^2 l R_a} \sin\left(\frac{l\pi z_a}{L}\right) \delta(r - R_a). \end{aligned} \quad (7.48)$$

Where I_a is the antenna current, z_a is the axial position of the loop, R_a is the antenna radius, and L is the cavity length. The Fourier modes of the Nagoya III antenna are written as follows,

$$\begin{aligned} j_{\theta,ml}(r) &= -\frac{2iI_a}{mL\pi} \cos\left(\frac{l\pi z_a}{L}\right) \sin\left(\frac{l\pi a}{L}\right) \delta(r - R_a), \\ j_{z,ml}(r) &= \frac{2I_a}{\pi^2 l R_a} \cos\left(\frac{l\pi z_a}{L}\right) \sin\left(\frac{l\pi a}{L}\right) \delta(r - R_a). \end{aligned} \quad (7.49)$$

The last antenna considered here is the half-turn helical antenna, and the expressions for its Fourier modes are listed next,

$$\begin{aligned} j_{\theta,ml}(r) &= \frac{l\pi R_a j_{z,ml}(r)}{imL}, \text{ with} \\ j_{z,ml}(r) &= \frac{I_a \delta(r - R_a)}{\pi L \sqrt{1 + \left(\frac{\pi N_t R_a}{a}\right)^2} \left(m^2 \sin^2 \beta - \left(\frac{l\pi R_a}{L}\right)^2 \cos^2 \beta\right)} \times \\ &\quad \{im \sin \beta (\cos(k_l z_2) \exp(-2imN_t\pi) - \cos(k_l z_1)) \\ &\quad - k_l R_a \cos \beta (\sin(k_l z_2) \exp(-2imN_t\pi) - \sin(k_l z_1))\} \end{aligned} \quad (7.50)$$

For the Nagoya and helical antennae, a is the antenna half length, and z_1, z_2 their extrema location, $z_1 = z_a - a$, and $z_2 = z_a + a$. N_t is the number of turns of the Helical antenna, i.e. $N_t = 0.5$ for the half-turn one. $\beta = \tan(N_t \pi R_a / a)$ is the helical angle. For all antennae considered here only odd m modes are excited, including negative m that come from the complex conjugate terms in Eq. 7.18, $m = \pm 1, \pm 3, \pm 5, \dots$. Taking into account all these assumptions, elec-

tromagnetic fields must be solved for at least the low $m - l$ modes contributing substantially to the absorbed power.

7.2.2 Absorbed power and plasma impedance analyses

Here, the plasma-antenna equivalent impedance, and the power absorbed by the plasma are computed according to the electromagnetic field patterns. Apart from determining the aforementioned parameters, these methods allow checking the consistency of the current 1D model.

The first method evaluates the absorbed power P_a within the plasma, as the time-averaged electric work developed by the induced currents and electric field, $\mathbf{j} \cdot \mathbf{E}$. Note that this current $\mathbf{j}$ appears due to the electromagnetic excitation of the plasma and there is no relation with currents developed due to plasma magnetization. After some easy algebra, the expression yielding P_a is

$$P_a = \frac{1}{2} \int_V \Re(\mathbf{j} \cdot \mathbf{E}^*) dV, \quad (7.51)$$

which is an integral over all the plasma volume ($V = \pi R^2 L$). Here, superscript $(*)$ means the complex conjugate of the involved variable, $\mathbf{j} = \bar{\sigma} \cdot \mathbf{E} = -i\omega\epsilon_0\bar{\epsilon}_r \cdot \mathbf{E}$, the plasma conductivity being $\bar{\sigma} = -i\omega\epsilon_0\bar{\epsilon}_r$. An alternative method evaluates the flow of power is based on the flux of the Poynting Vector through a surface surrounding the plasma volume, $\partial V = 2\pi RL$

$$P_a = \frac{1}{2} \mu_0 \int_{\partial V} \Re(\mathbf{E} \times \mathbf{B}^*)_{r_{\partial V}} d\partial V. \quad (7.52)$$

Eqs. 7.51 and Eq. 7.52 must yield the same result. The last method evaluates the potential drop through the antenna U , and considers that the impedance Z is proportional to the ratio of this drop over the antenna current I_a . U is computed as follows,

$$-U = \oint_{l_a} \mathbf{E}_a \cdot d\mathbf{l}_a. \quad (7.53)$$

$\mathbf{E}_a(\mathbf{r}_a)$ is the electric field vector evaluated at each point of the antenna, l_a is the closed line defined by the antenna and $d\mathbf{l}_a$ its differential length vector. Then, the plasma-antenna resistance is $\Re(Z) = \Re(U) / I_a$, and the absorbed power is $P_a = \Re(Z) I_a^2$.

7.2.3 Uniform approximation

In this subsection the plasma resistance $\Re(Z)$ is computed according to the methodology presented above, for the following geometry and some of the main design parameters selected in subsection 7.1.4. The plasma radius or tube radius, $R_p = 0.0735$ m, the aspect ratio $L/R = 3,5$ (two cases will be evaluated), applied static field $B_0 = 450$ G, operational frequency $\omega_{RF} =$

Figure 7.17 – Contribution of each longitudinal l mode (adding all azimuthal m modes) to the absorbed power P_a for the loop, Nagoya and helical antennae. (a) is for $L/R = 3$ and (b) for $L/R = 5$.

13.56 MHz, electron temperature about $T_e = 20$ eV, uniform plasma density $n_0 = 5.6 \cdot 10^{18} \text{ m}^{-3}$, with an equivalent electron collision frequency of $\nu_e = 3.2 \cdot 10^6 \text{ s}^{-1}$.

Although results are not presented, a preliminary analysis has determined that the radius of the resonator cage, R_w , does not have any effect on the plasma-wall interaction and resulting impedance, for this reason, this is kept constant in all simulations $R_w = 2R_p$. Contrary to this behaviour, R_a presents a strong influence on the obtained results, the larger the ratio R_a/R_p is, the worse the antenna performances are. Consequently, a realistic radius of the antenna loop is considered about $R_a = 1.05R_p$. In the case of the Nagoya and half-turn helical antennae, their lengths are set at $L_a = L/2$, and are centered at half the tube length, $z_a = L/2$. Different lengths and positions are further analyzed in [50].

For the assortment of antennae introduced previously (Sec. 7.2.1), the absorbed power and the plasma resistance are computed. For each longitudinal l mode, all azimuthal modes in $m \in [-51, 51]$ are accounted in the computation. It is not required to include more modes because the power contribution of modes with large values of $\|m\|$ rapidly decays. l modes from 1 to 42 are computed in all antennae considered here.

Results of the plasma resistance are summarized in Table 7.4. Apart from this, the contribution of each l mode, adding the contribution of all m modes, is explored in Figure 7.17 in terms of the contribution to the absorbed power, P_a , concluding that not always the lowest l mode is the one with the highest contribution to P_a . This is because too small wavenumbers could be out of the He propagation regime ($k_l = l\pi/L < k_{\parallel, \min}$) as depicted in Fig. 7.1 (Eq. 7.14). Note also that only odd modes are accounted in the loop antenna, for the Nagoya III only $l = 2, 6, 10, \dots$ and for the half-turn Helical antenna only $l = 1, 3, 4, 5, 7, 8, 9, 11, \dots$. All other

Figure 7.18 – Variation of the plasma resistance against the plasma density for the half turn Helical antenna and $L/R = 5$. All other parameters are kept constant, $B_0 = 450$ G, $T_e = 20$ eV.

Antenna type	$\Re(Z)$, ($L/R = 3$)	$\Re(Z)$, ($L/R = 5$)
Half-loop	1.10	0.67
Nagoya	1.43	1.75
Half-turn Helical	3.74	3.10

Table 7.4 – Plasma resistance for the Half-loop, Nagoya, and Half-turn Helical antennae, under the hypotheses of the uniform $n_0 - B_0$ approximation.

antenna current modes are zero, $j_{\theta}^{(l,m)} = j_z^{(l,m)} = 0$, due to axial symmetries.

The half turn helical antenna is the one which offers the best performances, coupling more RF power to the plasma, thus presenting a higher resistance in both length scenarios.

To conclude this uniform approximation, the sensitivity of the plasma resistance against the variation of the plasma density keeping constant all other parameters, such as the plasma temperature, the RF frequency, the magnetic field strength, is analysed for the half-turn helical antenna configuration. Results are depicted in Figure 7.18, and point out the existence of a plasma density threshold, below which the inductive behaviour of the plasma increases, i.e. lower resistance and less power is effectively coupled to the plasma.

7.2.4 Non uniform approximation

The radial variation of density and magnetic field introduces important changes in the local dielectric tensor of the plasma. This means that the wave dispersion relation at a given radial position is modified as well. Because the dispersion relation is not uniform, the wave propagation, and of course, the plasma resistance and power deposition will be also altered by

these radial effects, including the possibility that HE mode could not properly propagate.

In this subsection four cases with radial variation of n_0 and B_0 are explored. First, by using the radial fluid model of Chapter 3, plasma density and magnetic profiles are determined. The main parameters are listed in Table 7.5. Among them, the drop of density at the sheath entrance n_s/n_0 and the effective magnetic field on the axis $B_0(r=0)$ give a clear picture of how strong is the radial variation of these properties.

The antenna selected here for all the scenarios is the half-turn helical antenna, with length $L_a = 0.5L$, radius $R_a = 1.05R_p$ and centred at $z_a = 0.4L$. The tube aspect ratio is $L/R = 5$.

Non-uniform cases are compared against an equivalent uniform scenario, at constant n_0 and B_0 . For them, the density at the axis is taken as the reference density. The first 50 l modes and the odd m modes comprised between -21 and 21 are included in the computation of the plasma resistance. Plasma resistance results are summarized in Table 7.6. Error values are for the non-uniform scenarios and compare the resistance obtained by using the P_a method (Eq. 7.51) against the one computed by applying the U method (Eq. 7.53).

For all analysed cases, the computed plasma resistance accounting for radial variations of density and magnetic field is higher than in the uniform scenario, although any trend between these results has not been identified. Concerning the differences reported between cases of identical applied field, A vs C and B vs D, the resulting resistance is lower in C and D, both scenarios present a stronger depletion of the magnetic field due to the induced effect, caused by a non-negligible β_0 . So, at these levels of B_0 , the induced field damages, apart from the plasma confinement, the rf-wave plasma coupling, reducing the equivalent plasma resistance. The contribution of each l mode to the total resistance is plotted in the bar charts of Figure 7.19. For each l mode, the number of m modes to include varies, but the solver always ensures the convergence of the plasma resistance.

After reviewing the previous figures of resistance contribution, it seems that the number of longitudinal modes that really contribute to the resistance is lower in the non-uniform cases. This could be explained by comparing this behaviour to the 0D model. The variation of density and magnetic field modifies the dispersion relation as already mentioned. Let us consider the 0D model locally at a given radius within the plasma column, it is easy to conclude that the bounds of the space in which HE mode propagates are modified. Close to the plasma edge, where density drops abruptly, limits $k_{\parallel,min}$, $k_{\parallel,max}$ define a narrower gap, which translated to the 1D model means that the number of l modes contributing to the plasma resistance is reduced, and higher modes can be disregarded. However, these results do

Figure 7.19– Contribution of each longitudinal l mode (adding all azimuthal m modes) to the absorbed power P_a for the non uniform cases listed in Table 7.5. Red bars are the solution of the non-uniform approximation, which is compared with the uniform case in blue.

