Capturing yield surface evolution with a multilinear anisotropic kinematic hardening model

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ABSTRACT

Many authors have observed experimentally that the macroscopic yield surface changes substantially its shape during plastic flow, specially in metals which suffer significant work hardening. The evolution is frequently characterized by a corner effect in the stress direction of loading, and a flatter shape in the opposite direction. In order to incorporate this effect many constitutive models for yield surface evolution have been proposed in the literature. In this work we perform some numerical predictions for experiments similar to the ones performed in the literature using a multilayer kinematic hardening model which employs the associative Prager's translation rule. Using this model we prescribe offsets of probing plastic strain, so apparent yield surfaces can be determined in a similar way as it is performed in the actual experiments. We show that similar shapes to those reported in experiments are obtained. From the simulations we can conclude that a relevant part of the apparent yield surface evolution may be related to the anisotropic kinematic hardening field.

1. Introduction

Classical phenomenological theories of plasticity for metals are based on the existence of an elastic domain characterized by a yield surface. For polycrystal isotropic metals, the Maxwell-von Mises yield criterion has been verified by a number of authors, starting with the tension-torsion experiments of Taylor and Quinney (1932). This yield surface is a circle in the \((\sigma - \sqrt{3}\tau)\) tension stress-\(\tau\) torsion stress \(\tau\) plane and in the deviatoric stress \(\tau\) plane. However, for at least some hardening materials, upon plastic straining in one direction the measured yield surface not only translates due to kinematic hardening, but also changes its shape. This change of shape has been observed by many authors in different metals, see Theocaris and Hazell (1965), Kuwabara et al. (2000), Ishikawa (1997), Ishikawa and Sasaki (1989), Khan et al. (2009, 2010a, 2010b); Hu et al. (2015); 2014, Wu and Yeh (1991), Wu (2003), Sung et al. (2011), Kim et al. (2009), Rousset (1985); Rousset and Marquis (1985), Benallal and Marquis (1987), among others. As observed in these experiments, the actual shape of the measured yield surface depends on several factors as the material itself, the amount of prestress, and the permanent plastic strain (probing strain) after which the onset of plasticity (i.e., the limit of the elastic domain) is established. The relevance of this change of shape is unquestionable because it largely affects non-proportional loading and the springback behavior.

Many experiments show similar conclusions on the evolution of the measured shape of the yield surface. Upon prestressing in one direction in the \(\sigma - \sqrt{3}\tau\) (axial-torsion) plane, the yield surface shows a "nose" in that direction and an almost flat line in the opposite direction (Khan et al., 2009, 2010a, 2010b; Hu et al., 2014, 2015, Wu and Yeh, 1991; Rousset, 1985; Rousset and Marquis, 1985; Benallal and Marquis, 1987), resulting in an often named "egg" effect (Lemaitre and Desmorat, 2005). This nose (and the opposite flat part) changes according to new substantial prestressing. Some experiments have also observed that in the direction perpendicular to the prestressing one (in the axial-torsion plane) the measured elastic domain becomes wider than in the direction of pre-loading (Ishikawa, 1997; Ishikawa and Sasaki, 1989; Khan et al., 2009, 2010a, 2010b; Hu et al., 2014, 2015, 2016, Wu and Yeh, 1991; Rousset, 1985; Rousset and Marquis, 1985; Benallal and Marquis, 1987). Furthermore, although it is rarely accentuated (usually neglected) some experiments also show symmetric slightly concave parts in the surface behind the nose, an effect clearly seen in the experimental data of Wu and Yeh (1991) and also present in some of the tests of Khan et al. (2009, 2010a, 2010b).

Because of the major importance of all these effects, many material models have been proposed or extended in order to account for the shape evolution of the yield surface. Some of the models are phenomenological (Helling and Miller, 1987, 1988; Kuryyka and Zyczkowski, 1996; Voyiadjis and Foroozesh, 1999; François, 2001; Liu et al., 2011; Wu and Hong, 2011; Lee et al., 2012; Radi and Abdul-Latif, 2012; Barlat et al., 2013; Shi et al. (2014)) and some of
them micromechanically-based or motivated (Zattarin et al., 2004; Kabirian and Khan, 2015; Yoshida et al., 2014; Hu et al., 2015). These models are substantially more complex than traditional models (Lemaître and Desmorat, 2005) and despite of their complexity, they are usually not able to capture some details. Probably a recent crystal plasticity model is the first one to capture small concavities sometimes present in experiments (Hu et al., 2015).

