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**PARAMETER IDENTIFICATION AND
MODELING OF CONTACT PROPERTIES
FOR ROBOTIC APPLICATIONS**

PHD THESIS

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Abstract

The sense of touch is especially important in task performance in unstructured environments. Many of the tasks we perform normally would be problematic, if not impossible, without sensory information of this type. Daily tasks that we take for granted, such as chewing or walking require the use of tactile information (pressure, exerted force, etc.) to be done. Thus, it is logical to think that in order to build robots capable of performing tasks in unstructured environments, it is necessary to provide them with the same information.

A research area that would benefit from the inclusion of tactile information, and one that is of particular interest to robotics, is biomimetic locomotion. Improving biomimetic locomotion can impact fields such as service robotics, rehabilitation robotics, and search and rescue robots. For this reason, we will focus on biomimetic locomotion, but we will keep in other applications that can use haptic perception.

Animals change their apparent leg stiffness when changing from a rigid surface to a softer one. This change is done because animals and humans maintain the same center of mass trajectory in different surfaces; thus, a change on the apparent stiffness must occur. The realization of this change requires information on the contact properties of the environment. How to extract this information is a question that still needs much research. Because of this, we have done an experimental study that compares the performance of different system identification techniques when they are applied to contact modeling. During the evaluation of the results it was found that the recursive least squares method has the best performance for haptic applications. Based on the results of this study we have selected the recursive least squares method and the spring dashpot model. With this design parameters, we devised an algorithm to be used in a robotic leg.

The algorithm was implemented on a 3-degree-of-freedom (DoF) underactuated leg and then it was tested using four different terrains. The results show that the algorithm was capable of differentiating between terrains according to their stiffness and that the convergence time was under the average time a human runner is in contact with the ground. Nevertheless, the algorithm was not able to differentiate the damping coefficient in granular media. This result is to be expected because granular media are extremely complex and can exhibit highly nonlinear behavior. To solve this problem, we proposed a new contact model based on the conservation of momentum and the similarities of granular media with liquids. The model was tested under different experimental conditions, varying the dynamic and geometric properties of the contact with satisfactory results. However, it was found that changing the geometric properties has an impact in the obtained parameters, because it changes the dynamics of the interaction between the actuator and the surface. A solution for this problem might be to use pressure instead of exerted force as an input to the model

It is important to note that the results of this work have been published in five articles in Journal Citation Report indexed journals and 13 publication in International Scientific Index proceedings. Furthermore, to continue with the research of this thesis a possible line is the reformulation of the proposed model to use pressure instead of force as an input, and to implement the proposed identification algorithm in an exoskeleton or a walking robot to select the appropriate impedance of the legs at each step.

Resumen

El sentido del tacto es de especial importancia en el desarrollo de tareas en ambientes no estructurados. Muchas de las tareas que realizamos normalmente serían problemáticas, por no decir imposibles, sin información sensorial de este tipo. Desde masticar hasta caminar requieren el uso de información táctil (fuerza ejercida, presión, etc.) para poder realizarse. Del mismo modo, resulta lógico pensar que para construir robots capaces de realizar tareas en ambientes no estructurados con similar destreza que sus contrapartes biológicas es necesario dotarlos de la misma información del entorno de la que estos últimos disponen.

Un área de investigación que se beneficiaría de esto, y que es de especial interés para la robótica, es la locomoción biomimética. Mejorando la locomoción biomimética se pueden mejorar otros campos como la robótica de servicio, la robótica de rescate y la robótica de rehabilitación. Por esta razón, en esta tesis nos enfocaremos en la locomoción biomimética pero sin olvidar otras importantes áreas de aplicación como lo es la percepción háptica.

Los animales realizan cambios en la rigidez aparente de sus extremidades al cambiar de una superficie rígida a una suave y viceversa. Esto se debe a que tanto animales como humanos mantienen el mismo movimiento en el centro de masa sin importar la rigidez de la superficie, y en consecuencia mantienen una rigidez equivalente para todas las superficies. La realización de este cambio requiere información de las propiedades de contacto de dichas superficies. El cómo extraer esta información es una pregunta que aún requiere mucha investigación. Por esta razón, en esta Tesis Doctoral se ha realizado un estudio experimental entre distintas técnicas de identificación de sistemas aplicadas al modelado de las propiedades de contacto. En la evaluación de las técnicas se ha encontrado que los mínimos cuadrados recursivos son los que presentan mejores prestaciones para su utilización en aplicaciones hápticas. Como consecuencia de este resultado, se ha desarrollado e implementado un algoritmo para la identificación de las propiedades de contacto en tiempo real con robots caminantes.

El algoritmo implementado es capaz de diferenciar entre distintos tipos de terreno; presenta un tiempo de convergencia acorde al tiempo promedio de contacto con el suelo observado en humanos; sin embargo, no es capaz de diferenciar con precisión el coeficiente de amortiguación entre dos medios granulares. Este resultado se debe a que los medios granulares son extremadamente complejos y exhiben comportamientos altamente no lineales. Para resolver este problema, se planteó un modelo de contacto basado en la conservación del momento en las colisiones y en las similitudes de los medios granulares con los líquidos. Dicho modelo se ha probado variando distintas condiciones de experimentación como las propiedades dinámicas y geométricas del robot usado para los experimentos con resultados satisfactorios. Aunque se ha detectado que los cambios geométricos influyen en

las propiedades estimadas por el modelo, estos cambios también afectan la dinámica de la interacción entre actuador y el entorno. Se ha determinado que la estrategia para evitar esto y mejorar la capacidad de diferenciación entre terrenos es utilizar la presión en lugar de la fuerza como entrada al sistema.

Este trabajo de investigación ha dado lugar a cinco artículos en revistas indexadas en el Journal Citation Report y 13 publicaciones en conferencias internacionales International Scientific Index. Por otra parte, como continuación de esta investigación se sugiere la reformulación del modelo utilizando como entrada la presión en vez de fuerza y la implementación del algoritmo propuesto en exoesqueletos de rodilla y en robots caminantes para seleccionar la impedancia en la pisada.

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Chapter 1

Introduction

Almost all animals, including humans, interact with and discover the environment through the sense of touch. Tactile information is fundamental to many of our daily tasks, even though it is taken for granted. However, tasks such as walking, chewing, and grabbing objects can be very problematic without the sense of touch [Gallace and Spence, 2014]. Performing tasks in unstructured or completely unknown environments is a relatively easy thing to do for a human or an animal. For example, when manipulating an egg, if the applied force is too little, the egg slips. Contrarily, if the force applied is too great, the egg will break. A precise force is applied and constant feedback of the measured applied forces keeps the egg intact [Tiwana et al., 2012].

There are many applications and scenarios where a robot is not likely to possess contextual information of objects a priori, or a completely detailed map of where obstacles are. In an attempt to solve this issue, there has been an intensive activity on exteroceptive sensing being used in robotics, such as those based on cameras, microphones, radars, lasers, etc. However, the sense of touch is key for a wide range of applications. Given that tactile information is such an important part of our sensory input it is logical to try to include this feedback in robotic applications as well. There has been an increasing amount of work on the subject in the past decade, specially in the robotic manipulation and teleoperation fields [Dahiya et al., 2010, Ferre, 2008]. However, contact modeling and characterization has not received as much attention as the development of new sensors, even though this area is crucial to many applications, such as service, exploratory and surgical robots.

1.1 Applications

As stated above, there are many robotics applications that can benefit from haptic information. These applications can be classified in several ways; however, for our purposes we will separate them into interactive and autonomous applications:

1.1.1 Interactive Robotics Applications

We will call interactive all those robotics applications which main objective is human-machine interaction.

- **Medical robotics** An important application of haptics in the field of medicine are haptic simulators that are used and studied as training tools [Coles et al., 2011]. The idea behind this field of research is to eliminate the need for medical personnel to train while performing real surgeries; thus, reducing the risk for patients which is a key factor in the medical field. In Fig. 1.1 some of the medical training tools that are in use today are shown. The main challenge in this particular application is the fidelity in which tactile sensations need to be reproduced to provide effective training.

Another important example of haptic perception in medical robotics is the application to minimally invasive surgery [Westebring-Van Der Putten et al., 2008]. Minimally invasive surgery is traditionally performed via long instruments that can distort the natural haptic feedback a surgeon has when he uses his hands; the use of robots with haptic feedback aims at reducing this distortion and enhance eye-hand coordination. Some examples of surgical robots are the daVinci system (Intuitive Surgical Inc., Mountain View, CA) and the Zeus system (Computer Motion Inc., Santa Barbara, CA) (see Fig. 1.2).

- **Gaming** There are some factors that contribute to the gaming experience that keep a player coming back for more [Varalakshmi et al., 2012]: compelling storyline, challenging scenarios, etc. Tactile feedback can be crucial to reveal information that cannot be seen. Moreover, it enhances the sense of user immersion [Hayward et al., 2004]. Another very interesting research prospect is the case of dexterity games; earlier this type of games were made in fixed form; i.e. they could only be solved in very specific ways. Now, with haptic elements they can be made infinitely programmable. An application that is already in commercial stage is flight and driving simulation [Hayward et al., 2004].
- **Tele-manipulation** This application consists of using a haptic interface to move an object with a robot in a remote location. It can be useful in cases where it is necessary to move objects in hazardous environments (e.g. a nuclear reactor, outer space, underwater, etc.). To introduce haptic feedback in this application the concept of passivity (see glossary of terms in appendix A) is key [Hayward et al., 2004]. For that reason, the use of Port-Hamiltonian system theory is very useful because it allows the modeling of physical systems in a very general way and has proven useful in many studies [Secchi et al., 2007, Stramigioli et al., 2002, Secchi et al., 2006]. Note that surgical robots are a special case of tele-manipulation with augmented safety issues and subject to considerably more regulation; therefore all the challenges in surgical robots apply to tele-manipulation.

1.1.2 Autonomous applications

We call autonomous all the applications which main objective is not to reproduce or translate the sense of touch to a human but to use that information to better achieve their objective.

Figure 1.1: Robotic medical training tools: (a) SimQuest's burr hole drilling simulation hardware (Two 3DOFF Falcon devices arranged to give 5DOFF feedback to a single drill handle) [Coles et al., 2011]. (b) Modified Novint Falcon force feedback device for a pulse palpation simulation (Istituto Italiano di Tecnologia and Bangor University) [Coles et al., 2011]. (c) BIGNePSi, Bangor University's ultrasound-guided needle puncture simulator using two Omni force feedback devices [Coles et al., 2011]. (d) A haptic suturing simulator with modified PHANTOM Premium 1.5 end effector, mounted upside down to increased range of motion for task [Coles et al., 2011].

Figure 1.2: Surgical robots: Da Vinci system (left). Zeus system (right).

- Rehabilitation Robotics** This application consists on using a robot to make a patient follow specific patterns of motion to help him regain strength or heal injuries. This example that might be confusing as it clearly involves interaction with humans. However, from a haptic perception point of view, the objective of this kind of robot is not to reproduce a sensation, but to determine how much force a human is exerting to help him establish a normal pattern. In other words, in a rehabilitation application it is more about making the patient follow a specific pattern of motion than providing reliable force feedback. To do this, the robot needs tactile information such as muscle tone, exerted force (to and by the robot) to adjust its actuators to exert the required torque to help patients perform a task. In this moment, there are several robotic applications and studies that try to exploit virtual reality and haptic feedback in therapy [Holden, 2005, Hesse et al., 2003]. Fig. 1.3 shows two examples of rehabilitation robotics that use haptic feedback to train motor skills on the patient; by gradually reducing the support the robot gives to the patient.

Figure 1.3: Rehabilitation robots: MIT-Manus (left). Lokomat system (right).

- Assembly and cooperative robotics**

This application consists of assembling a product such as a car or a piece of furniture. This can usually needs to be done by multiple robots because of the shapes of the pieces. The case of assembly and specifically cooperating assembly is a very difficult task full of uncertainties and very unstructured. To this date, it is not fully automated for that reason. However, new research is emerging to overcome the issues presented in this application using tactile information, although currently more focused in object recognition, it can lead to great improvements in the industrial world [Suwanratchatamane et al., 2010, Liu et al., 2014, Ilonen et al., 2013]. One interesting development in this area is the use of Scaling Series to solve the full global six DoF localization problem using only the sense of touch [Petrovskaya and Khatib, 2011].

- Locomotion**

Walking might not seem to be a task that requires much tactile information, and that statement is somewhat true if the gait pattern is statically stable; as long as the center

of mass is located inside the support polygon there is no risk of falling. However, in the case of dynamic locomotion, reaction forces can become cause of destabilization and a correcting action needs to be taken to avoid falling. Moreover, as we said before with a statically stable gait the lack of need of force feedback is only somewhat accurate; tactile information is very helpful in the assessment of the quality of a selected foothold: Using haptics to evaluate the local surface properties before the weight of the robot is shifted onto the corresponding foot will help deciding whether a new and different placement of this foot is indicated, and hence improve the overall stability of locomotion in rough and highly unstructured terrain [Hoepflinger et al., 2010]. Secondly, the obtained haptic information can be included in a geometric map of the environment where it can be combined with data from other sensors.

Additionally, as it will be discussed in detail further below, haptic information can help reduce energy consumption, adapt the step to the terrain and increasing overall adaptability to different terrains.

1.2 Motivation for this Thesis

Better contact models and identification methods can improve the quality of the information that a robot receives from the environment. From texture differentiation to obstacle avoidance, specially in environments where visual feedback is not possible. It also can improve the feedback given to an operator, e.g. a surgeon, who is performing delicate operations remotely using a robotic device. Moreover, high quality tactile feedback is a must when a robot is in unstructured environments, specially if humans are present, for safety reasons.

In order to make a robot safely interact with its environment, the robot's mechanical behavior should be selected according to the environment with which it will be in contact [Arevalo and Garcia, 2012]. One topic in which this is of particular importance is biomimetic locomotion. Improvements in this area can help in developing better search and rescue robots and improving gait assistance for the disabled to name a few applications. Thus, in this thesis we will focus on this topic.

1.2.1 Biomimetic Locomotion

In the literature, it is generally accepted that a Spring-Loaded Inverted Pendulum (SLIP) approach [Alexander, 1988] models the basic dynamics of running remarkably well 1.4. Although the SLIP approach models the system merely as a point mass on a simple Hookean spring, it explains the mechanics of running of several animals, including humans [Alexander, 1988, Cavagna et al., 1988, Blickhan, 1989, Blickhan and Full, 1993, Farley et al., 1993, McMahon, 1985, Ferris et al., 1998]. The most important feature of this model is that with only 3 parameters (stiffness, mass and length) it can model a wide variety of gaits. In fact, in [Blickhan and Full, 1993] it is stated that over a hundred and fifty species of animals (with different leg number, skeletal type and body form) follow the same function of the minimal cost of energy. Although many of them are capable of performing different

gait patterns, all of these patterns can be modeled using a virtual monopod (SLIP); Figure 1.4 illustrates the basic behavior of the SLIP model.

Figure 1.4: (a) The bouncing monopod as model. (b) Ground-reaction force observed for running, trotting and hopping animals. This figure was taken from [Blickhan and Full, 1993]

To adjust this model to different animals, the mass and the length need to be changed; while for various gait patterns, it suffices to adjust the spring (leg) stiffness in the model and for the same mass and leg length. This effect is illustrated in [Blickhan and Full, 1993] with Fig. 1.5.

An interesting fact is that it was found that the relative compression of the SLIP was gait dependent, but did not depend on size. Thus, the SLIP represents a reasonable approximation of the dynamics used by many legged animals for terrestrial locomotion and it can predict the gait dependent differences from initial conditions. The gait difference for runners, trotters and hoppers is illustrated in Fig. 1.6.

Furthermore, the SLIP has been used to model the dynamic behavior of running robots [Raibert, 1990, Full and Koditschek, 1999, Altendorfer et al., 2001]. There are many works that use a bouncing gait as a starting point to design efficient runners from Raibert's robots that balance [Raibert, 1986] to the MIT Cheetah [Seok et al., 2013]. However, what is even more interesting is that a hexapod robot like Rhex [Altendorfer et al., 2001] which design has little (visual) resemblance to a cockroach exhibits a bouncing gait just like a monopod [Full and Koditschek, 1999] (see Fig. 1.7). Furthermore, Srinivasan and Ruina [Srinivasan, 2006] performed a numerical study using an inverted pendulum with a telescopic leg which found that the minimum energy expenditure gait for rapid locomotion followed the mechanics of a SLIP.

This suggests that the same conclusions derived from biomechanics studies can be applied to robots as well. In particular, a very important fact to energy economy and adaptability is: Animals make stiffness changes to accommodate different surfaces. One reason added to energy economy is that if the same leg stiffness is used on every surface, the body can be put out of balance whenever an abrupt change in the surface stiffness occurs; however, adjusting the stiffness offers a smooth transition between surfaces [Ferris et al.,

Figure 1.5: Path of the planar monopod and time course of the ground reaction force during one complete step. (A) Slow run with low vertical landing. (B) Running on a soft spring. (C) Fast hop. (D) A low angle of attack. This figure was taken from [Blickhan and Full, 1993]

Figure 1.6: Motion of the center of mass and time course of total energy of the center of mass for a trotting, running and hopping monopode gait. This figure was taken from [Blickhan and Full, 1993]

1998]. In Fig. 1.8 it can be seen how different runners adjust their apparent leg stiffness to accommodate different terrains and how each runner maintained the same vertical displacement of their center of mass no matter how much displacement the surface had. A second reason is that adjusting the leg stiffness to accommodate different terrains can lead to a more efficient running because increasing leg stiffness as terrain stiffness decreases leads to less mechanical work done by the leg and more mechanical work done by the terrain [Ferris et al., 1998]. In other words, by adjusting the leg stiffness to accommodate the terrain adaptability, energy efficiency can be enhanced.

1.2.2 Challenges in Biomimetic Locomotion

To extrapolate the knowledge from the biomechanics studies to legged robots, one important consideration is how to know when the stiffness adjustment must be performed. Animals, at least birds [Clark, 1988] and humans [Ferris et al., 1998], change their leg stiffness before stepping on the new surface when they are knowingly changing surfaces

Figure 1.7: Comparison between a SLIP and Rhex data (top). Rhex robot (bottom). This figure was taken from [Altendorfer et al., 2001]

(see Fig. 1.8). Nonetheless, the question remains as to how they adjust leg stiffness the first time when the surface is unknown. This question leads to the idea that some sort of information on the ground stiffness must be acquired directly from the surface (at least) the first time they step on it while running.

Furthermore, when a robot is in contact with its environment, the robot's mechanical behavior should be selected according to the environment with which it will be in contact [Arevalo and Garcia, 2012]. Additionally, to be able to predict the reaction forces and to gain adaptability to different contact surfaces, it is necessary either to have previous knowledge of the properties of the surfaces or to be able to identify those surfaces' parameters. Figure 1.9 shows a flow chart of a generic robotic application where the robot is in contact with its environment. Note that in the chart the process is iterative because the environment can change, thus it is necessary to identify those changes and adapt the model to select the appropriate interaction control strategy.

It is important to note that extracting this information is a real challenge because one of the assumptions made in the system identification literature is that the system is well excited [Ljung, 1999]. This statement means that the data gained from the outputs of

Figure 1.8: Effective vertical stiffness and leg stiffness versus surface stiffness (left). Surface displacement and total vertical displacement versus surface stiffness (right). Figure taken from [Ferris et al., 1998]

Figure 1.9: Flow chart of a generic application of a robot in contact with the environment

the system need to be sufficient to describe the properties in the close neighborhood of the measured frequency band. Furthermore, in the case of an unknown system, it is necessary to excite all of the frequency bands of interest, and this situation calls for a specially designed excitation signal [Simon and Schoukens, 2000]. This possibility may not be feasible in some applications (e.g. walking robots or tele-operated robots) because the robot may not be able to deliver arbitrary signals, such as a sine wave or a chirp signal. Moreover, the environment can change substantially from one task to the other which calls for the use of adaptive algorithms to be able to detect changes in the terrain. These scenarios suggest that an online approach should be used; because it can extract the needed information without the need of stopping the current task.

Another important issue to consider is to what extent it is necessary to know the environment properties. There are many models that can be used to describe certain properties of the contact force [Giraldi and Dharm, 2002, Marhefka and Orin, 1999, Hunt and Crossley, 1975, Kim, 1999] of environment. Nevertheless, from a system identification point of view the most important feature of the models is that they can be either linear or nonlinear. This characteristic is important because the majority of the methods used for system identification assume a linear model of the system; however, in the case of contact, a linear model can leave out some important characteristics of the phenomenon, such as plastic deformation [Goldsmith, 1960]. Nevertheless, linear models are very appealing for their simplicity and because there are many algorithms that show good performance when identifying the parameters of such models [Gustafsson and Gustafsson, 2000]. A more detailed description of these issues will be given in the next chapter.

Despite the fact that the work on biomimetic locomotion is increasing, we have identified three specific problems that need to be addressed:

- There is no meaningful comparison between existing system identification methods to be applied on-line to the modeling of contact forces
- To the best of our knowledge, there is no algorithm capable of differentiating between contact surfaces to adapt the stiffness of the leg.
- There is a lack of suitable models to be used in highly dissipative terrains such as granular media

1.3 Thesis Objectives

With the aforementioned problems in mind we have set the following objectives for this PhD Thesis:

- Evaluate the existing identification techniques to extract contact properties information from position and force measurements.
- Propose an algorithm to extract contact properties of the terrain online using the existing framework.

- Implement and evaluate the performance of the algorithm and identify possible improvements
- Propose a new model and identification method to improve the existing models and techniques.

1.4 Thesis Outline

This thesis is organized as follows:

Chapter 2 presents a literature review of the topics involved in this thesis. These works come from independent disciplines such as contact modeling and system identification. In this chapter we provide sufficient information to make this thesis self contained. However, given the extensive literature existent in both contact modeling and system identification a complete treatment of the topics is impossible; thus, we refer to the appropriate literature to further expand on the specific areas.

Chapter 3 describes a model of a robot in contact with its environment and describes the approaches used in the literature of haptic perception to extract the contact properties. These approaches are divided in passive and active sensing, although a combination of the two can be used.

Chapter 4 presents an experimental investigation which includes a meaningful comparison between several system identification techniques applied to contact modeling. Even though, there are many authors that work on both disciplines, a study such as this does not exist in the present literature and is part of the novel contributions of this thesis.

Chapter 5 introduces a general algorithm to extract the contact properties of the ground for a haptic application with special focus on legged locomotion. The algorithm takes into consideration robot structure and different ground conditions. Furthermore, the algorithm is implemented on a 3 DOF underactuated robotic leg and its performance is evaluated.