CASE	$n(r=0)$ (m ⁻³)	B_0 (G)	β_0	n_s/n_0	$B(r=0)/B_0$
A	4e19	200	0.40	1.63e-2	0.47
B	8e19	300	0.36	1.19e-2	0.55
C	6e19	200	0.60	8.90e-2	0.06
D	1e20	300	0.45	2.14e-2	0.38

Table 7.5 – Summary of fluid properties for the cases analysed with the 1D model of wave propagation with radial variable properties. Density at the axis and applied magnetic fields, first and second column respectively. Third column is the β_0 parameter computed according to $n_0 - B_0$. The fourth column lists the ratio of the density at the sheath transition over the density at the axis, n_s/n_0 , and last column is for the magnetic field at the axis after subtracting the counter-effect of the induced field.

CASE	$\Re(Z)$ (Ω)	Error	$\Re(Z)$ (Ω) Uniform Case
A	3.89	-0.41 %	2.52
B	3.83	0.16 %	2.27
C	2.74	-0.36 %	1.92
D	3.25	0.54 %	1.88

Table 7.6 – Plasma resistance for the cases in Table 7.5. The error compares the plasma resistance obtained using the volumetric P_a method against the resistance based on the computation of the potential drop along the antenna U , as discussed in Section 7.2.2. The third column is for the resistance obtained by applying the uniform approximation and considering an equivalent uniform scenario.

not allow to conclude that the resistance would be larger in the non-uniform scenario, since this depends completely on how the average values are taken for the uniform case. The main conclusion of this study is that the radial variations of n_0 and B_0 should be always included in the resolution of the electromagnetic fields, an issue that was not considered in previous works [147, 148, 151].

7.3 Assessment of the HPT performances

In this section a detailed analysis of the design point set in Section 7.1.4 is carried out in order to determine more precisely the fluid-dynamic response of the plasma within the HPS, and thus assessing its efficiencies. A preliminary characterization of thermal loads is also included. Finally, by assuming some hypotheses on the MN stage, the performances of a 15 kW HPT are predicted and discussed.

7.3.1 Description of internal fluid dynamics and source efficiencies

The analysis is only performed for the 20 eV case of Table 7.3. The theoretical model used here is the one detailed in Chapter 3, and it considers the magnetic screening of any possible plasma flowing towards the rear wall, which is crucial to obtain an efficient HPS.

In this scenario the main parameters are: $T_e = 20$ eV, $B_0 = 450$ G, $n_0 = 5.6 \cdot 10^{18} \text{ m}^{-3}$, which corresponds to $R = 0.0735$ m and $\dot{m} = 43.9$ mg/s of Argon, and $L/R = 10$. According to the simulation carried out, the average Hall parameter is about $\chi_H \simeq 27$, which is indeed more favourable than the one taken by design in Section 7.1. This larger value agrees with the fact that in the preliminary study the fraction of neutrals was overestimated, and consequently v_e had been taken higher than it actually is. The high degree of ionization obtained in the simulations is the reason why electron-ion collisions are dominant against electron-neutral collisions, along most of the dielectric chamber length, and consequently v_e is lower than in that simplified model. Under these conditions, ionization occurs in a short fraction of the tube, localized close to the rear wall, a fact that is coherent with the mean free path of ionization $\delta_{ion} \sim u_{n0}/v_{ion} \ll L$. Another consequence of this high magnetized regime is that $v_w/v_{ion} \ll 1$ is kept almost constant along the tube, which indicates that there is a balance between plasma wall recombination and plasma production.

Concerning HPS efficiencies, the model reaches complete ionization of the neutral gas $\eta_u \sim 1$. Probably, this indicates that the tube could be shortened without penalising this figure. The production efficiency is also high $\eta_p = 0.99$, but this optimum result benefits from the hypothesis that almost no plasma impinges either the rear wall (magnetic cusp) or the lateral wall (high radial magnetization). These two figures are extremely optimistic in comparison to most of the experimental results, including those presented in Chapter 6. The only experimental work that provided proven evidence of this high propellant utilization was the VASIMR. Absolute power losses to the lateral wall are very low as well, only about 73 W. Main power expenses are attributed to ionization and other inelastic losses, such as excitation/radiation, and are rated at 3.5 kW. Consequently, the beam power or useful power for 13.5 kW of power deposited into the plasma is about 9.9 kW, so the chamber efficiency is $\eta_{cham} = P_{beam}/P_a = 73.5$ %.

Based on these results it is feasible to reduce both, the length of the tube and the magnetic field strength. Figure 7.20 compares $\eta_p(B_0)$ and $\eta_{cham}(B_0)$ variation against the applied field and for two different chamber aspect ratios $L/R = 10$ and $L/R = 5$. The weaker the magnetic field is the higher are the losses to the wall, thus reducing η_p and η_{cham} but since the length of the tube has been reduced by a factor of 2, the total amount of losses has also been reduced

Figure 7.20 – (a) HPS production efficiency $\eta_p(B_0)$ and (b) chamber efficiency $\eta_{cham}(B_0)$, both as functions of the applied magnetic field. Solid line is for the original design length $L/R = 10$ and the dashed one is for a shorter tube $L/R = 5$.

Figure 7.21 – (a) Radiative heat flux at the rear wall, $q_{rad}(r, z = -L)$. (b) Radiative heat flux at the lateral wall, $q_{rad}(r = R, z)$. 15 kW HPS at $T_e = 20$ eV design point.

approximately by 2. Consequently, all source performances are kept high enough, even below 450 G.

7.3.2 Characterization of thermal loads

This topic is of special concern from the point of view of the impact of the HPT at subsystem, and at platform level, since thermal loads must be evacuated or re-used in other subsystems. Two contributions to these loads are considered. First, the plasma energy flux (internal energy and heat flux) impinging the walls. Under the assumption of a fully screened rear wall, only the flux onto the lateral wall is accounted for. Second, the heat flux emitted by the plasma when it is considered as a radiative source.

Regarding the heat flux to the lateral wall, this can be written as a function of the axial coordinate and only the radial component must be computed because it is the only one that contributes to the thermal flux at the lateral wall,

$$q_r(z) = \left(2 + \frac{1}{2} \log \frac{m_i}{2\pi m_e}\right) kT_e n_s(z) c_s. \quad (7.54)$$

Integrating this flux over all the wall surface, the total power is estimated about $Q_{wall} = 105,4$ W, at the design point scenario. The second contribution, the heat power emitted by radiation from a differential volume of plasma is dP_{rad} , and it is evaluated according to the fluid model results,

$$dP_{rad}(r, z) = (\alpha_{ion} - 1) E_{ion} n n_n R_{ion}, \quad (7.55)$$

with α_{ion} as in Eq B.6 (B), and E_{ion} the ionization energy.

The plasma column of length L is divided in n_d infinitesimal discs of width dz_i , and volume $dV(z_i)$. Each one emits by radiation $dP_{rad,i} = P_{rad}(z_i) dV(z_i)$. This is supposed to be emitted from the geometrical centre of the disc, on the axis line. Let consider a differential surface dS_j on the dielectric tube wall, and assuming that radiation is isotropic, the radiation flux through dS_j is,

$$q_{rad,j} = \frac{1}{4\pi dS_j} \sum_{i=1}^{n_d} P_{rad}(z_i) dV(z_i) d\Omega_{i,j}. \quad (7.56)$$

$d\Omega_{i,j}$ is the solid angle that sweep dS_j from a reference frame located at the centre of the i disc ($z = z_i, r = 0$). Figure 7.21 shows the distribution of q_{rad} along the rear and lateral walls.

By integrating the heat flux on the rear (B) and lateral (L) walls, and also at the tube exit section (E), the total power through each surface is $Q_B = 904.1$ W, $Q_L = 918.8$ W, $Q_E = 18.7$ W respectively.

It is concluded that the radiative heat is larger than the energy flux of plasma impinging on the walls. In future work much more attention should be focused on the excitation/radiation models, since the thermal loads due to these phenomena are quite large. These radiative losses are sensitive to the propellant used. Argon is particularly bad in this sense, because of the many lines that can be excite. Furthermore, the lower the plasma temperature is, the larger the radiative loads are, as described by the Dugan model [152], which has been used to characterize the ionization/inelastic losses in Chapter 3.

7.3.3 HPT Performances

The assessment of HPT performances consists mainly of the determination of the thrust efficiency, $\eta = F^2/2\dot{m}P$, which can be split into different partial contributions,

$$\begin{aligned}\eta &= \eta_u \eta_{RF} \eta_{cham} \eta_{noz} \frac{F^2}{2\dot{m}_i P_{iz}}, \text{ with} \\ \eta_u &= \dot{m}_i / \dot{m} \\ \eta_{RF} &= P_a / P_{RF} \\ \eta_{cham} &= P_{beam} / P_a \\ \eta_{noz} &= \eta_{exp} \eta_{div} P_{iz} / P_{beam}, \text{ with, } \eta_{exp} = P_i / P_{beam}, \text{ and } \eta_{div} = P_{iz} / P_i,\end{aligned}\tag{7.57}$$

The ratio $F^2/2\dot{m}_i P_{iz} \sim 1$ is simplified by considering a mono-energetic ion beam far downstream. From previous sections, the propellant utilization and chamber efficiency could be taken as optimistic as $\eta_u \sim 1$ and $\eta_{cham} = 0.74$ respectively. η_{RF} depends basically on how much power is coupled to the plasma in comparison to the power emitted by the antenna, and also how efficient is the RF power generator, and its circuitry: matching box, transmission lines, etc... The first phenomenon is an issue not solved yet. Although this Thesis has dealt with a 1D model of the RF-wave propagation into a magnetized plasma column in order to estimate the plasma resistance, this model assumes that all the power emitted by the antenna is effectively coupled to the plasma, which is far from being true. The second issue depends on the efficiency of RF technology state of the art. Based again on VASIMR experience it could be rated at 0.95 (see Table A.9). So, in this case $\eta_{RF} = 0.9$ is taken. The assessment of η_{noz} is another factor that nowadays is still not solved, in spite of the effort invested on the development of magnetized plasma plumes models [17, 123, 153, 154, 155, 156], there are still some uncertainties about the physics involved. The lack of knowledge about the electron thermodynamics along the expansion precludes proper determination of η_{exp} , for this reason a first attempt to solve this problem, even simplified to a one dimensional paraxial geometry, has been carried out in Chapter 5. η_{div} is highly affected by, among other phenomena, the plasma detachment mechanism. Acceptable values for these efficiencies are, according to some researchers [146], $\eta_{div} = 0.9$ and $\eta_{exp} = 0.8$, resulting an overall nozzle efficiency, $\eta_{noz} = 0.72$. Considering all these partial efficiencies, the theoretical thrust efficiency will not be higher than $\eta = 0.48$.