The purpose of this paper is to perform some predictions of experiments to detect apparent yield surfaces employing a special multilayer (or multilayer) nested yield surfaces model. The model is based on the original ideas of Iwan (1967) and Mroz (1969), Montáns (2000) of employing several nested yield surfaces that discretize the uniaxial stress-strain curve. The procedure does not require any parameter-fitting procedure; the prescribed stress–strain data are exactly captured in the uniaxial case in a similar way as in our hyperelastic (Latorre and Montáns, 2013; Latorre and Montáns, 2014b) and damage models (Miñano and Montáns, 2015). From a theoretical standpoint, there is a clear and remarkable difference of our model with the Mroz proposal. In our case the outer surfaces are not yield surfaces, but only hardening surfaces; i.e. they are simply a tool to compute the effective anisotropic hardening modulus. The actual yield surface is always the innermost one. The plastic strains are always normal to yield surface and the hardening direction of the yield surface follows Prager’s associative hardening rule. From a computational standpoint, whereas for the Mroz model there are some relevant restrictions when formulating a fully implicit closest point projection algorithm (Montáns, 2000; Caminero and Montáns, 2006; Montáns and Caminero, 2007), in the case of Prager’s rule a closest point projection algorithm is possible without restriction, and this algorithm reduces to the solution of a nonlinear scalar function (Montáns, 2001; Montáns, 2004). Furthermore, it is remarkable that in the case of linear kinematic hardening, the model exactly reduces to classical J2-plasticity regardless of the number of surfaces employed (Montáns and Caminero, 2007) not only from a theoretical but also from a computational point of view (i.e. the global iterations are the same up to round-off errors).

Therefore, during the predictions given below, we note that the actual (analytical) yield surface is always the same, i.e. the innermost von Mises surface. Both the plastic flow and the hardening (i.e. translation of the yield surface) follows associative rules. The stress–driven simulations have been performed using a fully implicit algorithm (Montáns, 2001; Montáns, 2004) but also from a computational point of view (i.e. the global iterations are the same up to round-off errors).

2. Summary of the model

The main objective of the model is to account for multiaxial nonlinear anisotropic kinematic hardening during nonproportional loading. In order to meet this goal, several nested (initially concentric) surfaces are employed. The innermost one is the yield surface, the boundary of the elastic domain, taken as the von Mises one

\[
\sigma \leq \sigma^\text{M}
\]

where \(\sigma^\text{M}\) is the deviatoric stress tensor, \(\sigma_1\) is the backstress tensor, \(\|\|\) is the Euclidean norm and \(r_1 = \sqrt{2/3\sigma_v}\) is the radius of the yield surface for the corresponding yield stress \(\sigma_v\).

We apply the principle of maximum dissipation and assume associativity of both the plastic flow and of the hardening, i.e.

\[
\dot{\varepsilon} = \dot{\gamma} \frac{\partial f_1}{\partial \sigma_1} = \dot{\gamma} \hat{n} \text{ and } \alpha_1 = -\lambda \frac{\partial f_1}{\partial \sigma_1} = \lambda \hat{n} \text{ with } \hat{n} := \frac{\sigma^\text{M} - \sigma_1}{\|\sigma^\text{M} - \sigma_1\|}
\]

where \(\dot{\varepsilon}\) is the plastic strain rate and \(\alpha_1\) is the rate of the backstress, \(\dot{\gamma}\) and \(\lambda\) are computed from the hardening pattern and the consistency condition. Let \(H\) be the effective hardening modulus. Then we have the usual relation

\[
\dot{\gamma} = \frac{2}{3} H \dot{\gamma} \text{ so } \hat{n} = \frac{\lambda}{\frac{2}{3} H} = \frac{||\alpha_1||}{\frac{2}{3} H}
\]

and Prager’s rule results in

\[
\alpha_1 = \frac{2}{3} H \dot{\gamma} \hat{n}
\]

