OPTIMAL WELFARE PRICE IN A HIGHWAY COMPETING WITH AN UNTOLLED ALTERNATIVE: THE INFLUENCE OF INCOME DISTRIBUTION

First and corresponding author: Alejandro Ortega Hortelano
Affiliation: Research Fellow, Transportation Research Group, University of Southampton
Address: Faculty of Engineering and the Environment
Boldwood Campus
Southampton S016 7QF.
Phone: (+44-0) 23 8059 9575
Fax: (+44-0) 23 8059 3152
e-mail: a.ortega-hortelano@soton.ac.uk

Second author: Jose Manuel Vassallo
Affiliation: Associate Professor, Transportation Department, Universidad Politecnica de Madrid.
Address: Escuela Tecnica Superior de Ingenieros de Caminos, Canales y Puertos
Calle Profesor Aranguren s/n,
28040 Madrid, Spain.
Phone: (+34) 91 336 66 55
Fax: (+34) 91 336 53 62
e-mail: josemanuel.vassallo@upm.es

Third author: Juan Ignacio Perez
Affiliation: Senior Lecturer, Environmental and Hydraulic Department, Universidad Politecnica de Madrid.
Address: Escuela Tecnica Superior de Ingenieros de Caminos, Canales y Puertos
Calle Profesor Aranguren s/n,
28040 Madrid, Spain.
Phone: (+34) 91 336 67 05
Fax: (+34) 91 336 53 62
e-mail: ji.perez@upm.es
ABSTRACT

In some countries it is fairly common to see two roads with the same origin and destination competing in the same corridor. One of them is usually a toll highway that offers a better quality to the users compared to its alternative: a free parallel single road. The users thus have to decide whether it is worth paying the toll for the advantages offered. This problem, known as the “untolled alternative”, has been largely studied in the academic literature. Particular attention has been paid to calculate the optimal welfare toll that maximizes economic efficiency. However, there is a gap in the academic literature regarding how income distribution affects the optimal toll. The main objective of the paper is to add knowledge on the topic by analyzing the influence of the distribution of the values of travel time (VTT) of the users of this corridor— which is closely related to their income distribution— on the optimal toll price. To solve this problem, we define a mathematical model aimed at obtaining the optimal welfare price for this kind of corridor under the hypothesis that drivers decide over the expectation of free flow conditions. The results show that the higher the average VTT the higher the optimal price, and the higher the dispersion (variance) of this VTT the lower the optimal price.
1. INTRODUCTION

The problem of road pricing has been largely studied. It is well acknowledged that in order to achieve the maximum social welfare, users must internalize the externalities they produce and do not perceive. This is usually achieved through a toll or a tax. However, that toll can harm low income users. Depending on the objective function to be optimized (e.g. maximize welfare, maximize social equity, maximize income, etc.) the optimal toll might vary substantially. The academic literature about pricing and welfare is vast and diverse. However, as far as we are concerned, there is a gap in the literature regarding the optimal price in a corridor where a toll highway competes with a free conventional road for different distributions of the values of travel time (VTT). Particularly we did not find any analytical research estimating the optimal welfare price when varying the conditions of the average value of travel time (VTT) and its dispersion. Apart from the introduction, the paper contains five more sections. In section 2, we conduct an extensive literature review on the subject and define the objectives of the paper. In section 3, we set the objective function to be optimized in this research, describe the methodology and explain the adopted assumptions. In section 4, we define the parameters and variables to apply to a specific case study. In section 5, we show the results responding to the objectives previously established. Finally, in section 6, we offer a set of conclusions and lessons.

2. LITERATURE REVIEW

The problem of the untolled alternative has been widely studied. One of the first research works dealing with the topic is the mathematical demonstration carried out by Marchand (1). He focuses on the case in which it is impossible to get the first best. The author obtained a theoretical solution to the problem through the lagrangian approach. The model is static so it does not consider peak hour's demand. Verhoef, Nijkamp and Rietveld (2) updated and improved the model previously mentioned by defining two functions to optimize: one to maximize welfare, and the other to get the maximum revenue from the toll. For the welfare case they found that if the cost for driving in the toll road is really low then setting any toll in just one of the roads turns out to be totally inefficient.

Braid (3) conducted a similar approach focusing on congestion. He assumed that users always travel through the less costly option for them, and their cost depends on three parameters: schedule delay cost, the value of time waiting in the queue to access to the highway, and the toll. He proved that when the schedule delay cost is a V-shaped function and both roads have the same capacity, then two out of three users will drive through the toll highway and the gains of welfare will be the two thirds of the hypothetical gains with the first best solution.