Regarding the thrust generated by the HPT, this is split into the force generated by the source and the thrust gain due to the plasma expansion at the magnetic nozzle. HPS thrust is based on measuring the plasma momentum at the source exit section, which at the design point is estimated about $F_{cham} = 611$ mN, resulting a source thrust compactness of $F_{cham}/\pi R^2 = 36$ mN/m². Based on the expected thrust efficiency, the total thrust should be around $F =$

795 mN, the specific impulse $I_{sp} = 1846$ s, and the thrust power ratio of 53 mN/kW, which is quite promising.

7.4 Conclusions

The pre-design of a 15 kW HPT has been carried out, following some criteria based on simplifications or scaling laws derived from 0D, 1D, and 2D models. Fluid criteria stated mainly the need to magnetically confine the plasma inside the source, in order to avoid or reduce wall losses, this requires keeping a high Hall parameter, $\chi_H \gg 1$. This criterion also yields almost full ionization of the injected gas for a long enough discharge chamber, $L/L^* \gg 1$. Concerning the wave criteria it has been based on a 0D RF wave model. The main criterion has consisted in studying the propagation of TG-HE modes within the plasma column at different regimes of $n_0 - B_0$. The determination of the design point: chamber radius, mass flow rate, magnetic strength has been set satisfying these criteria or without penalizing them to much.

In a second detailed study of the RF-wave plasma interaction, an extended 1D-radial model has been implemented in order to understand the plasma response, when it is excited by different antennae. The amount of power that is actually dumped onto the plasma and the resulting plasma impedance have been determined. It has been concluded that the half-turn helical antenna offers a better response, since it provides the higher resistance, i.e. more RF-power is coupled onto the plasma. The roles of plasma density and magnetic field non-uniformities have been also addressed, concluding that uniform 1D models used so far do not provide a good quantitative approximation to the plasma impedance.

With the goal of determining the HPT efficiencies, a more precise study of the plasma generation and its internal dynamics has been carried out with the 2D model of Chapter 3 at the design point conditions. Results have been rather consistent with the previous estimations, thus validating this procedure. However, some results may point to remaining inadequacies of the models. First, the model indicates that neutral depletion, or ionization, occurs within a very short length at the back of chamber, leaving a quasi-sonic flow in the rest of the chamber, and an equilibrium between plasma generation and wall recombination. This response seems to be quite idealized and would require further analysis and experimental verification. It also suggests that the chamber length can be much shorter than the one selected initially, and $L/R = 5$ is the final choice for the chamber aspect ratio. Second, it is concluded that magnetic strength for a good confinement has been overestimated, and a reduction from 1000 G to 450 G has been proposed.

Regarding the HPT propulsive performances, some hard assumptions have been made on the prediction of the nozzle efficiency according to the existent literature. Uncertainties on the thermal/kinetic energy conversion, plasma detachment, and electron cooling are still not solved by the community. So, Chapter 5 sets the basis to explore as a future work, the role of the electron cooling on determining the theoretical limits of the expansion efficiency. With all this assumptions and results derived from aforementioned models (chamber efficiency, and propellant utilization), the thrust efficiency is expected not higher than the 50%, unless other effects not considered here could improve it. Based on this efficiency the HPT thrust would be around 795 mN, the specific impulse 1846 s and the thrust/power ratio 36 mN/m^2 . These values must be interpreted just as a first approximation to HPT figures of merit, and require a further analyses of the different physical processes, and of course, experimental validation.

8 Conclusions

The review of the existing HPT prototypes and general experiments on HPSs has provided a broad insight on their performances and propulsive capabilities. From the pioneers, such as Chen and Boswell [18, 19], till today, published results about the propulsive capabilities have shown some controversy and have been usually disconnected from theoretical models. This has pointed out the need of developing novel models in order to simplify the understanding of the complex phenomena that take place in a HPT.

Concerning the two dimensional fluid model of the HPS, main physical processes have been analysed based on the design and operational parameters: chamber dimensions, mass flow rate of argon, magnetic field strength, and power deposition. The optimal values of design and operational parameters that maximize propellant utilization and production efficiency have been determined. Overall HPT performances have been described after matching this HPS model with the results of the magnetic nozzle model DIMAGO: thrust, power balance, and thruster efficiencies, assuming isothermal electrons, have been estimated. Thermal, electric and magnetic contributions to thrust have been identified, based on the plasma momentum equation. Analogously in terms of the energy balance, it has been determined for several operational regimes how the absorbed power splits into beam power, ionization losses and wall losses. This study has determined as well the theoretical thrust efficiency about 48 %, the main losses being the lack of screening of the chamber walls, and the incomplete plasma expansion on the nozzle stage.

In Chapter 4 a hybrid model of the plasma flow through a diverging magnetic nozzle has been developed, based on the EP2 heritage of *HallMa*, which was oriented to the simulation of HETs performances. Different magnetization regimes of the electrons species have been analysed with this tool. In the highly magnetized regime, or near collisionless regime, the model

matches the results of other fully magnetized models, for instance DIMAGNO [17], because of the resemblance of hypotheses concerning electron thermodynamics. As a consequence, the same uncertainties of these models affect the obtained results.

This model has illustrated the differences between the highly magnetized and demagnetized regimes. Ions detach inwardly from the magnetic lines in the highly magnetized regime obtaining a more collimated beam. On the other hand, for the weakly magnetized regime, ion streamlines diverge more than the magnetic lines, i.e. there is outward detachment. Some difficulties on the closure of the charge conservation law in the low plasma density regions have been identified and further investigation on this is required to improve the current model.

The current uncertainties on the electrons thermodynamics evolution throughout the MN has motivated the work performed in Chapter 5, being this a key factor to determine the propulsive capabilities of a MN. The one-dimensional and steady solution of a fully magnetized and collisionless plasma flowing along a magnetic channel has been determined, under the hypotheses of quasi-neutrality and current-free. The total potential drop, and its parametric dependence on the mass ratio and the ion energy has been satisfactorily analysed. This result is more realistic than the unbounded potential drop of models based on the Boltzmann equilibrium, or limited potentials attributed to polytropic models. The potential barrier causes the partial depletion of the EDF, yielding kinetic electron cooling, the loss of the isotropy, and the formation of collisionless heat fluxes. This result is also similar to the solution of the potential drop through an electrostatic Debye sheath.

Electrons have been split into different groups according to their effective potential. The contribution of each population to the moments of the EDF has been analysed in depth identifying the population that dominates in each region. The trapped population has been identified as the one that is responsible for sustaining the plasma quasi-neutrality on the divergent side for acceptable values of the dimensionless parameters. This also keeps the entire confined population isotropic but not isothermal. The role of the high energy electrons consists on ensuring the zero net current condition, self-neutralizing the ion beam. These are also responsible for the asymptotic trend of the electron temperature. Asymptotic laws at the far field have been derived for the electric potential, density and temperatures.

Model limits have been explored as well. By decreasing the mass ratio $m_i/m_e \rightarrow 1$, the ion-electron particle dynamics are more similar, and consequently most of the trapped population is depleted, without penalizing on the fulfilment of the quasi-neutrality and current-free conditions.

Future actions on this topic are proposed next. First, to explore the effect of considering a superposition of two electron populations at different temperatures, say cold and suprathermal electrons. This should help to understand the formation of quasi-double layers. Second, the postulation of bi-Maxwellian electrons/ions distributions at the plasma reservoir is more consistent with the distribution in some devices as the ECR or the VASIMR. Third, in the current work, transient effects and collisions have been pointed to as possible mechanisms to trap electrons. Both require to update the current model to more advanced models, e.g. Vlasov equation solver including the time-dependent terms.

The test campaign carried out with the modified HPT at the EPPDyL can be considered of a great success. The experiments have determined that the flux to the back wall is not comparable to the forward plasma flux as predicted by the theoretical model of Chapter 3. Regarding the use of a permanent magnet to screen the back wall from plasma impinging, it requires further assessment according to the experimentally obtained results. Furthermore, the permanent magnet modifies the whole discharge and HPT performances.

The axial structure of the plasma density measured within the source at the EPPDyL does not fit quantitatively the theoretical predictions of Chapter 3. The different magnetic topology of the experiment in comparison to the theoretical model and the general simplicity of the theory could explain the experimental-theoretical mismatch. Consequently, the HPS model should be carefully improved to overcome some limitations such as the one dimensional magnetic topology, the absence of double-charged ions or metastable atoms, etc,. However, the use of hybrid schemes, instead of fluid ones, is highly recommended to deal with these limitations, following a similar strategy as in Chapter 4.

Finally, in Chapter 7 the pre-design of a 15 kW HPT has been performed. Different fluid and wave criteria have been established in order to determine the chamber radius, mass flow rate, magnetic strength, etc,. Concerning the fluid effects, the most restrictive criterion requires to keep a high Hall parameter in order to maintain the plasma confined, to reduce wall losses, and to reach almost the full ionization of the injected gas. Wave criteria have been preliminarily analysed with a uniform unbounded model of wave propagation, ensuring that TG-HE modes satisfactorily propagate for a given density and applied field.

The wave criterion has been extended to the implementation of a one dimensional radial model of the wave propagation through the plasma column. The influence of the plasma density and the magnetic field non-uniformities have been assessed, concluding that uniform 1D models used so far do not provide a good quantitative approximation to the plasma

impedance. Different antennas have been studied. The half-turn helical antenna has offered the best response in all analysed cases (higher resistance, thus larger absorbed power).

At the design point, HPT performances have been computed by using the model of Chapter 3. Although results are consistent with preliminary estimations, some of them are also questionable. Ionization is very localized at the back part of the HPS, indicating that the chamber length could be shortened, an idealized result which has not been observed experimentally, neither in Chapter 6 nor in the literature review. Regarding the HPT propulsive performances, some hard assumptions have been taken on the prediction of the nozzle efficiency according to the existent literature. So, the thrust efficiency is expected not higher than 50 % at 15 kW of power. Based on this efficiency the HPT thrust would be around 795 mN, the specific impulse 1846 s and the thrust/power ratio 36 mN/m^2 .

To summarize, this Thesis has reached the goal of identifying the important phenomena of the internal plasma fluid dynamics, which are crucial to assess the HPS performances. It also has provided a clear image of the plasma response in the MN from the high to the weak magnetization regimes. The self-consistent determination of the ambipolar field and the evolution of the EDF through the MN has been addressed with a simple kinetic paraxial model, which indeed states propulsive performances limits (finite drop of the potential) and the evolution of electron temperature and its associated cooling. This Thesis has tried to experimentally validate some of the theoretical results. However, these indicate that some of the models developed in this Thesis are too ideal and tied to strong hypotheses, which have been identified, and different solutions have been proposed as a future work.

A Summary of HPS and HPT prototypes characteristics

This annex summarizes the main properties of HPS general experiments and HPT prototypes analysed in Chapter [2](#). All the information is tabulated in order to facilitate the comparison between the achievements of each institution.