From the constitutive equation for the deviatoric stress rate, using Eq. (2)

\[
\dot{\sigma}^\text{M} = 2\mu \left(\dot{\varepsilon}^\text{M} - \dot{\varepsilon}^\text{P}\right) = 2\mu \dot{\varepsilon}^\text{M} - 2\mu \dot{\gamma} \hat{n}
\]

where \(\dot{\varepsilon}^\text{M}\) are the deviatoric strain rates and \(\mu\) is the shear modulus. The consistency conditions are

\[
\begin{align*}
\dot{f}_1 &= 0, \quad \dot{f}_1 = 0 \text{ if } \dot{\gamma} > 0 \\
\dot{f}_1 &= 0 \text{ if } \dot{\gamma} = 0
\end{align*}
\]

Using \(\dot{f}_1 / \partial \sigma = -\sigma_1 / \partial \sigma_1 = \hat{n}\), we readily obtain the consistency parameter

\[
\dot{f}_1 = 0 \Rightarrow \hat{n} : (\dot{\sigma}^\text{M} - \sigma_1) = 0 \Rightarrow \dot{\gamma} = \frac{2\mu \hat{n} : \dot{\varepsilon}}{2\mu + \frac{2}{3} H}
\]

The elastoplastic tangent moduli \(C^\text{EP}\) relate stress rates \(\dot{\varepsilon}\) with total strain rates \(\varepsilon\) by \(\text{by } \sigma = C^\text{EP} : \dot{\varepsilon}\). These moduli are obtained from the same classical expression employing Eq. (8) in the constitutive equation of the deviatoric stress rates \(\dot{\sigma}^\text{M}\)

\[
\dot{\sigma}^\text{M} = 2\mu \dot{\varepsilon} - 2\mu \dot{\gamma} \hat{n} = 2\mu \dot{\varepsilon} - 2\mu \frac{\dot{\gamma}^2 \hat{n}}{2\mu + \frac{2}{3} H}
\]

Fig. 1. Geometric relations of the model in the deviatoric plane. Left: stress tensor \(\sigma^\text{M}\), flow direction \(\hat{n}\), hardening surfaces \(f_1\), contact points and translation directions \(\hat{m}_1\). Right: equivalent surfaces \(f_2\).
so—cf. Eq. (2.3.9) of Ref. Simó and Hughes [1998]

\[
C^{ep} = K \mathbb{1} + 2 \mu \left( \mathbb{1} - \frac{1}{3} \mathbb{1} \right) - \frac{2 \mu}{2 \mu + \frac{4}{3} H} \hat{\mathbf{n}} \otimes \hat{\mathbf{n}}
\]

(10)

where \( \mathbb{1} \) is the fourth order fully symmetric identity tensor, \( \mathbb{1} \) is the second order one and \( K \) is the bulk modulus.

From the previous equations it is apparent that the present model is the widely known classical kinematically hardened J2-plasticity (Jubelin, 1990). In fact, the only difference with the usually employed model is how we compute the effective hardening \( H \), a procedure we explain now. In the case \( H \) is constant, there is not a single theoretical or computational difference with the classical formulation. However, we want \( H \) to change according to the load level and load path, preserving Masing's rules and also describing the hardening field for the case of nonproportional loading. We further want \( H \) to be explicitly given and determined by a uniaxial test. To this end, we employ the idea of Mroz of using several concentric surfaces to describe the hardening field. However, in our case these surfaces are just a tool to compute the effective hardening \( H \), they are not successive yield surfaces and they do not change the hardening translation rule of the yield surface; the translation rule is still Prager's rule.

Then, in order to compute the effective \( H \) which accounts for kinematic hardening, several outer surfaces are employed which can be written as

\[
f_i := \| \mathbf{a} - \mathbf{a}_i \| - r_i \quad \text{with} \quad i > 1
\]

(11)

where \( \mathbf{a} \) is the stress tensor at the contact point with the inner surface \( i = 1 \) and the actual stresses for \( i > 1 \). An alternative equivalent expression (which differs from Mroz's setting but is arguably better for developing the formulation and which could allow for the interpretation of the surfaces as yield surfaces for internal variables) is see Fig. 1

\[
f_i := \| \mathbf{a}_i - \mathbf{a}_i - \mathbf{r}_i \| - (r_i - r_{i-1}) \leq 0 \quad \text{with} \quad i > 1
\]

(12)