Verhoef and Small (4) focused on the consequences of pricing in the second best option with heterogeneity between public and private managers of infrastructure. The transportation network selected by the authors was made up of two stretches. The first stretch was divided into two paths which had the same origin and destination but with different alternatives. The second stretch was the same for both alternatives. Their research concludes that considering heterogeneity in users, i.e. drivers with different VTT, helps avoiding underestimation of the benefits stemming from second best pricing of one parallel link.

Liu and MacDonald (5) studied the problem by adding two periods of time: the peak and the off-peak hour to a case study consisting of adding two toll lanes to an existing bridge made up of four lanes. They found welfare gains ninety percent greater when the whole bridge
was tolled than when only two lanes were tolled. In other words, as expected, the results were better with the first best case.

The same authors (6) improved their model by including a relationship between the peak and off-peak hour periods. They compared the behavior of a transportation network in three scenarios: without any toll, completely tolled (first best), and tolled only in some stretches (second best). They found important differences between the scenario without any toll and the second best scenario in the second best, as a decrease of total traffic or a switch from the toll roads to the free roads. They also obtained welfare loses when comparing this solution with the first best, i.e. completely tolled.

Verhoef (7) set up a new model by introducing linear functions of cost and demand. This methodology is applicable to any network when looking for both the first best and the second best solution. The main conclusions he got were that more efficient results are achieved with the first best scheme.

Ferrari (8) studied welfare variations after introducing a toll. The author did it through a theoretical model for three types of users: users of public transportation, car users and users who would be shifted from their cars to public transportation. The author demonstrated that in some scenarios a toll can be profitable for the society. Nevertheless, with the combination of both no congestion and many users captive from the public transport, then the toll would imply a loss of welfare.

Yang and Zhang (9) developed a numerical model applied to two examples of transport networks. Due to the impossibility of achieving the first best solution, they focused on the second best one. They obtained the sections of the network where the toll should be imposed, to whom and the price of it. In a theoretical urban network with 43 links the maximum welfare can be achieved through marginal cost pricing, but with the toll in just 10 links welfare was found to be almost identical.

Salas, Robusté and Sauri (10) found the optimal price in the metropolitan area of Barcelona by calculating the social cost for several scenarios of traffic reduction. Verhoef, Koh and Shepherd (11) also studied generalized costs in congested transport networks.

Like the authors of this paper, Du and Wang (12) have been unable to find any research related to welfare and heterogeneous users. They considered park and ride facilities in the accesses to a city and introduced travel time reliability and heterogeneous commuters. However, they only tested the model with four different types of users, and did not look into differences in the optimal price for different VTT distributions.

Although a more detailed review can be seen in Ortega (13), it is noteworthy that there is a gap in the literature regarding the optimal welfare price in a tolled highway competing with a free road for different VTT distributions of the potential users. In other words, many studies have been surrounding the question, but none of them have directly solved it through an analytic model. Thus, the objective of the paper is to add knowledge in the topic by defining a model to obtain the optimal toll price in terms of the VTT distribution for an interurban corridor where a toll highway and an untolled conventional road compete. This type of network is common in many interurban corridors, for instance in Spain, where there are two options for travelling in a particular origin–destination pair. Instead of the theoretical lagrangian approach used in the majority of the above mentioned researches, we prefer to use an engineering approach to the problem since it is more logical and easy to follow for readers without a mathematician or economic background.
3. ASSUMPTIONS AND METHODOLOGY

In order to pursue the objective of this paper, an optimization problem has been formulated which is aimed at calculating the optimal toll price that maximizes the social welfare (i.e. minimize the social cost) in an interurban corridor of the above-mentioned characteristics, for a given number of potential users and a distribution of their VTT. For formulating this problem, some hypotheses were assumed which are described below:

The potential users will be divided into 100 groups. Each group will have a different VTT and a daily expenditure limit for transport related to their income. Each potential user is equivalent to one car, so if the group of potential users is made of one hundred users, that means one hundred vehicles.

The potential users are supposed not to be familiar with the traffic conditions in the corridor. Therefore, they will decide whether they will travel or not, and through which road they will do it, on the basis of the expected travel time, their VTT, the gasoline cost expected under free flow conditions and the toll in the highway.

Due to the second assumption, the extra-costs that the users may end up facing if travel conditions are not as they originally expected are not known by them at the time of making the travel decision. However, once they decide to travel, they will automatically internalize the majority of this extra costs because they will be directly affected by them. This assumption is taken over two basis. Firstly, only daily travelers are well informed about travel time reliability (14). Secondly, it is remarkable that even that type of users do not have accurate travel time information about theirs trips (15) and their travel options are greatly influenced by their subjective perception (16). In other words, an assumption of perfectly well informed users would be less realistic than the assumption already taken.

We want to note that our model is valid for interurban networks where users are not familiar with the congestion in the highways. For the model to be applicable to commuters in an urban transport network, it would be necessary to change this assumption.