Chapter 6 introduces a linear contact model capable of capturing some of the most important properties of highly dissipative and nonlinear granular materials, such as energy dissipation and plastic deformation and gives an example of how to integrate the contact model in the model of a robot. Meanwhile Chapter 7 gives an experimental validation of the proposed model using two different granular materials and variations of the actuator dynamic and geometric properties.

Finally, in Chapter 8 the conclusions are drawn, the contributions are summarized and a few lines of future work are proposed.

Chapter 2

State of the Art

In the previous chapter we have discussed the importance of haptic perception for animals and robots when they are performing tasks in unstructured environments, giving special attention to legged locomotion as the motivation for this thesis. In addition, we have identified three key problems that require special attention. In this chapter we will explore the concepts related to the identification of the contact properties of the environment because they are the basis for the development of the solution to the aforementioned problems. These concepts come from a variety of disciplines ranging from optimization to rigid body dynamics. The goal of this chapter is to provide the reader with enough knowledge to tackle the identification of contact properties. To do this, we first review the literature on contact models; then we move towards system identification and goodness-of-fit evaluation.

However, before going into the details of the mathematical formulation let us present the nomenclature that will be used throughout the thesis.

2.1 Nomenclature

In this thesis we will make use of many symbols that are conventionally used to denote some quantities. Table 2.1 presents the symbols of functions and constants that are used throughout the thesis.

Table 2.1: Symbols that are used to denote specific quantities in the literature

Symbol	Definition
x	Displacement in the direction of movement
F	Force; sub-indexes are used to specify different forces
J	General cost function
u	Input of a system; in general $u \in \mathbb{R}$
y	Output of a system; in general $y \in \mathbb{R}$
e	Modeling error
θ	Vector of parameters used to predict the output of a system from the inputs

φ	Regressor. Vector of the organized inputs and outputs of the system to form a linear regression
a	Numeric constant; in general $a \in \mathbb{R}$
b	Numeric constant; in general $b \in \mathbb{R}$
t	Time; in general $t \in \mathbb{R} \geq 0$
η	Noise; in general $\eta \in \mathbb{R}$
n	Numeric constant; in general $n \in \mathbb{N}$
m	Numeric constant; in general $m \in \mathbb{N}$
λ	Forgetting factor; a design constant that satisfies $0 \leq \lambda \leq 1$
R	Correlation matrix; by definition $R \geq 0$
q	Vector containing the positions of the joints of a robot
v	Vector containing the velocities of the end effector of a robot
τ	Vector containing the joint torques of a robot
s	General contact surface
δ	General deformation along the vector of a surface s
$\mathbf{J}$	Jacobian matrix
r	Vector containing the residuals of an identification; i.e. the errors at each time instant
K	Elastic constant of a Hookean spring
B	Damping coefficient of a mechanical element
h	Sampling period of a signal
Φ	Packing fraction
V	Volume
N	Arbitrary natural number
f	Arbitrary function
d	Euclidean distance between two points

In addition, we will use some symbols that are commonly used to denote some mathematical expressions. These symbols are shown in Table. 2.2.

Table 2.2: Symbols that denote mathematical operations

Operator	Definition
$\dot{f}$	Denotes time derivative of a f
$\ddot{f}$	Denotes the second time derivative of f
G^T	Denotes the transpose of G
f^*	Denotes the minimum value of f according to some cost function

$\hat{f}$	Denotes the estimated value of f
$\bar{f}$	Denotes the mean value of f
$\ f\ $	Denotes the norm of f
$E(f g)$	Denotes the expected of f given g
$Var(f g)$	Denotes the variance of f given g
$\ln(f)$	Denotes the natural logarithm of f
∇f	Denotes the gradient of f

To complete the terms and symbols given in the tables above, a glossary of terms is given in appendix A.

2.2 Contact Models

A robot that is required to interact with its environment can be modeled as a robot with an unequally constrained motion [Featherstone, 2008], because this interaction often involves both contact and free space motion. This means that, when the distance d between the robot's end-effector and a contact surface is greater than zero the force F is equal to zero (see Fig. 2.1); and when the distance d is zero then $F > 0$.

Figure 2.1: Unequally constrained robot. Free motion (left). Contact (right)

Before going any further it is important to differentiate between contact and impact, as these two phenomena usually have very different treatments in the literature. Impact is a complex phenomenon that occurs when two or more bodies undergo a collision [Giraldi and Dharm, 2002] and it is characterized by very short duration, rapid energy dissipation, large accelerations (positive and negative) and very high force levels. On the other hand,

contact is a term used to describe situations where two or more bodies come in touch with each other and it implies a continuous process that takes place over a finite time [Giraldi and Dharm, 2002]. Note that contact may include an impact as the beginning of the process, but an impact is always described as a very short event. For this reason, we review the description of impact along with contact, but we emphasize models that can include both.

In the related literature, there are basically two types of approaches to describe impact. The first type is referred as impulse-momentum methods, and the second as force based methods [Kim, 1999]. The first class of methods cannot be easily applied to robot interaction tasks, although recent studies have successfully used this approach in simulation through the solution of complementarity problems [Todorov, 2010, Stewart and Trinkle, 2000, Todorov et al., 2012]. The impulse-momentum problem for two rigid bodies is described by the following set of equations:

$$m_1(\mathbf{v}_1 - \mathbf{v}_1 |_{t_0}) = \mathbf{I} \quad (2.1)$$

$$m_2(\mathbf{v}_2 - \mathbf{v}_2 |_{t_0}) = -\mathbf{I} \quad (2.2)$$

$$\mathbf{L}_1 - \mathbf{L}_1 |_{t_0} = \mathbf{d}_1 \times \mathbf{I} + \mathbf{A} \quad (2.3)$$

$$\mathbf{L}_2 - \mathbf{L}_2 |_{t_0} = \mathbf{d}_2 \times \mathbf{I} - \mathbf{A} \quad (2.4)$$

where the sub-indexes indicate the body 1 and 2; m is the mass; $\mathbf{v}$ is the velocity; $\mathbf{I}$ is the impulse; $\mathbf{L}$ is the angular momentum; $\mathbf{d}$ is the distance from the center of mass to the contact point; and $\mathbf{A}$ is the angular impulse.

One of the problems to use this formulation is that it requires additional relations and constraints to be imposed in order to solve the problem: e.g. zero velocity in the tangential direction. Another problem, and maybe more important, is that the formulation is based on the perfect rigid body assumption, which does not hold in some cases, especially in high-velocity impacts where deformation is a common phenomenon. Additionally, when the number of bodies is very high, such as with granular media, the complexity of the problem can be difficult to manage. Finally, when using impulse-momentum methods, friction can only be taken into consideration using Coulomb's friction, because when analyzing impact with impulse-momentum methods the velocity can only be evaluated before and after the impact. On the other hand, if for any application (such as material identification) there is the need for a more complete friction model that describes friction as a continuous function, it cannot be computed with impulse-momentum methods. For those reasons, we are interested in force based methods which generally describe the contact force (which includes impact force as well) as:

$$\mathbf{F}_c = \mathbf{F}_v(\dot{x}) + \mathbf{F}_d(x) \quad (2.5)$$

where $\mathbf{F}_c$ is a function of the local deformation, which is modeled as a displacement x . $\mathbf{F}_v$ and $\mathbf{F}_d$ are the reaction forces proportional to the indentation and the velocity of indentation respectively. Figure 2.2 shows a general depiction of the force based methods. Within this class of methods, there are three main models in the literature:

Figure 2.2: General force based methods

- Spring-dashpot model [Goldsmith, 1960]

$$F_c = b\dot{x} + kx \quad (2.6)$$

In this model, the contact force is modeled as a linear spring and a linear damper connected in parallel, in order to ensure elastic behavior and energy dissipation [Goldsmith, 1960]. Fig. 2.3 shows a diagram of the model.

Figure 2.3: Depiction of the spring-dashpot model

- Hertz's model [Abrate, 1998]

$$F_c = F_{c,max} \left(\frac{x - x_p}{x_{max} - x_p} \right) \quad (2.7)$$

where $F_{c,max}$ and x_{max} are the maximum normal force and indentation respectively, x is the local indentation and x_p is the permanent indentation. It is important to note that although it is referred as Hertz's model, this model is an adaptation of Hertz's original model to include permanent deformation, which was not considered by Hertz. Figure 2.4 shows a diagram of Hertz's model.

- Non-linear damping model [Hunt and Crossley, 1975, Hertz et al., 1896]

$$F_c = bx^p \dot{x}^q + kx^n \quad (2.8)$$

Note that this model is very similar to the spring-dashpot model, with the particularity that the speed depends non linearly on the speed and the position.

Figure 2.4: Depiction of the Hertz model

Throughout this work; the velocity, position and force vectors are considered positive in the direction of the penetration of the robot into the surface.

For the moment, we will focus on the spring-dashpot model because it models impact forces sufficiently well (at least on linear surfaces) [Giraldi and Dharf, 2002], but in the following chapters a more complete model will be introduced. Nonetheless, for a detailed explanation on the other models we refer to the works of Giraldi [Giraldi and Dharf, 2002], Stewart [Stewart, 2000], and Goldsmith [Goldsmith, 1960].

The spring-dashpot model as described by equation (2.6) has its shortcomings [Marhefka and Orin, 1999]. The biggest is that the force can be discontinuous because the model is linear in the velocity and when the velocity is greater than zero at the instant of impact, the damping term will produce a discontinuous change in the force. Another important weakness of this model is that as the objects separate, the relative velocity between them tends to be negative; therefore there is a negative force holding the objects together, which is not the case when the surfaces in contact have no adhesion. Finally, the coefficient of restitution (a quantity that expresses the energy loss caused by the impact), in this model, is independent from the impact velocity which was contradicted by experimental foundation [Goldsmith, 1960]. To better illustrate this, consider the coefficient of restitution defined as [Nagurka and Huang, 2004]:

$$C_r = \frac{\|\dot{x}|_{tf}\|}{\|\dot{x}|_{t0}\|} \quad (2.9)$$

where C_r is the restitution coefficient; tf and $t0$ are the final and initial instants of a collision. If this equation is solved for the spring-dashpot model it results in [Nagurka and Huang, 2004, Schwager and Pöschel, 2007]:

$$C_r = \exp \left(-\frac{\pi}{\sqrt{\frac{2Km}{B^2} - 1}} \right) \quad (2.10)$$

Nevertheless, this model provides a reasonably good approximation of the energy dissipation associated with the normal forces without explicitly considering plastic deformation issues [Giraldi and Dharm, 2002]. Moreover, many authors have successfully implemented this model for manipulation tasks [Vukobratovic and Pottonjak, 1999, Howard and Kumar, 1993, Mirza et al., 1993] and for legged locomotion [Waldron et al., 2008].

Once we have a description of the contact force, let us look at how to estimate the parameters of the model that describe it.

2.3 System Identification

System identification is one of the many applications of mathematical optimization, a mature mathematical discipline used in most areas of scientific research [Todorov, 2006] to solve minimization problems; from planetary landing [Cogley, 2002] to economics [Sibley et al., 2010]. The problem of system identification can be defined as the following minimization problem:

$$\min \sum_0^T J(e, \theta) \quad (2.11)$$

where J is a cost function of the modeling error defined as [Gustafsson and Gustafsson, 2000]:

$$e = y - \hat{y}(\theta, u) \quad (2.12)$$

where y is the measured output of the system, $\hat{y}$ is the estimated value of the output which is a function of the input (u) of the system and a parameter vector θ . The goal of system identification is to obtain the true model of the system [Gustafsson and Gustafsson, 2000]. Nevertheless, it is always good to keep in mind that every model is unrealistic [Duindam et al., 2009], which implies that the model should always be expected to be sub-optimal or unexplained phenomena.

To solve this optimization problem, first it is necessary to find a function that can estimate the output of the system in terms of the input. There are multiple methods to do this, however, in general, most of the system identification methods use linear regression [Weisberg, 2005].

Within this family of models, one of the most popular is the Auto-Regressive Moving Average (ARMA) and it has many variants, e.g. Auto-Regressive Moving-Average Model with Exogenous Inputs (ARMAX) and Auto-Regressive-Integrated-Moving-Average model

(ARIMA) [Ljung, 1999, Box et al., 1976]. The general ARMA model for a system is given by:

$$b_0 u_t + b_1 u_{t-1} + \dots + b_m u_{t-m} = y_t + a_1 y_{t-1} + \dots + a_n y_{t-n} \quad (2.13)$$

where u and y are the input and the output of the system, respectively, and b_i and a_j are the parameters of the model, with $i = 1, 2, \dots, m$ and $j = 0, 1, 2, \dots, n$. Therefore, the system can be represented as:

$$y_t = \varphi^T \theta \quad (2.14)$$

where

$$\begin{aligned} \varphi^T &= [-y_{t-1}, -y_{t-2}, \dots, -y_{t-n}, u_t, \dots, u_{t-m}] \\ \theta &= [a_1, a_2, \dots, a_n, b_0, \dots, b_m]^T \end{aligned}$$

Once a process is described as a linear regression, all that remains to solve the identification problem is to define a cost function [Betts, 2010]. In order to do so, it is necessary to define a modeling error in terms of the regression; thus, rewriting equation (2.12) the modeling error is given by:

$$e_t = y_t - \varphi^T \theta. \quad (2.13)$$

A particularly popular loss function is e_t^2 , which has given its name to a family of methods called the Least Squares (LS) methods. There are many parameter identification methods within this family, and some of these methods are intended to be used off-line, such as the Ordinary Least Squares (OLS) method and the Stochastic Least Squares (SLS) method [Gustafsson and Gustafsson, 2000]. As an alternative to the aforementioned methods, there are also some adaptive approaches based on LS methods that can be used to estimate the parameters in a model. In the next sections we review the most common methods in the literature, classifying them into off-line and on-line methods. Off-line methods are those which compute the parameters using the output of a complete data sample; as opposed to on-line approaches which compute the parameters while the samples are being acquired from the environment.

2.3.1 Off-line Methods

Off-line methods use a set of system input and output measurements to compute the parameters of the model. Typically the data set should be large and the system needs to excite all the frequencies of interest, that is the input signal has to have a frequency range that can produce all the behaviors of interest in the system (e.g. in a seismic test, the input signal needs to resemble as closely as possible a real earthquake).

We will review two main methods of this type: Ordinary Least Squares and Windowed Least Squares.

Ordinary Least Squares

The Least Squares method can be written as follows:

$$\| J_N \| = \frac{1}{N} \sum_{t=1}^N (y_t - \varphi^T \theta)^2 \quad (2.14)$$

where the parameters are defined in table 2.1. It is important to note that equation (2.14) has a unique feature: it can be solved analytically, if and only if the matrix inverse of the inside of the sum exists. This feature is a consequence of the fact that the parameterization is linear and that the cost function is quadratic on the parameter vector. The solution of equation (2.14) yields the LS estimate:

$$\hat{\theta}_N = \operatorname{argmin} \| J_N \| = \left[\frac{1}{N} \sum_{t=1}^N \varphi_t \varphi_t^T \right]^{-1} \frac{1}{N} \sum_{t=1}^N \varphi_t y_t \quad (2.15)$$

Note that in the absence of noise and assuming that the parameterization describes the system dynamics perfectly, the LS estimation converges to the true model of the system. However, to investigate the convergence properties we may write:

$$Q(N) = \frac{1}{N} \sum_{t=1}^N \varphi_t \varphi_t^T$$

and let:

$$y_t^* = \varphi_t^T \theta_0 + \eta_t \quad (2.15)$$

where η_t is a term that represents the noise and the modeling error in the system and θ_0 is the true model of the system. If we solve equation (2.14) using y_t^* we have:

$$\hat{\theta}_N = \theta_0 + Q_N^{-1} \frac{1}{N} \sum_{t=1}^N \varphi_t \eta_t \quad (2.16)$$

thus, as $N \rightarrow \infty$ then $\hat{\theta}_N$ converges to θ_0 which is the true model of the system.

Weighted Least Squares

When there are measurements with different degrees of reliability, one strategy is to assign different weights to each measurement. This weight can be for different degrees of noise or because there are elements of the system that are not as representative as others. Let the vector β be the vector of weights, then (2.14) can be rewritten as:

$$\|J_N\| = \frac{1}{N} \sum_{t=1}^N \beta (y_t - \varphi^T \theta)^2 \quad (2.17)$$

then, solving the minimization problem the resulting estimate is:

$$\hat{\theta}_N = \operatorname{argmin} \|J_N\| = \left[\frac{1}{N} \sum_{t=1}^N \beta \varphi_t \varphi_t^T \right]^{-1} \frac{1}{N} \sum_{t=1}^N \beta \varphi_t y_t \quad (2.18)$$

A convergence analysis analogous to the case of the Ordinary Least Squares case reveals that it has the same convergence properties, with the exception that it is multiplied by the weight vector.

It is important to note that these off-line approaches require the system to be well excited; therefore, the matrices used are very large and they require special excitation signals to be acquired, thus they are not adequate for their use on-line. Additionally, these methods lack adaptability in the sense that they compute only one set of parameters to fit the data.

2.3.2 On-line Methods

The limitations of the off-line methods for some applications have lead to the development of on-line methods based on adaptive filtering [Ljung, 1999, Box et al., 1976, Gustafsson and Gustafsson, 2000].

Recursive Least Squares (RLS)

The RLS algorithm minimizes the loss function [Ljung, 1999, Gustafsson and Gustafsson, 2000]:

$$f(\theta) = \sum_{k=1}^t \lambda^{k-t} (y_k - \hat{y}_k)^2 \quad (2.19)$$

where λ is a design parameter called the forgetting factor (see Glossary for a definition of terms). From the equation, it can be seen that λ is multiplied by the error and gives a weight with a decreasing exponent at each time instant. Then, since $0 < \lambda \leq 1$ it gives exponentially less weight to older error samples. To minimize this function, there are two main variants: one variant uses the covariance matrix R of the measurements, and the other variant computes its inverse using the matrix inversion lemma [Ljung, 1999]. The algorithm is summarized as follows:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + L_k (y_k - \varphi_k^T \hat{\theta}_{k-1}) \quad (2.20)$$

$$L_k = \frac{P_{k-1} \varphi_k}{\lambda + \varphi_k^T P_{k-1} \varphi_k} \quad (2.21)$$

$$P_k = \frac{1}{\lambda} \left(P_{k-1} - \frac{P_{k-1} \varphi_k \varphi_k^T P_{k-1}}{\lambda + \varphi_k^T P_{k-1} \varphi_k} \right) \quad (2.22)$$

Note that as this method is a Least Square estimate its asymptotic properties will converge to the properties of the Ordinary Least Square method. In addition, since we can make use of the matrix inversion Lemma this algorithm is highly efficient when using large data samples, as it does not require matrix inversion.

Windowed Least Squares (WLS)

The WLS algorithm is another recursive variant of the LS method that aims to minimize the squared error but over a finite window instead of only one iteration [Choi and Bien, 1989]:

$$f(\theta) = \sum_{k=t-N+1}^t \lambda^{k-t} (y_k - \hat{y}_k)^2 \quad (2.23)$$

In this case, two updates are applied for each sample, which increase the complexity by a factor of 2 [Gustafsson and Gustafsson, 2000]. Additionally, it is necessary to store N measurements which can be another important drawback if N is too large. In this case the algorithm can be summarized in the following steps:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + F_k (e_k^0 \varphi_{k-1} - \lambda^{N-1} e_k^N \varphi_{k-N-2}) \quad (2.24)$$

$$e_k^0 = y_k - \varphi_{k-1}^T \hat{\theta}_{k-1} \quad (2.25)$$

$$e_k^N = y_{k-N-1} - \varphi_{k-N-2}^T \hat{\theta}_{k-1} \quad (2.26)$$

$$C_k = \frac{1}{\lambda} \left(F_{k-1} - \frac{F_{k-1} \varphi_{k-1} \varphi_{k-1}^T F_{k-1}}{\lambda + \varphi_{k-1}^T F_{k-1} \varphi_{k-1}} \right) \quad (2.27)$$

$$F_k = C_k + \frac{C_k \varphi_{k-N-2} \varphi_{k-N-2}^T C_k}{1/\lambda^{N-2} - \varphi_{k-N-2}^T C_k \varphi_{k-N-2}} \quad (2.28)$$

where e^0 and e^N are the estimation errors in the current iteration and in the past N th iteration, respectively, C and F are two adaptive gain matrices. It is worth noticing that the initial value of the matrix F should be large [Choi and Bien, 1989] to allow the algorithm to correct for big deviations at the beginning of the iteration.

2.3.3 Estimating Parameters for Nonlinear Models

As stated previously, the methods described above are aimed at identifying the parameters of linear models. To estimate the parameters of nonlinear models, there are two main alternatives [Ljung, 1999, Box et al., 1976]. The first alternative is to try to find a known nonlinear relationship for the measurement vector while leaving the parameters constant and then to treat the system as a linear model. Specifically, this method finds a description of the model in the following manner:

$$\begin{aligned} b_0 f_0(u_t) + \\ b_1 f_1(u_{t-1}) + \dots + b_m f_m(u_{t-m}) = & y_t + a_1 g_1(y_{t-1}) + \\ & \dots + a_n g_n(y_{t-n}) \end{aligned} \quad (2.29)$$

where f_i and g_j , with $i = 0, 1, \dots, m$ and $j = 1, 2, \dots, n$ are nonlinear functions of the measurements; while b_i and a_j are constant parameters. Note that if the system is remodeled in this way, it is still a linear regression because the nonlinearity affects only the measurements, not the parameters. Nevertheless, finding such a representation may be impossible or extremely difficult and requires knowledge of the underlying physics of the system, which may be unavailable.

Another possibility is to use a recursive method to find a solution to the original minimization problem (i.e., minimize the modeling error). In the optimal control literature, there are many methods, with different convergence properties and computational costs, to find solutions to this problem. The interested reader is referred to the work of Betts [Betts, 2010] and Dennis [Dennis and Schnabel, 1996] for a good introduction on the subject and a practical treatment on various methods.

The most popular choices for solving minimization problems are the so-called Newton methods, which ensure quadratic convergence [Betts, 2010, Dennis and Schnabel, 1996]. Nevertheless, these methods require computation of the Jacobian and Hessian matrices (see Glossary) to find the solutions, which can be a severe computational burden and may not exist. Another alternative is the Steepest Descent Minimization method [Gustafsson and Gustafsson, 2000] that only requires computation of the gradient of the problem and is, thus, computationally less expensive at the expense of losing the quadratic convergence.

Steepest Descent Minimization (SDM)

The Steepest Descent algorithm [Gustafsson and Gustafsson, 2000] is defined by:

$$\hat{\theta}_k = \hat{\theta}_{k-1} - \mu \left. \frac{df(\theta)}{d\theta} \right|_{\theta=\hat{\theta}_{k-1}} \quad (2.30)$$

where $f(\theta)$ is some loss function and μ is a design constant. This equation can be applied to any general loss function, and, in this study, we chose to apply it to the squared modeling error.