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		Australian National University
Prototype		Helicon Double Layer Thruster (HDLT)
Thruster Performances	Power	•200-800 W [27] •250-650 W [29]
	Thrust	•Max: 2 mN in Mode A & 800 W [27] •Max: 6 mN in Mode B & 800 W [27] •40-50 % of thrust is magnetic in [27] •1-2.8 mN [29]
	I_{sp}	800 s (mode B of [27])
	Propellant	Ar, Kr, N_2 , CH_4 , NH_3 , N_2O [29, 73]
	$\dot{m}$	•0.75 mg/s •1 mg/s of Kr in [29]
	η_t	•1 % (mode A of [27]) •3 % (mode B of [27])
	η_u	25-35 % [29]
Plasma Parameters	Density	$4 - 5 \cdot 10^{17} \text{ m}^{-3}$
	Temperature	4-6 eV
Magnetic field	Strength	100-200 G
	Circuit type	•Mode A: 2 solenoids, near-axial field [27] . •Mode B: only 1 solenoid is active, magnetic throat inside chamber [27].
RF Subsystem	Antenna Type	•Double Saddle Antenna [71, 34] •Double Loop Antenna [27]
	Frequency	13.56 MHz.
	Antenna geometry	•Double-loop, centred on the source, $D = 11 \text{ cm}$ •Double saddle, $L = 18 \text{ cm}$, [71, 29]
	η_{RF}	90 % [157]
	Impedance	$R_a = 0.76 \Omega$ (double saddle), $R_{MNx} = 50 \Omega$
Chamber	Dimensions	•$L = 25 \text{ cm}$, $D = 9 \text{ cm}$ [27]. •$L = 17.2 \text{ cm}$, $D = 8 \text{ cm}$ [29]
	Material	Pyrex glass
	Weight	N/A
Other Properties	PPU details	π -type matching network (RLC circuit) with 2 variable vacuum capacitors .
	References	•Direct thrust measurements: [98, 27, 29] •Indirect thrust measurements: [71, 157]. •Magnetic field steering: [74]. •DL: [34, 66]

Table A.1 – Helicon Double Layer Thruster (HDLT) at the Australian National University (ANU).

Research group		University of California, Los Angeles (UCLA) Principal Researcher: Francis Chen and ANU		
Experiment main reference		[158]	[46]	[47]
Source Performances	Power	≤ 800 W	≤ 2 kW	≤ 2 kW
	Propellant	Argon		
	Pressure	4 mTorr	12 mTorr	2 mTorr
Plasma Parameters	Density	$6 \cdot 10^{18} \text{ m}^{-3}$ (100 G)	$1.4 - 2 \cdot 10^{19} \text{ m}^{-3}$ (peaks measured downstream)	$1.2 \cdot 10^{18} \text{ m}^{-3}$ (1kW, 360G)
	Temperature	N/A	2-5 eV	3.5 eV (Maxwellian) & 15-25 eV (hot tail)
Magnetic field	Topology	Axial		
	Strength	0-1000 G	0 – 900 G	0-400 G
	Circuit type	Electromagnets		
RF Subsystem	Frequency	31.2 MHz	27.12 MHz	13.56 MHz
	Antenna Type	Half-wave length antenna	Right hand polarized Helical antenna & Nagoya III	Nagoya III
	Antenna geometry	$L_a = 12 \text{ cm}$	N/A	$L_a = 20 \text{ cm}$
	Efficiency	99 %	90 %	N/A
	Impedance	N/A		2.8 Ω
Chamber	Length	90 mm	170 mm	108 mm
	Diameter	20 mm	25 mm	100 mm
	Material	Quartz		

Table A.2 – HPS experiments at the UCLA.

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		West Virginia University, with the support of Princeton Plasma Physics Laboratory (PPPL) and ANU	
Prototype		MNX magnetic nozzle experiment [58]	HeLIX, Helicon Experiment [52]
Source Performances	Power	≤ 2 kW	
	I_{sp}	1100 s (30 eV ion beam)	920 s
	Propellant	Argon	
	$\dot{m}$	0.4-30 mTorr	3.5-8.6 sccm
Plasma Parameters	Density	$2 \cdot 10^{19} \text{ m}^{-3}$ (peak)	$2.5 \cdot 10^{17} \text{ m}^{-3}$
	Temperature	4-9 eV	7-10 eV
Magnetic field	Topology	Axial + MN	Axial
	Strength	•300-1200 G (Helmholtz coil) • ≤ 2000 G (MN throat)	0 – 1300 G
	Circuit type	Helmholtz coil (wrapped to source) + MN coil ($D_i = 2$ cm, $L = 3$ cm, $I_B = 400$ A)	10 Electro-magnets.
RF Subsystem	Antenna Type	Double-saddle	Half wave, helical antenna
	Frequency	27 MHz	6-18 MHz
	Antenna geometry	$L_a = 10$ cm	$L_a = 19$ cm
Chamber	Geometry	$L = 300$ mm, $D = 40$ mm	$L = 610$ mm, $D = 100$ mm
	Material	Pyrex glass	
Other Properties	Diaphragm	1 cm (metal disk, floating or biased)	N/A
	Copper rings	15 internal copper rings, 4 cm diameter, 8 of them electrically biased at the expansion region	N/A
	DL issues	All these studies discuss the role of a DL on plasma properties. DL affects the plasma properties upstream and downstream of itself.	
	Other References	[55, 101, 53, 159]	

Table A.3 – MNX and HeLIX experiments at the Princeton Plasma Physics Laboratory and the West Virginia University.

Research group		Wisconsin University Group		
Experiment main reference		[60]	[61]	[64]
Source Performances	Power	≤ 3 kW	≤ 1.4 kW	≤ 1 kW
	Propellant	Argon		
	$\dot{m}$	22-150 sccm	1 mTorr	3 mTorr
	η_u	0.35-0.92	N/A	
Plasma Parameters	Density	$1.6 \cdot 10^{19} \text{ m}^{-3}$ (max)	$1 - 2 \cdot 10^{19} \text{ m}^{-3}$ (640-1120 G)	•Inductive mode: 10^{17} m^{-3} ($\leq 650 \text{ W}, B_0 \leq 360 \text{ G}$) •Transition mode: 10^{18} m^{-3} (650 – 900 W, $B_0 = 200 - 400 \text{ G}$) •Helicon mode: 10^{19} m^{-3} ($\geq 700 \text{ W}, B_0 \geq 400 \text{ G}$)
	Temperature	1.7-5 eV	$T_{e\parallel} = 3 - 5 \text{ eV}, T_{e\perp} = 3.5 - 7 \text{ eV}$	3.5-5.5 eV (5-35 eV hot tail)
Magnetic field	Topology	Axial		
	Strength	≤ 1500 G	≤ 1120 G	≤ 1200 G
	Circuit type	6 electromagnets: $L = 7 \text{ cm}$ $D_{in} = 20 \text{ cm}$ $D_{out} = 50 \text{ cm}$		Coils N/A
RF Subsystem	Frequency	13.56 MHz		
	Antenna Type	Half-turn double-helix	Nagoya Type III	Half-turn double-helix antenna
	Antenna geometry	$L_a = 18 \text{ cm}$	$L_a = 15 \text{ cm}$	$L_a = 20 \text{ cm}$
	Efficiency	98 %	99 %	99 %
	Impedance	N/A	$Z = 0.92 + j11 \Omega$	N/A
Chamber	Length	150 cm	122 cm	175 cm
	Diameter	10 cm		
	Material	Pyrex glass		
Other	References	[160, 62]		

Table A.4 – WOMBAT and other HPS experiments at the Wisconsin University.

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		MIT - Space Propulsion Laboratory
Prototype		mini-Helicon Thruster Experiment (mHTX)
Thruster Performances	Power	700 – 1100 W [10]
	Thrust	10-20 mN [10]
	I_{sp}	1000-2000 s (Ar), 2000-4000 s (N ₂) [10]
	Propellant	Ar, N ₂ , Ne, Xe, gas mixtures including air [10]
	$\dot{m}$	10 – 50 sccm, [10]
	η_t	20 % (Ar), 18 % (N ₂), [10]
	η_u	> 90 %, [10]
Plasma Parameters	Density	$10^{19} - 10^{20} \text{ m}^{-3}$
	Temperature	$\geq 7 \text{ eV}$
Magnetic field	Topology	Convergent-Divergent
	Strength	1500 G – 1800 G
	Circuit type	• 162 turns copper solenoid [65] . • $L_s = 120 \text{ mm}$, $D_{(s,in)} = 35 \text{ mm}$, $t_s = 66 \text{ mm}^2$ [65] • $R_s = 0.03 \Omega$, $I_{max} = 180 \text{ A}$, $P_s = 970 \text{ W}$ [65] • Helmholtz coil were also proposed
RF Subsystem	Antenna Type	Right-hand polarized, half-helical antenna
	Frequency	13.56 MHz [10]
	Antenna geometry	$L_a = 100 \text{ mm}$, [65]
Chamber	Source Geometry	Cylindrical: $L = 300 \text{ mm}$, $D = 20 \text{ mm}$, $t = 1.5 \text{ mm}$ [65].
	Material	Fused silica quartz tube
	Weight	N/A
Other Properties	PPU details	Advanced Energy RFPP-10 1.2 kW [10]
	References	[161]
	Magnetic Mirror	Located at the back of the chamber tube, using a NdFeB permanent magnet, which creates a magnetic cusped-field topology
	Comments	Experiment cancelled in 2009.

Table A.5 – mini-Helicon Thruster Experiment (mHTX) at the Electric Propulsion Laboratory (MIT).

Research group		Georgia Tech, HPEPL	
Prototype		Cylindrical Helicon Plasma Thruster (CHPT)	
Thruster Performances	Power	0-1.5 kW	215-840 W
	Thrust	N/A	Max 6.2 mN ($I_{sp} = 142$ s)
	I_{sp}	1500 s	Max 377 s (5.55 mN)
	Propellant	Argon	
	$\dot{m}$	0.74 – 4.45 mg/s	1.5-4.5 mg/s
	η_t	N/A	1.37 %
	η_u	N/A	< 46 % (indirectly estimated)
Plasma Parameters	Density	$8 \cdot 10^{18} \text{ m}^{-3}$ (1 KW, 1100 G, 7 MHz, 3 mg/s)	N/A
	Temperature	1-8 eV	N/A
Magnetic field	Topology	Axial inside the tube, and divergent outside.	
	Strength	0-1100 G	150-450 G
	Circuit type	Solenoid	• 2 x 725 turn solenoids, spaced 197 mm apart, centred on the source. • $L_s = 76 \text{ mm}$ $D_{(s,in)} = 197 \text{ mm}$
RF Subsystem	Antenna Type	Half helical	Double saddle antenna
	Frequency	2-15 MHz	11.9-13.56 MHz
	Antenna geometry	$L_a = 132.8 \text{ mm}$, $D_a = 49 \text{ mm}$	$L_a = 200 \text{ mm}$, $D_a = 150 \text{ mm}$
Chamber	Source Geometry	$L = 600 \text{ mm}$, $D = 45 \text{ mm}$	$L = 273 \text{ mm}$, $D = 140 \text{ mm}$
	Material	Pyrex glass	
Other Properties	PPU details	• 60 kW EMHP DC power supply (solenoids) • ACOM 2000 A, 1.5 kW, linear RF amplifier • Bird 43 Thruline wattmeter • AT5K 3.5 kW L-Type matching network • YAESU FT-540 HF Transceiver	
	P_{RF} cable losses	N/A	15.7 % (11.9 MHz) - 28.8 % (13.56 MHz)
	Main Reference	[14]	[28]

Table A.6 – Cylindrical Helicon Plasma Thruster at Georgia Tech.