In the selected transport network the total cost for the society is defined as follows:

\[ SC = UC + EC + HOB + GB \]  \hspace{1cm} (1)

Where:

- \( SC \) is the total social cost in €. This is the objective function that has to be minimized.
- \( UC \) is the total cost that the users bear per trip in €. It is divided into four terms which are travel time, toll, fuel cost and maintenance of the vehicle.
- \( EC \) represents the externalities produced by the vehicles in €. They are the summation of environmental cost – i.e. gas emissions, noise and so on – plus accidents.
- \( HOB \) is the net operating balance for the road operator either public or private in €. It consists of the tolls paid in the highway minus the maintenance cost in the toll highway.
- \( GB \) is the result for the Government responsible for maintaining the conventional road in €. It is calculated as the taxes recovered from fuel minus the maintenance cost of the road.

Congestion costs do not appear explicitly. According to the methodology they automatically appear when traffic increases and the speed in the road decreases.
The potential users of the corridor are divided into groups according to their income and this is depicted by the index $i = 1, \ldots, 100$. The index $j$ represents the different options for the potential users and it can be $H$ (Toll Highway) or $R$ (Conventional Road). The potential users have a daily expenditure limit for transport, which is defined as follows:

\[ T^j + GC^j > \theta \times I_i \forall j, \text{ users of group } i \text{ do not travel} \]
\[ N_{i,\text{exp}} = NPU_i \text{ and } Ni = 0; \text{ otherwise } N_{i,\text{exp}} = 0 \text{ and } Ni = NPU_i \]  

Where $T^j$ is the toll to be paid in € in the option $j$, $GC^j$ is the expected gasoline cost in € under free flow conditions in the option $j$, $\theta$ is the limit of expenditure in transport (17) in per unit values, $I_i$ is the daily income expressed in € of the group $i$, $NPU_i$ is the number of potential users of group $i$, $N_i$ is the number of users of group $i$ who decide to travel, and $N_{i,\text{exp}}$ is the number of users of group $i$ which are “pushed out” of the corridor because the cost they have to bear to travel is too high for them. All groups of users decide first whether they travel or not, and if they do it, then they decide through which road they will do it according to the cost they expect to bear in each alternative (3):

\[ DC_i^j = T^j + \varphi \times GC^j + VTT_i \times ETT^j \]  

Where $DC_i^j$ is cost estimated by the group $i$ for the option $j$ expressed in €. $VTT_i$ is the VTT in €/hour for the group of users $i$ that is strongly related to their income, $I_i$ (18). In order to simplify the model, $VTT_i \times 8 = I_i$, i.e. the daily income is equivalent to eight hours which is the usual working day. $\varphi$ is a coefficient that expresses the user's perception with respect to the cost of gasoline (19). Finally, $ETT^j$ is the expected travel time under free-flow traffic conditions in the road $j$. $DC_i^j$ is calculated for each group of potential users who have decided to travel; each of them will travel through the road with the lowest $DC$. The total cost for the users (UC) is calculated as the summation of the real cost for each group of users, $RC_i$ (4), which is in turn calculated from (5-8).

\[ UC = \sum_i RC_i \]  

\[ RC_i = RC_i^H + RC_i^R + RC_{i,\text{exp}}, \forall i \]  

\[ RC_i^H = [T^H + VTT_i \times ETT^H + (1 + \sigma) \times VTT_i \times (RTT^H - ETT^H)] \times N_i \times (1 - U_i), \forall i \]  

\[ RC_i^R = [VTT_i \times ETT^R + (1 + \sigma) \times VTT_i \times (RTT^R - ETT^R) + RGC^R + MCV^R] \times N_i \times U_i, \forall i \]  

\[ RC_{i,\text{exp}} = N_{i,\text{exp}} \times \left( DC_{i,\text{exp}} + \frac{1}{2\Delta T_{i,\text{exp}}} \right), \forall i \]
$RC_i$ is the real cost the users have to bear in €, $\sigma$ is a coefficient which penalizes the exceeding travel time above free flow (20), $RGC^j$ is the real gasoline cost in the road $j$ in €, and $MCV^j$ indicates the vehicle maintenance cost in the option $j$ in €, which depends on the length of the road $j$ as well as on the kilometer maintenance cost (MCV). $RC_{i,exp}$ is the real cost to be assigned to the users who do not travel and $DC_{i,exp}$ is the decision cost of these users. The expression (8) has been added in order to take into account the social cost for the users who do not travel. In other words, this is the so called “rule of a half” in the CBA but adapted to users who decide not to travel instead of their usual induced demand. $U_i$ is a binary variable which takes the value 1 if the users travel through the conventional road or 0 if the users choose the toll highway; in those cases where the users of group $i$ decide not to travel, $RC^j_i = RC^R_i = 0$, regardless the value of $U_i$ (2). $RTT^j$ is the real travel time in the road $j$, which according to the Bureau of Public Roads is calculated as follows:

$$RTT^j = ETT^j \times \left[ 1 + \alpha^j x \left( \frac{N^j}{CAP^j} \right)^{\beta^j} \right]$$  \hspace{1cm} (9)$$

Where $CAP^j$ is the capacity of the road $j$ and $\alpha^j$ and $\beta^j$ are necessary coefficients to calculate the real travel time.