2.4 Evaluation of Estimated Model

The evaluation of the estimated model is a very important step in system identification and it has become a discipline of study by its own right [Weisberg, 2005]. The idea behind the evaluation is to decide if the model is adequate or not according to certain objective criteria. One useful piece of information that can be used to develop such criteria is the residuals of the model defined as:

$$r = y - \hat{y} \quad (2.31)$$

where y is the measured output of the system and $\hat{y}$ is the estimated value of the output. This quantity differs from the modeling error defined previously because, here, $\hat{y}$ is computed using the estimated vector of parameters with each input, that is:

$$\hat{y} = u\hat{\theta} \quad (2.32)$$

Additionally, as the modeling error is a random variable with zero mean, the residuals have the following properties:

$$E(r) = 0 \quad (2.33)$$

$$Var(r) = \sigma^2(1 - \hat{H}) \quad (2.34)$$

$$\hat{y} = \hat{H}y \quad (2.35)$$

$$\hat{H} = \begin{bmatrix} h_{11}, h_{12}, \dots, h_{1n} \\ h_{21}, h_{22}, \dots, h_{2n} \\ \vdots, \vdots, \ddots, \vdots \\ h_{n1}, h_{n2}, \dots, h_{nn} \end{bmatrix} \quad (2.36)$$

Once we have this quantity we can use their autocorrelation (see Glossary in appendix A) or the cumulative periodogram (see appendix A) to perform diagnostic checks on the model. There are several tools available in the literature to use residuals as indicators of goodness-of-fit [Betts, 2010, Weisberg, 2005, Ljung, 1999]. One useful tool that can help evaluate the performance of the model in a qualitative manner is the scatter plot [Weisberg, 2005].

2.4.1 Scatter Plots

These plots are residuals against estimated values, time, and input (see Fig. 2.5). Scatter plots are used because although they are somewhat informal, they provide a powerful tool to assess the quality of the model. Scatter plots can be produced with a variety of combinations of the residuals and other quantities of the model, such as the outputs, the inputs, and the estimates. Particularly, there are three typical residual plots that help us give insight on the performance of the model [Weisberg, 2005]. To explain how the scatter plots can be useful, it is necessary to introduce the concept of a mean function (see appendix A). Thus, let us denote a mean function as:

$$E(y | u) = f(u) \quad (2.36)$$

That is a mean function is the expectancy (see appendix A) of an output y given a certain input u as a function of the input. Now, if we look at the definition of an ARMA process and the expectancy is taken in both sides, it can be seen that the model of the system is a mean function.

Residuals are used in scatter plots of r against U . Where, U is one of the terms in the mean function or some linear combination of the terms [Weisberg, 2005], such as inputs, outputs and estimates. The key features of these residual plots when the correct model is fit are as follows:

- The mean function is $E(r | U) = 0$. This means that the scatter plot of residuals on the horizontal axis versus any linear combination of the inputs, outputs or estimates should have a constant mean function equal to 0.
- Since $Var(r|U) = \sigma^2(1 - h_{ii})$ even if the fitted model is correct, the variance of the residuals is not quite constant. The variability will be smaller for high-leverage cases with h_{ii} close to 1. Where h_{ii} is the diagonal of the matrix $\hat{H}$ that transforms y into $\hat{y}$:
- The residuals are correlated, but this correlation is generally unimportant and not visible in residual plots.

These features mean that when the model is correct, residual plots should look like null plots [Weisberg, 2005] (see Fig. 2.5 a). However, when the model is built under incorrect assumptions, scatter plots can be used as an indicator of the problem, because the plot of the residual against one or more of the parameters of the mean function will not be a null plot. Figure 2.5 b through h show typical plots of incorrectly estimated models.

The subplots in Figure 2.5 from (b) to (d) indicate a nonconstant variance of the quantity on the horizontal axis. The curves from (e) to (h) show an incorrect mean function (or model in the ARMA case); with the last two cases indicating that nonconstant variance is also present. These conclusions can be inferred for simple regressions, however for more complex regressions, non-null scatter plots indicate a problem but they do not necessarily indicate where it is.

To extract more information some quantitative evaluation is required. Nevertheless, there is no universally accepted goodness-of-fit criterion in the literature.

2.4.2 Quantitative Evaluation

In the extensive literature on the subject of assessing goodness of fit [Ljung, 1999, Kvålseth, 1985, Motulsky and Christopoulos, 2004], one good indicator of the goodness of fit is the autocorrelation of the residuals to check for independence between the current and past predictions [Ljung, 1999]. If the model is perfect, R should be Dirac's delta (see appendix A). In other words, the predictions should be independent from each other. It is important to note that, because many models depend on the past inputs and outputs, some correlation is to be expected, but it should decrease rapidly over time.

Another logical method to measure the goodness of fit quantitatively is the mean squared sum of the residuals, defined as:

$$MSE = \frac{\sqrt{\sum (y_i - \hat{y}_i)^2}}{N} \quad (2.37)$$

which should be as small as possible.

Note that both of the tests described up to this point may seem logical, but they have a very important disadvantage: They present us with unanswered questions: How small is considered a good fit? How fast should R decrease to zero? To answer this questions there are statistical tests with intuitive interpretation that have been proposed [Vonesh et al.,

Figure 2.5: Residual plots: (a) null plot; (b) right-opening megaphone; (c) left-opening megaphone; (d) double outward box; (e)(f) nonlinearity; (g)(h) combinations of nonlinearity and nonconstant variance function (Figure taken from [Weisberg, 2005])

1996, Spiess and Neumeyer, 2010], such as the likelihood ratio [Vonesh et al., 1996], the chi-square test [Magee, 1990], etc. One of the most widely used approach to evaluate quantitatively the goodness of fit is the use of the coefficient of determination (R^2).

The R^2 Statistic

There are many different definitions of the R^2 statistic [Kvålseth, 1985]. Three of the most popular definitions are as follows:

$$R_1^2 = 1 - \sum (y - \hat{y})^2 / \sum (y - \bar{y})^2 \quad (2.38)$$

$$R_2^2 = 1 - \sum (y - \hat{y})^2 / \sum (y)^2 \quad (2.39)$$

$$R_3^2 = \sum (\hat{y})^2 / \sum (y)^2 \quad (2.40)$$

where $\hat{y}$ is the predicted value of y and $\bar{y}$ denotes its mean.

Recommendations to choose from the different definitions can be found in [Kvålseth, 1985]. Nonetheless, the decision of which R^2 to choose depends on:

- The type of model used.
- The fitting technique.
- The properties of R^2 considered desirable.

In general, an R^2 statistic should satisfy the following four key conditions:

- R^2 must have an intuitive interpretation.
- R^2 has to be independent of the units of measurement.
- R^2 has to be bound by defined endpoints, i.e. $0 \leq R^2 \leq 1$ where 1 is perfect fit.
- Positive and negative residuals should be weighted equally by R^2 .

From the definition (2.40) it can be inferred that R_3^2 may take values larger than 1 for nonlinear models. R_2^2 has been shown to be generally different from the other two definitions [Montgomery et al., 1982]. Meanwhile, R_1^2 satisfies almost all of the conditions with the exception of the lower bound. Nevertheless, R_1^2 becomes negative in two situations: when the model is grossly misspecified or when a robust fitting algorithm is used with highly contaminated data. However, both cases are easily identifiable in practice.

One weakness of the R^2 statistic is that it should be used as a comparative index to check if there is an improvement over the null hypothesis [Kvålseth, 1985] or with respect to other models, because there is no universally accepted cut-off criteria to accept or reject a model [Kvålseth, 1985].

For that reason, to complement the R^2 statistic, other goodness-of-fit criteria have been proposed [Kvålseth, 1985, Vonesh et al., 1996].

The r_c Index

This index was introduced [Vonesh et al., 1996] as a modification of the concordance relation between two variables described in [Lawrence and Lin, 1989] and is given by:

$$r_c = 1 - \frac{\Sigma(y - \hat{y})}{\Sigma(y - \bar{y}\mathbf{1})^2 + \Sigma(\hat{y} - \bar{\hat{y}}\mathbf{1})^2 + N(\bar{y} - \bar{\hat{y}})^2} \quad (2.41)$$

where $\mathbf{1}$ is a vector of ones of the appropriate size and the rest of the notation is the same as the used in the definition of R^2 .

The index r_c takes values from -1 to 1, again with 1 indicating a perfect fit and $r_c \leq 0$ indicating lack of fit. As the authors point out this index has some of the disadvantages of R^2 , nonetheless it can be used without the need for a null hypothesis in contrast with R^2 .

2.5 Summary

The literature on contact models distinguishes between two main methods: force based and the impulse-momentum methods. The first of the two is of special interest to this work, because it aims at describing the impact/contact force as a continuum. The second, although it has had many successful implementations, analyzes the impact force after the impact has occurred. Furthermore including a continuous contact force in the formulation is complex and makes it very difficult to formulate a system identification procedure.

Nonetheless, if we select a model such as the spring dashpot, it is possible to use many of the currently available system identification methods such as: Ordinary Least Squares, Recursive Least Squares and Windowed Least Squares. Furthermore, whenever a linear model is not capable of showing the main characteristics of a complex phenomenon such as impact and a nonlinear model needs to be used; there are some methods that can be used to estimate the parameters of such models: One possibility is to find a linear combination of a non linear function of the parameters, which allows the use of any of the linear identification methods. A second approach is to use a method capable of identifying nonlinear parameters such as Newton methods or the Steepest Descent Minimization.

When we combine contact modeling and system identification to extract the properties of an object via sensors in a robot we are in the field of haptic perception. This field has received increasing attention over the last few years, as it is one of the most advanced skills that animals possess to perform highly complex tasks in unknown environments.

Chapter 3

Modeling and Identification of Robot Contact

Before going into more depth on system identification applied to contact modeling it is convenient to discuss the model of a robot in contact with the environment. Then, using the model as a starting point, the specific issues of the identification of contact properties will be discussed. In this chapter, a model of a robot in contact with the environment using the spring-dashpot model is given, for the sake of simplicity, however more complex models can be included in the model following this procedure. The environment can be any dynamic system working with the robot: e.g. another robot, a support, the ground, a rotating table, etc. as long as it can be described using the chosen model.

3.1 Model of a Robot in Contact with the Environment

Note that when a robot interacts with the environment, this interaction is typically described in task-space forces. It is important to point out that by forces we mean a six dimensional vector containing linear forces in 3D and the torques expressed in euler angles, in other words a generalized force vector [Featherstone, 2008]. However, as stated before, we are interested in environments that are not structured and unknown. Thus, we will start with the robot in joint space which is normally known a priori and we will model the robot in terms of joint coordinates, and map the task space forces to the joint space torques by means of the Jacobian matrix.

In general, the dynamic model of a robot in joint coordinates is given by:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} \quad (3.1)$$

where $\mathbf{M}(\mathbf{q})$ is the inertia matrix; $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is a matrix grouping the Coriolis, centrifugal and gravitational components; $\boldsymbol{\tau}$ is the vector of torques for each joint; and $\mathbf{q}$ is the vector of positions of each joint.

To map the task space forces to the joint torques we will use the following relationship [Siciliano and Villani, 1999]:

Figure 3.1: Robot in contact with the environment

$$\mathbf{v} = \mathbf{J} \cdot \dot{\mathbf{q}} \quad (3.2)$$

where $\mathbf{v}$ is the vector of task space velocities and $\mathbf{J}$ is the Jacobian matrix of the robot. From this relationship and the definition of mechanical power we can derive the mapping between forces in task space and torques in joint space as follows:

$$\mathbf{T}^T \cdot \mathbf{v} = \tau^T \cdot \dot{\mathbf{q}} \quad (3.3)$$

$$\mathbf{T}^T \cdot \mathbf{J} \dot{\mathbf{q}} = \tau^T \cdot \dot{\mathbf{q}} \quad (3.4)$$

$$\mathbf{J}^T \cdot \mathbf{F} = \tau \quad (3.5)$$

where $\mathbf{T}$ is the matrix of generalized forces in task space.

Now, consider a robot with n DoF, which is in contact with the environment in surface s^c and is able to move freely along the surface (see Fig. 3.1). Such a robot can be described by the following expression:

$$\mathbf{M} \ddot{\mathbf{q}} + \mathbf{C} = \tau + \mathbf{J}^{cT} \mathbf{F}_c + \mathbf{J}^{fT} \mathbf{F}_f \quad (3.6)$$

where, $\mathbf{J}^c$ is the Jacobian matrix in the constraint coordinate (see appendix A), $\mathbf{J}^f$ is the Jacobian matrix in the free motion coordinate (see appendix A), $\mathbf{F}_c$ is the contact force and $\mathbf{F}_f$ is the friction force. Moreover, the end effector is considered to be deformable, and the environment is modeled by the spring-dashpot contact model. The model of the end effector is taken from [Vukobratovic and Pottonjak, 1999] and it resembles the model used by Alexander in [Alexander, 1988] to describe the paws in some mammals, such as dogs. The complete model is shown in Fig. 3.2.

Based on the assumption that the friction force is present in the directions where no deformation is allowed, (e.g. the leg's walking direction), all the equations for the contact

Figure 3.2: Robot/environment contact model

motion will be derived without the friction force without loss of generality and afterwards the friction will be added for completeness.

3.1.1 Model without Friction

Let the robot coordinates be referenced to a coordinate system fixed to the contact surface (see Fig. 3.2). If the surface and the robot are considered perfectly rigid, the contact is described by a geometrical constraint and it has the following form:

$$s^c = 0 \quad (3.7)$$

where s^c is the surface of the constraint.

However, since the surfaces are assumed to be deformable, s^c should describe the deformation along the coordinates where deformation is allowed. Nevertheless, when there is no contact force (3.7) holds, but when the contact force exists s^c is described by:

$$s^c = \delta_e + \delta_{ri} + \delta_{re} \quad (3.8)$$

where δ_{ri} is the robot's internal deformation, δ_{re} is the robot's external deformation and δ_e is the environment's deformation. The external deformation is the local deformation at the contact point, while the internal deformation is used to model the energy loss due to the transmission of stress waves in the body.

Now, the environment reaction force can be given by the spring-dashpot model (2.6) and can be rewritten as:

$$\mathbf{F}_e = \mathbf{K}\delta_e + \mathbf{B}\dot{\delta}_e, \quad (3.9)$$

considering that the environment has infinite mass after the spring-damper system. The relationship between the robot deformation and the joint motion can be described as:

$$\ddot{\mathbf{s}}^c = \ddot{\delta}_{ri} + \ddot{\delta}_{re} = \mathbf{J}^c \ddot{\mathbf{q}} + \dot{\mathbf{J}}^c \dot{\mathbf{q}} \quad (3.10)$$

At this point, to completely describe the system (without including friction) the equations relating the environment's and robot's contact force are to be included. These equations are: the equation of motion for the robot's external deformation accelerations, and the force equilibrium equation for the environment:

$$\mathbf{F}_e - \mathbf{F}_c = \mathbf{0} \quad (3.11)$$

$$\mathbf{M}_r \ddot{\delta}_{re} = -(\mathbf{F}_c - \mathbf{F}_{cre}), \quad (3.12)$$

$$\mathbf{F}_{cre} = -\mathbf{K}_{re}\delta_{re} - \mathbf{B}_{re}\dot{\delta}_{re} \quad (3.13)$$

And finally, the internal robot's deformation can be described by the following equation:

$$\mathbf{F}_{c_{ri}} = -\mathbf{K}_{ri}\delta_{ri} - \mathbf{B}_{ri}\dot{\delta}_{ri} \quad (3.14)$$

Equations (3.6) through (3.14) define the system very well and these equations can be solved for the robot's accelerations ($\ddot{\mathbf{q}}$) and forces ($\mathbf{F}_c$) with the exception of the friction force.

3.1.2 Adding Friction

Friction is a very complex phenomenon that occurs when two or more bodies are in contact and in relative motion. It depends on many parameters such as velocity, material properties, temperature, etc. There has been extensive research effort in modeling friction; however the models proposed concentrate on describing the main properties of the phenomenon.

As with contact, there are two main types of approaches to friction modeling: static and dynamic. Static models, due to their simplicity, are used in many applications [Vukobratovic and Pottonjak, 1999, Chiaverini et al., 1999], however they do not explain several properties in systems with friction such as the Dahl effect, frictional lag, stick-slip motion and pre-sliding displacement [Canudas de Wit and Kelly, 2007] which can be relevant for certain applications and are considered in most dynamic models [Olsson et al., 1998, Armstrong-Helouvry and Canudas de Wit, 1994].

In this work, a modification of Coulomb's friction model is used, since it sufficiently describes the effects of friction for many applications such as walking and grasping, therefore the friction force is as follows:

$$F = \begin{cases} \mu_d F_c v, & \text{if } v \neq 0 \\ \mu_s F_c, & \text{if } v = 0 \end{cases} \quad (3.15)$$

where F is the friction force, F_c is the contact force and v is a unit vector parallel to the relative velocity between the foot and the ground. μ_c is the Coulomb friction coefficient and μ_s is the static friction coefficient.

In order to include the friction force in the dynamic model of the robot in contact with the environment, what is left to define is the space where the friction force acts and relate it to the robot coordinates. Since friction exists in the direction of motion, the coordinates where friction occurs are unconstrained; the mapping between the workspace and joint coordinates is as follows:

$$\dot{s} = J^f \dot{q} \quad (3.15)$$

With this equation, the dynamics of the system can be modeled, since the unit vector v depends on the direction of motion. It is important to note that if a constant friction coefficient (μ) is used, the contact forces are assumed to always be greater than the friction force. For non constant coefficients when the relative velocity between the robot and the environment is zero, it is necessary to introduce the following condition [Vukobratovic and Pottonjak, 1999]:

$$\ddot{s}^f = J^f \ddot{q} + \dot{J}^f \dot{q} \quad (3.15)$$

This scenario is especially useful for walking robots, since it is essential that no relative motion between the foot and the environment during the support phase exists for the leg to propel the body during walking and avoid slipping.

3.2 Recognizing Environment Properties

Once the robot is modeled, system identification can be performed to extract the environment properties that are relevant to a particular application. Haptic perception is important to recognize local and dynamic properties of the environment and the physical constraints imposed on the end-effector [Kikuuwe and Yoshikawa, 2005]. In general, to implement this function in a robot, it is necessary to obtain data of position and force, then recognizing constraint patterns in the data and infer the mechanisms that realize those constraints.

There are several studies that aim at discovering restrictions imposed at end-effectors by using position and force measurements [Yoshikawa et al., 1996, Erickson et al., 2003, Kikuuwe and Yoshikawa, 2005]. Those studies use a variety of techniques that can be divided in two main categories: active sensing (also referred as active probing) and passive sensing. The former consists on using signals specifically designed to excite the system and obtain parameter estimation. While the latter uses information obtained by the robot during its normal operation. The choice between the two approaches depends on the system set-up; on the available sensors and their placement; on the relative stiffness of the robot and the environment; and if free motion is allowed, etc.

3.2.1 Active Sensing

Active sensing is a method where the identification process is an integral part of the application, for example, a probing operation to locate objects in space, and then pick the object and put it in another location. This type of sensing method has the advantage of producing precise and accurate estimates under the existence of disturbances, such as uncertainties and noise. In addition, it allows efficient data collection because it is part of the control system. Nevertheless, it complicates low-level control problems and objective planning as the probing operation is a part of the robot control (see Fig. 3.3). Furthermore, when a probing operation is performed, the main objective of the control could be suspended, as it is the case of [Yoshikawa et al., 1996, MacLean, 1996, De Schutter et al., 1999, Emura and Tachi, 1997].

Figure 3.3: Active sensing block diagram

Using this type of approach, Erickson, Weber and Sharf [Erickson et al., 2003] used an impedance control law with four different algorithms to extract the stiffness and damping coefficient of a flexible wall. In the experiments, they used the spring-dashpot model and both persistent and non persistent excitation. After the experiments they came to the conclusion that the stiffness coefficient can be estimated with relatively good accuracy with and without persistent excitation. However, their experiments indicate that estimating a damping coefficient accurately without persistent excitation is not possible. Another important contribution to haptic perception is the work of Mouri et al. In [Mouri et al., 1999] they were able to successfully identify the contact type of a robot grasping an object of unknown form by minimizing a squared cost function on the contact moment.

3.2.2 Passive Sensing

Passive sensing is a method where, the perception part of the application is independent of the control objective. This type of sensing is independent from the control law (see Fig. 3.4). Although it is less efficient as it implies the implementation of a completely separated module from the control, it has a broader range of potential applications [Kikuuwe and Yoshikawa, 2005]. Using this approach, it is possible to monitor interactions from

human operators with an environment and then use this information in a learn by example algorithm to train a robot [Skubic et al., 2000, Hovland et al., 1996].

Figure 3.4: Passive sensing block diagram

Moreover, Kikuuwe and Yoshikawa proposed a passive sensing algorithm to estimate the stiffness of a contact surface with a Weighted Least Squares method. This algorithm was able to detect discontinuities in the surface and to give an evaluation of uncertainty. While Spyros and Costas [Spyros and Costas, 2010] used a passive sensing approach to reflect the perceived impedance by a slave robot to a master robot in contact with a human operator.

3.2.3 Combined Approach

It is important to note that passive and active sensing are not mutually exclusive. For example, one could use an active approach to find objects in space by performing a probing operation and then use a passive sensing approach while carrying the object to classify it. In [Kikuuwe and Yoshikawa, 2005], the authors extended the method proposed in [Kikuuwe and Yoshikawa, 2003] to recognize the shape and friction coefficient of the contact surface using a combination of passive and active sensing.

There is an increasing effort in the field of haptic perception and most of these new works employ a combination of passive and active sensing approaches. In the medical field, haptics is used to create simulators to train for minimally invasive surgery [Goksel et al., 2011]. Additionally, there are many works towards the creation of tactile maps [Varalakshmi et al., 2012] to help improve robots' haptic perception [Chu et al., 2015, Takamuku et al., 2007, Denei et al., 2015] so that they can manipulate and discriminate between objects based solely on the sense of touch.

As we have discussed, both passive and active sensing can be used alone or in combination to expand the capabilities of a robot. Active sensing has the advantage that it is more accurate, but it is embedded in the control algorithm. On the other hand, passive sensing can be applied to a wider range of applications as it is independent of robot control.

3.3 Summary

Modeling a robot in contact with the environment, from a control engineering point of view implies the conversion between task and joint coordinates. This transformation can be done in either the contact forces of the environment or the torques generated by the robot's actuators. In this chapter we have done the latter, because the information of the robot is, in principle, available a priori and we are interested in extracting the information of the environment, which is not.