As mentioned, these outer surfaces are merely a tool to compute the effective nonlinear kinematic hardening preserving Masing's cyclic behavior and allowing for consistent nonproportional loading (Montans and Caminero, 2007). The translation of the hardening surfaces follow their specific rule which may be derived from the condition \( df_i/dt = 0 \) (i.e., they do not overlap) when they are active. Taking the derivative of Eq. (12),

\[
\frac{d f_i}{d t} = -\hat{\mathbf{m}}_i \quad \text{with} \quad \hat{\mathbf{m}}_i = \frac{\mathbf{a}_i - \mathbf{a}_i}{\| \mathbf{a}_i - \mathbf{a}_i \|} \quad \text{and} \quad \hat{\mathbf{m}}_0 = \hat{\mathbf{n}}
\]

(13)

so for any active surface we obtain the following geometric expression from the non-overlapping condition

\[
\frac{d f_i}{d t} = \hat{\mathbf{m}}_i : (\mathbf{a}_i - \mathbf{a}_i) = 0 \iff \| \mathbf{a}_i \| = \| \mathbf{a}_i \| (\hat{\mathbf{m}}_i : \hat{\mathbf{m}}_i)
\]

(14)

where (\cdot) is the Macaulay bracket function. The Lagrange multiplier \( \gamma \) is computed from the hardening multipliers \( \lambda_i \) of the active surfaces \( i = 1, \ldots, a \). These values are computed from the projection of the translation of the surfaces on the hardening direction \( \hat{\mathbf{n}} \) given by Prager's rule

\[
\lambda_i = \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}} \|
\]

(15)

For each \( \lambda_i \), the contribution \( \gamma_i \) to \( \gamma \) is then—see Eq. (4)

\[
\gamma_i = \frac{\lambda_i \rho_i}{H_i} = \frac{\| \hat{\mathbf{a}}_i \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}}}{H_i} = \frac{\| \hat{\mathbf{m}}_i : \hat{\mathbf{n}} \|}{H_i} \frac{\| \hat{\mathbf{a}}_i \|}{H_i} \frac{\| \hat{\mathbf{m}}_i : \hat{\mathbf{m}}_{i-1} \|}{H_i} - j = 1
\]

(16)

where \( H_i \) is the hardening modulus associated to surface \( i \). In the previous expression we have used repeatedly Eq. (14) and defined for notational comfort \( \hat{\mathbf{m}}_i = \hat{\mathbf{n}} \). Then

\[
\gamma = \sum_{i=1}^a \gamma_i = \sum_{i=1}^a \frac{\| \hat{\mathbf{a}}_i \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}}}{H_i} = \frac{\| \hat{\mathbf{m}}_i : \hat{\mathbf{n}} \|}{H_i} \frac{\| \hat{\mathbf{a}}_i \|}{H_i} \frac{\| \hat{\mathbf{m}}_i : \hat{\mathbf{m}}_{i-1} \|}{H_i} - j = 1
\]

(17)

so by comparison with Eq. (4) we arrive at the expression of the effective hardening moduli to be employed in Eqs. (8) and (10), i.e., in the formulation of classical J2-plasticity model

\[
\frac{d \gamma}{d t} = \sum_{i=1}^a \frac{\| \hat{\mathbf{a}}_i \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}}}{H_i} \| \hat{\mathbf{a}}_i \| \| \hat{\mathbf{m}}_i : \hat{\mathbf{m}}_{i-1} \| - j = 1
\]

(19)

In a uniaxial test, the tangent modulus \( E^{ep}_{ij} \) is obtained as always by

\[
E^{ep}_{ij} = \frac{\| \hat{\mathbf{a}}_i \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}}}{H_i} \| \hat{\mathbf{a}}_i \| \| \hat{\mathbf{m}}_i : \hat{\mathbf{m}}_{i-1} \| - j = 1
\]

(20)

Given a uniaxial stress–strain curve \( \sigma(s) \), some points \( \mathbf{\sigma}_i, \mathbf{\epsilon}_i \) may be taken as the discretization of such curve, where \( \mathbf{\sigma}_0 = \mathbf{\sigma}_0 = 0 \) and \( \mathbf{\sigma}_i = \mathbf{\sigma}_i \). Then the parameters \( \mathbf{\gamma}_i \) are uniquely and explicitly computed in a recursive form

\[
E^{ep}_{ij} = \frac{\| \hat{\mathbf{a}}_i \| \hat{\mathbf{m}}_i : \hat{\mathbf{n}}}{H_i} \| \hat{\mathbf{a}}_i \| \| \hat{\mathbf{m}}_i : \hat{\mathbf{m}}_{i-1} \| - j = 1
\]

(21)

where \( E^{ep}_{ij} = E \) and \( E^{ep}_{ij} = E \) (where \( E \) is the total number of surfaces) is given directly by the user as the residual hardening for \( s \rightarrow \infty \).