$RGC^j$ has been obtained according to the following expression (21):

$$RGC^j = \left[ a_0 + \frac{a_1}{DIST^j/RTT^j} \right] + a_2 \times \left( \frac{DIST^j}{RTT^j} \right)^2 \times RGF^j \times GP \times \frac{DIST^j}{1000 \times Dens.\,Gas.}$$  \hspace{1cm} (10)$$

Where (10) $DIST^j$ is the length of the road $j$ in km, $RGF^j$ is a factor that takes into account the additional gasoline consumption when the road $j$ does have any slope, $GP$ is the gasoline price with taxes in € and $Dens.\,Gas$ is the gasoline density. The number 1000 has been added in order to convert grams into kilograms.

The cost of externalities is obtained from:

$$EC = \sum_{i=1}^{n} (1 - U_i) \times N_i \times \frac{RGC^H}{GP \times DIST^H} \times ExternalityCost + AccidentCost \times DIST^H$$  \hspace{1cm} (11)$$
\[
\sum_{i=1}^{n} U_i \times N_i \times \left[ \frac{RGC^R}{GP \times DISTR^R} \times ExternalityCost + \frac{GP \times DISTR^R}{GP \times DISTR^R} \times AccidentCost \right] \times DIST^R
\]

Where \( ExternalityCost \) is the kilometer cost of the externality due to the gasoline consumption, noise and so on, and \( AccidentCost \) is the cost of accidents in €/Km.

The net operating profit for the road operator is calculated as follows:

\[
HOB = \sum_{i=1}^{n} (1 - U_i) \times N_i \times (CC^H \times DIST^H - T^H)
\]

(12)

Where \( CC^H \) is the maintenance cost of the toll highway in €/km. Those maintenance costs that do not depend on the traffic level have been removed from the calculation since they are constant and therefore would not have any influence in the optimization.

Finally, the balance for the government which manages the untolled conventional road is obtained according to (13):

\[
GB = \sum_{i=1}^{n} U_i \times N_i \times CC^R \times DIST^R - \sum_{i=1}^{n} (1 - U_i) \times N_i \times \delta \times RGC^H
\]

- \( \sum_{i=1}^{n} U_i \times N_i \times (\delta \times RGC^R) \)

(13)

Where \( \delta \) refers to the percentage of taxes paid due to fuel consumption (22). In the last two equations, (13) and (12), the terms related to the toll and gasoline taxes are expressed with a negative sign. However, these terms are included with a positive sign in the equations of the real cost of users (5 - 8), and therefore they are considered as a mere money transfer among different stakeholders of the society.

It is noteworthy that the problem of finding the toll that minimizes SC is nonlinear and nonconvex (23), and that therefore the use of commercial solvers based on mixed integer nonlinear programming does not guarantee that a global optimum can be obtained. Indeed, for the case study presented in section 4, it was found that the optimal solution obtained with BONMIN and COUENNE algorithms, both available in GAMS and developed by IBM in collaboration with Carnegie Mellon University (see http://www.coin-or.org/index.html for more information), varies significantly depending on the initial toll from which these algorithms begin the search of the optimal solution. For this reason, and considering the moderate size of the problem under study, the resolution by simulating several scenarios is presented as a practical and rigorous option. Therefore, in order to analyze the influence of the average VTT and the dispersion of the VTT on the optimal toll price that maximizes the social welfare, the above-described problem has been solved for different values of potential users
and distributions of VTT through a realistic case study, whose characteristics are described in section 4.

4. VARIABLES FOR THE CASE STUDY

In this section we define the characteristics of the variables of the case study where we apply our methodology. We distinguish three different types of variables: first, a set of input variables —VTT distribution, tolls, and total number of potential users (TNPU)— that we study in greater detail; second, some parameters that we assume constant to facilitate the analysis —e.g. the slope of the roads fuel taxes, etc.—; and third, a set of output variables —SC, number of users who do not travel and traffic distribution in the corridor— that result from the model. The results are obtained for 25 Lognormal income distributions. Although we have focused our analysis on studying the effects of the VTT distribution and the TNPU on the optimal toll price, the methodology presented in the paper can be easily adapted to study other variables. In Table 1 below, we show the ranges selected for the input variables, and the parameters that remain constant in the analysis:

<table>
<thead>
<tr>
<th>Table 1. Parameters and variables selected for the case study</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>$VTT_i$</td>
</tr>
<tr>
<td>$T^H$</td>
</tr>
<tr>
<td>$TNPU$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th><strong>Definition</strong></th>
<th><strong>Value</strong></th>
<th><strong>Source</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$DIST^H$</td>
<td>Distance in the highway</td>
<td>90 km</td>
<td>(34)</td>
</tr>
<tr>
<td>$DIST^R$</td>
<td>Distance in the road</td>
<td>100 km</td>
<td>(34)</td>
</tr>
<tr>
<td>$ETT^H$</td>
<td>Expected Travel Time in the highway</td>
<td>120 km/h</td>
<td>(33)</td>
</tr>
<tr>
<td>$ETT^R$</td>
<td>Expected Travel Time in the road</td>
<td>100 km/h</td>
<td>(33)</td>
</tr>
<tr>
<td>( CAP^H )</td>
<td>Highway capacity</td>
<td>2,400 users/lane/hour, 2 lanes</td>
<td>(33)</td>
</tr>
<tr>
<td>----------</td>
<td>-----------------</td>
<td>-------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>( CAP^R )</td>
<td>Road capacity</td>
<td>1,700 users/lane/hour, 1 lane</td>
<td>(33)</td>
</tr>
<tr>
<td>( ExternalityCost )</td>
<td>Kilometer cost of the externality due to the gasoline consumption, noise and so on</td>
<td>0.05€/veh – km with a fuel consumption of 11.4 liters/100Km</td>
<td>(35)</td>
</tr>
<tr>
<td>( AccidentCost )</td>
<td>Cost of accidents</td>
<td>0.03 €/veh – km</td>
<td>(35)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Penalization of the exceeding travel time above free flow</td>
<td>0.3</td>
<td>(36); (37); (20); (38); (39)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Percentage of taxes paid due to fuel consumption</td>
<td>45 %</td>
<td>(22)</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>Parameter to calculate gasoline consumption</td>
<td>54.70</td>
<td>(21)</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>Parameter to calculate gasoline consumption</td>
<td>495.88</td>
<td>(21)</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>Parameter to calculate gasoline consumption</td>
<td>-0.54</td>
<td>(21)</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>Parameter to calculate gasoline consumption</td>
<td>0.004</td>
<td>(21)</td>
</tr>
<tr>
<td>( RGF^H, RGF^R )</td>
<td>Parameter which takes into account the slope of the roads in gasoline consumption</td>
<td>1 (flat roads)</td>
<td>(21) for a slope between -1% and 1%</td>
</tr>
<tr>
<td>( Dens.Gas )</td>
<td>Gasoline density</td>
<td>0.753 kg/liter</td>
<td>(40)</td>
</tr>
<tr>
<td>( CC^H )</td>
<td>Maintenance cost of the toll highway</td>
<td>1.8E-04 €/veh - km</td>
<td>(41); (42)</td>
</tr>
<tr>
<td>( CC^R )</td>
<td>Maintenance cost of the road</td>
<td>6.12E-05 €/veh - km</td>
<td>(43); (42)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value</td>
<td>Source(s)</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
<td>-----------</td>
</tr>
<tr>
<td>$\alpha^H$</td>
<td>Highway parameter to calculate the real travel time</td>
<td>1</td>
<td>(44); (45); (46)</td>
</tr>
<tr>
<td>$\alpha^R$</td>
<td>Road parameter to calculate the real travel time</td>
<td>1</td>
<td>(44); (45); (46)</td>
</tr>
<tr>
<td>$\beta^H$</td>
<td>Highway parameter to calculate the real travel time</td>
<td>4</td>
<td>(44); (45); (46)</td>
</tr>
<tr>
<td>$\beta^R$</td>
<td>Road parameter to calculate the real travel time</td>
<td>2</td>
<td>(44); (45); (46)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>User’s perception with respect to the cost of gasoline</td>
<td>0.9</td>
<td>(47); (19)</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Limit of expenditure in transport</td>
<td>0.2</td>
<td>(17)</td>
</tr>
<tr>
<td>$MCV$</td>
<td>Vehicle maintenance cost</td>
<td>0.037 €/Km</td>
<td>(48)</td>
</tr>
<tr>
<td>$GP$</td>
<td>Gasoline price</td>
<td>1.5 €/liter</td>
<td>(22)</td>
</tr>
</tbody>
</table>