Once this choice is made, what remains to model a robot in contact with the environment is to write the force balance equations according to the imposed restrictions. The restrictions can be used to separate the coordinates where free motion (with or without friction) is allowed and where contact is applied. A good advantage of this type of formulation is that it allows to change the coordinates where free motion or contact is, just by changing the definition of the Jacobian matrices.

Now, to tackle the problem of extracting the contact properties of the environment there are two main approaches: active and passive sensing. Even though active sensing has many advantages in terms of accuracy, passive sensing is more appropriate for our application as it does not require to stop the robot operation to perform identification specific actions.

In the next chapter an experimental analysis of the system identification methods described in the previous chapters will be presented. This analysis includes a comparison to evaluate the performance of each method when applied to contact modeling.

Chapter 4

Comparison Between System Identification Methods

The previous chapter gives an introduction on contact modeling and system identification. In this chapter we will select one linear and one nonlinear model described in Chapter 2 and an experimental evaluation will be presented. The idea is to evaluate, experimentally, the performance of the system identification methods described in Section 2.3 when they are used for the estimation of contact properties. The experiments are performed with four different environments and afterwards their performance is analyzed using scatter plots and goodness of fit criteria.

4.1 Experimental Set-up

Figure 4.1: Experimental set-up [Arevalo et al., 2013a]. Depiction (left). Schematic diagram (right).

The ultimate goal of this work is to devise a system identification procedure that can be used in robotic applications online. For that purpose, an experimental set-up has been

conceived to resemble a generic robot contacting various materials. The experimental set-up depicted in Fig. 4.1 was built; it is very simple and consists of a Series Elastic Actuator (SEA) [Pratt et al., 2002] that pushes against four wooden boxes containing different materials (Fig. 4.4). The SEA was chosen because it includes one position encoder at the spring, allowing it to indirectly measure the exerted force, and another encoder measures the displacement of the actuator's end effector (see Fig. 4.1 right). Furthermore, the spring between the end effector and the rest of the actuator decouples the dynamics of the motor and the ground. The first three materials used for testing are polyurethane-low-density flexible foam, gravel, river sand. These materials were chosen, because sand and gravel are commonly encountered in unstructured walking scenarios, while the polyurethane-low-density flexible foam was chosen because it is a highly nonlinear material, and it is commonly used to wrap objects while packing them. The fourth material is a variable impedance box. The box was designed to resemble as closely as possible the behavior of the linear spring dash-pot model (see equation 2.6). The design of the box is depicted in Fig. 4.2. It consists of a metal surface attached to four compression springs with the same elastic constant arranged in parallel fixed to a base. The box has four sliding guides so the surface is able to move in parallel with the base and push two pneumatic dampers arranged antagonistically; thus giving a damping constant.

Figure 4.2: CAD model of the variable impedance box

To perform the experiments a simple force controller (see Fig. 4.3) was implemented on a National Instruments PXI industrial measurement system with an execution time of $500 \mu\text{s}$. The data collection of force and position was performed with the same period as the control algorithm.

Figure 4.3: Diagram of the control scheme

4.2 Testing Procedure

The experiments consist on setting the actuator at an initial position touching the environment (i.e., the surface of any of the materials used to fill the box) and then applying a 60 N force downwards for 15 seconds; this force is applied to ensure that the contact is held in a range of forces from 0 to 100 N. The constant force is then followed by the application of a series of 10 steps of ± 10 N around the offset force with a frequency of 0.75 Hz and a duty factor of 50% for 10 seconds (Fig. 4.5) to simulate a soft pushing or squeezing. At this time, the force pattern amplitude is changed to 40 N for another 10 s (Fig. 4.5) to simulate a hard push or squeeze. The frequency, duty factor and duration of the signal were selected to obtain large quantities of data and explore the performance of each identification method. This test was performed 10 times for each of the four materials to acquire enough data for a statistical evaluation.

Before applying the estimation algorithms to the obtained data, let us first observe the response obtained from each force pattern for each of the materials used. Fig. 4.6 shows the displacement (using the interface between the actuator and the ground as the zero position) at the actuator's tip needed to exert the force pattern described above for the variable-impedance box (upper left), the gravel (upper right), the river sand (bottom left) and the low-density polyurethane foam (bottom right). It is important to note that Fig. 4.6 only shows the results for one experiment. In the figure, it can be observed that the river sand and the polyurethane foam clearly exhibit a nonlinear response. This behavior includes some sort of deformation that is evidenced by the mean displacement which increases with time. Thus, the behavior of the material cannot be completely explained using a linear model, such as (2.6).

Nevertheless, since we are applying an adaptive filtering approach to the problem, this nonlinearity can be modeled by a variation of the linear parameters. In other words, the algorithm will identify the parameters of a collection of models for certain operating points. This approach can be useful if the identification is applied online. However if one is interested in modeling a certain type of environment and then use it in simulation or in a control application without online identification it would be better to use a nonlinear model, because a nonlinear model can capture richer dynamic behavior than a collection of linear models.

On the other hand, even though the gravel and the variable-impedance box responses appear to be linear, they still exhibit some nonlinear behavior since the displacements vary for each force value. Nevertheless, this variation is more related to the geometrical structure of the contact (in the case of the gravel) and to separation (in the case of the box) than to

Figure 4.4: Variable-impedance box (up-left), gravel (up-right), river sand (bottom-left) and low-density semi-rigid foam (bottom-right) used for the experiments.

Figure 4.5: Force reference for the experiments from $t = 18$ s to $t = 41$ s

deformation. This separation is produced because the SEA has a faster dynamic response than the variable impedance box; thus it is capable of moving away faster than the springs

of the variable impedance box can react.

Note that by geometrical structure we refer to, in the case of the gravel, the rearrangement of the boulders that may change making them closer to each other. In addition, because the boulders have irregular shapes, this rearrangement translates into different contact surfaces between them. In turn, the different surfaces can increase (in the case of bigger contact surface) or decrease (in the case of smaller contact surface) the apparent stiffness of the material. In the case of the variable impedance box, the separation might generate an impact force when the actuator pushes down as it increases the force.

Figure 4.6: Variable-impedance box displacement (up-left), gravel displacement (up-right), river sand (bottom-left) and low density semi-rigid foam (bottom-right) displacement and mean displacement against time for the force pattern previously described.

4.3 Model Estimation

In this section, the performance of the online methods described in Section 2.3.2 is evaluated both quantitatively and qualitatively. The offline methods are not considered in this evaluation because, as we have seen in the previous section, the materials exhibit nonlinear behavior. Thus, using a linear model with an offline method that lacks adaptability does not lead to any significant result.

Now, before applying the identification algorithms to the obtained experimental data there are a few issues that need to be addressed. The first is that the system response presented quantization errors (see appendix A); such as the error produced when translating a continuous signal such as the actuator position to a digital number. Quantization can be a problem because it causes the system to appear to be at an equilibrium point (the system response shows no variation for a certain amount of samples), which can affect the performance of the aforementioned adaptive filtering methods, and for this reason, it was necessary to reduce the sampling rate by a factor of five, thus, increasing the sampling time from 0.5 to 2.5 ms. The other issues will be addressed further below as they are particular to the type of estimation (linear or non linear).

4.3.1 Linear Model Estimation

In this section we evaluate the RLS and the WLS methods. To apply the identification algorithms, it is necessary to find a linear regression model of the system. In this case, because the equation of the linear spring-dashpot model (2.6) is linear, the procedure is straightforward. Let us rewrite equation (2.6) in discrete time by means of finite differences as:

$$F_k = Kx_k + B \frac{x_k - x_{k-1}}{h} \quad (4.1)$$

where h is the sampling period; $i = 1, 2, \dots, N$ and N is the number of samples; K is the elastic constant; and B is the damping coefficient.

$$x = \varphi^T \theta \quad (4.2)$$

where,

$$\varphi^T = [-x_{k-1}, F] \quad (4.3)$$

$$\theta^T = [B/(hK + B), h/(hK + B)] \quad (4.4)$$

Note that equation (4.1) can be solved for both x and F . Nevertheless, we choose to use the force as input and the position as the output. This choice is motivated by the following analysis:

Let η be the vector of the unknown modeling errors and the noise in the regression (4.2); the complete estimation can be written as:

$$F = Kx + B\dot{x} + \eta \quad (4.5)$$

$$x = \frac{1}{K}(F - B\dot{x} - \eta) \quad (4.6)$$

Now let us denote θ_1 the parameter vector of equation (4.5) and θ_2 the parameter vector of equation (4.6). Applying the least squares estimation to the above equations; plugging (4.5) and (4.6) in (2.15) yields:

Figure 4.7: Residuals obtained with the variable-impedance box (first row), Gravel (second row), river sand (third row) and low-density flexible foam (last row) against estimated values (first column), input values (second column) and time (third column) using the RLS algorithm and the linear spring-dashpot model.

$$\hat{\theta}_1 = \theta_1 + Q(\varphi_1)\eta \quad (4.7)$$

$$\hat{\theta}_2 = \theta_2 + Q(\varphi_2)\eta \quad (4.8)$$

where $Q(\varphi_i)$ is a matrix of the inputs and outputs of the system, defined in equation (2.14). Then, taking the norm at both sides and finding the largest singular value σ of the estimates yields:

$$\| \hat{\theta}_1 - \theta_1 \| = \sigma(Q(\varphi_1)) \| \eta \| \quad (4.9)$$

$$\| \hat{\theta}_2 - \theta_2 \| = \sigma(Q(\varphi_2)) \frac{1}{K} \| \eta \| \quad (4.10)$$

Note that $1/K$ is taken out of the singular value function because it is a constant that multiplies all inputs of the system (see equation 4.6). This analysis suggests that using the force as the input should lead to a better estimation as $K \gg 1$.

To analyze the performance of the methods, for an initial qualitative evaluation, we make use of their scatter plots described in Section 2.4.1. In this and the following section, all of the scatter plots have been computed using the mean values of the residuals and the fitted values based on the 10 tests performed for each material. Recall the definition of residuals given by equation (2.31):

$$r = y - \hat{y} \quad (4.11)$$

where y is the measured output of the system and $\hat{y}$ is the estimated value of the output. In this case, the output y is considered to be the ground penetration x .

Now, let us analyze the behavior of the two approaches separately for each of the materials used in the experiments. Fig. 4.7 shows the scatter plots of the residuals obtained with the RLS algorithm for the variable-impedance box (top), gravel (second row), river sand (third row) and polyurethane foam (bottom row). The first column shows the residuals against the estimated values of $\hat{x}$, the second column shows them against the inputs F , and the third column shows the residuals against time. In Fig. 4.7, there is no discernible pattern other than a horizontal band on the plots against time. This is an indicator that the model produces a good fit for the data [Draper and Smith, 2014].

Figure 4.8 shows the scatter plots of the residuals of the WLS algorithm in the same layout as the previous figure. Here, in contrast to Fig. 4.7 for the variable impedance box a curved arc can be observed in the plots of the residuals against $\hat{x}$ and F . As mentioned in Section 2.4.1, this indicates that the model needs to include extra terms [Draper and Smith, 2014]. Furthermore, in all the other plots either an ascending or descending band can be observed; this pattern can be indicative of a missing first-order term in time [Draper and Smith, 2014], that there is an error in the calculation or that a zero crossing term is missing from the model [Weisberg, 2005]. These patterns indicate that the windowed least squares might not be a good procedure to estimate the parameters of the model.

After analyzing the scatter plots, it seems evident that the RLS method is more accurate than the WLS method. However, scatter plots are an informal tool. Thus, let us verify the conclusion by using an R^2 statistic to compare their performance.

Because the model is linear it makes no difference which of the definitions given in Section 2.4.2 is used. Nonetheless, we will use R^2 defined as:

$$R^2 = 1 - \Sigma(y - \hat{y})^2 / \Sigma(y - \bar{y})^2 \quad (4.12)$$

Figure 4.8: Residuals obtained with the variable-impedance box (first row), Gravel (second row), river sand (third row) and low-density flexible foam (last row) against estimated values (first column), input values (second column) and time (third column) using the WLS algorithm and the linear spring-dashpot model.

The reason of this choice is that to make a meaningful comparison with the nonlinear models the same R^2 statistic should be used and the other definitions given in Section 2.4.2 are not recommended in the nonlinear case. Moreover, the index r_c will be used to give a more clear criteria for choosing the best performing method, because r_c tests the model against the output and does not require a null hypothesis.

Table 4.1 shows the best and the worst values of R^2 and table 4.2 shows the best and the worst r_c obtained for the 10 repetitions and for each material using RLS and WLS.

Table 4.1: Best and worst R^2 results for the linear model

Ground	RLS		WLS	
	Best R^2	Worst R^2	Best R^2	Worst R^2
V.I. Box	0.9810	0.9734	0.8022	0.5092
Gravel	0.9880	0.9496	0.9105	0.1656
River sand	0.9336	0.9270	0.2765	0.0263
Foam	0.8220	0.7781	0.3870	0.0519

Table 4.2: Best and worst r_c results for the linear model

Ground	RLS		WLS	
	Best r_c	Worst r_c	Best r_c	Worst r_c
V.I. Box	0.9934	0.9868	0.9140	0.7447
Gravel	0.9940	0.9755	0.9630	0.2703
River-sand	0.9920	0.9650	0.5042	0.2342
Foam	0.9670	0.8982	0.6235	0.5092

Table 4.3: Linear model mean squared error

Ground	RLS Error		WLS Error	
	Mean	Std	Mean	Std
V.I. Box	7.14E-4	4.48E-4	2.11E-3	2.34E-4
Gravel	2.97E-3	5.77E-3	4.10E-3	8.05E-3
River-sand	3.72E-3	1.42E-3	5.14E-3	1.78E-3
Foam	6.32E-3	4.48E-4	3.99E-3	5.31E-4

Looking at tables 4.1 and 4.2 it can be seen that, as suspected, the performance of the WLS algorithm is much worse than the RLS for every material available. Nonetheless, in the case of the variable impedance box, both algorithms perform in an acceptable manner since both R^2 and r_c are close to 1. Note that r_c indicates a better fit than R^2 does; however, both indexes agree in which fitting is better for all the cases presented in this work. To further corroborate this results, table 4.3 shows the averages of mean square error for each experiment taken over the ten repetitions as well as their standard deviations.

The errors reported in the table are smaller for the RLS method, nonetheless the standard deviation is high. This indicates that the overall fit is better, but the response of the model is not smooth. In contrast, the WLS method produces a worse fit, but the response is much smoother, which could be important in some applications.

4.3.2 Nonlinear Model Estimation

In this section, we attempt to fit the experimental data using the nonlinear spring-dashpot model (2.8). To do this, the SDM and NRLS methods explained in Section 2.3 are used. Nevertheless, as mentioned above we need to address certain issues. First, to identify the parameters of the nonlinear spring-dashpot model, we first try to find a linear regression model for the equation (2.8) and then treat the model as a linear system [Ljung, 1999]. To find such a representation, it is useful to rewrite (2.8) using the definition of exponentiation function: $a^b = e^{b \ln(a)}$, where $\ln$ is the natural logarithm and a and $b \in \mathbb{C}$ (i.e., they are complex numbers):

$$F = B e^{p \ln(x)} e^{q \ln(\dot{x})} + K e^{n \ln(x)} \quad (4.13)$$

If a Taylor series expansion of the exponential function is performed, it is possible to find a linear regression form for the problem.

It is important to note that this model has five parameters to be identified (see equation (2.8)). This fact is important because if one is interested in obtaining the actual physical parameters (b , k , q , p and n), it would be necessary to have at least five equations relating the new parameters to those of the model; otherwise, it would be impossible to determine values for the parameters without performing another minimization. For this reason, the use of a 1st-order Taylor approximation for each variable (x and $\dot{x}$), which provides only four equations, is impractical. Conversely, the use of a 2nd-order expansion for all of the variables would lead to nine equations; thus, the original parameters would be over-determined. This may not be a problem when finding the relationship between the new parameters and the parameters of the model, however it is undesirable from a modeling point of view. Thus, to simplify the equation, there are three options:

- i Using a 2nd-order expansion for each variable which would (in principle) increase the estimation accuracy at the cost of increasing the number of parameters.
- ii Using a 2nd-order Taylor series expansion in either x or $\dot{x}$ and a 1st-order expansion on the other variable.
- iii Trying to find simplifications to the original problem.

The third option is chosen, because, as stated previously, the nonlinear spring-dashpot model has been studied by various authors, and there is a standard choice for the parameters, namely $n = p$ and $q = 1$, which can be very useful in developing a regression model. Rewriting equation (4.13) with the aforementioned parameters yields:

$$F = B e^{n \ln(x)} \dot{x} + K e^{n \ln(x)} \quad (4.14)$$

This parameter choice has reduced the dimensionality of the problem from five to three. Moreover, the performance of a 1st-order series expansion yields four equations with which to map the new parameters to the original model:

$$\begin{aligned} F &= B(1 + n \ln x)\dot{x} + K(1 + n \ln x) \\ &= B + Bn \ln x \dot{x} + K + Kn \ln x \end{aligned} \quad (4.14)$$

which can now be rewritten as:

$$F = \theta \varphi = \theta_1 \dot{x} + \theta_2 \ln x \dot{x} + \theta_3 + \theta_4 \ln x \quad (4.15)$$

$$\varphi^T = [\dot{x}, \ln x \dot{x}, 1, \ln x] \quad (4.16)$$

with $b = \theta_1$, $k = \theta_3$, $n = \theta_2/\theta_1 = \theta_4/\theta_2$. Now, with this representation, it is possible to apply the RLS method to the problem. The decision to use the RLS method and not the WLS method is based on the fact that the former exhibited a better performance in the previous experiments.

The other method used in this section is the SDM. The use of this method requires only the computation of the gradient, which can be calculated analytically or estimated using finite differences. In this case, the analytical calculation of the gradient is very easy, and performing the calculation increases the accuracy of the method and reduces the computational cost. Computing the gradient of the squared modeling error and inserting the result in (2.30) yields:

$$\theta_k = \theta_{k-1} - \mu \nabla f(\theta) \quad (4.17)$$

where,

$$\begin{aligned} \nabla f(\theta) &= \begin{bmatrix} -2x^p \dot{x}^q (F + Kx^n - x^p \dot{x}^q B) \\ 2x^n (F + Kx^n - x^p \dot{x}^q B) \\ -2x^p \dot{x}^q B \ln(\dot{x}) (F + Kx^n - x^p \dot{x}^q B) \\ -2x^p \dot{x}^q B \ln(x) (F + Kx^n - x^p \dot{x}^q B) \\ 2Kx^n \ln(x) (F + Kx^n - x^p \dot{x}^q B) \end{bmatrix} \\ \theta &= [K, B, q, p, n] \end{aligned}$$

In both algorithms, the logarithm function is present. This fact is important for two reasons. First, because the deformation is defined in a manner such that $x \geq 0$ and $\ln(0) = -\infty$, it is necessary to regularize the logarithm to avoid the logarithm becoming infinite. In particular, we chose to use $\ln(x) \approx \ln(x + \epsilon)$ with $\epsilon \ll 1$. The second reason is that in the steepest descent minimization, $\dot{x}$ is an argument of the logarithm. $\dot{x} \in \mathbb{R}$ so it can have negative values and the results of the logarithm can therefore be complex. This situation is perfectly possible in practice, nevertheless if we resort to the standard choices in the parameters and choose to set $q = 1$, and the gradient is rewritten as:

$$\begin{aligned} \nabla f(\theta) &= \begin{bmatrix} -2x^p \dot{x} (F + Kx^n - x^p \dot{x} B) \\ 2x^n (F + Kx^n - x^p \dot{x} B) \\ -2x^p \dot{x} B \ln(x) (F + Kx^n - x^p \dot{x} B) \\ 2Kx^n \ln(x) (F + Kx^n - x^p \dot{x} B) \end{bmatrix} \\ \theta &= [K, B, p, n] \end{aligned}$$

Figure 4.9: Residuals obtained with the variable-impedance box (first row), Gravel (second row), river sand (third row) and low density flexible foam (last row) against estimated values (first column), input values (second column) and time (third column) using the RLS algorithm and the nonlinear spring-dashpot model.

Fig. 4.9 shows the scatter plots ordered in the same manner as the previous methods, that is: the first row corresponds to the variable-impedance box, the second row to the gravel, the third row shows the river-sand and the final row the flexible foam. Then, the first column shows the residuals r against the estimated values $\hat{F}$, against the input x (second column) and against time (last column). Note that in this case the force is considered the output and the position is considered the input. The reason for this is that it was impossible to find a closed form expression for the displacement in terms of the input force. In Fig. 4.9

Figure 4.10: Residuals obtained with the variable-impedance box (first row), Gravel (second row), river sand (third row) and low density flexible foam (last row) against estimated values (first column), input values (second column) and time (third column) using the SDM algorithm and the nonlinear spring-dashpot model.

it can be observed that for all the materials the residuals against time describe a horizontal pattern, which indicates a good fit. Nonetheless, an ascending band is observed when the residuals are plotted against the input while a descending band is observed in the residuals plotted against $\hat{F}$. This indicates that there are first-order terms missing in time or that the model is lacking a zero order term.

On the other hand, if we look at Fig. 4.10 which shows the scatter plots obtained SDM algorithm, it shows curved bands instead of ascending or descending lines. This is an

indication of missing terms both of first and second-order in time or that the output needs to be transformed. However, the residuals plotted against time show no significant problems as it was the case of the RLS method applied to the nonlinear model.

Table 4.4: Best and worst R^2 results for the nonlinear model

Ground	NRLS		Steepest Descent	
	Best R^2	Worst R^2	Best R^2	Worst R^2
V.I. Box	0.7853	0.7491	0.6983	0.6646
Gravel	0.7756	0.6547	0.7081	0.6670
River sand	0.7175	0.6351	0.6938	0.5990
Foam	0.4408	0.4120	0.3792	0.3191

Table 4.5: Best and worst r_c results for the nonlinear model

Ground	RLS		WLS	
	Best r_c	Worst r_c	Best r_c	Worst r_c
V.I. Box	0.8822	0.8605	0.8612	0.8484
Gravel	0.8770	0.7950	0.8642	0.8438
River-sand	0.8395	0.7808	0.8577	0.8161
Foam	0.6035	0.5817	0.7114	0.6836

To evaluate quantitatively the fit produced by each method, we use the same two goodness-of-fit indexes as before. Table 4.4 and table 4.5 show the best and the worst R^2 and r_c for each material and each method. In the tables, it can be seen that the RLS method performs better than the steepest minimization algorithm in accordance with the scatter plots. However, it is interesting to note that the SDM presented less variation of values for both indexes; this is an indicative of a greater robustness to the nonlinear behavior over the NRLS method. A possible improvement would be to use higher order Taylor series term and switch to a Newton method to estimate the parameters of the model directly.