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		Georgia Tech, High-Power Electric Propulsion Laboratory
Prototype		Annular Helicon Plasma Source
Source Performances	Power	1 W
	Propellant	Argon, Xenon
	$\dot{m}$	2.97 mg/s
	η_u	80 – 90 %
Plasma Parameters	Density	• 10^{17} m^{-3} peak for Ar • $2.4 \cdot 10^{17} \text{ m}^{-3}$ peak for Xe
	Temperature	3.7 eV – 20 eV
Magnetic field	Topology	Axial inside the source
	Strength	450 G
	Circuit type	Set of 5 Solenoids (300-turn)
RF Subsystem	Antenna Type	2 left-twist helical pitch antenna. One wrapped outside the outer tube, and the other wrapped inside the inner tube.
	Frequency	13.56 MHz
	Antenna Geometry	• $L_{(a,in)} = 192.7 \text{ mm}$; $D_{(a,in)} = 110 \text{ mm}$ • $L_{(a,out)} = 192.7 \text{ mm}$; $D_{(a,out)} = 186 \text{ mm}$
	Impedance	$R = 3\Omega$
	Efficiency	99.5 %
Chamber	Source Geometry	2 concentric tubes: • $L_{in} = 1626 \text{ mm}$; $D_{in} = 120 \text{ mm}$ • $L_{out} = 1626 \text{ mm}$; $D_{out} = 171 \text{ mm}$
	Material	Pyrex glass
Other Properties	PPU details	The same of the cylindrical version (Table A.6)
	References	[82, 79, 80, 162]
	HET	This HPS experiment is aimed to interface a 5 kW HET, working as an ionization stage.
	Two antennas	Depending on the antenna current direction they distinguished the parallel or the anti-parallel antenna current configurations.

Table A.7 – Annular Helicon Plasma Source experiments at Georgia Tech.

Research group		University of Washington
Prototype		High Power Helicon Thruster (HPHT)
Thruster Performances	Power	20 - 50 kW [22, 163]
	Thrust	1-2 N [22]
	I_{sp}	2000 s (Ar), 3000 s (N_2), 5000 s (H_2) [22]
	Propellant	Ar, N_2 , H_2
	$\dot{m}$	160 sccm [22]
	η_t	55 % (for 1 N) [22]
	η_u	~ 100 %
Plasma Parameters	Density	$10^{18} - 2 \cdot 10^{20} \text{ m}^{-3}$
	Temperature	25 eV
Magnetic field	Topology	Conventional configuration: axial inside the source, and MN downstream.
	Strength	Average: 150 G [22]
	Circuit type	6 coils spaced 22 mm apart. Each coil with a diameter $D_c = 73 \text{ mm}$ [22] .
RF Subsystem	Antenna Type	Half helical (helicon coil)
	Frequency	0.5-1 MHz
	Antenna geometry	$L_a = 120 \text{ mm}$, $D_a = 70 \text{ mm}$
Chamber	Source Geometry	Cylindrical: $L = 150 \text{ mm}$, $D = 70 \text{ mm}$
	Material	Quartz glass
	Weight	20 kg including RF matching network (and PPU) [22]
Other Properties	PPU details	•Intensity (I): 2000 Amp. [22] •PPU uses IGBT (Insulated-gate bipolar transistor). [163] •Solid state power supply. [163]
	References	[22, 163, 85, 84, 164]
	Igniter	It needs an igniter to create a seed plasma [163].
	Pulsed operation	RF operates during $\Delta t_{RF} = 2 \cdot 10^{-4} \text{ s}$, Igniter $\Delta t_{ign} = 25 \cdot 10^{-6} \text{ s}$, Gas Injection $\Delta t_{inj} = 3 \text{ ms}$. [163]

Table A.8 – High Power Helicon Thruster (HPHT) at the University of Washington.

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		Ad-Astra Rocket Company
Prototype		VASIMR (ICRH off)
Thruster Performances	Power	28 kW
	Thrust	500 mN (indirectly estimated)
	I_{sp}	1900 s
	Propellant	Ar, D, Ne, Xe. (Presented results are for Argon)
	$\dot{m}$	25 mg/s
	η_t	16 %
	η_u	95 %
Plasma Parameters	Density	10^{20} m^{-3}
	Temperature	9 eV (max)
Magnetic field	Topology	Axial inside the HPS, and MN outside. A “magnetic choke” reduces plasma diameter between HPS and ICRH.
	Strength	1700 G (in the HPS)
	Circuit type	Superconducting magnets.
RF Subsystem	Antenna Type	Quad-phased helical antenna, m=1 mode [88]
	Frequency	6.78 MHz
	Antenna geometry	N/A
	η_{RF}	95 %
	Impedance	$R_{plasma} = 2 \Omega$, $R_{antenna} = 0.1 \Omega$
Chamber	Source Geometry	Cylindrical: $D = 90 \text{ mm}$ (VX100 experiments)
	Material	Quartz glass
	Weight	• Specific mass of HPS RF generator $\sim 1 \text{ kg/kW}$ • Specific mass of ICRH RF generator $\sim 0.5 \text{ kg/kW}$
Other Properties	PPU details	• Solid State RF generators (Nautel Limited of Canada), P_{RF} up to 40kW, 6.78MHz, efficiency 92%, input voltage 375 VDC. • PPU uses IGBT (Insulated-gate bipolar transistor). [163] • Solid state power supply. [163]
	References	[90, 92, 165, 166, 88, 87]
	VASIMR (ICRH on)	$\eta_t > 50 \%$
	HS RF startup	• Tuning time $< 100 \text{ ms}$
	Magnets Power	• Magnet power supply: 500 W • Cryocoolers: 2 kW
	Vacuum conditions	$P_{cham} < 2 \cdot 10^{-5} \text{ Torr}$, $n_{background} \sim 10^{15} \text{ m}^{-3}$

Table A.9 – VASIMR-HPSO (ICRH off) designed by Ad-Astra Rocket Company.

Research group		FP7 Consortium (coordinated by Univ. Padua, CISAS)
Prototype		Helicon Plasma Hydrazine. COmbined Micro (HPHCOM)
Thruster Performances	Power	•8 W, qualification model (QM1) at KhAI •0-90 W (experimental model)
	Thrust	0.5 mN, QM1 at KhAI (1.4 mN extrapolated at 45 W)
	I_{sp}	422 s (1300 s extrapolated at 45 W; QM1-KhAI)
	Propellant	Argon
	$\dot{m}$	0.1-0.2 mg/s. Optimum: 0.12 mg/s (QM1-KhAI)
	η_t	13 % (QM1-KhAI)
	η_u	90 % (experimental model)
Plasma Parameters	Density	$1.4 - 2.1 \cdot 10^{18} \text{ m}^{-3}$ (50-90 W, 0.225 mg/s CISAS experimental model)
	Temperature	Two electron populations claimed at QM1-KhAI experiments, 1 – 3 eV, 10 – 25 eV (9 eV max, EM experiments)
Magnetic field	Topology	Peaked inside the source, sharp divergence and inverted field close to the exit section..
	Strength	400 G (QM-KhAI), 1100G (QM2 at CISAS)
	Circuit type	•4 PMEA (Permanent Magnets Electro-Analogous) •QM2 : combines a ring of axially-oriented-magnets coupled with a ring of radially-oriented ones. Each magnet is a cube of 20mm side)
RF Subsystem	Antenna Type	•4-6 turns spiral antenna •Compact resonant antenna (s-helicon antenna, QM models)
	Frequency	•13.56 MHz, few experiments at 3-10 MHz
	Antenna geometry	•Inner coil: 24 turns, $D_{(a,in)} = 23 \text{ mm}$, $t_{(a,in)} = 1.5 \text{ mm}$ (s-helicon antenna) •Outer coil: 3 turns, $D_{(a,out)} = 34 \text{ mm}$, $t_{(a,out)} = 2 \text{ mm}$
	η_{RF}	80 %, experimental model
Chamber	Source Geometry	Cylindrical: $L = 55 \text{ mm}$, $D = 18 \text{ mm}$
	Material	Quartz glass or alumina
	Weight	668 g (QM at KhAI) 1990 g (QM2 at CISAS, PM assembly: 1395g)
Other Properties	PPU details	Discrete matching box MFJ-993B, MFJ-943E.
	References	[30, 94]
	Diaphragm	•Ceramic, located at the source exit section • $D_{diaphragm} = 4 \text{ mm}$
	Injector material	Ceramic
	Nozzle screen	•Fibre insulator, aperture angle 60 degrees •Authors claim it increases ion current and decreases RF losses
	Thruster Dimensions	$L_{thruster} = 157 \text{ mm}$

Table A.10 – Helicon Plasma Hydrazine, COmbined Micro (HPHCOM) coordinated by Padua University, funded by 7th Framework Programm (EU).

Appendix A. Summary of HPS and HPT prototypes characteristics

Research group		Institute for Nuclear Research National Academy of Sciences (INR-NAS), Ukraine
Prototype		Compact Helicon Plasma Thruster
Thruster Performances	Power	600-1000 W
	Thrust, η_t , I_{sp}	N/A
	Propellant	Argon
	η_u	$\sim 100\%$, depending on the magnetic circuit [13].
Plasma Parameters	Density	•Max: $4 - 5 \cdot 10^{18} \text{ m}^{-3}$ ($P_{Ar} = 3 - 5 \text{ mTorr}$, AF+EM, 600 W) •$1 - 2 \cdot 10^{18} \text{ m}^{-3}$ ($P_{Ar} = 0.7 \text{ mTorr}$ and 3-5 mTorr respectively, AF) •AF + CFA configuration produces plasma densities between AF+EM and single AF configurations.
	Temperature	•9 eV – 15 eV ($P_{Ar} = 5 - 0.7 \text{ mTorr}$, CFA+AF)[13] •6 - 8 eV and 14 - 18eV ($P_{Ar} = 0.7 \text{ mTorr}$ and 3-5 mTorr respectively, AF)[96]
Magnetic field	Topology	•CFA (cylindrical ferrite assembly, radial magnetization) produces a magnetic cusp (inside the source), although the field is purely axial within it. MN topology outside the source. •AF (annular ferrite, axial magnetization) produces two cusps. The 1st one is close to the source outlet. The 2nd is inside the source, where the magnetic field is approximately axial. •CFA+AF. Here, the 2nd cusp produced by AF within the source vanishes, and the one close to the source outlet is weaker. [13]
	Strength	•CFA, 150 G (peak) near CFA ends. 0 G at the CFA centre point. •AF, 460 G (peak) at the AF centre point. Decreases to 0 G, 3 cm away from the centre.
	Circuit type	Permanent magnets dimensions: •$L_{CFA} = 120 \text{ mm}$, $D_{CFA,in} = 120 \text{ mm}$, $t_{CFA} = 15 \text{ mm}$ •$L_{AF} = 18 \text{ mm}$, $D_{AF,out} = 135 \text{ mm}$, $D_{AF,in} = 55 \text{ mm}$
RF Subsystem	Antenna Type	Double or triple-turn
	Frequency	13.56 MHz
	Antenna geometry	Circular, wrapped around the source.
Chamber	Source Geometry	$L = 200 - 300 \text{ mm}$, $D = 45 \text{ mm}$
	Material	Quartz
Other Properties	Double Layer	Observed in some magnetic configurations, also depending on vacuum chamber pressure conditions. DL near source outlet in the CFA+AF magnetic configuration.
	References	[13, 95, 96]
	Others	When using exclusively PM, AF+CFA is the most efficient configuration in terms of plasma production and beam acceleration.