It is important to note that the total capacity of the corridor is 6,500 vehicles per hour. The conventional road does have the 26.15% of this total capacity. The ranges for the three variables, $VTT$, $T^H$ and $TNPU$ have been selected following the recommendations by several researches. Each $VTT$ distribution is characterized by its average ($\mu$), variance ($\sigma^2$) and shape. Firstly, the average was chosen. For the case of long distance travelers in the California State Route 91 values of time between 7.09$/\text{hour}$ and 29.42$/\text{hour}$ were found to be plausible (21). There were a high variability in the results which is explained by the kind of survey conducted, i.e. Stated Preferences or Revealed Preferences. In (25) the case of the Interstate 15 in California was studied, and the values in the peak hour ranged from 20$/\text{hour}$ to 40$/\text{hour}$. As in the previous research a high variability was found. For the case of Cost Benefit Analysis in Spain, in (23) a range from 9.18$/\text{hour}$ to 22.34$/\text{hour}$ is recommended. Secondly, regarding the variance of the distribution, in the case of the Spanish regions this can vary up and down to 30% from its average (22). Furthermore, the GINI index can range from around 0.20 to almost 0.70 across different countries (28). Thirdly, with respect to the distribution’s shape this must be a Lognormal (24), and the variances of this lognormal might range from 10 to 1,000 (29). Finally, in an update of the valuations conducted by (26) and (27) similar ranges, values and shape are recommended. The Spanish tax return analyzed in the paper is classified as low average and high variance within the $VTT$ distributions selected, which is sensible taking into account its economic situation across Europe. Therefore, the 25 lognormal VTT distributions are considered as a realistic sample for our case study.
5. RESULTS

This section is split into five subsections. The first one explains how the model calculates the optimal toll, and shows the evolution of social cost for different tolls and traffic levels. The second one analyzes the effect of the average VTT on the optimal toll. The third one studies the effect of the variance of VTT on the optimal price. The fourth analyses the combination of the three input variables of the model: average VTT (μ), variance of VTT (σ²) and TNPU. Finally, the fifth subsection tests the model with a VTT distribution resembling the income characteristics of Spain.

5.1. Evolution of social cost and traffic

Figure 1 shows the evolution of SC as a function of TH, for a particular VTT distribution. The lognormal VTT distribution chosen for this section does have an average μ = 19 €/hour and a variance σ² = 249.28 in the middle of the range previously selected. This distribution does not have any users pushed out.

![Figure 1. Evolution of social cost (SC) for different toll prices (TH) and levels of potential users (TNPU)](image)

Figure 1 shows that the larger the potential traffic the greater the social cost and also the optimal toll. This figure also demonstrates that for higher levels of potential traffic if the toll is far from the optimum, the social cost will increase a lot. In Figure 2 we can see how the larger the potential traffic the greater the road share stemming from the application of the optimal toll. For instance, the optimal toll for 3,000 potential traffic in the corridor is €2 and the share absorbed by the road is 16% of the total traffic. However, if the potential traffic is 7,000 then the optimal toll will be €2.6 and the traffic share of the road will rise to 35%. This result makes sense, given the fact that more users in the corridor mean greater congestion and therefore a higher cost of fuel, travel time, road maintenance and externalities.
5.2. Effect of average VTT

In this section we analyse the effect of the average VTT on the optimal toll. Figure 3 shows the optimal toll corresponding to five different VTT distributions —all with the same variance ($\sigma^2=249.28$) but with different averages $\mu$— as a function of the potential users.

Before getting into Figure 3, it is necessary to clarify that for the distributions with $\mu=13\,\text{€/hour}$ and $\mu=16\,\text{€/hour}$ there is, respectively, a percentage of 38% and 15% of potential users who decide not to travel because this is too costly for them. However, the other three distributions —corresponding to wealthier people— do not have any group of potential users pushed out. From the preceding figure some conclusions can be drawn. First, the larger the potential traffic the higher the optimal toll. This conclusion is in line with previous findings identified in the literature review. Second, it is noteworthy that ceteris paribus, the higher the value of $\mu$ the higher the optimal toll. However, this increase in the toll is not proportional to
μ, since the optimum toll rises at a lower rate than μ. Third, when the traffic is near to the capacity of the corridor, the share of the road is also close to its relative capacity in the corridor although slightly above it. And fourth, there is a discontinuity in the model for low level of potential users and low values of μ, where the optimal toll is zero. This result is related to the total number of potential users in the corridor. When the users of the corridor are below a certain threshold, there is no congestion and the externalities of using the highway are lower than using the road, so it is better not to set any toll and all users will drive through the highway. This threshold depends on both the number of potential users and the percentage of users who decide not to travel.

5.3. Effect of the variance of VTT

In this section we examine the effect of the VTT variance on the optimal tolls. For this purpose, in Figure 4 we present the optimal toll corresponding to five different VTT distributions all with the same average (μ=19 €/hour), but with a different variance (see Table 1). For each VTT distribution, Figure 4 shows the evolution of the optimal toll as a function of the traffic level.