Table 4.6: Nonlinear model mean squared error

V.I. Ground	RLS Error		SDM Error	
	Mean	Std	Mean	Std
Box	1.59E-2	9.38E-5	2.38E-3	3.99E-5
Gravel	1.61E-2	1.15E-4	2.52E-3	6.92E-5
River-Sand	1.59E-2	6.02E-5	2.79E-3	1.04E-4
Foam	1.57E-2	1.92E-5	4.24E-3	5.46E-5

Moreover, as with the linear model, we show the average of the mean square error for each material over the 10 tests in Table 4.6. In this table, the SDM algorithm exhibits a

superior performance compared with the RLS algorithm. A very interesting result is that both of the algorithms applied to the nonlinear case are outperformed by the RLS algorithm used for the linear model according to R^2 , r_c and the mean squared error. This implies that they are not well suited for on-line use, although the nonlinear identification offers more robustness in addressing plastic deformation, as expected and indicated by lesser standard deviation. Another important pitfall of the nonlinear model is that, in some cases, it may have complex parameters, which can be undesirable if applied online. However, this model may be useful if one is interested in modeling the environmental behavior using simulations because the nonlinear models can model plastic deformation without a change in the parameters of the model.

4.4 Summary

In this chapter, two continuous contact models have been reviewed, along with four possibilities for identifying the parameters of the models (two for the linear model and two for the nonlinear model). For the linear model, the qualitative analysis revealed that the RLS algorithm has superior performance, while the WLS algorithm offers a smoother response at the expense of accuracy. Moreover, plastic deformation, which is unexplained by the linear model, can be effectively accounted for by changing the parameters of the linear model. Additionally, the quantitative evaluation of the linear parameter estimation method showed that the RLS algorithm has superior performance compared with the WLS algorithm, as shown by the qualitative evaluation using scatter plots.

Regarding the nonlinear model, two alternatives were explored. The first alternative is finding a linear regression representation of the nonlinear model and then treating it as a linear model. The second is directly using the equation of the nonlinear model and solving the optimization problem of minimizing the modeling error. The qualitative evaluation showed that both methods require improvements. However, these methods offer a very low standard deviation which is indicative of robustness. Thus, after some improvements they may be used to extract contact parameters to be used in the simulation.

Finally, an overall comparison of the methods shows that the RLS method outperforms all of the other methods, even though it was applied to a linear model. This shows that plastic deformation can be modeled as a variation of the parameters in each time step or at least in some instants of time.

In the next chapter, we will introduce a procedure to identify the properties of the terrain using a three DoF underactuated robotic leg.

Chapter 5

Event Driven Identification

Up until this chapter we have described various system identification techniques and compared their performance with different models and materials. In the previous chapter, we came to the conclusion that the RLS method with the linear spring-dashpot model is the best suited for an online haptic application. In this chapter, we will focus on legged locomotion, because this application has motivated this thesis. Here, we present an algorithm to recognize ground impedance using a three DoF planar leg. Then, we analyze the performance of the algorithm with four different types of ground: a rigid ground, the variable impedance box, river sand and gravel.

5.1 Identification Procedure

The identification procedure needs to address certain questions that are general for every haptic application: When should the identification start? How are the samples taken? Is it necessary or even practical to use the system identification approach during the entire operation? The goal of this section is to give an answer to the aforementioned questions.

5.1.1 Basic Considerations

From the discussion in Section 3.2 we know that there are two possible approaches to implement an environment identification procedure, namely active and passive sensing. Active sensing, although more precise, is not suitable to be used in legged robots because using a specialized control method for the identification would disrupt the robot running behavior, which might cause destabilization.

Moreover, active sensing methods are continuously performing the system identification algorithm as it is an integral part of the robot control. On the other hand, the system identification literature establishes that identifying a system in equilibrium is useless because information about its dynamic behavior cannot be obtained. This implies that the identification should start as soon as the foot touches the ground and stop when the foot leaves the ground or the contact has been stabilized. Therefore, passive sensing is the logical choice. Considering this last remark, one possible state machine diagram of the identification procedure can be envisaged (see Fig. 5.1).

Figure 5.1: State machine diagram of the system identification method

This state machine consists of two basic states: (a) the “idle” state, in which all of the matrices and vectors of the identification method are reset to their original values and wait until the leg contacts the ground; in this case, it goes to the next state; and (b) the “identify ground impedance” state, which performs the system identification according to the RLS method described in the previous section.

Note that the “idle” state is said to reset the system identification algorithm. This feature is not strictly necessary if the robot steps on the same surface more than once. In fact, using the estimates of a prior identification with a given surface might enhance the accuracy of the estimation. Nevertheless, we chose to reset the algorithm here to prepare the robot for a new surface, as using the estimates of another surface as the starting point might affect convergence, because the initial guess of the estimate can be too far off for the algorithm to compensate.

Another reason to choose a passive sensing approach is the sampling frequency. When controlling a system, the sampling frequency should be as high as possible to maintain good tracking error of the reference trajectories, regardless of the control method used. Nevertheless, in this case the control loop frequency may be too high and produce quantization errors in the identification algorithm. These errors, as discussed in the previous chapter might lead the system identification algorithm to fail because the system will appear to be in equilibrium. Thus, the sampling frequency for the identification algorithm should be chosen accordingly to reflect the changes in the system response. As a conclusion, the sampling frequency of the identification algorithm might be different to that of the control loop depending on the material to be identified.

5.1.2 Robot’s Mechanical Structure Considerations

Another important point is the effect of the robot’s mechanical structure on the identification process. Let us consider a robot during a bouncing gait in contact with the ground (see Fig. 5.2). The behavior of the robot should resemble a SLIP [Full and Koditschek, 1999, Raibert, 1986]. Nevertheless, many robots have rigid links, and the spring compres-

sion at the lowest part of the stance phase could be achieved by flexing a rigid leg. In this case, there is no need to consider deformation on the robot's links, and the state diagram described in the above section might be sufficient.

Figure 5.2: Graphical representation of a robot's behavior during the stance phase of a bouncing gait

In contrast, the robot might not be entirely constructed of rigid links, as is the case with many robots designed to run [Raibert et al., 1986, Raibert et al., 2008, Nichol et al., 2004, Semini, 2010, Garcia et al., 2011, Grizzle et al., 2009, Hurst and Rizzi, 2008]. In this case, when computing the ground penetration (x), the compression of the robot's links must be considered to obtain an accurate measure.

Figure 5.3: Graphical representation of the robot's sinking

Moreover, whenever the robot changes from one surface to another, there can be a different amount of sinking (if no prior stiffness adjustment is performed), and this information also has to be considered. By sinking we mean that the robot's support leg penetrates

the ground; changing the distance of the COM to the ground's surface (see Fig. 5.3). The reason to consider the sinking difference is that when identifying the impedance of the surface using only the interface (the contact surface) between the two systems, the computed impedance will be the series combination of the impedances of the robot and ground, as illustrated by Fig. 5.2. If sinking is not considered, then the identification will overestimate the stiffness of the system because the algorithm would perceive less ground penetration than the actual value for the same amount of force.

Figure 5.4: State machine diagram of the system identification method considering the robot structure

With these last considerations a modified version of the state machine diagram in Fig. 5.1 with a wider range of applicability can be designed (see Fig. 5.4). In this diagram, two new states have been included: an “initialization” state and “compute COM vertical position” state. The initialization state computes the initial leg position, its maximum length, and the COM vertical position when the algorithm starts. Then, the “compute COM vertical position” state is included to consider sinking. This state needs to compute the COM’s vertical displacement to pass the information to the next identification state at the beginning of the stance phase. Then, the “identify ground impedance” state can compute the COM’s vertical displacement from the moment the foot contacts the ground; thus, the ground penetration will be equal to the difference between the COM’s displacement and the leg’s compression, that is:

$$x = Z_{foot} - Z_{COM} \quad (5.1)$$

Note that to increase the adaptability to the terrain, the ultimate goal of the identification approach is not to estimate the ground properties as accurately as possible but to estimate the robot-ground interaction properties. Thus, the leg stiffness can be adjusted to hold the total robot-ground impedance constant, which is desirable because biomechanical studies in humans [Ferris et al., 1998] show that humans adjust their leg stiffness to maintain constant the total impedance, leading to more energy-efficient running.

In the next section, the system identification procedure is implemented on a robotic leg prototype [Garcia et al., 2011, Arevalo et al., 2012] and used to identify the impedance of the leg in contact with different types of ground.

5.2 Experimental Set-up

The experimental set-up consists of a three DoF robotic leg, the HADE leg (see appendix B) fixed to the wall by a sliding rail, so horizontal movements of its COM are restricted, but it is capable of moving vertically (see Fig. 5.7). Moreover, the vertical movement is limited by a mechanical top at the bottom to prevent falling when the leg is not in contact with the ground. Incremental encoders are used to measure the position of each joint and to indirectly measure the joint torque by measuring the spring deflection at each actuator and the fetlock spring. In addition, the leg implements a position control scheme based on inverse kinematics and proportional-integral-derivative (PID) controllers at each active joint (hip and knee), while ground contact is detected by monitoring the ankle spring torque. The control system is depicted in Fig. 5.5 and it was implemented using a National Instruments PXI-1024Q acquisition and control unit (Austin, TX, USA) and had an execution time of 0.5 ms per cycle; while the identification algorithm had an execution time of 5 ms per cycle. The complete set-up is depicted in a schematic diagram in Fig. 5.6.

Figure 5.5: Schematic diagram of the control law for the leg

Figure 5.6: Schematic diagram of the experimental set-up used in the experiments

Figure 5.7 shows a picture of the experimental set-up using two different surfaces. The foot was placed over each surface and then commanded to move upwards until it was not in contact with the surface. Then, it was commanded to stay in that position for 0.5 s. Afterwards, it was commanded to return to its initial position (in contact with the ground) and stay there for another 0.5 s. This procedure was repeated twice for each experiment.

Figure 5.7: Experimental set-up. Leg stepping over the variable-impedance box (right) and sand (left)

Moreover, the experiments were performed on the four different types of terrain that were used in the preceding chapter.

5.2.1 Implementing the System Identification Algorithm on the HADE Leg

Prior to the implementation on the HADE leg, it is necessary to choose values for the forgetting factor λ and the sampling period. To do this, we have applied the RLS algorithm to the data obtained in Chapter 4 with different values of λ and different sampling periods. Figure 5.8 and 5.9 show the mean square modeling error against the sampling time for the variable impedance box and the river sand respectively. The results in Fig. 5.8 and 5.9 were obtained by resampling the data using sampling periods between 0.5 ms and 40 ms; then applying the RLS algorithm to 10 data collections for each terrain; and finally taking the average MSE with each value of the sampling period.

Figure 5.8 shows that, generally, as the sampling frequency decreases (sampling time increases) the modeling error rises, except for the sampling periods between 0.5 ms and 3 ms where the error decreases. This is explained by the quantization error introduced by the

Figure 5.8: Mean Square Error vs sampling time for the variable impedance box

encoders measuring position. Nevertheless, when the algorithm was applied to the river sand, the modeling error always increases as the sampling frequency is reduced. The difference is because the river sand is less stiff, thus there are larger variations in the position for the same force; therefore, the quantization error has less influence in the result.

Figure 5.9: Mean Square Error vs sampling time for river sand

As the algorithm is intended to be used with different types of terrain, we have chosen a sampling period of 5 ms partly because it presented near minimum modeling error when used in a terrain that resembles our model. Moreover, when the algorithm was implemented in the PXI-1024Q unit higher sampling frequencies interfered with the execution of the

control loop. The problems here explained could be solved by using a computer with more computational power and with the use of higher resolution encoders.

To choose the forgetting factor, the algorithm was applied to the same data as the sampling frequency experiment, using values of λ between 0.890 and 1. Fig. 5.10 and 5.11 show the mean squared modeling error vs. λ for the variable impedance box and the river sand respectively.

Figure 5.10: Mean Square Error vs λ for the variable impedance box

In Fig. 5.10 it can be seen that values of λ between 0.89 and 0.94 lead to more or less a constant modeling error, nevertheless as λ increases from 0.940 to 0.990 the modeling error decreases until it reaches its minimum around $\lambda = 0.980$ and then increases a little between $\lambda = 0.990$ to $\lambda = 1$. In Fig. 5.11 a similar behavior is observed, although the constant behavior of the modeling error is from $\lambda = 0.890$ through $\lambda = 0.970$, then the error increases sharply and starts decreasing until it reaches its minimum at $\lambda = 0.996$ and increasing towards $\lambda = 1$.

Considering the results of the previous experiments, we have chosen a value of $\lambda = 0.996$ as it presented an error value near minimum with the variable impedance box and the minimum error value with the river-sand.

Once the values for λ and the sampling frequency were chosen, we have tested the algorithm with the same data for different forms of equation (2.6). The results of this implementation are shown in table 5.1. As it can be seen in the recursion, the implementations where direct measures are used to compute the modeling error perform similarly while in the case where indirect measures are used the performance is much worse. This results verify the theoretical discussion in Section 4.3.1 on the choice of the regression equation.

Once the parameters of the identification algorithm were chosen, to implement the system identification on the HADE leg, given the leg characteristics, it was necessary to use the state machine depicted in Fig. 5.4. Here, during the “initialization” state, it was both the total extended length of the ankle spring and the offset force had to be computed to obtain

Figure 5.11: Mean Square Error vs λ for the river sand

Table 5.1: Normalized mean square error for different forms of equation (2.6)

Parameterization	Normalized error
$x = 1/KF - B/K\dot{x}$	0.0014
$F = Kx + B\dot{x}$	0.0107
$\dot{x} = 1/BF - K/Bx$	2.66

an accurate measure. The latter is a consequence of the incremental encoders mounted on the leg and the fact that the spring starts precompressed because of the leg's weight. This implementation has been also presented in [Arevalo et al., 2013b].

To compute the ground penetration as equation (5.1), it was necessary to compute the ankle's vertical position Z_{ankle} and the foot's vertical position Z_{foot} . The ground penetration δ will be equal to the difference between these positions (see Fig. 5.12). At this moment, a limitation for testing the identification algorithm on different types of terrain at the same time arose. Because it was not possible to compute the COM displacement directly due to the limitations of the HADE leg set-up, sinking could not be considered. To bypass this problem, the algorithm was tested using only one type of soil at a time, and the leg's initial position was computed while resting on the surface to be identified, so the same amount of sinking was assumed in the calculations.

As it can be seen in Fig. 5.12, x can vary only due to flexing of the foot, even without observable ground penetration. Nevertheless, as our primary objective is to identify the total impedance of the ground-robot system this is not an issue. In other words, if the ground is perfectly rigid (or stiff enough) the algorithm will identify the leg's impedance which is a logical result as the total stiffness will converge to the leg's stiffness, because when computing the equivalent stiffness of series springs:

Figure 5.12: Identification parameters for the HADE leg

$$\lim_{K_2 \rightarrow \infty} (1/K_{eq}) = \lim_{K_2 \rightarrow \infty} (1/K_1 + 1/K_2) = 1/K_1 \quad (5.2)$$

Figure 5.13: Scaled Gaussian bell of the estimation results for the stiffness (left) and damping (right) coefficients for the V.I. box (continuous line) and the rigid ground (dashed line).

The exerted foot force was computed using the force measurements on the ankle spring only. This force was a good approximation because the torque transmitted by the spring to the ankle was the same as the torque transmitted by the ankle to the ground, as they were connected to each other through a rigid link. Nevertheless, the arm from the spring to the ankle was variable; therefore, this arm had to be computed for each force measurement. Technically, the arm from the foot to the ankle was also variable, as it depended on the area

of contact of the foot, but this variation was small because the foot touched the ground at approximately the same contact point every time.

One last thing to note about the implementation of the identification algorithm on the HADE leg is the detection of contact. As the HADE leg has no force or contact sensor at the sole of the foot, it was necessary to monitor the ankle spring. If the spring is sufficiently rigid that it does not oscillate when the leg is in swing phase, it suffices to monitor only the spring and launch the identification algorithm when its value is below zero (the force of the spring has the opposite sign from the contact force because of the manner in which it is attached to the leg). This is the case for the HADE leg, so contact is detected whenever the value of the force becomes negative.

Nevertheless, note that if the spring does oscillate, then the contact can be detected when the force reaches some threshold. The value of the threshold will depend on the oscillation, and it can also be computed in the “compute COM position” state.

In our case, the threshold was computed when the foot left the ground. This is because the encoders used were incremental and the foot started resting on the ground. This caused an offset in the force measurements when the leg was lifted.

5.3 Results

The results of the identification performed on the rigid ground and variable impedance box are presented in Fig. 5.13, which consists of the scaled Gaussian bell of the estimated total stiffness (left) with the variable impedance box (continuous line) and rigid ground (dashed line), as well as the Gauss bell of the damping coefficient (right) for the variable impedance box (continuous line) and rigid ground (dashed line). The scaling was performed as follows:

$$G = \frac{\Sigma(f)}{\Sigma(n)} f \quad (5.3)$$

where f is the Gaussian function obtained from the data; n is the distribution of the values of the histogram of the data; and G is the Gaussian bell to be plotted.

Scaled Gaussian bells were chosen to offer an intuitive representation of the results. Though they do not offer a quantitative result, the figure shows that the majority of the estimated values lay at approximately 2,750 N/m and 3,500 N/m for the variable impedance box and rigid ground respectively.

Moreover, from the curves it can be seen that the results of both grounds are differentiated, as the majority of the curves' areas do not overlap with each other. In addition, to offer a quantitative evaluation, Table 5.2 shows the mean value, the standard deviation of the estimated values, the theoretically calculated value, and the mean of the estimation error.

Note that in the case of the rigid ground, the identified stiffness corresponds to the leg's stiffness, as the series combination of two series springs tends to the less stiff (see equation 5.2). Moreover, in the experiment, the final position of the leg on the ground was the extended position, which meant that the leg's stiffness was approximately equal to the

Table 5.2: Estimation results for the stiffness of the variable impedance box and rigid ground

Ground	Theoretical (N/m)	Estimated value		Error
		Mean (N/m)	STD	Mean (%)
V.I. Box	2,639	2,801.5	68.45	9.35
Rigid	3,113	3,392.6	161.37	6.16

Table 5.3: Estimation results for the damping coefficient of the variable impedance box and rigid ground

Ground	Estimated value	
	Mean (Ns/m)	STD
V.I. Box	2.0533	0.0453
Rigid	1.7471	0.0862

ankle spring because the series elastic actuator's elastic constant was 15 times higher than that of the ankle spring.

The theoretical values given in Table 5.2 were computed with the stiffness values supplied by the manufacturer, so the tolerance error also affected the identification. The damping constant was not included in the table because this value was unknown for both the leg and the variable impedance box, and no theoretical value could be used to evaluate the performance of the identification method. However, the estimated values of the damping coefficient and their standard deviation are shown in Table 5.3.

Figure 5.14 shows the results of the identification for the sand and rocks. The figure shows the scaled Gaussian bells of the estimated values of the stiffness (left) and damping coefficients (right) for the rocks (dashed line) and the sand (continuous line). In the curves, it can be observed that the two types of ground have well-differentiated stiffness. In the case of the rocks, the majority of the points were concentrated at approximately 2,500 N/m, while in the case of the sand they were concentrated at approximately 1,100 N/m. Note that both of these terrains present nonlinear behavior, such as plastic deformation (they can be compressed due to geometrical rearrangement of the rocks or the grains of sand). Thus, the dispersion standard deviation of the estimation should be larger than in the previous case.

Regarding the damping coefficients, it can be inferred from the curves that the damping coefficients of both types of terrain were relatively close. In the case of the rocks, most of the estimated values were approximately 300 Ns/m (see Fig. 5.14 right), while in the case of the sand, the values were concentrated at approximately 500 Ns/m. To provide quantitative results, the mean and standard deviation of the estimated values for the rocks and sand are presented in Table 5.4.

Table 5.4 demonstrates that the standard deviation of all quantities was high compared to their respective mean values. This result indicates unmodeled behavior, which was to be expected because the model used in this work was linear, whereas terrain behavior is not. Nonetheless, they had well-differentiated values of stiffness. Expressed mathematically,

Table 5.4: Estimation results for the sand and rocks

Ground	Estimated stiffness		Estimated damping	
	Mean (N/m)	Std	Mean (Ns/m)	Std
Rocks	2,436.9	761.81	281.01	84.13
Sand	1,181.1	276.60	528.22	124.85

Figure 5.14: Scaled Gaussian bell of the estimation results for the stiffness (left) and damping (right) coefficients for the rocks (continuous line) and the river-sand (dashed line).

$$\overline{K}_{sand} + std(K_{sand}) < \overline{K}_{rocks} - std(K_{rocks}) \quad (5.4)$$

where $\overline{K}$ indicates mean value of the samples and $std(K)$ indicates standard deviation. In contrast, the damping coefficients did not appear useful for discriminating between the terrains. Nevertheless, these coefficients did indicate the energy losses during locomotion, so they can be used to compensate for that energy.

The value of the standard deviation for the rigid ground and adjustable-impedance box was approximately 2% of the mean, while the values for the rocks and sand were between 23% and 30%. The standard deviation was relatively high in both cases because there was unmodeled behavior due to motor transmissions, joint friction, and other factors that are intrinsic to the leg's mechanical system. Nonetheless, the method still presents more accurate results with the linear terrains than with the nonlinear terrains.

5.3.1 Convergence Time

Another important aspect of identifying the terrain contact properties with a legged robot is the convergence time. This parameter is important because the identification should be performed completely within the stance phase of any particular leg.

Figure 5.15 shows the average values of the components of the vector of parameters used in the identification of sand versus time. Note that the figure shows the values of $\theta = [1/K, B/K]$, which was intentional because the vector of parameters was initialized with $\theta_1 = 1/K = 0$, implying that $K \rightarrow \text{inf}$, which might have been confusing for representation purposes.

Figure 5.15: Estimated values of the parameters $1/K$ (dashed line) and B/K (solid line) obtained by applying the identification algorithm on sand versus time.

The parameters $1/K$ and B/K reach convergence approximately at $t = 0.3s$. In contrast, biomechanical studies show that the contact time of humans when running on rigid and soft elastic surfaces at a speed of 3.7 m/s is approximately 0.2 s [Kerdok et al., 2002]. McMahon in [McMahon and Greene, 1979] reported that the contact time for a human running over pillows at a speed of 4 m/s was over 0.3 s. Although it is true that contact time decreases with increasing speed, these results show that the algorithm can be used for running at moderate speeds (on viscous terrains). In addition, it is important to note that the average velocity of legged robots is below 2 m/s (with the exception of Boston Dynamics' Cheetah).