Table A.11 – Compact Helicon Plasma Thruster (COHPT) at the Institute for Nuclear Research National Academy of Sciences (INR-NAS), Ukraine

Research group		Iwate University, Marioka (Japan)
Prototype		Permanent Magnet Expanding Plasma
Thruster Performances	Power	• ≤ 725 W • ≤ 2 kW [99]
	Thrust	• 0.5 mN (140 W) - 3 mN (725 W) [98] • 2.5-6 mN (200-800 W) [27] • 15 mN (2 kW) [99]
	I_{sp}	• 500 s • 2000 s [99]
	Propellant	Argon
	$\dot{m}$	25 sccm (0.6 mg/s)
	η_t	1 % [98], 3 % [27], 7 % [99]
	η_u	< 50 %
Plasma Parameters	Density	$3.5 \cdot 10^{17} - 1.5 \cdot 10^{18} \text{ m}^{-3}$
	Temperature	5.5 eV – 8 eV
Magnetic field	Topology	Different solutions: uniform-divergent, convergent-divergent
	Strength	~ 200 G within the source, ~ 750 G with nozzle electromagnet
	Circuit type	Permanent magnets, double concentric arrays of neodymium iron boron magnets (NdFeB)
RF Subsystem	Antenna Type	Double turn loop, centred on the source (where B is maximum).
	Frequency	13.56 MHz
Chamber	Source Geometry	$L = 80 - 200$ mm (extensible), $D = 65$ mm
	Material	Pyrex glass
Other Properties	PPU details	π -type matching network
	References	[97, 98, 27, 99, 100]
	Others	Reported an inductively coupled mode discharge [98]

Table A.12 – Permanent Magnet expanding Plasma. Iwate University, Marioka (Japan).

B Argon collision rates

The expressions proposed below for the different collision rates are reasonable approximations for the purposes of this work. The rates for ionization, electron-neutral collisions, electron-ion collisions, and ion-neutral collisions are, respectively,

$$R_{ion}(T_e) = c_e \sqrt{\frac{8}{\pi}} \sigma_{ion} \left(1 + \frac{T_e E_{ion}}{(T_e + E_{ion})^2} \right) \exp \left(-\frac{E_{ion}}{T_e} \right), \quad (\text{B.1})$$

$$R_{en}(T_e) = \sqrt{\frac{8T_e}{\pi m_e}} \sigma_{en}, \quad (\text{B.2})$$

$$R_{ei}(T_e, n_e) = \left(\frac{T_e}{1 \text{ eV}} \right)^{-3/2} \ln \Lambda(T_e, n_e) \cdot 9.2 \cdot 10^{-14} \text{ m}^3 \text{ s}^{-1}, \quad (\text{B.3})$$

with E_{ion} the first ionization energy. For ion-neutral collisions and $c_{in} = |\mathbf{u}_i - \mathbf{u}_n|$, we have

$$R_{in}(c_{in}) = c_{in} (k_2 - k_1 \log_{10} c_{in})^2, \quad (\text{B.4})$$

which can be expressed as,

$$R_{in}(c_{in}) = R_{in,s}(T_e) \hat{c}_{in} (1 - a_1 \log_{10} \hat{c}_{in})^2, \quad (\text{B.5})$$

with $\hat{c}_{in} = c_{in}/c_s$, $c_s = \sqrt{T_e/m_i}$, $R_{in,s} = R_{in}(c_s)$, and $a_1 = k_1(k_2 - k_1 \log_{10} c_s)^{-1}$. The constants involved in the above expressions are gas-dependent. For argon, we will take $E_{ion} = 15.76 \text{ eV}$, $\sigma_{ion} = 2.8 \cdot 10^{-20} \text{ m}^2$. σ_{en} is the electron-neutral cross section, although it depends on the electron energy, its average value is taken, $\sigma_{en} = 1.5 \cdot 10^{-19} \text{ m}^{-2}$ for Argon and $\sigma_{en} = 3 \cdot 10^{-19} \text{ m}^{-2}$ for Xenon [125]. Other constants are $k_2 = 10.5 \cdot 10^{-10} \text{ m}$, and $k_1 = 1.67 \cdot 10^{-10} \text{ m}$.

For constant T_e and a given gas, R_{ion} , R_{en} , and $R_{in,s}$ are constant; R_{ei} is a constant too if an average value is used for $\ln \Lambda(n_e, T_e)$, which is the Coulomb logarithm. Observe that the non-linear expression used for R_{in} correspond to the high-pressure case of Fruchtman et al. [103], but, even for this case, ion-neutral collisions will be found negligible in the desired operation range.

Excitation collisions are taken into account through the effective ionization energy $E'_{ion}(T_e)$

Figure B.1 – Ionization and collision rates, $R_j(m^3/s)$, for $j = ei, en, ion$ and (in, s) .

$= E_{ion} \alpha_{ion}(T_e)$ and α_{ion} the ionization cost factor. From Dugan [152], a fitting formula for Argon is

$$\alpha_{ion}(T_e) \approx 1.4 + 0.4 \exp(0.7 E_{ion} / T_e). \quad (B.6)$$

Collisional rates for these processes, R_k ($k = ion, en, ei, in$), are plotted in Figure B.1 to show their dependence on the temperature T_e .

C Plasma kinetic approach

The kinetic theory of plasmas studies the behaviour and interaction of a system of charged particles, such as ions and electrons, and non-charged particles, as neutrals. Each specie is defined through its distribution function in a dimensional space of seven dimensions, $f_\alpha = f_\alpha(\mathbf{x}, \mathbf{u}, t)$. The distribution function indicates the probability of finding a particle in a certain position $\mathbf{x}$ with a given velocity $\mathbf{u}$ at the time instant t . Bittencourt [167] summarizes in his work the fundamentals of this theory.

However, the distribution function is usually unknown because it requires to solve the Boltzmann equation, that is typically unaffordable, or someone of its simplification indeed, such as the Vlasov equation, which is its collisionless limit.

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{u} \cdot \nabla f_\alpha + \mathbf{a} \cdot \nabla_{\mathbf{u}} f_\alpha = 0. \quad (\text{C.1})$$

In the previous equation $\mathbf{a}$ is the particle acceleration. Each component of $\mathbf{x}, \mathbf{u}$ together with the time t are the seven independent variables, and the distribution function f is the unique unknown. son los vectores velocidad y aceleración de la partícula respectivamente, que conjuntamente con el tiempo constituyen el grupo de 7 variables independientes, mientras que la incógnita es única siendo la propia función de distribución.

C.1 Moments of a distribution function

If the distribution function were known, all macroscopic properties would be determined taking moments of this function as presented in the following equations. In all of them, the integral is over the velocity space $\Omega_{\mathbf{u}}$.

Appendix C. Plasma kinetic approach

- Particle density:

$$n_\alpha(\mathbf{x}, t) = \int_{\Omega_u} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.2})$$

- Flux of particles:

$$\mathbf{g}_\alpha(\mathbf{x}, t) = \int_{\Omega_u} \mathbf{u} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\Omega_u \quad (\text{C.3})$$

- Momentum flux tensor:

$$\bar{\bar{P}}_\alpha(\mathbf{x}, t) = \int_{\Omega_u} m_\alpha \mathbf{u} \mathbf{u} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.4})$$

- Total energy density:

$$W_\alpha(\mathbf{x}, t) = \int_{\Omega_u} (1/2) m_\alpha u^2 f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.5})$$

- Total energy flux:

$$\mathbf{e}_\alpha(\mathbf{x}, t) = \int_{\Omega_u} (1/2) m_\alpha u^2 \mathbf{u} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.6})$$

With the expressions above, other properties can be deduced:

- Average velocity vector:

$$\mathbf{u}_\alpha(\mathbf{x}, t) = \frac{\mathbf{g}_\alpha}{n_\alpha} \quad (\text{C.7})$$

- The difference among the average velocity and the particle velocity $\mathbf{u}_\alpha$ is $\mathbf{c} = \mathbf{u} - \mathbf{u}_\alpha$. Thus, the pressure tensor is defined as following,

$$\bar{\bar{p}}_\alpha(\mathbf{x}, t) = \bar{\bar{P}}_\alpha - m_\alpha n_\alpha \mathbf{u}_\alpha \mathbf{u}_\alpha = \int_{\Omega_u} m_\alpha \mathbf{c} \mathbf{c} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.8})$$

- The trace of the pressure tensor allows determining the scalar pressure:

$$p_\alpha(\mathbf{x}, t) = (1/3) \text{tr}(\bar{\bar{p}}_\alpha) = \int_{\Omega_u} (1/3) m_\alpha c^2 f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.9})$$

- Temperature is defined in each direction i through the pressure tensor.

$$n_\alpha k T_{\alpha i}(\mathbf{x}, t) = \bar{\bar{p}}_{\alpha ii} \quad (\text{C.10})$$

- The absolute temperature is the average of the temperature in each direction:

$$p_\alpha = n_\alpha k T_\alpha \quad (\text{C.11})$$

k is the Boltzmann constant.

- Finally, the heat flux is determined:

$$\mathbf{q}_\alpha(\mathbf{x}, t) = \int_{\Omega_u} (1/2) m_\alpha c^2 \mathbf{c} f_\alpha(\mathbf{x}, \mathbf{u}, t) d\mathbf{u} \quad (\text{C.12})$$

- This allows to split the total energy flux:

$$\mathbf{e}_\alpha(\mathbf{x}, t) = W_\alpha \mathbf{u}_\alpha + \mathbf{u}_\alpha \cdot \bar{\bar{P}}_\alpha + \mathbf{q}_\alpha \quad (\text{C.13})$$

- And the total energy density is redefined as:

$$W_\alpha = (1/2) m_\alpha n_\alpha u_\alpha^2 + (3/2) p_\alpha \quad (\text{C.14})$$

C.2 Cylindrical symmetry and fully magnetized plasma

In Chapter 5 the moments of the distribution function are computed on the energy - magnetic moment space $(E - \mu)$, which are the particle movement invariants of that model, see for instance Eqs. 5.12, 5.16. The derivation of these expressions is summarized here.