Note that if \( H_i \rightarrow \infty \) (or \( E^{ep}_{ij} = E \)) then \( \gamma_i \rightarrow 0 \) and surface \( i \) has no influence on \( H \) (it is like it does not exist), so the predictions employing different arbitrary discretizations are consistent (Montans and Caminero, 2007) and the case of classical kinematically hardened J2-plasticity is recovered for bilinear stress–strain curves as a particular case. Because of the simple structure of the model, it is possible to develop a fully implicit closest point projection algorithm in which the local algorithm reduces to solving a nonlinear scalar equation as in the case of classical J2 plasticity with mixed hardening. This equation is

\[
R(\Delta \gamma) = \Delta \gamma - \sum_{i=1}^a \gamma_i \Delta \gamma_i \rightarrow 0
\]

(22)

where \( \Delta (\cdot) \) is the increment during the finite step. Note also that once \( \gamma_i \) is determined, the update of the surfaces is readily given by Eq. (5) and by Eq. (14) along with Eq. (13). Of course when developing a fully implicit, radial return computational algorithm the derivatives of \( H \) must also be taken into account. This is the major algorithmic difference with the usual algorithm of J2-plasticity.

Further details on the model, an implicit computational algorithm and an example showing the ability to model multiaxial nonproportional loading in soils can be found in Montans (2001). A plane stress projected algorithm can be found in Montans (2004). A bounding surface model following the same principles with simulations in nonproportional multiaxial loading of soils during an earthquake can also be found in Montans and Borja (2002). The consistency of the multiaxial behavior and predictions of the (Lamba and Sidebottom, 1978) nonproportional experiments in metals can be found in Montans and Caminero (2007). The purpose of the next sections is to show that the model is capable of predicting some aspects of the yield surface evolution observed in many experiments if those experiments are simulated numerically employing the model.
3. The experiments of Wu and Yeh (1991)

As mentioned in the introduction many experiments have measured the yield surface evolution in different materials, some at small strains and some at large strains. We have considered only small strains so there is no relevant difference among strain measures and questionable shear effects (Latorre and Montans, 2014a). From those we have selected the experiments of Wu and Yeh (1991) performed on 304 stainless steel, one of the most versatile and widely used stainless steels.

Wu and Yeh (1991) performed tests with pre-straining in different directions, but the reported measured yield surface evolution is similar for all cases. Fig. 2 shows some of their experimental results, redrawn from Fig. 6 of their paper. Measured surfaces have a nose in the pre-loading direction and a more flat part in the opposite direction. The width of the yield surface seems to be always larger in the direction perpendicular to the preloading direction than in that direction. Many of the surfaces seem to have a small, slightly concave zone behind the nose if experimental dots are to be trusted, a zone that Wu and Yeh (as most authors) have neglected when tracing a continuous yield function probably because yield surfaces should be convex due to Drucker stability (Lubliner, 1980). However this experimental observation is repeated in most of their figures, and also found in some of the experiments in Khan et al. (2009, 2010a, 2010b).

The experimental observations of Wu and Yeh have been obtained using probing paths perpendicular to the pre-loading direction, as shown in Fig. 2. The probing paths are relevant because the reported experimental observations change slightly for different probing paths (Khan et al., 2009, 2010a, 2010b; Hu et al., 2014, 2015). Usually when the probing path is radial, the flat part of the surface seems to become slightly curved (Hu et al., 2014, 2015).

4. Numerical predictions

Wu and Yeh do not report the actual stress–strain curve of their experiments, although some points can be obtained from the published results. Hence we have adapted the uniaxial data from Ishikawa and Sasaki (1989) so the stress–strain data have similar values as those that can be inferred from the Wu and Yeh experiments. Since our target is not to study this specific material but to simulate the evolution of the yield surface in general and to relate it to non-linear kinematic hardening, the accuracy of the material data is not very important because we are mostly interested in the overall effects. The stress–strain curve employed in the experiments, obtained via a digitalization software, is shown in Fig. 3. In Fig. 3 we also show the specific hardening surfaces employed in the simulations which corresponds to the different marks in the curve. The size of the actual yield surface has been estimated from the minimum yield stresses measured by Wu and Yeh for the minimum probing strain of 5 µm and from the different reported yield surfaces. However, Wu and Yeh note that the actual plastic strain incurred before a yield point can be confirmed is approximately 10 µm. Then, we note that the experimental determination of the actual yield surface (for zero probing strain) is impossible and can only be estimated.