Moreover, the convergence time differed as the impedance varied (i.e., with different systems). Figure 5.16 shows the values of the estimated parameters with the rigid ground. The convergence time was approximately 0.15 s, which was lower than the value measured by Kerdok et al. [Kerdok et al., 2002]. This value varies with speed, so the proposed algorithm can be used at relatively high speeds as long as the contact time is higher than 0.15 s.

The above discussion is valid with an ankle spring with $K_{spring} = 3,113$ N/m and with the execution time of 5 ms. If the sampling frequency is increased then the convergence time can be reduced. However, as it was stated in previous sections this would require encoders with higher resolution.

Figure 5.16: Estimated values of the parameters $1/K$ (dashed line) and B/K (solid line) obtained by applying the identification algorithm on a rigid ground vs time.

5.4 Summary

The algorithm presented on this chapter identifies the combined impedance of the leg and ground simultaneously. Moreover, it considers several aspects, such as ground modeling, the effect of the mechanical structure of the robot, and sampling frequency. It was implemented on the HADE leg (an underactuated 3-DoF planar mechanism) and it was experimentally evaluated.

The results of the evaluation are encouraging, as the algorithm offers good convergence and tracking when applied to elastic terrains (a rigid ground and adjustable impedance box). Moreover, the convergence time of the algorithm was analyzed, comparing the convergence time with the published data of contact time in running humans. This comparison shows that the algorithm is faster than the contact time of human running (3.7 m/s) in rigid grounds. Thus, for rigid surfaces it can be used at relatively high velocities if the contact time is higher than 0.15 s.

In the case of unknown dissipative terrains (sand and rocks), the algorithm is capable of producing well-differentiated estimations of their respective stiffness. However, the results of the damping coefficient need further improvement. Moreover, even though it is capable of determining that the terrains are different the linear model does not capture much of the non linear phenomena that occur in those terrains. This is explained by the nature of these environments, although our results produce differentiated estimations, the assumptions of the model are incorrect; sand and rocks are granular media. Thus they exhibit some properties of solid matter, which can be captured by the spring-dashpot model and some properties that are characteristic of liquid matter, which cannot be described by our model. To address some of these issues, in the next chapter a new contact model is proposed.

Chapter 6

Proposed Contact Model

Over the last two chapters we have seen that it is possible to approximate the nonlinear behavior of some materials using the RLS method and the linear spring-dashpot model with a good degree of accuracy. Nevertheless, there is still unexplained behavior. In this chapter we propose a new contact model that is capable of modeling many of the nonlinear effects of some of the terrains used in this thesis. As mentioned before, the unexplained behavior is due to the granular nature of rocks and sand. In this chapter, we review some of the literature on granular media and give a linear model capable of capturing phenomena of special interest to haptic perception in general and to the walking application specifically.

6.1 Granular Media

As we mentioned before, the spring dashpot model has a wide range of applications. Nonetheless, many of the objects that we find in everyday life can be classified or modeled as granular media, e.g. salt, sugar, seeds, talk, etc. For that reason, throughout this section we will attempt to build a model that can be applied to such type of material. Granular media exhibit extremely complex behavior that resembles different states of matter, i.e. solid, liquid and gas. Although granular media has many engineering applications it remains poorly understood [Andreotti et al., 2013]. There is no theoretical framework capable of describing all of the behaviors exhibited, even in the case of a granular medium composed of identical spherical particles. In [Andreotti et al., 2013] the authors give a few clues that can explain why these materials are so difficult to describe:

- **Granular media are composed of a large number of particles.** With the help of today computers, the simulation of a large number of particles is not a strong barrier. Nevertheless, consider that a single teaspoon of sugar contains close to 10^6 particles; therefore a much larger scale event such as an avalanche poses a great challenge even with present-day computing technology.
- **Thermal fluctuations are negligible.** Many materials with large number of particles have been analyzed and are well understood (gases and fluids). Furthermore, the number of molecules present on such materials is much higher than the number of particles in most granular media applications. Nevertheless, the size of the particles

in a granular media is too large to experience Brownian motion (see appendix A) and statistical averaging is not possible [Andreotti et al., 2013].

- **Lack of scale separation.** The size of the particles in a typical granular medium makes it difficult to separate the microscopic scale (grain size) from the macroscopic scale (size of the flow). In other words, it is difficult to differentiate between the size of grain and the distance traveled by the grains moving in a flow; contrary to liquids where the size of the molecules is small enough to consider the flow a separate phenomenon. In this case, the sizes of the particles play a role in the behavior of the material.
- **Interactions between grains are complex.** The interactions between grains involve highly nonlinear phenomena such as inelastic shocks, friction and capillary forces.
- **Granular media easily dissipate energy.** When an object collides with granular media the object's kinetic energy is dissipated almost instantly, which highly differs from classical systems.
- **Granular media can exhibit different states of matter.** Granular media can behave in different ways depending on its handling. It can flow, it can sustain stresses or it can become a gas when strongly agitated. Furthermore, all these regimes can coexist in a single configuration.

Although granular media can exhibit gas like behavior, we are more interested in the intermediate between solid and liquid. Because this characteristic is shared by other disordered materials (foams, emulsions and pastes) formed by mesoscopic elements (see appendix A) such that the thermal fluctuation is negligible compared to particle interaction [Andreotti et al., 2013].

As mentioned before, particle interaction is extremely complex because many types of forces are present and their dominance depends on many factors. There are solid contact forces such as elastic contact, solid friction, collisions both elastic and inelastic; cohesion forces such as electrostatic effects, adhesion, capillary cohesion, the formation of solid bridges; flow-like forces such as viscous damping, Archimedes force, the Magus force (see appendix A). Furthermore, the previous list is by no means comprehensive (see [Andreotti et al., 2013]).

Nonetheless, there are a few works that show that it is possible to model granular media as a continuum, at least in some configurations [Andreotti et al., 2013, Jiang and Liu, 2007, Jiang and Liu, 2009]. Most of these works are based on the notion of an averaging function that has an integral over all the space equal to one, has a maximum at zero and vanishes outside a given volume. One averaging function commonly used is a Gaussian function with zero mean and width W [Andreotti et al., 2013] to be an averaging function:

$$G = \frac{1}{\pi W^2} e^{-\frac{\|\delta\|^2}{W^2}} \quad (6.1)$$

where δ is the vector of positions of the particles. Then, after writing the equilibrium equations of each grain the integral is taken around the point of interest weighting each

individual equation with the averaging function. This process leaves us with a density and a velocity of the equivalent continuum medium of the form [Andreotti et al., 2013]:

$$\rho(\delta, t) = m \sum_{i=1}^N G(\delta - \delta^i(t)) \quad (6.2)$$

$$\rho \mathbf{v}(\delta, t) = m \sum_{i=1}^N \mathbf{v}^i(t) G(\delta - \delta^i(t)) \quad (6.3)$$

where N is the number of particles; the super-index i denotes each individual particle; $\mathbf{v}$ is the velocity of the particles; m is the mass; δ^i and $\mathbf{v}^i$ are the position and speed of each particle COM [Andreotti et al., 2013].

This result is very important because it allows treating a granular medium as a single element rather than a collection of a large number of particles.

Throughout the various studies, there is one quantity used extensively that can be used to give an idea of what type of behavior should be expected. The volume fraction Φ is defined as the ratio of the volume occupied by the grains to the total volume occupied by the packing:

$$\Phi = \frac{V_{grains}}{V_{total}} \quad (6.4)$$

Note that Φ is used here to denote the packing fraction to avoid confusion with the notation used in other chapters. However, in the literature the letter ϕ is more commonly used to denote this quantity and there are other equivalent quantities defined in different fields.

The packing fraction, although purely geometric by definition, can serve as an indicator of which forces are dominant in a granular packing. It has values from zero to one, where one means that the volume is totally occupied by the grains (e.g. cubical particles stacked). At higher values of Φ the grains are closer together; thus contact and capillary forces are more important than collisions for example. Moreover, in [Li et al., 2009] Li et al. discovered a dependence of the motion of a legged robot on the packing fraction.

Taking as a starting point the equations of conservation of energy, momentum and matter, and using Φ to define a switching point some theoretical frameworks have been developed to describe granular media [Andreotti et al., 2013] as a continuum. One example is the granular solid hydrodynamics [Jiang and Liu, 2009]. These works are of extreme interest because some parallelism with solids and fluids can be traced.

An important parallelism is the observation of lift forces and Archimedes' law. Guillard et al. [Guillard et al., 2014] observed lift forces that increased linearly with depth that scale in the same manner as buoyancy with a large amplification. On the other hand, using an excluded volume model the Archimedes' law was verified in [Maes and Thomas, 2011]. Moreover, before the fluid limit of the medium, buoyancy was also observed. Nevertheless, some memory properties needed to be added to Archimedes law. In other words, the motion of an object subject to a lift force in a granular medium depends on its past position [Maes and Thomas, 2011].

With the characteristics described above, in the following section we propose a model that can exhibit some of the most relevant properties of granular media.

6.2 Proposed Model

The model proposed here is developed parting from the following idea: The stiffness and viscosity of a granular medium depends on the volume fraction [Andreotti et al., 2013]. However, the packing fraction changes according to the different stimuli a granular medium can receive (vibration, compaction, shear stress, etc.). In particular, we are interested in the behavior of the packing fraction in response to compression. The change of the volume fraction as a response to compression can be modeled by Heckel's law [Heckel, 1961]:

$$\ln \left(\frac{1}{1 - \Phi} \right) = QP + C \quad (6.5)$$

where Q and C are constants and P is the pressure. Since Heckel's law is an empirical law, let us take $C = 0$ and Q as the area of contact scaled by some scalar α ; thus $QP = \alpha F$. Taking the exponential function at both sides we have:

$$\frac{1}{1 - \Phi} = e^{\alpha F} \quad (6.6)$$

$$\Phi = 1 - e^{-\alpha F} \quad (6.7)$$

To find a linear approximation of the exponential term in equation (6.7) we take the Taylor series expansion, which yields:

$$\Phi = 1 - 1 + \alpha F - \frac{(\alpha F)^2}{2!} + \dots + \frac{(\alpha F)^n}{n!} \quad (6.8)$$

$$\Phi \approx \alpha F \quad (6.9)$$

Now, recall that the apparent mechanical impedance of the granular packing depends proportionally on the change of Φ . To model the contact properties, let us divide the ground penetration x in two components: a linear contribution given by the elasticity of the material, i.e. a Hookean spring; and a nonlinear part that includes all the other forces that come into play in granular media. As we have seen before, granular media is capable of dissipating energy very rapidly and that dissipation is dependent on Φ . Moreover, the energy dissipation implies a rearrangement of the particles to accommodate the object that is in contact with the medium. In other words, it behaves like a liquid when an object is thrown at its surface. Now, if we consider the original surface of the granular material this penetration can be modeled as a permanent indentation that is a function of Φ .

$$x = \frac{F}{K} + y \quad (6.10)$$

$$y = f(\Phi) \quad (6.11)$$

There is no strict definition in the literature of $f(\Phi)$, but there are certain considerations that can help in the choice of the function. First, by definition $0 \leq \Phi \leq 1$; at the lower limit ($\Phi = 0$) there are no particles to stop the penetration in the medium, which is a case that has no relevance in this study. Therefore we will consider $0 < \Phi \leq 1$.

On the other hand, if we take $\Phi = 1$ we would be in presence of a perfectly occupied space by all the particles. This means that we are in presence of a solid. Consider for a moment that the granular media is made of steel; when an object collides with the medium it could make a dent on the surface meaning that some energy is lost in terms of plastic deformation; or it could bounce and behave as a perfectly elastic collision. On the contrary, if the granular medium is made of rubber particles, then the behavior would be completely different, the energy loss during the impact would be because of a damping effect, or it could not be energy loss at all. The point of this discussion is the following, $f(\Phi)$ should contain a term that depends on the material. Furthermore, the geometry of the particles also needs to be taken into account, as it would be more difficult to perfectly occupy the space (or as close as possible) if the particles are spheres with different sizes or perfect cubes.

Taking into consideration the previous discussion, a first approximation of y can be constructed as follows:

$$y = \frac{\beta}{\Phi} f(E) \quad (6.12)$$

In the above equation β denotes the material-dependent constant and $f(E)$ denotes the energy loss. The variable y represents the nonlinear part of the contact in the model; thus, the logic behind including an energy term is the following:

When a force is applied to a granular medium it can suffer a variety of changes. At low stress levels, the grains undergo a few rearrangements, which leads to a small amount of compaction. When the stress increases, the grains begin to deform and that changes the packing fraction. When the force is low, the deformations are initially elastic. If the force continues to increase, then the grains undergo large plastic deformation and may even break; this leads to more rearrangements and a significant amount of compaction, i.e. an increase in Φ [Andreotti et al., 2013]. The rearrangements described before imply changes in the energy of the individual particles, and so does the plastic deformation and the breakage.

To include the aforementioned effects we need to include a quantity that is capable of representing the changes on the velocity during a collision. According to classical dynamics such quantity is the impulse and it is related to the force by the Impulse-Momentum Theorem. The reason for choosing the impulse and not other movement based quantity such as the kinetic energy is that the kinetic energy is not preserved in a collision, and in our case the objects actually interact for a long time in a very complex manner. Moreover, as the impulse is by definition the time integral of the force in a period of time, it can give us the memory effect present in granular media [Guillard et al., 2014]. Furthermore, using the impulse it is possible to obtain an estimation of the energy lost in the collision by means of the velocity.

Taking into account the previous discussion, we write the packing fraction as:

$$f(\Phi) = a \int_{t_0}^t F dt \quad (6.13)$$

where a is $\frac{\beta}{\Phi}$. Moreover, it is also important to mention that we could have chosen to use a closed integral of the force in terms of the displacement to use the mechanical work required to compress the material. However, in previous experiments performed with the data from Chapter 4 this has proven to be ineffective in terms of goodness-of-fit, because the material is not confined [Andreotti et al., 2013] and the movement of the particles outside the area of contact makes it difficult to use a closed integral as a measure of energy loss. Taking the trapezoidal rule to convert (6.13) into discrete time, we have:

$$f(\Phi)_{k+1} = f(\Phi)_k + \frac{ah}{2}(F_k + F_{k-1}) \quad (6.14)$$

To complete the model, we plug the value of $f(\Phi)$ into (6.11), which yields:

$$y_{k+1} = y_k + \frac{ah}{2}(F_k + F_{k-1}) \quad (6.15)$$

$$x_{k+1} = bF_k + y_k \quad (6.16)$$

Now that we have completed the model, we will perform a preliminary evaluation using some heuristics to help with the fitting. A more complete evaluation of the performance will be given in the next chapter.

6.3 Preliminary Evaluation of the Proposed Model

In Chapter 4 we applied the force pattern shown in Fig. 4.5 to different materials, two of them obvious granular media (rocks and river sand) and a third classified as porous media (polyurethane foam). Now, polyurethane foam, although not strictly a granular material, exhibits a behavior that is very similar [Andreotti et al., 2013, Li et al., 2009]. After applying the force pattern, a recursive least squares method was used to estimate the contact properties of the materials according to the spring-dashpot model. The recursive method showed good results in terms of data-fit, however, when Ordinary Least Squares were used to fit the data of the whole experiment to the spring-dashpot model, the data-fit was poor.

Figure 6.1 shows the ground deformation (solid line) and the predicted deformation (dashed line) obtained with the LS method when applied to river sand. As it can be seen on the figure the measured deformation exhibits a highly nonlinear behavior. This non-linear behavior has a very complex explanation, as we have detailed in the previous section; and it cannot be accurately captured by the spring-dashpot model [Giraldi and Dharf, 2002].

Now, to apply the proposed model, let us assume that the sand in the box is jammed, i.e. it does not flow; thus elastic forces and friction between grains are more important than flow-like forces. In [Andreotti et al., 2013] it is shown that the apparent stiffness of a granular packing depends on Φ , and as it increases, the apparent stiffness of the packing also

Figure 6.1: Measured ground deformation (solid line) and predicted deformation (dashed line) against time using Least Squares method and the spring dashpot model

increases. Nevertheless, to apply the model (see equation 6.16) to the data used to perform the comparison in 4 it was necessary to include a switching threshold on the parameters of the model because the difference in the input between $t < 30$ s and $t \geq 30$ s is too big; thus creating very different behaviors in the material. This evidenced that the model can be used for a given pushing force, but that further study is needed to extend it to different forces. By including this variable as follows, the fitting improves drastically:

$$\begin{aligned}
 y_k &= y_{k-1} + a_1(F_k - y_{k-1}) & F_k < 90 \\
 x_k &= b_1 F_k + y_k \\
 y_k &= y_{k-1} + a_2(F_k - y_{k-1}) & F_k \geq 90 \\
 x_k &= b_2 F_k + y_k
 \end{aligned} \tag{6.15}$$

Notice that the switching threshold at $F = 90N$ is arbitrarily defined, for the moment, because of characteristics of the input signal used. As it can be seen in Fig. 4.5, the force switches from a maximum of $70N$ to $100N$ and as the force increases, the packing fraction of the sand under the actuator is changed; thus, the apparent impedance of the medium changes.

Figure 6.1 shows the data-fit using regular least squares with the spring-dashpot model (dashed line) overlapped with the ground penetration (solid line); while Fig. 6.2 shows the data fit using the proposed model (dashed line) overlapped with the measured output (solid line). The ground penetration was measured as the position of the end effector of the actuator. It is important to note that this approximation is valid in this case because the experiments were conducted with the actuator starting in contact with the material. Notice

Figure 6.2: Measured ground deformation (solid line) and predicted deformation (dashed line) against time using the model in equation (6.15)

that, although the model is linear, by including the impulse as a measure of the energy loss we are able to model the plastic deformation with a certain degree of accuracy.

Furthermore, let us compare the proposed model with the linear spring dashpot. Note that both models are constructed following a very similar idea: they have an elastic constant that takes charge of the storage and restitution of energy and a term that dissipates energy. To compare both models, we will take a control-systems approach because it allows to intuitively see the similarities without explicitly solving the equations with no loss of generality. Let us plug equation (6.11) into (6.10) and find the transfer function of F for our model:

$$a \int_{t_0}^t F dt = x - \frac{F}{K} \quad (6.16)$$

$$aF + \frac{1}{K} \frac{dF}{dt} = \dot{x} \quad (6.17)$$

taking Laplace transform and using F as the output:

$$F \left(a + \frac{1}{K} s \right) = x s \quad (6.18)$$

$$\frac{F}{x} = \frac{K s}{K a + s} \quad (6.19)$$

$$(6.20)$$

while the spring-dashpot transfer function is:

$$\frac{F}{x} = K + B s \quad (6.21)$$

According to control theory a system such as the ideal spring dashpot cannot be constructed as it requires knowledge of the future or infinite energy [Ogata and Yang, 1970] (it has more zeroes than poles). In other words, the ideal spring dashpot has an increasing gain over all the possible frequencies. However the transfer function of our model, not only can be constructed, but it is, in fact, the transfer function of what would be a realizable differentiator system. The point of this discussion is that even though both models were devised with the same idea, the model proposed here resembles more a system that can be found in practice whereas the linear spring-dashpot is purely theoretical.

6.4 Including the Proposed Contact Model in the Model of a Robot

Before further assessing the validity of the model, let us give an example of how to include it in a robotic application analog to the one depicted in Fig. 3.2. Recall from Section 3.1 that the model of a robot without friction is given by:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C} = \boldsymbol{\tau} + \mathbf{J}^c \mathbf{F}_c \quad (6.22)$$

Thus, to use the contact model here presented, for a perfectly rigid robot, it suffices with substitute F_c with the contact model in continuous form:

$$F_c = (x_e - y)K_e \quad (6.23)$$

$$y = \frac{\beta}{\Phi} \int_{t_0}^t F_c dt \quad (6.24)$$

where the subindex e indicates environment. This equation can be solved numerically when the constraint $x_e = x_r$ is imposed. In the case that the robot is not perfectly rigid, it is necessary to follow a similar analysis to the one presented in Chapter 3. Instead of using the contact model here presented to define F_c it is necessary to include an environment contact force F_e that satisfies (see Fig. 6.3):

$$F_c - F_e = 0 \quad (6.25)$$

where F_e is now the force of the proposed model and F_c is described by equations (3.10) through (3.14). This set of equations can be solved for the accelerations and the contact forces numerically.

Figure 6.3: Robot/environment contact model with granular media

6.5 Summary

Granular media are highly nonlinear and exhibit solid, fluid and gas like behavior. Additionally, many of the simplifications that are done to treat gases and fluids as continuum mechanisms do not hold in the case of a granular material. However, as we have seen along this chapter, it is possible to consider granular media as a continuum.

Nevertheless, as the interactions between the particles are so complex, to describe the behavior of such materials is extremely difficult, even in the case of spherical particles. Because granular media has so many applications to engineering and science, there has been much effort into predicting its behavior, specially in the case of a granular packing. One interesting development that helps treat a granular medium as a continuum is the Packing Fraction. Using this quantity and a few considerations based on the literature of granular media and mechanics of continuum, we have proposed a model based on the impulse and the packing fraction to approximate the behavior of a granular material in contact with a robot.

However, even though this model has proven to approximate quite well the behavior of river sand, it still needs some work to be able to describe a wide range of forces. The next chapter will be dedicated to explore the dependency of the model on the force and to give a complete model with a meaningful comparison using different actuators and materials.

Chapter 7

Experimental Validation

In this chapter we will first validate the proposed contact model for a wide range of forces by performing goodness-of-fit evaluations with different forces. Then, a system identification algorithm will be used to extract the properties of the model for each individual force value. After performing the identification, we will give a parameter function dependent on the force to extend the validity of the model to a range of forces. Finally, the model will be tested under different conditions: two different granular materials, two different end-effectors and two different actuator's dynamics.

7.1 Experimental Validation on River Sand

7.1.1 Experimental Setup

To verify the behavior of the model under a wide range of forces, we have performed experiments under forces from 10 N to 400 N on river sand. For the data acquisition we have used the same set-up as in Chapter 4: a Yobotics Inc. series elastic actuator (SEA) with a Force PID controller implemented (see Fig. 7.1). The actuator has two linear encoders with a resolution of 500 counts per inch; resulting in 2000 counts per inch in quadrature encoding. The actuator has a spring between the ball-screw and the end effector to decouple the motor inertia to the load. The SEA is fixed to a vertical column so the end effector is touching the surface of the sand. Then, a series of square force pulses is commanded for five seconds with amplitudes from 10 N to 400 N in increments of 10 N.

Recall that, as seen in the previous chapter, sand exhibits highly non-linear behavior. That behavior is correlated to the applied force; thus, we have fitted a simple linear model, for each force amplitude, in the experiments.