First, consider that the problem is axially symmetric, so

$$d^3 w = 2\pi w_\perp dw_\perp dw_\parallel. \quad (\text{C.15})$$

Second, velocities have to be expressed in terms of the invariants,

$$w_\perp^2 = \frac{2B\mu}{m_\alpha}, \quad (\text{C.16})$$

$$w_\parallel^2 = \frac{2}{m_\alpha} (E - \mu B + q_\alpha \phi). \quad (\text{C.17})$$

In the integral expressions $dw_\perp dw_\parallel$ must be replaced by $|J| dE d\mu$, with J the Jacobian of the reference frame transformation.

$$|J| = \frac{1}{m_\alpha |w_\parallel|} \frac{B}{m_\alpha w_\perp}. \quad (\text{C.18})$$

So, any moment of the IDF or EDF is computed as,

$$\begin{aligned} \langle \chi_\alpha \rangle &= \frac{1}{n_\alpha} \iiint_{\Omega_{w_\alpha}} \chi_\alpha f_\alpha d^3 w_\alpha = \frac{1}{n_\alpha} \int_{\Omega_{w_{\alpha,\perp}}} \int_{\Omega_{w_{\alpha,\parallel}}} \chi_\alpha f_\alpha 2\pi w_{\alpha,\perp} dw_{\alpha,\perp} dw_{\alpha,\parallel} \\ &= \frac{1}{n_\alpha} \frac{2\pi B}{m_\alpha^2} \int_{\Omega_{E_\alpha}} \int_{\Omega_{\mu_\alpha}} \frac{\chi_\alpha(E_\alpha, \mu_\alpha)}{|w_{\alpha,\parallel}|} f_\alpha(E_\alpha, \mu_\alpha) d\mu_\alpha dE_\alpha. \end{aligned} \quad (\text{C.19})$$

D Wave propagation into a magnetized plasma

In this annex further details on the 0D and 1D models of the wave propagation within a magnetized plasma are described.

D.1 0D model

For the derivation of the wave dispersion relation, Eq. 7.11, let consider first the definition of the dielectric tensor of a magnetized plasma $\bar{\epsilon}_r$. This requires to determine the relation between the electric displacement field $\mathbf{D}$, and the electric field $\mathbf{E}$, $\dot{\mathbf{D}} = \mathbf{j} + \epsilon_0 \dot{\mathbf{E}}$. Taking, $\mathbf{j}_\alpha = q_\alpha n_\alpha \mathbf{u}_\alpha$, it is necessary to determine $\mathbf{u}$ through the momentum equation of a particle immersed in an axial magnetic field $B_0 \mathbf{1}_z$,

$$\frac{d\mathbf{u}_\alpha}{dt} = \frac{q_\alpha}{m_\alpha} (\mathbf{E} + B_0 \mathbf{u}_\alpha \times \mathbf{1}_z). \quad (\text{D.1})$$

Assuming that $\{\mathbf{u}_\alpha, \mathbf{E}\} = \{\mathbf{u}_\alpha, \mathbf{E}\} \exp(i\omega t)$, each component of the velocity vector become,

$$u_{\alpha,z} = \frac{q_\alpha}{i\omega m_\alpha} E_z, \quad (\text{D.2})$$

$$u_{\alpha,x} = \frac{1}{1 - \frac{\omega_{\alpha,c}^2}{\omega^2}} \frac{q_\alpha}{m_\alpha} \left(\frac{E_x}{i\omega} - \frac{\omega_{\alpha,c}}{\omega^2} E_y \right), \quad (\text{D.3})$$

$$u_{\alpha,y} = \frac{1}{1 - \frac{\omega_{\alpha,c}^2}{\omega^2}} \frac{q_\alpha}{m_\alpha} \left(\frac{E_y}{i\omega} + \frac{\omega_{\alpha,c}}{\omega^2} E_x \right), \quad (\text{D.4})$$

in which $\omega_{\alpha,c} = eB_0/m_\alpha$.

By applying the following relation,

$$\mathbf{D} = \frac{\mathbf{j}}{i\omega} + \epsilon_0 \mathbf{E}, \quad (\text{D.5})$$

Appendix D. Wave propagation into a magnetized plasma

the dielectric tensor ϵ_r , $\mathbf{D} = \epsilon_0 \bar{\bar{\epsilon}}_r \mathbf{E}$ becomes:

$$\mathbf{D} = \epsilon_0 \left\{ \begin{array}{ccc} 1 + \sum_{\alpha} \frac{\omega_{\alpha,p}^2}{\omega_{\alpha,c}^2 - \omega^2} & - \sum_{\alpha} i \frac{\omega_{\alpha,p}^2}{\omega} \frac{\omega_{\alpha,c}}{\omega_{\alpha,c}^2 - \omega^2} & 0 \\ \sum_{\alpha} i \frac{\omega_{\alpha,p}^2}{\omega} \frac{\omega_{\alpha,c}}{\omega_{\alpha,c}^2 - \omega^2} & 1 + \sum_{\alpha} \frac{\omega_{\alpha,p}^2}{\omega_{\alpha,c}^2 - \omega^2} & 0 \\ 0 & 0 & 1 - \sum_{\alpha} \frac{\omega_{\alpha,p}^2}{\omega} \end{array} \right\} \mathbf{E}. \quad (\text{D.6})$$

$\omega_{\alpha,p} = n_{\alpha} q_{\alpha}^2 / \epsilon_0 m \alpha$ is the particle plasma frequency.

Now, taking again Eqs. 7.9-7.10, with electromagnetic fields $\mathbf{E}, \mathbf{B}$ of the shape $\{\mathbf{E}, \mathbf{B}\} \exp(i\mathbf{k}\mathbf{x} - i\omega t)$, the set of equations becomes,

$$\mathbf{B} = \frac{\mathbf{k} \times \mathbf{E}}{\omega}, \quad (\text{D.7})$$

$$\mathbf{k} \times \mathbf{B} = -\frac{\omega}{c^2} \bar{\bar{\epsilon}}_r \mathbf{E}. \quad (\text{D.8})$$

$\mathbf{k}$ is the wavenumber vector of a planar wave, $\mathbf{k} = k(\sin\theta \mathbf{1}_{\perp} + \cos\theta \mathbf{1}_{\parallel})$.

Combining both previous equation, and looking for non trivial solutions of $\mathbf{E}$, the following condition is found,

$$\left| \left(\frac{d_e k}{\omega} \right)^2 \bar{\bar{X}} - \bar{\bar{\epsilon}}'_r \right| = 0, \text{ with} \quad (\text{D.9})$$

$$\bar{\bar{X}} = \bar{\bar{I}} - \frac{\mathbf{k} \cdot \mathbf{k}}{k^2}, \text{ and} \quad (\text{D.10})$$

$$\bar{\bar{\epsilon}}'_r = \left(\frac{\omega}{\omega_{ep}} \right)^2 \bar{\bar{\epsilon}}_r. \quad (\text{D.11})$$

$d_e = c/\omega_{ep}$ is the collisionless plasma skin depth. And the modified dielectric tensor $\bar{\bar{\epsilon}}'_r$, only considering electrons, results as follow,

$$\bar{\bar{\epsilon}}'_r = \begin{pmatrix} \epsilon'_{\perp} & \epsilon'_H & 0 \\ -\epsilon'_H & \epsilon'_{\perp} & 0 \\ 0 & 0 & \epsilon'_{\parallel} \end{pmatrix} \quad (\text{D.12})$$

with,

$$\epsilon'_{\perp} = \frac{\omega^2}{\omega_{ep}^2} + \frac{\omega^2}{\omega_{ec}^2 - \omega^2}, \quad (\text{D.13})$$

$$\epsilon'_{\parallel} = \frac{\omega^2}{\omega_{ep}^2} - 1, \quad (\text{D.14})$$

$$\epsilon'_H = -i \frac{\omega_{ec} \omega}{\omega_{ec}^2 - \omega^2}. \quad (\text{D.15})$$

A biquadratic equation for $U = d_e k$ is determined from Eq. D.9 according to previous expressions,

$$\alpha U^4 - \beta U^2 + \gamma = 0 \quad (\text{D.16})$$

with,

$$\alpha = \epsilon'_{\parallel} \cos^2 \theta + \epsilon'_{\perp} \sin^2 \theta, \quad (\text{D.17})$$

$$\beta = \epsilon'_{\perp} \epsilon'_{\parallel} (1 + \cos^2 \theta) + \sin^2 \theta (\epsilon'^2_H + \epsilon'^2_{\perp}), \quad (\text{D.18})$$

$$\gamma = \epsilon'_{\parallel} (\epsilon'^2_H + \epsilon'^2_{\perp}). \quad (\text{D.19})$$

Assuming the ordering, $\omega_{ep} \gg \omega_{ec}, \omega$, then the unique valid solution for this equation is,

$$U^2 = \frac{1}{\frac{\omega_{ec}}{\omega} \cos \theta - 1}, \quad (\text{D.20})$$

which corresponds to Eq. 7.12. If electrons collisions were included in Eq. D.1, $-\nu_e \mathbf{u}_e$, then the previous equation yields,

$$U^2 = \frac{1}{\frac{\omega_{ec}}{\omega} \cos \theta - \left(1 + i \frac{\nu}{\omega}\right)}. \quad (\text{D.21})$$

D.2 1D model, uniform

Let consider first the set of equations that determine $\mathbf{E}$, $\mathbf{B}$ within the plasma $r < r_p$, Eqs. 7.36-7.37,

$$\hat{B}_{z,p} = \sum_j a_j J_m(p_j \hat{r}), \quad (\text{D.22})$$

$$\hat{E}_{z,p} = \sum_j a_j e_j J_m(p_j \hat{r}), \quad (\text{D.23})$$

$$\hat{B}_{\phi,p} = \sum_j a_j B_{\phi,j}, \quad (\text{D.24})$$

$$\hat{E}_{\phi,p} = \sum_j a_j E_{\phi,j}, \quad (\text{D.25})$$

for $j = 1, 2$ and

$$B_{\phi,j} = \frac{\hat{k}_l m}{\epsilon_2^2 - (\epsilon_1 - \hat{k}_l^2)^2 \hat{r}} (\epsilon_1 - \hat{k}_l^2 + \hat{k}_l \epsilon_2 e_j) J_m(p_j \hat{r}) + \frac{1}{\epsilon_2^2 - (\epsilon_1 - \hat{k}_l^2)^2} (\hat{k}_l \epsilon_2 + \epsilon_1 \alpha e_j) J'_m(p_j \hat{r}), \quad (\text{D.26})$$

$$E_{\phi,j} = \frac{m}{\epsilon_2^2 - (\epsilon_1 - \hat{k}_l^2)^2 \hat{r}} (\hat{k}_l (\epsilon_1 - \hat{k}_l^2) e_j + \epsilon_2) J_m(p_j \hat{r}) + \frac{1}{\epsilon_2^2 - (\epsilon_1 - \hat{k}_l^2)^2} (\epsilon_1 - \hat{k}_l^2 + \hat{k}_l \epsilon_2 e_j) J'_m(p_j \hat{r}), \quad (\text{D.27})$$

$$e_j = \frac{\alpha - p_j^2}{\gamma}, \quad (\text{D.28})$$

Appendix D. Wave propagation into a magnetized plasma

and p_j determined from Eq. 7.38, with its coefficients,

$$p^4 - (\alpha + \beta)p^2 + \alpha\beta - \gamma\delta = 0, \text{ with} \quad (\text{D.29})$$

$$\alpha = \epsilon_1 - \frac{\epsilon_2^2}{\epsilon_1} - \hat{k}_l^2, \quad \beta = \epsilon_3 \left(1 - \frac{\hat{k}_l^2}{\epsilon_1}\right), \quad \gamma = \frac{\epsilon_2 \epsilon_3 \hat{k}_l}{\epsilon_1}, \quad \delta = \frac{\epsilon_2 \hat{k}_l}{\epsilon_1}.$$