In a numerical model, we of course know the yield surface and then we need no probing plastic strain. In fact in our model it is always the innermost surface. However, the point is that in experiments the yield surface is detected after some plastic strain has already occurred, and that the apparent yield surface does not coincide necessarily with the analytical one. In this case the definition of the equivalent plastic strain we use is

\[
\dot{\varepsilon}^{p} = \int \sqrt{\frac{2}{3} \| \dot{\varepsilon}^{p} \|} \, dt
\]

\[
\rightarrow \sqrt{(\varepsilon^{p})^2 + (\gamma^{p})^2/3} \text{ under proportional loading} \tag{23}
\]

where \( \dot{\varepsilon}^{p} \) is the plastic deformation rate tensor, \( \varepsilon^{p} \) is the axial plastic strain and \( \gamma^{p} \) is the engineering plastic torsional strain. For the reverse yield point at the direction of pre-loading, a smaller offset plastic strain value is selected in order to guarantee that the detection of other yield points does not exceed the area of the actual yield surface at the direction of pre-loading. For 304 stainless steel, we chose 10 µm to be the analytical value for \( \varepsilon^{p} \) (experimental measurements are usually between 5 µm and 10 µm) in addition to 3.5 µm, which is the value for \( \varepsilon^{p} \) used to find the reverse yield point (Wu and Yeh, 1991).
The results of the simulations are shown in Fig. 4. From the figure, we can observe that the main characteristics of the evolution of the apparent yield surface are captured. For instance, following the same probing path than Wu and Yeh, a nose is obtained in the direction of preloading and, in the opposite direction, a flat zone is also predicted. Interestingly, the model predicts symmetric small concave zones behind the nose connecting the nose and the flatter part. These concave zones may vary depending on the pre-loading amount, the size of the actual yield surface, and on the nonlinear hardening. Note also that the predicted yield surface is also wider in the perpendicular direction respect to the preloading direction. The results employing different preloading paths are similar, and within the same level of agreement to the experiments of Wu and Yeh as it can be easily inferred. It is also noticeable a sharper point in the loading direction than that predicted by the model, specially in the middle figure. This disagreement may be possibly attributable to several factors, as to a smaller 0-offset yield surface or to viscous effects and yield stress relaxation at the prestress point. The latter effects are frequently found in experiments on this material and we have not included them in the numerical simulations.

Fig. 5 (a) shows the actual position of the hardening surfaces for one of the cases. From this figure the reason for the predictions obtained can be easily deduced. First we must emphasize again that the actual yield surface is the innermost one. The effective hardening modulus in the direction of the preloading is much lower than the hardening modulus in the opposite or normal directions (see below). Hence, in the preloading direction the probing plastic strain is obtained with a very small increment in the stress, i.e. the offset from the yield surface is very small. However, in the direction perpendicular to the preloading one, a larger stress offset value from the yield surface is needed for the same plastic strain.

The size and shape of these detected yield surfaces are largely decided by the definition of yield, the amount of prestressing and partially by the probing direction. When the offset equivalent plastic strain $\varepsilon^p$ is larger, the yield surfaces will be extended as one could obviously expect. This is also observed in the experiments of Hu et al. (2014). With this model, if the $\varepsilon^p$ value is increased not only the size of the yield surface along the probing direction will be larger, the shape will also be sharper, as shown in Fig. 5(b). If the $\varepsilon^p$ value is very small, the detected yield surface is closer to the innermost
surface being eliminated

hardening surface. Furthermore, the actual procedure to define the probing plastic strain may also have a relevant influence in the form and shape of the measured yield surface.