![Figure 4](image)

**FIGURE 4. Optimal toll for different levels of potential users (TNPU) and different variances of VTT (σ2)**

Figure 4 proves that the higher the VTT variance (or dispersion), the lower the optimal toll. In other words, for the same average VTT (μ) and potential traffic (TNPU), the larger the Gini index the lower the optimal toll. This means that ceteris paribus tolls in regions with a widespread income distribution should be lower than in regions with a concentrated income distribution. The higher the variance there are more “rich” users in the distribution, but there are also more “poor” users, and as a result the toll must be lower because with a lower toll more people travel on the road. As in the previous figures, the greater the potential traffic the higher the toll. Finally, it is remarkable that along with the increase in traffic there is also an increase in the share of the conventional road until it reaches a traffic level slightly above its relative capacity in the corridor.

5.4. Combined effect of μ and σ2

In the previous two subsections, we have analyzed the effect of the average VTT (μ) and the variance of (σ2) separately. However the combined effect of both variables has not been considered yet. In order to study this effect, in Figure 6 we present the optimal toll
corresponding to all VTT distributions (see Table 1). The Figure contains four graphs. In each graph, the y-axis indicates the optimal toll in euros and the x-axis represents different average VTT (μ). Each graph corresponds to a different potential traffic level.

**FIGURE 5. Optimal toll for different levels of potential users (TNPU) and different VTT distributions (μ and σ2)**

The trends previously observed are confirmed in Figure 5. First, the greater the potential traffic, the higher the toll. Second, the higher the average VTT (μ) the higher the optimal toll. Third, the higher the VTT dispersion (σ2) the lower the optimal toll.

The discontinuity observed in Figure 3 and in the upper left graph of Figure 5 can be better understood in Figure 6, where the road share corresponding to the optimal toll is depicted for different VTT distributions. The horizontal black line in Figure 6 shows the relative capacity of the road in the corridor. This figure shows that the larger the potential traffic the greater the share of the conventional road. In fact, when the potential traffic reaches the capacity of the corridor, the traffic share in the conventional road is, in most cases, around 30/32%. Nevertheless, for distributions with low μ the road share lies between 24% and 27%. This is likely caused by the high percentage of potential users who do not travel when the average VTT of the distribution is low.

As can be observed in Figures 5 and 6, for low average VTT and low potential traffic, both the optimal toll and the road share are null. This result seems to be reasonable because with low traffic levels there are almost free flow conditions and there is no need to internalize the externalities through a toll. Finally the higher share of the road above its capacity is due to the fact that, according to Table 1, a higher β has been imposed on the toll highway.
FIGURE 6. Road share for different VTT distributions and different levels of potential users, TNPU

These findings can be applied to traffic management. Under free flow congestion in the corridor the toll must be null, which is if there are less than 2,500 users the toll must be null. From that point, the toll must be increased and also the share of the road until the traffic in the road is higher than the capacity of the road and the congestion in the corridor is unavoidable.

5.5. The Spanish Case

Finally, we have calculated the optimal toll for different levels of potential users, with a VTT distribution resembling the socioeconomic characteristics of Spain. With these results, we intend to provide insight on the differences between the optimal toll according to our methodology and the current toll levels in a particular place. The income distribution has been obtained from the National Bureau of Statistics of Spain. The shape of the distribution is very similar to a lognormal. The main results of this analysis are summarized in Table 2.

TABLE 2. Optimal toll and road share for different levels of potential users (TNPU) considering the income distribution of Spain

<table>
<thead>
<tr>
<th>Kind of distribution</th>
<th>Potential Traffic (Number of potential users)</th>
<th>Optimal price in the toll highway (€)/ (€/km)</th>
<th>Share of traffic in the conventional road (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proxilognormal</td>
<td>3,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>4,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>5,000</td>
<td>1.7/(0.018)</td>
<td>18.84</td>
</tr>
<tr>
<td></td>
<td>6,000</td>
<td>1.9/(0.021)</td>
<td>26.08</td>
</tr>
<tr>
<td></td>
<td>7,000</td>
<td>1.9/(0.021)</td>
<td>26.08</td>
</tr>
</tbody>
</table>

The results for the case of Spain are similar to the ones obtained for low average (μ) and high variance (σ²) distributions. With free flow traffic conditions, the optimal tolls will be zero. By contrast, insofar as the potential traffic increases, the optimal toll will increase to efficiently distribute traffic to each road. When the potential traffic reaches the capacity of the
corridor, the traffic share in the road will be around 26%. As previously explained, this is due to the high percentage of users who decide not to travel (31%). Finally, comparing the optimal tolls obtained through our methodology with the current tolls in Spain’s highways with similar characteristics, we note that current tolls are around four times higher than the optimal ones.

6. CONCLUSIONS AND POLICY LESSONS

This paper presents a model to obtain the toll that maximizes the social welfare in a corridor where a toll highway competes with a parallel conventional road. The model is devised for an interurban corridor with non-recurrent users who take their decisions assuming free-flow traffic conditions. The results of the paper shed new light on the topic. The most important contribution is that optimal tolls are not the same for different VTT distributions and consequently for users with different income distributions. The paper demonstrates that, for the type of corridor studied, the higher the VTT average the higher the optimal price, the higher the VTT dispersion the lower the optimal price and finally, the larger the number of potential users the higher the optimal toll. Consequently, tolls should be higher in regions with lower Gini indexes than in regions with higher Gini indexes.