Additionally, note that the basic system identification problem can be written as follows [Ljung, 1999]:

$$\theta = \underset{\theta \in \Theta}{\operatorname{argmin}} J(\hat{x}(\theta, u), x) \quad (7.1)$$

where $J(\hat{x}(\theta, u), x)$ is a cost function that depends on the observed values x and the predicted values $\hat{x}(\theta, u)$; which in turn depends on some parameters θ and the inputs of the

Figure 7.1: Experimental set-up consisting on a SEA pushing on a wooden box filled with river sand

system u (although it can depend on the current and past values of x as well).

The approach we followed in the first chapters to system identification was to write x as an auto-regressive process and minimize a quadratic cost for a given data set. However, to try to obtain a response that works reasonably well for all the experiments in different initial conditions, we have decided to modify slightly the cost function of the basic system identification problem to perform the identification on several tests at once. This idea was taken from [Mortdatch et al., 2015] where a similar set-up was used to synthesize trajectories of a biped robot with an unreliable model. Following this idea the cost function used in this work can be written as follows:

$$J = \sum_{i=1}^N \| X^i - \hat{X}^i \|_2^2 \quad (7.2)$$

where X^i is the vector of the observed ground penetrations of the $i - th$ experiment for a given force amplitude; and $\hat{X}^i$ is the vector containing the estimated values for the same given experiment. Mathematically:

$$\hat{X}^i = [\hat{x}_0^i, \hat{x}_1^i, \dots, \hat{x}_T^i] \quad (7.2)$$

where $\hat{x}_k^i$ is computed using F_k according to equation (6.16). To perform the validation of the model, we have run ten repetitions for each value of force, giving a total of 400 tests. Of these experiments, we used eight randomly selected tests (320 total) to perform the numerical optimization according to equation (7.2) and a curve fitting using Matlab's curve-fitting toolbox to obtain a dependency on the force of the parameters a and b . The remaining two (80 total) to evaluate the results with goodness-of-fit criteria.

7.1.2 Curve Fitting

As we have said before, the model proposed in Chapter 6 is valid for only an operating point, i.e. a given force value. To extend the model to different force values we have performed a curve fitting on the result of the above explained minimization over a range of forces. Fig. 7.2 shows parameter a of the contact model Vs. input force (dots), with a curve fitting overlapped. Recall that:

$$a = \frac{\beta}{\Phi} \quad (7.3)$$

From the figure it can be observed that the behavior of a is as expected: When a higher force is applied to the medium, the packing fraction Φ comes closer to 1; thus, a decreases, which in turn reveals an increase of the apparent stiffness and a decrease in the energy dissipation due to the rearrangement of the materials. This rearrangement is not purely geometric, although a geometric arrangement is indeed happening, it also includes changes in other factors, such as more contact surface which implies more friction, plastic deformation of the grains, formation of solid bridges, breakage of the particles, etc.

Figure 7.2: Parameter a of the proposed contact model against values of F_{max}

Figure 7.3: Parameter b of the proposed contact model against values of F_{max}

In Fig. 7.3, the parameter b is shown. Intuitively, it should decrease as the force increases. To analyze the behavior of this parameter, let us rewrite equation (6.16) as:

$$b = \frac{x - y}{F} \quad (7.4)$$

If we look at the graph of the position difference $\|x - y\|_2$ against input force (see Fig. 7.4) a quantitative explanation can be found. From $F = 10$ N to $F = 100$ N the position difference is from the order of $1e-8$ (three orders of magnitude lower than the value of x) which means that the value of b is not detectable by the minimization algorithm; or in other words, the behavior of the contact model is dominated by the packing fraction term. Now, recall that b is in fact the inverse of the Hookean part of the contact model:

$$b = \frac{1}{K} \quad (7.5)$$

Then, as the force increases, the packing fraction also increases which means that there is more volume occupied by the granular particles. This causes the effect of the spring K to become more important, because there is more contact surface. This increase behaves non-linearly until it reaches approximately $F = 210$ N. From this point onwards, the position difference between $x - y$ increases almost linearly with an increase of the force. This explains the behavior of b in Fig. 7.3 where the parameter increases and then stabilizes at a certain value.

It is important to note that, in this particular case, the SEA spring is connected in series with the spring K in the contact model. Therefore, what we are measuring is the series combination of both the SEA spring and the spring in our model. However, as the SEA spring constant is known, K is easy to compute from the value of b .

Figure 7.4: Squared module 2 of the difference between x and y

Table 7.1 shows the polynomial structure of each parameter of the model while Table 7.2 shows the parameters obtained by the tool. With the results of the curve fitting we can write the complete contact model that includes the hardening effect of the sand:

Table 7.1: Equations of the curve fitting for each parameter of the proposed model

a	$p_1 F^4 + p_2 F^3 + p_3 F^2 + p_4 F + p_5$
b	$q_1 F^2 + q_2 F + q_3$

$$a = p_1 F_{max}^4 + p_2 F_{max}^3 + p_3 F_{max}^2 + p_4 F_{max} + p_5 \quad (7.5)$$

$$c = q_1 F_{max}^2 + q_2 F_{max} + q_3 \quad (7.6)$$

$$y_k = y_{k-1} + a(F_k - F_{k-1}) \quad (7.7)$$

$$x = bF_k + y \quad (7.8)$$

7.1.3 Goodness-of-Fit

In Chapter 2 we have reviewed some of the extensive literature on the subject of assessing goodness of fit [Ljung, 1999, Kvålseth, 1985, Vonesh et al., 1996, Spiess and Neumeyer, 2010, Motulsky and Christopoulos, 2004]. As a first measure of goodness-of-fit, we use the autocorrelation (R) of the residuals to check for independence between the current and past predictions [Ljung, 1999].

Table 7.2: Estimated parameters by Matlab's curve fitting tool

p_1	2.180e-6
p_2	-1.770e-6
p_3	-4.797e-6
p_4	6.765e-6
p_5	4.972e-6
q_1	-5.504e-11
q_2	3.385e-8
q_3	-1.400e-7

Figure 7.5: Autocorrelation of the residuals against time delay

Figure 7.5 shows R for values of $F_{max} = 10N, 200N, 400N$. In the figure, it can be seen that for low values of F_{max} , autocorrelation does not decrease; thus the model performs less successfully. However, as F increases the residuals begin being independent of each other at $t \approx 0.02s$, which means that as F increases the fit improves. The reason why the model does not perform very good at low forces is because granular media resembles a liquid and the apparent surface tension is easy to break with slow movements; given that the input forces have the exact same frequency, lower forces exhibit lower accelerations and, in consequence, low velocity. Moreover, at very low forces the dynamics of the actuator could be playing an important role in the measures.

To corroborate the result provided by the autocorrelation, we make use of the R^2 criterion [Kvålseth, 1985]. This quantity is more useful when it is used to compare different models. Here, we use the null hypothesis as a comparison, i.e. a model that has no relation between input and output; thus, in our case R^2 is always zero for the null hypothesis. As we have seen in Chapter 2, there are several implementations of the R^2 criterion, here we

make use of the following three [Kvålseth, 1985]:

$$R_1^2 = 1 - \frac{\sum (x - \hat{x})^2}{\sum (x - \bar{x})^2} \quad (7.9)$$

$$R_2^2 = 1 - \frac{\sum (x - \hat{x})^2}{\sum (x)^2} \quad (7.10)$$

$$R_3^2 = \frac{\sum (\hat{x})^2}{\sum (x)^2} \quad (7.11)$$

where $\hat{x}$ is the predicted value of x and $\bar{x}$ denotes its mean.

Fig. 7.6 shows the value of R_i^2 against the value of F_{max} . This graph establishes an intuitive measure of the confidence region of the fit. Moreover, it corroborates the previous statement that the model improves as F_{max} increases (the closer R^2 is to 1 the better the fit is). More importantly, from $F_{max} = 100N$ onward all three implementations of R^2 imply that the confidence interval of our results is high.

Figure 7.6: Three implementations of the R^2 criterion against values of F_{max} . R_1^2 (circles), R_2^2 (squares), R_3^2 (dots)

As a last measure of the goodness of fit Fig. 7.7 shows the mean square error as a percentage of the maximum displacement x against F_{max} . This computation was performed using this formula:

$$mse = \frac{\sum (x - \hat{x})^2 / N}{\sum (x)^2 / N} \quad (7.12)$$

In Fig. 7.7, it can be seen that for $F < 100N$ the error is high. We have given a possible explanation for this, but given the extreme complexity of the medium, further studies are needed to reduce it. Nonetheless, as our objective is to give a model that can be used for haptic perception, an error of 12% is acceptable in many applications. Note that in the case

Figure 7.7: Average of the Mean squared error over the 10 repetitions of each experiment against value of F_{max}

of a walking robot, the model can be used with river sand whenever the robot weights more than 21 Kg (or produces more than 210 N of force). However, as we will discuss further below, this limitation depends on the material and robot structure.

Figure 7.8: Autocorrelation for glass beads (left) and calibrated sand (right) of a square wave

7.2 Performance with Different Materials

To evaluate the validity of the model to general granular media, we have repeated the above experiments with two different materials: calibrated sand and glass beads.

Fig. 7.8 shows the autocorrelation using a square wave of three different values of force (10 N, 200 N and 400 N) for glass beads (left) and calibrated sand (right). In general, the performance of the method is quite similar: as the force increases, the correlation between residuals decreases. However, the calibrated sand shows a more intuitive behavior than the glass beads. This can be explained because the grain interactions have more friction which allows the material to exert more resistance to the actuator as it is pushing the surface of the material.

Figure 7.9: Test force patterns used to test the model. Square wave (dashed line). Sine wave (solid line)

Note that the glass beads, although they exhibit a lower correlation when using a 400 N force than when using a 200 N force the value does not change as significantly as the sand. This behavior is explained again by the interaction forces between grains. The beads have much less friction than the sand grains; thus if you think of the materials as liquids, it can be seen as a non-Newtonian fluid vs. a Newtonian fluid (see appendix A). In this case, the non-Newtonian fluid is the sand. In other words, as the friction coefficient is higher for the sand grains, it can exert more resistance to shear forces than the glass beads. To further explore this behavior, in the next section, we perform experiments with different end effector diameters (to change the shear forces to compression forces).

7.3 Performance with Different Waveforms

Before exploring in detail the behavior of sand and glass beads with respect to actuator geometry and dynamics, we will further analyze the validity of the model and the identification method. To do this, we have repeated the valid experiments performed with glass and a square wave with a sine wave in the same amplitude pattern: a 1 Hz sine wave with amplitude from 10N to 300N in steps of 10N. The patterns used are shown in Fig. 7.9 for

the 10 N case, the other cases are the same but with the amplitude multiplied. After doing the experiments, we have applied the same identification algorithm to eight randomly selected experiments for each force value and compared the obtained parameters.

Figure 7.10: Autocorrelation for glass beads (top-left) and calibrated sand (bottom-left) of a square wave and autocorrelation for glass (top-right) beads and calibrated sand (bottom-right) with a sine wave

Figure 7.10 shows the autocorrelation using a square wave of three different values of force (10 N, 200 N and 400 N) for glass beads (top-left) and calibrated sand (bottom-left); and the autocorrelation using a sine wave of three different values of maximum force (10 N, 200 N and 400 N) for glass beads (top-right) and calibrated sand (bottom-right). When using a sine wave, as it happened with the square wave, the autocorrelation of the residuals decreases as the maximum value of the force increases. However, when looking at the time difference between the residuals, the sine wave decreases much more slowly than the square wave.

Again, let us draw a parallelism with a liquid: if you try to submerge your hand in a liquid very slowly the resistance encountered is very low; nevertheless, if you violently hit

the water, there will be a noticeable resistance. In our experiment, the sine wave increases the force smoothly from 0 N to the maximum value resembling our first scenario; thus it encounters less resistance from the material than the square wave, which changes from 0 N to the maximum value in a very short time. Now, if you recall that in the model we separated the ground penetration x in two components: a linear contribution given by the elasticity of the material, i.e. a Hookean spring; and a nonlinear part that includes all the other forces that come into play in granular media:

$$y_{k+1} = y_k + \frac{\beta h}{\Phi 2}(F_k + F_{k-1}) \quad (7.13)$$

$$x_{k+1} = \frac{1}{K}F_k + y_k \quad (7.14)$$

This suggest that the different waveforms should affect more the viscous part represented by parameter $a = \frac{\beta h}{\Phi 2}$. Figures 7.11 show the mean and the standard deviation of the parameters a and b obtained with the sine wave and the square wave. As it was conjectured, the difference between parameter a is higher than the difference in parameter b . In fact, note that the standard deviation being very high for parameter a in many values of force; and more than twice the value of the parameter in some cases. As a consequence we conclude that using a sine wave to excite the media to extract their properties is not useful, at least with this actuator.

Figure 7.11: Mean and standard deviation of parameters a (left) and b (right) obtained with a sine wave and a square wave vs force value

However, in the next section we will continue to use the sine wave to explore the effects of changing the actuator characteristics, both dynamic and geometric.

7.4 Performance with Different Actuator Characteristics

In order to analyze the dependency of the model on the actuator characteristics, in this section we repeat the experiments performed in Sections 7.2 and 7.3 with a rigid actuator (recall that up to this point we have used a series elastic actuator); and we will change the diameter of the end effector to reduce its size and to increase it.

7.4.1 Different Actuator Dynamics

Figure 7.12: Experimental set-up with different dynamics. Setup with the SEA (left). Setup with the blocked spring (right).

To explore the behavior of the model with different actuator dynamics we repeated the force pattern applied in Sections 7.1.1 and 7.3 with a rigid actuator. The data acquisition is very similar to the one used up until this point, with the difference that we use a load cell instead of an indirect measure of the SEA spring. The new set-up is shown in Fig. 7.12.

Fig. 7.13 shows R for values of $F_{max} = 10N, 200N, 400N$ of the residuals of the model, fitted with data obtained using a rigid actuator and the same end effector as before. The autocorrelation for the glass beads with a square wave is shown at the top-left; the bottom-left curve shows the autocorrelation for the calibrated sand with a square wave; at the bottom the autocorrelation of the glass beads (right) and the sand (left) obtained with a sine wave.

A comparison between Fig. 7.13 and Fig. 7.10 reveals that the behavior for the sine wave is practically the same, the only noteworthy difference is that the autocorrelation drops faster. Nonetheless, this is explained by the fact that the actuator spring is no longer coupled with the Hookean part of the model. Now, looking at the square wave, it is interesting to note that the 10N wave now produces a much more accurate modeling of the

Figure 7.13: Autocorrelation for glass beads (top-left) and calibrated sand (bottom-left) of a square wave and autocorrelation for glass (top-right) beads and calibrated sand (bottom-right) with a sine wave using a rigid actuator

Figure 7.14: Different end-effectors used in the thesis.

parameters. The reason for this is that since now the elastic element of the motor is not present a more direct measure can be taken of the exerted force giving as a result a more accurate estimation. However a difference such as this is to be expected, because when the dynamics of the actuator is changed the overall system dynamics (robot and surface) is changed as well.

7.4.2 Different End-effectors

Figure 7.15: Autocorrelation for an actuator with a circular end effector of 65 mm diameter with glass beads (top-left) and calibrated sand (top-right) and for an actuator with a circular end effector of 100 mm diameter with glass beads (bottom-left) and calibrated sand (bottom-right)

Now, let us explore the behavior of the model under different geometric conditions: a change in the end effector diameter. For this experiment, we have used the series elastic actuator to provide meaningful comparison. Then we have switched the end effector with two different circular metal plates with different diameters: 65 mm and 100 mm (see Fig.

7.14). The end effector used until now was a circular metal plate with 75 mm diameter. The experiments performed here have the same structure as the ones we have performed until now; and so is the case of the data acquisition system.

Figure 7.15 shows the autocorrelation obtained using the aforementioned set-up. The top curves show the data obtained with the end effector of 65 mm for the sand (right) and the glass (left). As it can be seen in the figure, the glass beads response resembles the behavior obtained with the sinusoidal wave. Meanwhile, in the case of the sand we can see that the correlation of the residuals decreases with the force, however it is much slower than the previous end effector (see Fig. 7.8). On the contrary when the diameter of the end effector is increased (Fig. 7.15 bottom) the behavior resembles the behavior obtained with the 75 mm diameter end effector but it is much faster in the case of the glass beads.

One conclusion that can be drawn of these results is that a change in the area of the contact surface can greatly influence the behavior of the model. This does not have important consequences when using a particular robot to extract the properties of the ground and then using the results to alter the behavior of that same robot. However, if the obtained parameters are planned to be used in a different robot the behavior can change drastically.

To explain this behavior we go back once more to the similitude between granular media and liquids. Note that we have applied the same force at the surface of the glass and the sand; while we have changed the area of the contact. Since the shear stress is defined as:

$$\tau_s = \frac{F}{A} \quad (7.15)$$

where τ_s is the shear stress F is the force and A is the area.

If we think of the glass beads and the sand as a liquid what we have been doing is essentially varying the shear stress making it higher when reducing the contact area. And since liquids cannot sustain shear stress the higher it is the easier it is to penetrate the material. Nevertheless as the contact area increases the shear stress is lower and making it harder to penetrate the material. Therefore, an improvement that can be done to the model proposed here is to include the pressure as a variable to account for another aspect of the liquid like behavior of granular media.

7.5 Summary

In this Chapter, we have used a Yobotics inc. Series Elastic Actuator to experimentally validate the contact model proposed in Chapter 6. We have applied forces ranging from 10 N to 400 N to river sand and then fitted the obtained data using a square error cost over 10 repetitions for each input force; resulting in an array of 40 different estimated parameters. These parameters, in turn were used to obtain an expression dependent on the value of the maximum force applied using Matlab's curve fitting tool.

Moreover, a qualitative and quantitative evaluation of the results of the previous experiments is given using various methods (R^2 , mean squared error, and autocorrelation). The results are very good for this particular material and after the evaluation, the most intuitive

criterion for the analysis of the performance is the autocorrelation (given that the mean squared error is small).

To further analyze the behavior of the model we have used it in two more granular media: glass beads and calibrated sand. The results for both materials using the same set-up as with the river sand were as expected. On the glass beads the model performs slightly less good as on the calibrated sand because of the lower friction coefficient of the glass particles. Afterwards, we proceeded to change the force pattern applied to the materials to compare the behavior of the model. Here, it turns out that the autocorrelation is not a good indicator because the gradual pushing of a sine wave masks the independence of the results.

A revision of the estimated parameters shows that the linear part of the model is correctly estimated using both signals. However, the smoothness of the sine wave makes the identification algorithm to severely underestimate the viscosity of the material.

As a final test for the model, we have changed the actuator properties, both dynamic and geometric. A dynamic change has an influence on the estimated parameters, but it does not significantly affect the behavior of the model. Nevertheless, the geometry has an important effect in the performance of the model.

The explanation of this result can be found if we consider the fact that a granular media exhibits some of the characteristics of a liquid. When the contact area is reduced the shear stress increases; and since liquids do not sustain shear stresses it is easier for the actuator to penetrate the ground. This drives us to the conclusion that the model can be used with similar sized contact areas, otherwise the parameters needs to be estimated again. However, one way to solve this problem may be to rewrite the model to consider the contact area and estimate the parameters, but these solution is left for future work.

Finally, an interesting conclusion that can be drawn from the fact that the contact surface affects so much the behavior of the granular media is that when a robot is going to be in contact with granular materials such as sand, the larger the contact surface of the end-effector the better the contact will be. In other words, as the contact area increases, the behavior of the granular media will be closer to the behavior of a solid than to a liquid and it will be able to sustain larger forces. In the case of a walking robot, the larger the area of the foot, the less it will be affected by the energy dissipation of the granular material.

Chapter 8

Conclusions, Contributions and Future Work

8.1 Conclusions

Haptic perception is a very important skill to efficient task performance in unknown environments, such as the manipulation of unspecified objects under uncertain conditions. This field of study has been receiving increasing attention from the robotic community in the recent years. Although there are relatively few works on this specific subject, there is a vast amount of knowledge on the two main disciplines that involve the implementation of haptic perception in robotics: Contact modeling and system identification.

In the contact modeling literature, there are two main methods: force based and the impulse-momentum methods. The latter is computationally effective as it can be modeled as a dynamic programming problem. However, the formulation is very complex and it treats the impact force as a discontinuous function. The force based methods on the other hand, describe the impact and contact force as a continuous function, which makes it suitable to apply a system identification procedure to model this interaction.

Nevertheless, the challenge for the extraction of the contact properties is much more complex than selecting between force based and impulse momentum contact models. Different materials have distinct properties, and many of them are nonlinear. However, in this thesis we have demonstrated that a linear model such as the spring dashpot model can be used in combination with an adaptive filtering technique to differentiate between two non linear terrains.

In fact, an experimental setup to obtain data on the contact behavior for four different materials (a metal box featuring compression springs and pneumatic dampers, gravel, river sand and polyurethane semi-rigid foam) was built and used for an experimental evaluation of the performance of linear and nonlinear identification methods. Then they were evaluated both qualitatively and quantitatively using scatter plots and two statistical goodness-of-fit criteria. Surprisingly, the RLS method in combination with the linear spring dashpot model proved to be the most accurate of the system identification algorithms even in presence of nonlinear terrains such as river sand.

After selecting the RLS method combined with the linear spring dashpot model, we

have proposed and implemented an algorithm to extract the properties of the ground using a walking robot. The goal of this algorithm is to identify the combined impedance of the leg and ground simultaneously. Moreover, it considers several aspects, such as ground modeling, the structure of the robot, and sampling frequency.

The implementation of the algorithm was done using the HADE leg (an underactuated 3-DOF planar mechanism) with a National Instruments PXI running at a $500 \mu\text{s}$ control loop. The algorithm was evaluated using four different terrains: river sand, gravel, a rigid ground, and a the variable impedance box). The procedure offers a convergence time that is coherent with the biomechanical studies of humans running. Nevertheless, we found some issues when the algorithm was applied to highly dissipative terrains such as sand.

Although capable of producing differentiated estimations of the terrain stiffness, to produce a differentiated estimate of the damping coefficient, the algorithm needs further improvement. Moreover, the spring dashpot model is not capable of capturing some of the behaviors that sand exhibits. The reason for this is that the river sand and the gravel are granular media. Thus, they exhibit some properties of solid matter, which can be captured by the spring-dashpot model and some properties that are characteristic of liquid matter, which cannot be described by a linear model.

Granular media are highly nonlinear materials that exhibit solid, fluid and gas like behavior. The interactions between particles are so complex that there is no model that can account for all the exhibited behaviors, even in the case of spherical particles. Part of the reason for this is the fact that much of the simplifications that are done to treat gases and fluids as continuum mechanisms do not hold in the case of a granular material.