The shape and size of the yield surface also depend on the probing path, and even on the probing sequence and number of problings performed. If the probing path is starting from the center of the yield surface and going at radial directions, and the other conditions are the same, as shown in Fig. 5(c), the yield surface will be less distorted and slightly more expanded in the pre-loading direction. Even though probing at the radial directions is also used in experiments, and these experiments seem to show a more rounded surface (Hu et al., 2014, 2015), the perpendicular probing path used by Wu and Yeh (among others) seems to be preferred because this perpendicular probing path can guarantee that the yield surface detected doesn't exceed the actual yield surface at the direction of pre-loading, see discussion in Wu and Yeh (1991). Note that in Fig. 5(c) accumulation of plastic strains also swifit the original pre-loading point and the reverse one. Furthermore, the influence of an increased probing value for detecting the reverse yield has also an effect on the roundness of that part and on the size of the yield surface.

One of the questions that may be raised is the consistency of the predictions given by the model when the number and size of surfaces change. Precisely, this is one of the attributes of the model. In Fig. 5(d) we show the differences in results when the fifth innermost surface is eliminated. Because the number of surfaces to discretize the nonlinear stress-strain curve was adequate and because surfaces are just a tool to compute the effective multiaxial hardening, the results obtained are very similar. The elimination of one surface simply brings a less accurate equivalent hardening, but does not change the overall anisotropy, the flow or the translation rule of the yield surface. This is a property that has been proven to be critical in the consistency of the predictions and which is not found in other multisurface models (Montáns and Caminero, 2007).

Fig. 5. (a) Relative position of the hardening surfaces and apparent yield surface for 1 με and 3.5 με detected reverse yield points. (b) Influence of the offset microstrain used to detect the yield surfaces: 5 με, 10 με and 20 με (from inside to outside), (c) Influence of the loading path and of the microstrain used to detect the reverse yield point: perpendicular (thin line) and radial (thick dotted line), (d) Influence of the elimination of one surface in the predicted yield surfaces.
yield surface, see details in Ishikawa and Sasaki (1989). Then they usually very complex and have different success in capturing some
tant. Some of the distinctive aspects of these yield surfaces in some
be due to the accuracy in the determination of the starting point for
(1989) in their results in order to make them comparable. There are
cartesian from 0° to 180° in steps of 30°, see Fig. 6(b). We have numeri­
ments small concave zones behind the nose.
material is: the presence of a nose or corner effect in the loading
evolution of the yield surface when kinematic hardening is impor­
V3r) plane at an angle away
ned in a different direction in the (σ, √3r) plane at an angle away
from the preloading direction. Different tests were conducted at an­
gles from 0° to 180° in steps of 30°, see Fig. 6(b). We have numeri­
forced the initial reloading point the center of the theoretical yield surface. From a
comparison of both sides of the figure, one can deduce that, in gen­
Figure 6. Simulation of the experiments of Fig. 6 of Ishikawa and Sasaki (1989). (a) Experimental results redrawn from Ishikawa and Sasaki (1989), which consist of stress-strain
curves obtained when loading at different angles to the preloading direction from the assumed center of the yield surface. (b) Results of the numerical simulations following the
same loading paths from the center of the theoretical yield surface.
reloading.

5. Conclusions

Many experiments performed in metals measure a characteristic
evolution of the yield surface when kinematic hardening is impor­
tant. Some of the distinctive aspects of these yield surfaces in some
materials are: the presence of a nose or corner effect in the loading
direction when a substantial preloading is applied, a usually flatter
zone in the opposite direction, wider measured yield surfaces in the
direction perpendicular to the preloading one, and in some experi­
ments small concave zones behind the nose.

Many constitutive models have been proposed or enhanced in or­
der to take into account the observed distortion. These models are
usually very complex and have different success in capturing some
of the experimentally observed details. In this work we show some
numerical predictions using a simple multilayer model for nonlinear
kinematic hardening. To obtain these predictions we have followed
the usual experimental procedures. The actual yield surface of the
model is always a von Mises circle and the model can be considered
traditional J2-plasticity with kinematic hardening in which the effec­
tive multiaxial kinematic hardening modulus is computed employ­ing
several surfaces as a tool.

However, the results presented herein show that with this model
similar shapes to those measured in experiments may be obtained if
the numerical experiments are performed in a similar way as those
experiments. These apparent yield surfaces obtained in the numerical
simulation can only be attributed to the developed anisotropic hard­
ening. Therefore we can conclude that anisotropic kinematic hardening
itself may be one of the main players (among others) in the observed
phenomena.

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