The radial components of $\mathbf{E}$, $\mathbf{B}$ are just the following algebraic expressions, Eqs. 7.25-7.26,

$$\hat{E}_r = \frac{\epsilon_2}{\epsilon_1} \hat{E}_\phi + \frac{\hat{k}_l}{\epsilon_1} \hat{B}_\phi - \frac{m}{\hat{r} \epsilon_1} \hat{B}_z, \quad (\text{D.30})$$

$$\hat{B}_r = \frac{m}{\hat{r}} \hat{E}_z - \hat{k}_l \hat{E}_\phi. \quad (\text{D.31})$$

Second, equations for the wave propagation through the vacuum gap until the antenna $r_p < r < r_a$, Eqs. 7.41-7.42,

$$\hat{B}_{z,v} = a_3 I_m(\kappa \hat{r}) + a_4 K_m(\kappa \hat{r}), \quad (\text{D.32})$$

$$\hat{E}_{z,v} = a_5 I_m(\kappa \hat{r}) + a_6 K_m(\kappa \hat{r}), \quad (\text{D.33})$$

$$\hat{B}_{\phi,v} = \frac{1}{\hat{k}_l^2 - 1} \left(\frac{\hat{k}_l m}{\hat{r}} \hat{B}_{z,v} + \hat{E}'_{z,v} \right), \quad (\text{D.34})$$

$$\hat{E}_{\phi,v} = \frac{1}{\hat{k}_l^2 - 1} \left(\frac{\hat{k}_l m}{\hat{r}} \hat{E}_{z,v} + \hat{B}'_{z,v} \right), \quad (\text{D.35})$$

with $\kappa = \hat{k}_l^2 - 1$. $\hat{E}_r$, $\hat{B}_r$ are the same as in Eqs. D.30-D.31. From the antenna to the wall, $r_a < r < r_w$, the four previous equations apply, but replacing unknown coefficients $a_3 - a_6$ by $a_7 - a_{10}$ respectively. The solution for $\mathbf{E}$, $\mathbf{B}$ at each side of the antenna will be distinguished by v^- and v^+ . Third, consider also the forcing terms of the antenna excited currents,

$$\delta B_z = -\mu_0 j_\theta \delta r, \quad (\text{D.36})$$

$$\delta B_\theta = \mu_0 j_z \delta r. \quad (\text{D.37})$$

There are 10 constants to be determined, $a_1 - a_{10}$. Matching conditions at $r = r_p$, provides 4 equations,

$$\hat{E}_{z,p}(\hat{r} = 1) = \hat{E}_{z,v^-}(\hat{r} = 1), \quad (\text{D.38})$$

$$\hat{B}_{z,p}(\hat{r} = 1) = \hat{B}_{z,v^-}(\hat{r} = 1) \quad (\text{D.39})$$

$$\hat{E}_{\phi,p}(\hat{r} = 1) = \hat{E}_{\phi,v^-}(\hat{r} = 1), \quad (\text{D.40})$$

$$\hat{B}_{\phi,p}(\hat{r} = 1) = \hat{B}_{\phi,v^-}(\hat{r} = 1) \quad (\text{D.41})$$

4 equations more are given by matching conditions at $r = r_a$,

$$\hat{E}_{z,v^-}(\hat{r} = r_a/r_p) = \hat{E}_{z,v^+}(\hat{r} = r_a/r_p), \quad (\text{D.42})$$

$$\hat{E}_{\phi,v^-}(\hat{r} = r_a/r_p) = \hat{E}_{\phi,v^+}(\hat{r} = r_a/r_p), \quad (\text{D.43})$$

$$\hat{B}_{z,v^-}(\hat{r} = r_a/r_p) = \hat{B}_{z,v^+}(\hat{r} = r_a/r_p) - \delta \hat{B}_z, \quad (\text{D.44})$$

$$\hat{B}_{\phi,v^-}(\hat{r} = r_a/r_p) = \hat{B}_{\phi,v^+}(\hat{r} = r_a/r_p) - \delta \hat{B}_\phi, \quad (\text{D.45})$$

finally, 2 extra equations belong to the wall boundary conditions at $r = r_w$,

$$\hat{E}_{z,v^+}(\hat{r} = r_w/r_p) = 0, \quad (\text{D.46})$$

$$\hat{E}_{\phi,v^+}(\hat{r} = r_w/r_p) = 0. \quad (\text{D.47})$$

Just, as a redundant information, the dielectric tensor $\bar{\bar{\epsilon}}_r$ of an axially magnetized plasma column (only considering electron dynamics) is,

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}, \text{ with} \quad (\text{D.48})$$

$$\epsilon_1 = 1 - \frac{\omega_{ep}^2(\omega + i\nu_j)}{\omega[(\omega + i\nu_e)^2 - \omega_{ec}^2]}, \quad (\text{D.49})$$

$$\epsilon_2 = + \frac{\omega_{ec}\omega_{ep}^2}{\omega[(\omega + i\nu_e)^2 - \omega_{ec}^2]}, \quad (\text{D.50})$$

$$\epsilon_3 = 1 - \sum_j \frac{\omega_{ep}^2}{\omega(\omega + i\nu_e)^2}. \quad (\text{D.51})$$

D.3 1D model, non-uniform

For the non-uniform scenario, with $n(r)$, $B_0(r)$ non constant within the plasma column, the vacuum shape of the electromagnetic fields are exactly the same as in previous section D.2.

The differences arise only within the plasma, where the fields were numerically integrated as introduced in Chapter 7. Let consider first the expansion of Maxwell equations (7.21 - 7.24) at the axis $\hat{r} \rightarrow 0$, and assume that each component of $\mathbf{E}$, $\mathbf{B}$ is regular there. Then, these components behave as indicated in Eq. 7.43,

$$\hat{E}_\phi = \alpha_1 \hat{r}^{p-1}, \quad \hat{E}_z = \alpha_2 \hat{r}^p, \quad \hat{B}_\phi = \alpha_3 \hat{r}^{p-1}, \quad \hat{B}_z = \alpha_4 \hat{r}^p. \quad (\text{D.52})$$

Looking for non-trivial solutions of $\mathbf{E}$, $\mathbf{B}$, at $\hat{r} \rightarrow 0$ the following condition was found,

$$\begin{vmatrix} m\frac{\epsilon_2}{\epsilon_1} - p & 0 & \frac{\hat{k}_l m}{\epsilon_1} & -\frac{m^2}{\epsilon_1} \\ \hat{k}_l \frac{\epsilon_2}{\epsilon_1} & -p & \frac{\hat{k}_l^2}{\epsilon_1} - 1 & \frac{\hat{k}_l m}{\epsilon_1} \\ \hat{k}_l m & -m^2 & -p & 0 \\ \hat{k}_l^2 + \frac{\epsilon_2^2}{\epsilon_1} - \epsilon_1 & -m\hat{k}_l & \hat{k}_l \frac{\epsilon_2}{\epsilon_1} & -m\frac{\epsilon_2}{\epsilon_1} - p \end{vmatrix} = 0. \quad (\text{D.53})$$

This yields to $(p^2 - m^2)^2 = 0$, and then, Eqs. 7.44 - 7.46 can be easily deduced,

$$m > 0, p = m \begin{cases} m\alpha_2 & = \hat{k}_l \alpha_1 - \alpha_3 \\ m\alpha_4 & = (\epsilon_2 - \epsilon_1) \alpha_1 + \hat{k}_l \alpha_3 \end{cases} \quad (\text{D.54})$$

Appendix D. Wave propagation into a magnetized plasma

$$m < 0, p = -m \begin{cases} m\alpha_2 = \hat{k}_l \alpha_1 + \alpha_3 \\ m\alpha_4 = (\epsilon_2 + \epsilon_1) \alpha_1 + \hat{k}_l \alpha_3 \end{cases} \quad (\text{D.55})$$

$$m = 0 \begin{cases} \alpha_2 = \frac{2}{\epsilon_3} \alpha_3 \\ \alpha_4 = 2\alpha_1 \end{cases} \quad (\text{D.56})$$

Only two constants among $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are undetermined. Thus, two modes per each component of the electromagnetic field $\{\hat{E}_{z,p}, \hat{B}_{z,p}, \hat{E}_{\phi,p}, \hat{B}_{\phi,p}\}$ can be distinguished, the first one $\{\hat{E}_{z1}, \hat{E}_{\phi1}, \hat{B}_{z1}, \hat{B}_{\phi1}\}$ took $\alpha_1 = 1, \alpha_3 = 0$, and the second one $(\hat{E}_{z2}, \hat{E}_{\phi2}, \hat{B}_{z2}, \hat{B}_{\phi2})$ took $\alpha_1 = 0, \alpha_3 = 1$. Each mode is numerically integrated from the axis until the plasma edge. The solution within the plasma agrees the superposition principle due to the linearity of the problem, but scaling each mode by an unknown constant that depends again on the boundary conditions. To meet the same nomenclature as in the uniform case, the solution within the plasma can be written as follows,

$$\hat{E}_{z,p} = a_1 \hat{E}_{z1} + a_2 \hat{E}_{z2}, \quad (\text{D.57})$$

$$\hat{B}_{z,p} = a_1 \hat{B}_{z1} + a_2 \hat{B}_{z2}, \quad (\text{D.58})$$

$$\hat{E}_{\phi,p} = a_1 \hat{E}_{\phi1} + a_2 \hat{E}_{\phi2}, \quad (\text{D.59})$$

$$\hat{B}_{\phi,p} = a_1 \hat{B}_{\phi1} + a_2 \hat{B}_{\phi2}. \quad (\text{D.60})$$

The same matching equations presented above, [D.38](#) - [D.47](#), are still valid in the non-uniform scenario. Note that the role of a_1 and a_2 , and the reconstruction of the fields within the plasma is different in each scenario.

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