However, using a quantity known as packing fraction and making some energy considerations, it is possible to treat granular media as a continuum. Making use of this results, and parting from some parallelism drawn with liquid and solid matter in the literature, we have developed a contact model based on the impulse and the packing fraction to approximate the behavior of a granular material in contact with a robot. Then, we performed a preliminary evaluation of the model using the data from previous experiments with river sand with very good fitting results.

For the assessment of the proposed contact model, after the preliminary evaluation, we have applied forces ranging from 10 N to 400 N to river sand and then fitted the obtained data using a square error cost over eight repetitions for each input force; resulting in an array of 40 different estimated parameters. This was done to introduce a dependency of the model parameter on the maximum applied force (using Matlab's curve fitting tool). The result of this experiment was evaluated with different criteria giving satisfactory results.

To further analyze the behavior of the model we have performed a variety of tests; we have used it in two more granular media: glass beads and calibrated sand; changed the force pattern applied to the materials; changed the actuator's dynamic and geometric characteristics. With these tests we have found that a smooth waveform cannot be used to test this model with granular media as it underestimates the viscous term of the model; because of the similarities of the material with liquids. A change in the geometric properties of the actuator can have very important consequences (again because of the similarities with liquids). A change in the dynamic of the actuator does not significantly change the behavior of the model. The model performs similarly with different materials.

These results show that our contact model can exhibit a part of the highly nonlinear behavior of granular media, such as apparent plastic deformation or rearrangement of the particles, in a satisfactory manner. Moreover, the model is not affected by the dynamic properties of the actuator used (note that the parameters do change, which is expected as the system dynamics is changed). However, a change in the waveform and in the geometric properties of the actuator can have important effects in the performance. These issues are present because of the similarities of the granular media, nonetheless one possible improvement to solve the problem is offered.

8.2 Contributions

The work performed in these thesis has produced the following contributions to the State of the Art:

- An experimental evaluation of the most common system identification algorithms when they are applied to contact modeling
- The introduction of an algorithm to extract the properties of the environment, focusing on the extraction of the properties of the terrain for walking robots
- The implementation and evaluation of the aforementioned algorithm using a 3 DOF underactuated robotic leg
- The development of a new linear model capable of capturing some of the nonlinear effects exhibited by granular media such as plastic deformation; and giving a estimate of the energy dissipated at impact.
- A thorough evaluation of the proposed model using statistical goodness-of-fit criteria and testing its behavior under various different conditions

These contributions have given place to five publications in indexed scientific journals and 13 publications in ISI proceedings:

Scientific Journals (JCR indexed)

- Juan Carlos Arevalo, Daniel Sanz-Merodio, Manuel Cestari, and Elena Garcia. Identifying Ground-Robot Impedance to Improve Terrain Adaptability in Running Robots. **International Journal of Advanced Robotic Systems**, Vol. 12, No. 1. (2015)
- Manuel Cestari, Daniel Sanz-Merodio, Juan Carlos Arevalo and Elena Garcia. An Adjustable Compliant Joint for Lower-Limb Exoskeletons. **IEEE/ASME Transactions on Mechatronics**, Vol. 20, No 2, p. 889-898, (2015)
- Juan Carlos Arevalo, Manuel Cestari, Daniel Sanz-Merodio and Elena Garcia. On the Necessity of Including Joint Passive Dynamics in the Impedance Control of Robotic Legs. **International Journal of Advanced Robotic Systems: Robot Control**. Vol. 11, No. 1, (2015).

- Daniel Sanz-Merodio, Manuel Cestari, Juan Carlos Arevalo and Elena Garcia. Exploiting joint synergy for actuation in a lower-limb active orthosis. **Industrial Robot: An International Journal**, Vol. 40, No. 3, pp. 224-228, (2013)
- Juan Carlos Arevalo and Elena Garcia, Impedance Control: An Insight into the Concepts Involved. **IEEE Transactions on Systems, Man and Cybernetics: Part C: Applications & Reviews**, Vol. 42, No. 6, p. 1400-1411, (2012)

Proceedings (ISI)

- Juan Carlos Arevalo, Manuel Cestari, Daniel Sanz-Merodio and Elena Garcia. Event Driven Ground-Impedance Identification for Legged Robots. **16th International Conference on Climbing and Walking Robots**. Sydney, Australia, 2013.
- Manuel Cestari, Daniel Sanz-Merodio, Juan Carlos Arevalo, Xavier Carrillo and Elena Garcia. Actuator with Adjustable-Rigidity and Embedded Sensor for an Active Orthosis Knee Joint. **16th International Conference on Climbing and Walking Robots**. Sydney, Australia, 2013.
- Daniel Sanz-Merodio, Manuel Cestari, Juan Carlos Arevalo y Elena Garcia. Implementation of an Adjustable Compliant Knee Joint in a Lower-Limb Exoskeletons. **16th International Conference on Climbing and Walking Robots**. Sydney, Australia, 2013.
- Juan Carlos Arevalo, Xavier Alberto Carrillo, Manuel Cestari, Daniel Sanz-Merodio and Elena Garcia. System Identification Applied to Contact Modeling: An Experimental Investigation. **IEEE International Conference on Robotics and Automation**. Karlsruhe, Germany, 2013.
- Juan Carlos Arevalo, Daniel Sanz-Merodio, Manuel Cestari and Elena Garcia. Parameterized Inverted and Double Pendulum Model for Controlling Lower-limb Active Orthosis. **IEEE International Conference on Robotics and Biomimetics (ROBIO 2012)**. Shenzhen, China, 2012.
- Daniel Sanz-Merodio, Manuel Cestari, Juan Carlos Arevalo and Elena Garcia. A lower-limb exoskeleton for gait assistance in quadriplegia. **IEEE International Conference on Robotics and Biomimetics (ROBIO 2012)** Shenzhen, China, 2012. (Best Paper in Robotics Finalist)
- Juan Carlos Arevalo, Manuel Cestari, Daniel Sanz-Merodio and Elena Garcia. Impedance Control for a Bioinspired Underactuated Leg. **15th International Conference on Climbing and Walking Robots**. Baltimore, Maryland, 2012. (Highly Commended - Best Technical Paper Award)
- Daniel Sanz-Merodio, Manuel Cestari, Juan Carlos Arevalo and Elena Garcia. Strategies to reduce energy expenditure in the motion control of lower-limb orthoses. **15th International Conference on Climbing and Walking Robots**. Baltimore, Maryland, 2012. (Highly Commended - Innovation Award 2012).

- Daniel Sanz-Merodio, Manuel Cestari, Juan Carlos Arevalo and Elena Garcia. Taking Advantage of the Synergy for the Actuation in a Lower-Limb Active Orthosis. **15th International Conference on Climbing and Walking Robots.** Baltimore, Maryland, 2012.
- Gustavo Munoz, Manuel Cestari, Juan Carlos Arevalo, Daniel Sanz-Merodio and Elena Garcia. Benefits and Drawbacks of MRF Knee Damping in Legged Robots. **15th International Conference on Climbing and Walking Robots.** Baltimore, USA, 2012. (Runner up - Best Technical Paper Award).
- Manuel Cestari, Daniel Sanz-Merodio, Juan Carlos Arevalo and Elena Garcia. Development of Synergic Multiarticular Actuation in a Lower-limb Active Orthosis. **13th Mechatronics Forum International Conference.** Linz, Austria, 2012.
- Elena Garcia, Daniel Sanz-Merodio, Fernando Sanchez, Juan Carlos Arevalo and Pablo Gonzalez de Santos. Development of the ATLAS lower-limb active orthosis. **14th International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines, Field Robotics.** Paris, France, 2011.
- Juan Carlos Arevalo, Jesus Pestana, Fernando Sanchez, Javier Francisco Sarria and Elena Garcia. Impedance Control of an Agile-locomotion Robotic Leg. **13 International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines (CLAWAR 2010),** Nagoya, Japan 31, 2010.

8.3 Future Work

To continue the work performed in this thesis there are three possible lines of research:

First, the formulation of the proposed contact model using pressure instead of force as the input to account for variations in the geometrical structure of the actuators, and further exploration in the buoyancy effects observed in granular media, for example consider using the contact area to develop a displaced volume relationship with the reaction forces at the moment of impact.

Second, the implementation of the proposed algorithm to the on-line identification of the contact properties of the ground in a walking robot. This, with the objective of choosing the appropriate value of the impedance of the robotic legs at the moment of each step; thus, increasing the adaptability to different terrains.

Third, we are planning to implement the algorithm presented here and the variable impedance approach to a knee exoskeleton to improve the user experience and help develop more natural and adaptable gait patterns. Moreover, there is a search and rescue robot being developed that would greatly benefit of haptic perception capabilities to explore unknown environments.

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Appendix A

Glossary of Terms

Adhesion

Is the tendency of dissimilar particles or surfaces to cling to one another. The inter-molecular forces responsible for the function of various kinds of stickers and sticky tape fall into the categories of chemical adhesion, dispersive adhesion, and diffusive adhesion [Andreotti et al., 2013].

Archimedes force

Archimedes force or buoyancy is an upward force exerted by a fluid that opposes the weight of an immersed object. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid. Thus the pressure at the bottom of a column of fluid is greater than that at the top of the column. Similarly, the pressure at the bottom of an object submerged in a fluid is greater than at the top of the object. This pressure difference results in a net upwards force on the object [Halliday et al., 2010].

Autocorrelation

Autocorrelation is the similarity between observations as a function of the time lag between them. It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal obscured by noise, or identifying the missing fundamental frequency in a signal implied by its harmonic frequencies. It is often used in signal processing for analyzing functions or series of values, such as time domain signals [Landau and Zito, 2005].

$$R(\tau) = \frac{E [(X_t - \mu_t)(X_{t+\tau} - \mu)]}{\sigma^s} \quad (\text{A.1})$$

Brownian motion

A Brownian motion is the random motion of particles suspended in a fluid (a liquid or a gas) resulting from their collision with the fast-moving atoms or molecules in the gas or liquid [Feynman et al., 2013].

Coefficient of restitution

The coefficient of restitution (COR) is a measure of the “restitution” of a collision between two objects: how much of the kinetic energy remains for the objects to rebound from one another vs. how much is lost as heat, or work done deforming the objects [Halliday et al., 2010].

The coefficient is defined as the ratio of relative speeds after and before an impact, taken along the line of the impact.

COM

The center of mass (COM) in space is the unique point where the weighted relative position of the distributed mass sums to zero or the point where if a force is applied causes it to move in direction of force without rotation. The definition of COM is as follows:

If the mass distribution is continuous with the density (ρ) within a solid Q , then the integral of the weighted position coordinates of the points in this volume relative to the center of mass R over the volume V is zero, that is

$$\iiint_Q \rho(\mathbf{r})(\mathbf{r} - \mathbf{R})dV = 0 \quad (\text{A.2})$$

where Q is the solid, ρ is the density, R is the center of mass and V is the volume [Halliday et al., 2010].

Complementarity problem

Given a real matrix M and vector q , the linear complementarity problem $\text{LCP}(M, q)$ seeks vectors z and w which satisfy the following constraints [Giraldi and Dharg, 2002]:

$$w, z \geq 0 \quad (\text{A.3})$$

$$z^T w = 0 \quad (\text{A.4})$$

$$w = Mz + q \quad (\text{A.5})$$

From the equations, it follows that each component of w and z is non-negative, and that $\sum_i w_i z_i = 0$. This is the complementarity condition, since it implies that at most one of each pair $\{w_i, z_i\}$ can be positive.

Constraint

A constraint is a condition of an optimization problem that the solution must satisfy. There are several types of constraints primarily equality constraints, inequality constraints, and integer constraints [Betts, 2010].

Constraint coordinate

A constraint coordinate is the coordinate where a rigid body encounters a restriction to its movement [Featherstone, 2008].

Coriolis Force

The Coriolis force is an inertial force (also called a fictitious force) that acts on objects that are in motion relative to a rotating reference frame [Barrientos et al., 1997].

Cost Function

In mathematical optimization, statistics, decision theory and machine learning, a loss function or cost function is a function that maps an event or values of one or more variables onto a real number intuitively representing some "cost" associated with the event. An optimization problem seeks to minimize a loss function [Todorov, 2006].

Cumulative periodogram

The cumulative periodogram is a direct application of the periodogram for testing the hypothesis that a particular time series is a white noise sequence. The cumulative periodogram is an effective diagnostic tool for residuals [Ljung, 1999]. The cumulative periodogram is defined as:

$$\hat{F}(\omega_k) = \frac{\sum_{j=1}^k \hat{f}(\omega_j)}{\sum_{j=1}^q \hat{f}(\omega_j)} \quad (\text{A.6})$$

with $k = 1, \dots, q$ where $q = n/2 + 1$

Dahl effect

The so-called Dahl effect models the behavior of sticking. It is based on the curve of the friction force vs displacement in the Dahl friction model [Olsson et al., 1998]:

$$\frac{dF}{dx} = \sigma \left(1 - \frac{F}{F_c} \text{sgn}(v) \right)^\alpha \quad (\text{A.7})$$

where σ is the stiffness coefficient and α is a parameter that determines the shape of the curve (see fig. A.1).

Dirac's delta

A Dirac's delta is a function with value 1 at $t = 0$ and 0 when $t \neq 0$ [Landau and Zito, 2005].

Figure A.1: Friction force as a function of displacement for Dahl's model [Olsson et al., 1998].

Expectancy

if X is a random variable defined on a probability space (Ω, Σ, P) , then the expected value of X , denoted by $E[X]$, is defined as the Lebesgue integral [Johnson et al., 2000].

$$E[X] = \int_{\Omega} X \, dP = \int_{\Omega} X(\omega) P(d\omega) \quad (\text{A.8})$$

When this integral exists, it is defined as the expectation of X . Not all random variables have a finite expected value, since the integral may not converge absolutely.

Free coordinate

A free coordinate is the coordinate where a rigid body can move without any restrictions [Featherstone, 2008].

Gaussian bell

A Gaussian bell or function is a function of the form [Rudin, 1964]:

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}} \quad (\text{A.9})$$

for arbitrary real constants a , b and c .

Generalized force

In the study of the rigid body dynamics of multibody systems, the term generalized coordinates refers to the parameters that describe the configuration of the system relative to some reference configuration. These parameters must uniquely define the configuration of the system relative to the reference configuration. The generalized forces are the forces of a system in the generalized coordinates [Featherstone, 2008].

Hamiltonian

The Hamiltonian is a quantity that provides a more abstract understanding of classical mechanics. It was developed using a different formalism as Lagrangian mechanics but offers the same results. In terms of the Lagrangian, the Hamiltonian is defined as [Todorov, 2006]:

$$H(x, p) = p^T \dot{x} - L(x, \dot{x}) \quad (\text{A.10})$$

where p is the generalized momentum, x is the position of the system and L is the Lagrangian.

Hessian matrix

In mathematics, the Hessian matrix or Hessian is a square matrix of second-order partial derivatives of a scalar-valued function, or scalar field. It describes the local curvature of a function of many variables [Rudin, 1964].

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_2} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix} \quad (\text{A.11})$$

Indentation

A deep recess or notch on the edge or surface of something [Dictionary, 2004].

Jacobian Matrix

The Jacobian matrix is the matrix of all first-order partial derivatives of a vector-valued function. When the matrix is a square matrix, both the matrix and its determinant are referred to as the Jacobian in literature [Rudin, 1964].

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (\text{A.12})$$

Joint space

Joint space is the space defined by the organized collection of all the articular positions of a robot. These positions can be angular or linear [Barrientos et al., 1997].

Linear regression

Given a data set $\{y_i, x_{i1}, \dots, x_{ip}\}_{i=1}^n$ of n statistical units, a linear regression model assumes that the relationship between the dependent variable y_i and the p -vector of regressors x_i is linear. In other words is any model that can be expressed as [Ljung, 1999]:

$$\mathbf{y} = \mathbf{x}\theta \quad (\text{A.13})$$

where θ is a vector of constant parameters, i.e. $\mathbf{y}$ is a linear combination of $\mathbf{x}$

Mean function

A mean function is defined by:

$$E(Y \mid X = x) = f(x) \quad (\text{A.14})$$

which means that the expected value of the response Y when the predictor X is fixed at a value x is a function of the value of x [Weisberg, 2005].

Magnus force

The Magnus force is the commonly observed effect in which a spinning ball curves away from its principal flowing path [Andreotti et al., 2013].

Mesoscopic element

A mesoscopic element is an element which scale can be described as being between the size of a quantity of atoms (such as a molecule) and of materials measuring micrometers [Andreotti et al., 2013].

Newtonian fluid

A Newtonian fluid is a fluid in which the viscous stresses arising from its flow, at every point, are linearly proportional to the local strain rate or the rate of change of its deformation over time [Batchelor, 2000].

Non-Newtonian fluid

A Non-Newtonian fluid is a fluid which properties are different to those of a Newtonian fluid [Batchelor, 2000].

Passivity

Passivity is frequently used in control systems to design stable control systems or to show stability in control systems. This is especially important in the design of large, complex control systems [Dirks and Scherpen, 2010].

Standard deviation

The standard deviation is a measure that is used to quantify the amount of variation or dispersion of a set of data values; it is defined as [Johnson et al., 2000]:

$$\sigma = \sqrt{\frac{1}{N} [(x_1 - \mu)^2 + (x_2 - \mu)^2 + \cdots + (x_N - \mu)^2]} \quad (\text{A.15})$$

where $\mu = \frac{1}{N}(x_1 + \cdots + x_N)$

Quantization

Quantization is the process of mapping a continuum or large set of input values to a smaller set (i.e.) discrete. Rounding and truncation are typical examples of quantization processes [Landau and Zito, 2005].

Sagittal plane

The sagittal plane is the plane parallel to the direction of motion of a rigid body in free space [Siciliano and Villani, 1999].

Stick-slip

Stick-slip is a phenomenon observed when two solids slide over each other; instead of moving smoothly, the objects alternately stick together and slip past each other [Thompson and Robbins, 1990].

Task space

Task space is the space defined by the organized collection of all the positions of a robot relative to a task's frame of reference. These positions can be angular or linear [Barrientos et al., 1997].

Taylor series

Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. The definition of the Taylor series of a function f is [Rudin, 1964]:

$$f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3 + \cdots \quad (\text{A.16})$$

Underactuated mechanism

Underactuation is a technical term used in robotics and control theory to describe mechanical systems that cannot be commanded to follow arbitrary trajectories in configuration space. This condition can occur for a number of reasons, the simplest of which is when the system has a lower number of actuators than degrees of freedom [Bergerman and Xu, 1996].

Variance

The variance of a random variable X is the expected value of the squared deviation from the mean of X , $\mu = E(X)$ [Johnson et al., 2000]:

$$Var(X) = E((X - \mu)^2) \tag{A.17}$$

Appendix B

HADE Leg

The HADE leg prototype (see Figure B.1) is a planar, three-degree-of-freedom (DOF), underactuated mechanism featuring the characteristics of a horse leg [Garcia et al., 2011]. It is actuated by two Series Elastic Actuators (see fig. B.2), one at the hip and one at the knee, while the ankle joint is passively driven by an extension spring with a stiffness constant $K_s = 3,113$ N/m. The hip and knee actuators characteristics are given in table B.1, in addition, they have a spring with an encoder attached that acts as a force sensor with a resolution of 1.2 N per count and an encoder for measuring the position with a resolution of $50.\mu\text{m}$ per count. In turn, the ankle has a linear encoder with resolution $224 \mu\text{m}$ per count which is equivalent as 0.69 N per count.

Figure B.1: The HADE leg prototype. Denavit-Hartenberg parameters (left). Real prototype (right).

The leg direct kinematics provides foot position and orientation from joint angles as

Figure B.2: Schematic diagram of the Serial Elastic Actuator [Pratt et al., 2002].

Table B.1: Series Elastic Actuator characteristics

Model	SEA23-23
Weight (kg)	1.1
Diameter (mm)	58
Length (mm)	150 + stroke
Maximum Stroke (mm)	150
Ball-screw pitch (mm)	2
Motor	Moog Silencer BN23-23ZL-03LH
Rotor inertia (gcm ²)	106
Operating voltage (V)	48
Maximum current (A)	20
Maximum speed (m/s)	0.27
Cont. force at max. speed (N)	566
Continuous power (W)	166
Int. force at max. speed (N)	1,300
Intermittent power (W)	629
Int. Power to weight (W/kg)	571
Closed-loop bandwidth (Hz)	35
Saturation bandwidth (Hz)	7.5
Efficiency (%)	90
Spring Century Die Spring	1222-A
Spring stiffness (N/mm)	4 78.4
Encoder	USDigital EM1-0-500
Pos. resolution (lines/mm)	19.68

follows:

$$x = a_1 \cos(q_1) + a_2 \cos(q_1 + q_2) + a_3 \cos(q_1 + q_2 + q_3) \quad (\text{B.1})$$

$$y = a_1 \sin(q_1) + a_2 \sin(q_1 + q_2) + a_3 \sin(q_1 + q_2 + q_3) \quad (\text{B.2})$$

$$\phi = q_1 + q_2 + q_3 \quad (\text{B.3})$$

where (x, y, ϕ) are foot x and y position and orientation respectively in the leg's base reference frame, while expression C_{ijk} means $\cos(\theta_i + \theta_j + \theta_k)$.

Regarding inverse kinematics, joint position can be obtained from cartesian foot posi-

tion and orientation as follows:

$$\theta_1 = \text{atan2}(S_1, C_1) \quad (\text{B.4})$$

$$\theta_2 = \text{atan2}(S_2, C_2) \quad (\text{B.5})$$

$$\theta_3 = \phi - \theta_1 - \theta_2 \quad (\text{B.6})$$

being

$$C_2 = \frac{pw_x^2 + pw_y^2 - a_1^2 - a_2^2}{2a_1a_2} \quad (\text{B.7})$$

$$S_2 = \pm \sqrt{1 - C_2^2} \quad (\text{B.8})$$

$$C_1 = \frac{(a_1 + a_2S_2)pw_x - a_2S_2pw_y}{pw_x^2 + pw_z^2} \quad (\text{B.9})$$

$$S_1 = \frac{(a_1 + a_2S_2)pw_y - a_2S_2pw_x}{pw_x^2 + pw_z^2} \quad (\text{B.10})$$

where

$$pw_x = x_0 - a_3C_\phi \quad (\text{B.11})$$

$$pw_y = y_0 - a_3S_\phi \quad (\text{B.12})$$

The Jacobian matrix relating joint speeds and foot cartesian speed has the following expression:

$$J = \begin{bmatrix} -a_1S_1 - a_2S_{12} - a_3S_{123} & -a_2S_{12} - a_3S_{123} & -a_3S_{123} \\ a_1C_1 + a_2C_{12} + a_3C_{123} & a_2C_{12} + a_3C_{123} & a_3C_{123} \\ 1 & 1 & 1 \end{bmatrix} \quad (\text{B.13})$$