The latest constitutes the main policy lesson added on the field. For instance, a country such as Brazil—with low inequality-distributed income per capita—should have ceteris paribus optimal tolls lower than a country like Sweden—with high equally-distributed income per capita. The methodology presented in the paper can be easily applied to any other corridor of similar characteristics located in any other region.

The model shows that the optimal toll always triggers a traffic share according to the capacities of both roads. Under free flow conditions the optimal toll is the one that makes users travel only through the highway. As a general rule the greater the potential traffic the higher the traffic share in the conventional road. When the capacity of the corridor is reached, this share is slightly above its relative capacity.

This research may be extended with new research topics such as the use of a logit model for the users’ decision making, the derivation of an analytical expression, or the definition of a set of necessary and sufficient conditions for the optimal solution of the problem formulated in the paper through a Lagrangian approach. Moreover, it would be interested to design a new model for recurrent users (commuters) who perfectly know the traffic in the corridor, and consequently decide on the basis of both real travel time and monetary cost.
7. REFERENCES


INTRODUCTION

- In some countries it is fairly common to see two roads with the same origin and destination competing in the same corridor. One of them is usually a toll highway that offers a better quality to the users compared to its alternative: a free parallel single road. The users thus have to decide whether it is worth paying the toll for the advantages offered. This problem, known as the “untolled alternative”, has been largely studied in the academic literature. Particular attention has been paid to calculate the optimal welfare toll that maximizes economic efficiency.
- It is well acknowledged that in order to achieve the maximum social welfare, users must internalize the externalities they produce and do not perceive. This is usually achieved through a toll or a tax. However, that toll can harm low income users. Depending on the objective function to be optimized (e.g. maximize welfare, maximize social equity, maximize income, etc.) the optimal toll might vary substantially.

OBJECTIVES OF THE PAPER

- Add knowledge in the area by defining a model to obtain the optimal toll price in terms of the VTT distribution for an interurban corridor where a toll highway and an untolled conventional road compete.

METHODLOGY

Two hypotheses were assumed:

- The potential users will be divided into 100 groups. Each group will have a different VTT and a daily expenditure limit for transport related to their income. Each potential user is equivalent to one car, so the group of potential users is made of one hundred users, that means one hundred vehicles.
- The potential users are supposed not to be familiar with the traffic conditions in the corridor. Therefore, they will decide whether they will travel or not, and through which road they will do it, on the basis of the expected travel time, their VTT, the gasoline cost under free flow conditions and the toll in the highway. It is well acknowledged that in order to achieve the maximum social welfare, users must internalize the externalities they produce and do not perceive. This is usually achieved through a toll or a tax. However, that toll can harm low income users.
- The objective function that has to be minimized:

\[ SC = UC + EC + HOB + GB \]

- UC is the total cost that the users bear per trip in €. It is divided into four terms which are travel time, toll, fuel cost and maintenance of the vehicle.
- HOB is the net operating balance for the road operator either public or private in €. It consists of the tolls paid in the highway minus the maintenance cost in the toll highway.

- GB is the result for the Government responsible for maintaining the conventional road in €. It is calculated as the taxes recovered from fuel minus the maintenance cost of the road.
- EC represents the externalities produced by the vehicles in €. They are the summation of environmental cost – e.g. gas emissions, noise and so on – plus accidents.

CASE STUDY

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>Average VTT</td>
<td>Kraemer et al. (2004)</td>
</tr>
<tr>
<td>σ²</td>
<td>Variance VTT</td>
<td>Kraemer et al. (2004)</td>
</tr>
<tr>
<td>TNPU</td>
<td>Potential Users</td>
<td>Kraemer et al. (2004)</td>
</tr>
</tbody>
</table>

CONCLUSIONS

- Optimal tolls are not the same for different VTT distributions and consequently for users with different income distributions. The paper demonstrates that, for the type of corridor studied, the higher the VTT the average the higher the optimal price, the higher the VTT dispersion the lower the optimal price and finally, the larger the number of potential users the higher the optimal toll. Consequently, tolls should be higher in regions with lower Gini indexes than in regions with higher Gini indexes.
- The optimal toll always triggers a traffic share according to the capacities of both roads. Under free flow conditions the optimal toll is the one that makes users travel only through the highway. When the capacity of the corridor is reached, this share is slightly above its relative capacity.
- Further research topics: the use of a logit model for the users’ decision making, the derivation of an analytical expression, or a new model for recurrent users (commuters) who perfectly know the traffic in the corridor.